AiryAi

Notations

Traditional name

Airy function Ai

Traditional notation

$\text{Ai}(z)$

Mathematica StandardForm notation

`AiryAi[z]`

Primary definition

03.05.02.0001.01

$$\text{Ai}(z) = \frac{1}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_0F_1\left(;\frac{2}{3};\frac{z^3}{9}\right) - \frac{z}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\,{}_0F_1\left(;\frac{4}{3};\frac{z^3}{9}\right)$$

Specific values

Values at fixed points

03.05.03.0001.01

$$\text{Ai}(0) = \frac{1}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}$$

Values at infinities

03.05.03.0002.01

$$\lim_{x\to\infty} \text{Ai}(x) = 0$$

03.05.03.0003.01

$$\lim_{x\to-\infty} \text{Ai}(x) = 0$$

General characteristics

Domain and analyticity

$\text{Ai}(z)$ is an entire, and so analytic, function of z, which is defined in the whole complex z-plane.

03.05.04.0001.01

$z \longrightarrow \mathrm{Ai}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Mirror symmetry

03.05.04.0002.01

$\mathrm{Ai}(\bar{z}) = \overline{\mathrm{Ai}(z)}$

Periodicity

No periodicity

Poles and essential singularities

The function $\mathrm{Ai}(z)$ has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

03.05.04.0003.01

$\mathcal{Sing}_z(\mathrm{Ai}(z)) = \{\{\tilde{\infty}, \infty\}\}$

Branch points

The function $\mathrm{Ai}(z)$ does not have branch points.

03.05.04.0004.01

$\mathcal{BP}_z(\mathrm{Ai}(z)) = \{\}$

Branch cuts

The function $\mathrm{Ai}(z)$ does not have branch cuts.

03.05.04.0005.01

$\mathcal{BC}_z(\mathrm{Ai}(z)) = \{\}$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

03.05.06.0028.01

$$\mathrm{Ai}(z) \propto \mathrm{Ai}(z_0) + \mathrm{Ai}'(z_0)(z - z_0) + \frac{z_0}{2}\mathrm{Ai}(z_0)(z - z_0)^2 + \ldots \,/;\, (z \to z_0)$$

03.05.06.0029.01

$$\mathrm{Ai}(z) \propto \mathrm{Ai}(z_0) + \mathrm{Ai}'(z_0)(z - z_0) + \frac{z_0}{2}\mathrm{Ai}(z_0)(z - z_0)^2 + O\left((z - z_0)^3\right)$$

03.05.06.0030.01

$$\mathrm{Ai}(z) \propto \frac{\mathrm{Ai}(z_0)}{2} + \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{z_0^{-k}}{2} \left(\sum_{s=0}^{k} \sum_{j=0}^{s} \sum_{i=0}^{s-1} \frac{(-1)^{j+s-1}\, i\, (-i+s-1)!\, (-3\,i+3\,s-1)\, (-3\,j-k+3\,s+1)_k \left(-\frac{1}{3}\right)_s}{i!\, j!\, (s-j)!\, (s-2\,i)! \left(\frac{2}{3}\right)_i \left(\frac{4}{3}-s\right)_i} \left(-\frac{z_0^3}{9}\right)^i - \sum_{s=0}^{k} \sum_{j=0}^{s} \sum_{i=0}^{s} \frac{(-1)^{j+s-1}\, (s-i)!\, (-3\,j+3\,s+1)\, (-3\,j-k+3\,s+2)_{k-1} \left(\frac{1}{3}\right)_s}{i!\, j!\, (s-j)!\, (s-2\,i)! \left(\frac{1}{3}\right)_i \left(\frac{2}{3}-s\right)_i} \left(-\frac{z_0^3}{9}\right)^i \right) \mathrm{Ai}(z_0) + \frac{z_0^{1-k}}{2} \left(\sum_{s=0}^{k} \sum_{j=0}^{s} \sum_{i=0}^{s-1} \frac{(-1)^{j+s-1}\, (-i+s-1)!\, (-3\,j+3\,s+1)\, (-3\,j-k+3\,s+2)_{k-1} \left(\frac{1}{3}\right)_s}{i!\, j!\, (s-j)!\, (-2\,i+s-1)! \left(\frac{4}{3}\right)_i \left(\frac{2}{3}-s\right)_i} \left(-\frac{z_0^3}{9}\right)^i - \sum_{s=0}^{k} \sum_{j=0}^{s} \sum_{i=0}^{s-1} \frac{(-1)^{j+s-1}\, (-i+s-1)!\, (-3\,j-k+3\,s+1)_k \left(-\frac{1}{3}\right)_s}{i!\, j!\, (s-j)!\, (-2\,i+s-1)! \left(\frac{2}{3}\right)_i \left(\frac{4}{3}-s\right)_i} \left(-\frac{z_0^3}{9}\right)^i \right) \mathrm{Ai}'(z_0) \right) (z-z_0)^k$$

03.05.06.0031.01

$$\mathrm{Ai}(z) \propto \sum_{k=0}^{\infty} \frac{1}{k!} \left(3^{2/3}\, \Gamma\!\left(\frac{1}{3}\right) {}_2\tilde{F}_3\!\left(\frac{1}{3}, 1; \frac{1-k}{3}, \frac{2-k}{3}, 1-\frac{k}{3}; \frac{z_0^3}{9}\right) - z_0\, \Gamma\!\left(\frac{2}{3}\right) {}_2\tilde{F}_3\!\left(\frac{2}{3}, 1; \frac{2-k}{3}, 1-\frac{k}{3}, \frac{4-k}{3}; \frac{z_0^3}{9}\right) \right) (z-z_0)^k$$

03.05.06.0032.01

$$\mathrm{Ai}(z) \propto \mathrm{Ai}(z_0)\, (1 + O(z-z_0))$$

Expansions at $z = 0$

For the function itself

03.05.06.0001.02

$$\mathrm{Ai}(z) \propto \frac{1}{3^{2/3}\, \Gamma\!\left(\frac{2}{3}\right)} \left(1 + \frac{z^3}{6} + \frac{z^6}{180} + \ldots\right) - \frac{z}{\sqrt[3]{3}\, \Gamma\!\left(\frac{1}{3}\right)} \left(1 + \frac{z^3}{12} + \frac{z^6}{504} + \ldots\right) /;\ (z \to 0)$$

03.05.06.0033.01

$$\mathrm{Ai}(z) \propto \frac{1}{3^{2/3}\, \Gamma\!\left(\frac{2}{3}\right)} \left(1 + \frac{z^3}{6} + \frac{z^6}{180} + O\!\left(z^9\right)\right) - \frac{z}{\sqrt[3]{3}\, \Gamma\!\left(\frac{1}{3}\right)} \left(1 + \frac{z^3}{12} + \frac{z^6}{504} + O\!\left(z^9\right)\right)$$

03.05.06.0002.01

$$\mathrm{Ai}(z) = \frac{1}{3^{2/3}\, \Gamma\!\left(\frac{2}{3}\right)} \sum_{k=0}^{\infty} \frac{1}{\left(\frac{2}{3}\right)_k k!} \left(\frac{z^3}{9}\right)^k - \frac{z}{\sqrt[3]{3}\, \Gamma\!\left(\frac{1}{3}\right)} \sum_{k=0}^{\infty} \frac{1}{\left(\frac{4}{3}\right)_k k!} \left(\frac{z^3}{9}\right)^k$$

03.05.06.0003.01

$$\mathrm{Ai}(z) = \frac{1}{3^{2/3}\, \Gamma\!\left(\frac{2}{3}\right)}\, {}_0F_1\!\left(; \frac{2}{3}; \frac{z^3}{9}\right) - \frac{z}{\sqrt[3]{3}\, \Gamma\!\left(\frac{1}{3}\right)}\, {}_0F_1\!\left(; \frac{4}{3}; \frac{z^3}{9}\right)$$

03.05.06.0034.01

$$\mathrm{Ai}(z) = \frac{1}{3^{2/3}\, \pi} \sum_{k=0}^{\infty} \frac{\Gamma\!\left(\frac{k+1}{3}\right) \sin\!\left(\frac{2\,\pi\,(k+1)}{3}\right)}{k!} \left(\sqrt[3]{3}\, z\right)^k$$

03.05.06.0004.02

$$\mathrm{Ai}(z) \propto \frac{1}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)} + \frac{z}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)} + O\left(z^3\right)$$

03.05.06.0035.01

$$\mathrm{Ai}(z) = F_\infty(z) \;/;$$
$$\left(\left(F_n(z) = \frac{1}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)} \sum_{k=0}^{n} \frac{\left(\frac{z^3}{9}\right)^k}{\left(\frac{2}{3}\right)_k k!} - \frac{z}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)} \sum_{k=0}^{n} \frac{\left(\frac{z^3}{9}\right)^k}{\left(\frac{4}{3}\right)_k k!} = \mathrm{Ai}(z) - \frac{1}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)(n+1)!\left(\frac{2}{3}\right)_{n+1}} \left(\frac{z^3}{9}\right)^{n+1} {}_1F_2\left(1;\, n+2,\, n+\frac{5}{3};\, \frac{z^3}{9}\right) + \frac{z\left(\frac{z^3}{9}\right)^{n+1}}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)(n+1)!\left(\frac{4}{3}\right)_{n+1}} {}_1F_2\left(1;\, n+2,\, n+\frac{7}{3};\, \frac{z^3}{9}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

Expansions inside Stokes sectors

In exponential form

03.05.06.0014.01

$$\mathrm{Ai}(z) \propto \frac{1}{2\sqrt{\pi}\,\sqrt[4]{z}}\, e^{-\frac{2}{3} z^{3/2}} \left(1 - \frac{5}{48\, z^{3/2}} + \frac{385}{4608\, z^3} + O\left(\frac{1}{z^{9/2}}\right)\right) /;\ |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.05.06.0015.01

$$\mathrm{Ai}(z) \propto \frac{e^{-\frac{2}{3} z^{3/2}}}{2\sqrt{\pi}\,\sqrt[4]{z}} \left(\sum_{k=0}^{n} \frac{\left(\frac{1}{6}\right)_k \left(\frac{5}{6}\right)_k}{k!} \left(-\frac{3}{4\, z^{3/2}}\right)^k + O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right) /;\ |\arg(z)| < \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.05.06.0016.01

$$\mathrm{Ai}(z) \propto \frac{e^{-\frac{2}{3} z^{3/2}}}{2\sqrt{\pi}\,\sqrt[4]{z}} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{6}\right)_k \left(\frac{5}{6}\right)_k}{k!} \left(-\frac{3}{4\, z^{3/2}}\right)^k /;\ |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.05.06.0036.01

$$\mathrm{Ai}(z) \propto \frac{e^{-\frac{1}{3}(2 z^{3/2})}}{2\sqrt{\pi}\,\sqrt[4]{z}} \left(\sum_{k=0}^{n} \frac{\left(\frac{1}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{9}{4 z^3}\right)^k}{\left(\frac{1}{2}\right)_k k!} + O\left(\frac{1}{z^{3(n+1)}}\right)\right) - \frac{5\, e^{-\frac{1}{3}(2 z^{3/2})}}{96\sqrt{\pi}\, z^{7/4}} \left(\sum_{k=0}^{n} \frac{\left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{17}{12}\right)_k \left(\frac{9}{4 z^3}\right)^k}{k! \left(\frac{3}{2}\right)_k} + O\left(\frac{1}{z^{3(n+1)}}\right)\right) /;\ |\arg(z)| < \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.05.06.0005.01

$$\mathrm{Ai}(z) \propto \frac{1}{2\sqrt{\pi}\,\sqrt[4]{z}}\, e^{-\frac{2}{3} z^{3/2}}\, {}_2F_0\left(\frac{1}{6}, \frac{5}{6};;\, -\frac{3}{4\, z^{3/2}}\right) /;\ |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.05.06.0006.01

$$\operatorname{Ai}(z) \propto \frac{1}{2\sqrt{\pi}\,\sqrt[4]{z}}\, e^{-\frac{2}{3} z^{3/2}} \left(1 + O\!\left(\frac{1}{z^{3/2}}\right)\right) /;\ |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

In trigonometric form

03.05.06.0017.01

$$\operatorname{Ai}(-z) \propto \frac{1}{\sqrt{\pi}\,\sqrt[4]{z}} \left(\sin\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 - \frac{385}{4608 z^3} + \frac{37\,182\,145}{127\,401\,984 z^6} + O\!\left(\frac{1}{z^9}\right)\right) - \frac{5}{48 z^{3/2}} \cos\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 - \frac{17\,017}{13\,824 z^3} + \frac{1\,078\,282\,205}{127\,401\,984 z^6} + O\!\left(\frac{1}{z^9}\right)\right)\right) /;\ |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.05.06.0018.01

$$\operatorname{Ai}(-z) \propto \frac{1}{\sqrt{\pi}\,\sqrt[4]{z}} \left(\sin\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{9}{4 z^3}\right)^k + O\!\left(\frac{1}{z^{3n+3}}\right)\right) - \frac{5}{48 z^{3/2}} \cos\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{9}{4 z^3}\right)^k + O\!\left(\frac{1}{z^{3n+3}}\right)\right)\right) /;\ |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.05.06.0019.01

$$\operatorname{Ai}(-z) \propto \frac{1}{\sqrt{\pi}\,\sqrt[4]{z}} \left(\sin\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{9}{4 z^3}\right)^k - \frac{5}{48 z^{3/2}} \cos\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{9}{4 z^3}\right)^k\right) /;\ |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.05.06.0007.01

$$\operatorname{Ai}(-z) \propto \frac{1}{\sqrt{\pi}\,\sqrt[4]{z}} \left(\sin\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) {}_4F_1\!\left(\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}; \frac{1}{2}; -\frac{9}{4 z^3}\right) - \frac{5}{48 z^{3/2}} \cos\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) {}_4F_1\!\left(\frac{7}{12}, \frac{11}{12}, \frac{13}{12}, \frac{17}{12}; \frac{3}{2}; -\frac{9}{4 z^3}\right)\right) /;\ |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.05.06.0008.01

$$\operatorname{Ai}(-z) \propto \frac{1}{\sqrt{\pi}\,\sqrt[4]{z}} \left(\sin\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 + O\!\left(\frac{1}{z^3}\right)\right) - \frac{5}{48 z^{3/2}} \cos\!\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 + O\!\left(\frac{1}{z^3}\right)\right)\right) /;\ |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-containing arguments

03.05.06.0020.01

$$\mathrm{Ai}(z) \propto \frac{1}{2\sqrt{3\pi}\,(-z^3)^{5/12}}\left(\sqrt[12]{-1}\left(\sqrt[3]{-z^3}-\sqrt[3]{-1}\,z\right)e^{\frac{1}{3}(-2)\,i\sqrt{-z^3}}\left(1+\frac{5\,i}{48\sqrt{-z^3}}+\frac{385}{4608\,z^3}+O\!\left(\frac{1}{z^{9/2}}\right)\right)-(-1)^{11/12}\left(\sqrt[3]{-z^3}+(-1)^{2/3}z\right)e^{\frac{2}{3}i\sqrt{-z^3}}\left(1-\frac{5\,i}{48\sqrt{-z^3}}+\frac{385}{4608\,z^3}+O\!\left(\frac{1}{z^{9/2}}\right)\right)\right)/;\ (|z|\to\infty)$$

03.05.06.0021.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12}}{2\sqrt{3\pi}}\left(\sqrt[12]{-1}\,e^{\frac{1}{3}(-2)\,i\sqrt{-z^3}}\left(\sqrt[3]{-z^3}-\sqrt[3]{-1}\,z\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{6}\right)_k\left(\frac{5}{6}\right)_k}{k!}\left(\frac{3\,i}{4\sqrt{-z^3}}\right)^k+O\!\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)+\frac{1}{\sqrt[12]{-1}}\,e^{\frac{2}{3}i\sqrt{-z^3}}\left(\sqrt[3]{-z^3}+(-1)^{2/3}z\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{6}\right)_k\left(\frac{5}{6}\right)_k}{k!}\left(-\frac{3\,i}{4\sqrt{-z^3}}\right)^k+O\!\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)\right)/;\ (|z|\to\infty)\land n\in\mathbb{N}$$

03.05.06.0022.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12}}{2\sqrt{3\pi}}\left(\sqrt[12]{-1}\,e^{\frac{1}{3}(-2)\,i\sqrt{-z^3}}\left(\sqrt[3]{-z^3}-\sqrt[3]{-1}\,z\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{6}\right)_k\left(\frac{5}{6}\right)_k}{k!}\left(\frac{3\,i}{4\sqrt{-z^3}}\right)^k+\frac{1}{\sqrt[12]{-1}}\,e^{\frac{2}{3}i\sqrt{-z^3}}\left(\sqrt[3]{-z^3}+(-1)^{2/3}z\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{6}\right)_k\left(\frac{5}{6}\right)_k}{k!}\left(-\frac{3\,i}{4\sqrt{-z^3}}\right)^k\right)/;\ (|z|\to\infty)$$

03.05.06.0009.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12}}{2\sqrt{3\pi}}\left(\sqrt[12]{-1}\,e^{-\frac{2i}{3}\sqrt{-z^3}}\left(\sqrt[3]{-z^3}-\sqrt[3]{-1}\,z\right){}_2F_0\!\left(\frac{1}{6},\frac{5}{6};;\frac{3\,i}{4\sqrt{-z^3}}\right)+\frac{1}{\sqrt[12]{-1}}\,e^{\frac{2i}{3}\sqrt{-z^3}}\left(\sqrt[3]{-z^3}+(-1)^{2/3}z\right){}_2F_0\!\left(\frac{1}{6},\frac{5}{6};;-\frac{3\,i}{4\sqrt{-z^3}}\right)\right)/;\ (|z|\to\infty)$$

03.05.06.0037.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12}\sqrt[4]{-1}}{4\sqrt{3\pi}}\left(\left(e^{-\frac{2i}{3}\sqrt{-z^3}}\left(\left(-i+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(i+\sqrt{3}\right)z\right)+i\,e^{\frac{2i}{3}\sqrt{-z^3}}\left(\left(-i+\sqrt{3}\right)z-\left(i+\sqrt{3}\right)\sqrt[3]{-z^3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\!\left(\frac{1}{z^{3n+3}}\right)\right)+\frac{5}{48\sqrt{-z^3}}\left(i\,e^{-\frac{2i}{3}\sqrt{-z^3}}\left(\left(-i+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(-i-\sqrt{3}\right)z\right)+e^{\frac{2i}{3}\sqrt{-z^3}}\left(\left(-i+\sqrt{3}\right)z-\left(i+\sqrt{3}\right)\sqrt[3]{-z^3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\!\left(\frac{1}{z^{3n+3}}\right)\right)\right)/;\ (|z|\to\infty)\land n\in\mathbb{N}$$

03.05.06.0038.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12} \sqrt[4]{-1}}{4 \sqrt{3\pi}} \Bigg(\Bigg(e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) \sqrt[3]{-z^3} - \left(i+\sqrt{3}\right) z \right) + i\, e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) z - \left(i+\sqrt{3}\right) \sqrt[3]{-z^3} \right) \Bigg) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{k! \left(\frac{1}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k + \frac{5}{48 \sqrt{-z^3}} \Bigg(i\, e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) \sqrt[3]{-z^3} + \left(-i-\sqrt{3}\right) z \right) + e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) z - \left(i+\sqrt{3}\right) \sqrt[3]{-z^3} \right) \Bigg) \sum_{k=0}^{\infty} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{k! \left(\frac{3}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k \Bigg) /;\ (|z| \to \infty)$$

03.05.06.0039.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12} \sqrt[4]{-1}}{4 \sqrt{3\pi}} \Bigg(\Bigg(e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) \sqrt[3]{-z^3} - \left(i+\sqrt{3}\right) z \right) + i\, e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) z - \left(i+\sqrt{3}\right) \sqrt[3]{-z^3} \right) \Bigg) {}_4F_1\!\left(\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}; \frac{1}{2}; \frac{9}{4 z^3}\right) + \frac{5}{48 \sqrt{-z^3}} \Bigg(i\, e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) \sqrt[3]{-z^3} + \left(-i-\sqrt{3}\right) z \right) + e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) z - \left(i+\sqrt{3}\right) \sqrt[3]{-z^3} \right) \Bigg) {}_4F_1\!\left(\frac{7}{12}, \frac{11}{12}, \frac{13}{12}, \frac{17}{12}; \frac{3}{2}; \frac{9}{4 z^3}\right) \Bigg) /;\ (|z| \to \infty)$$

03.05.06.0010.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12}}{2 \sqrt{3\pi}} \left(\frac{1}{\sqrt[12]{-1}} e^{\frac{2i}{3}\sqrt{-z^3}} \left(\sqrt[3]{-z^3} + (-1)^{2/3} z \right) \left(1 + O\!\left(\frac{1}{z^{3/2}}\right) \right) + \sqrt[12]{-1}\, e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\sqrt[3]{-z^3} - \sqrt[3]{-1}\, z \right) \left(1 + O\!\left(\frac{1}{z^{3/2}}\right) \right) \right) /;\ (|z| \to \infty)$$

Using exponential function with branch cut-free arguments

03.05.06.0040.01

$$\mathrm{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{(-z^3)^{-5/12}}{\sqrt{2}\,z^{3/2}}\left(e^{-\frac{2z^{3/2}}{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}-\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\left(1+\frac{385}{4608\,z^3}+\frac{37\,182\,145}{127\,401\,984\,z^6}+\frac{5\,849\,680\,962\,125}{1\,761\,205\,026\,816\,z^9}+O\left(\frac{1}{z^{12}}\right)\right)+\frac{5}{48\sqrt{2}\,(-z^3)^{17/12}}\left(e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{-\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}-\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z+\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\left(1+\frac{17\,017}{13\,824\,z^3}+\frac{1\,078\,282\,205}{127\,401\,984\,z^6}+\frac{253\,541\,886\,272\,675}{1\,761\,205\,026\,816\,z^9}+O\left(\frac{1}{z^{12}}\right)\right)\right)\,/;\,(|z|\to\infty)$$

03.05.06.0041.01

$$\mathrm{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{(-z^3)^{-5/12}}{\sqrt{2}\,z^{3/2}}\left(e^{-\frac{2z^{3/2}}{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}-\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)+\frac{5}{48\sqrt{2}\,(-z^3)^{17/12}}\left(e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{-\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}-\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z+\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)\right)\,/;\,(|z|\to\infty)\land n\in\mathbb{N}$$

03.05.06.0042.01

$$\operatorname{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{(-z^3)^{-5/12}}{\sqrt{2}\,z^{3/2}}\left(e^{-\frac{2z^{3/2}}{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}-\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+\frac{5}{48\sqrt{2}\,(-z^3)^{17/12}}\left(e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{-\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}-\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z+\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\right)\;/;\;(|z|\to\infty)$$

03.05.06.0043.01

$$\operatorname{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{(-z^3)^{-5/12}}{\sqrt{2}\,z^{3/2}}\left(e^{-\frac{2z^{3/2}}{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}-\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\,{}_4F_1\!\left(\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{11}{12};\frac{1}{2};\frac{9}{4z^3}\right)+\frac{5}{48\sqrt{2}\,(-z^3)^{17/12}}\left(e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{-\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}-\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z+\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\,{}_4F_1\!\left(\frac{7}{12},\frac{11}{12},\frac{13}{12},\frac{17}{12};\frac{3}{2};\frac{9}{4z^3}\right)\right)\;/;\;(|z|\to\infty)$$

03.05.06.0044.01

$$\mathrm{Ai}(z) \propto \frac{1}{4\sqrt{3}\pi}\left(\frac{(-z^3)^{-5/12}}{\sqrt{2}\,z^{3/2}}\left(e^{-\frac{2z^{3/2}}{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}-\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right)+\frac{5}{48\sqrt{2}\,(-z^3)^{17/12}}\left(e^{\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}+\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z-\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)-e^{-\frac{2z^{3/2}}{3}}\left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\,z^{3/2}+\left(-1+\sqrt{3}\right)z^{5/2}-\left(1+\sqrt{3}\right)\sqrt{-z^3}\,z+\left(-1+\sqrt{3}\right)(-z^3)^{5/6}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right)\right)\;/;\;(|z|\to\infty)$$

03.05.06.0045.01

$$\mathrm{Ai}(z) \propto \begin{cases} \frac{e^{-\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}}-\frac{i\,e^{\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} & \arg(z)\le-\frac{2\pi}{3} \\ \frac{e^{-\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} & -\frac{2\pi}{3}<\arg(z)\le\frac{2\pi}{3} \\ \frac{e^{-\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}}+\frac{i\,e^{\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} & \text{True} \end{cases}\;/;\;(|z|\to\infty)$$

Expansions for any *z* in trigonometric form

Using trigonometric functions with branch cut-containing arguments

03.05.06.0023.01

$$\mathrm{Ai}(z) \propto \frac{1}{2\sqrt{3}\pi\,(-z^3)^{5/12}}\left(\left(\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\sqrt{3}\left(\sqrt[3]{-z^3}-z\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(1+\frac{385}{4608\,z^3}+\frac{37\,182\,145}{127\,401\,984\,z^6}+O\left(\frac{1}{z^9}\right)\right)+\frac{5}{48\sqrt{-z^3}}\left(\sqrt{3}\left(z-\sqrt[3]{-z^3}\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(1+\frac{17\,017}{13\,824\,z^3}+\frac{1\,078\,282\,205}{127\,401\,984\,z^6}+O\left(\frac{1}{z^9}\right)\right)\right)\;/;\;(|z|\to\infty)$$

03.05.06.0024.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12}}{2\sqrt{3}\,\pi}\left(\left(\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\sqrt{3}\left(\sqrt[3]{-z^3}-z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k + \frac{5}{48\sqrt{-z^3}}\left(\sqrt{3}\left(z-\sqrt[3]{-z^3}\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\right) /;\ (|z|\to\infty)\land n\in\mathbb{N}$$

03.05.06.0025.01

$$\mathrm{Ai}(z) \propto \frac{(-z^3)^{-5/12}}{2\sqrt{3}\,\pi}\left(\left(\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\sqrt{3}\left(\sqrt[3]{-z^3}-z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k + \frac{5}{48\sqrt{-z^3}}\left(\sqrt{3}\left(z-\sqrt[3]{-z^3}\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\right) /;\ (|z|\to\infty)$$

03.05.06.0026.01

$$\mathrm{Ai}(z) \propto \frac{1}{2\sqrt{3}\,\pi}(-z^3)^{-5/12}\left(\left(\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\sqrt{3}\left(\sqrt[3]{-z^3}-z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right){}_4F_1\left(\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{11}{12};\frac{1}{2};\frac{9}{4z^3}\right) + \frac{5}{48\sqrt{-z^3}}\left(\sqrt{3}\left(z-\sqrt[3]{-z^3}\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right){}_4F_1\left(\frac{7}{12},\frac{11}{12},\frac{13}{12},\frac{17}{12};\frac{3}{2};\frac{9}{4z^3}\right)\right) /;\ (|z|\to\infty)$$

03.05.06.0027.01

$$\mathrm{Ai}(z) \propto \frac{1}{2\sqrt{3}\,\pi\,(-z^3)^{5/12}}\left(\left(\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\sqrt{3}\left(\sqrt[3]{-z^3}-z\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right) + \frac{5}{48\sqrt{-z^3}}\left(\sqrt{3}\left(z-\sqrt[3]{-z^3}\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\left(\sqrt[3]{-z^3}+z\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right)\right) /;\ (|z|\to\infty)$$

Using trigonometric functions with branch cut-free arguments

03.05.06.0046.01

$$\mathrm{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{\sqrt{2}\,(-z^3)^{-5/12}}{z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right)\left(1+\frac{385}{4608z^3}+\frac{37182145}{127401984z^6}+\frac{5849680962125}{1761205026816z^9}+O\left(\frac{1}{z^{12}}\right)\right)-\frac{5}{24\sqrt{2}\,(-z^3)^{17/12}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right)\left(1+\frac{17017}{13824z^3}+\frac{1078282205}{127401984z^6}+\frac{253541886272675}{1761205026816z^9}+O\left(\frac{1}{z^{12}}\right)\right)\right)\;/;\;(|z|\to\infty)$$

03.05.06.0047.01

$$\mathrm{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{\sqrt{2}\,(-z^3)^{-5/12}}{z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)-\frac{5}{24\sqrt{2}\,(-z^3)^{17/12}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)\right)\;/;\;(|z|\to\infty)\land n\in\mathbb{N}$$

03.05.06.0048.01

$$\mathrm{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{\sqrt{2}\,(-z^3)^{-5/12}}{z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k-\frac{5}{24\sqrt{2}\,(-z^3)^{17/12}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\right)\;/;\;(|z|\to\infty)$$

03.05.06.0049.01

$$\mathrm{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{\sqrt{2}\,(-z^3)^{-5/12}}{z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\cosh\left(\frac{2\,z^{3/2}}{3}\right)+\sqrt{-z^3}\,\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2\,z^{3/2}}{3}\right)\right){}_4F_1\left(\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{11}{12};\frac{1}{2};\frac{9}{4\,z^3}\right)-\frac{5}{24\sqrt{2}\,(-z^3)^{17/12}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\sinh\left(\frac{2\,z^{3/2}}{3}\right)+\sqrt{-z^3}\,\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2\,z^{3/2}}{3}\right)\right){}_4F_1\left(\frac{7}{12},\frac{11}{12},\frac{13}{12},\frac{17}{12};\frac{3}{2};\frac{9}{4\,z^3}\right)\right)\,/;\,(|z|\to\infty)$$

03.05.06.0050.01

$$\mathrm{Ai}(z) \propto \frac{1}{4\sqrt{3}\,\pi}\left(\frac{\sqrt{2}\,(-z^3)^{-5/12}}{z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\cosh\left(\frac{2\,z^{3/2}}{3}\right)+\sqrt{-z^3}\,\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2\,z^{3/2}}{3}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right)-\frac{5}{24\sqrt{2}\,(-z^3)^{17/12}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\sinh\left(\frac{2\,z^{3/2}}{3}\right)+\sqrt{-z^3}\,\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2\,z^{3/2}}{3}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right)\right)\,/;\,(|z|\to\infty)$$

03.05.06.0051.01

$$\mathrm{Ai}(z) \propto \begin{cases} -\frac{(-1)^{3/4}}{\sqrt{2\pi}\,\sqrt[4]{z}}\left(\cosh\left(\frac{2\,z^{3/2}}{3}\right)-i\sinh\left(\frac{2\,z^{3/2}}{3}\right)\right) & \arg(z)\le-\frac{2\pi}{3} \\ \frac{1}{2\sqrt{\pi}\,\sqrt[4]{z}}\left(\cosh\left(\frac{2\,z^{3/2}}{3}\right)-\sinh\left(\frac{2\,z^{3/2}}{3}\right)\right) & -\frac{2\pi}{3}<\arg(z)\le\frac{2\pi}{3} \\ \frac{\sqrt[4]{-1}}{\sqrt{2\pi}\,\sqrt[4]{z}}\left(\cosh\left(\frac{2\,z^{3/2}}{3}\right)+i\sinh\left(\frac{2\,z^{3/2}}{3}\right)\right) & \text{True} \end{cases}$$

Moment expansions

03.05.06.0013.01

$$\mathrm{Ai}(x) = \delta(x)+\sum_{k=1}^{\infty}\frac{(-1)^k}{3^k\,k!}\,\frac{\partial^{3k}\delta(x)}{\partial x^{3k}}\,/;\,x\in\mathbb{R}$$

Residue representations

03.05.06.0011.01

$$\mathrm{Ai}(z) = \frac{1}{2\pi\sqrt[6]{3}}\left(\sum_{j=0}^{\infty}\mathrm{res}_s\left(\left(\Gamma\left(s+\frac{1}{3}\right)\left(3^{-2/3}\,z\right)^{-3s}\right)\Gamma(s)\right)(-j)+\sum_{j=0}^{\infty}\mathrm{res}_s\left(\Gamma(s)\left(3^{-2/3}\,z\right)^{-3s}\left(\Gamma\left(s+\frac{1}{3}\right)\right)\right)\left(-j-\frac{1}{3}\right)\right)$$

03.05.06.0012.01

$$\mathrm{Ai}(z) = \frac{\pi}{3^{2/3}}\left(\sum_{j=0}^{\infty}\mathrm{res}_s\left(\frac{\left(\frac{z^3}{9}\right)^{-s}}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{2}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s)\right)(-j) - \frac{z}{3^{2/3}}\sum_{j=0}^{\infty}\mathrm{res}_s\left(\frac{\left(\frac{z^3}{9}\right)^{-s}}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{4}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s)\right)(-j)\right)$$

Integral representations

On the real axis

Of the direct function

03.05.07.0001.01

$$\mathrm{Ai}(z) = \frac{1}{\pi}\int_0^{\infty}\cos\left(\frac{t^3}{3}+z\,t\right)dt\,/;\ \mathrm{Im}(z) = 0$$

03.05.07.0008.01

$$\mathrm{Ai}(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\left(\frac{t^3}{3}+z\,t\right)}dt\,/;\ \mathrm{Im}(z) = 0$$

Involving the direct function

03.05.07.0002.01

$$\mathrm{Ai}(x)^2 = \frac{1}{4\pi\sqrt{3}}\int_0^{\infty}t\,J_0\left(\frac{t^3}{12}+x\,t\right)dt\,/;\ x > 0$$

Involving related functions

03.05.07.0003.01

$$\mathrm{Ai}(x)^2 + \mathrm{Bi}(x)^2 = \frac{1}{\pi^{3/2}}\int_0^{\infty}\frac{1}{\sqrt{t}}\,e^{x\,t-\frac{t^3}{12}}\,dt$$

Contour integral representations

03.05.07.0004.01

$$\mathrm{Ai}(z) = \frac{1}{2\pi i}\int_{\infty\,e^{-\frac{\pi i}{3}}}^{\infty\,e^{\frac{\pi i}{3}}}e^{\frac{t^3}{3}-z\,t}\,dt$$

03.05.07.0005.01

$$\mathrm{Ai}(z) = \frac{1}{\left(2\pi\sqrt[6]{3}\right)2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\Gamma(s)\,\Gamma\left(s+\frac{1}{3}\right)\left(3^{-2/3}z\right)^{-3s}ds\,/;\ 0 < \gamma$$

03.05.07.0006.01

$$\mathrm{Ai}(z) = \frac{1}{\left(2\pi\sqrt[6]{3}\right)2\pi i}\int_{\mathcal{L}}\Gamma(s)\,\Gamma\left(s+\frac{1}{3}\right)\left(3^{-2/3}z\right)^{-3s}ds$$

03.05.07.0007.01

$$\mathrm{Ai}(z) = \frac{\pi}{3^{2/3}}\left(\frac{1}{2\pi i}\int_{\mathcal{L}}\frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{2}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\left(\frac{z^3}{9}\right)^{-s}ds - \frac{z}{3^{2/3}}\,\frac{1}{2\pi i}\int_{\mathcal{L}}\frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{4}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\left(\frac{z^3}{9}\right)^{-s}ds\right)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.05.13.0001.01

$$w''(z) - z\,w(z) = 0\ /;\ w(z) = \mathrm{Ai}(z) \bigwedge w(0) = \frac{1}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)} \bigwedge w'(0) = -\frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}$$

03.05.13.0002.01

$$w''(z) - z\,w(z) = 0\ /;\ w(z) = \mathrm{Ai}(z)\,c_1 + c_2\,\mathrm{Bi}(z)$$

03.05.13.0003.01

$$W_z(\mathrm{Ai}(z), \mathrm{Bi}(z)) = \frac{1}{\pi}$$

03.05.13.0004.01

$$W_z\left(\mathrm{Ai}(z), \mathrm{Ai}\left(z\,e^{\frac{2\pi i}{3}}\right)\right) = \frac{1}{2\pi}\,e^{-\frac{\pi i}{6}}$$

03.05.13.0005.01

$$W_z\left(\mathrm{Ai}(z), \mathrm{Ai}\left(z\,e^{-\frac{1}{3}(2\pi i)}\right)\right) = \frac{1}{2\pi}\,e^{\frac{\pi i}{6}}$$

03.05.13.0006.01

$$W_z\left(\mathrm{Ai}\left(z\,e^{-\frac{1}{3}(2\pi i)}\right), \mathrm{Ai}\left(z\,e^{\frac{2\pi i}{3}}\right)\right) = \frac{1}{2\pi i}$$

03.05.13.0012.01

$$g'(z)\,w''(z) - g''(z)\,w'(z) - g(z)\,g'(z)^3\,w(z) = 0\ /;\ w(z) = c_1\,\mathrm{Ai}(g(z)) + c_2\,\mathrm{Bi}(g(z))$$

03.05.13.0013.01

$$W_z(\mathrm{Ai}(g(z)), \mathrm{Bi}(g(z))) = \frac{g'(z)}{\pi}$$

03.05.13.0014.01

$$g'(z)\,h(z)^2\,w''(z) - (2\,g'(z)\,h'(z) + h(z)\,g''(z))\,h(z)\,w'(z) + \left(-g(z)\,h(z)^2\,g'(z)^3 + 2\,h'(z)^2\,g'(z) - h(z)\,h''(z)\,g'(z) + h(z)\,h'(z)\,g''(z)\right)w(z) = 0\ /;\ w(z) = c_1\,h(z)\,\mathrm{Ai}(g(z)) + c_2\,h(z)\,\mathrm{Bi}(g(z))$$

03.05.13.0015.01

$$W_z(h(z)\,\mathrm{Ai}(g(z)), h(z)\,\mathrm{Bi}(g(z))) = \frac{h(z)^2\,g'(z)}{\pi}$$

03.05.13.0016.01

$$z^2\,w''(z) + z\,(1 - r - 2\,s)\,w'(z) + \left(-a^3\,r^2\,z^{3r} + s^2 + r\,s\right)w(z) = 0\ /;\ w(z) = c_1\,z^s\,\mathrm{Ai}(a\,z^r) + c_2\,z^s\,\mathrm{Bi}(a\,z^r)$$

03.05.13.0017.01

$$W_z(z^s\,\mathrm{Ai}(a\,z^r), z^s\,\mathrm{Bi}(a\,z^r)) = \frac{a\,r\,z^{r+2s-1}}{\pi}$$

03.05.13.0018.01

$$w''(z) - (\log(r) + 2\log(s))\,w'(z) + \left(-a^3\log^2(r)\,r^{3z} + \log^2(s) + \log(r)\log(s)\right)w(z) = 0\ /;\ w(z) = c_1\,s^z\,\mathrm{Ai}(a\,r^z) + c_2\,s^z\,\mathrm{Bi}(a\,r^z)$$

03.05.13.0019.01

$$W_z(s^z \operatorname{Ai}(a r^z), s^z \operatorname{Bi}(a r^z)) = \frac{a r^z s^{2z} \log(r)}{\pi}$$

Involving related functions

03.05.13.0007.01

$$w^{(3)}(z) - 4 z w'(z) - 2 w(z) = 0 /; w(z) = c_1 \operatorname{Ai}(z)^2 + c_2 \operatorname{Bi}(z) \operatorname{Ai}(z) + c_3 \operatorname{Bi}(z)^2$$

03.05.13.0008.01

$$W_z\left(\operatorname{Ai}(z)^2, \operatorname{Ai}(z) \operatorname{Bi}(z), \operatorname{Bi}(z)^2\right) = \frac{2}{\pi^3}$$

03.05.13.0009.01

$$w^{(3)}(z) - 4 z w'(z) - 2 w(z) = 0 /; w(z) = w_1(z) w_2(z) \land w_1''(z) - z w_1(z) = 0 \land w_2''(z) - z w_2(z) = 0$$

Ordinary nonlinear differential equations

03.05.13.0010.01

$$w'(z) + w(z)^2 - z = 0 /; w(z) = \frac{\operatorname{Bi}'(z) + c_1 \operatorname{Ai}'(z)}{\operatorname{Bi}(z) + c_1 \operatorname{Ai}(z)}$$

Riccati form of differential equation

03.05.13.0011.01

$$256 w'(z)^2 z^5 - 128 w'(z) w^{(3)}(z) z^4 + 16 w^{(3)}(z)^2 z^3 + 192 w'(z) w''(z) z^3 - 80 w'(z)^2 z^2 - 48 w''(z) w^{(3)}(z) z^2 + 36 w''(z)^2 z + 16 w'(z) w^{(3)}(z) z + w^{(3)}(z)^2 - 36 w'(z) w''(z) = 0 /; w(z) = \operatorname{Ai}(z) \operatorname{Ai}'(z)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.05.16.0001.01

$$\operatorname{Ai}(c (d z^n)^m) = \frac{1}{2} \left(\frac{(d z^n)^m}{d^m z^{m n}} + 1\right) \operatorname{Ai}(c d^m z^{m n}) - \frac{1}{2\sqrt{3}} \left(\frac{(d z^n)^m}{d^m z^{m n}} - 1\right) \operatorname{Bi}(c d^m z^{m n}) /; 3 m \in \mathbb{Z}$$

03.05.16.0002.01

$$\operatorname{Ai}\left(\sqrt[3]{z^3}\right) = \frac{1}{2} \left(\frac{\sqrt[3]{z^3}}{z} + 1\right) \operatorname{Ai}(z) - \frac{1}{2\sqrt{3}} \left(\frac{\sqrt[3]{z^3}}{z} - 1\right) \operatorname{Bi}(z)$$

03.05.16.0003.01

$$\operatorname{Ai}\left((-1)^{2/3} z\right) = \frac{1}{4} \left(1 + i \sqrt{3}\right) (\operatorname{Ai}(z) - i \operatorname{Bi}(z))$$

03.05.16.0004.01

$$\operatorname{Ai}\left(-\left(\sqrt[3]{-1}\, z\right)\right) = \frac{1}{4} \left(1 - i \sqrt{3}\right) (\operatorname{Ai}(z) + i \operatorname{Bi}(z))$$

Identities

Functional identities

03.05.17.0001.01

$$e^{\frac{2\pi i}{3}} \mathrm{Ai}\left(z\, e^{\frac{2\pi i}{3}}\right) + e^{-\frac{2\pi i}{3}} \mathrm{Ai}\left(z\, e^{-\frac{2\pi i}{3}}\right) + \mathrm{Ai}(z) == 0$$

Identities involving determinants

03.05.17.0002.01

$$\frac{\partial^2 w(z)}{\partial z^2} == 2\, w(z)^3 + 2\, z\, w(z) + (2\, n + 1) \,/;\, w(z) == \frac{\partial \log\left(\frac{\tau_{n+1}(z)}{\tau_n(z)}\right)}{\partial z} \bigwedge \tau_n(z) == \left| \left(\frac{\partial^{k+l} \mathrm{Ai}(z)}{\partial z^{k+l}} \right)_{\substack{0 \le k \le n-1 \\ 0 \le l \le n-1}} \right|$$

Complex characteristics

Real part

03.05.19.0001.01

$$\mathrm{Re}(\mathrm{Ai}(x + i\, y)) == \frac{1}{2} \left(\mathrm{Ai}\left(x - x \sqrt{-\frac{y^2}{x^2}} \right) + \mathrm{Ai}\left(x + x \sqrt{-\frac{y^2}{x^2}} \right) \right)$$

Imaginary part

03.05.19.0002.01

$$\mathrm{Im}(\mathrm{Ai}(x + i\, y)) == \frac{x}{2\, y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Ai}\left(x - x \sqrt{-\frac{y^2}{x^2}} \right) - \mathrm{Ai}\left(x + x \sqrt{-\frac{y^2}{x^2}} \right) \right)$$

Absolute value

03.05.19.0003.01

$$|\mathrm{Ai}(x + i\, y)| == \sqrt{ \mathrm{Ai}\left(x - x \sqrt{-\frac{y^2}{x^2}} \right) \mathrm{Ai}\left(x + x \sqrt{-\frac{y^2}{x^2}} \right) }$$

Argument

03.05.19.0004.01

$$\arg(\mathrm{Ai}(x + i\, y)) == \tan^{-1}\left(\frac{1}{2} \left(\mathrm{Ai}\left(x - x \sqrt{-\frac{y^2}{x^2}} \right) + \mathrm{Ai}\left(x + x \sqrt{-\frac{y^2}{x^2}} \right) \right), \frac{x}{2\, y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Ai}\left(x - x \sqrt{-\frac{y^2}{x^2}} \right) - \mathrm{Ai}\left(x + x \sqrt{-\frac{y^2}{x^2}} \right) \right) \right)$$

Conjugate value

03.05.19.0005.01

$$\overline{\mathrm{Ai}(x + i\, y)} == \frac{1}{2} \left(\mathrm{Ai}\left(\sqrt{-\frac{y^2}{x^2}}\, x + x \right) + \mathrm{Ai}\left(x - x \sqrt{-\frac{y^2}{x^2}} \right) \right) - \frac{i\, x}{2\, y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Ai}\left(x - x \sqrt{-\frac{y^2}{x^2}} \right) - \mathrm{Ai}\left(\sqrt{-\frac{y^2}{x^2}}\, x + x \right) \right)$$

Signum value

03.05.19.0006.01

$$\operatorname{sgn}(\operatorname{Ai}(x+i\,y)) = \frac{\frac{i}{y}\sqrt{-\frac{y^2}{x^2}}\,x\left(\operatorname{Ai}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Ai}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)\right)+\operatorname{Ai}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)+\operatorname{Ai}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)}{2\sqrt{\operatorname{Ai}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Ai}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)}}$$

Differentiation

Low-order differentiation

03.05.20.0001.01

$$\frac{\partial \operatorname{Ai}(z)}{\partial z} = \operatorname{Ai}'(z)$$

03.05.20.0002.01

$$\frac{\partial^2 \operatorname{Ai}(z)}{\partial z^2} = z\operatorname{Ai}(z)$$

Symbolic differentiation

03.05.20.0005.01

$$\frac{\partial^n \operatorname{Ai}(z)}{\partial z^n} = \frac{1}{2}\operatorname{Ai}(z)\,\delta_n + \frac{1}{2}z^{-n}\left(\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k-1}\,i\,(-i+k-1)!\,(-3\,i+3\,k-1)\,(-3\,j+3\,k-n+1)_n\left(-\frac{1}{3}\right)_k}{i!\,j!\,(k-j)!\,(k-2\,i)!\left(\frac{2}{3}\right)_i\left(\frac{4}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i - \sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k}\frac{(-1)^{j+k-1}\,(k-i)!\,(-3\,j+3\,k+1)\,(-3\,j+3\,k-n+2)_{n-1}\left(\frac{1}{3}\right)_k}{i!\,j!\,(k-j)!\,(k-2\,i)!\left(\frac{1}{3}\right)_i\left(\frac{2}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i\right)\operatorname{Ai}(z) + \frac{1}{2}z^{1-n}\left(\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k-1}\,(-i+k-1)!\,(-3\,j+3\,k+1)\,(-3\,j+3\,k-n+2)_{n-1}\left(\frac{1}{3}\right)_k}{i!\,j!\,(k-j)!\,(-2\,i+k-1)!\left(\frac{4}{3}\right)_i\left(\frac{2}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i - \sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k-1}\,(-i+k-1)!\,(-3\,j+3\,k-n+1)_n\left(-\frac{1}{3}\right)_k}{i!\,j!\,(k-j)!\,(-2\,i+k-1)!\left(\frac{2}{3}\right)_i\left(\frac{4}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i\right)\operatorname{Ai}'(z)\,/;\,n\in\mathbb{N}$$

03.05.20.0003.02

$$\frac{\partial^n \operatorname{Ai}(z)}{\partial z^n} = 3^{n-\frac{4}{3}}\,z^{-n}\left(3^{2/3}\,\Gamma\left(\frac{1}{3}\right){}_2\tilde{F}_3\left(\frac{1}{3},1;\frac{1-n}{3},\frac{2-n}{3},1-\frac{n}{3};\frac{z^3}{9}\right)-z\,\Gamma\left(\frac{2}{3}\right){}_2\tilde{F}_3\left(\frac{2}{3},1;\frac{2-n}{3},1-\frac{n}{3},\frac{4-n}{3};\frac{z^3}{9}\right)\right)/;\,n\in\mathbb{N}$$

Fractional integro-differentiation

03.05.20.0004.01

$$\frac{\partial^\alpha \operatorname{Ai}(z)}{\partial z^\alpha} = 3^{\alpha-\frac{4}{3}}\,z^{-\alpha}\left(3^{2/3}\,\Gamma\left(\frac{1}{3}\right){}_2\tilde{F}_3\left(\frac{1}{3},1;\frac{1-\alpha}{3},\frac{2-\alpha}{3},1-\frac{\alpha}{3};\frac{z^3}{9}\right)-z\,\Gamma\left(\frac{2}{3}\right){}_2\tilde{F}_3\left(\frac{2}{3},1;\frac{2-\alpha}{3},1-\frac{\alpha}{3},\frac{4-\alpha}{3};\frac{z^3}{9}\right)\right)$$

Integration

Indefinite integration

Involving only one direct function

03.05.21.0001.01

$$\int \operatorname{Ai}(a\,z)\,dz = \frac{z\,\Gamma\left(\frac{1}{3}\right)}{3\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\,{}_1F_2\left(\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{a^3 z^3}{9}\right) - \frac{a\,z^2\,\Gamma\left(\frac{2}{3}\right)}{9\sqrt[3]{3}\,\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right)}\,{}_1F_2\left(\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{a^3 z^3}{9}\right)$$

03.05.21.0002.01

$$\int \operatorname{Ai}(z)\,dz = \frac{z}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_1F_2\left(\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{z^3}{9}\right) - \frac{\sqrt[6]{3}}{4\pi}\,z^2\,\Gamma\left(\frac{2}{3}\right)\,{}_1F_2\left(\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{z^3}{9}\right)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear arguments

03.05.21.0003.01

$$\int z^{\alpha-1}\operatorname{Ai}(a\,z)\,dz = \frac{z^{\alpha}\,\Gamma\left(\frac{\alpha}{3}\right)}{3\,3^{2/3}}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3};\frac{2}{3},\frac{\alpha}{3}+1;\frac{a^3 z^3}{9}\right) - \frac{a\,z^{\alpha+1}\,\Gamma\left(\frac{\alpha}{3}+\frac{1}{3}\right)}{9\sqrt[3]{3}}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3}+\frac{1}{3};\frac{4}{3},\frac{\alpha}{3}+\frac{4}{3};\frac{a^3 z^3}{9}\right)$$

03.05.21.0004.01

$$\int z^{\alpha-1}\operatorname{Ai}(z)\,dz = \frac{\Gamma\left(\frac{\alpha}{3}\right)}{3\,3^{2/3}}\,z^{\alpha}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3};\frac{2}{3},\frac{\alpha}{3}+1;\frac{z^3}{9}\right) - \frac{\Gamma\left(\frac{\alpha}{3}+\frac{1}{3}\right)}{9\sqrt[3]{3}}\,z^{\alpha+1}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3}+\frac{1}{3};\frac{4}{3},\frac{\alpha}{3}+\frac{4}{3};\frac{z^3}{9}\right)$$

03.05.21.0005.01

$$\int z^{n+3}\operatorname{Ai}(z)\,dz = -(n+2)\operatorname{Ai}(z)\,z^{n+1} + \operatorname{Ai}'(z)\,z^{n+2} + (n+1)(n+2)\int z^{n}\operatorname{Ai}(z)\,dz\;/;\;n\in\mathbb{N}$$

03.05.21.0006.01

$$\int z\operatorname{Ai}(z)\,dz = \operatorname{Ai}'(z)$$

03.05.21.0007.01

$$\int z^2\operatorname{Ai}(z)\,dz = z\operatorname{Ai}'(z) - \operatorname{Ai}(z)$$

03.05.21.0008.01

$$\int \sqrt{z}\operatorname{Ai}(z)\,dz = \frac{z^{3/2}}{9\,3^{2/3}}\left(\frac{6}{\Gamma\left(\frac{2}{3}\right)}\,{}_1F_2\left(\frac{1}{2};\frac{2}{3},\frac{3}{2};\frac{z^3}{9}\right) - \frac{\sqrt[3]{3}\,z\,\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{11}{6}\right)}\,{}_1F_2\left(\frac{5}{6};\frac{4}{3},\frac{11}{6};\frac{z^3}{9}\right)\right)$$

Power arguments

03.05.21.0009.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(a\,z^r)\,dz = \frac{z^\alpha\,\Gamma\left(\frac{\alpha}{3r}\right)}{3\,3^{2/3}\,r}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3r};\,\frac{2}{3},\,\frac{\alpha}{3r}+1;\,\frac{1}{9}\,a^3\,z^{3r}\right) - \frac{a\,z^{r+\alpha}\,\Gamma\left(\frac{\alpha}{3r}+\frac{1}{3}\right)}{9\sqrt[3]{3}\,r}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3r}+\frac{1}{3};\,\frac{4}{3},\,\frac{\alpha}{3r}+\frac{4}{3};\,\frac{1}{9}\,a^3\,z^{3r}\right)$$

Involving exponential function

Involving exp

Linear argument

03.05.21.0010.01

$$\int e^{\frac{1}{3}(-2)(a\,z)^{3/2}}\,\mathrm{Ai}(a\,z)\,dz = \frac{z}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_2F_2\left(\frac{1}{6},\,\frac{2}{3};\,\frac{1}{3},\,\frac{5}{3};\,\frac{1}{3}\,(-4)\,(a\,z)^{3/2}\right) - \frac{a\,z^2}{6\sqrt[3]{3}\,\Gamma\left(\frac{4}{3}\right)}\,{}_2F_2\left(\frac{5}{6},\,\frac{4}{3};\,\frac{5}{3},\,\frac{7}{3};\,\frac{1}{3}\,(-4)\,(a\,z)^{3/2}\right)$$

03.05.21.0011.01

$$\int e^{\frac{2}{3}(a\,z)^{3/2}}\,\mathrm{Ai}(a\,z)\,dz = \frac{z}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_2F_2\left(\frac{1}{6},\,\frac{2}{3};\,\frac{1}{3},\,\frac{5}{3};\,\frac{4}{3}\,(a\,z)^{3/2}\right) - \frac{a\,z^2}{6\sqrt[3]{3}\,\Gamma\left(\frac{4}{3}\right)}\,{}_2F_2\left(\frac{5}{6},\,\frac{4}{3};\,\frac{5}{3},\,\frac{7}{3};\,\frac{4}{3}\,(a\,z)^{3/2}\right)$$

Power arguments

03.05.21.0012.01

$$\int e^{\frac{1}{3}(-2)(a\,z^r)^{3/2}}\,\mathrm{Ai}(a\,z^r)\,dz = \frac{1}{3\,3^{2/3}\,(r+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}$$
$$\left(z\left(3\,(r+1)\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_2\left(\frac{1}{6},\,\frac{2}{3r};\,\frac{1}{3},\,1+\frac{2}{3r};\,\frac{1}{3}\,(-4)\,(a\,z^r)^{3/2}\right) - \sqrt[3]{3}\,a\,z^r\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_2\left(\frac{5}{6},\,\frac{2}{3}+\frac{2}{3r};\,\frac{5}{3},\,\frac{5}{3}+\frac{2}{3r};\,\frac{1}{3}\,(-4)\,(a\,z^r)^{3/2}\right)\right)\right)$$

03.05.21.0013.01

$$\int e^{\frac{2}{3}(a\,z^r)^{3/2}}\,\mathrm{Ai}(a\,z^r)\,dz = \frac{z}{3\,3^{2/3}\,(r+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}$$
$$\left(3\,(r+1)\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_2\left(\frac{1}{6},\,\frac{2}{3r};\,\frac{1}{3},\,1+\frac{2}{3r};\,\frac{4}{3}\,(a\,z^r)^{3/2}\right) - \sqrt[3]{3}\,a\,z^r\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_2\left(\frac{5}{6},\,\frac{2}{3}+\frac{2}{3r};\,\frac{5}{3},\,\frac{5}{3}+\frac{2}{3r};\,\frac{4}{3}\,(a\,z^r)^{3/2}\right)\right)$$

Involving exponential function and a power function

Involving exp and power

Linear arguments

03.05.21.0014.01

$$\int z^{\alpha-1} e^{-\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}(a z) \, dz = \frac{1}{3\, 3^{2/3} \alpha (\alpha+1) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z^\alpha \left(3 (\alpha+1) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2\alpha}{3}; \frac{1}{3}, \frac{2\alpha}{3}+1; \frac{1}{3}(-4)(a z)^{3/2}\right) - \sqrt[3]{3}\, a z \alpha \, \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2\alpha}{3}+\frac{2}{3}; \frac{5}{3}, \frac{2\alpha}{3}+\frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)$$

03.05.21.0015.01

$$\int \sqrt{z}\, e^{\frac{1}{3}(-2)(a z)^{3/2}} \operatorname{Ai}(a z) \, dz = \frac{1}{15 a^2 \sqrt{z}\, \Gamma\left(\frac{2}{3}\right)} \left(2 e^{\frac{1}{3}(-2)(a z)^{3/2}} \left(3 a^2 \operatorname{Ai}(a z) \Gamma\left(\frac{2}{3}\right) z^2 + \sqrt{a z} \left(-a z\, I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{2}{3}\right) \left(a^{3/2} z^{3/2}\right)^{2/3} + \sqrt[3]{3}\, e^{\frac{2}{3}(a z)^{3/2}} + \frac{a^3 z^3 \, \Gamma\left(\frac{2}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right)\right)$$

03.05.21.0016.01

$$\int z^{\alpha-1} e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}(a z) \, dz = \frac{z^\alpha}{3\, 3^{2/3} \alpha (\alpha+1) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(3 (\alpha+1) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2\alpha}{3}; \frac{1}{3}, \frac{2\alpha}{3}+1; \frac{4}{3}(a z)^{3/2}\right) - \sqrt[3]{3}\, a z \alpha \, \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2\alpha}{3}+\frac{2}{3}; \frac{5}{3}, \frac{2\alpha}{3}+\frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right)\right)$$

03.05.21.0017.01

$$\int \sqrt{z}\, e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}(a z) \, dz = \frac{1}{15 a^2 \sqrt{z}\, \Gamma\left(\frac{2}{3}\right)} \left(6 a^2 e^{\frac{2}{3}(a z)^{3/2}} z^2 \operatorname{Ai}(a z) \Gamma\left(\frac{2}{3}\right) - 2\sqrt{a z} \left(-a e^{\frac{2}{3}(a z)^{3/2}} z\, I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{2}{3}\right) \left(a^{3/2} z^{3/2}\right)^{2/3} + \sqrt[3]{3} + \frac{a^3 \, \Gamma\left(\frac{2}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} e^{\frac{2}{3}(a z)^{3/2}} z^3 I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right)$$

Power arguments

03.05.21.0018.01

$$\int z^{\alpha-1} e^{-\frac{2}{3}(a z^r)^{3/2}} \operatorname{Ai}(a z^r) \, dz = \frac{1}{3\, 3^{2/3} \alpha (r+\alpha) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z^\alpha \left(3 (r+\alpha) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2\alpha}{3r}; \frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - \sqrt[3]{3}\, a z^r \alpha \, \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2\alpha}{3r}+\frac{2}{3}; \frac{5}{3}, \frac{2\alpha}{3r}+\frac{5}{3}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.05.21.0019.01

$$\int z^{\alpha-1} e^{\frac{2}{3}(a z^r)^{3/2}} \operatorname{Ai}(a z^r) \, dz = \frac{z^\alpha}{3\, 3^{2/3} \alpha (r+\alpha) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(3 (r+\alpha) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2\alpha}{3r}; \frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{4}{3}(a z^r)^{3/2}\right) - \sqrt[3]{3}\, a z^r \alpha \, \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2\alpha}{3r}+\frac{2}{3}; \frac{5}{3}, \frac{2\alpha}{3r}+\frac{5}{3}; \frac{4}{3}(a z^r)^{3/2}\right)\right)$$

Involving hyperbolic functions

Involving sinh

Linear argument

03.05.21.0020.01

$$\int \sinh\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Ai}(a z)\, dz = \frac{1}{12\, 3^{2/3}\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z \left(6\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right) - 6\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) + \sqrt[3]{3}\, a z\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right)\right)\right)\right)$$

03.05.21.0021.01

$$\int \sinh\left(\frac{2}{3}(a z)^{3/2} + b\right) \mathrm{Ai}(a z)\, dz = -\frac{1}{12\, 3^{2/3}\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(e^{-b} z \left(-6\, e^{2b}\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right) + 6\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - \sqrt[3]{3}\, a z\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - e^{2b}\, {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right)\right)\right)\right)$$

Power arguments

03.05.21.0022.01

$$\int \sinh\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Ai}(a z^r)\, dz = \frac{1}{6\, 3^{2/3}\, (r+1)\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z \left(\sqrt[3]{3}\, a\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right)\right) z^r + 3(r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) - 3(r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.05.21.0023.01

$$\int \sinh\left(\frac{2}{3}(a z^r)^{3/2} + b\right) \mathrm{Ai}(a z^r)\, dz = -\frac{1}{6\, 3^{2/3}\, (r+1)\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(e^{-b} z \left(-\sqrt[3]{3}\, a\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - e^{2b}\, {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right)\right) z^r - 3\, e^{2b} (r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + 3(r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

Involving cosh

Linear argument

03.05.21.0024.01

$$\int \cosh\left(\frac{2}{3}(a z)^{3/2}\right) \operatorname{Ai}(a z)\, dz = -\frac{1}{12\, 3^{2/3}\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z\left(-6\,\Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right) - 6\,\Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) + \sqrt[3]{3}\, a z\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right) + {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)\right)$$

03.05.21.0025.01

$$\int \cosh\left(\frac{2}{3}(a z)^{3/2} + b\right) \operatorname{Ai}(a z)\, dz = -\frac{1}{12\, 3^{2/3}\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(e^{-b} z\left(-6\, e^{2b}\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right) - 6\,\Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) + \sqrt[3]{3}\, a z\, \Gamma\left(\frac{2}{3}\right) \left(e^{2b}\, {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right) + {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)\right)$$

Power arguments

03.05.21.0026.01

$$\int \cosh\left(\frac{2}{3}(a z^r)^{3/2}\right) \operatorname{Ai}(a z^r)\, dz = \frac{1}{6\, 3^{2/3}\, (r+1)\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z\left(-\sqrt[3]{3}\, a\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right) z^r + 3(r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + 3(r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.05.21.0027.01

$$\int \cosh\left(\frac{2}{3}(a z^r)^{3/2} + b\right) \operatorname{Ai}(a z^r)\, dz = \frac{1}{6\, 3^{2/3}\, (r+1)\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(e^{-b} z\left(-\sqrt[3]{3}\, a\, \Gamma\left(\frac{2}{3}\right) \left(e^{2b}\, {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right) z^r + 3\, e^{2b}\, (r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + 3(r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

Involving hyperbolic functions and a power function

Involving sinh and power

Linear argument

03.05.21.0028.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Ai}(a\,z)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(\alpha+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(z^{\alpha}\left(3\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3};\frac{1}{3},\frac{2\,\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)-3\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3};\frac{1}{3},\frac{2\,\alpha}{3}+1;\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)+\sqrt[3]{3}\,a\,z\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)-{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3}+\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)\right)\right)\right)$$

03.05.21.0029.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z)^{3/2}+b\right)\mathrm{Ai}(a\,z)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(\alpha+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(3\,e^{2\,b}\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3};\frac{1}{3},\frac{2\,\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)-3\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3};\frac{1}{3},\frac{2\,\alpha}{3}+1;\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)+\sqrt[3]{3}\,a\,z\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)-e^{2\,b}\,{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3}+\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)\right)\right)\right)$$

Power arguments

03.05.21.0030.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z^r)^{3/2}\right)\mathrm{Ai}(a\,z^r)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(r+\alpha)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(z^{\alpha}\left(\sqrt[3]{3}\,a\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3\,r}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3\,r}+\frac{5}{3};-\frac{4}{3}(a\,z^r)^{3/2}\right)-{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3\,r}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3\,r}+\frac{5}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)\right)z^r+3\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3\,r};\frac{1}{3},\frac{2\,\alpha}{3\,r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)-3\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3\,r};\frac{1}{3},\frac{2\,\alpha}{3\,r}+1;-\frac{4}{3}(a\,z^r)^{3/2}\right)\right)\right)$$

03.05.21.0031.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z^r)^{3/2}+b\right)\mathrm{Ai}(a\,z^r)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(r+\alpha)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(\sqrt[3]{3}\,a\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3\,r}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3\,r}+\frac{5}{3};-\frac{4}{3}(a\,z^r)^{3/2}\right)-e^{2\,b}\,{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3\,r}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3\,r}+\frac{5}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)\right)z^r+3\,e^{2\,b}\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3\,r};\frac{1}{3},\frac{2\,\alpha}{3\,r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)-3\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3\,r};\frac{1}{3},\frac{2\,\alpha}{3\,r}+1;-\frac{4}{3}(a\,z^r)^{3/2}\right)\right)\right)$$

Involving cosh and power

Linear argument

03.05.21.0032.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Ai}(a\,z)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(\alpha+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)} \left(z^{\alpha}\left(3\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3};\frac{1}{3},\frac{2\,\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)+3\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3};\frac{1}{3},\frac{2\,\alpha}{3}+1;\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)-\sqrt[3]{3}\;a\,z\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3}+\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)\right)\right)\right)$$

03.05.21.0033.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z)^{3/2}+b\right)\mathrm{Ai}(a\,z)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(\alpha+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)} \left(e^{-b}\,z^{\alpha}\left(3\,e^{2\,b}\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3};\frac{1}{3},\frac{2\,\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)+3\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3};\frac{1}{3},\frac{2\,\alpha}{3}+1;\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)-\sqrt[3]{3}\;a\,z\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left(e^{2\,b}\,{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3}+\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)\right)\right)\right)$$

Power arguments

03.05.21.0034.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z^r)^{3/2}\right)\mathrm{Ai}(a\,z^r)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(r+\alpha)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)} \left(z^{\alpha}\left(-\sqrt[3]{3}\;a\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3\,r}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3\,r}+\frac{5}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3\,r}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3\,r}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z^r)^{3/2}\right)\right)z^r+3\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3\,r};\frac{1}{3},\frac{2\,\alpha}{3\,r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+3\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3\,r};\frac{1}{3},\frac{2\,\alpha}{3\,r}+1;\frac{1}{3}(-4)\,(a\,z^r)^{3/2}\right)\right)\right)$$

03.05.21.0035.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z^r)^{3/2}+b\right)\mathrm{Ai}(a\,z^r)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(r+\alpha)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)} \left(e^{-b}\,z^{\alpha}\left(-\sqrt[3]{3}\;a\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left(e^{2\,b}\,{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3\,r}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3\,r}+\frac{5}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2\,\alpha}{3\,r}+\frac{2}{3};\frac{5}{3},\frac{2\,\alpha}{3\,r}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z^r)^{3/2}\right)\right)z^r+3\,e^{2\,b}\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3\,r};\frac{1}{3},\frac{2\,\alpha}{3\,r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+3\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\,\alpha}{3\,r};\frac{1}{3},\frac{2\,\alpha}{3\,r}+1;\frac{1}{3}(-4)\,(a\,z^r)^{3/2}\right)\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

Linear arguments

03.05.21.0036.01

$$\int \mathrm{Ai}(a\,z)^2\, dz = z\,\mathrm{Ai}(a\,z)^2 - \frac{\mathrm{Ai}'(a\,z)^2}{a}$$

03.05.21.0037.01

$$\int \frac{1}{\mathrm{Ai}(a\,z)^2}\, dz = \frac{\pi\,\mathrm{Bi}(a\,z)}{a\,\mathrm{Ai}(a\,z)}$$

Power arguments

03.05.21.0038.01

$$\int \mathrm{Ai}(a\,z^r)^2\, dz = \frac{z}{6\,2^{2/3}\sqrt[3]{3}\,\pi^{3/2}\,r}\, G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\ \frac{1}{3}\ \middle|\ \begin{matrix} 1-\frac{1}{3r},\ \frac{5}{6} \\ 0,\ \frac{1}{3},\ \frac{2}{3},\ -\frac{1}{3r} \end{matrix}\right)$$

Involving products of the direct function

Linear arguments

03.05.21.0039.01

$$\int \mathrm{Ai}(-a\,z)\,\mathrm{Ai}(a\,z)\, dz = \frac{1}{4\sqrt[3]{2}\,3^{2/3}\,a\,\pi^{3/2}}\, G_{1,5}^{3,1}\!\left(\frac{a\,z}{\sqrt[3]{2}\,3^{2/3}},\ \frac{1}{6}\ \middle|\ \begin{matrix} 1 \\ \frac{1}{6},\ \frac{1}{2},\ \frac{5}{6},\ 0,\ \frac{1}{3} \end{matrix}\right)$$

Power arguments

03.05.21.0040.01

$$\int \mathrm{Ai}(-a\,z^r)\,\mathrm{Ai}(a\,z^r)\, dz = \frac{z}{12\,2^{2/3}\sqrt[3]{3}\,\pi^{3/2}\,r}\, G_{2,6}^{4,1}\!\left(\frac{a\,z^r}{\sqrt[3]{2}\,3^{2/3}},\ \frac{1}{6}\ \middle|\ \begin{matrix} 1-\frac{1}{6r},\ \frac{1}{6} \\ 0,\ \frac{1}{6},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{6},\ -\frac{1}{6r} \end{matrix}\right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

Linear arguments

03.05.21.0041.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(a\,z)^2\, dz = \frac{z^{\alpha}}{6\,2^{2/3}\sqrt[3]{3}\,\pi^{3/2}}\, G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} 1-\frac{\alpha}{3},\ \frac{5}{6} \\ 0,\ \frac{1}{3},\ \frac{2}{3},\ -\frac{\alpha}{3} \end{matrix}\right)$$

03.05.21.0042.01

$$\int z\,\mathrm{Ai}(a\,z)^2\, dz = \frac{1}{3\,a^2}\left(a^2\,z^2\,\mathrm{Ai}(a\,z)^2 + \mathrm{Ai}'(a\,z)\,\mathrm{Ai}(a\,z) - a\,z\,\mathrm{Ai}'(a\,z)^2\right)$$

03.05.21.0043.01

$$\int z^2\,\mathrm{Ai}(a\,z)^2\,dz = \frac{1}{5\,a^3}\left(\left(a^3\,z^3-1\right)\mathrm{Ai}(a\,z)^2+2\,a\,z\,\mathrm{Ai}'(a\,z)\,\mathrm{Ai}(a\,z)-a^2\,z^2\,\mathrm{Ai}'(a\,z)^2\right)$$

03.05.21.0044.01

$$\int z^3\,\mathrm{Ai}(a\,z)^2\,dz = \frac{1}{7\,a^4}\left(a^4\,\mathrm{Ai}(a\,z)^2\,z^4+3\,a^2\,\mathrm{Ai}(a\,z)\,\mathrm{Ai}'(a\,z)\,z^2-\left(a^3\,z^3+3\right)\mathrm{Ai}'(a\,z)^2\right)$$

Power arguments

03.05.21.0045.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(a\,z^r)^2\,dz = \frac{z^\alpha}{6\,2^{2/3}\,\sqrt[3]{3}\,\pi^{3/2}\,r}\,G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3}a\,z^r,\ \frac{1}{3}\,\middle|\,\begin{array}{c}1-\frac{\alpha}{3\,r},\ \frac{5}{6}\\ 0,\ \frac{1}{3},\ \frac{2}{3},\ -\frac{\alpha}{3\,r}\end{array}\right)$$

Involving products of the direct function and a power function

Linear arguments

03.05.21.0046.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(-a\,z)\,\mathrm{Ai}(a\,z)\,dz = \frac{z^\alpha}{12\,2^{2/3}\,\sqrt[3]{3}\,\pi^{3/2}}\,G_{1,5}^{3,1}\!\left(\frac{a\,z}{\sqrt[3]{2}\,3^{2/3}},\ \frac{1}{6}\,\middle|\,\begin{array}{c}1-\frac{\alpha}{6}\\ 0,\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{6},\ -\frac{\alpha}{6}\end{array}\right)$$

Power arguments

03.05.21.0047.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(-a\,z^r)\,\mathrm{Ai}(a\,z^r)\,dz = \frac{z^\alpha}{12\,2^{2/3}\,\sqrt[3]{3}\,\pi^{3/2}\,r}\,G_{2,6}^{4,1}\!\left(\frac{a\,z^r}{\sqrt[3]{2}\,3^{2/3}},\ \frac{1}{6}\,\middle|\,\begin{array}{c}1-\frac{\alpha}{6\,r},\ \frac{1}{6}\\ 0,\ \frac{1}{6},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{6},\ -\frac{\alpha}{6\,r}\end{array}\right)$$

Involving direct function and Bessel-type functions

Involving Bessel functions

Involving Bessel I

Linear argument

03.05.21.0048.01

$$\int I_\nu\!\left(\frac{2}{3}\,(a\,z)^{3/2}\right)\mathrm{Ai}(a\,z)\,dz = \frac{2^{\nu-2}\,3^{-\nu-\frac{1}{2}}\left((a\,z)^{3/2}\right)^\nu}{a\,\pi^{3/2}}\,G_{3,5}^{2,3}\!\left(\left(\frac{2}{3}\right)^{2/3}a\,z,\ \frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1-\nu}{2},\ 1-\frac{\nu}{2},\ 1-\frac{\nu}{2}\\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{3}-\nu,\ \frac{2}{3}-\nu,\ -\frac{\nu}{2}\end{array}\right)$$

Power arguments

03.05.21.0049.01

$$\int I_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right)\operatorname{Ai}(a z^r)\,dz = \frac{2^{\nu-\frac{4}{3}}\,3^{-\nu-\frac{7}{6}}\,z\left((a z^r)^{3/2}\right)^\nu}{\pi^{3/2}\,r}\,G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{6}(1-3\nu),\ \frac{1}{6}(4-3\nu),\ -\frac{\nu}{2}-\frac{1}{3r}+1\\ 0,\ \frac{1}{3},\ \frac{1}{3}-\nu,\ -\nu,\ -\frac{3r\nu+2}{6r}\end{matrix}\right)$$

Involving Bessel I and power

Linear argument

03.05.21.0050.01

$$\int z^{\alpha-1} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right)\operatorname{Ai}(a z)\,dz = \frac{2^{\nu-\frac{4}{3}}\,3^{-\nu-\frac{7}{6}}\,z^\alpha\left((a z)^{3/2}\right)^\nu}{\pi^{3/2}}\,G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{6}(1-3\nu),\ \frac{1}{6}(4-3\nu),\ \frac{1}{6}(-2\alpha-3\nu+6)\\ 0,\ \frac{1}{3},\ \frac{1}{6}(-2\alpha-3\nu),\ \frac{1}{3}-\nu,\ -\nu\end{matrix}\right)$$

03.05.21.0051.01

$$\int z^{3/2} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right)\operatorname{Ai}(a z)\,dz = \frac{2^{\nu-\frac{4}{3}}\,3^{-\nu-\frac{7}{6}}\,z^{5/2}\left((a z)^{3/2}\right)^\nu}{\pi^{3/2}}\,G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{6}(1-3\nu),\ \frac{1}{6}(1-3\nu),\ \frac{1}{6}(4-3\nu)\\ 0,\ \frac{1}{3},\ \frac{1}{6}(-3\nu-5),\ \frac{1}{3}-\nu,\ -\nu\end{matrix}\right)$$

03.05.21.0052.01

$$\int z^{-3/2} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right)\operatorname{Ai}(a z)\,dz = \frac{2^{\nu-\frac{4}{3}}\,3^{-\nu-\frac{7}{6}}\left((a z)^{3/2}\right)^\nu}{\pi^{3/2}\sqrt{z}}\,G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} a z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{6}(4-3\nu),\ \frac{1}{6}(7-3\nu)\\ 0,\ \frac{1}{3},\ \frac{1}{3}-\nu,\ -\nu\end{matrix}\right)$$

Power arguments

03.05.21.0053.01

$$\int z^{\alpha-1} I_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right)\operatorname{Ai}(a z^r)\,dz = \frac{2^{\nu-\frac{4}{3}}\,3^{-\nu-\frac{7}{6}}\,z^\alpha\left((a z^r)^{3/2}\right)^\nu}{\pi^{3/2}\,r}\,G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{6}(1-3\nu),\ \frac{1}{6}(4-3\nu),\ -\frac{\alpha}{3r}-\frac{\nu}{2}+1\\ 0,\ \frac{1}{3},\ \frac{1}{3}-\nu,\ -\nu,\ -\frac{2\alpha+3r\nu}{6r}\end{matrix}\right)$$

Involving Bessel K

Linear argument

03.05.21.0054.01

$$\int K_\nu\left(\frac{2}{3}(a z)^{3/2}\right)\operatorname{Ai}(a z)\,dz = -\frac{1}{a\sqrt{\pi}}\left(2^{-\nu-3}\,3^{-\nu-\frac{1}{2}}\left((a z)^{3/2}\right)^{-\nu}\csc(\pi\nu)\right.$$

$$\left.\left(4^\nu\left((a z)^{3/2}\right)^{2\nu} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1-\nu}{2},\ 1-\frac{\nu}{2},\ 1-\frac{\nu}{2}\\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{3}-\nu,\ \frac{2}{3}-\nu,\ -\frac{\nu}{2}\end{matrix}\right) - 9^\nu\, G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{\nu+1}{2},\ \frac{\nu+2}{2},\ \frac{\nu+2}{2}\\ \frac{1}{3},\ \frac{2}{3},\ \frac{\nu}{2},\ \nu+\frac{1}{3},\ \nu+\frac{2}{3}\end{matrix}\right)\right)\right)$$

03.05.21.0055.01

$$\int K_0\left(\frac{2}{3}(a z)^{3/2}\right)\operatorname{Ai}(a z)\,dz =$$

$$\frac{1}{8\sqrt{3}\,a\,\pi^{3/2}}\left(2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z,\ \frac{1}{3}\ \middle|\ \begin{matrix}1,\ \frac{1}{2},\ 1\\ \frac{1}{3},\ \frac{1}{3},\ \frac{2}{3},\ \frac{2}{3},\ 0\end{matrix}\right) + \left(3\log(a z)-2\log\left((a z)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{2},\ 1,\ 1\\ \frac{1}{3},\ \frac{2}{3},\ 0,\ \frac{1}{3},\ \frac{2}{3}\end{matrix}\right)\right)$$

03.05.21.0056.01

$$\int K_1\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Ai}(a z)\,dz = \frac{1}{8\sqrt{3}\,a\,\pi^{3/2}}\left(2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3}\,\middle|\, \begin{matrix} 1, \frac{1}{2}, 1 \\ -\frac{1}{6}, \frac{1}{6}, \frac{5}{6}, \frac{7}{6}, 0 \end{matrix}\right) + \left(2\log\left((a z)^{3/2}\right) - 3\log(a z)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1, 1 \\ \frac{5}{6}, \frac{7}{6}, -\frac{1}{6}, 0, \frac{1}{6} \end{matrix}\right)\right)$$

03.05.21.0057.01

$$\int K_2\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Ai}(a z)\,dz = \frac{1}{8\sqrt{3}\,a\,\pi^{3/2}}\left(2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3}\,\middle|\, \begin{matrix} 1, \frac{1}{2}, 1 \\ -\frac{2}{3}, -\frac{1}{3}, \frac{4}{3}, \frac{5}{3}, 0 \end{matrix}\right) + \left(3\log(a z) - 2\log\left((a z)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1, 1 \\ \frac{4}{3}, \frac{5}{3}, -\frac{2}{3}, -\frac{1}{3}, 0 \end{matrix}\right)\right)$$

Power arguments

03.05.21.0058.01

$$\int K_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Ai}(a z^r)\,dz = -\frac{1}{\sqrt{\pi}\,r}\left(2^{-\nu-\frac{7}{3}}\,3^{-\nu-\frac{7}{6}}\,z\left((a z^r)^{3/2}\right)^{-\nu}\csc(\pi\nu)\left(4^\nu\left((a z^r)^{3/2}\right)^{2\nu} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{6}(1-3\nu), \frac{1}{6}(4-3\nu), -\frac{\nu}{2}-\frac{1}{3r}+1 \\ 0, \frac{1}{3}, \frac{1}{3}-\nu, -\nu, -\frac{3r\nu+2}{6r} \end{matrix}\right) - 9^\nu\,G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{\nu}{2}-\frac{1}{3r}+1, \frac{1}{6}(3\nu+1), \frac{1}{6}(3\nu+4) \\ 0, \frac{1}{3}, \nu, \nu+\frac{1}{3}, \frac{3r\nu-2}{6r} \end{matrix}\right)\right)\right)$$

03.05.21.0059.01

$$\int K_0\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Ai}(a z^r)\,dz = \frac{1}{12\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}\,r}\left(z\left(2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} 1-\frac{1}{3r}, \frac{1}{6}, \frac{2}{3} \\ 0, 0, \frac{1}{3}, \frac{1}{3}, -\frac{1}{3r} \end{matrix}\right) + \left(3\log(a z^r) - 2\log\left((a z^r)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r} \\ 0, \frac{1}{3}, 0, \frac{1}{3}, -\frac{1}{3r} \end{matrix}\right)\right)\right)$$

03.05.21.0060.01

$$\int K_1\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Ai}(a z^r)\,dz = \frac{1}{12\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}\,r}\left(z\left(2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} 1-\frac{1}{3r}, \frac{1}{6}, \frac{2}{3} \\ -\frac{1}{2}, -\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, -\frac{1}{3r} \end{matrix}\right) + \left(2\log\left((a z^r)^{3/2}\right) - 3\log(a z^r)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r} \\ \frac{1}{2}, \frac{5}{6}, -\frac{1}{2}, -\frac{1}{6}, -\frac{1}{3r} \end{matrix}\right)\right)\right)$$

03.05.21.0061.01

$$\int K_2\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Ai}(a z^r)\,dz = \frac{1}{12\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}\,r}\left(z\left(2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} 1-\frac{1}{3r}, \frac{1}{6}, \frac{2}{3} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{1}{3r} \end{matrix}\right) + \left(3\log(a z^r) - 2\log\left((a z^r)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r} \\ 1, \frac{4}{3}, -1, -\frac{2}{3}, -\frac{1}{3r} \end{matrix}\right)\right)\right)$$

Involving Bessel K and power

Linear argument

03.05.21.0062.01

$$\int z^{\alpha-1} K_\nu\left(\frac{2}{3}(a\,z)^{3/2}\right) \mathrm{Ai}(a\,z)\,dz = -\frac{1}{\sqrt{\pi}}\left(2^{-\nu-\frac{7}{3}}\,3^{-\nu-\frac{7}{6}}\,z^\alpha\left((a\,z)^{3/2}\right)^{-\nu}\csc(\pi\,\nu)\left(4^\nu\left((a\,z)^{3/2}\right)^{2\,\nu} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{6}(1-3\,\nu),\ \frac{1}{6}(4-3\,\nu),\ \frac{1}{6}(-2\,\alpha-3\,\nu+6) \\ 0,\ \frac{1}{3},\ \frac{1}{6}(-2\,\alpha-3\,\nu),\ \frac{1}{3}-\nu,\ -\nu \end{matrix}\right) - 9^\nu\, G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{6}(3\,\nu+1),\ \frac{1}{6}(3\,\nu+4),\ \frac{1}{6}(-2\,\alpha+3\,\nu+6) \\ 0,\ \frac{1}{3},\ \nu,\ \nu+\frac{1}{3},\ \frac{1}{6}(3\,\nu-2\,\alpha) \end{matrix}\right)\right)\right)$$

03.05.21.0063.01

$$\int z^{\alpha-1} K_0\left(\frac{2}{3}(a\,z)^{3/2}\right) \mathrm{Ai}(a\,z)\,dz = \frac{z^\alpha}{12\,\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}}\left(2\,\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} 1-\frac{\alpha}{3},\ \frac{1}{6},\ \frac{2}{3} \\ 0,\ 0,\ \frac{1}{3},\ \frac{1}{3},\ -\frac{\alpha}{3} \end{matrix}\right) + \left(3\log(a\,z)-2\log\left((a\,z)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{6},\ \frac{2}{3},\ 1-\frac{\alpha}{3} \\ 0,\ \frac{1}{3},\ 0,\ \frac{1}{3},\ -\frac{\alpha}{3} \end{matrix}\right)\right)$$

03.05.21.0064.01

$$\int z^{\alpha-1} K_1\left(\frac{2}{3}(a\,z)^{3/2}\right) \mathrm{Ai}(a\,z)\,dz = \frac{1}{12\,\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}}\left(z^\alpha\left(2\,\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} 1-\frac{\alpha}{3},\ \frac{1}{6},\ \frac{2}{3} \\ -\frac{1}{2},\ -\frac{1}{6},\ \frac{1}{2},\ \frac{5}{6},\ -\frac{\alpha}{3} \end{matrix}\right) + \left(2\log\left((a\,z)^{3/2}\right)-3\log(a\,z)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{6},\ \frac{2}{3},\ 1-\frac{\alpha}{3} \\ \frac{1}{2},\ \frac{5}{6},\ -\frac{1}{2},\ -\frac{1}{6},\ -\frac{\alpha}{3} \end{matrix}\right)\right)\right)$$

03.05.21.0065.01

$$\int z^{\alpha-1} K_2\left(\frac{2}{3}(a\,z)^{3/2}\right) \mathrm{Ai}(a\,z)\,dz = \frac{1}{12\,\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}}\left(z^\alpha\left(2\,\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} 1-\frac{\alpha}{3},\ \frac{1}{6},\ \frac{2}{3} \\ -1,\ -\frac{2}{3},\ 1,\ \frac{4}{3},\ -\frac{\alpha}{3} \end{matrix}\right) + \left(3\log(a\,z)-2\log\left((a\,z)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{6},\ \frac{2}{3},\ 1-\frac{\alpha}{3} \\ 1,\ \frac{4}{3},\ -1,\ -\frac{2}{3},\ -\frac{\alpha}{3} \end{matrix}\right)\right)\right)$$

03.05.21.0066.01

$$\int z^{3/2} K_2\left(\frac{2}{3}(a\,z)^{3/2}\right) \mathrm{Ai}(a\,z)\,dz = \frac{1}{12\,\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}}\left(z^{5/2}\left(2\,\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{6},\ \frac{1}{6},\ \frac{2}{3} \\ -1,\ -\frac{2}{3},\ 1,\ \frac{4}{3},\ -\frac{5}{6} \end{matrix}\right) + \left(3\log(a\,z)-2\log\left((a\,z)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{6},\ \frac{1}{6},\ \frac{2}{3} \\ 1,\ \frac{4}{3},\ -1,\ -\frac{5}{6},\ -\frac{2}{3} \end{matrix}\right)\right)\right)$$

03.05.21.0067.01

$$\int z^{-3/2} K_2\left(\frac{2}{3}(a\,z)^{3/2}\right) \operatorname{Ai}(a\,z)\,dz = \frac{1}{12\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}\sqrt{z}} \left(2\,\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\frac{1}{3}\,\middle|\, \begin{matrix} \frac{7}{6},\frac{1}{6},\frac{2}{3} \\ -1,-\frac{2}{3},1,\frac{4}{3},\frac{1}{6}\end{matrix}\right) + \left(3\log(a\,z) - 2\log\left((a\,z)^{3/2}\right)\right) G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\frac{1}{3}\,\middle|\, \begin{matrix} \frac{2}{3},\frac{7}{6} \\ 1,\frac{4}{3},-1,-\frac{2}{3}\end{matrix}\right)\right)$$

Power arguments

03.05.21.0068.01

$$\int z^{\alpha-1} K_\nu\left(\frac{2}{3}(a\,z^r)^{3/2}\right) \operatorname{Ai}(a\,z^r)\,dz = \frac{1}{2}\pi\csc(\pi\,\nu)\left(\frac{2^{-\nu-\frac{4}{3}}\,3^{\nu-\frac{7}{6}}\,z^\alpha\left((a\,z^r)^{3/2}\right)^{-\nu}}{\pi^{3/2}\,r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\frac{1}{3}\,\middle|\, \begin{matrix} -\frac{\alpha}{3r}+\frac{\nu}{2}+1,\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4) \\ 0,\frac{1}{3},\frac{\nu}{2}-\frac{\alpha}{3r},\nu,\nu+\frac{1}{3}\end{matrix}\right) - \frac{2^{\nu-\frac{4}{3}}\,3^{-\nu-\frac{7}{6}}\,z^\alpha\left((a\,z^r)^{3/2}\right)^{\nu}}{\pi^{3/2}\,r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{6}(1-3\nu),\frac{1}{6}(4-3\nu),-\frac{\alpha}{3r}-\frac{\nu}{2}+1 \\ 0,\frac{1}{3},\frac{1}{3}-\nu,-\nu,-\frac{2\alpha+3r\nu}{6r}\end{matrix}\right)\right)$$

03.05.21.0069.01

$$\int z^{\alpha-1} K_0\left(\frac{2}{3}(a\,z^r)^{3/2}\right) \operatorname{Ai}(a\,z^r)\,dz = \frac{1}{12\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}\,r} \left(z^\alpha\left(2\,\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\frac{1}{3}\,\middle|\, \begin{matrix} 1-\frac{\alpha}{3r},\frac{1}{6},\frac{2}{3} \\ 0,0,\frac{1}{3},\frac{1}{3},-\frac{\alpha}{3r}\end{matrix}\right) + \left(3\log(a\,z^r) - 2\log\left((a\,z^r)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{6},\frac{2}{3},1-\frac{\alpha}{3r} \\ 0,\frac{1}{3},0,\frac{1}{3},-\frac{\alpha}{3r}\end{matrix}\right)\right)\right)$$

03.05.21.0070.01

$$\int z^{\alpha-1} K_1\left(\frac{2}{3}(a\,z^r)^{3/2}\right) \operatorname{Ai}(a\,z^r)\,dz = \frac{1}{2\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}} \left(\frac{z^\alpha}{3\,r} G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\frac{1}{3}\,\middle|\, \begin{matrix} 1-\frac{\alpha}{3r},\frac{1}{6},\frac{2}{3} \\ -\frac{1}{2},-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{\alpha}{3r}\end{matrix}\right) - \frac{z^\alpha\left(\frac{3}{2}\log(a\,z^r) - \log\left((a\,z^r)^{3/2}\right)\right)}{3\,\pi\,r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{6},\frac{2}{3},1-\frac{\alpha}{3r} \\ \frac{1}{2},\frac{5}{6},-\frac{1}{2},-\frac{1}{6},-\frac{\alpha}{3r}\end{matrix}\right)\right)$$

03.05.21.0071.01

$$\int z^{\alpha-1} K_2\left(\frac{2}{3}(a\,z^r)^{3/2}\right) \operatorname{Ai}(a\,z^r)\,dz = \frac{1}{2\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}} \left(\frac{z^\alpha}{3\,r} G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\frac{1}{3}\,\middle|\, \begin{matrix} 1-\frac{\alpha}{3r},\frac{1}{6},\frac{2}{3} \\ -1,-\frac{2}{3},1,\frac{4}{3},-\frac{\alpha}{3r}\end{matrix}\right) + \frac{z^\alpha\left(\frac{3}{2}\log(a\,z^r) - \log\left((a\,z^r)^{3/2}\right)\right)}{3\,\pi\,r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{6},\frac{2}{3},1-\frac{\alpha}{3r} \\ 1,\frac{4}{3},-1,-\frac{2}{3},-\frac{\alpha}{3r}\end{matrix}\right)\right)$$

Definite integration

For the direct function itself

03.05.21.0072.01

$$\int_{-\infty}^{\infty} \mathrm{Ai}(t)\, dt = 1$$

03.05.21.0073.01

$$\int_{0}^{\infty} \mathrm{Ai}(t)\, dt = \frac{1}{3}$$

03.05.21.0074.01

$$\int_{-\infty}^{0} \mathrm{Ai}(t)\, dt = \frac{2}{3}$$

03.05.21.0075.01

$$\int_{0}^{\infty} t^{\alpha-1}\, \mathrm{Ai}(t)\, dt = \frac{1}{2\pi}\, 3^{\frac{4\alpha-7}{6}}\, \Gamma\left(\frac{\alpha}{3}\right) \Gamma\left(\frac{\alpha+1}{3}\right) /; \mathrm{Re}(\alpha) > 0$$

03.05.21.0080.01

$$\int_{-\infty}^{\infty} t^{\alpha-1}\, \mathrm{Ai}(t)\, dt = \frac{1}{2}\, 3^{\frac{1}{3}(2\alpha-5)} \left(\frac{\left(\sqrt{3}\, \csc\left(\frac{\pi\alpha}{3}\right) - 2(-1)^{\alpha}\right) \Gamma\left(\frac{\alpha+1}{3}\right)}{\Gamma\left(1-\frac{\alpha}{3}\right)} - \frac{2(-1)^{\alpha}\, \Gamma\left(\frac{\alpha}{3}\right)}{\Gamma\left(\frac{2}{3}-\frac{\alpha}{3}\right)} \right) /; \mathrm{Re}(\alpha) > 0$$

Involving the direct function

03.05.21.0076.01

$$\int_{0}^{\infty} \mathrm{Ai}(t)^2\, dt = -\frac{1}{12\sqrt[3]{2}\, \sqrt[6]{3}\, \pi^{3/2}}\, \Gamma\left(-\frac{1}{6}\right)$$

03.05.21.0077.01

$$\int_{0}^{\infty} t^{\alpha-1}\, \mathrm{Ai}(t)^2\, dt = \frac{1}{\pi^{3/2}\, \Gamma\left(\frac{1}{3}-\frac{\alpha}{3}\right)}\, 2^{-\frac{2\alpha+5}{3}}\, 3^{-\frac{2\alpha+11}{6}}\, \Gamma\left(\frac{1}{6}-\frac{\alpha}{3}\right) \left(3^{\alpha}\, \Gamma\left(\frac{\alpha}{3}\right) \Gamma\left(\frac{\alpha+1}{3}\right) - 2\sqrt{3}\, \Gamma\left(\frac{1}{3}-\frac{\alpha}{3}\right) \Gamma(\alpha) \sin\left(\frac{\pi\alpha}{3}\right)\right) /; \mathrm{Re}(\alpha) > 0$$

03.05.21.0084.01

$$\int_{-\infty}^{\infty} \frac{\mathrm{Ai}(x)}{x-z}\, dx = i\pi \left(\mathrm{Ai}(z) + \frac{i}{6\pi} \left(-6\, {}_0F_1\left(;\frac{4}{3};\frac{z^3}{9}\right) {}_1F_2\left(\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{z^3}{9}\right) z^2 + 3\, {}_0F_1\left(;\frac{2}{3};\frac{z^3}{9}\right) {}_1F_2\left(\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{z^3}{9}\right) z^2 + 2\pi\, \mathrm{Bi}(z) \right) \right) /;$$
$$\mathrm{Im}(z) > 0$$

03.05.21.0081.01

$$\int_{-\infty}^{\infty} \frac{\mathrm{Ai}(x)^2}{x+z}\, dx = i\pi\, \mathrm{Ai}(z)\, (\mathrm{Ai}(z) + i\, \mathrm{Bi}(z)) /; \mathrm{Im}(z) > 0$$

Involving the direct function and derivatives

03.05.21.0082.01

$$\mathrm{Ai}^{(m+n-2)}\left(\frac{x+y}{\sqrt[3]{t}}\right) = t^{\frac{1}{3}(m+n-1)} \int_{0}^{t} \frac{\mathrm{Ai}^{(n)}\left(\frac{x}{\sqrt[3]{\tau}}\right) \mathrm{Ai}^{(m)}\left(\frac{y}{\sqrt[3]{t-\tau}}\right)}{\tau^{\frac{n+1}{3}}\, (t-\tau)^{\frac{m+1}{3}}}\, d\tau /; n \in \mathbb{Z} \wedge m \in \mathbb{Z}$$

03.05.21.0083.01

$$\int_{0}^{t} \frac{\mathrm{Ai}^{(n)}\left(\frac{x}{\sqrt[3]{\tau}}\right) \mathrm{Ai}^{(m)}\left(\frac{y}{\sqrt[3]{t-\tau}}\right)}{\tau^{\frac{n+1}{3}}\, (t-\tau)^{\frac{m+1}{3}}}\, d\tau = t^{-\frac{1}{3}(m+n-1)}\, \mathrm{Ai}^{(m+n-2)}\left(\frac{x+y}{\sqrt[3]{t}}\right) /; n \in \mathbb{Z} \wedge m \in \mathbb{Z}$$

Multiple integration

03.05.21.0078.01

$$\int_0^x \int_0^x \operatorname{Ai}(t)\,dt\,dx = \operatorname{Ai}'(0) - \operatorname{Ai}'(x) + x \int_0^x \operatorname{Ai}(t)\,dt$$

03.05.21.0079.01

$$\int_0^\infty \underbrace{\int_t^\infty \ldots \int_t^\infty}_{n-\text{times}} \operatorname{Ai}(-t)\,dt\,dt \ldots dt\,dt = \frac{2\, 3^{-\frac{n+2}{3}}}{\Gamma\left(\frac{n+2}{3}\right)} \cos\left(\frac{1}{3}(n-1)\pi\right)$$

Integral transforms

Fourier exp transforms

03.05.22.0001.01

$$\mathcal{F}_t[\operatorname{Ai}(t)](z) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{i z^3}{3}}$$

03.05.22.0009.01

$$\mathcal{F}_t[\operatorname{Ai}(t)\operatorname{Ai}(t+w)](z) = \frac{e^{\frac{1}{2} i \left(\frac{z^3}{6} + w z - \frac{w^2}{2z}\right)}}{2\sqrt{2\pi}\,\sqrt{i \pi z}}$$

Inverse Fourier exp transforms

03.05.22.0002.01

$$\mathcal{F}_t^{-1}[\operatorname{Ai}(t)](z) = \frac{1}{48\sqrt{2}\,\pi^{3/2}} \left(\sqrt[6]{3}\left(9\,\Gamma\left(\frac{5}{3}\right) + 2\,\Gamma\left(-\frac{1}{3}\right)\right) {}_1F_2\left(1; \frac{7}{6}, \frac{5}{3}; -\frac{z^6}{36}\right) z^4 + 48\pi\, e^{\frac{i z^3}{3}}\right)$$

Fourier cos transforms

03.05.22.0003.01

$$\mathcal{F}c_t[\operatorname{Ai}(t)](z) = \frac{1}{12\sqrt{2}\,\pi^{3/2}} \left(3\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right) {}_1F_2\left(1; \frac{7}{6}, \frac{5}{3}; -\frac{z^6}{36}\right) z^4 - 2\, 3^{5/6}\,\Gamma\left(\frac{1}{3}\right) {}_1F_2\left(1; \frac{5}{6}, \frac{4}{3}; -\frac{z^6}{36}\right) z^2 + 8\pi \cos\left(\frac{z^3}{3}\right)\right)$$

Fourier sin transforms

03.05.22.0004.01

$$\mathcal{F}s_t[\operatorname{Ai}(t)](z) = \frac{1}{\sqrt{2\pi}} \left(\frac{9\, 3^{5/6}\,\Gamma\left(\frac{10}{3}\right) z^5}{280\pi} {}_1F_2\left(1; \frac{4}{3}, \frac{11}{6}; -\frac{z^6}{36}\right) + \frac{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right) z}{\pi} {}_1F_2\left(1; \frac{2}{3}, \frac{7}{6}; -\frac{z^6}{36}\right) - \frac{2}{3}\sin\left(\frac{z^3}{3}\right)\right)$$

Laplace transforms

03.05.22.0005.01

$$\mathcal{L}_t[\operatorname{Ai}(t)](z) = \frac{1}{12\pi}\, e^{-\frac{z^3}{3}} \left(\left(3 i + \sqrt{3}\right) \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{2}{3}, -\frac{z^3}{3}\right) - 2(-1)^{2/3}\sqrt{3}\,\Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{1}{3}, -\frac{z^3}{3}\right)\right)$$

03.05.22.0006.01

$$\mathcal{L}_t[\operatorname{Ai}(-t)](z) = \frac{1}{18\pi z^2}\, e^{\frac{z^3}{3}} \left(3^{5/6} E_{\frac{1}{3}}\left(\frac{z^3}{3}\right) \Gamma\left(\frac{1}{3}\right) z^4 + 3\sqrt[6]{3}\, E_{\frac{2}{3}}\left(\frac{z^3}{3}\right) \Gamma\left(\frac{2}{3}\right) z^3 - 6\pi\left(z\sqrt[3]{z^3} + \left(z^3\right)^{2/3} - 2z^2\right)\right)$$

Mellin transforms

03.05.22.0007.01

$$\mathcal{M}_t[\text{Ai}(t)](z) = \frac{1}{2\pi} 3^{\frac{4z-7}{6}} \Gamma\left(\frac{z}{3}\right) \Gamma\left(\frac{z+1}{3}\right) /; \text{Re}(z) > 0$$

Hankel transforms

03.05.22.0008.01

$$\mathcal{H}_{t;\nu}[\text{Ai}(t)](z) = 2^{-\nu-6} 3^{-\frac{\nu+1}{3}} z^{\nu+\frac{1}{2}} \left(\frac{1}{\pi\,\Gamma(\nu+1)} \left(32\, 3^{\nu+\frac{1}{6}} \Gamma\left(\frac{\nu}{3}+\frac{1}{2}\right) \Gamma\left(\frac{\nu}{3}+\frac{5}{6}\right) {}_4F_5\left(\frac{\nu}{6}+\frac{1}{4}, \frac{\nu}{6}+\frac{5}{12}, \frac{\nu}{6}+\frac{3}{4}, \frac{\nu}{6}+\frac{11}{12}; \frac{1}{3}, \frac{2}{3}, \frac{\nu}{3}+\frac{1}{3}, \frac{\nu}{3}+\frac{2}{3}, \frac{\nu}{3}+1; -\frac{z^6}{36}\right)\right) + \frac{1}{\pi\,\Gamma(\nu+3)} \left(3^{\nu+\frac{17}{6}} z^4\, \Gamma\left(\frac{\nu}{3}+\frac{11}{6}\right) \Gamma\left(\frac{\nu}{3}+\frac{13}{6}\right) {}_4F_5\left(\frac{\nu}{6}+\frac{11}{12}, \frac{\nu}{6}+\frac{13}{12}, \frac{\nu}{6}+\frac{17}{12}, \frac{\nu}{6}+\frac{19}{12}; \frac{4}{3}, \frac{5}{3}, \frac{\nu}{3}+1, \frac{\nu}{3}+\frac{4}{3}, \frac{\nu}{3}+\frac{5}{3}; -\frac{z^6}{36}\right)\right) - \frac{1}{\Gamma\left(\frac{\nu}{3}+1\right) \Gamma\left(\frac{\nu+2}{3}\right) \Gamma\left(\frac{\nu+4}{3}\right)} \left(16 z^2\, \Gamma\left(\frac{\nu}{3}+\frac{7}{6}\right) \Gamma\left(\frac{\nu}{3}+\frac{3}{2}\right) {}_4F_5\left(\frac{\nu}{6}+\frac{7}{12}, \frac{\nu}{6}+\frac{3}{4}, \frac{\nu}{6}+\frac{13}{12}, \frac{\nu}{6}+\frac{5}{4}; \frac{2}{3}, \frac{4}{3}, \frac{\nu}{3}+\frac{2}{3}, \frac{\nu}{3}+1, \frac{\nu}{3}+\frac{4}{3}; -\frac{z^6}{36}\right)\right)\right) /; z > 0 \bigwedge \text{Re}(\nu) > -\frac{3}{2}$$

Representations through more general functions

Through hypergeometric functions

Involving $_0F_1$

03.05.26.0001.01

$$\text{Ai}(z) = \frac{1}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\, {}_0F_1\left(; \frac{2}{3}; \frac{z^3}{9}\right) - \frac{z}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\, {}_0F_1\left(; \frac{4}{3}; \frac{z^3}{9}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

03.05.26.0002.01

$$\text{Ai}(z) = \frac{\pi}{3^{2/3}} \left(G_{1,3}^{1,0}\left(\frac{z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \frac{1}{3}, \frac{1}{2} \end{matrix}\right) - \frac{z}{3^{2/3}} G_{1,3}^{1,0}\left(\frac{z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, -\frac{1}{3}, \frac{1}{2} \end{matrix}\right)\right)$$

03.05.26.0024.01

$$\text{Ai}(z) = \frac{1}{2\pi\sqrt[6]{3}}\, G_{0,2}^{2,0}\left(\frac{z^3}{9} \,\middle|\, 0, \frac{1}{3}\right) /; -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

Classical cases involving exp

03.05.26.0025.01

$$e^{-\frac{1}{3}(2z^{3/2})}\,\text{Ai}(z) = \frac{1}{2^{2/3}\,\sqrt[6]{3}\,\sqrt{\pi}}\,G_{1,2}^{2,0}\!\left(\frac{4z^{3/2}}{3}\,\middle|\,\begin{matrix}\frac{5}{6}\\ 0,\,\frac{2}{3}\end{matrix}\right) /;\, -\frac{2\pi}{3} < \arg(z) \leq \frac{2\pi}{3}$$

03.05.26.0026.01

$$e^{\frac{2z^{3/2}}{3}}\,\text{Ai}(z) = \frac{1}{2\,2^{2/3}\,\sqrt[6]{3}\,\pi^{3/2}}\,G_{1,2}^{2,1}\!\left(\frac{4z^{3/2}}{3}\,\middle|\,\begin{matrix}\frac{5}{6}\\ 0,\,\frac{2}{3}\end{matrix}\right) /;\, -\frac{2\pi}{3} < \arg(z) \leq \frac{2\pi}{3}$$

03.05.26.0027.01

$$e^{-z}\,\text{Ai}\!\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \frac{1}{2^{2/3}\,\sqrt[6]{3}\,\sqrt{\pi}}\,G_{1,2}^{2,0}\!\left(2z\,\middle|\,\begin{matrix}\frac{5}{6}\\ 0,\,\frac{2}{3}\end{matrix}\right)$$

03.05.26.0028.01

$$e^{z}\,\text{Ai}\!\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \frac{1}{2\,2^{2/3}\,\sqrt[6]{3}\,\pi^{3/2}}\,G_{1,2}^{2,1}\!\left(2z\,\middle|\,\begin{matrix}\frac{5}{6}\\ 0,\,\frac{2}{3}\end{matrix}\right)$$

Classical cases involving ${}_0F_1$

03.05.26.0003.01

$$\text{Ai}(z)\,{}_0F_1\!\left(;b;\frac{z^3}{9}\right) = \frac{2^{b-\frac{7}{3}}\,\Gamma(b)}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\!\left(\frac{4z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\,\frac{1}{6}(7-3b)\\ 0,\,\frac{1}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right) /;\, -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

03.05.26.0022.01

$$\text{Ai}\!\left(3^{2/3}\,\sqrt[3]{z}\right){}_0F_1(;b;z) = \frac{\Gamma(b)\,2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\,\frac{1}{6}(7-3b)\\ 0,\,\frac{1}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right)$$

Classical cases involving ${}_0\tilde{F}_1$

03.05.26.0004.01

$$\text{Ai}(z)\,{}_0\tilde{F}_1\!\left(;b;\frac{z^3}{9}\right) = \frac{2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\!\left(\frac{4z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\,\frac{1}{6}(7-3b)\\ 0,\,\frac{1}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right) /;\, -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

03.05.26.0023.01

$$\text{Ai}\!\left(3^{2/3}\,\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z) = \frac{2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\,\frac{1}{6}(7-3b)\\ 0,\,\frac{1}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right)$$

Generalized cases for the direct function itself

03.05.26.0005.01

$$\text{Ai}(z) = \frac{1}{2\pi\,\sqrt[6]{3}}\,G_{0,2}^{2,0}\!\left(3^{-2/3}\,z,\,\frac{1}{3}\,\middle|\,0,\,\frac{1}{3}\right)$$

Generalized cases involving exp

03.05.26.0006.01

$$\exp\left(-\frac{2 z^{3/2}}{3}\right) \mathrm{Ai}(z) = \frac{1}{2^{2/3}\sqrt{\pi}\sqrt[6]{3}} G_{1,2}^{2,0}\left(3^{-2/3} 2^{4/3} z, \frac{2}{3} \,\middle|\, \begin{matrix} \frac{5}{6} \\ 0, \frac{2}{3} \end{matrix}\right)$$

03.05.26.0007.01

$$\exp\left(\frac{2 z^{3/2}}{3}\right) \mathrm{Ai}(z) = \frac{1}{2\, 2^{2/3}\sqrt[6]{3}\, \pi^{3/2}} G_{1,2}^{2,1}\left(\frac{2\sqrt[3]{2}\, z}{3^{2/3}}, \frac{2}{3} \,\middle|\, \begin{matrix} \frac{5}{6} \\ 0, \frac{2}{3} \end{matrix}\right)$$

Generalized cases involving cosh

03.05.26.0008.01

$$\cosh\left(\frac{2 z^{3/2}}{3}\right) \mathrm{Ai}(z) = \sqrt[6]{\frac{2}{3}}\, \pi\, G_{2,4}^{2,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{5}{12}, \frac{11}{12} \\ 0, \frac{1}{3}, \frac{1}{2}, \frac{5}{6} \end{matrix}\right)$$

03.05.26.0029.01

$$\cosh(z)\, \mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \sqrt[6]{\frac{2}{3}}\, \pi\, G_{2,4}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{5}{12}, \frac{11}{12} \\ 0, \frac{1}{3}, \frac{1}{2}, \frac{5}{6} \end{matrix}\right)$$

Generalized cases involving sinh

03.05.26.0009.01

$$\sinh\left(\frac{2 z^{3/2}}{3}\right) \mathrm{Ai}(z) = -\sqrt[6]{\frac{2}{3}}\, \pi\, G_{2,4}^{2,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{5}{12}, \frac{11}{12} \\ \frac{1}{2}, \frac{5}{6}, 0, \frac{1}{3} \end{matrix}\right)$$

03.05.26.0030.01

$$\sinh(z)\, \mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = -\sqrt[6]{\frac{2}{3}}\, \pi\, G_{2,4}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{5}{12}, \frac{11}{12} \\ \frac{1}{2}, \frac{5}{6}, 0, \frac{1}{3} \end{matrix}\right)$$

Generalized cases for powers of Ai

03.05.26.0010.01

$$\mathrm{Ai}(z)^2 = \frac{1}{2\, 2^{2/3}\sqrt[3]{3}\, \pi^{3/2}} G_{1,3}^{3,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{5}{6} \\ 0, \frac{1}{3}, \frac{2}{3} \end{matrix}\right)$$

Generalized cases involving Ai′

03.05.26.0011.01

$$\mathrm{Ai}(z)\, \mathrm{Ai}'(z) = -\frac{1}{4\pi^{3/2}} G_{1,3}^{3,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \frac{1}{3}, \frac{2}{3} \end{matrix}\right)$$

Generalized cases involving Bi

03.05.26.0012.01

$$\mathrm{Ai}(z)\, \mathrm{Bi}(z) = \frac{1}{2\, 2^{2/3}\sqrt[3]{3}\, \pi^{3/2}} G_{1,3}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3} \end{matrix}\right)$$

Generalized cases involving Bi′

03.05.26.0013.01

$$\operatorname{Ai}(z)\operatorname{Bi}'(z) = \frac{1}{2\pi} - \frac{1}{4\pi^{3/2}} G_{1,3}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{2} \\ \frac{1}{3}, \frac{2}{3}, 0 \end{matrix}\right)$$

Generalized cases involving ${}_0F_1$

03.05.26.0014.01

$$\operatorname{Ai}(z)\, {}_0F_1\left(; b; \frac{z^3}{9}\right) = \frac{2^{b-\frac{7}{3}}\,\Gamma(b)}{\sqrt[6]{3}\,\pi^{3/2}} G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b) \\ 0, \frac{1}{3}, 1-b, \frac{4}{3}-b \end{matrix}\right)$$

03.05.26.0031.01

$$\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right) {}_0F_1(; b; z) = \frac{2^{b-\frac{7}{3}}\,\Gamma(b)}{\sqrt[6]{3}\,\pi^{3/2}} G_{2,4}^{2,2}\left(2^{2/3}\sqrt[3]{z}, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b) \\ 0, \frac{1}{3}, 1-b, \frac{4}{3}-b \end{matrix}\right)$$

Generalized cases involving ${}_0\tilde{F}_1$

03.05.26.0015.01

$$\operatorname{Ai}(z)\, {}_0\tilde{F}_1\left(; b; \frac{z^3}{9}\right) = \frac{2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}} G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b) \\ 0, \frac{1}{3}, 1-b, \frac{4}{3}-b \end{matrix}\right)$$

03.05.26.0032.01

$$\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right) {}_0\tilde{F}_1(; b; z) = \frac{2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}} G_{2,4}^{2,2}\left(2^{2/3}\sqrt[3]{z}, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b) \\ 0, \frac{1}{3}, 1-b, \frac{4}{3}-b \end{matrix}\right)$$

Generalized cases involving Bessel I

03.05.26.0016.01

$$\operatorname{Ai}(z)\, I_\nu\left(\frac{2z^{3/2}}{3}\right) = \frac{z^{-\frac{3\nu}{2}}\left(z^{3/2}\right)^\nu}{2\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}} G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3} \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+2), -\frac{\nu}{2}, \frac{1}{6}(2-3\nu) \end{matrix}\right)$$

03.05.26.0033.01

$$\operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) I_\nu(z) = \frac{1}{2\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}} G_{2,4}^{2,2}\left(z^{2/3}, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3} \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+2), -\frac{\nu}{2}, \frac{1}{6}(2-3\nu) \end{matrix}\right)$$

Generalized cases Bessel involving K

03.05.26.0017.01

$$\operatorname{Ai}(z)\, K_\nu\left(\frac{2z^{3/2}}{3}\right) = \frac{1}{2\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}} G_{2,4}^{4,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3} \\ -\frac{\nu}{2}, \frac{1}{6}(2-3\nu), \frac{\nu}{2}, \frac{1}{6}(3\nu+2) \end{matrix}\right) /; -\frac{1}{3}(2\pi) < \arg(z) \leq \frac{2\pi}{3}$$

03.05.26.0034.01

$$\operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) K_\nu(z) = \frac{1}{2\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}} G_{2,4}^{4,0}\left(z^{2/3}, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3} \\ -\frac{\nu}{2}, \frac{1}{6}(2-3\nu), \frac{\nu}{2}, \frac{1}{6}(3\nu+2) \end{matrix}\right)$$

Through other functions

Involving Bessel functions

03.05.26.0018.01

$$\mathrm{Ai}(z) = \frac{1}{3}\sqrt{-z}\left(J_{\frac{1}{3}}\left(\frac{2}{3}(-z)^{3/2}\right) + J_{-\frac{1}{3}}\left(\frac{2}{3}(-z)^{3/2}\right)\right) /;\ \mathrm{Re}(z) \leq 0$$

03.05.26.0019.01

$$\mathrm{Ai}(z) = \frac{1}{3}\sqrt{z}\left(I_{-\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right) - I_{\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right)\right) /;\ \mathrm{Re}(z) \geq 0$$

03.05.26.0020.01

$$\mathrm{Ai}(z) = \frac{1}{3}\left(\sqrt[3]{z^{3/2}}\; I_{-\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right) - z\left(z^{3/2}\right)^{-\frac{1}{3}} I_{\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right)\right)$$

03.05.26.0021.01

$$\mathrm{Ai}(z) = \frac{1}{\pi}\sqrt{\frac{z}{3}}\; K_{\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right) /;\ \mathrm{Re}(z) \geq 0$$

Representations through equivalent functions

With related functions

03.05.27.0001.01

$$e^{\frac{i\pi}{3}} \mathrm{Ai}\left(e^{\frac{2 i \pi}{3}} z\right) + \mathrm{Ai}\left(e^{-\frac{2 i \pi}{3}} z\right) = e^{\frac{i\pi}{6}} \mathrm{Bi}(z)$$

03.05.27.0002.01

$$\mathrm{Ai}\left(z\, e^{\frac{2\pi i}{3}}\right) = \frac{1}{2} e^{\frac{\pi i}{3}} (\mathrm{Ai}(z) - i\, \mathrm{Bi}(z))$$

03.05.27.0003.01

$$\mathrm{Ai}\left(z\, e^{-\frac{2\pi i}{3}}\right) = \frac{1}{2} e^{-\frac{\pi i}{3}} (\mathrm{Ai}(z) + i\, \mathrm{Bi}(z))$$

Zeros

03.05.30.0001.01

$$\mathrm{Ai}(z) = 0 /;\ z = z_k \bigwedge k \in \mathbb{N} \bigwedge z_k = f\left(\frac{3}{8}\pi(4k-1)\right) \bigwedge$$

$$f(d) = -d^{2/3}\left(1 + \frac{5}{48 d^2} - \frac{5}{36 d^4} + \frac{77125}{82944 d^6} - \frac{108056875}{6967296 d^8} + \frac{162375596875}{334430208 d^{10}} - \frac{1622671914671875}{66217181184 d^{12}} + \frac{150126478779573265625}{82639042117632 d^{14}} - \frac{644932726927939889453125}{3470839768940544 d^{16}} + \frac{13042116997445589075044921875}{520200964553048064 d^{18}} - \frac{569789860268573944980176052734375}{132083753999696658432 d^{20}}\right)$$

03.05.30.0002.01

$$\mathrm{Im}(z_k) = 0 \bigwedge \mathrm{Re}(z_k) < 0 /;\ \mathrm{Ai}(z_k) = 0$$

On the real axis, Ai(z) has an infinite number of zeros, all of which are negative. In the complex plane, Ai(z) has no other zeros.

Theorems

The solution of the time-dependent free particle Schrödinger equation

The solution $\psi(x, t) == \text{Ai}\left(c\left(x - c^3 t^2\right)\right) \exp\left(i\, c^3 t\left(x - 2/3\, c^3 t^2\right)\right)$ to the time-dependent free particle Schrödinger equation $-\frac{\partial^2 \psi(x,t)}{\partial x^2} == i \frac{\partial \psi(x,t)}{\partial t}$ evolves with constant acceleration and without distortion or spreading.

The solution of the time-independent Schrödinger equation

The delta function normalizable solution of the time-independent Schrödinger equation with a linear potential $-y''(x) + x\, y(x) == \varepsilon\, y(x)$ has the form: $y(x) == \text{Ai}(x - \varepsilon)$.

Edge scaling limit

The probability that the largest eigenvalue of a Gaussian Unitary Ensemble of dimension d can be expressed using the integral kernel $\left(\text{Ai}(x)\, \text{Ai}'(y) - \text{Ai}'(x)\, \text{Ai}(y)\right) / (x - y)$.

Linearized Korteweg–de Vries equation

The solution of the initial value problem $\frac{\partial u(x,t)}{\partial t} + \frac{\partial^3 u(x,t)}{\partial x^3} = 0$, $u(x, 0) = f(x)$ is given by the Airy transform $u(x, t) = (3\, t)^{-1/3} \int_{-\infty}^{\infty} \text{Ai}\left((x - y)\, (3\, t)^{-1/3}\right) f(y)\, dy$.

History

– G. B. Airy (1838), H. Jeffreys (1928, 1942)
– J. C. P. Miller (1946) suggested the notations Ai, Bi

Applications of Ai include quantum mechanics of linear potential, electrodynamics, combinatorics, analysis of the complexity of algorithms, optical theory of the rainbow, solid state physics and semiconductors in electric fields.

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