ArcTan2

Notations

Traditional name

Two-argument inverse tangent

Traditional notation

$\tan^{-1}(x, y)$

Mathematica StandardForm notation

`ArcTan[x, y]`

Primary definition

01.15.02.0001.01

$$\tan^{-1}(x, y) = -i \log\left(\frac{x + i y}{\sqrt{x^2 + y^2}}\right)$$

Specific values

Specialized values

For fixed *x*

01.15.03.0006.01

$$\tan^{-1}(x, 0) = (1 - \theta(x))\,\pi \;/;\; x \in \mathbb{R} \land x \neq 0$$

For fixed *y*

01.15.03.0001.01

$$\tan^{-1}(x, y) = \tan^{-1}\left(\frac{y}{x}\right) \;/;\; y \in \mathbb{R} \bigwedge -\frac{\pi}{2} < \arg(x) \leq \frac{\pi}{2}$$

01.15.03.0002.01

$$\tan^{-1}(x, y) = \tan^{-1}\left(\frac{y}{x}\right) + (2\,\theta(y) - 1)\,\pi \;/;\; y \in \mathbb{R} \bigwedge \operatorname{Re}(x) < 0$$

01.15.03.0003.01

$$\tan^{-1}(x, y) = \frac{\pi\left(2\sqrt{x^2} - x\right)}{4\,y} \;/;\; x^2 = y^2$$

01.15.03.0004.01

$$\tan^{-1}(\infty, y) == 0$$

01.15.03.0005.01

$$\tan^{-1}(-\infty, y) == (2\,\theta(\mathrm{Re}(y)) - 1)\,\pi$$

General characteristics

Domain and analyticity

$\tan^{-1}(x, y)$ is an analytical function of x and y, which is defined over $\mathbb{C}^2$.

01.15.04.0001.01

$$(x * y) \longrightarrow \tan^{-1}(x, y) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\tan^{-1}(x, y)$ is an even function.

01.15.04.0002.01

$$\tan^{-1}(-x, -y) == \tan^{-1}(x, y) \;/;\; y \in \mathbb{R} \bigwedge -\frac{\pi}{2} < \arg(x) \leq \frac{\pi}{2}$$

Mirror symmetry

01.15.04.0003.01

$$\tan^{-1}(\overline{x}, \overline{y}) == \overline{\tan^{-1}(x, y)}$$

Periodicity

No periodicity

Homogeneity

01.15.04.0020.01

$$\tan^{-1}(c\,x, c\,y) = \tan^{-1}(x, y) \;/;\; c > 0$$

01.15.04.0021.01

$$\tan^{-1}(c\,x, c\,y) = \tan^{-1}(x, y) \;/;\; c < 0 \bigwedge y \in \mathbb{R} \bigwedge -\frac{\pi}{2} < \arg(x) \leq \frac{\pi}{2}$$

Poles and essential singularities

With respect to *x*

The function $\tan^{-1}(x, y)$ does not have poles and essential singularities with respect to x.

01.15.04.0004.01

$$\mathcal{Sing}_x\left(\tan^{-1}(x, y)\right) == \{\}$$

With respect to *y*

The function $\tan^{-1}(x, y)$ does not have poles and essential singularities with respect to y.

01.15.04.0005.01

$$\mathcal{S}ing_y\left(\tan^{-1}(x, y)\right) == \{\}$$

Branch points

With respect to *y*

For fixed x the function $\tan^{-1}(x, y)$ has three branch points: $y = \pm i\, x,\ y == \tilde{\infty}$.

01.15.04.0006.01

$$\mathcal{BP}_y\left(\tan^{-1}(x, y)\right) == \{-i\, x, i\, x, \tilde{\infty}\}$$

01.15.04.0007.01

$$\mathcal{R}_y\left(\tan^{-1}(x, y), -i\, x\right) == \log$$

01.15.04.0008.01

$$\mathcal{R}_y\left(\tan^{-1}(x, y), i\, x\right) == \log$$

01.15.04.0009.01

$$\mathcal{R}_y\left(\tan^{-1}(x, y), \tilde{\infty}\right) == \log$$

With respect to *x*

For fixed y the function $\tan^{-1}(x, y)$ has three branch points: $x = \pm i\, y,\ x == \tilde{\infty}$.

01.15.04.0010.01

$$\mathcal{BP}_x\left(\tan^{-1}(x, y)\right) == \{-i\, y, i\, y, \tilde{\infty}\}$$

01.15.04.0011.01

$$\mathcal{R}_x\left(\tan^{-1}(x, y), -i\, y\right) == \log$$

01.15.04.0012.01

$$\mathcal{R}_x\left(\tan^{-1}(x, y), i\, y\right) == \log$$

01.15.04.0013.01

$$\mathcal{R}_x\left(\tan^{-1}(x, y), \tilde{\infty}\right) == \log$$

Branch cuts

With respect to *y*

For fixed x, the branch cuts lie along three curves: $\left(y == \sqrt{t - x^2},\ -\infty < t \leq 0\right)$, $\left(y == -\sqrt{t - x^2},\ -\infty < t \leq 0\right)$, and $(y == \theta(-\text{Re}(x))\, i\, x\, t,\ -1 \leq t \leq 1)$.

The function $\tan^{-1}(x, y)$ is continuous from the left for $y > -2$ and continuous from the right for $y < -2$ for $x \in (-\infty, 0)$.

01.15.04.0014.01

$$\mathcal{BC}_y\left(\tan^{-1}(x, y)\right) == \{\{(-i\, \infty, i\, \infty), -1\}\} /; x < 0 \bigwedge y > -2$$

01.15.04.0015.01

$$\mathcal{BC}_y\left(\tan^{-1}(x, y)\right) == \{\{(-i\, \infty, i\, \infty), 1\}\} /; x < 0 \bigwedge y < -2$$

The function $\tan^{-1}(x, y)$ is continuous from the right for $y > 2$ and continuous from the left for $y < -2$ for $x \in (0, \infty)$.

01.15.04.0016.01

$$\mathcal{BC}_y\left(\tan^{-1}(x, y)\right) = \{\{(-i\infty, i\infty), 1\}\} /; x < 0 \bigwedge y > 2$$

01.15.04.0017.01

$$\mathcal{BC}_y\left(\tan^{-1}(x, y)\right) = \{\{(-i\infty, i\infty), -1\}\} /; x < 0 \bigwedge y < -2$$

01.15.04.0018.01

$$\lim_{\epsilon \to +0} \tan^{-1}(x, \epsilon) = \tan^{-1}(x, 0) = \pi /; x < 0$$

01.15.04.0019.01

$$\lim_{\epsilon \to +0} \tan^{-1}(x, -\epsilon) = -\pi /; x < 0$$

With respect to *x*

For fixed y, the branch cuts lie along three curves: $\left(x = \sqrt{t - y^2},\ -\infty < t \le 0\right)$, $\left(x = -\sqrt{t - y^2},\ -\infty < t \le 0\right)$, and $(x = \operatorname{sign}(\operatorname{Im}(y))\, i\, y\, t,\ 1 \le t < \infty)$.

Analytic continuations

The analytic continuation of $\tan^{-1}$ has infinitely many sheets; the values of $\tilde{\tan}^{-1}$ are $\tilde{\tan}^{-1}(z) = \tan^{-1}(z) + k\pi /; k \in \mathbb{Z}$.

Series representations

Generalized power series

Expansions at $x = 0$

For the function itself

01.15.06.0001.02

$$\tan^{-1}(x, y) \propto -i \log\left(\frac{i y}{\sqrt{y^2}}\right) - \frac{x}{y} + \frac{x^3}{3 y^3} - \frac{x^5}{5 y^5} + \ldots /; \left(\frac{x}{y} \to 0\right) \bigwedge y \notin \mathbb{R} \bigwedge i y \notin \mathbb{R}$$

01.15.06.0017.01

$$\tan^{-1}(x, y) \propto -i \log\left(\frac{i y}{\sqrt{y^2}}\right) - \frac{x}{y} + \frac{x^3}{3 y^3} - \frac{x^5}{5 y^5} + O\left(\frac{x^7}{y^7}\right) /; y \notin \mathbb{R} \bigwedge i y \notin \mathbb{R}$$

01.15.06.0002.01

$$\tan^{-1}(x, y) = -i \log\left(\frac{i y}{\sqrt{y^2}}\right) - \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \left(\frac{x}{y}\right)^{2k+1} /; |x| < |y| \bigwedge y \notin \mathbb{R} \bigwedge i y \notin \mathbb{R}$$

01.15.06.0003.01

$$\tan^{-1}(x, y) = -i \log\left(\frac{i y}{\sqrt{y^2}}\right) - \frac{x}{y}\, {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{x^2}{y^2}\right) /; |x| < |y| \bigwedge y \notin \mathbb{R} \bigwedge i y \notin \mathbb{R}$$

01.15.06.0004.02

$$\tan^{-1}(x, y) \propto -i \log\left(\frac{i\,y}{\sqrt{y^2}}\right) - \frac{x}{y} + O\left(\frac{x^3}{y^3}\right) /; y \notin \mathbb{R} \bigwedge i\,y \notin \mathbb{R}$$

Expansions at $x == \infty$

For the function itself

01.15.06.0005.02

$$\tan^{-1}(x, y) \propto -i \log\left(\frac{x}{\sqrt{x^2}}\right) + \frac{y}{x} - \frac{y^3}{3\,x^3} + \frac{y^5}{5\,x^5} - \ldots /; \left(\frac{y}{x} \to 0\right) \bigwedge y \in \mathbb{R} \bigwedge \operatorname{Re}(x) \geq 0$$

01.15.06.0018.01

$$\tan^{-1}(x, y) \propto -i \log\left(\frac{x}{\sqrt{x^2}}\right) + \frac{y}{x} - \frac{y^3}{3\,x^3} + \frac{y^5}{5\,x^5} + O\left(\frac{y^7}{x^7}\right) /; y \in \mathbb{R} \bigwedge \operatorname{Re}(x) \geq 0$$

01.15.06.0006.01

$$\tan^{-1}(x, y) == -i \log\left(\frac{x}{\sqrt{x^2}}\right) + \sum_{k=0}^{\infty} \frac{(-1)^k}{2\,k+1} \left(\frac{y}{x}\right)^{2\,k+1} /; |y| < |x| \bigwedge y \in \mathbb{R} \bigwedge \operatorname{Re}(x) \geq 0$$

01.15.06.0007.01

$$\tan^{-1}(x, y) == -i \log\left(\frac{x}{\sqrt{x^2}}\right) + \frac{y}{x}\, {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{y^2}{x^2}\right) /; |y| < |x| \bigwedge y \in \mathbb{R} \bigwedge \operatorname{Re}(x) \geq 0$$

01.15.06.0008.02

$$\tan^{-1}(x, y) \propto -i \log\left(\frac{x}{\sqrt{x^2}}\right) + \frac{y}{x} + O\left(\frac{y^3}{x^3}\right) /; y \in \mathbb{R} \bigwedge \operatorname{Re}(x) \geq 0$$

Expansions at $y == 0$

For the function itself

01.15.06.0009.02

$$\tan^{-1}(x, y) \propto \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\;x} - 1\right)(\log(-y) - \log(y)) + \frac{y}{x} - \frac{y^3}{3\,x^3} + \frac{y^5}{5\,x^5} - \ldots /; \left(\frac{y}{x} \to 0\right) \bigwedge y \in \mathbb{R}$$

01.15.06.0019.01

$$\tan^{-1}(x, y) \propto \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\;x} - 1\right)(\log(-y) - \log(y)) + \frac{y}{x} - \frac{y^3}{3\,x^3} + \frac{y^5}{5\,x^5} + O\left(\frac{y^7}{x^7}\right) /; y \in \mathbb{R}$$

01.15.06.0010.01

$$\tan^{-1}(x, y) == \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\left(\frac{y}{x}\right)^{2k+1} /; |y| < |x| \bigwedge y \in \mathbb{R}$$

01.15.06.0011.01

$$\tan^{-1}(x, y) == \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \frac{y}{x}\, {}_2F_1\!\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{y^2}{x^2}\right) /; |y| < |x| \bigwedge y \in \mathbb{R}$$

01.15.06.0012.02

$$\tan^{-1}(x, y) \propto \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \frac{y}{x} + O\!\left(\frac{y^3}{x^3}\right) /; y \in \mathbb{R}$$

Expansions at $y == \infty$

For the function itself

01.15.06.0013.01

$$\tan^{-1}(x, y) \propto \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \frac{\pi x}{2y}\sqrt{\frac{y^2}{x^2}} - \frac{x}{y} + \frac{x^3}{3y^3} - \frac{x^5}{5y^5} + \ldots /; \left(\frac{x}{y} \to 0\right) \bigwedge y \in \mathbb{R}$$

01.15.06.0020.01

$$\tan^{-1}(x, y) \propto \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \frac{\pi x}{2y}\sqrt{\frac{y^2}{x^2}} - \frac{x}{y} + \frac{x^3}{3y^3} - \frac{x^5}{5y^5} + O\!\left(\frac{x^7}{y^7}\right) /; y \in \mathbb{R}$$

01.15.06.0014.01

$$\tan^{-1}(x, y) == \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \frac{\pi x}{2y}\sqrt{\frac{y^2}{x^2}} - \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\left(\frac{x}{y}\right)^{2k+1} /; |y| > |x| \bigwedge y \in \mathbb{R}$$

01.15.06.0015.01

$$\tan^{-1}(x, y) == \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \frac{\pi x}{2y}\sqrt{\frac{y^2}{x^2}} - \frac{x}{y}\, {}_2F_1\!\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{x^2}{y^2}\right) /; |y| > |x| \bigwedge y \in \mathbb{R}$$

01.15.06.0016.02

$$\tan^{-1}(x, y) \propto \frac{i}{2}\left(\frac{1}{x\sqrt{\frac{1}{x^2}}} - 1\right)(\log(-y) - \log(y)) + \sqrt{\frac{y^2}{x^2}}\,\frac{\pi x}{2y} - \frac{x}{y} + O\left(\frac{x^3}{y^3}\right) /; y \in \mathbb{R}$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

01.15.13.0001.01

$$\left(z_1^2 + z_2^2\right)\frac{\partial^2 w(z_1, z_2)}{\partial z_1^2} + 2\frac{\partial w(z_1, z_2)}{\partial z_1} z_1 == 0 /; w(z_1, z_2) == \tan^{-1}(z_1, z_2)$$

01.15.13.0002.01

$$\left(z_1^2 + z_2^2\right)\frac{\partial^2 w(z_1, z_2)}{\partial z_2^2} + 2\frac{\partial w(z_1, z_2)}{\partial z_2} z_2 == 0 /; w(z_1, z_2) == \tan^{-1}(z_1, z_2)$$

Partial differential equations

01.15.13.0003.01

$$\frac{\partial^2 w(z_1, z_2)}{\partial z_1^2} + \frac{\partial^2 w(z_1, z_2)}{\partial z_2^2} == 0 /; w(z_1, z_2) == \tan^{-1}(z_1, z_2)$$

01.15.13.0004.01

$$z_1\frac{\partial w(z_1, z_2)}{\partial z_2} - z_2\frac{\partial w(z_1, z_2)}{\partial z_1} - 1 == 0 /; w(z_1, z_2) == \tan^{-1}(z_1, z_2)$$

Transformations

Transformations and argument simplifications

01.15.16.0001.01

$$\tan^{-1}(c\,x, c\,y) = \tan^{-1}(x, y) /; c > 0$$

01.15.16.0002.01

$$\tan^{-1}(c\,x, c\,y) = \tan^{-1}(x, y) /; c < 0 \bigwedge y \in \mathbb{R} \bigwedge -\frac{\pi}{2} < \arg(x) \leq \frac{\pi}{2}$$

Complex characteristics

Real part

01.15.19.0001.01

$$\operatorname{Re}\left(\tan^{-1}(x, y)\right) = \tan^{-1}\left(\left((\operatorname{Im}(x)+\operatorname{Re}(y))\sin\left(\frac{1}{2}\tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\right)\right)\right)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right) + \left(\cos\left(\frac{1}{2}\tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\right)\right)(\operatorname{Re}(x)-\operatorname{Im}(y))\right)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right), \left(\cos\left(\frac{1}{2}\tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\right)\right)(\operatorname{Im}(x)+\operatorname{Re}(y))\right)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right) - \left((\operatorname{Re}(x)-\operatorname{Im}(y))\sin\left(\frac{1}{2}\tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\right)\right)\right)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right)\right)$$

Imaginary part

01.15.19.0002.01

$$\operatorname{Im}\left(\tan^{-1}(x, y)\right) = -\log\left(\frac{\sqrt{(\operatorname{Re}(x)-\operatorname{Im}(y))^2+(\operatorname{Im}(x)+\operatorname{Re}(y))^2}}{\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}}\right)$$

Absolute value

01.15.19.0003.01

$$\left|\tan^{-1}(x, y)\right| = \sqrt{\left(\tan^{-1}\left(\left((\mathrm{Im}(x)+\mathrm{Re}(y))\sin\left(\frac{1}{2}\tan^{-1}\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2,\ 2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y)\right)\right)\right)\Big/\left(\sqrt[4]{(2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y))^2+\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2\right)^2}\right)+\left(\cos\left(\frac{1}{2}\tan^{-1}\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2,\ 2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y)\right)\right)(\mathrm{Re}(x)-\mathrm{Im}(y))\right)\Big/\left(\sqrt[4]{(2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y))^2+\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2\right)^2}\right),\ \left(\cos\left(\frac{1}{2}\tan^{-1}\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2,\ 2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y)\right)\right)(\mathrm{Im}(x)+\mathrm{Re}(y))\right)\Big/\left(\sqrt[4]{(2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y))^2+\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2\right)^2}\right)-\left((\mathrm{Re}(x)-\mathrm{Im}(y))\sin\left(\frac{1}{2}\tan^{-1}\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2,\ 2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y)\right)\right)\right)\Big/\left(\sqrt[4]{(2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y))^2+\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2\right)^2}\right)\right)^2+\log^2\left(\frac{\sqrt{(\mathrm{Re}(x)-\mathrm{Im}(y))^2+(\mathrm{Im}(x)+\mathrm{Re}(y))^2}}{\sqrt[4]{(2\,\mathrm{Im}(x)\,\mathrm{Re}(x)+2\,\mathrm{Im}(y)\,\mathrm{Re}(y))^2+\left(-\mathrm{Im}(x)^2-\mathrm{Im}(y)^2+\mathrm{Re}(x)^2+\mathrm{Re}(y)^2\right)^2}}\right)\right)}$$

01.15.19.0004.01

$$\left|\tan^{-1}(x+i\,y)\right| = \frac{1}{2}\sqrt{\left(\tan^{-1}(1-y,\ x)-\tan^{-1}(y+1,\ -x)\right)^2+\frac{1}{4}\left(\log\left(x^2+(y-1)^2\right)-\log\left(x^2+(y+1)^2\right)\right)^2}$$

Argument

01.15.19.0005.01

$$\arg\left(\tan^{-1}(x, y)\right) = \tan^{-1}\left(\tan^{-1}\left(\left((\operatorname{Im}(x)+\operatorname{Re}(y)) \sin\left(\frac{1}{2} \tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y)\right)\right)\right) \Big/ \left(\sqrt[4]{(2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right) + \left(\cos\left(\frac{1}{2} \tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y)\right)\right) (\operatorname{Re}(x)-\operatorname{Im}(y))\right) \Big/ \left(\sqrt[4]{(2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right),\ \left(\cos\left(\frac{1}{2} \tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y)\right)\right) (\operatorname{Im}(x)+\operatorname{Re}(y))\right) \Big/ \left(\sqrt[4]{(2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right) - \left((\operatorname{Re}(x)-\operatorname{Im}(y)) \sin\left(\frac{1}{2} \tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y)\right)\right)\right) \Big/ \left(\sqrt[4]{(2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right)\right),\ -\log\left(\frac{\sqrt{(\operatorname{Re}(x)-\operatorname{Im}(y))^2+(\operatorname{Im}(x)+\operatorname{Re}(y))^2}}{\sqrt[4]{(2 \operatorname{Im}(x) \operatorname{Re}(x)+2 \operatorname{Im}(y) \operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}}\right)\right)$$

Conjugate value

01.15.19.0006.01

$$\overline{\tan^{-1}(x, y)} = \tan^{-1}\Bigg(\Bigg((\operatorname{Im}(x)+\operatorname{Re}(y))\sin\Big(\frac{1}{2}\tan^{-1}\big(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\big)\Big)\Bigg)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right) + \left(\cos\Big(\frac{1}{2}\tan^{-1}\big(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\big)\Big)(\operatorname{Re}(x)-\operatorname{Im}(y))\right)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right), \left(\cos\Big(\frac{1}{2}\tan^{-1}\big(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\big)\Big)(\operatorname{Im}(x)+\operatorname{Re}(y))\right)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right) - \left((\operatorname{Re}(x)-\operatorname{Im}(y))\sin\Big(\frac{1}{2}\tan^{-1}\big(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\big)\Big)\right)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right)\Bigg) + i\log\left(\frac{\sqrt{(\operatorname{Re}(x)-\operatorname{Im}(y))^2+(\operatorname{Im}(x)+\operatorname{Re}(y))^2}}{\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}}\right)$$

Signum value

01.15.19.0007.01

$$\operatorname{sgn}\left(\tan^{-1}(x, y)\right) = \Bigg(\tan^{-1}\Bigg(\frac{(\operatorname{Im}(x)+\operatorname{Re}(y))\sin\left(\frac{1}{2}\tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\right)\right)}{\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}} + \left(\cos\Big(\frac{1}{2}\tan^{-1}\big(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\big)\Big)(\operatorname{Re}(x)-\operatorname{Im}(y))\right)\Big/ \left(\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}\right), \frac{\cos\left(\frac{1}{2}\tan^{-1}\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\right)\right)(\operatorname{Im}(x)+\operatorname{Re}(y))}{\sqrt[4]{(2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y))^2+\left(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2\right)^2}} - \left((\operatorname{Re}(x)-\operatorname{Im}(y))\sin\Big(\frac{1}{2}\tan^{-1}\big(-\operatorname{Im}(x)^2-\operatorname{Im}(y)^2+\operatorname{Re}(x)^2+\operatorname{Re}(y)^2,\ 2\operatorname{Im}(x)\operatorname{Re}(x)+2\operatorname{Im}(y)\operatorname{Re}(y)\big)\Big)\right)\Big/$$

$$\left.\left.\left(\sqrt[4]{(2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y))^2+\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2\right)^2}\right)\right)\right)-$$

$$i\log\left(\frac{\sqrt{(\text{Re}(x)-\text{Im}(y))^2+(\text{Im}(x)+\text{Re}(y))^2}}{\sqrt[4]{(2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y))^2+\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2\right)^2}}\right)\Bigg)\Bigg/$$

$$\left(\sqrt{\left(\tan^{-1}\left(\left((\text{Im}(x)+\text{Re}(y))\sin\left(\frac{1}{2}\tan^{-1}\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2,\;2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y)\right)\right)\right)\right/\right.\right.}\right.$$

$$\left(\sqrt[4]{(2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y))^2+\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2\right)^2}\right)+$$

$$\left(\cos\left(\frac{1}{2}\tan^{-1}\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2,\;2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y)\right)\right)(\text{Re}(x)-\text{Im}(y))\right)\Bigg/$$

$$\left(\sqrt[4]{(2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y))^2+\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2\right)^2}\right),$$

$$\left(\cos\left(\frac{1}{2}\tan^{-1}\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2,\;2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y)\right)\right)(\text{Im}(x)+\text{Re}(y))\right)\Bigg/$$

$$\left(\sqrt[4]{(2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y))^2+\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2\right)^2}\right)-$$

$$\left((\text{Re}(x)-\text{Im}(y))\sin\left(\frac{1}{2}\tan^{-1}\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2,\;2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y)\right)\right)\right)\Bigg/$$

$$\left.\left.\left(\sqrt[4]{(2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y))^2+\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2\right)^2}\right)\right)\right)^2+$$

$$\left.\left.\left.\log^2\left(\frac{\sqrt{(\text{Re}(x)-\text{Im}(y))^2+(\text{Im}(x)+\text{Re}(y))^2}}{\sqrt[4]{(2\,\text{Im}(x)\,\text{Re}(x)+2\,\text{Im}(y)\,\text{Re}(y))^2+\left(-\text{Im}(x)^2-\text{Im}(y)^2+\text{Re}(x)^2+\text{Re}(y)^2\right)^2}}\right)\right)\right)\right)$$

Differentiation

Low-order differentiation

With respect to *x*

01.15.20.0001.01

$$\frac{\partial \tan^{-1}(x, y)}{\partial x} == -\frac{y}{x^2 + y^2}$$

01.15.20.0002.01

$$\frac{\partial^2 \tan^{-1}(x, y)}{\partial x^2} == \frac{2\, x\, y}{\left(x^2 + y^2\right)^2}$$

With respect to *y*

01.15.20.0003.01

$$\frac{\partial \tan^{-1}(x, y)}{\partial y} == \frac{x}{x^2 + y^2}$$

01.15.20.0004.01

$$\frac{\partial^2 \tan^{-1}(x, y)}{\partial y^2} == -\frac{2\, x\, y}{\left(x^2 + y^2\right)^2}$$

Symbolic differentiation

With respect to *x*

01.15.20.0009.01

$$\frac{\partial^n \tan^{-1}(x, y)}{\partial x^n} == \delta_n \tan^{-1}(x, y) - y\left(x^2 + y^2\right)^{-n} \sum_{k=0}^{n-1} \frac{(-1)^k\, k!\, (2\,k - n + 2)_{2\,(n-k)-2} \left(x^2 + y^2\right)^{-k+n-1}}{(-k + n - 1)!\, (2\,x)^{-2\,k+n-1}} \;/;\; n \in \mathbb{N}$$

01.15.20.0010.01

$$\frac{\partial^n \tan^{-1}(x, y)}{\partial x^n} == \frac{i\, (-1)^n\, (n-1)!}{2} \left((x + i\, y)^{-n} - (x - i\, y)^{-n}\right) \;/;\; n \in \mathbb{N}^+$$

01.15.20.0011.01

$$\frac{\partial^n \tan^{-1}(x, y)}{\partial x^n} == \frac{(n-1)!\, y^n}{\left(x^2 + y^2\right)^n} \sum_{k=0}^{n} \binom{n}{k} \cos\left(\frac{1}{2}\, \pi\, (k + n + 1)\right) \left(\frac{x}{y}\right)^k \;/;\; n \in \mathbb{N}^+$$

01.15.20.0005.02

$$\frac{\partial^n \tan^{-1}(x, y)}{\partial x^n} == -\frac{2^{n-1} \sqrt{\pi}\; x^{1-n}}{y}\, {}_3\tilde{F}_2\left(\frac{1}{2}, 1, 1;\, 1 - \frac{n}{2}, \frac{3-n}{2};\, -\frac{x^2}{y^2}\right) \;/;\; n \in \mathbb{N}$$

With respect to *y*

01.15.20.0012.01

$$\frac{\partial^n \tan^{-1}(x, y)}{\partial y^n} == \tan^{-1}(x, y)\, \delta_n + x\left(x^2 + y^2\right)^{-n} \sum_{k=0}^{n-1} \frac{\left((-1)^k\, k!\, (2\,k - n + 2)_{2\,(n-k)-2}\right) \left(x^2 + y^2\right)^{-k+n-1}}{(-k + n - 1)!\, (2\,y)^{-2\,k+n-1}} \;/;\; n \in \mathbb{N}$$

01.15.20.0013.01

$$\frac{\partial^n \tan^{-1}(x, y)}{\partial y^n} == \frac{i\, (-1)^n\, (n-1)!}{2} \left((y - i\, x)^{-n} - (i\, x + y)^{-n}\right) \;/;\; n \in \mathbb{N}^+$$

01.15.20.0014.01

$$\frac{\partial^n \tan^{-1}(x, y)}{\partial y^n} == -\frac{(n-1)!\, x^n}{\left(x^2+y^2\right)^n} \sum_{k=0}^{n} \binom{n}{k} \cos\left(\frac{1}{2}\pi\,(k+n+1)\right) \left(\frac{y}{x}\right)^k /;\ n \in \mathbb{N}^+$$

01.15.20.0006.02

$$\frac{\partial^n \tan^{-1}(x, y)}{\partial y^n} == \frac{2^{n-1}\sqrt{\pi}\ y^{1-n}}{x}\ {}_3\tilde{F}_2\left(\frac{1}{2}, 1, 1; 1-\frac{n}{2}, \frac{3-n}{2}; -\frac{y^2}{x^2}\right) /;\ n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to *x*

01.15.20.0007.01

$$\frac{\partial^\alpha \tan^{-1}(x, y)}{\partial x^\alpha} == -\frac{i}{\Gamma(1-\alpha)} \log\left(\frac{i\, y}{\sqrt{y^2}}\right) x^{-\alpha} - \frac{2^{\alpha-1}\sqrt{\pi}\ x^{1-\alpha}}{y}\ {}_3\tilde{F}_2\left(\frac{1}{2}, 1, 1; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{x^2}{y^2}\right) /;\ |x| < |y| \bigwedge y \notin \mathbb{R} \bigwedge i\, y \notin \mathbb{R}$$

With respect to *y*

01.15.20.0008.01

$$\frac{\partial^\alpha \tan^{-1}(x, y)}{\partial y^\alpha} == \frac{i\, y^{-\alpha}}{2\,\Gamma(1-\alpha)} \left(\frac{1}{\sqrt{\frac{1}{x^2}}\ x} - 1\right) (\log(-y) - \log(y)) + \frac{2^{\alpha-1}\sqrt{\pi}\ y^{1-\alpha}}{x}\ {}_3\tilde{F}_2\left(\frac{1}{2}, 1, 1; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{y^2}{x^2}\right) /;$$

$$|y| < |x| \bigwedge y \in \mathbb{R}$$

Integration

Indefinite integration

With respect to *x*

01.15.21.0001.01

$$\int \tan^{-1}(x, y)\, dx == x \tan^{-1}(x, y) + \frac{1}{2}\, y \log\left(x^2+y^2\right)$$

01.15.21.0002.01

$$\int x^{\alpha-1} \tan^{-1}(x, y)\, dx == \frac{x^\alpha}{\alpha} \tan^{-1}(x, y) + \frac{x^{\alpha+1}}{y\,\alpha\,(\alpha+1)}\ {}_2F_1\left(\frac{\alpha+1}{2}, 1; \frac{\alpha+3}{2}; -\frac{x^2}{y^2}\right)$$

With respect to *y*

01.15.21.0003.01

$$\int \tan^{-1}(x, y)\, dy == y \tan^{-1}(x, y) - \frac{1}{2}\, x \log\left(x^2+y^2\right)$$

01.15.21.0004.01

$$\int y^{\alpha-1} \tan^{-1}(x, y)\, dy == \frac{y^\alpha}{\alpha} \tan^{-1}(x, y) - \frac{y^{\alpha+1}}{x\,\alpha\,(\alpha+1)}\ {}_2F_1\left(\frac{\alpha+1}{2}, 1; \frac{\alpha+3}{2}; -\frac{y^2}{x^2}\right)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_2F_1$

01.15.26.0001.01

$$\tan^{-1}(x, y) = -i \log\left(\frac{i\, y}{\sqrt{y^2}}\right) - \frac{x}{y}\, {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{x^2}{y^2}\right) /; |x| < |y| \bigwedge y \notin \mathbb{R} \bigwedge i\, y \notin \mathbb{R}$$

01.15.26.0002.01

$$\tan^{-1}(x, y) = -i \log\left(\frac{x}{\sqrt{x^2}}\right) + \frac{y}{x}\, {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{y^2}{x^2}\right) /; |y| < |x| \bigwedge y \in \mathbb{R} \bigwedge \mathrm{Re}(x) \geq 0$$

01.15.26.0003.01

$$\tan^{-1}(x, y) = \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \frac{y}{x}\, {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{y^2}{x^2}\right) /; |y| < |x| \bigwedge y \in \mathbb{R}$$

01.15.26.0004.01

$$\tan^{-1}(x, y) = \frac{i}{2}\left(\frac{1}{\sqrt{\frac{1}{x^2}}\, x} - 1\right)(\log(-y) - \log(y)) + \frac{\pi\, x}{2\, y} \sqrt{\frac{y^2}{x^2}} - \frac{x}{y}\, {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{x^2}{y^2}\right) /; |y| > |x| \bigwedge y \in \mathbb{R}$$

Through other functions

01.15.26.0005.01

$$\tan^{-1}(x, y) = \arg(x + i\, y) /; x \in \mathbb{R} \bigwedge y \in \mathbb{R}$$

Representations through equivalent functions

With related functions

Involving log

01.15.27.0001.01

$$\tan^{-1}(x, y) = -i \log\left(\frac{x + i\, y}{\sqrt{x^2 + y^2}}\right)$$

01.15.27.0002.01

$$\tan^{-1}(x, y) = \arg\left(\frac{x + i\, y}{\sqrt{x^2 + y^2}}\right) - i \log\left(\frac{|x + i\, y|}{\sqrt{|x^2 + y^2|}}\right)$$

Involving $\tan^{-1}$

01.15.27.0006.01

$$\tan^{-1}(x,\,y) = \tan^{-1}\left(\frac{y}{x}\right) + \frac{\pi}{2}\,(1-\operatorname{sgn}(x))\operatorname{sgn}\left(\operatorname{sgn}(y)+\frac{1}{2}\right) /;\; x\in\mathbb{R} \bigwedge y\in\mathbb{R} \bigwedge x\neq 0$$

01.15.27.0003.01

$$\tan^{-1}(x,\,y) = \tan^{-1}\left(\frac{y}{x}\right) /;\; y\in\mathbb{R} \bigwedge -\frac{\pi}{2} < \arg(x) \leq \frac{\pi}{2}$$

01.15.27.0004.01

$$\tan^{-1}(x,\,y) = \tan^{-1}\left(\frac{y}{x}\right) + (2\,\theta(y)-1)\,\pi /;\; y\in\mathbb{R} \bigwedge \operatorname{Re}(x) < 0$$

01.15.27.0005.01

$$\tan^{-1}(x,\,y) = \tan^{-1}\left(\frac{y}{x}\right) + \frac{i}{2}\left(\frac{1}{x\sqrt{\frac{1}{x^2}}}-1\right)(\log(-y)-\log(y)) /;\; \operatorname{Re}(x)\neq 0 \bigwedge y\in\mathbb{R}$$

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