# AiryBi



## Notations

### Traditional name

Airy function Bi

### Traditional notation

$\mathrm{Bi}(z)$

### *Mathematica* StandardForm notation

`AiryBi[z]`

## Primary definition

03.06.02.0001.01

$$\mathrm{Bi}(z) = \frac{1}{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_0F_1\left(;\frac{2}{3};\frac{z^3}{9}\right) + \frac{z}{\Gamma\left(\frac{1}{3}\right)}\sqrt[6]{3}\,{}_0F_1\left(;\frac{4}{3};\frac{z^3}{9}\right)$$

## Specific values

### Values at fixed points

03.06.03.0001.01

$$\mathrm{Bi}(0) = \frac{1}{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}$$

### Values at infinities

03.06.03.0002.01

$$\lim_{x\to\infty} \mathrm{Bi}(x) = \infty$$

03.06.03.0003.01

$$\lim_{x\to-\infty} \mathrm{Bi}(x) = 0$$

## General characteristics

### Domain and analyticity

Bi($z$) is an entire analytical function of $z$, which is defined in the whole complex $z$-plane.

03.06.04.0001.01

$$z \longrightarrow \mathrm{Bi}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

## Symmetries and periodicities

### Mirror symmetry

03.06.04.0002.01

$$\mathrm{Bi}(\bar{z}) == \overline{\mathrm{Bi}(z)}$$

### Periodicity

No periodicity

## Poles and essential singularities

The function Bi($z$) has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

03.06.04.0003.01

$$\mathcal{S}ing_z(\mathrm{Bi}(z)) == \{\{\tilde{\infty}, \infty\}\}$$

## Branch points

The function Bi($z$) does not have branch points.

03.06.04.0004.01

$$\mathcal{BP}_z(\mathrm{Bi}(z)) == \{\}$$

## Branch cuts

The function Bi($z$) does not have branch cuts.

03.06.04.0005.01

$$\mathcal{BC}_z(\mathrm{Bi}(z)) == \{\}$$

# Series representations

## Generalized power series

### Expansions at generic point $z == z_0$

#### For the function itself

03.06.06.0031.01

$$\mathrm{Bi}(z) \propto \mathrm{Bi}(z_0) + \mathrm{Bi}'(z_0)\,(z - z_0) + \frac{1}{2}\, z_0\, \mathrm{Bi}(z_0)\,(z - z_0)^2 + \ldots \,/;\, (z \to z_0)$$

03.06.06.0032.01

$$\mathrm{Bi}(z) \propto \mathrm{Bi}(z_0) + \mathrm{Bi}'(z_0)\,(z - z_0) + \frac{1}{2}\, z_0\, \mathrm{Bi}(z_0)\,(z - z_0)^2 + O\big((z - z_0)^3\big)$$

03.06.06.0033.01

$$\mathrm{Bi}(z) = \frac{\mathrm{Bi}(z_0)}{2} + \sum_{k=0}^{\infty} \frac{1}{k!} \left( \frac{z_0^{-k}}{2} \left( \sum_{s=0}^{k} \sum_{j=0}^{s} \sum_{i=0}^{s-1} \frac{(-1)^{j+s-1}\, i\, (-i+s-1)!\, (-3i+3s-1)\, (-3j-k+3s+1)_k \left(-\frac{1}{3}\right)_s}{i!\, j!\, (s-j)!\, (s-2i)!\, \left(\frac{2}{3}\right)_i \left(\frac{4}{3}-s\right)_i} \left(-\frac{z_0^3}{9}\right)^i - \sum_{s=0}^{k} \sum_{j=0}^{s} \sum_{i=0}^{s} \frac{(-1)^{j+s-1}\, (s-i)!\, (-3j+3s+1)\, (-3j-k+3s+2)_{k-1} \left(\frac{1}{3}\right)_s}{i!\, j!\, (s-j)!\, (s-2i)!\, \left(\frac{1}{3}\right)_i \left(\frac{2}{3}-s\right)_i} \left(-\frac{z_0^3}{9}\right)^i \right) \mathrm{Bi}(z_0) + \frac{z_0^{1-k}}{2} \left( \sum_{s=0}^{k} \sum_{j=0}^{s} \sum_{i=0}^{s-1} \frac{(-1)^{j+s-1}\, (-i+s-1)!\, (-3j+3s+1)\, (-3j-k+3s+2)_{k-1} \left(\frac{1}{3}\right)_s}{i!\, j!\, (s-j)!\, (-2i+s-1)!\, \left(\frac{4}{3}\right)_i \left(\frac{2}{3}-s\right)_i} \left(-\frac{z_0^3}{9}\right)^i - \sum_{s=0}^{k} \sum_{j=0}^{s} \sum_{i=0}^{s-1} \frac{(-1)^{j+s-1}\, (-i+s-1)!\, (-3j-k+3s+1)_k \left(-\frac{1}{3}\right)_s}{i!\, j!\, (s-j)!\, (-2i+s-1)!\, \left(\frac{2}{3}\right)_i \left(\frac{4}{3}-s\right)_i} \left(-\frac{z_0^3}{9}\right)^i \right) \mathrm{Bi}'(z_0) \right) (z-z_0)^k$$

03.06.06.0034.01

$$\mathrm{Bi}(z) = \sum_{k=0}^{\infty} \frac{3^{k-\frac{5}{6}}\, z_0^{-k}}{k!} \left( 3^{2/3}\, \Gamma\!\left(\frac{1}{3}\right) {}_2\tilde{F}_3\!\left(\frac{1}{3}, 1; \frac{1-k}{3}, \frac{2-k}{3}, 1-\frac{k}{3}; \frac{z_0^3}{9}\right) + \Gamma\!\left(\frac{2}{3}\right) {}_2\tilde{F}_3\!\left(\frac{2}{3}, 1; \frac{2-k}{3}, 1-\frac{k}{3}, \frac{4-k}{3}; \frac{z_0^3}{9}\right) z_0 \right) (z-z_0)^k$$

03.06.06.0035.01

$$\mathrm{Bi}(z) \propto \mathrm{Bi}(z_0)\, (1 + O(z-z_0))$$

### Expansions at $z = 0$

## For the function itself

03.06.06.0001.02

$$\mathrm{Bi}(z) \propto \frac{1}{\sqrt[6]{3}\, \Gamma\!\left(\frac{2}{3}\right)} \left(1 + \frac{z^3}{6} + \frac{z^6}{180} + \ldots\right) + \frac{\sqrt[6]{3}\, z}{\Gamma\!\left(\frac{1}{3}\right)} \left(1 + \frac{z^3}{12} + \frac{z^6}{504} + \ldots\right) /;\ (z \to 0)$$

03.06.06.0036.01

$$\mathrm{Bi}(z) \propto \frac{1}{\sqrt[6]{3}\, \Gamma\!\left(\frac{2}{3}\right)} \left(1 + \frac{z^3}{6} + \frac{z^6}{180} + O\!\left(z^9\right)\right) + \frac{\sqrt[6]{3}\, z}{\Gamma\!\left(\frac{1}{3}\right)} \left(1 + \frac{z^3}{12} + \frac{z^6}{504} + O\!\left(z^9\right)\right)$$

03.06.06.0002.01

$$\mathrm{Bi}(z) = \frac{1}{\sqrt[6]{3}\, \Gamma\!\left(\frac{2}{3}\right)} \sum_{k=0}^{\infty} \frac{1}{\left(\frac{2}{3}\right)_k k!} \left(\frac{z^3}{9}\right)^k + \frac{\sqrt[6]{3}}{\Gamma\!\left(\frac{1}{3}\right)} z \sum_{k=0}^{\infty} \frac{1}{\left(\frac{4}{3}\right)_k k!} \left(\frac{z^3}{9}\right)^k$$

03.06.06.0003.01

$$\mathrm{Bi}(z) = \frac{1}{\sqrt[6]{3}\, \Gamma\!\left(\frac{2}{3}\right)} {}_0F_1\!\left(; \frac{2}{3}; \frac{z^3}{9}\right) + \frac{\sqrt[6]{3}}{\Gamma\!\left(\frac{1}{3}\right)} z\, {}_0F_1\!\left(; \frac{4}{3}; \frac{z^3}{9}\right)$$

03.06.06.0037.01

$$\text{Bi}(z) = \frac{1}{\sqrt[6]{3}\,\pi} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{k+1}{3}\right) \left|\sin\left(\frac{2\pi(k+1)}{3}\right)\right|}{k!} \left(\sqrt[3]{3}\, z\right)^k$$

03.06.06.0004.02

$$\text{Bi}(z) \propto \frac{\sqrt[6]{3}\, z}{\Gamma\left(\frac{1}{3}\right)} + \frac{1}{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)} + O\left(z^3\right)$$

03.06.06.0038.01

$$\text{Bi}(z) = F_\infty(z) \,/;$$

$$\left(\left(F_n(z) = \frac{1}{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)} \sum_{k=0}^{n} \frac{\left(\frac{z^3}{9}\right)^k}{\left(\frac{2}{3}\right)_k k!} + \frac{\sqrt[6]{3}\, z}{\Gamma\left(\frac{1}{3}\right)} \sum_{k=0}^{n} \frac{\left(\frac{z^3}{9}\right)^k}{\left(\frac{4}{3}\right)_k k!} = \text{Bi}(z) - \frac{1}{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)(n+1)!\left(\frac{2}{3}\right)_{n+1}} \left(\frac{z^3}{9}\right)^{n+1} {}_1F_2\left(1; n+2, n+\frac{5}{3}; \frac{z^3}{9}\right) - \frac{\sqrt[6]{3}\, z}{\Gamma\left(\frac{1}{3}\right)(n+1)!\left(\frac{4}{3}\right)_{n+1}} \left(\frac{z^3}{9}\right)^{n+1} {}_1F_2\left(1; n+2, n+\frac{7}{3}; \frac{z^3}{9}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

## Asymptotic series expansions

### Expansions inside Stokes sectors

## In exponential form ||| In exponential form

03.06.06.0017.01

$$\text{Bi}(z) \propto \frac{e^{\frac{2z^{3/2}}{3}}}{\sqrt{\pi}\,\sqrt[4]{z}} \left(1 + \frac{5}{48\, z^{3/2}} + \frac{385}{4608\, z^3} + O\left(\frac{1}{z^{9/2}}\right)\right) /; |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty)$$

03.06.06.0018.01

$$\text{Bi}(z) \propto \frac{e^{\frac{2z^{3/2}}{3}}}{\sqrt{\pi}\,\sqrt[4]{z}} \left(\sum_{k=0}^{n} \frac{\left(\frac{1}{6}\right)_k \left(\frac{5}{6}\right)_k}{k!} \left(\frac{3}{4\, z^{3/2}}\right)^k + O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right) /; |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.06.06.0019.01

$$\text{Bi}(z) \propto \frac{e^{\frac{2z^{3/2}}{3}}}{\sqrt{\pi}\,\sqrt[4]{z}} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{6}\right)_k \left(\frac{5}{6}\right)_k}{k!} \left(\frac{3}{4\, z^{3/2}}\right)^k /; |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty)$$

03.06.06.0039.01

$$\mathrm{Bi}(z) \propto \frac{5\, e^{\frac{2 z^{3/2}}{3}}}{48 \sqrt{\pi}\, z^{7/4}} \left( \sum_{k=0}^{n} \frac{\left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{17}{12}\right)_k \left(\frac{9}{4 z^3}\right)^k}{\left(\frac{3}{2}\right)_k\, k!} + O\left(\frac{1}{z^{3(n+1)}}\right) \right) + \frac{e^{\frac{2 z^{3/2}}{3}}}{\sqrt{\pi}\, \sqrt[4]{z}} \left( \sum_{k=0}^{n} \frac{\left(\left(\frac{1}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k\right) \left(\frac{9}{4 z^3}\right)^k}{\left(\frac{1}{2}\right)_k\, k!} + O\left(\frac{1}{z^{3(n+1)}}\right) \right) /;\ |\arg(z)| < \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.06.06.0005.01

$$\mathrm{Bi}(z) \propto \frac{1}{\sqrt{\pi}\, \sqrt[4]{z}}\, e^{\frac{2}{3} z^{3/2}}\, {}_2F_0\left(\frac{1}{6}, \frac{5}{6};; \frac{3}{4 z^{3/2}}\right) /;\ |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty)$$

03.06.06.0006.01

$$\mathrm{Bi}(z) \propto \frac{1}{\sqrt{\pi}\, \sqrt[4]{z}}\, e^{\frac{2}{3} z^{3/2}} \left(1 + O\left(\frac{1}{z^{3/2}}\right)\right) /;\ |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty)$$

## In trigonometric form ||| In trigonometric form

03.06.06.0020.01

$$\mathrm{Bi}(-z) \propto \frac{1}{\sqrt{\pi}\, \sqrt[4]{z}} \left( \cos\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \left(1 - \frac{385}{4608 z^3} + \frac{37\,182\,145}{127\,401\,984 z^6} + O\left(\frac{1}{z^9}\right)\right) + \frac{5}{48 z^{3/2}} \sin\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \left(1 - \frac{17\,017}{13\,824 z^3} + \frac{1\,078\,282\,205}{127\,401\,984 z^6} + O\left(\frac{1}{z^9}\right)\right) \right) /;\ |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.06.06.0021.01

$$\mathrm{Bi}(-z) \propto \frac{1}{\sqrt{\pi}\, \sqrt[4]{z}} \left( \cos\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \left( \sum_{k=0}^{n} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{\left(\frac{1}{2}\right)_k\, k!} \left(-\frac{9}{4 z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right) \right) + \frac{5}{48 z^{3/2}} \sin\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \left( \sum_{k=0}^{n} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{\left(\frac{3}{2}\right)_k\, k!} \left(-\frac{9}{4 z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right) \right) \right) /;\ |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.06.06.0022.01

$$\mathrm{Bi}(-z) \propto \frac{1}{\sqrt{\pi}\, \sqrt[4]{z}} \left( \cos\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{\left(\frac{1}{2}\right)_k\, k!} \left(-\frac{9}{4 z^3}\right)^k + \frac{5}{48 z^{3/2}} \sin\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{\left(\frac{3}{2}\right)_k\, k!} \left(-\frac{9}{4 z^3}\right)^k \right) /;\ |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.06.06.0007.01

$$\mathrm{Bi}(-z) \propto \frac{1}{\sqrt{\pi}\,\sqrt[4]{z}}\left(\cos\left(\frac{2 z^{3/2}}{3}+\frac{\pi}{4}\right){}_4F_1\left(\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{11}{12};\frac{1}{2};-\frac{9}{4 z^3}\right)+\frac{5}{48 z^{3/2}}\,{}_4F_1\left(\frac{7}{12},\frac{11}{12},\frac{13}{12},\frac{17}{12};\frac{3}{2};-\frac{9}{4 z^3}\right)\sin\left(\frac{2 z^{3/2}}{3}+\frac{\pi}{4}\right)\right)/;\ |\arg(z)|<\frac{2\pi}{3}\bigwedge(|z|\to\infty)$$

03.06.06.0008.01

$$\mathrm{Bi}(-z) \propto \frac{1}{\sqrt{\pi}\,\sqrt[4]{z}}\left(\cos\left(\frac{2 z^{3/2}}{3}+\frac{\pi}{4}\right)\left(1+O\left(\frac{1}{z^3}\right)\right)+\frac{5}{48 z^{3/2}}\sin\left(\frac{2 z^{3/2}}{3}+\frac{\pi}{4}\right)\left(1+O\left(\frac{1}{z^3}\right)\right)\right)/;\ |\arg(z)|<\frac{2\pi}{3}\bigwedge(|z|\to\infty)$$

03.06.06.0009.01

$$\mathrm{Bi}\left(e^{\frac{\pi i}{3}} z\right) \propto \frac{1}{\sqrt[4]{z}}\,e^{\frac{i\pi}{6}}\sqrt{\frac{2}{\pi}}\left(\sin\left(\frac{2 z^{3/2}}{3}-\frac{1}{2} i\log(2)+\frac{\pi}{4}\right){}_4F_1\left(\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{11}{12};\frac{1}{2};-\frac{9}{4 z^3}\right)-\frac{5}{48 z^{3/2}}\cos\left(\frac{2 z^{3/2}}{3}-\frac{1}{2} i\log(2)+\frac{\pi}{4}\right){}_4F_1\left(\frac{7}{12},\frac{11}{12},\frac{13}{12},\frac{17}{12};\frac{3}{2};-\frac{9}{4 z^3}\right)\right)/;\ |\arg(z)|<\frac{2\pi}{3}\bigwedge(|z|\to\infty)$$

03.06.06.0010.01

$$\mathrm{Bi}\left(e^{\frac{\pi i}{3}} z\right) \propto \frac{1}{\sqrt[4]{z}}\,e^{\frac{i\pi}{6}}\sqrt{\frac{2}{\pi}}\left(\sin\left(\frac{2 z^{3/2}}{3}-\frac{1}{2} i\log(2)+\frac{\pi}{4}\right)\left(1+O\left(\frac{1}{z^3}\right)\right)-\frac{5}{48 z^{3/2}}\cos\left(\frac{2 z^{3/2}}{3}-\frac{1}{2} i\log(2)+\frac{\pi}{4}\right)\left(1+O\left(\frac{1}{z^3}\right)\right)\right)/;\ |\arg(z)|<\frac{2\pi}{3}\bigwedge(|z|\to\infty)$$

03.06.06.0011.01

$$\mathrm{Bi}\left(e^{-\frac{\pi i}{3}} z\right) \propto \frac{1}{\sqrt[4]{z}}\,e^{-\frac{i\pi}{6}}\sqrt{\frac{2}{\pi}}\left(\sin\left(\frac{2 z^{3/2}}{3}+\frac{1}{2} i\log(2)+\frac{\pi}{4}\right){}_4F_1\left(\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{11}{12};\frac{1}{2};-\frac{9}{4 z^3}\right)-\frac{5}{48 z^{3/2}}\cos\left(\frac{2 z^{3/2}}{3}+\frac{1}{2} i\log(2)+\frac{\pi}{4}\right){}_4F_1\left(\frac{7}{12},\frac{11}{12},\frac{13}{12},\frac{17}{12};\frac{3}{2};-\frac{9}{4 z^3}\right)\right)/;\ |\arg(z)|<\frac{2\pi}{3}\bigwedge(|z|\to\infty)$$

03.06.06.0012.01

$$\mathrm{Bi}\left(e^{-\frac{\pi i}{3}} z\right) \propto \frac{1}{\sqrt[4]{z}}\,e^{-\frac{i\pi}{6}}\sqrt{\frac{2}{\pi}}\left(\sin\left(\frac{2 z^{3/2}}{3}+\frac{1}{2} i\log(2)+\frac{\pi}{4}\right)\left(1+O\left(\frac{1}{z^3}\right)\right)-\frac{5}{48 z^{3/2}}\cos\left(\frac{2 z^{3/2}}{3}+\frac{1}{2} i\log(2)+\frac{\pi}{4}\right)\left(1+O\left(\frac{1}{z^3}\right)\right)\right)/;\ |\arg(z)|<\frac{2\pi}{3}\bigwedge(|z|\to\infty)$$

### Expansions for any $z$ in exponential form

## Using exponential function with branch cut-containing arguments

03.06.06.0023.01

$$\mathrm{Bi}(z) \propto \frac{\left(-z^3\right)^{-5/12}}{2\sqrt{\pi}}\left((-1)^{5/12} e^{\frac{1}{3}(-2) i\sqrt{-z^3}}\left(\frac{1}{\sqrt[3]{-1}}\sqrt[3]{-z^3}+z\right)\left(1+\frac{5 i}{48\sqrt{-z^3}}+\frac{385}{4608 z^3}+O\left(\frac{1}{z^{9/2}}\right)\right)-(-1)^{7/12} e^{\frac{2}{3} i\sqrt{-z^3}}\left(\sqrt[3]{-1}\,\sqrt[3]{-z^3}+z\right)\left(1-\frac{5 i}{48\sqrt{-z^3}}+\frac{385}{4608 z^3}+O\left(\frac{1}{z^{9/2}}\right)\right)\right)/;\ (|z|\to\infty)$$

03.06.06.0024.01

$$\text{Bi}(z) \propto \frac{1}{2\sqrt{\pi}}\left(-z^3\right)^{-5/12}\left((-1)^{5/12} e^{\frac{1}{3}(-2)i\sqrt{-z^3}}\left(\frac{1}{\sqrt[3]{-1}}\sqrt[3]{-z^3}+z\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{6}\right)_k\left(\frac{5}{6}\right)_k}{k!}\left(\frac{3i}{4\sqrt{-z^3}}\right)^k+O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)-\right.$$
$$\left.(-1)^{7/12} e^{\frac{2}{3}i\sqrt{-z^3}}\left(\sqrt[3]{-1}\sqrt[3]{-z^3}+z\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{6}\right)_k\left(\frac{5}{6}\right)_k}{k!}\left(-\frac{3i}{4\sqrt{-z^3}}\right)^k+O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)\right)/;(|z|\to\infty)\land n\in\mathbb{N}$$

03.06.06.0025.01

$$\text{Bi}(z) \propto \frac{\left(-z^3\right)^{-5/12}}{2\sqrt{\pi}}\left((-1)^{5/12} e^{\frac{1}{3}(-2)i\sqrt{-z^3}}\left(\frac{1}{\sqrt[3]{-1}}\sqrt[3]{-z^3}+z\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{6}\right)_k\left(\frac{5}{6}\right)_k}{k!}\left(\frac{3i}{4\sqrt{-z^3}}\right)^k-\right.$$
$$\left.(-1)^{7/12} e^{\frac{2}{3}i\sqrt{-z^3}}\left(\sqrt[3]{-1}\sqrt[3]{-z^3}+z\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{6}\right)_k\left(\frac{5}{6}\right)_k}{k!}\left(-\frac{3i}{4\sqrt{-z^3}}\right)^k\right)/;(|z|\to\infty)$$

03.06.06.0013.01

$$\text{Bi}(z) \propto \frac{1}{2\sqrt{\pi}}\left(-z^3\right)^{-5/12}\left((-1)^{5/12} e^{-\frac{2i}{3}\sqrt{-z^3}}\left(\frac{1}{\sqrt[3]{-1}}\sqrt[3]{-z^3}+z\right){}_2F_0\left(\frac{1}{6},\frac{5}{6};;\frac{3i}{4\sqrt{-z^3}}\right)-\right.$$
$$\left.(-1)^{7/12} e^{\frac{2i}{3}\sqrt{-z^3}}\left(\sqrt[3]{-1}\sqrt[3]{-z^3}+z\right){}_2F_0\left(\frac{1}{6},\frac{5}{6};;-\frac{3i}{4\sqrt{-z^3}}\right)\right)/;(|z|\to\infty)$$

03.06.06.0040.01

$$\text{Bi}(z) \propto \frac{\left(-z^3\right)^{-5/12}\sqrt[4]{-1}}{4\sqrt{\pi}}\left(\left(e^{-\frac{2i}{3}\sqrt{-z^3}}\left(\left(i+\sqrt{3}\right)z+\left(-i+\sqrt{3}\right)\sqrt[3]{-z^3}\right)+i\,e^{\frac{2i}{3}\sqrt{-z^3}}\left(-\left(i+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(-i+\sqrt{3}\right)z\right)\right)\right.$$
$$\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)+$$
$$\frac{5}{48\sqrt{-z^3}}\left(i\,e^{-\frac{2i}{3}\sqrt{-z^3}}\left(\left(-i+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(i+\sqrt{3}\right)z\right)-e^{\frac{2i}{3}\sqrt{-z^3}}\left(\left(i+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(-i+\sqrt{3}\right)z\right)\right)$$
$$\left.\left(\sum_{k=0}^{n}\frac{\left(\frac{7}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)\right)/;(|z|\to\infty)\land n\in\mathbb{N}$$

03.06.06.0041.01

$$\mathrm{Bi}(z) \propto \frac{\left(-z^3\right)^{-5/12} \sqrt[4]{-1}}{4\sqrt{\pi}} \Bigg( \left( e^{-\frac{2i}{3}\sqrt{-z^3}} \left( \left(i+\sqrt{3}\right) z + \left(-i+\sqrt{3}\right)\sqrt[3]{-z^3} \right) + i\, e^{\frac{2i}{3}\sqrt{-z^3}} \left( -\left(i+\sqrt{3}\right)\sqrt[3]{-z^3} - \left(-i+\sqrt{3}\right) z \right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + \frac{5}{48\sqrt{-z^3}} \left( i\, e^{-\frac{2i}{3}\sqrt{-z^3}} \left( \left(-i+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(i+\sqrt{3}\right) z \right) - e^{\frac{2i}{3}\sqrt{-z^3}} \left( \left(i+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-i+\sqrt{3}\right) z \right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k \Bigg) /;\ (|z| \to \infty)$$

03.06.06.0042.01

$$\mathrm{Bi}(z) \propto \frac{\left(-z^3\right)^{-5/12} \sqrt[4]{-1}}{4\sqrt{\pi}} \Bigg( \left( e^{-\frac{2i}{3}\sqrt{-z^3}} \left( \left(i+\sqrt{3}\right) z + \left(-i+\sqrt{3}\right)\sqrt[3]{-z^3} \right) + i\, e^{\frac{2i}{3}\sqrt{-z^3}} \left( -\left(i+\sqrt{3}\right)\sqrt[3]{-z^3} - \left(-i+\sqrt{3}\right) z \right) \right) {}_4F_1\!\left(\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}; \frac{1}{2}; \frac{9}{4z^3}\right) + \frac{5}{48\sqrt{-z^3}} \left( i\, e^{-\frac{2i}{3}\sqrt{-z^3}} \left( \left(-i+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(i+\sqrt{3}\right) z \right) - e^{\frac{2i}{3}\sqrt{-z^3}} \left( \left(i+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-i+\sqrt{3}\right) z \right) \right) {}_4F_1\!\left(\frac{7}{12}, \frac{11}{12}, \frac{13}{12}, \frac{17}{12}; \frac{3}{2}; \frac{9}{4z^3}\right) \Bigg) /;\ (|z| \to \infty)$$

03.06.06.0014.01

$$\mathrm{Bi}(z) \propto \frac{\left(-z^3\right)^{-5/12}}{2\sqrt{\pi}} \left( (-1)^{5/12} e^{-\frac{2i}{3}\sqrt{-z^3}} \left( \frac{1}{\sqrt[3]{-1}} \sqrt[3]{-z^3} + z \right) \left( 1 + O\!\left(\frac{1}{z^{3/2}}\right) \right) - (-1)^{7/12} e^{\frac{2i}{3}\sqrt{-z^3}} \left( \sqrt[3]{-1}\sqrt[3]{-z^3} + z \right) \left( 1 + O\!\left(\frac{1}{z^{3/2}}\right) \right) \right) /;\ (|z| \to \infty)$$

## Using exponential function with branch cut-free arguments

03.06.06.0043.01

$$\operatorname{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{8\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} - \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z - \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) \right) \left( 1 + \frac{385}{4608\, z^3} + \frac{37\,182\,145}{127\,401\,984\, z^6} + \frac{5\,849\,680\,962\,125}{1\,761\,205\,026\,816\, z^9} + O\left(\frac{1}{z^{12}}\right) \right) + \frac{5}{24\sqrt{2}\, z^3} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( -\left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} - \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) \right) \left( 1 + \frac{17\,017}{13\,824\, z^3} + \frac{1\,078\,282\,205}{127\,401\,984\, z^6} + \frac{253\,541\,886\,272\,675}{1\,761\,205\,026\,816\, z^9} + O\left(\frac{1}{z^{12}}\right) \right) \right) /;\ (|z| \to \infty)$$

03.06.06.0044.01

$$\operatorname{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{8\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} - \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z - \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) \right) \left( \sum_{k=0}^{n} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{k!\, \left(\frac{1}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right) \right) + \frac{5}{24\sqrt{2}\, z^3} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( -\left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} - \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) \right) \left( \sum_{k=0}^{n} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{k!\, \left(\frac{3}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right) \right) \right) /;\ (|z| \to \infty) \land n \in \mathbb{N}$$

03.06.06.0045.01

$$\operatorname{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{8\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} - \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z - \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{k! \left(\frac{1}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + \frac{5}{24\sqrt{2}\, z^3} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( -\left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} - \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{k! \left(\frac{3}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k \right) /; (|z| \to \infty)$$

03.06.06.0046.01

$$\operatorname{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{8\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} - \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z - \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) \right) {}_4F_1\!\left(\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}; \frac{1}{2}; \frac{9}{4z^3}\right) + \frac{5}{24\sqrt{2}\, z^3} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( -\left(1+\sqrt{3}\right) \sqrt[3]{-z^3}\, z^{3/2} - \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right) \sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right) (-z^3)^{5/6} \right) \right) {}_4F_1\!\left(\frac{7}{12}, \frac{11}{12}, \frac{13}{12}, \frac{17}{12}; \frac{3}{2}; \frac{9}{4z^3}\right) \right) /; (|z| \to \infty)$$

03.06.06.0047.01

$$\mathrm{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{8\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)(-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} - \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z - \left(-1+\sqrt{3}\right)(-z^3)^{5/6} \right) \right) \left(1 + O\left(\frac{1}{z^3}\right)\right) + \frac{5}{24\sqrt{2}\, z^3} \left( e^{\frac{2z^{3/2}}{3}} \left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)(-z^3)^{5/6} \right) + e^{-\frac{2z^{3/2}}{3}} \left( -\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} - \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)(-z^3)^{5/6} \right) \right) \left(1 + O\left(\frac{1}{z^3}\right)\right) \right) /; (|z| \to \infty)$$

03.06.06.0048.01

$$\mathrm{Bi}(z) \propto \begin{cases} -\frac{i\, e^{-\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} + \frac{e^{\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} & \arg(z) \le -\frac{2\pi}{3} \\ -\frac{i\, e^{-\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} + \frac{e^{\frac{2z^{3/2}}{3}}}{\sqrt{\pi}\sqrt[4]{z}} & -\frac{2\pi}{3} < \arg(z) \le 0 \\ \frac{i\, e^{-\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} + \frac{e^{\frac{2z^{3/2}}{3}}}{\sqrt{\pi}\sqrt[4]{z}} & 0 < \arg(z) \le \frac{2\pi}{3} \\ \frac{i\, e^{-\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} + \frac{e^{\frac{2z^{3/2}}{3}}}{2\sqrt{\pi}\sqrt[4]{z}} & \text{True} \end{cases} /; (|z| \to \infty)$$

### Expansions for any $z$ in trigonometric form

## Using trigonometric functions with branch cut-containing arguments

03.06.06.0026.01

$$\mathrm{Bi}(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{5/12}} \left( \left( \left(\sqrt[3]{-z^3} - z\right)\cos\left(\frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4}\right) + \sqrt{3}\left(\sqrt[3]{-z^3} + z\right)\cos\left(\frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3}\right) \right) \left(1 + \frac{385}{4608\, z^3} + \frac{37\,182\,145}{127\,401\,984\, z^6} + O\left(\frac{1}{z^9}\right)\right) + \frac{5}{48\sqrt{-z^3}} \left( \left(\sqrt[3]{-z^3} - z\right)\cos\left(\frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3}\right) - \sqrt{3}\left(\sqrt[3]{-z^3} + z\right)\cos\left(\frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4}\right) \right) \left(1 + \frac{17\,017}{13\,824\, z^3} + \frac{1\,078\,282\,205}{127\,401\,984\, z^6} + O\left(\frac{1}{z^9}\right)\right) \right) /; (|z| \to \infty)$$

03.06.06.0027.01

$$\mathrm{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{2\sqrt{\pi}} \left( \left( \left( \sqrt[3]{-z^3} - z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4} \right) + \sqrt{3} \left( \sqrt[3]{-z^3} + z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} - \frac{\pi}{4} \right) \right) \left( \sum_{k=0}^{n} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{k! \left(\frac{1}{2}\right)_k} \left( \frac{9}{4z^3} \right)^k + O\left( \frac{1}{z^{3n+3}} \right) \right) + \frac{5}{48\sqrt{-z^3}} \left( \left( \sqrt[3]{-z^3} - z \right) \cos\left( \frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3} \right) - \sqrt{3} \left( \sqrt[3]{-z^3} + z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4} \right) \right) \left( \sum_{k=0}^{n} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{k! \left(\frac{3}{2}\right)_k} \left( \frac{9}{4z^3} \right)^k + O\left( \frac{1}{z^{3n+3}} \right) \right) \right) /; (|z| \to \infty) \land n \in \mathbb{N}$$

03.06.06.0028.01

$$\mathrm{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{2\sqrt{\pi}} \left( \left( \left( \sqrt[3]{-z^3} - z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4} \right) + \sqrt{3} \left( \sqrt[3]{-z^3} + z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} - \frac{\pi}{4} \right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{k! \left(\frac{1}{2}\right)_k} \left( \frac{9}{4z^3} \right)^k + \frac{5}{48\sqrt{-z^3}} \left( \left( \sqrt[3]{-z^3} - z \right) \cos\left( \frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3} \right) - \sqrt{3} \left( \sqrt[3]{-z^3} + z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4} \right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{k! \left(\frac{3}{2}\right)_k} \left( \frac{9}{4z^3} \right)^k \right) /; (|z| \to \infty)$$

03.06.06.0029.01

$$\mathrm{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{2\sqrt{\pi}} \left( \left( \left( \sqrt[3]{-z^3} - z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4} \right) + \sqrt{3} \left( \sqrt[3]{-z^3} + z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} - \frac{\pi}{4} \right) \right) {}_4F_1\left( \frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}; \frac{1}{2}; \frac{9}{4z^3} \right) + \frac{5}{48\sqrt{-z^3}} \left( \left( \sqrt[3]{-z^3} - z \right) \cos\left( \frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3} \right) - \sqrt{3} \left( \sqrt[3]{-z^3} + z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4} \right) \right) {}_4F_1\left( \frac{7}{12}, \frac{11}{12}, \frac{13}{12}, \frac{17}{12}; \frac{3}{2}; \frac{9}{4z^3} \right) \right) /; (|z| \to \infty)$$

03.06.06.0030.01

$$\mathrm{Bi}(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{5/12}} \left( \left( \left( \sqrt[3]{-z^3} - z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4} \right) + \sqrt{3} \left( \sqrt[3]{-z^3} + z \right) \cos\left( \frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3} \right) \right) \left( 1 + O\left( \frac{1}{z^9} \right) \right) + \frac{5}{48\sqrt{-z^3}} \left( \left( \sqrt[3]{-z^3} - z \right) \cos\left( \frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3} \right) - \sqrt{3} \left( \sqrt[3]{-z^3} + z \right) \cos\left( \frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4} \right) \right) \left( 1 + O\left( \frac{1}{z^9} \right) \right) \right) /; (|z| \to \infty)$$

## Using trigonometric functions with branch cut-free arguments

03.06.06.0049.01

$$\operatorname{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{4\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( z^{3/2}\left( (1+\sqrt{3})\sqrt[3]{-z^3} + (-1+\sqrt{3})z \right) \cosh\left(\frac{2z^{3/2}}{3}\right) + \sqrt{-z^3}\left( (-1+\sqrt{3})\sqrt[3]{-z^3} + (1+\sqrt{3})z \right)\sinh\left(\frac{2z^{3/2}}{3}\right) \right) \left( 1 + \frac{385}{4608 z^3} + \frac{37182145}{127401984 z^6} + \frac{5849680962125}{1761205026816 z^9} + O\left(\frac{1}{z^{12}}\right) \right) + \frac{5}{24\sqrt{2}\, z^3} \left( z^{3/2}\left( (1+\sqrt{3})\sqrt[3]{-z^3} + (-1+\sqrt{3})z \right)\sinh\left(\frac{2z^{3/2}}{3}\right) + \sqrt{-z^3}\left( (-1+\sqrt{3})\sqrt[3]{-z^3} + (1+\sqrt{3})z \right)\cosh\left(\frac{2z^{3/2}}{3}\right) \right) \left( 1 + \frac{17017}{13824 z^3} + \frac{1078282205}{127401984 z^6} + \frac{253541886272675}{1761205026816 z^9} + O\left(\frac{1}{z^{12}}\right) \right) \right) /; (|z| \to \infty)$$

03.06.06.0050.01

$$\operatorname{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{4\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( z^{3/2}\left( (1+\sqrt{3})\sqrt[3]{-z^3} + (-1+\sqrt{3})z \right) \cosh\left(\frac{2z^{3/2}}{3}\right) + \sqrt{-z^3}\left( (-1+\sqrt{3})\sqrt[3]{-z^3} + (1+\sqrt{3})z \right)\sinh\left(\frac{2z^{3/2}}{3}\right) \right) \left( \sum_{k=0}^{n} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right) \right) + \frac{5}{24\sqrt{2}\, z^3} \left( z^{3/2}\left( (1+\sqrt{3})\sqrt[3]{-z^3} + (-1+\sqrt{3})z \right)\sinh\left(\frac{2z^{3/2}}{3}\right) + \sqrt{-z^3}\left( (-1+\sqrt{3})\sqrt[3]{-z^3} + (1+\sqrt{3})z \right)\cosh\left(\frac{2z^{3/2}}{3}\right) \right) \left( \sum_{k=0}^{n} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right) \right) \right) /; (|z| \to \infty) \land n \in \mathbb{N}$$

03.06.06.0051.01

$$\mathrm{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{4\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( z^{3/2}\left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-1+\sqrt{3}\right) z \right) \cosh\left(\frac{2 z^{3/2}}{3}\right) + \sqrt{-z^3} \left( \left(-1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(1+\sqrt{3}\right) z \right) \sinh\left(\frac{2 z^{3/2}}{3}\right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k}{k! \left(\frac{1}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k + \frac{5}{24\sqrt{2}\, z^3} \left( z^{3/2}\left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-1+\sqrt{3}\right) z \right) \sinh\left(\frac{2 z^{3/2}}{3}\right) + \sqrt{-z^3} \left( \left(-1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(1+\sqrt{3}\right) z \right) \cosh\left(\frac{2 z^{3/2}}{3}\right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{7}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{17}{12}\right)_k}{k! \left(\frac{3}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k \right) /; (|z| \to \infty)$$

03.06.06.0052.01

$$\mathrm{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{4\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( z^{3/2}\left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-1+\sqrt{3}\right) z \right) \cosh\left(\frac{2 z^{3/2}}{3}\right) + \sqrt{-z^3} \left( \left(-1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(1+\sqrt{3}\right) z \right) \sinh\left(\frac{2 z^{3/2}}{3}\right) \right) {}_4F_1\left(\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}; \frac{1}{2}; \frac{9}{4 z^3}\right) + \frac{5}{24\sqrt{2}\, z^3} \left( z^{3/2}\left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-1+\sqrt{3}\right) z \right) \sinh\left(\frac{2 z^{3/2}}{3}\right) + \sqrt{-z^3} \left( \left(-1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(1+\sqrt{3}\right) z \right) \cosh\left(\frac{2 z^{3/2}}{3}\right) \right) {}_4F_1\left(\frac{7}{12}, \frac{11}{12}, \frac{13}{12}, \frac{17}{12}; \frac{3}{2}; \frac{9}{4 z^3}\right) \right) /; (|z| \to \infty)$$

03.06.06.0053.01

$$\mathrm{Bi}(z) \propto \frac{(-z^3)^{-5/12}}{4\sqrt{\pi}} \left( \frac{\sqrt{2}}{z^{3/2}} \left( z^{3/2}\left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-1+\sqrt{3}\right) z \right) \cosh\left(\frac{2 z^{3/2}}{3}\right) + \sqrt{-z^3} \left( \left(-1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(1+\sqrt{3}\right) z \right) \sinh\left(\frac{2 z^{3/2}}{3}\right) \right) \left(1 + O\left(\frac{1}{z^3}\right)\right) + \frac{5}{24\sqrt{2}\, z^3} \left( z^{3/2}\left( \left(1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-1+\sqrt{3}\right) z \right) \sinh\left(\frac{2 z^{3/2}}{3}\right) + \sqrt{-z^3} \left( \left(-1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(1+\sqrt{3}\right) z \right) \cosh\left(\frac{2 z^{3/2}}{3}\right) \right) \left(1 + O\left(\frac{1}{z^3}\right)\right) \right) /; (|z| \to \infty)$$

03.06.06.0054.01

$$\text{Bi}(z) \propto \begin{cases} -\frac{(-1)^{3/4}}{\sqrt{2\pi}\sqrt[4]{z}}\left(\cosh\left(\frac{2z^{3/2}}{3}\right)+i\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & \arg(z) \leq -\frac{2\pi}{3} \\ -\frac{i}{2\sqrt{\pi}\sqrt[4]{z}}\left((1+2i)\cosh\left(\frac{2z^{3/2}}{3}\right)-(1-2i)\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & -\frac{2\pi}{3} < \arg(z) \leq 0 \\ \frac{i}{2\sqrt{\pi}\sqrt[4]{z}}\left((1-2i)\cosh\left(\frac{2z^{3/2}}{3}\right)-(1+2i)\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & 0 < \arg(z) \leq \frac{2\pi}{3} \\ \frac{\sqrt[4]{-1}}{\sqrt{2\pi}\sqrt[4]{z}}\left(\cosh\left(\frac{2z^{3/2}}{3}\right)-i\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & \text{True} \end{cases} /; (|z| \to \infty)$$

## Residue representations

03.06.06.0015.01

$$\text{Bi}(z) = \frac{2\pi}{\sqrt[6]{3}} \left(\sum_{j=0}^{\infty} \text{res}_s\left(\frac{\Gamma\left(s+\frac{1}{3}\right)\left(3^{-2/3}z\right)^{-3s}}{\Gamma\left(s+\frac{1}{6}\right)\Gamma\left(s+\frac{2}{3}\right)\Gamma\left(\frac{5}{6}-s\right)\Gamma\left(\frac{1}{3}-s\right)}\Gamma(s)\right)(-j) + \sum_{j=0}^{\infty} \text{res}_s\left(\frac{\Gamma(s)\left(3^{-2/3}z\right)^{-3s}}{\Gamma\left(s+\frac{1}{6}\right)\Gamma\left(s+\frac{2}{3}\right)\Gamma\left(\frac{5}{6}-s\right)\Gamma\left(\frac{1}{3}-s\right)}\Gamma\left(s+\frac{1}{3}\right)\right)\left(-j-\frac{1}{3}\right)\right)$$

03.06.06.0016.01

$$\text{Bi}(z) = \frac{\pi}{3^{5/6}}\left(z\sum_{j=0}^{\infty}\text{res}_s\left(\frac{\left(\frac{z^3}{9}\right)^{-s}}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{4}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s)\right)(-j) + 3^{2/3}\sum_{j=0}^{\infty}\text{res}_s\left(\frac{\left(\frac{z^3}{9}\right)^{-s}}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{2}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s)\right)(-j)\right)$$

# Integral representations

## On the real axis

### Of the direct function

03.06.07.0001.01

$$\text{Bi}(z) = \frac{1}{\pi}\int_0^{\infty}\left(\sin\left(\frac{t^3}{3}+z\,t\right)+e^{z\,t-\frac{t^3}{3}}\right)dt \,/;\, z<0$$

### Involving related functions

03.06.07.0002.01

$$\text{Ai}(x)^2+\text{Bi}(x)^2 = \frac{1}{\pi^{3/2}}\int_0^{\infty}\frac{1}{\sqrt{t}}\,e^{x\,t-\frac{t^3}{12}}\,dt$$

## Contour integral representations

03.06.07.0003.01

$$\text{Bi}(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty e^{\frac{\pi i}{3}}} e^{\frac{t^3}{3}-z\,t}\,dt + \frac{1}{2\pi}\int_{-\infty}^{\infty e^{-\frac{\pi i}{3}}} e^{\frac{t^3}{3}-z\,t}\,dt$$

03.06.07.0004.01

$$\text{Bi}(z) = \frac{i}{\sqrt[6]{3}}\int_{\mathcal{L}}\frac{\Gamma(s)\,\Gamma\left(s+\frac{1}{3}\right)\left(3^{-2/3}z\right)^{-3s}}{\Gamma\left(s+\frac{1}{6}\right)\Gamma\left(s+\frac{2}{3}\right)\Gamma\left(\frac{5}{6}-s\right)\Gamma\left(\frac{1}{3}-s\right)}\,ds$$

03.06.07.0005.01

$$\operatorname{Bi}(z) == \frac{\pi}{3^{5/6}} \left( \frac{z}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{4}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)} \left(\frac{z^3}{9}\right)^{-s} ds + \frac{3^{2/3}}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{2}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)} \left(\frac{z^3}{9}\right)^{-s} ds \right)$$

## Differential equations

### Ordinary linear differential equations and wronskians

#### For the direct function itself

03.06.13.0001.01

$$w''(z) - z\,w(z) == 0 \;/;\; w(z) == \operatorname{Bi}(z) \bigwedge w(0) == \frac{1}{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)} \bigwedge w'(0) == \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}$$

03.06.13.0002.01

$$w''(z) - z\,w(z) == 0 \;/;\; w(z) == \operatorname{Ai}(z)\,c_1 + c_2 \operatorname{Bi}(z)$$

03.06.13.0003.01

$$W_z(\operatorname{Ai}(z), \operatorname{Bi}(z)) == \frac{1}{\pi}$$

03.06.13.0008.01

$$g'(z)\,w''(z) - g''(z)\,w'(z) - g(z)\,g'(z)^3\,w(z) = 0 \;/;\; w(z) = c_1 \operatorname{Ai}(g(z)) + c_2 \operatorname{Bi}(g(z))$$

03.06.13.0009.01

$$W_z(\operatorname{Ai}(g(z)), \operatorname{Bi}(g(z))) == \frac{g'(z)}{\pi}$$

03.06.13.0010.01

$$g'(z)\,h(z)^2\,w''(z) - (2\,g'(z)\,h'(z) + h(z)\,g''(z))\,h(z)\,w'(z) + \left(-g(z)\,h(z)^2\,g'(z)^3 + 2\,h'(z)^2\,g'(z) - h(z)\,h''(z)\,g'(z) + h(z)\,h'(z)\,g''(z)\right) w(z) = 0 \;/;\; w(z) = c_1\,h(z) \operatorname{Ai}(g(z)) + c_2\,h(z) \operatorname{Bi}(g(z))$$

03.06.13.0011.01

$$W_z(h(z) \operatorname{Ai}(g(z)), h(z) \operatorname{Bi}(g(z))) == \frac{h(z)^2\,g'(z)}{\pi}$$

03.06.13.0012.01

$$z^2\,w''(z) + z\,(1 - r - 2\,s)\,w'(z) + \left(-a^3\,r^2\,z^{3r} + s^2 + r\,s\right) w(z) = 0 \;/;\; w(z) = c_1\,z^s \operatorname{Ai}(a\,z^r) + c_2\,z^s \operatorname{Bi}(a\,z^r)$$

03.06.13.0013.01

$$W_z(z^s \operatorname{Ai}(a\,z^r), z^s \operatorname{Bi}(a\,z^r)) == \frac{a\,r\,z^{r+2s-1}}{\pi}$$

03.06.13.0014.01

$$w''(z) - (\log(r) + 2\log(s))\,w'(z) + \left(-a^3 \log^2(r)\,r^{3z} + \log^2(s) + \log(r)\log(s)\right) w(z) = 0 \;/;\; w(z) = c_1\,s^z \operatorname{Ai}(a\,r^z) + c_2\,s^z \operatorname{Bi}(a\,r^z)$$

03.06.13.0015.01

$$W_z(s^z \operatorname{Ai}(a\,r^z), s^z \operatorname{Bi}(a\,r^z)) == \frac{a\,r^z\,s^{2z}\log(r)}{\pi}$$

#### Involving related functions

03.06.13.0004.01

$$w^{(3)}(z) - 4 z\, w'(z) - 2 w(z) = 0 /; w(z) = c_1 \operatorname{Ai}(z)^2 + c_2 \operatorname{Bi}(z) \operatorname{Ai}(z) + c_3 \operatorname{Bi}(z)^2$$

03.06.13.0005.01

$$w^{(3)}(z) - 4 z\, w'(z) - 2 w(z) = 0 /; w(z) = w_1(z)\, w_2(z) \bigwedge w_1''(z) - z\, w_1(z) = 0 \bigwedge w_2''(z) - z\, w_2(z) = 0$$

03.06.13.0006.01

$$W_z\left(\operatorname{Ai}(z)^2, \operatorname{Ai}(z) \operatorname{Bi}(z), \operatorname{Bi}(z)^2\right) = \frac{2}{\pi^3}$$

### Ordinary nonlinear differential equations

03.06.13.0007.01

$$w(z)^2 - z + w'(z) = 0 /; w(z) = \frac{\operatorname{Bi}'(z) + c_1 \operatorname{Ai}'(z)}{\operatorname{Bi}(z) + c_1 \operatorname{Ai}(z)}$$

Riccati form of differential equation

## Transformations

### Transformations and argument simplifications

#### Argument involving basic arithmetic operations

03.06.16.0001.01

$$\operatorname{Bi}(c\,(d\, z^n)^m) = \frac{1}{2}\left(\sqrt{3}\left(1 - \frac{(d\, z^n)^m}{d^m\, z^{m\,n}}\right) \operatorname{Ai}(c\, d^m\, z^{m\,n}) + \left(\frac{(d\, z^n)^m}{d^m\, z^{m\,n}} + 1\right) \operatorname{Bi}(c\, d^m\, z^{m\,n})\right) /; 3\, m \in \mathbb{Z}$$

03.06.16.0002.01

$$\operatorname{Bi}\left(\sqrt[3]{z^3}\right) = \frac{1}{2}\left(\sqrt{3}\left(1 - \frac{\sqrt[3]{z^3}}{z}\right) \operatorname{Ai}(z) + \left(\frac{\sqrt[3]{z^3}}{z} + 1\right) \operatorname{Bi}(z)\right)$$

03.06.16.0003.01

$$\operatorname{Bi}\left((-1)^{2/3} z\right) = \frac{1}{4}\left(-i + \sqrt{3}\right)(3 \operatorname{Ai}(z) + i \operatorname{Bi}(z))$$

03.06.16.0004.01

$$\operatorname{Bi}\left(-\left(\sqrt[3]{-1}\, z\right)\right) = \frac{1}{4}\left(i + \sqrt{3}\right)(3 \operatorname{Ai}(z) - i \operatorname{Bi}(z))$$

## Identities

### Functional identities

03.06.17.0001.01

$$\operatorname{Bi}(z) + e^{\frac{2\pi i}{3}} \operatorname{Bi}\left(z\, e^{\frac{2\pi i}{3}}\right) + e^{-\frac{2\pi i}{3}} \operatorname{Bi}\left(z\, e^{-\frac{2\pi i}{3}}\right) = 0$$

## Complex characteristics

### Real part

03.06.19.0001.01

$$\operatorname{Re}(\operatorname{Bi}(x+i\,y)) = \frac{1}{2}\left(\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Bi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

### Imaginary part

03.06.19.0002.01

$$\operatorname{Im}(\operatorname{Bi}(x+i\,y)) = \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Bi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

### Absolute value

03.06.19.0003.01

$$|\operatorname{Bi}(x+i\,y)| = \sqrt{\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Bi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)}$$

### Argument

03.06.19.0004.01

$$\arg(\operatorname{Bi}(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Bi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Bi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

### Conjugate value

03.06.19.0005.01

$$\overline{\operatorname{Bi}(x+i\,y)} = \frac{1}{2}\left(\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Bi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)-\frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Bi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

### Signum value

03.06.19.0006.01

$$\operatorname{sgn}(\operatorname{Bi}(x+i\,y)) = \frac{\frac{i}{y}\sqrt{-\frac{y^2}{x^2}}\,x\left(\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Bi}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)\right)+\operatorname{Bi}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)+\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)}{2\sqrt{\operatorname{Bi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Bi}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)}}$$

## Differentiation

### Low-order differentiation

03.06.20.0001.01

$$\frac{\partial \mathrm{Bi}(z)}{\partial z} = \mathrm{Bi}'(z)$$

03.06.20.0002.01

$$\partial_{\{z,2\}} \mathrm{AiryBi}[z] = z\, \mathrm{AiryBi}[z]$$

### Symbolic differentiation

03.06.20.0005.01

$$\frac{\partial^n \mathrm{Bi}(z)}{\partial z^n} = \frac{1}{2} \mathrm{Bi}(z)\, \delta_n + \frac{1}{2} z^{-n} \left( \sum_{k=0}^{n} \sum_{j=0}^{k} \sum_{i=0}^{k-1} \frac{(-1)^{j+k-1}\, i\, (-i+k-1)!\, (-3i+3k-1)\, (-3j+3k-n+1)_n \left(-\frac{1}{3}\right)_k}{i!\, j!\, (k-j)!\, (k-2i)! \left(\frac{2}{3}\right)_i \left(\frac{4}{3}-k\right)_i} \left(-\frac{z^3}{9}\right)^i - \sum_{k=0}^{n} \sum_{j=0}^{k} \sum_{i=0}^{k} \frac{(-1)^{j+k-1}\, (k-i)!\, (-3j+3k+1)\, (-3j+3k-n+2)_{n-1} \left(\frac{1}{3}\right)_k}{i!\, j!\, (k-j)!\, (k-2i)! \left(\frac{1}{3}\right)_i \left(\frac{2}{3}-k\right)_i} \left(-\frac{z^3}{9}\right)^i \right) \mathrm{Bi}(z) + \frac{1}{2} z^{1-n} \left( \sum_{k=0}^{n} \sum_{j=0}^{k} \sum_{i=0}^{k-1} \frac{(-1)^{j+k-1}\, (-i+k-1)!\, (-3j+3k+1)\, (-3j+3k-n+2)_{n-1} \left(\frac{1}{3}\right)_k}{i!\, j!\, (k-j)!\, (-2i+k-1)! \left(\frac{4}{3}\right)_i \left(\frac{2}{3}-k\right)_i} \left(-\frac{z^3}{9}\right)^i - \sum_{k=0}^{n} \sum_{j=0}^{k} \sum_{i=0}^{k-1} \frac{(-1)^{j+k-1}\, (-i+k-1)!\, (-3j+3k-n+1)_n \left(-\frac{1}{3}\right)_k}{i!\, j!\, (k-j)!\, (-2i+k-1)! \left(\frac{2}{3}\right)_i \left(\frac{4}{3}-k\right)_i} \left(-\frac{z^3}{9}\right)^i \right) \mathrm{Bi}'(z) \;/;\; n \in \mathbb{N}$$

03.06.20.0003.02

$$\frac{\partial^n \mathrm{Bi}(z)}{\partial z^n} = 3^{n-\frac{5}{6}} z^{-n} \left( 3^{2/3}\, \Gamma\!\left(\frac{1}{3}\right) {}_2\tilde{F}_3\!\left(\frac{1}{3}, 1; \frac{1-n}{3}, \frac{2-n}{3}, 1-\frac{n}{3}; \frac{z^3}{9}\right) + z\, \Gamma\!\left(\frac{2}{3}\right) {}_2\tilde{F}_3\!\left(\frac{2}{3}, 1; \frac{2-n}{3}, 1-\frac{n}{3}, \frac{4-n}{3}; \frac{z^3}{9}\right)\right) /;\; n \in \mathbb{N}$$

### Fractional integro-differentiation

03.06.20.0004.01

$$\frac{\partial^\alpha \mathrm{Bi}(z)}{\partial z^\alpha} = 3^{\alpha-\frac{5}{6}} z^{-\alpha} \left( 3^{2/3}\, \Gamma\!\left(\frac{1}{3}\right) {}_2\tilde{F}_3\!\left(\frac{1}{3}, 1; \frac{1-\alpha}{3}, \frac{2-\alpha}{3}, 1-\frac{\alpha}{3}; \frac{z^3}{9}\right) + z\, \Gamma\!\left(\frac{2}{3}\right) {}_2\tilde{F}_3\!\left(\frac{2}{3}, 1; \frac{2-\alpha}{3}, 1-\frac{\alpha}{3}, \frac{4-\alpha}{3}; \frac{z^3}{9}\right)\right)$$

## Integration

### Indefinite integration

#### Involving only one direct function

03.06.21.0001.01

$$\int \mathrm{Bi}(a\, z)\, dz = \frac{a\, \Gamma\!\left(\frac{2}{3}\right) z^2}{3\; 3^{5/6}\, \Gamma\!\left(\frac{4}{3}\right) \Gamma\!\left(\frac{5}{3}\right)}\, {}_1F_2\!\left(\frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{a^3 z^3}{9}\right) + \frac{\Gamma\!\left(\frac{1}{3}\right) z}{3 \sqrt[6]{3}\; \Gamma\!\left(\frac{2}{3}\right) \Gamma\!\left(\frac{4}{3}\right)}\, {}_1F_2\!\left(\frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{a^3 z^3}{9}\right)$$

03.06.21.0002.01

$$\int \mathrm{Bi}(z)\, dz = \frac{z}{\sqrt[6]{3}\; \Gamma\!\left(\frac{2}{3}\right)}\, {}_1F_2\!\left(\frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{z^3}{9}\right) + \frac{3^{2/3}}{4\pi} z^2\, \Gamma\!\left(\frac{2}{3}\right) {}_1F_2\!\left(\frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{z^3}{9}\right)$$

### Involving one direct function and elementary functions

## Involving power function

### Involving power

## Linear arguments

03.06.21.0003.01

$$\int z^{\alpha-1}\,\mathrm{Bi}(a\,z)\,dz = \frac{\Gamma\left(\frac{\alpha}{3}\right) z^{\alpha}}{3\,\sqrt[6]{3}}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3};\,\frac{2}{3},\,\frac{\alpha}{3}+1;\,\frac{a^3 z^3}{9}\right) + \frac{a\,\Gamma\left(\frac{\alpha}{3}+\frac{1}{3}\right) z^{\alpha+1}}{3\,3^{5/6}}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3}+\frac{1}{3};\,\frac{4}{3},\,\frac{\alpha}{3}+\frac{4}{3};\,\frac{a^3 z^3}{9}\right)$$

03.06.21.0004.01

$$\int z^{\alpha-1}\,\mathrm{Bi}(z)\,dz = \frac{z^{\alpha}}{3\,3^{5/6}}\left(3^{2/3}\,\Gamma\left(\frac{\alpha}{3}\right)\,{}_1\tilde{F}_2\left(\frac{\alpha}{3};\,\frac{2}{3},\,\frac{\alpha}{3}+1;\,\frac{z^3}{9}\right) + z\,\Gamma\left(\frac{\alpha+1}{3}\right)\,{}_1\tilde{F}_2\left(\frac{\alpha+1}{3};\,\frac{4}{3},\,\frac{\alpha+4}{3};\,\frac{z^3}{9}\right)\right)$$

03.06.21.0005.01

$$\int z^{n+3}\,\mathrm{Bi}(z)\,dz = -(n+2)\,z^{n+1}\,\mathrm{Bi}(z) + z^{n+2}\,\mathrm{Bi}'(z) + (n+1)\,(n+2)\int z^{n}\,\mathrm{Bi}(z)\,dz \;/;\; n\in\mathbb{N}$$

03.06.21.0006.01

$$\int z\,\mathrm{Bi}(z)\,dz = \mathrm{Bi}'(z)$$

03.06.21.0007.01

$$\int z^2\,\mathrm{Bi}(z)\,dz = z\,\mathrm{Bi}'(z) - \mathrm{Bi}(z)$$

03.06.21.0008.01

$$\int \sqrt{z}\,\mathrm{Bi}(z)\,dz = \frac{2\,z^{3/2}}{3\,\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_1F_2\left(\frac{1}{2};\,\frac{2}{3},\,\frac{3}{2};\,\frac{z^3}{9}\right) + \frac{\Gamma\left(\frac{5}{6}\right) z^{5/2}}{3\,3^{5/6}\,\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{11}{6}\right)}\,{}_1F_2\left(\frac{5}{6};\,\frac{4}{3},\,\frac{11}{6};\,\frac{z^3}{9}\right)$$

## Power arguments

03.06.21.0009.01

$$\int z^{\alpha-1}\,\mathrm{Bi}(a\,z^r)\,dz = \frac{z^{\alpha}}{3\,3^{5/6}\,r}\left(a\,\Gamma\left(\frac{r+\alpha}{3\,r}\right)\,{}_1\tilde{F}_2\left(\frac{r+\alpha}{3\,r};\,\frac{4}{3},\,\frac{1}{3}\left(\frac{\alpha}{r}+4\right);\,\frac{1}{9}\,a^3 z^{3\,r}\right) z^r + 3^{2/3}\,\Gamma\left(\frac{\alpha}{3\,r}\right)\,{}_1\tilde{F}_2\left(\frac{\alpha}{3\,r};\,\frac{2}{3},\,\frac{\alpha}{3\,r}+1;\,\frac{1}{9}\,a^3 z^{3\,r}\right)\right)$$

## Involving exponential function

### Involving exp

## Linear argument

03.06.21.0010.01

$$\int e^{-\frac{2}{3}(a\,z)^{3/2}}\,\mathrm{Bi}(a\,z)\,dz = \frac{z}{2\,3^{5/6}}\left(\frac{2\,3^{2/3}}{\Gamma\left(\frac{2}{3}\right)}\,{}_2F_2\left(\frac{1}{6},\,\frac{2}{3};\,\frac{1}{3},\,\frac{5}{3};\,\frac{1}{3}(-4)\,(a\,z)^{3/2}\right) + \frac{a\,z}{\Gamma\left(\frac{4}{3}\right)}\,{}_2F_2\left(\frac{5}{6},\,\frac{4}{3};\,\frac{5}{3},\,\frac{7}{3};\,\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)\right)$$

03.06.21.0011.01

$$\int e^{\frac{2}{3}(a z)^{3/2}} \mathrm{Bi}(a z)\, dz == \frac{z}{2\, 3^{5/6}} \left( \frac{2\, 3^{2/3}}{\Gamma\left(\frac{2}{3}\right)} {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{4}{3} (a z)^{3/2}\right) + \frac{a z}{\Gamma\left(\frac{4}{3}\right)} {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{4}{3} (a z)^{3/2}\right)\right)$$

## Power arguments

03.06.21.0012.01

$$\int e^{-\frac{2}{3}(a z^r)^{3/2}} \mathrm{Bi}(a z^r)\, dz == \frac{1}{3^{5/6} (r+1) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z \left(a \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3 r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3 r}; \frac{1}{3} (-4) (a z^r)^{3/2}\right) z^r + 3^{2/3} (r+1) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3 r}; \frac{1}{3}, 1 + \frac{2}{3 r}; \frac{1}{3} (-4) (a z^r)^{3/2}\right)\right)\right)$$

03.06.21.0013.01

$$\int e^{\frac{2}{3}(a z^r)^{3/2}} \mathrm{Bi}(a z^r)\, dz == \frac{z}{3^{5/6} (r+1) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(a \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3 r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3 r}; \frac{4}{3} (a z^r)^{3/2}\right) z^r + 3^{2/3} (r+1) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3 r}; \frac{1}{3}, 1 + \frac{2}{3 r}; \frac{4}{3} (a z^r)^{3/2}\right)\right)$$

## Involving exponential function and a power function

### Involving exp and power

## Linear arguments

03.06.21.0014.01

$$\int z^{\alpha-1} e^{-\frac{2}{3}(a z)^{3/2}} \mathrm{Bi}(a z)\, dz == \frac{1}{3^{5/6} \alpha (\alpha+1) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z^{\alpha} \left(3^{2/3} (\alpha+1) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2\alpha}{3}; \frac{1}{3}, \frac{2\alpha}{3} + 1; \frac{1}{3} (-4) (a z)^{3/2}\right) + a z \alpha \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2\alpha}{3} + \frac{2}{3}; \frac{5}{3}, \frac{2\alpha}{3} + \frac{5}{3}; \frac{1}{3} (-4) (a z)^{3/2}\right)\right)\right)$$

03.06.21.0015.01

$$\int \sqrt{z}\, e^{\frac{1}{3}(-2)(a z)^{3/2}} \mathrm{Ai}(a z)\, dz == \int \sqrt{z}\, e^{-\frac{2}{3}(a z)^{3/2}} \mathrm{Bi}(a z)\, dz == \frac{1}{15 a^2 \sqrt{z}\, \Gamma\left(\frac{2}{3}\right)} \left(2 e^{\frac{1}{3}(-2)(a z)^{3/2}} \left(3 a^2 \mathrm{Bi}(a z) \Gamma\left(\frac{2}{3}\right) z^2 + \sqrt{3} \sqrt{a z} \left(a z I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{2}{3}\right) \left(a^{3/2} z^{3/2}\right)^{2/3} + \sqrt[3]{3}\, e^{\frac{2}{3}(a z)^{3/2}} + \frac{a^3 z^3 \Gamma\left(\frac{2}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right)\right)$$

03.06.21.0016.01

$$\int z^{\alpha-1} e^{\frac{2}{3}(a z)^{3/2}} \mathrm{Bi}(a z)\, dz == \frac{z^{\alpha}}{3^{5/6} \alpha (\alpha+1) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(3^{2/3} (\alpha+1) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2\alpha}{3}; \frac{1}{3}, \frac{2\alpha}{3} + 1; \frac{4}{3} (a z)^{3/2}\right) + a z \alpha \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2\alpha}{3} + \frac{2}{3}; \frac{5}{3}, \frac{2\alpha}{3} + \frac{5}{3}; \frac{4}{3} (a z)^{3/2}\right)\right)$$

03.06.21.0017.01

$$\int \sqrt{z}\, e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Bi}(a z)\, dz = -\frac{1}{15 a^2 \sqrt{z}\, \Gamma\left(\frac{2}{3}\right)} \left(2 \left(\sqrt{3} \sqrt{a z} \left(a e^{\frac{2}{3}(a z)^{3/2}} z I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{2}{3}\right) \left(a^{3/2} z^{3/2}\right)^{2/3} + \sqrt[3]{3} + \frac{a^3 \Gamma\left(\frac{2}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} e^{\frac{2}{3}(a z)^{3/2}} z^3 I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right) - 3 a^2 e^{\frac{2}{3}(a z)^{3/2}} z^2 \operatorname{Bi}(a z) \Gamma\left(\frac{2}{3}\right)\right)\right)$$

## Power arguments

03.06.21.0018.01

$$\int z^{\alpha-1} e^{-\frac{2}{3}(a z^r)^{3/2}} \operatorname{Bi}(a z^r)\, dz = \frac{1}{3^{5/6} \alpha (r+\alpha) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z^\alpha \left(a \alpha \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2\alpha}{3r} + \frac{2}{3}; \frac{5}{3}, \frac{2\alpha}{3r} + \frac{5}{3}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) z^r + 3^{2/3} (r+\alpha) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2\alpha}{3r}; \frac{1}{3}, \frac{2\alpha}{3r} + 1; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.06.21.0019.01

$$\int z^{\alpha-1} e^{\frac{2}{3}(a z^r)^{3/2}} \operatorname{Bi}(a z^r)\, dz = \frac{z^\alpha}{3^{5/6} \alpha (r+\alpha) \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(a \alpha \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(\frac{5}{6}, \frac{2\alpha}{3r} + \frac{2}{3}; \frac{5}{3}, \frac{2\alpha}{3r} + \frac{5}{3}; \frac{4}{3}(a z^r)^{3/2}\right) z^r + 3^{2/3} (r+\alpha) \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2\alpha}{3r}; \frac{1}{3}, \frac{2\alpha}{3r} + 1; \frac{4}{3}(a z^r)^{3/2}\right)\right)$$

## Involving hyperbolic functions

### Involving sinh

## Linear argument

03.06.21.0020.01

$$\int \sinh\left(\frac{2}{3}(a z)^{3/2}\right) \operatorname{Bi}(a z)\, dz = \frac{1}{4\, 3^{5/6} \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z \left(2\, 3^{2/3} \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right) - 2\, 3^{2/3} \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) + a z \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right) - {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)\right)$$

03.06.21.0021.01

$$\int \sinh\left(\frac{2}{3}(a z)^{3/2} + b\right) \mathrm{Bi}(a z)\, dz = \frac{1}{4\, 3^{5/6}\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(e^{-b} z \left(2\, 3^{2/3} e^{2b}\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right) - 2\, 3^{2/3}\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - a z\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - e^{2b}\, {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right)\right)\right)\right)$$

## Power arguments

03.06.21.0022.01

$$\int \sinh\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}(a z^r)\, dz = -\frac{1}{2\, 3^{5/6} (r+1)\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z \left(a\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right)\right) z^r - 3^{2/3} (r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + 3^{2/3} (r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.06.21.0023.01

$$\int \sinh\left(\frac{2}{3}(a z^r)^{3/2} + b\right) \mathrm{Bi}(a z^r)\, dz = \frac{1}{2\, 3^{5/6} (r+1)\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(e^{-b} z \left(-a\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - e^{2b}\, {}_2F_2\left(\frac{5}{6}, \frac{2}{3} + \frac{2}{3r}; \frac{5}{3}, \frac{5}{3} + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right)\right) z^r + 3^{2/3} e^{2b} (r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) - 3^{2/3} (r+1)\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3r}; \frac{1}{3}, 1 + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

# Involving cosh

## Linear argument

03.06.21.0024.01

$$\int \cosh\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}(a z)\, dz = \frac{1}{4\, 3^{5/6}\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \left(z \left(2\, 3^{2/3}\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right) + 2\, 3^{2/3}\, \Gamma\left(\frac{4}{3}\right) {}_2F_2\left(\frac{1}{6}, \frac{2}{3}; \frac{1}{3}, \frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) + a z\, \Gamma\left(\frac{2}{3}\right) \left({}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right) + {}_2F_2\left(\frac{5}{6}, \frac{4}{3}; \frac{5}{3}, \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)\right)$$

03.06.21.0025.01

$$\int \cosh\left(\frac{2}{3}(a\,z)^{3/2}+b\right)\mathrm{Bi}(a\,z)\,dz = \frac{1}{4\,3^{5/6}\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(e^{-b}\,z\left(2\,3^{2/3}\,e^{2b}\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2}{3};\frac{1}{3},\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)+2\,3^{2/3}\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2}{3};\frac{1}{3},\frac{5}{3};\frac{1}{3}(-4)(a\,z)^{3/2}\right)+a\,z\,\Gamma\left(\frac{2}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{5}{6},\frac{4}{3};\frac{5}{3},\frac{7}{3};\frac{4}{3}(a\,z)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{4}{3};\frac{5}{3},\frac{7}{3};\frac{1}{3}(-4)(a\,z)^{3/2}\right)\right)\right)\right)$$

## Power arguments

03.06.21.0026.01

$$\int \cosh\left(\frac{2}{3}(a\,z^r)^{3/2}\right)\mathrm{Bi}(a\,z^r)\,dz = \frac{1}{2\,3^{5/6}\,(r+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(z\left(a\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2}{3}+\frac{2}{3r};\frac{5}{3},\frac{5}{3}+\frac{2}{3r};\frac{4}{3}(a\,z^r)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2}{3}+\frac{2}{3r};\frac{5}{3},\frac{5}{3}+\frac{2}{3r};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)z^r+3^{2/3}(r+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2}{3r};\frac{1}{3},1+\frac{2}{3r};\frac{4}{3}(a\,z^r)^{3/2}\right)+3^{2/3}(r+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2}{3r};\frac{1}{3},1+\frac{2}{3r};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)\right)$$

03.06.21.0027.01

$$\int \cosh\left(\frac{2}{3}(a\,z^r)^{3/2}+b\right)\mathrm{Bi}(a\,z^r)\,dz = \frac{1}{2\,3^{5/6}\,(r+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(e^{-b}\,z\left(a\,\Gamma\left(\frac{2}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{5}{6},\frac{2}{3}+\frac{2}{3r};\frac{5}{3},\frac{5}{3}+\frac{2}{3r};\frac{4}{3}(a\,z^r)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2}{3}+\frac{2}{3r};\frac{5}{3},\frac{5}{3}+\frac{2}{3r};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)z^r+3^{2/3}\,e^{2b}\,(r+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2}{3r};\frac{1}{3},1+\frac{2}{3r};\frac{4}{3}(a\,z^r)^{3/2}\right)+3^{2/3}(r+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2}{3r};\frac{1}{3},1+\frac{2}{3r};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)\right)$$

## Involving hyperbolic functions and a power function

### Involving sinh and power

## Linear argument

03.06.21.0028.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}(a\,z)\,dz = -\frac{1}{2\,3^{5/6}\,\alpha\,(\alpha+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(z^\alpha\left(-3^{2/3}(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3};\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)+3^{2/3}(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3};\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)(a\,z)^{3/2}\right)+a\,z\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3}+\frac{5}{3};\frac{1}{3}(-4)(a\,z)^{3/2}\right)-{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3}+\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)\right)\right)\right)$$

03.06.21.0029.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z)^{3/2}+b\right)\operatorname{Bi}(a\,z)\,dz = \frac{1}{2\,3^{5/6}\,\alpha\,(\alpha+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(3^{2/3}\,e^{2b}\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3};\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)-3^{2/3}\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3};\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)-a\,z\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)-e^{2b}\,{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3}+\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)\right)\right)\right)$$

## Power arguments

03.06.21.0030.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z^r)^{3/2}\right)\operatorname{Bi}(a\,z^r)\,dz = -\frac{1}{2\,3^{5/6}\,\alpha\,(r+\alpha)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(z^{\alpha}\left(a\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3r}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3r}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z^r)^{3/2}\right)-{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3r}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3r}+\frac{5}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)\right)z^r-3^{2/3}\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3r};\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+3^{2/3}\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3r};\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)\,(a\,z^r)^{3/2}\right)\right)\right)$$

03.06.21.0031.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z^r)^{3/2}+b\right)\operatorname{Bi}(a\,z^r)\,dz = \frac{1}{2\,3^{5/6}\,\alpha\,(r+\alpha)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(-a\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3r}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3r}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z^r)^{3/2}\right)-e^{2b}\,{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3r}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3r}+\frac{5}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)\right)z^r+3^{2/3}\,e^{2b}\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3r};\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)-3^{2/3}\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3r};\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)\,(a\,z^r)^{3/2}\right)\right)\right)$$

# Involving cosh and power

## Linear argument

03.06.21.0032.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z)^{3/2}\right)\operatorname{Bi}(a\,z)\,dz = \frac{1}{2\,3^{5/6}\,\alpha\,(\alpha+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(z^{\alpha}\left(3^{2/3}\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3};\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)+3^{2/3}\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3};\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)+a\,z\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3}+\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3}+\frac{5}{3};\frac{1}{3}(-4)\,(a\,z)^{3/2}\right)\right)\right)\right)$$

03.06.21.0033.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z)^{3/2}+b\right)\mathrm{Bi}(a\,z)\,dz = \frac{1}{2\,3^{5/6}\,\alpha\,(\alpha+1)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(3^{2/3}\,e^{2b}\,(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3};\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)+3^{2/3}(\alpha+1)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3};\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)(a\,z)^{3/2}\right)+a\,z\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3}+\frac{5}{3};\frac{4}{3}(a\,z)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3}+\frac{5}{3};\frac{1}{3}(-4)(a\,z)^{3/2}\right)\right)\right)\right)$$

## Power arguments

03.06.21.0034.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z^r)^{3/2}\right)\mathrm{Bi}(a\,z^r)\,dz = \frac{1}{2\,3^{5/6}\,\alpha\,(r+\alpha)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(z^{\alpha}\left(a\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left({}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3r}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3r}+\frac{5}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3r}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3r}+\frac{5}{3};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)z^r+3^{2/3}(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3r};\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+3^{2/3}(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3r};\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)\right)$$

03.06.21.0035.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z^r)^{3/2}+b\right)\mathrm{Bi}(a\,z^r)\,dz = \frac{1}{2\,3^{5/6}\,\alpha\,(r+\alpha)\,\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(a\,\alpha\,\Gamma\left(\frac{2}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3r}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3r}+\frac{5}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)+{}_2F_2\left(\frac{5}{6},\frac{2\alpha}{3r}+\frac{2}{3};\frac{5}{3},\frac{2\alpha}{3r}+\frac{5}{3};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)z^r+3^{2/3}\,e^{2b}\,(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3r};\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+3^{2/3}(r+\alpha)\,\Gamma\left(\frac{4}{3}\right){}_2F_2\left(\frac{1}{6},\frac{2\alpha}{3r};\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)\right)$$

### Involving functions of the direct function

## Involving elementary functions of the direct function

### Involving powers of the direct function

## Linear arguments

03.06.21.0036.01

$$\int \mathrm{Bi}(a\,z)^2\,dz = z\,\mathrm{Bi}(a\,z)^2-\frac{\mathrm{Bi}'(a\,z)^2}{a}$$

## Power arguments

03.06.21.0037.01

$$\int \mathrm{Bi}(a z^r)^2 \, dz = \frac{2\sqrt{\pi}\, z \sqrt[3]{\frac{2}{3}}}{3r} G_{2,4}^{1,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} 1-\frac{1}{3r}, \frac{5}{6} \\ \frac{1}{3}, 0, \frac{2}{3}, -\frac{1}{3r} \end{matrix}\right) + \frac{z}{2\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2} r} G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} 1-\frac{1}{3r}, \frac{5}{6} \\ 0, \frac{1}{3}, \frac{2}{3}, -\frac{1}{3r} \end{matrix}\right)$$

Involving products of the direct function

## Linear arguments

03.06.21.0038.01

$$\int \mathrm{Bi}(-a z)\, \mathrm{Bi}(a z) \, dz = \frac{\sqrt[3]{\frac{3}{2}}}{4 a \pi^{3/2}} G_{1,5}^{3,1}\left(\frac{a z}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \middle| \begin{matrix} 1 \\ \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, 0, \frac{1}{3} \end{matrix}\right) - \frac{a^4 z^5 \Gamma\left(\frac{5}{6}\right)}{81 \Gamma\left(\frac{4}{3}\right) \Gamma\left(\frac{5}{3}\right) \Gamma\left(\frac{11}{6}\right)} {}_1F_4\left(\frac{5}{6}; \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}; -\frac{1}{324} a^6 z^6\right)$$

## Power arguments

03.06.21.0039.01

$$\int \mathrm{Bi}(-a z^r)\, \mathrm{Bi}(a z^r) \, dz =$$

$$\frac{z}{4\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2} r} G_{2,6}^{4,1}\left(\frac{a z^r}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \middle| \begin{matrix} 1-\frac{1}{6r}, \frac{1}{6} \\ 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{1}{6}, -\frac{1}{6r} \end{matrix}\right) - \frac{\sqrt[3]{\frac{2}{3}} \sqrt{\pi}\, z}{3r} G_{1,5}^{1,1}\left(\frac{a z^r}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \middle| \begin{matrix} 1-\frac{1}{6r} \\ \frac{2}{3}, 0, \frac{1}{6}, \frac{1}{3}, -\frac{1}{6r} \end{matrix}\right)$$

**Involving functions of the direct function and elementary functions**

## Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

## Linear arguments

03.06.21.0040.01

$$\int z^{\alpha-1} \mathrm{Bi}(a z)^2 \, dz =$$

$$\frac{1}{36} z^{\alpha} \left(16 a z \Gamma\left(\frac{\alpha+1}{3}\right) {}_2\tilde{F}_3\left(\frac{1}{2}, \frac{\alpha+1}{3}; \frac{2}{3}, \frac{4}{3}, \frac{\alpha+4}{3}; \frac{4 a^3 z^3}{9}\right) + \frac{3 \sqrt[3]{2}\, 3^{2/3}}{\pi^{3/2}} G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} 1-\frac{\alpha}{3}, \frac{5}{6} \\ 0, \frac{1}{3}, \frac{2}{3}, -\frac{\alpha}{3} \end{matrix}\right)\right)$$

03.06.21.0041.01

$$\int z\, \mathrm{Bi}(a z)^2 \, dz = \frac{1}{3 a^2} \left(a^2 z^2 \mathrm{Bi}(a z)^2 + \mathrm{Bi}'(a z)\, \mathrm{Bi}(a z) - a z\, \mathrm{Bi}'(a z)^2\right)$$

03.06.21.0042.01

$$\int z^2 \mathrm{Bi}(a z)^2 \, dz = \frac{1}{5 a^3} \left(\left(a^3 z^3 - 1\right) \mathrm{Bi}(a z)^2 + 2 a z\, \mathrm{Bi}'(a z)\, \mathrm{Bi}(a z) - a^2 z^2\, \mathrm{Bi}'(a z)^2\right)$$

03.06.21.0043.01

$$\int z^3 \operatorname{Bi}(a z)^2 \, dz = \frac{1}{7 a^4} \left(a^4 \operatorname{Bi}(a z)^2 z^4 + 3 a^2 \operatorname{Bi}(a z) \operatorname{Bi}'(a z) z^2 - \left(a^3 z^3 + 3\right) \operatorname{Bi}'(a z)^2\right)$$

## Power arguments

03.06.21.0044.01

$$\int z^{\alpha-1} \operatorname{Bi}(a z^r)^2 \, dz = \frac{2 \sqrt{\pi} \, z^\alpha \sqrt[3]{\frac{2}{3}}}{3 r} G_{2,4}^{1,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} 1 - \frac{\alpha}{3 r}, \frac{5}{6} \\ \frac{1}{3}, 0, \frac{2}{3}, -\frac{\alpha}{3 r} \end{matrix}\right) + \frac{z^\alpha}{2 \, 2^{2/3} \sqrt[3]{3} \, \pi^{3/2} r} G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} 1 - \frac{\alpha}{3 r}, \frac{5}{6} \\ 0, \frac{1}{3}, \frac{2}{3}, -\frac{\alpha}{3 r} \end{matrix}\right)$$

Involving products of the direct function and a power function

## Linear arguments

03.06.21.0045.01

$$\int z^{\alpha-1} \operatorname{Bi}(-a z) \operatorname{Bi}(a z) \, dz = \frac{1}{648 \pi^{3/2}} \left(z^\alpha \left(27 \sqrt[3]{2} \, 3^{2/3} G_{1,5}^{3,1}\left(\frac{a z}{\sqrt[3]{2} \, 3^{2/3}}, \frac{1}{6} \middle| \begin{matrix} 1 - \frac{\alpha}{6} \\ 0, \frac{1}{3}, \frac{2}{3}, \frac{1}{6}, -\frac{\alpha}{6} \end{matrix}\right) - 4 a^4 \pi^2 z^4 \Gamma\left(\frac{\alpha+4}{6}\right) {}_1\tilde{F}_4\left(\frac{\alpha+4}{6}; \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{\alpha+10}{6}; -\frac{1}{324} a^6 z^6\right)\right)\right)$$

## Power arguments

03.06.21.0046.01

$$\int z^{\alpha-1} \operatorname{Bi}(-a z^r) \operatorname{Bi}(a z^r) \, dz = \frac{z^\alpha}{12 \, 2^{2/3} \sqrt[3]{3} \, \pi^{3/2} r} \left(3 G_{2,6}^{4,1}\left(\frac{a z^r}{\sqrt[3]{2} \, 3^{2/3}}, \frac{1}{6} \middle| \begin{matrix} 1 - \frac{\alpha}{6 r}, \frac{1}{6} \\ 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{1}{6}, -\frac{\alpha}{6 r} \end{matrix}\right) - 8 \pi^2 G_{1,5}^{1,1}\left(\frac{a z^r}{\sqrt[3]{2} \, 3^{2/3}}, \frac{1}{6} \middle| \begin{matrix} 1 - \frac{\alpha}{6 r} \\ \frac{2}{3}, 0, \frac{1}{6}, \frac{1}{3}, -\frac{\alpha}{6 r} \end{matrix}\right)\right)$$

**Involving direct function and Bessel-type functions**

## Involving Bessel functions

Involving Bessel *I*

## Linear argument

03.06.21.0047.01

$$\int I_\nu\left(\frac{2}{3} (a z)^{3/2}\right) \operatorname{Bi}(a z) \, dz = \frac{2^\nu 3^{-\nu-\frac{1}{2}} \sqrt{\pi} \left((a z)^{3/2}\right)^\nu}{a} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1-\nu}{2}, 1 - \frac{\nu}{2}, 1 - \frac{\nu}{2}, \frac{1}{2}, 1 \\ \frac{1}{3}, \frac{2}{3}, \frac{1}{2}, 1, \frac{1}{3} - \nu, \frac{2}{3} - \nu, -\frac{\nu}{2} \end{matrix}\right)$$

## Power arguments

03.06.21.0048.01

$$\int I_\nu\left(\frac{2}{3}\left(a\,z^r\right)^{3/2}\right)\mathrm{Bi}(a\,z^r)\,dz = \frac{2^{\nu+\frac{2}{3}}\,3^{-\nu-\frac{7}{6}}\sqrt{\pi}\;z\left(\left(a\,z^r\right)^{3/2}\right)^\nu}{r}\,G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(1-3\nu),\,\frac{1}{6}(4-3\nu),\,-\frac{\nu}{2}-\frac{1}{3r}+1,\,\frac{1}{6},\,\frac{2}{3}\\ 0,\,\frac{1}{3},\,\frac{1}{6},\,\frac{2}{3},\,\frac{1}{3}-\nu,\,-\nu,\,-\frac{3r\nu+2}{6r}\end{matrix}\right)$$

### Involving Bessel $I$ and power

## Linear argument

03.06.21.0049.01

$$\int z^{\alpha-1}\,I_\nu\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}(a\,z)\,dz = 2^{\nu+\frac{2}{3}}\,3^{-\nu-\frac{7}{6}}\sqrt{\pi}\;z^\alpha\left((a\,z)^{3/2}\right)^\nu G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(1-3\nu),\,\frac{1}{6}(4-3\nu),\,\frac{1}{6}(-2\alpha-3\nu+6),\,\frac{1}{6},\,\frac{2}{3}\\ 0,\,\frac{1}{3},\,\frac{1}{6},\,\frac{2}{3},\,\frac{1}{6}(-2\alpha-3\nu),\,\frac{1}{3}-\nu,\,-\nu\end{matrix}\right)$$

03.06.21.0050.01

$$\int z^{3/2}\,I_\nu\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}(a\,z)\,dz = 2^{\nu+\frac{2}{3}}\,3^{-\nu-\frac{7}{6}}\sqrt{\pi}\;z^{5/2}\left((a\,z)^{3/2}\right)^\nu G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(1-3\nu),\,\frac{1}{6}(1-3\nu),\,\frac{1}{6}(4-3\nu),\,\frac{1}{6},\,\frac{2}{3}\\ 0,\,\frac{1}{3},\,\frac{1}{6},\,\frac{2}{3},\,\frac{1}{6}(-3\nu-5),\,\frac{1}{3}-\nu,\,-\nu\end{matrix}\right)$$

03.06.21.0051.01

$$\int z^{-3/2}\,I_\nu\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}(a\,z)\,dz = \frac{2^{\nu+\frac{2}{3}}\,3^{-\nu-\frac{7}{6}}\sqrt{\pi}\left((a\,z)^{3/2}\right)^\nu}{\sqrt{z}}\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3\nu),\,\frac{1}{6}(7-3\nu),\,\frac{1}{6},\,\frac{2}{3}\\ 0,\,\frac{1}{3},\,\frac{1}{6},\,\frac{2}{3},\,\frac{1}{3}-\nu,\,-\nu\end{matrix}\right)$$

## Power arguments

03.06.21.0052.01

$$\int z^{\alpha-1}\,I_\nu\left(\frac{2}{3}\left(a\,z^r\right)^{3/2}\right)\mathrm{Bi}(a\,z^r)\,dz = \frac{2^{\nu+\frac{2}{3}}\,3^{-\nu-\frac{7}{6}}\sqrt{\pi}\;z^\alpha\left(\left(a\,z^r\right)^{3/2}\right)^\nu}{r}\,G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(1-3\nu),\,\frac{1}{6}(4-3\nu),\,-\frac{\alpha}{3r}-\frac{\nu}{2}+1,\,\frac{1}{6},\,\frac{2}{3}\\ 0,\,\frac{1}{3},\,\frac{1}{6},\,\frac{2}{3},\,\frac{1}{3}-\nu,\,-\nu,\,-\frac{2\alpha+3r\nu}{6r}\end{matrix}\right)$$

### Involving Bessel $K$

## Linear argument

03.06.21.0053.01

$$\int K_\nu\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}(a\,z)\,dz = \frac{1}{2\,a}\left(\pi\csc(\pi\,\nu)\left(2^{-\nu}\,3^{\nu-\frac{1}{2}}\sqrt{\pi}\,\left((a\,z)^{3/2}\right)^{-\nu} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{\nu+1}{2},\ \frac{\nu+2}{2},\ \frac{\nu+2}{2},\ \frac{1}{2},\ 1\\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ 1,\ \frac{\nu}{2},\ \nu+\frac{1}{3},\ \nu+\frac{2}{3}\end{matrix}\right) - 2^{\nu}\,3^{-\nu-\frac{1}{2}}\sqrt{\pi}\,\left((a\,z)^{3/2}\right)^{\nu} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1-\nu}{2},\ 1-\frac{\nu}{2},\ 1-\frac{\nu}{2},\ \frac{1}{2},\ 1\\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ 1,\ \frac{1}{3}-\nu,\ \frac{2}{3}-\nu,\ -\frac{\nu}{2}\end{matrix}\right)\right)\right)$$

03.06.21.0054.01

$$\int K_0\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}(a\,z)\,dz = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,a}\left(\sqrt{\pi}\left(\left(\frac{3}{2}\log(a\,z)-\log\left((a\,z)^{3/2}\right)\right) G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}1,\ \frac{1}{2},\ 1\\ \frac{1}{3},\ \frac{2}{3},\ 0,\ \frac{1}{3},\ \frac{2}{3}\end{matrix}\right)\sqrt[3]{\frac{2}{3}} + \frac{1}{2\,2^{2/3}\,\sqrt[3]{3}\,\pi^3} G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{2},\ 1,\ 1\\ \frac{1}{3},\ \frac{1}{3},\ \frac{2}{3},\ \frac{2}{3},\ 0\end{matrix}\right) + \frac{1}{2\,2^{2/3}\,\sqrt[3]{3}\,\pi} G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{2},\ 1,\ 1,\ \frac{1}{4},\ \frac{3}{4}\\ \frac{1}{3},\ \frac{1}{3},\ \frac{2}{3},\ \frac{2}{3},\ 0,\ \frac{1}{4},\ \frac{3}{4}\end{matrix}\right)\right)\right)$$

03.06.21.0055.01

$$\int K_1\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}(a\,z)\,dz = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,a}\left(\sqrt{\pi}\left(\left(\frac{3}{2}\log(a\,z)-\log\left((a\,z)^{3/2}\right)\right) G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}1,\ \frac{1}{2},\ 1\\ \frac{5}{6},\ \frac{7}{6},\ -\frac{1}{6},\ 0,\ \frac{1}{6}\end{matrix}\right)\sqrt[3]{\frac{2}{3}} + \frac{1}{2\,2^{2/3}\,\sqrt[3]{3}\,\pi} G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{2},\ 1,\ 1,\ \frac{1}{4},\ \frac{3}{4}\\ -\frac{1}{6},\ \frac{1}{6},\ \frac{5}{6},\ \frac{7}{6},\ 0,\ \frac{1}{4},\ \frac{3}{4}\end{matrix}\right) - \frac{1}{2\,2^{2/3}\,\sqrt[3]{3}\,\pi^3} G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{2},\ 1,\ 1\\ -\frac{1}{6},\ \frac{1}{6},\ \frac{5}{6},\ \frac{7}{6},\ 0\end{matrix}\right)\right)\right)$$

03.06.21.0056.01

$$\int K_2\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}(a\,z)\,dz = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,a}\left(\sqrt{\pi}\left(\left(\frac{3}{2}\log(a\,z)-\log\left((a\,z)^{3/2}\right)\right) G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}1,\ \frac{1}{2},\ 1\\ \frac{4}{3},\ \frac{5}{3},\ -\frac{2}{3},\ -\frac{1}{3},\ 0\end{matrix}\right)\sqrt[3]{\frac{2}{3}} + \frac{1}{2\,2^{2/3}\,\sqrt[3]{3}\,\pi^3} G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{2},\ 1,\ 1\\ -\frac{2}{3},\ -\frac{1}{3},\ \frac{4}{3},\ \frac{5}{3},\ 0\end{matrix}\right) + \frac{1}{2\,2^{2/3}\,\sqrt[3]{3}\,\pi} G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{2},\ 1,\ 1,\ \frac{1}{4},\ \frac{3}{4}\\ -\frac{2}{3},\ -\frac{1}{3},\ \frac{4}{3},\ \frac{5}{3},\ 0,\ \frac{1}{4},\ \frac{3}{4}\end{matrix}\right)\right)\right)$$

## Power arguments

03.06.21.0057.01

$$\int K_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}(a z^r)\,dz = -\frac{1}{r}\left(2^{-\nu-\frac{1}{3}}\,3^{-\nu-\frac{7}{6}}\,\pi^{3/2}\,z\left((a z^r)^{3/2}\right)^{-\nu}\csc(\pi\nu)\left(4^\nu\left((a z^r)^{3/2}\right)^{2\nu} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}(1-3\nu), \frac{1}{6}(4-3\nu), -\frac{\nu}{2}-\frac{1}{3r}+1, \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{1}{3}-\nu, -\nu, -\frac{3r\nu+2}{6r} \end{array}\right) - 9^\nu\, G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{\nu}{2}-\frac{1}{3r}+1, \frac{1}{6}(3\nu+1), \frac{1}{6}(3\nu+4), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \nu, \nu+\frac{1}{3}, \frac{3r\nu-2}{6r} \end{array}\right)\right)\right)$$

03.06.21.0058.01

$$\int K_0\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}(a z^r)\,dz = \frac{1}{6\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{5/2}\,r}\left(z\left(G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r} \\ 0, 0, \frac{1}{3}, \frac{1}{3}, -\frac{1}{3r} \end{array}\right) + \pi^2\left(2\pi\left(3\log(a z^r) - 2\log\left((a z^r)^{3/2}\right)\right) G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r}, \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, -\frac{1}{3r} \end{array}\right) + G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r}, -\frac{1}{12}, \frac{5}{12} \\ 0, 0, \frac{1}{3}, \frac{1}{3}, -\frac{1}{12}, \frac{5}{12}, -\frac{1}{3r} \end{array}\right)\right)\right)\right)$$

03.06.21.0059.01

$$\int K_1\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}(a z^r)\,dz = \frac{1}{6\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{5/2}\,r}\left(z\left(\pi^2\left(2\pi\left(2\log\left((a z^r)^{3/2}\right) - 3\log(a z^r)\right) G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r}, \frac{2}{3}, \frac{7}{6} \\ \frac{1}{2}, \frac{5}{6}, -\frac{1}{2}, -\frac{1}{6}, \frac{2}{3}, \frac{7}{6}, -\frac{1}{3r} \end{array}\right) + G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r}, -\frac{1}{12}, \frac{5}{12} \\ -\frac{1}{2}, -\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, -\frac{1}{12}, \frac{5}{12}, -\frac{1}{3r} \end{array}\right)\right) - G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r} \\ -\frac{1}{2}, -\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, -\frac{1}{3r} \end{array}\right)\right)\right)$$

03.06.21.0060.01

$$\int K_2\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}(a z^r)\,dz = \frac{1}{6\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{5/2}\,r}\left(z\left(G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{1}{3r} \end{array}\right) + \pi^2\left(2\pi\left(3\log(a z^r) - 2\log\left((a z^r)^{3/2}\right)\right) G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r}, \frac{7}{6}, \frac{5}{3} \\ 1, \frac{4}{3}, -1, -\frac{2}{3}, \frac{7}{6}, \frac{5}{3}, -\frac{1}{3r} \end{array}\right) + G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3}, 1-\frac{1}{3r}, -\frac{1}{12}, \frac{5}{12} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{1}{12}, \frac{5}{12}, -\frac{1}{3r} \end{array}\right)\right)\right)\right)$$

## Involving Bessel $K$ and power

### Linear argument

03.06.21.0061.01

$$\int z^{\alpha-1} K_\nu\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}(a z)\, dz = -2^{-\nu-\frac{1}{3}}\, 3^{-\nu-\frac{7}{6}}\, \pi^{3/2} z^\alpha \left((a z)^{3/2}\right)^{-\nu} \csc(\pi \nu) \left(4^\nu \left((a z)^{3/2}\right)^{2\nu} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(1-3\nu), \frac{1}{6}(4-3\nu), \frac{1}{6}(-2\alpha-3\nu+6), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{1}{6}(-2\alpha-3\nu), \frac{1}{3}-\nu, -\nu \end{matrix}\right) - 9^\nu G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(3\nu+1), \frac{1}{6}(3\nu+4), \frac{1}{6}(-2\alpha+3\nu+6), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \nu, \nu+\frac{1}{3}, \frac{1}{6}(3\nu-2\alpha) \end{matrix}\right)\right)$$

03.06.21.0062.01

$$\int z^{\alpha-1} K_0\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}(a z)\, dz = \frac{1}{6\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{5/2}} \left(z^\alpha \left(2\pi^3 \left(3\log(a z) - 2\log\left((a z)^{3/2}\right)\right) G_{4,5}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{2}{3}, 1-\alpha, \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, 0, \frac{1}{3}, -\frac{\alpha}{3} \end{matrix}\right) + G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3} \\ 0, 0, \frac{1}{3}, \frac{1}{3}, -\frac{\alpha}{3} \end{matrix}\right) + \pi^2 G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3}, -\frac{1}{12}, \frac{5}{12} \\ 0, 0, \frac{1}{3}, \frac{1}{3}, -\frac{1}{12}, \frac{5}{12}, -\frac{\alpha}{3} \end{matrix}\right)\right)\right)$$

03.06.21.0063.01

$$\int z^{\alpha-1} K_1\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}(a z)\, dz = \frac{1}{6\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{5/2}} \left(z^\alpha \left(2\pi^3 \left(3\log(a z) - 2\log\left((a z)^{3/2}\right)\right) G_{4,5}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{2}{3}, 1-\alpha, \frac{1}{6}, \frac{2}{3} \\ \frac{1}{2}, \frac{5}{6}, -\frac{1}{2}, -\frac{1}{6}, -\frac{\alpha}{3} \end{matrix}\right) - G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3} \\ -\frac{1}{2}, -\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, -\frac{\alpha}{3} \end{matrix}\right) + \pi^2 G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3}, -\frac{1}{12}, \frac{5}{12} \\ -\frac{1}{2}, -\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, -\frac{1}{12}, \frac{5}{12}, -\frac{\alpha}{3} \end{matrix}\right)\right)\right)$$

03.06.21.0064.01

$$\int z^{\alpha-1} K_2\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}(a z)\, dz = \frac{1}{6\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{5/2}} \left(z^\alpha \left(2\pi^3 \left(3\log(a z) - 2\log\left((a z)^{3/2}\right)\right) G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} 1-\frac{\alpha}{3}, \frac{1}{6}, \frac{2}{3} \\ 1, \frac{4}{3}, -1, -\frac{2}{3}, -\frac{\alpha}{3} \end{matrix}\right) + G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{\alpha}{3} \end{matrix}\right) + \pi^2 G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3}, -\frac{1}{12}, \frac{5}{12} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{1}{12}, \frac{5}{12}, -\frac{\alpha}{3} \end{matrix}\right)\right)\right)$$

03.06.21.0065.01

$$\int z^{3/2} K_2\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}(a z)\, dz = \frac{1}{6 \sqrt[3]{2} \sqrt[6]{3} \pi^{5/2}} \left(z^{5/2} \left(2 \pi^3 \left(3 \log(a z) - 2 \log\left((a z)^{3/2}\right)\right) G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{1}{6}, \frac{2}{3} \\ 1, \frac{4}{3}, -1, -\frac{5}{6}, -\frac{2}{3} \end{matrix}\right) + G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{1}{6}, \frac{2}{3} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{5}{6} \end{matrix}\right) + \pi^2 G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{1}{6}, \frac{2}{3}, -\frac{1}{12}, \frac{5}{12} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{5}{6}, -\frac{1}{12}, \frac{5}{12} \end{matrix}\right)\right)\right)$$

03.06.21.0066.01

$$\int z^{-3/2} K_2\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}(a z)\, dz = \frac{1}{6 \sqrt[3]{2} \sqrt[6]{3} \pi^{5/2} \sqrt{z}} \left(2 \pi^3 \left(2 \log\left((a z)^{3/2}\right) - 3 \log(a z)\right) G_{2,4}^{2,0}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{2}{3}, \frac{7}{6} \\ 1, \frac{4}{3}, -1, -\frac{2}{3} \end{matrix}\right) + G_{2,4}^{4,2}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{2}{3}, \frac{7}{6} \\ -1, -\frac{2}{3}, 1, \frac{4}{3} \end{matrix}\right) + \pi^2 G_{4,6}^{4,2}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{2}{3}, \frac{7}{6}, -\frac{1}{12}, \frac{5}{12} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{1}{12}, \frac{5}{12} \end{matrix}\right)\right)$$

## Power arguments

03.06.21.0067.01

$$\int z^{\alpha-1} K_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}(a z^r)\, dz = \frac{1}{2} \pi \csc(\pi \nu) \left(\frac{2^{\frac{2}{3}-\nu} 3^{\nu-\frac{7}{6}} \sqrt{\pi}\, z^\alpha \left((a z^r)^{3/2}\right)^{-\nu}}{r} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} -\frac{\alpha}{3 r} + \frac{\nu}{2} + 1, \frac{1}{6}(3 \nu + 1), \frac{1}{6}(3 \nu + 4), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{\nu}{2} - \frac{\alpha}{3 r}, \nu, \nu + \frac{1}{3} \end{matrix}\right) - \frac{2^{\nu+\frac{2}{3}} 3^{-\nu-\frac{7}{6}} \sqrt{\pi}\, z^\alpha \left((a z^r)^{3/2}\right)^{\nu}}{r} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(1 - 3 \nu), \frac{1}{6}(4 - 3 \nu), -\frac{\alpha}{3 r} - \frac{\nu}{2} + 1, \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{1}{3} - \nu, -\nu, -\frac{2 \alpha + 3 r \nu}{6 r} \end{matrix}\right)\right)$$

03.06.21.0068.01

$$\int z^{\alpha-1} K_0\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}(a z^r)\, dz = \frac{1}{6 \sqrt[3]{2} \sqrt[6]{3} \pi^{5/2} r} \left(z^\alpha \left(G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1 - \frac{\alpha}{3 r} \\ 0, 0, \frac{1}{3}, \frac{1}{3}, -\frac{\alpha}{3 r} \end{matrix}\right) + \pi^2 \left(2 \pi \left(3 \log(a z^r) - 2 \log\left((a z^r)^{3/2}\right)\right) G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1 - \frac{\alpha}{3 r}, \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, -\frac{\alpha}{3 r} \end{matrix}\right) + G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1 - \frac{\alpha}{3 r}, -\frac{1}{12}, \frac{5}{12} \\ 0, 0, \frac{1}{3}, \frac{1}{3}, -\frac{1}{12}, \frac{5}{12}, -\frac{\alpha}{3 r} \end{matrix}\right)\right)\right)\right)$$

03.06.21.0069.01

$$\int z^{\alpha-1} K_1\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}(a z^r)\, dz = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}}\left(\sqrt{\pi}\left(-\frac{z^\alpha}{6\pi^3 r} G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3r} \\ -\frac{1}{2}, -\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, -\frac{\alpha}{3r} \end{matrix}\right) - \frac{2 z^\alpha \left(\frac{3}{2}\log(a z^r) - \log\left((a z^r)^{3/2}\right)\right)}{3r} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3r}, \frac{2}{3}, \frac{7}{6} \\ \frac{1}{2}, \frac{5}{6}, -\frac{1}{2}, -\frac{1}{6}, \frac{2}{3}, \frac{7}{6}, -\frac{\alpha}{3r} \end{matrix}\right) + \frac{z^\alpha}{6\pi r} G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3r}, -\frac{1}{12}, \frac{5}{12} \\ -\frac{1}{2}, -\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, -\frac{1}{12}, \frac{5}{12}, -\frac{\alpha}{3r} \end{matrix}\right)\right)\right)$$

03.06.21.0070.01

$$\int z^{\alpha-1} K_2\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}(a z^r)\, dz = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}}\left(\sqrt{\pi}\left(\frac{z^\alpha}{6\pi^3 r} G_{3,5}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3r} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{\alpha}{3r} \end{matrix}\right) + \frac{2 z^\alpha \left(\frac{3}{2}\log(a z^r) - \log\left((a z^r)^{3/2}\right)\right)}{3r} G_{5,7}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3r}, \frac{7}{6}, \frac{5}{3} \\ 1, \frac{4}{3}, -1, -\frac{2}{3}, \frac{7}{6}, \frac{5}{3}, -\frac{\alpha}{3r} \end{matrix}\right) + \frac{z^\alpha}{6\pi r} G_{5,7}^{4,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}, \frac{2}{3}, 1-\frac{\alpha}{3r}, -\frac{1}{12}, \frac{5}{12} \\ -1, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{1}{12}, \frac{5}{12}, -\frac{\alpha}{3r} \end{matrix}\right)\right)\right)$$

## Involving other Airy functions

### Involving Ai

### Linear arguments

03.06.21.0071.01

$$\int \mathrm{Ai}(a z)\, \mathrm{Bi}(-a z)\, dz = \frac{1}{4\sqrt[3]{2}\; 3^{2/3}\, a\, \pi^{3/2}} G_{1,5}^{3,1}\left(\frac{a z}{\sqrt[3]{2}\; 3^{2/3}}, \frac{1}{6} \middle| \begin{matrix} 1 \\ \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, 0, \frac{5}{6} \end{matrix}\right)$$

03.06.21.0072.01

$$\int \mathrm{Ai}(a z)\, \mathrm{Bi}(a z)\, dz = z\, \mathrm{Ai}(a z)\, \mathrm{Bi}(a z) - \frac{\mathrm{Ai}'(a z)\, \mathrm{Bi}'(a z)}{a}$$

03.06.21.0073.01

$$\int \frac{1}{\mathrm{Ai}(z)\, \mathrm{Bi}(z)}\, dz = \pi \log\left(\frac{\mathrm{Bi}(z)}{\mathrm{Ai}(z)}\right)$$

03.06.21.0086.01

$$\int \frac{\mathrm{Bi}(z)^n}{\mathrm{Ai}(z)^{n+2}}\, dz = \frac{\pi}{n+1}\left(\frac{\mathrm{Bi}(z)}{\mathrm{Ai}(z)}\right)^{n+1} /;\ n \in \mathbb{N}$$

03.06.21.0087.01

$$\int \frac{1}{\mathrm{Ai}(z)^2 + \mathrm{Bi}(z)^2}\, dz = \pi \tan^{-1}\left(\frac{\mathrm{Bi}(z)}{\mathrm{Ai}(z)}\right)$$

03.06.21.0088.01

$$\int \frac{1}{\mathrm{Ai}(z)^2} f\left(\frac{\mathrm{Bi}(z)}{\mathrm{Ai}(z)}\right) dz = \pi F\left(\frac{\mathrm{Bi}(z)}{\mathrm{Ai}(z)}\right) /; F'(\zeta) = f(\zeta)$$

03.06.21.0089.01

$$\int \frac{1}{\mathrm{Bi}(z)^2} f\left(\frac{\mathrm{Ai}(z)}{\mathrm{Bi}(z)}\right) dz = -\pi F\left(\frac{\mathrm{Ai}(z)}{\mathrm{Bi}(z)}\right) /; F'(\zeta) = f(\zeta)$$

03.06.21.0074.01

$$\int \frac{\mathrm{Ai}(z)\,\mathrm{Bi}(z)}{\left(\mathrm{Ai}(z)^2+\mathrm{Bi}(z)^2\right)^2} dz = -\frac{\pi\,\mathrm{Ai}(z)^2}{2\left(\mathrm{Ai}(z)^2+\mathrm{Bi}(z)^2\right)}$$

## Power arguments

03.06.21.0075.01

$$\int \mathrm{Ai}(a z^r)\,\mathrm{Bi}(-a z^r)\, dz = -\frac{z}{12\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2}\, r} G_{2,6}^{4,1}\left(\frac{a z^r}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \middle| \begin{array}{c} 1-\frac{1}{6r}, -\frac{1}{3} \\ 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{1}{6r} \end{array}\right)$$

03.06.21.0076.01

$$\int \mathrm{Ai}(a z^r)\,\mathrm{Bi}(a z^r)\, dz = \frac{z}{6\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2}\, r} G_{3,5}^{3,2}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{array}{c} \frac{5}{6}, 1-\frac{1}{3r}, \frac{1}{3} \\ 0, \frac{1}{3}, \frac{2}{3}, \frac{1}{3}, -\frac{1}{3r} \end{array}\right)$$

# Involving Ai and power

## Linear arguments

03.06.21.0077.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(a z)\,\mathrm{Bi}(-a z)\, dz = \frac{z^\alpha}{12\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2}} G_{1,5}^{3,1}\left(\frac{a z}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \middle| \begin{array}{c} 1-\frac{\alpha}{6} \\ 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, -\frac{\alpha}{6} \end{array}\right)$$

03.06.21.0078.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(a z)\,\mathrm{Bi}(a z)\, dz = \frac{z^\alpha}{6\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2}} G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} \frac{5}{6}, 1-\frac{\alpha}{3} \\ 0, \frac{2}{3}, \frac{1}{3}, -\frac{\alpha}{3} \end{array}\right)$$

03.06.21.0079.01

$$\int z\,\mathrm{Ai}(a z)\,\mathrm{Bi}(a z)\, dz = \frac{1}{6 a^2}\left(\mathrm{Ai}(a z)\left(2 a^2\,\mathrm{Bi}(a z)\, z^2 + \mathrm{Bi}'(a z)\right) + \mathrm{Ai}'(a z)\left(\mathrm{Bi}(a z) - 2 a z\,\mathrm{Bi}'(a z)\right)\right)$$

03.06.21.0080.01

$$\int z^2\,\mathrm{Ai}(a z)\,\mathrm{Bi}(a z)\, dz = \frac{1}{5 a^3}\left(\mathrm{Ai}(a z)\left(\left(a^3 z^3 - 1\right)\mathrm{Bi}(a z) + a z\,\mathrm{Bi}'(a z)\right) + a z\,\mathrm{Ai}'(a z)\left(\mathrm{Bi}(a z) - a z\,\mathrm{Bi}'(a z)\right)\right)$$

03.06.21.0081.01

$$\int z^3\,\mathrm{Ai}(a z)\,\mathrm{Bi}(a z)\, dz = \frac{1}{14 a^4}\left(\mathrm{Ai}(a z)\left(2 a^4\,\mathrm{Bi}(a z)\, z^4 + 3 a^2\,\mathrm{Bi}'(a z)\, z^2\right) + \mathrm{Ai}'(a z)\left(3 a^2 z^2\,\mathrm{Bi}(a z) - 2\left(a^3 z^3 + 3\right)\mathrm{Bi}'(a z)\right)\right)$$

## Power arguments

03.06.21.0082.01

$$\int z^{\alpha-1} \operatorname{Ai}(a z^r) \operatorname{Bi}(-a z^r) \, dz = -\frac{z^\alpha}{12\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2}\, r} G_{2,6}^{4,1}\left(\frac{a z^r}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \,\middle|\, \begin{matrix} 1-\frac{\alpha}{6r}, -\frac{1}{3} \\ 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{\alpha}{6r} \end{matrix}\right)$$

03.06.21.0083.01

$$\int z^{\alpha-1} \operatorname{Ai}(a z^r) \operatorname{Bi}(a z^r) \, dz = \frac{z^\alpha}{6\, 2^{2/3} \sqrt[3]{3}\, \pi^{3/2}\, r} G_{3,5}^{3,2}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{5}{6}, 1-\frac{\alpha}{3r}, \frac{1}{3} \\ 0, \frac{1}{3}, \frac{2}{3}, \frac{1}{3}, -\frac{\alpha}{3r} \end{matrix}\right)$$

### Definite integration

#### For the direct function itself

03.06.21.0084.01

$$\int_{-\infty}^{0} \operatorname{Bi}(t) \, dt = 0$$

#### Involving direct function and Bessel-type functions

03.06.21.0090.01

$$\int_0^\infty \frac{1}{(\operatorname{Ai}(x) - i \operatorname{Bi}(x))^2} \, dx = \frac{1}{4} \pi \left(i - \sqrt{3}\right)$$

03.06.21.0091.01

$$\int_0^\infty \frac{\operatorname{Ai}(x) \operatorname{Bi}(x)}{\left(\operatorname{Ai}(x)^2 + \operatorname{Bi}(x)^2\right)^2} \, dx = \frac{\pi}{8}$$

#### Multiple integration

03.06.21.0085.01

$$\int_0^x \int_0^x \operatorname{Bi}(t) \, dt \, dx = \operatorname{Bi}'(0) - \operatorname{Bi}'(x) + x \int_0^x \operatorname{Bi}(t) \, dt$$

## Integral transforms

### Fourier exp transforms

03.06.22.0001.01

$$\mathcal{F}_t[\operatorname{Bi}(t)](z) = \frac{i}{60 \sqrt{2} \sqrt[3]{3}\, \pi^{3/2}} \left(40\, \Gamma\!\left(-\frac{4}{3}\right) {}_1F_2\!\left(1; \frac{2}{3}, \frac{7}{6}; -\frac{z^6}{36}\right) - 3\, 3^{2/3} z^4\, \Gamma\!\left(\frac{1}{3}\right) {}_1F_2\!\left(1; \frac{4}{3}, \frac{11}{6}; -\frac{z^6}{36}\right)\right) \left(z - \sqrt{z^2} \operatorname{sgn}(z)\right)$$

### Inverse Fourier exp transforms

03.06.22.0002.01

$$\mathcal{F}_t^{-1}[\operatorname{Bi}(t)](z) = \frac{i}{60 \sqrt{2} \sqrt[3]{3}\, \pi^{3/2}} \left(3\, 3^{2/3} z^4\, \Gamma\!\left(\frac{1}{3}\right) {}_1F_2\!\left(1; \frac{4}{3}, \frac{11}{6}; -\frac{z^6}{36}\right) - 40\, \Gamma\!\left(-\frac{4}{3}\right) {}_1F_2\!\left(1; \frac{2}{3}, \frac{7}{6}; -\frac{z^6}{36}\right)\right) \left(z - \sqrt{z^2} \operatorname{sgn}(z)\right)$$

### Laplace transforms

03.06.22.0003.01

$$\mathcal{L}_t[\mathrm{Bi}(t)](z) = \frac{e^{-\frac{z^3}{3}}}{2\pi z^2}\left(-z\,\Gamma\left(\frac{1}{3}\right)\left(\Gamma\left(\frac{2}{3}\right)-\Gamma\left(\frac{2}{3},-\frac{z^3}{3}\right)\right)\sqrt[3]{-z^3}-\left(-z^3\right)^{2/3}\Gamma\left(\frac{2}{3}\right)\left(\Gamma\left(\frac{1}{3}\right)-\Gamma\left(\frac{1}{3},-\frac{z^3}{3}\right)\right)\right)$$

03.06.22.0004.01

$$\mathcal{L}_t[\mathrm{Bi}(-t)](z) = \frac{1}{2\pi z^2}e^{\frac{z^3}{3}}\sqrt[3]{z^3}\left(\Gamma\left(\frac{2}{3}\right)\left(\Gamma\left(\frac{1}{3}\right)-\Gamma\left(\frac{1}{3},\frac{z^3}{3}\right)\right)\sqrt[3]{z^3}+z\,\Gamma\left(\frac{1}{3}\right)\left(\Gamma\left(\frac{2}{3},\frac{z^3}{3}\right)-\Gamma\left(\frac{2}{3}\right)\right)\right)$$

# Representations through more general functions

## Through hypergeometric functions

### Involving $_0F_1$

03.06.26.0001.01

$$\mathrm{Bi}(z) = \frac{1}{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_0F_1\left(;\frac{2}{3};\frac{z^3}{9}\right)+\frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}\,z\,{}_0F_1\left(;\frac{4}{3};\frac{z^3}{9}\right)$$

## Through Meijer *G*

### Classical cases for the direct function itself

03.06.26.0002.01

$$\mathrm{Bi}(z) = \frac{\pi}{3^{5/6}}\left(3^{2/3}\,G_{1,3}^{1,0}\left(\frac{z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,\frac{1}{3},\frac{1}{2}\end{matrix}\right)+z\,G_{1,3}^{1,0}\left(\frac{z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,-\frac{1}{3},\frac{1}{2}\end{matrix}\right)\right)$$

03.06.26.0026.01

$$\mathrm{Bi}(z) = 2\pi\frac{1}{\sqrt[6]{3}}\,G_{2,4}^{2,0}\left(\frac{z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{6},\frac{2}{3}\\ 0,\frac{1}{3},\frac{1}{6},\frac{2}{3}\end{matrix}\right)\;/;\;-\frac{\pi}{3}<\arg(z)\leq\frac{\pi}{3}$$

### Classical cases involving exp

03.06.26.0027.01

$$e^{-\frac{1}{3}\left(2z^{3/2}\right)}\,\mathrm{Bi}(z) = \frac{1}{2^{2/3}\sqrt[6]{3}\,\sqrt{\pi}}\,G_{2,3}^{2,1}\left(\frac{4z^{3/2}}{3}\,\middle|\,\begin{matrix}\frac{5}{6},\frac{1}{3}\\ 0,\frac{2}{3},\frac{1}{3}\end{matrix}\right)\;/;\;-\frac{2\pi}{3}<\arg(z)\leq\frac{2\pi}{3}$$

03.06.26.0028.01

$$e^{\frac{2z^{3/2}}{3}}\,\mathrm{Bi}(z) = \frac{\sqrt[3]{2}\,\sqrt{\pi}}{\sqrt[6]{3}}\,G_{2,3}^{2,0}\left(\frac{4z^{3/2}}{3}\,\middle|\,\begin{matrix}\frac{1}{3},\frac{5}{6}\\ 0,\frac{2}{3},\frac{1}{3}\end{matrix}\right)\;/;\;-\frac{2\pi}{3}<\arg(z)\leq\frac{2\pi}{3}$$

03.06.26.0029.01

$$e^{-z}\,\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right) = \frac{1}{2^{2/3}\sqrt[6]{3}\,\sqrt{\pi}}\,G_{2,3}^{2,1}\left(2z\,\middle|\,\begin{matrix}\frac{5}{6},\frac{1}{3}\\ 0,\frac{2}{3},\frac{1}{3}\end{matrix}\right)$$

03.06.26.0030.01

$$e^z\,\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \frac{\sqrt[3]{2}\,\sqrt{\pi}}{\sqrt[6]{3}}\, G_{2,3}^{2,0}\left(2\,z \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3} \end{array}\right)$$

### Classical cases involving ${}_0F_1$

03.06.26.0003.01

$$\mathrm{Bi}(z)\,{}_0F_1\left(;b;\frac{z^3}{9}\right) = \frac{2^{b-\frac{1}{3}}\sqrt{\pi}\,\Gamma(b)}{\sqrt[6]{3}}\, G_{4,6}^{2,2}\left(\frac{4\,z^3}{9} \,\middle|\, \begin{array}{c} \frac{1}{6}(4-3\,b), \frac{1}{6}(7-3\,b), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, 1-b, \frac{4}{3}-b \end{array}\right) /; -\frac{\pi}{3} < \arg(z) \le \frac{\pi}{3}$$

03.06.26.0022.01

$$\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z) = \frac{2^{b-\frac{1}{3}}\sqrt{\pi}\,\Gamma(b)}{\sqrt[6]{3}}\, G_{4,6}^{2,2}\left(4\,z \,\middle|\, \begin{array}{c} \frac{1}{6}(4-3\,b), \frac{1}{6}(7-3\,b), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, 1-b, \frac{4}{3}-b \end{array}\right)$$

### Classical cases involving ${}_0\tilde{F}_1$

03.06.26.0004.01

$$\mathrm{Bi}(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right) = \frac{2^{b-\frac{1}{3}}\sqrt{\pi}}{\sqrt[6]{3}}\, G_{4,6}^{2,2}\left(\frac{4\,z^3}{9} \,\middle|\, \begin{array}{c} \frac{1}{6}(4-3\,b), \frac{1}{6}(7-3\,b), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, 1-b, \frac{4}{3}-b \end{array}\right) /; -\frac{\pi}{3} < \arg(z) \le \frac{\pi}{3}$$

03.06.26.0023.01

$$\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z) = \frac{2^{b-\frac{1}{3}}\sqrt{\pi}}{\sqrt[6]{3}}\, G_{4,6}^{2,2}\left(4\,z \,\middle|\, \begin{array}{c} \frac{1}{6}(4-3\,b), \frac{1}{6}(7-3\,b), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, 1-b, \frac{4}{3}-b \end{array}\right)$$

### Generalized cases for the direct function itself

03.06.26.0005.01

$$\mathrm{Bi}(z) = \frac{2\,\pi}{\sqrt[6]{3}}\, G_{2,4}^{2,0}\left(3^{-2/3}\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3} \end{array}\right)$$

### Generalized cases involving exp

03.06.26.0006.01

$$\exp\left(-\frac{2}{3}z^{3/2}\right)\mathrm{Bi}(z) = \frac{1}{2^{2/3}\sqrt[6]{3}\,\sqrt{\pi}}\, G_{2,3}^{2,1}\left(\frac{2\sqrt[3]{2}\,z}{3^{2/3}}, \frac{2}{3} \,\middle|\, \begin{array}{c} \frac{5}{6}, \frac{1}{3} \\ 0, \frac{2}{3}, \frac{1}{3} \end{array}\right)$$

03.06.26.0007.01

$$\exp\left(\frac{2}{3}z^{3/2}\right)\mathrm{Bi}(z) = \frac{\sqrt[3]{2}\,\sqrt{\pi}}{\sqrt[6]{3}}\, G_{2,3}^{2,0}\left(3^{-2/3}\,2^{4/3}\,z, \frac{2}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3} \end{array}\right)$$

### Generalized cases involving cosh

03.06.26.0008.02

$$\cosh\left(\frac{2\,z^{3/2}}{3}\right)\mathrm{Bi}(z) = \sqrt[6]{\frac{2}{3}}\,\pi\, G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{5}{12}, \frac{11}{12}, \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6} \end{array}\right)$$

03.06.26.0031.01

$$\cosh(z)\,\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \sqrt[6]{\frac{2}{3}}\,\pi\,G_{4,6}^{2,2}\left(z, \frac{1}{2}\,\middle|\, \begin{matrix} \frac{5}{12}, \frac{11}{12}, \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6} \end{matrix}\right)$$

## Generalized cases involving sinh

03.06.26.0009.02

$$\sinh\left(\frac{2\,z^{3/2}}{3}\right)\mathrm{Bi}(z) = -\sqrt[6]{\frac{2}{3}}\,\pi\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{5}{12}, \frac{11}{12}, \frac{1}{6}, \frac{2}{3} \\ \frac{1}{2}, \frac{5}{6}, 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3} \end{matrix}\right)$$

03.06.26.0032.01

$$\sinh(z)\,\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = -\sqrt[6]{\frac{2}{3}}\,\pi\,G_{4,6}^{2,2}\left(z, \frac{1}{2}\,\middle|\, \begin{matrix} \frac{5}{12}, \frac{11}{12}, \frac{1}{6}, \frac{2}{3} \\ \frac{1}{2}, \frac{5}{6}, 0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3} \end{matrix}\right)$$

## Generalized cases for powers of Bi

03.06.26.0010.01

$$\mathrm{Bi}(z)^2 = \sqrt[3]{\frac{2}{3}}\,\sqrt{\pi}\left(G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{5}{6}, \frac{1}{2}, 1 \\ \frac{1}{3}, \frac{2}{3}, 0, \frac{1}{2}, 1 \end{matrix}\right) + G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{5}{6}, \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{2}{3} \end{matrix}\right)\right)$$

## Generalized cases involving Ai

03.06.26.0011.01

$$\mathrm{Ai}(z)\,\mathrm{Bi}(z) = \frac{1}{2\,2^{2/3}\sqrt[3]{3}\,\pi^{3/2}}\,G_{1,3}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3} \end{matrix}\right)$$

## Generalized cases involving Ai′

03.06.26.0012.01

$$\mathrm{Ai}'(z)\,\mathrm{Bi}(z) = -\frac{1}{4\,\pi^{3/2}}\,G_{1,3}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2} \\ \frac{1}{3}, \frac{2}{3}, 0 \end{matrix}\right) - \frac{1}{2\,\pi}$$

03.06.26.0013.01

$$\mathrm{Ai}'(z)\,\mathrm{Bi}(z) = \frac{\sqrt{3}}{4\,\pi^{3/2}}\,G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ -\frac{2}{3}, 0, \frac{2}{3}, \frac{1}{3} \end{matrix}\right) - 2\,G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{3}, 1, 0, \frac{2}{3} \end{matrix}\right)$$

03.06.26.0033.01

$$\mathrm{Ai}'(z)\,\mathrm{Bi}(z) = \frac{1}{4\,\pi^{3/2}}\,G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{3}, \frac{2}{3}, 0, 1 \end{matrix}\right) - \frac{1}{\sqrt{3}\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)}$$

## Generalized cases involving Bi′

03.06.26.0014.01

$$\mathrm{Bi}(z)\,\mathrm{Bi}'(z) = \frac{3}{4\,\pi^{3/2}}\,G_{1,3}^{3,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \frac{1}{3}, \frac{2}{3} \end{matrix}\right) + 2\sqrt{\pi}\,G_{2,4}^{2,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3}\,\middle|\, \begin{matrix} 1, \frac{1}{2} \\ \frac{2}{3}, \frac{1}{3}, 0, 1 \end{matrix}\right)$$

## Generalized cases involving ${}_0F_1$

03.06.26.0015.01

$$\mathrm{Bi}(z)\,{}_0F_1\left(;b;\frac{z^3}{9}\right)=\frac{2^{b-\frac{1}{3}}\sqrt{\pi}\,\Gamma(b)}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b),\frac{1}{6},\frac{2}{3}\\ 0,\frac{1}{3},\frac{1}{6},\frac{2}{3},1-b,\frac{4}{3}-b\end{matrix}\right)$$

03.06.26.0034.01

$$\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z)=\frac{2^{b-\frac{1}{3}}\sqrt{\pi}\,\Gamma(b)}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b),\frac{1}{6},\frac{2}{3}\\ 0,\frac{1}{3},\frac{1}{6},\frac{2}{3},1-b,\frac{4}{3}-b\end{matrix}\right)$$

### Generalized cases involving ${}_0\tilde{F}_1$

03.06.26.0016.01

$$\mathrm{Bi}(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=\frac{2^{b-\frac{1}{3}}\sqrt{\pi}}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b),\frac{1}{6},\frac{2}{3}\\ 0,\frac{1}{3},\frac{1}{6},\frac{2}{3},1-b,\frac{4}{3}-b\end{matrix}\right)$$

03.06.26.0035.01

$$\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z)=\frac{2^{b-\frac{1}{3}}\sqrt{\pi}}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b),\frac{1}{6},\frac{2}{3}\\ 0,\frac{1}{3},\frac{1}{6},\frac{2}{3},1-b,\frac{4}{3}-b\end{matrix}\right)$$

### Generalized cases involving Bessel $I$

03.06.26.0017.01

$$\mathrm{Bi}(z)\,I_\nu\left(\frac{2z^{3/2}}{3}\right)=\frac{2^{2/3}\sqrt{\pi}\,z^{-\frac{3\nu}{2}}\left(z^{3/2}\right)^\nu}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6},\frac{2}{3},\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4)\\ \frac{\nu}{2},\frac{1}{6}(3\nu+2),-\frac{\nu}{2},\frac{1}{6}(2-3\nu),\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4)\end{matrix}\right)$$

03.06.26.0024.01

$$\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)I_\nu(z)=\frac{2^{2/3}\sqrt{\pi}}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(z^{2/3},\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6},\frac{2}{3},\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4)\\ \frac{\nu}{2},\frac{1}{6}(3\nu+2),-\frac{\nu}{2},\frac{1}{6}(2-3\nu),\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4)\end{matrix}\right)$$

### Generalized cases involving Bessel $K$

03.06.26.0018.01

$$\mathrm{Bi}(z)\,K_\nu\left(\frac{2z^{3/2}}{3}\right)=\frac{\pi^{3/2}\csc(\pi\nu)}{\sqrt[3]{2}\,\sqrt[6]{3}}\left(G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6},\frac{2}{3},\frac{1}{6}(1-3\nu),\frac{1}{6}(4-3\nu)\\ -\frac{\nu}{2},\frac{1}{6}(2-3\nu),\frac{\nu}{2},\frac{1}{6}(3\nu+2),\frac{1}{6}(1-3\nu),\frac{1}{6}(4-3\nu)\end{matrix}\right)-\right.$$
$$\left.G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6},\frac{2}{3},\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4)\\ \frac{\nu}{2},\frac{1}{6}(3\nu+2),-\frac{\nu}{2},\frac{1}{6}(2-3\nu),\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4)\end{matrix}\right)\right)\;/;\;-\frac{2\pi}{3}<\arg(z)\le\frac{2\pi}{3}$$

03.06.26.0025.01

$$\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)K_\nu(z)=\frac{\pi^{3/2}\csc(\pi\nu)}{\sqrt[3]{2}\,\sqrt[6]{3}}\left(G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{6},\frac{2}{3},\frac{1}{6}(1-3\nu),\frac{1}{6}(4-3\nu)\\ -\frac{\nu}{2},\frac{1}{6}(2-3\nu),\frac{1}{6}(1-3\nu),\frac{1}{6}(4-3\nu),\frac{\nu}{2},\frac{1}{6}(3\nu+2)\end{matrix}\right)-\right.$$
$$\left.G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{6},\frac{2}{3},\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4)\\ \frac{\nu}{2},\frac{1}{6}(3\nu+2),-\frac{\nu}{2},\frac{1}{6}(2-3\nu),\frac{1}{6}(3\nu+1),\frac{1}{6}(3\nu+4)\end{matrix}\right)\right)$$

## Through other functions

#### Involving Bessel functions

03.06.26.0019.01

$$\mathrm{Bi}(z) = \sqrt{-\frac{z}{3}} \left( J_{-\frac{1}{3}}\left(\frac{2}{3}(-z)^{3/2}\right) - J_{\frac{1}{3}}\left(\frac{2}{3}(-z)^{3/2}\right) \right) /; \mathrm{Re}(z) \le 0$$

03.06.26.0020.01

$$\mathrm{Bi}(z) = \frac{\sqrt{z}}{\sqrt{3}} \left( I_{\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right) + I_{-\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right) \right) /; \mathrm{Re}(z) \ge 0$$

03.06.26.0021.01

$$\mathrm{Bi}(z) = \frac{1}{\sqrt{3}} \left( I_{-\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right) \sqrt[3]{z^{3/2}} + z\, I_{\frac{1}{3}}\left(\frac{2 z^{3/2}}{3}\right) \left(z^{3/2}\right)^{-\frac{1}{3}} \right)$$

## Representations through equivalent functions

### With related functions

03.06.27.0001.01

$$\mathrm{Bi}(z) = e^{\frac{\pi i}{6}} \mathrm{Ai}\left(e^{\frac{2\pi i}{3}} z\right) + e^{-\frac{\pi i}{6}} \mathrm{Ai}\left(e^{-\frac{2\pi i}{3}} z\right)$$

03.06.27.0002.01

$$\mathrm{Bi}(z) = 2 \sqrt[6]{-1}\, \mathrm{Ai}\left((-1)^{2/3} z\right) - i\, \mathrm{Ai}(z)$$

03.06.27.0003.01

$$\mathrm{Bi}(z) = i\, \mathrm{Ai}(z) - 2 (-1)^{5/6} \mathrm{Ai}\left(-\sqrt[3]{-1}\, z\right)$$

## Zeros

03.06.30.0001.01

$$\mathrm{Bi}(z) = 0 /; z = z_k \bigwedge k \in \mathbb{N}$$

03.06.30.0002.01

$$\mathrm{Im}(z_k) = 0 \bigwedge \mathrm{Re}(z_k) < 0 /; \mathrm{Bi}(z_k) = 0$$

On the real axis, Bi$(z)$ has an infinite number of zeros, all of which are negative.

03.06.30.0003.01

$$\frac{\pi}{3} < |\arg(z_k)| < \frac{\pi}{2} /; \mathrm{Bi}(z_k) = 0$$

The equation Bi$(x) = 0$ has only negative real solutions and solutions in the sector $\frac{\pi}{3} < |\mathrm{Arg}(x)| < \frac{\pi}{2}$.

## Theorems

### The general solution of the time-independent Schrödinger equation

The general solution to the time-independent Schrödinger equation of a particle in a constant potential $-\psi''(x) - F\, x\, \psi(x) = \varepsilon\, \psi(x)$ is given by $\psi(x) = c_1\, \mathrm{Ai}\left(F^{-2/3} (\varepsilon + F\, x)\right) + c_2\, \mathrm{Bi}\left(F^{-2/3} (\varepsilon + F\, x)\right)$.

## History

– G. B. Airy (1838), H. Jeffreys (1928, 1942)
– J. C. P. Miller (1946) suggested the notations Ai, Bi.

Applications of Bi include quantum mechanics of linear potential, electrodynamics, combinatorics, analysis of the complexity of algorithms, optical theory of the rainbow, solid state physics, and semiconductors in electric fields.

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