BesselI

Notations

Traditional name

Modified Bessel function of the first kind

Traditional notation

$I_\nu(z)$

Mathematica StandardForm notation

`BesselI[ν, z]`

Primary definition

03.02.02.0001.01

$$I_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\nu+1)\, k!} \left(\frac{z}{2}\right)^{2k+\nu}$$

Specific values

Specialized values

For fixed ν

03.02.03.0001.01

$I_\nu(0) = 0 \;/;\; \mathrm{Re}(\nu) > 0 \lor \nu \in \mathbb{Z}$

03.02.03.0002.01

$I_\nu(0) = \tilde{\infty} \;/;\; \mathrm{Re}(\nu) < 0 \bigwedge \nu \notin \mathbb{Z}$

03.02.03.0003.01

$I_\nu(0) = ¿ \;/;\; \mathrm{Re}(\nu) = 0 \land \nu \neq 0$

For fixed z

Explicit rational ν

03.02.03.0015.01

$$I_{-\frac{14}{3}}(z) = \frac{1}{81\, 3^{5/6}\, z^{14/3}} \left(1760 \sqrt[3]{2} \left(9 z^{4/3} \left(\frac{9 z^2}{110}+1\right)\left(\sqrt{3}\, \text{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \text{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - 4 \sqrt[3]{2} \sqrt[6]{3} \left(\frac{81 z^4}{14080} + \frac{27 z^2}{88} + 1\right)\left(3 \text{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \sqrt{3}\, \text{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.02.03.0016.01

$$I_{-\frac{9}{2}}(z) = \sqrt{\frac{2}{\pi}} \frac{\left(z^4 + 45 z^2 + 105\right) \cosh(z) - 5 z \left(2 z^2 + 21\right) \sinh(z)}{z^{9/2}}$$

03.02.03.0017.01

$$I_{-\frac{13}{3}}(z) = -\frac{1}{27\, 3^{5/6}\, z^{13/3}} \left(1120\, 2^{2/3} \left(\sqrt[6]{3}\, z^{2/3} \left(\frac{9 z^2}{80}+1\right)\left(3 \text{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\, \text{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - \frac{81 z^4 + 3024 z^2 + 4480}{2240 \sqrt[3]{2}} \left(\sqrt{3}\, \text{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \text{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.02.03.0018.01

$$I_{-\frac{11}{3}}(z) = -\frac{1}{27\, 3^{5/6}\, z^{11/3}} \left(80 \sqrt[3]{2} \left(9 \left(\frac{9 z^2}{160}+1\right)\left(\sqrt{3}\, \text{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \text{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{4/3} + \frac{\sqrt[6]{3}\left(9 z^2 + 32\right)}{4\, 2^{2/3}} \left(\sqrt{3}\, \text{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3 \text{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.02.03.0019.01

$$I_{-\frac{7}{2}}(z) = \sqrt{\frac{2}{\pi}} \frac{z \left(z^2 + 15\right) \sinh(z) - 3 \left(2 z^2 + 5\right) \cosh(z)}{z^{7/2}}$$

03.02.03.0020.01

$$I_{-\frac{10}{3}}(z) = \frac{56\, 2^{2/3}}{9\, 3^{5/6}\, z^{10/3}} \left(\sqrt[6]{3}\, z^{2/3} \left(\frac{9 z^2}{112}+1\right)\left(3 \text{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\, \text{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - \frac{9 z^2 + 14}{7 \sqrt[3]{2}} \left(\sqrt{3}\, \text{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \text{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.02.03.0021.01

$$I_{-\frac{8}{3}}(z) = \frac{5 \sqrt[3]{2}}{9\, 3^{5/6}\, z^{8/3}} \left(9 \left(\sqrt{3}\, \text{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \text{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{4/3} + \frac{\sqrt[6]{3}\left(9 z^2 + 40\right)}{5\, 2^{2/3}} \left(\sqrt{3}\, \text{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3 \text{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.02.03.0022.01

$$I_{-\frac{5}{2}}(z) = \sqrt{\frac{2}{\pi}} \frac{\left(z^2 + 3\right) \cosh(z) - 3 z \sinh(z)}{z^{5/2}}$$

03.02.03.0023.01

$$I_{-\frac{7}{3}}(z) = -\frac{4\, 2^{2/3}}{3\, 3^{5/6}\, z^{7/3}} \left(\sqrt[6]{3}\, z^{2/3} \left(3 \text{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\, \text{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - \frac{9 z^2 + 16}{8 \sqrt[3]{2}} \left(\sqrt{3}\, \text{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \text{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.02.03.0024.01

$$I_{-\frac{5}{3}}(z) = -\frac{1}{3\,2^{2/3}\,3^{5/6}\,z^{5/3}}\left(9\left(\sqrt{3}\,\operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{4/3} + 4\sqrt[3]{2}\,\sqrt[6]{3}\left(\sqrt{3}\,\operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.02.03.0025.01

$$I_{-\frac{3}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{z\sinh(z) - \cosh(z)}{z^{3/2}}$$

03.02.03.0026.01

$$I_{-\frac{4}{3}}(z) = \frac{1}{\sqrt[3]{2}\,3^{5/6}\,z^{4/3}}\left(\sqrt[6]{3}\,z^{2/3}\left(3\operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\,\operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - 2^{2/3}\left(\sqrt{3}\,\operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.02.03.0010.01

$$I_{-\frac{2}{3}}(z) = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,z^{2/3}}\left(\operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \sqrt{3}\,\operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.02.03.0005.01

$$I_{-\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{\cosh(z)}{\sqrt{z}}$$

03.02.03.0008.01

$$I_{-\frac{1}{3}}(z) = \frac{1}{2^{2/3}\,\sqrt[3]{3}\,\sqrt[3]{z}}\left(3\operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\,\operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.02.03.0007.01

$$I_{\frac{1}{3}}(z) = \frac{1}{2^{2/3}\,\sqrt[3]{3}\,\sqrt[3]{z}}\left(\sqrt{3}\,\operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.02.03.0004.01

$$I_{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{\sinh(z)}{\sqrt{z}}$$

03.02.03.0009.01

$$I_{\frac{2}{3}}(z) = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,z^{2/3}}\left(\sqrt{3}\,\operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.02.03.0027.01

$$I_{\frac{4}{3}}(z) = \frac{1}{\sqrt[3]{2}\,3^{5/6}\,z^{4/3}}\left(\sqrt[6]{3}\left(\sqrt{3}\,\operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{2/3} + 2^{2/3}\left(\sqrt{3}\,\operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.02.03.0028.01

$$I_{\frac{3}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{z\cosh(z) - \sinh(z)}{z^{3/2}}$$

03.02.03.0029.01

$$I_{\frac{5}{3}}(z) = \frac{1}{3\,2^{2/3}\,3^{5/6}\,z^{5/3}}\left(9\,z^{4/3}\left(\sqrt{3}\,\operatorname{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \operatorname{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - 4\sqrt[3]{2}\,\sqrt[6]{3}\left(3\operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\,\operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.02.03.0030.01

$$I_{\frac{7}{3}}(z) = -\frac{4\,2^{2/3}}{3\,3^{5/6}\,z^{7/3}}\left(\sqrt[6]{3}\left(\sqrt{3}\,\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)-3\,\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)z^{2/3}+2^{2/3}\left(\frac{9\,z^2}{16}+1\right)\left(\sqrt{3}\,\mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)-\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)\right)$$

03.02.03.0031.01

$$I_{\frac{5}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{(z^2+3)\sinh(z)-3\,z\cosh(z)}{z^{5/2}}$$

03.02.03.0032.01

$$I_{\frac{8}{3}}(z) = -\frac{1}{9\,3^{5/6}\,z^{8/3}}\,5\sqrt[3]{2}\left(9\,z^{4/3}\left(\sqrt{3}\,\mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)+\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)-4\sqrt[3]{2}\,\sqrt[6]{3}\left(\frac{9\,z^2}{40}+1\right)\left(3\,\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)+\sqrt{3}\,\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)\right)$$

03.02.03.0033.01

$$I_{\frac{10}{3}}(z) = \frac{1}{9\,3^{5/6}\,z^{10/3}}\,56\,2^{2/3}\left(\sqrt[6]{3}\left(\frac{9\,z^2}{112}+1\right)\left(\sqrt{3}\,\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)-3\,\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)z^{2/3}+2^{2/3}\left(\frac{9\,z^2}{14}+1\right)\left(\sqrt{3}\,\mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)-\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)\right)$$

03.02.03.0034.01

$$I_{\frac{7}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{z\,(z^2+15)\cosh(z)-3\,(2\,z^2+5)\sinh(z)}{z^{7/2}}$$

03.02.03.0035.01

$$I_{\frac{11}{3}}(z) = \frac{1}{27\,3^{5/6}\,z^{11/3}}\,80\sqrt[3]{2}\left(9\,z^{4/3}\left(\frac{9\,z^2}{160}+1\right)\left(\sqrt{3}\,\mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)+\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)-4\sqrt[3]{2}\,\sqrt[6]{3}\left(\frac{9\,z^2}{32}+1\right)\left(3\,\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)+\sqrt{3}\,\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)\right)$$

03.02.03.0036.01

$$I_{\frac{13}{3}}(z) = -\frac{1}{27\,3^{5/6}\,z^{13/3}}\,1120\,2^{2/3}\left(\sqrt[6]{3}\left(\frac{9\,z^2}{80}+1\right)\left(\sqrt{3}\,\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)-3\,\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)z^{2/3}+2^{2/3}\left(\frac{81\,z^4}{4480}+\frac{27\,z^2}{40}+1\right)\left(\sqrt{3}\,\mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)-\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)\right)$$

03.02.03.0037.01

$$I_{\frac{9}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{(z^4+45\,z^2+105)\sinh(z)-5\,z\,(2\,z^2+21)\cosh(z)}{z^{9/2}}$$

03.02.03.0038.01

$$I_{\frac{14}{3}}(z) = -\frac{1}{81\,3^{5/6}\,z^{14/3}}\,1760\sqrt[3]{2}\left(9\,z^{4/3}\left(\frac{9\,z^2}{110}+1\right)\left(\sqrt{3}\,\mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)+\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)-4\sqrt[3]{2}\,\sqrt[6]{3}\left(\frac{81\,z^4}{14\,080}+\frac{27\,z^2}{88}+1\right)\left(3\,\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)+\sqrt{3}\,\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)\right)\right)$$

Symbolic rational ν

03.02.03.0006.01

$$I_\nu(z) = -\frac{1}{\sqrt{z}} e^{\frac{\pi i}{2}\left(\frac{1}{2}-\nu\right)} \sqrt{\frac{2}{\pi}} \left(\sinh\left(\frac{\pi i}{2}\left(\frac{1}{2}-\nu\right)-z\right) \sum_{k=0}^{\left\lfloor \frac{2|\nu|-1}{4} \right\rfloor} \frac{\left(|\nu|+2k-\frac{1}{2}\right)!}{(2k)!\left(|\nu|-2k-\frac{1}{2}\right)!(2z)^{2k}} + \cosh\left(\frac{\pi i}{2}\left(\frac{1}{2}-\nu\right)-z\right) \sum_{k=0}^{\left\lfloor \frac{2|\nu|-3}{4} \right\rfloor} \frac{\left(|\nu|+2k+\frac{1}{2}\right)!(2z)^{-2k-1}}{(2k+1)!\left(|\nu|-2k-\frac{3}{2}\right)!} \right) /; \nu-\frac{1}{2} \in \mathbb{Z}$$

03.02.03.0011.01

$$I_\nu(z) = \frac{2^{|\nu|-\frac{5}{3}} z^{-|\nu|} \Gamma\left(-\frac{1}{3}\right)}{3^{5/6} \Gamma(1-|\nu|)} \left(\sqrt[6]{3}\, z^{2/3} \left(\sqrt{3}\, \mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\,\mathrm{sgn}(\nu)\, \mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) \sum_{k=0}^{|\nu|-\frac{4}{3}} \frac{\left(|\nu|-k-\frac{4}{3}\right)!}{k!\left(|\nu|-2k-\frac{4}{3}\right)!\left(\frac{4}{3}\right)_k (1-|\nu|)_k} \left(-\frac{z^2}{4}\right)^k + 2^{2/3} \left(\mathrm{sgn}(\nu)\sqrt{3}\, \mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) \sum_{k=0}^{|\nu|-\frac{1}{3}} \frac{\left(|\nu|-k-\frac{1}{3}\right)!}{k!\left(|\nu|-2k-\frac{1}{3}\right)!\left(\frac{1}{3}\right)_k (1-|\nu|)_k} \left(-\frac{z^2}{4}\right)^k \right) /; |\nu|-\frac{1}{3} \in \mathbb{Z}$$

03.02.03.0012.01

$$I_\nu(z) = \frac{\mathrm{sgn}(\nu)\, 2^{|\nu|-\frac{7}{3}} z^{-|\nu|} \Gamma\left(-\frac{2}{3}\right)}{3\, 3^{5/6} \Gamma(1-|\nu|)} \left(9 z^{4/3} \left(\sqrt{3}\, \mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \mathrm{sgn}(\nu)\, \mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) \sum_{k=0}^{|\nu|-\frac{5}{3}} \frac{\left(|\nu|-k-\frac{5}{3}\right)!}{k!\left(|\nu|-2k-\frac{5}{3}\right)!\left(\frac{5}{3}\right)_k (1-|\nu|)_k} \left(-\frac{z^2}{4}\right)^k - 4\sqrt[3]{2}\, \sqrt[6]{3} \left(3\, \mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\, \mathrm{sgn}(\nu)\, \mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) \sum_{k=0}^{|\nu|-\frac{2}{3}} \frac{\left(|\nu|-k-\frac{2}{3}\right)!}{k!\left(|\nu|-2k-\frac{2}{3}\right)!\left(\frac{2}{3}\right)_k (1-|\nu|)_k} \left(-\frac{z^2}{4}\right)^k \right) /; |\nu|-\frac{2}{3} \in \mathbb{Z}$$

Values at fixed points

03.02.03.0013.01

$$I_0(0) = 1$$

Values at infinities

03.02.03.0014.01

$$\lim_{x\to\infty} I_\nu(x) = \infty$$

03.02.03.0039.01

$$\lim_{x\to-\infty} I_\nu(x) = (-1)^\nu \infty$$

03.02.03.0040.01

$$I_\nu\left(e^{i\lambda}\infty\right) = \begin{cases} 0 & |\lambda| = \frac{\pi}{2} \\ \tilde{\infty} & \text{True} \end{cases} /; \mathrm{Im}(\lambda) = 0$$

03.02.03.0041.01

$I_\nu(i\,\infty) = 0$

03.02.03.0042.01

$I_\nu(-i\,\infty) = 0$

General characteristics

Domain and analyticity

$I_\nu(z)$ is an analytical function of ν and z, which is defined in $\mathbb{C}^2$.

03.02.04.0001.01

$(\nu * z) \longrightarrow I_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

03.02.04.0002.01

$I_\nu(-z) == (-z)^\nu z^{-\nu} I_\nu(z)$

03.02.04.0003.01

$I_{-n}(z) == I_n(z) \;/;\; n \in \mathbb{Z}$

Mirror symmetry

03.02.04.0004.01

$I_{\bar{\nu}}(\bar{z}) == \overline{I_\nu(z)} \;/;\; z \notin (-\infty, 0)$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, the function $I_\nu(z)$ has an essential singularity at $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is a branch point for generic ν.

03.02.04.0005.01

$\mathcal{S}ing_z(I_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$

With respect to ν

For fixed z, the function $I_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

03.02.04.0006.01

$\mathcal{S}ing_\nu(I_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$

Branch points

With respect to z

For fixed noninteger ν, the function $I_\nu(z)$ has two branch points: $z == 0$, $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is an essential singularity.

03.02.04.0007.01

$$\mathcal{BP}_z(I_\nu(z)) == \{0, \tilde{\infty}\} /; \nu \notin \mathbb{Z}$$

03.02.04.0008.01

$$\mathcal{BP}_z(I_\nu(z)) == \{\} /; \nu \in \mathbb{Z}$$

03.02.04.0009.01

$$\mathcal{R}_z(I_\nu(z), 0) == \log /; \nu \notin \mathbb{Q}$$

03.02.04.0010.01

$$\mathcal{R}_z\left(I_{\frac{p}{q}}(z), 0\right) == q /; p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$$

03.02.04.0011.01

$$\mathcal{R}_z(I_\nu(z), \tilde{\infty}) == \log /; \nu \notin \mathbb{Q}$$

03.02.04.0012.01

$$\mathcal{R}_z\left(I_{\frac{p}{q}}(z), \tilde{\infty}\right) == q /; p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$$

With respect to ν

For fixed z, the function $I_\nu(z)$ does not have branch points.

03.02.04.0013.01

$$\mathcal{BP}_\nu(I_\nu(z)) == \{\}$$

Branch cuts

With respect to z

When ν is an integer, $I_\nu(z)$ is an entire function of z. For fixed noninteger ν, it has one infinitely long branch cut. For fixed noninteger ν, the function $I_\nu(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

03.02.04.0014.01

$$\mathcal{BC}_z(I_\nu(z)) == \{\{(-\infty, 0), -i\}\} /; \nu \notin \mathbb{Z}$$

03.02.04.0015.01

$$\mathcal{BC}_z(I_\nu(z)) == \{\} /; \nu \in \mathbb{Z}$$

03.02.04.0016.01

$$\lim_{\epsilon \to +0} I_\nu(x + i\,\epsilon) == I_\nu(x) /; x \in \mathbb{R} \land x < 0$$

03.02.04.0017.01

$$\lim_{\epsilon \to +0} I_\nu(x - i\,\epsilon) == e^{-2 i \pi \nu}\, I_\nu(x) /; x \in \mathbb{R} \land x < 0$$

With respect to ν

For fixed z, the function $I_\nu(z)$ is an entire function of ν and does not have branch cuts.

03.02.04.0018.01

$$\mathcal{BC}_\nu(I_\nu(z)) == \{\}$$

Series representations

Generalized power series

Expansions at $\nu == \pm n$

03.02.06.0021.01

$$I_\nu(z) \propto I_n(z) + \left((-1)^{n-1} K_n(z) + \frac{(-1)^n n!}{2} \left(\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{(-1)^k}{(n-k)\, k!} I_k(z) \left(\frac{z}{2}\right)^k\right)(\nu - n) + \ldots \,/;\, (\nu \to n) \bigwedge n \in \mathbb{N}$$

03.02.06.0022.01

$$I_\nu(z) \propto I_n(z) + \left(\frac{n!}{2}\left(-\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{(-1)^k}{(n-k)\, k!} I_k(z) \left(\frac{z}{2}\right)^k - (-1)^n K_n(z) + \sum_{k=0}^{n-1} \frac{(-1)^{n-k-1} (n-k-1)!}{k!} \left(\frac{z}{2}\right)^{2k-n} + \frac{1}{n!}\left(\frac{z}{2}\right)^n \sum_{j=1}^{n} \frac{1}{j}\, {}_1F_2\left(j;\, j+1,\, n+1;\, \frac{z^2}{4}\right)\right)(\nu + n) + \ldots \,/;\, (\nu \to -n) \bigwedge n \in \mathbb{N}^+$$

Expansions at generic point $z == z_0$

For the function itself

03.02.06.0023.01

$$I_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(I_\nu(z_0) + \left(\frac{\nu}{z_0} I_\nu(z_0) + I_{\nu+1}(z_0)\right)(z - z_0) + \frac{-I_{\nu+1}(z_0)\, z_0 + I_\nu(z_0)\left((\nu-1)\,\nu + z_0^2\right)}{2\, z_0^2} (z - z_0)^2 + \ldots\right) /;\, (z \to z_0)$$

03.02.06.0024.01

$$I_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(I_\nu(z_0) + \left(\frac{\nu}{z_0} I_\nu(z_0) + I_{\nu+1}(z_0)\right)(z - z_0) + \frac{-I_{\nu+1}(z_0)\, z_0 + I_\nu(z_0)\left((\nu-1)\,\nu + z_0^2\right)}{2\, z_0^2} (z - z_0)^2 + O\left((z - z_0)^3\right)\right)$$

03.02.06.0025.01

$$I_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{I_\nu^{(0,k)}(z_0)}{k!} (z - z_0)^k$$

03.02.06.0026.01

$$I_\nu(z) = \sqrt{\pi}\; \Gamma(\nu+1) \left(\frac{z_0}{4}\right)^\nu \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{z_0^{-k}\, 2^k}{k!}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1;\, \frac{1}{2}(\nu-k+1),\, \frac{1}{2}(\nu-k+2),\, \nu+1;\, \frac{z_0^2}{4}\right)(z - z_0)^k$$

03.02.06.0027.01

$$I_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^{-k}}{k!} \sum_{j=0}^{k} \binom{k}{j} I_{2j-k+\nu}(z_0)\, (z - z_0)^k$$

03.02.06.0028.01

$$I_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{z_0^{-k}}{k!} \sum_{m=0}^{k} (-1)^{k+m} \binom{k}{m} (-\nu)_{k-m} \sum_{i=0}^{m} \frac{(-1)^{i-1}\, 2^{2i-m}\, (-m)_{2(m-i)}\, (\nu)_i}{(m-i)!} \left(\frac{z_0}{2} \sum_{j=0}^{i-1} \frac{(i-j-1)!}{j!\,(i-2j-1)!\,(-i-\nu+1)_j\,(\nu)_{j+1}} \left(-\frac{z_0^2}{4}\right)^j I_{\nu-1}(z_0) - \sum_{j=0}^{i} \frac{(i-j)!}{j!\,(i-2j)!\,(-i-\nu+1)_j\,(\nu)_j} \left(-\frac{z_0^2}{4}\right)^j I_\nu(z_0)\right) (z-z_0)^k$$

03.02.06.0029.01

$$I_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} I_\nu(z_0)\,(1 + O(z-z_0))$$

Expansions on branch cuts

For the function itself

03.02.06.0030.01

$$I_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(I_\nu(x) + \left(\frac{\nu}{z_0} I_\nu(x) + I_{\nu+1}(x)\right)(z-x) + \frac{\left(x^2 + (\nu-1)\nu\right) I_\nu(x) - I_{\nu+1}(x)\, x}{2x^2} (z-x)^2 + \ldots\right) /;\ (z \to x) \land x \in \mathbb{R} \land x < 0$$

03.02.06.0031.01

$$I_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(I_\nu(x) + \left(\frac{\nu}{z_0} I_\nu(x) + I_{\nu+1}(x)\right)(z-x) + \frac{\left(x^2 + (\nu-1)\nu\right) I_\nu(x) - I_{\nu+1}(x)\, x}{2x^2} (z-x)^2 + O\left((z-x)^3\right)\right) /;\ x \in \mathbb{R} \land x < 0$$

03.02.06.0032.01

$$I_\nu(z) = 2^{-2\nu} \sqrt{\pi}\, \Gamma(\nu+1)\, e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} x^\nu \sum_{k=0}^{\infty} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1;\ \frac{1}{2}(\nu+1-k), \frac{1}{2}(\nu+2-k), \nu+1;\ \frac{x^2}{4}\right) \frac{2^k (z-x)^k}{x^k\, k!} /;$$
$$x \in \mathbb{R} \land x < 0$$

03.02.06.0033.01

$$I_\nu(z) = e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^{-k}}{k!} \sum_{j=0}^{k} \binom{k}{j} I_{2j-k+\nu}(x)\,(z-x)^k /;\ x \in \mathbb{R} \land x < 0$$

03.02.06.0034.01

$$I_\nu(z) = e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{x^{-k}}{k!} \sum_{m=0}^{k} (-1)^{k+m} \binom{k}{m} (-\nu)_{k-m} \sum_{i=0}^{m} \frac{(-1)^{i-1}\, 2^{2i-m}\, (-m)_{2(m-i)}\, (\nu)_i}{(m-i)!} \left(\frac{x}{2} \sum_{j=0}^{i-1} \frac{(i-j-1)!}{j!\,(i-2j-1)!\,(-i-\nu+1)_j\,(\nu)_{j+1}} \left(-\frac{x^2}{4}\right)^j I_{\nu-1}(x) - \sum_{j=0}^{i} \frac{(i-j)!}{j!\,(i-2j)!\,(-i-\nu+1)_j\,(\nu)_j} \left(-\frac{x^2}{4}\right)^j I_\nu(x)\right) (z-x)^k /;\ x \in \mathbb{R} \land x < 0$$

03.02.06.0035.01

$$I_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} I_\nu(x)\,(1 + O(z-x)) /;\ x \in \mathbb{R} \land x < 0$$

Expansions at $z == 0$

For the function itself

General case

03.02.06.0001.02

$$I_\nu(z) \propto \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu \left(1 + \frac{z^2}{4(\nu+1)} + \frac{z^4}{32(\nu+1)(\nu+2)} + \ldots\right) /; (z \to 0)$$

03.02.06.0036.01

$$I_\nu(z) \propto \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu \left(1 + \frac{z^2}{4(\nu+1)} + \frac{z^4}{32(\nu+1)(\nu+2)} + O\left(z^6\right)\right)$$

03.02.06.0002.01

$$I_\nu(z) == \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\nu+1)\, k!} \left(\frac{z}{2}\right)^{2k+\nu}$$

03.02.06.0037.01

$$I_\nu(z) == \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{z^{2k}}{4^k\, (\nu+1)_k\, k!}$$

03.02.06.0038.01

$$I_\nu(z) == \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu {}_0F_1\left(; \nu+1; \frac{z^2}{4}\right)$$

03.02.06.0003.01

$$I_\nu(z) == \left(\frac{z}{2}\right)^\nu {}_0\tilde{F}_1\left(; \nu+1; \frac{z^2}{4}\right)$$

03.02.06.0004.02

$$I_\nu(z) \propto \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu + O\left(z^{\nu+2}\right) /; -\nu \notin \mathbb{N}^+$$

03.02.06.0039.01

$$I_\nu(z) == F_\infty(z, \nu) /; \left(\left(F_n(z, \nu) == \sum_{k=0}^{n} \frac{\left(\frac{z}{2}\right)^{2k+\nu}}{\Gamma(k+\nu+1)\, k!} == I_\nu(z) - \frac{2^{-2n-\nu-2}\, z^{2n+\nu+2}}{\Gamma(n+\nu+2)\, (n+1)!}\, {}_1F_2\left(1; n+2, n+\nu+2; \frac{z^2}{4}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

03.02.06.0040.01

$$I_\nu(z) \propto \frac{1}{\Gamma(1-\nu)} \left(\frac{z}{2}\right)^{-\nu} \left(1 + \frac{z^2}{4(1-\nu)} + \frac{z^4}{32(1-\nu)(2-\nu)} + \ldots\right) /; (z \to 0) \bigwedge -\nu \in \mathbb{N}^+$$

03.02.06.0041.01

$$I_\nu(z) \propto \frac{1}{\Gamma(1-\nu)} \left(\frac{z}{2}\right)^{-\nu} \left(1 + \frac{z^2}{4(1-\nu)} + \frac{z^4}{32(1-\nu)(2-\nu)} + O\left(z^6\right)\right) /; (z \to 0) \bigwedge -\nu \in \mathbb{N}^+$$

03.02.06.0042.01

$$I_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k-\nu+1)\,k!} \left(\frac{z}{2}\right)^{2k-\nu} /;\ -\nu \in \mathbb{N}^+$$

03.02.06.0043.01

$$I_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+|\nu|+1)\,k!} \left(\frac{z}{2}\right)^{2k+|\nu|} /;\ \nu \in \mathbb{Z}$$

03.02.06.0044.01

$$I_\nu(z) = \frac{1}{\Gamma(1-\nu)} \left(\frac{z}{2}\right)^{-\nu} \sum_{k=0}^{\infty} \frac{z^{2k}}{4^k\,(1-\nu)_k\,k!} /;\ -\nu \in \mathbb{N}^+$$

03.02.06.0045.01

$$I_\nu(z) = \frac{1}{\Gamma(1-\nu)} \left(\frac{z}{2}\right)^{-\nu} {}_0F_1\!\left(;1-\nu;\frac{z^2}{4}\right) /;\ -\nu \in \mathbb{N}^+$$

03.02.06.0046.01

$$I_\nu(z) = \left(\frac{z}{2}\right)^{-\nu} {}_0\tilde{F}_1\!\left(;1-\nu;\frac{z^2}{4}\right) /;\ -\nu \in \mathbb{N}^+$$

03.02.06.0005.02

$$I_\nu(z) \propto \frac{1}{\Gamma(1-\nu)} \left(-\frac{z}{2}\right)^{-\nu} + O\!\left(z^{2-\nu}\right) /;\ -\nu \in \mathbb{N}^+$$

Generic formulas for main term

03.02.06.0047.01

$$I_\nu(z) \propto \begin{cases} \frac{1}{\Gamma(1-\nu)} \left(\frac{z}{2}\right)^{-\nu} & -\nu \in \mathbb{N}^+ \\ \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^{\nu} & \text{True} \end{cases} /;\ (z \to 0)$$

For small integer powers of the function

For the second power

03.02.06.0048.01

$$I_\nu(z)^2 \propto \frac{2^{-2\nu} z^{2\nu}}{\Gamma(\nu+1)^2} \left(1 + \frac{z^2}{2+2\nu} + \frac{(3+2\nu)\,z^4}{16\,(1+\nu)^2\,(2+\nu)} + \ldots\right) /;\ (z \to 0)$$

03.02.06.0049.01

$$I_\nu(z)^2 \propto \frac{2^{-2\nu} z^{2\nu}}{\Gamma(\nu+1)^2} \left(1 + \frac{z^2}{2+2\nu} + \frac{(3+2\nu)\,z^4}{16\,(1+\nu)^2\,(2+\nu)} + O\!\left(z^6\right)\right)$$

03.02.06.0050.01

$$I_\nu(z)^2 = \frac{z^{2\nu}}{2^{2\nu}\,\Gamma(\nu+1)^2} \sum_{k=0}^{\infty} \frac{\left(\nu+\frac{1}{2}\right)_k z^{2k}}{(\nu+1)_k\,(2\nu+1)_k\,k!}$$

03.02.06.0051.01

$$I_\nu(z)^2 = \frac{z^{2\nu}}{2^{2\nu}\,\Gamma(\nu+1)^2}\,{}_1F_2\left(\nu+\frac{1}{2};\nu+1,2\nu+1;z^2\right)$$

03.02.06.0052.01

$$I_\nu(z)^2 = \frac{\sec(\pi\nu)\sqrt{\pi}\,z^{2\nu}}{\Gamma\left(\frac{1}{2}-\nu\right)}\,{}_1\tilde{F}_2\left(\nu+\frac{1}{2};\nu+1,2\nu+1;z^2\right)$$

03.02.06.0053.01

$$I_\nu(z)^2 \propto \frac{2^{-2\nu}\,z^{2\nu}}{\Gamma(\nu+1)^2}\left(1+O\left(z^2\right)\right)$$

03.02.06.0054.01

$$I_\nu(z)^2 = F_\infty(z.\nu)\;/;\;\left(\left(F_n(z,\nu) = \frac{z^{2\nu}}{2^{2\nu}\,\Gamma(\nu+1)^2}\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k z^{2k}}{(\nu+1)_k\,(2\nu+1)_k\,k!} = \right.\right.$$
$$\left.\left. I_\nu(z)^2 - \frac{z^{2n+2\nu+2}\,\Gamma\left(n+\nu+\frac{3}{2}\right)}{\sqrt{\pi}\;\Gamma(n+\nu+2)\,\Gamma(n+2\nu+2)\,(n+1)!}\,{}_2F_3\left(1,n+\nu+\frac{3}{2};n+2,n+\nu+2,n+2\nu+2;z^2\right)\right)\bigwedge n\in\mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

Expansions inside Stokes sectors

Expansions containing $z \to \infty$

In exponential form ||| In exponential form

03.02.06.0055.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^z\left(1+\frac{1-4\nu^2}{8z}+\frac{16\nu^4-40\nu^2+9}{128z^2}+\ldots\right)-i\,e^{-z-i\pi\nu}\left(1-\frac{1-4\nu^2}{8z}+\frac{16\nu^4-40\nu^2+9}{128z^2}+\ldots\right)\right)\;/;$$
$$-\pi<\arg(z)<\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0056.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^z\left(1+\frac{1-4\nu^2}{8z}+\frac{9-40\nu^2+16\nu^4}{128z^2}+\ldots\right)+e^{i\pi\nu-z}\,i\left(1-\frac{1-4\nu^2}{8z}+\frac{9-40\nu^2+16\nu^4}{128z^2}+\ldots\right)\right)\;/;$$
$$-\frac{\pi}{2}<\arg(z)<\pi\bigwedge(|z|\to\infty)$$

03.02.06.0057.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^z\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(\frac{1}{2z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)-i\,e^{-z-i\pi\nu}\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(-\frac{1}{2z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)\right)\;/;$$
$$-\pi<\arg(z)<\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0058.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^z\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(\frac{1}{2z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)+e^{i\pi\nu-z}\,i\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(-\frac{1}{2z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)\right)/;\ -\frac{\pi}{2}<\arg(z)<\pi\bigwedge(|z|\to\infty)$$

03.02.06.0059.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^z\,{}_2F_0\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;;\frac{1}{2z}\right)-i\,e^{-z-i\pi\nu}\,{}_2F_0\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;;-\frac{1}{2z}\right)\right)/;\ -\pi<\arg(z)<\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0060.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^z\,{}_2F_0\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;;\frac{1}{2z}\right)+e^{i\pi\nu-z}\,i\,{}_2F_0\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;;-\frac{1}{2z}\right)\right)/;\ -\frac{\pi}{2}<\arg(z)<\pi\bigwedge(|z|\to\infty)$$

03.02.06.0006.01

$$I_\nu(z) \propto \frac{e^z}{\sqrt{2\pi z}}\left(1+O\left(\frac{1}{z}\right)\right)/;\ |\arg(z)|<\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0061.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^z\left(1+O\left(\frac{1}{z}\right)\right)-i\,e^{-z-i\pi\nu}\left(1+O\left(\frac{1}{z}\right)\right)\right)/;\ -\pi<\arg(z)<\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0062.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^z\left(1+O\left(\frac{1}{z}\right)\right)+e^{i\pi\nu-z}\,i\left(1+O\left(\frac{1}{z}\right)\right)\right)/;\ -\frac{\pi}{2}<\arg(z)<\pi\bigwedge(|z|\to\infty)$$

In hyperbolic form ||| In hyperbolic form

03.02.06.0063.01

$$I_\nu(z) \propto \frac{\sqrt{2}\,e^{-\frac{\pi i(2\nu+1)}{4}}}{\sqrt{\pi z}}\left(\cosh\left(z+\frac{\pi i(2\nu+1)}{4}\right)\left(1+\frac{16\nu^4-40\nu^2+9}{128z^2}+\frac{256\nu^8-5376\nu^6+31584\nu^4-51664\nu^2+11025}{98304z^4}+\ldots\right)+\frac{1-4\nu^2}{8z}\sinh\left(z+\frac{\pi i(2\nu+1)}{4}\right)\left(1+\frac{16\nu^4-136\nu^2+225}{384z^2}+\frac{256\nu^8-10496\nu^6+137824\nu^4-656784\nu^2+893025}{491520z^4}+\ldots\right)\right)/;\ -\pi<\arg(z)<\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0064.01

$$I_\nu(z) \propto \frac{\sqrt{2}}{\sqrt{\pi z}} e^{-\frac{\pi i (2\nu+1)}{4}} \left(\cosh\left(z + \frac{\pi i (2\nu+1)}{4}\right) \left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(1-2\nu)\right)_k \left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(2\nu+1)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right) \right) + \frac{1-4\nu^2}{8z} \sinh\left(z + \frac{\pi i (2\nu+1)}{4}\right) \left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(5-2\nu)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k \left(\frac{1}{4}(2\nu+5)\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right) \right) \right) /; -\pi < \arg(z) < \frac{\pi}{2} \bigwedge (|z| \to \infty)$$

03.02.06.0065.01

$$I_\nu(z) \propto \frac{1}{\sqrt{\pi z}} \left(\sqrt{2}\, e^{-\frac{\pi i (2\nu+1)}{4}} \left(\cosh\left(z + \frac{1}{4}\pi i (2\nu+1)\right) {}_4F_1\left(\frac{1}{4}(1-2\nu), \frac{1}{4}(3-2\nu), \frac{1}{4}(2\nu+1), \frac{1}{4}(2\nu+3); \frac{1}{2}; \frac{1}{z^2}\right) + \frac{1-4\nu^2}{8z} \sinh\left(z + \frac{1}{4}\pi i (2\nu+1)\right) {}_4F_1\left(\frac{1}{4}(3-2\nu), \frac{1}{4}(5-2\nu), \frac{1}{4}(2\nu+3), \frac{1}{4}(2\nu+5); \frac{3}{2}; \frac{1}{z^2}\right) \right) \right) /; -\pi < \arg(z) < \frac{\pi}{2} \bigwedge (|z| \to \infty)$$

03.02.06.0066.01

$$I_\nu(z) \propto \frac{\sqrt{2}\, e^{-\frac{\pi i (2\nu+1)}{4}}}{\sqrt{\pi z}} \left(\cosh\left(z + \frac{\pi i (2\nu+1)}{4}\right) \left(1 + O\left(\frac{1}{z^2}\right)\right) + \frac{1-4\nu^2}{8z} \sinh\left(z + \frac{\pi i (2\nu+1)}{4}\right) \left(1 + O\left(\frac{1}{z^2}\right)\right) \right) /; -\pi < \arg(z) < \frac{\pi}{2} \bigwedge (|z| \to \infty)$$

Expansions containing $z \to -\infty$

In exponential form ||| In exponential form

03.02.06.0067.01

$$I_\nu(z) \propto \frac{(i z)^{-\nu-\frac{1}{2}} z^\nu}{\sqrt{2\pi}} \left(e^{\frac{2\nu+1}{4}\pi i + z} \left(1 + \frac{1-4\nu^2}{8z} + \frac{16\nu^4 - 40\nu^2 + 9}{128 z^2} + \ldots\right) + e^{-\frac{2\nu+1}{4}\pi i - z} \left(1 - \frac{1-4\nu^2}{8z} + \frac{16\nu^4 - 40\nu^2 + 9}{128 z^2} + \ldots\right) \right) /; \arg(z) \neq \frac{\pi}{2} \bigwedge (|z| \to \infty)$$

03.02.06.0068.01

$$I_\nu(z) \propto \frac{e^{2 i \pi \nu}}{\sqrt{2\pi}\, \sqrt{-z}} \left(i\, e^z \left(\sum_{k=0}^{n} \frac{\left(\nu+\frac{1}{2}\right)_k \left(\frac{1}{2}-\nu\right)_k}{k!} \left(\frac{1}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) + e^{-z-i\pi\nu} \left(\sum_{k=0}^{n} \frac{\left(\nu+\frac{1}{2}\right)_k \left(\frac{1}{2}-\nu\right)_k}{k!} \left(-\frac{1}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) \right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.02.06.0069.01

$$I_\nu(z) \propto \frac{(i\,z)^{-\nu-\frac{1}{2}}\,z^\nu}{\sqrt{2\,\pi}}\left(e^{\frac{1}{4}(2\,\nu+1)\,\pi\,i+z}\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(\frac{1}{2\,z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)+ e^{-\frac{1}{4}(2\,\nu+1)\,\pi\,i-z}\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(-\frac{1}{2\,z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)\right) /;\ \arg(z)\neq\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0007.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\,\pi}}\,(i\,z)^{-\nu-\frac{1}{2}}\,z^\nu\left(\exp\left(z+\frac{i\,\pi\,(2\,\nu+1)}{4}\right){}_2F_0\left(\nu+\frac{1}{2},\,\frac{1}{2}-\nu;\,;\,\frac{1}{2\,z}\right)+\exp\left(-z-\frac{i\,\pi\,(2\,\nu+1)}{4}\right){}_2F_0\left(\nu+\frac{1}{2},\,\frac{1}{2}-\nu;\,;\,-\frac{1}{2\,z}\right)\right) /;\ |\arg(i\,z)|<\pi\bigwedge(|z|\to\infty)$$

03.02.06.0070.01

$$I_\nu(z) \propto \frac{(i\,z)^{-\nu-\frac{1}{2}}\,z^\nu}{\sqrt{2\,\pi}}\left(e^{\frac{2\,\nu+1}{4}\,\pi\,i+z}\left(1+O\left(\frac{1}{z}\right)\right)+e^{-\frac{2\,\nu+1}{4}\,\pi\,i-z}\left(1+O\left(\frac{1}{z}\right)\right)\right) /;\ \arg(z)\neq\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

In hyperbolic form ||| In hyperbolic form

03.02.06.0071.01

$$I_\nu(z) \propto \frac{\sqrt{2}\,(i\,z)^{-\nu}\,z^\nu}{\sqrt{\pi\,i\,z}}\left(\cosh\left(z+\frac{\pi\,i\,(2\,\nu+1)}{4}\right)\left(1+\frac{16\,\nu^4-40\,\nu^2+9}{128\,z^2}+\frac{256\,\nu^8-5376\,\nu^6+31\,584\,\nu^4-51\,664\,\nu^2+11\,025}{98\,304\,z^4}+\ldots\right)+ \frac{1-4\,\nu^2}{8\,z}\sinh\left(z+\frac{\pi\,i\,(2\,\nu+1)}{4}\right)\left(1+\frac{16\,\nu^4-136\,\nu^2+225}{384\,z^2}+ \frac{256\,\nu^8-10\,496\,\nu^6+137\,824\,\nu^4-656\,784\,\nu^2+893\,025}{491\,520\,z^4}+\ldots\right)\right) /;\ \arg(z)\neq\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0072.01

$$I_\nu(z) \propto \frac{\sqrt{2}\,(i\,z)^{-\nu}\,z^\nu}{\sqrt{\pi\,i\,z}}\left(\cosh\left(z+\frac{\pi\,i\,(2\,\nu+1)}{4}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{4}(1-2\,\nu)\right)_k\left(\frac{1}{4}(3-2\,\nu)\right)_k\left(\frac{1}{4}(2\,\nu+1)\right)_k\left(\frac{1}{4}(2\,\nu+3)\right)_k}{\left(\frac{1}{2}\right)_k\,k!}\left(\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2\,n+2}}\right)\right)+\frac{1-4\,\nu^2}{8\,z}\sinh\left(z+\frac{\pi\,i\,(2\,\nu+1)}{4}\right) \left(\sum_{k=0}^{n}\frac{\left(\frac{1}{4}(3-2\,\nu)\right)_k\left(\frac{1}{4}(5-2\,\nu)\right)_k\left(\frac{1}{4}(2\,\nu+3)\right)_k\left(\frac{1}{4}(2\,\nu+5)\right)_k}{\left(\frac{3}{2}\right)_k\,k!}\left(\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2\,n+2}}\right)\right)\right) /;\ \arg(z)\neq\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0073.01

$$I_\nu(z) \propto \frac{\sqrt{2}\,(i\,z)^{-\nu}\,z^\nu}{\sqrt{\pi\,i\,z}}\left(\cosh\left(z+\frac{\pi\,i\,(2\,\nu+1)}{4}\right){}_4F_1\left(\frac{1}{4}(1-2\,\nu),\,\frac{1}{4}(3-2\,\nu),\,\frac{1}{4}(2\,\nu+1),\,\frac{1}{4}(2\,\nu+3);\,\frac{1}{2};\,\frac{1}{z^2}\right)+ \frac{1-4\,\nu^2}{8\,z}\sinh\left(z+\frac{\pi\,i\,(2\,\nu+1)}{4}\right){}_4F_1\left(\frac{1}{4}(3-2\,\nu),\,\frac{1}{4}(5-2\,\nu),\,\frac{1}{4}(2\,\nu+3),\,\frac{1}{4}(2\,\nu+5);\,\frac{3}{2};\,\frac{1}{z^2}\right)\right) /;\ \arg(z)\neq\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0074.01

$$I_\nu(z) \propto \sqrt{\frac{2}{\pi}}\,(i\,z)^{-\frac{1}{2}-\nu}\,z^\nu\left(\cosh\left(z+\frac{\pi\,i\,(2\,\nu+1)}{4}\right)\left(1+O\left(\frac{1}{z^2}\right)\right)+\frac{1-4\,\nu^2}{8\,z}\sinh\left(z+\frac{\pi\,i\,(2\,\nu+1)}{4}\right)\left(1+O\left(\frac{1}{z^2}\right)\right)\right)/;\ \arg(z)\neq\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

03.02.06.0008.01

$$I_\nu(z) \propto \sqrt{\frac{2}{\pi}}\,(i\,z)^{-\nu-\frac{1}{2}}\,z^\nu\cosh\left(z+\frac{i\,\pi\,(2\,\nu+1)}{4}\right)\left(1+O\left(\frac{1}{z}\right)\right)/;\ |\arg(i\,z)|<\pi\bigwedge(|z|\to\infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-containing arguments

03.02.06.0009.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\,\pi}}\,z^\nu\left(-z^2\right)^{-\frac{1}{4}(2\,\nu+1)}\left(e^{-i\left(\sqrt{-z^2}-\frac{1}{4}(2\,\nu+1)\pi\right)}\left(1+\frac{i\,(1-4\,\nu^2)}{8\sqrt{-z^2}}+\frac{9-40\,\nu^2+16\,\nu^4}{128\,z^2}+\ldots\right)+e^{i\left(\sqrt{-z^2}-\frac{1}{4}(2\,\nu+1)\pi\right)}\left(1-\frac{i\,(1-4\,\nu^2)}{8\sqrt{-z^2}}+\frac{9-40\,\nu^2+16\,\nu^4}{128\,z^2}+\ldots\right)\right)/;\ (|z|\to\infty)$$

03.02.06.0075.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\,\pi}}\,z^\nu\left(-z^2\right)^{-\frac{1}{4}(2\,\nu+1)}\left(e^{i\left(\frac{1}{4}(2\,\nu+1)\pi-\sqrt{-z^2}\right)}\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(\frac{i}{2\sqrt{-z^2}}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)+e^{-i\left(\frac{1}{4}(2\,\nu+1)\pi-\sqrt{-z^2}\right)}\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(-\frac{i}{2\sqrt{-z^2}}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)\right)/;\ (|z|\to\infty)$$

03.02.06.0010.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\,\pi}}\,z^\nu\left(-z^2\right)^{-\frac{2\,\nu+1}{4}}\left(\exp\left(i\left(\frac{(2\,\nu+1)\,\pi}{4}-\sqrt{-z^2}\right)\right){}_2F_0\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;;\frac{i}{2\sqrt{-z^2}}\right)+\exp\left(-i\left(\frac{(2\,\nu+1)\,\pi}{4}-\sqrt{-z^2}\right)\right){}_2F_0\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;;-\frac{i}{2\sqrt{-z^2}}\right)\right)/;\ (|z|\to\infty)$$

03.02.06.0011.01

$$I_\nu(z) \propto \frac{1}{\sqrt{2\,\pi}}\,z^\nu\left(-z^2\right)^{-\frac{2\,\nu+1}{4}}\left(\exp\left(-i\left(\frac{(2\,\nu+1)\,\pi}{4}-\sqrt{-z^2}\right)\right)\left(1+O\left(\frac{1}{z}\right)\right)+\exp\left(i\left(\frac{(2\,\nu+1)\,\pi}{4}-\sqrt{-z^2}\right)\right)\left(1+O\left(\frac{1}{z}\right)\right)\right)/;\ (|z|\to\infty)$$

Using exponential function with branch cut-free arguments

03.02.06.0076.01

$$I_\nu(z) \propto \frac{\sqrt[4]{-1}\,(i\,z)^{-\nu}\,z^\nu\,i^\nu}{\sqrt{2\,\pi\,i\,z}}\left(e^z\left(1+\frac{1-4\,\nu^2}{8\,z}+\frac{9-40\,\nu^2+16\,\nu^4}{128\,z^2}+\ldots\right)+ e^{-z}\left(\frac{i\,\sqrt{z^2}}{z}\cos(\pi\,\nu)-\sin(\pi\,\nu)\right)\left(1+\frac{-1+4\,\nu^2}{8\,z}+\frac{9-40\,\nu^2+16\,\nu^4}{128\,z^2}+\ldots\right)\right)\,/;\,(|z|\to\infty)$$

03.02.06.0077.01

$$I_\nu(z) \propto \frac{\sqrt[4]{-1}\,(i\,z)^{-\nu}\,z^\nu\,i^\nu}{\sqrt{2\,\pi\,i\,z}}\left(e^z\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(\frac{1}{2\,z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)+ e^{-z}\left(\frac{i\,\sqrt{z^2}\,\cos(\pi\,\nu)}{z}-\sin(\pi\,\nu)\right)\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(-\frac{1}{2\,z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)\right)\,/;\,(|z|\to\infty)$$

03.02.06.0078.01

$$I_\nu(z) \propto \frac{\sqrt[4]{-1}\,(i\,z)^{-\nu}\,z^\nu\,i^\nu}{\sqrt{2\,\pi\,i\,z}}\left(e^z\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(\frac{1}{2\,z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)+ e^{-z}\left(\frac{i\,\sqrt{z^2}\,\cos(\pi\,\nu)}{z}-\sin(\pi\,\nu)\right)\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(-\frac{1}{2\,z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)\right)\,/;\,(|z|\to\infty)$$

03.02.06.0079.01

$$I_\nu(z) \propto \frac{z^\nu\left(-z^2\right)^{\frac{1}{4}(-2\,\nu-1)}}{\sqrt{4\,\pi}}\left(e^z\left(\left(1-\frac{z}{\sqrt{-z^2}}\right)\cos\left(\frac{\pi\,\nu}{2}\right)-\left(\frac{z}{\sqrt{-z^2}}+1\right)\sin\left(\frac{\pi\,\nu}{2}\right)\right){}_2F_0\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;;\frac{1}{2\,z}\right)+ e^{-z}\left(\left(\frac{z}{\sqrt{-z^2}}+1\right)\cos\left(\frac{\pi\,\nu}{2}\right)-\left(1-\frac{z}{\sqrt{-z^2}}\right)\sin\left(\frac{\pi\,\nu}{2}\right)\right){}_2F_0\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;;-\frac{1}{2\,z}\right)\right)\,/;\,(|z|\to\infty)$$

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03.02.06.0013.02

$$I_\nu(z) \propto \frac{\sqrt[4]{-1}\,(i\,z)^{-\nu}\,z^\nu\,i^\nu}{\sqrt{2\,\pi\,i\,z}}\left(e^z\left(1+O\left(\frac{1}{z}\right)\right)+e^{-z}\left(\frac{i\,\sqrt{z^2}}{z}\cos(\pi\,\nu)-\sin(\pi\,\nu)\right)\left(1+O\left(\frac{1}{z}\right)\right)\right)\,/;\,(|z|\to\infty)$$

03.02.06.0080.01

$$I_\nu(z) \propto \begin{cases} \frac{e^z-i\,e^{-z-i\,\pi\,\nu}}{\sqrt{2\pi}\,\sqrt{z}} & \arg(z)\le-\frac{\pi}{2} \\ \frac{e^z+i\,e^{i\,\pi\,\nu-z}}{\sqrt{2\pi}\,\sqrt{z}} & -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} \\ \frac{i\,e^{i\,\pi\,\nu-z}-e^{z+2\,i\,\pi\,\nu}}{\sqrt{2\pi}\,\sqrt{z}} & \text{True} \end{cases}\,/;\,(|z|\to\infty)$$

Expansions for any *z* in trigonometric form

Using trigonometric functions with branch cut-containing arguments

03.02.06.0014.01

$$I_\nu(z) \propto \frac{\sqrt{2}}{\sqrt{\pi}} z^\nu \left(-z^2\right)^{-\frac{2\nu+1}{4}} \left(\cos\left(\sqrt{-z^2} - \frac{\pi(2\nu+1)}{4}\right)\left(1 + \frac{9 - 40\nu^2 + 16\nu^4}{128 z^2} + \frac{11025 - 51664\nu^2 + 31584\nu^4 - 5376\nu^6 + 256\nu^8}{98304 z^4} + \ldots\right) + \frac{1 - 4\nu^2}{8\sqrt{-z^2}} \sin\left(\sqrt{-z^2} - \frac{\pi(2\nu+1)}{4}\right) \left(1 + \frac{225 - 136\nu^2 + 16\nu^4}{384 z^2} + \frac{893025 - 656784\nu^2 + 137824\nu^4 - 10496\nu^6 + 256\nu^8}{491520 z^4} + \ldots\right)\right) /; (|z| \to \infty)$$

03.02.06.0081.01

$$I_\nu(z) \propto \frac{\sqrt{2}\, z^\nu \left(-z^2\right)^{-\frac{1}{4}(2\nu+1)}}{\sqrt{\pi}} \left(\cos\left(\sqrt{-z^2} - \frac{\pi(2\nu+1)}{4}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(1-2\nu)\right)_k \left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(2\nu+1)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right)\right) + \frac{1-4\nu^2}{8\sqrt{-z^2}} \sin\left(\sqrt{-z^2} - \frac{\pi(2\nu+1)}{4}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(5-2\nu)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k \left(\frac{1}{4}(2\nu+5)\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right)\right)\right) /; (|z| \to \infty)$$

03.02.06.0015.01

$$I_\nu(z) \propto \frac{1}{\sqrt{\pi}} \left(\sqrt{2}\, z^\nu \left(-z^2\right)^{-\frac{2\nu+1}{4}} \left(\cos\left(\sqrt{-z^2} - \frac{\pi(2\nu+1)}{4}\right) {}_4F_1\left(\frac{1-2\nu}{4}, \frac{3-2\nu}{4}, \frac{2\nu+1}{4}, \frac{2\nu+3}{4}; \frac{1}{2}; \frac{1}{z^2}\right) + \frac{1-4\nu^2}{8\sqrt{-z^2}} {}_4F_1\left(\frac{3-2\nu}{4}, \frac{5-2\nu}{4}, \frac{2\nu+3}{4}, \frac{2\nu+5}{4}; \frac{3}{2}; \frac{1}{z^2}\right) \sin\left(\sqrt{-z^2} - \frac{\pi(2\nu+1)}{4}\right)\right)\right) /; (|z| \to \infty)$$

03.02.06.0016.01

$$I_\nu(z) \propto \frac{\sqrt{2}\, z^\nu \left(-z^2\right)^{-\frac{2\nu+1}{4}}}{\sqrt{\pi}} \left(\cos\left(\sqrt{-z^2} - \frac{\pi(2\nu+1)}{4}\right)\left(1 + O\left(\frac{1}{z^2}\right)\right) + \frac{1-4\nu^2}{8\sqrt{-z^2}} \sin\left(\sqrt{-z^2} - \frac{\pi(2\nu+1)}{4}\right)\left(1 + O\left(\frac{1}{z^2}\right)\right)\right) /; (|z| \to \infty)$$

Using trigonometric functions with branch cut-free arguments

03.02.06.0082.01

$$I_\nu(z) \propto \frac{(i\,z)^{-\nu}\,z^\nu}{\sqrt{2\,\pi}} \left(\left(\frac{1}{\sqrt{i\,z}} \left(1 - \frac{\sqrt{z^2}}{z}\right) \cosh\!\left(z + \frac{i\,\pi\,(2\,\nu+1)}{4}\right) + \frac{e^{i\,\pi\,\nu}}{\sqrt{-i\,z}} \left(\frac{\sqrt{z^2}}{z} + 1\right) \cosh\!\left(z - \frac{i\,\pi\,(2\,\nu+1)}{4}\right) \right) \left(1 + \frac{9 - 40\,\nu^2 + 16\,\nu^4}{128\,z^2} + \frac{11\,025 - 51\,664\,\nu^2 + 31\,584\,\nu^4 - 5376\,\nu^6 + 256\,\nu^8}{98\,304\,z^4} + \ldots\right) + \frac{1 - 4\,\nu^2}{8\,z} \left(\frac{1}{\sqrt{i\,z}} \left(1 - \frac{\sqrt{z^2}}{z}\right) \sinh\!\left(z + \frac{i\,\pi\,(2\,\nu+1)}{4}\right) + \frac{e^{i\,\pi\,\nu}}{\sqrt{-i\,z}} \left(\frac{\sqrt{z^2}}{z} + 1\right) \sinh\!\left(z - \frac{i\,\pi\,(2\,\nu+1)}{4}\right) \right) \left(1 + \frac{225 - 136\,\nu^2 + 16\,\nu^4}{384\,z^2} + \frac{893\,025 - 656\,784\,\nu^2 + 137\,824\,\nu^4 - 10\,496\,\nu^6 + 256\,\nu^8}{491\,520\,z^4} + \ldots\right) \right) /;\ (|z| \to \infty)$$

03.02.06.0083.01

$$I_\nu(z) \propto \frac{(i\,z)^{-\nu}\,z^\nu}{\sqrt{2\,\pi}} \left(\left(\frac{1}{\sqrt{i\,z}} \left(1 - \frac{\sqrt{z^2}}{z}\right) \cosh\!\left(z + \frac{i\,\pi\,(2\,\nu+1)}{4}\right) + \frac{e^{i\,\pi\,\nu}}{\sqrt{-i\,z}} \left(\frac{\sqrt{z^2}}{z} + 1\right) \cosh\!\left(z - \frac{i\,\pi\,(2\,\nu+1)}{4}\right) \right) \left(\sum_{k=0}^{n} \frac{\left(\frac{1-2\,\nu}{4}\right)_k \left(\frac{3-2\,\nu}{4}\right)_k \left(\frac{1+2\,\nu}{4}\right)_k \left(\frac{3+2\,\nu}{4}\right)_k}{\left(\frac{3}{2}\right)_k\, k!\, z^{2\,k}} + O\!\left(\frac{1}{z^{2\,n+2}}\right) \right) + \frac{1 - 4\,\nu^2}{8\,z} \left(\frac{1}{\sqrt{i\,z}} \left(1 - \frac{\sqrt{z^2}}{z}\right) \sinh\!\left(z + \frac{i\,\pi\,(2\,\nu+1)}{4}\right) + \frac{e^{i\,\pi\,\nu}}{\sqrt{-i\,z}} \left(\frac{\sqrt{z^2}}{z} + 1\right) \sinh\!\left(z - \frac{i\,\pi\,(2\,\nu+1)}{4}\right) \right) \left(\sum_{k=0}^{n} \frac{\left(\frac{3-2\,\nu}{4}\right)_k \left(\frac{5-2\,\nu}{4}\right)_k \left(\frac{3+2\,\nu}{4}\right)_k \left(\frac{5+2\,\nu}{4}\right)_k}{\left(\frac{3}{2}\right)_k\, k!\, z^{2\,k}} + O\!\left(\frac{1}{z^{2\,n+2}}\right) \right) \right) /;\ (|z| \to \infty)$$

03.02.06.0084.01

$$I_\nu(z) \propto \frac{(i\,z)^{-\nu}\,z^\nu}{\sqrt{2\,\pi}} \left(\left(\frac{1}{\sqrt{i\,z}} \left(1 - \frac{\sqrt{z^2}}{z}\right) \cosh\!\left(z + \frac{i\,\pi\,(2\,\nu+1)}{4}\right) + \frac{e^{i\,\pi\,\nu}}{\sqrt{-i\,z}} \left(\frac{\sqrt{z^2}}{z} + 1\right) \cosh\!\left(z - \frac{i\,\pi\,(2\,\nu+1)}{4}\right) \right) {}_4F_1\!\left(\frac{1-2\,\nu}{4}, \frac{3-2\,\nu}{4}, \frac{1+2\,\nu}{4}, \frac{3+2\,\nu}{4}; \frac{1}{2}; \frac{1}{z^2}\right) + \frac{1 - 4\,\nu^2}{8\,z} \left(\frac{1}{\sqrt{i\,z}} \left(1 - \frac{\sqrt{z^2}}{z}\right) \sinh\!\left(z + \frac{i\,\pi\,(2\,\nu+1)}{4}\right) + \frac{e^{i\,\pi\,\nu}}{\sqrt{-i\,z}} \left(\frac{\sqrt{z^2}}{z} + 1\right) \sinh\!\left(z - \frac{i\,\pi\,(2\,\nu+1)}{4}\right) \right) {}_4F_1\!\left(\frac{3-2\,\nu}{4}, \frac{5-2\,\nu}{4}, \frac{3+2\,\nu}{4}, \frac{5+2\,\nu}{4}; \frac{3}{2}; \frac{1}{z^2}\right) \right) /;\ (|z| \to \infty)$$

03.02.06.0085.01

$$I_\nu(z) \propto \frac{(i z)^{-\nu} z^\nu}{\sqrt{2\pi}} \left(\left(\frac{1}{\sqrt{i z}} \left(1 - \frac{\sqrt{z^2}}{z} \right) \cosh\left(z + \frac{i \pi (2\nu+1)}{4} \right) + \frac{e^{i\pi\nu}}{\sqrt{-i z}} \left(\frac{\sqrt{z^2}}{z} + 1 \right) \cosh\left(z - \frac{i\pi(2\nu+1)}{4} \right) \right) \left(1 + O\left(\frac{1}{z^2} \right) \right) + \frac{1 - 4\nu^2}{8 z} \left(\frac{1}{\sqrt{i z}} \left(1 - \frac{\sqrt{z^2}}{z} \right) \sinh\left(z + \frac{i\pi(2\nu+1)}{4} \right) + \frac{e^{i\pi\nu}}{\sqrt{-i z}} \left(\frac{\sqrt{z^2}}{z} + 1 \right) \sinh\left(z - \frac{i\pi(2\nu+1)}{4} \right) \right) \left(1 + O\left(\frac{1}{z^2} \right) \right) \right) /; (|z| \to \infty)$$

03.02.06.0086.01

$$I_\nu(z) \propto \begin{cases} -\frac{(-1)^{3/4}\sqrt{2}}{i^\nu \sqrt{\pi}\sqrt{z}} \cosh\left(z + \frac{1}{4} i \pi (2\nu+1)\right) & \arg(z) \le -\frac{\pi}{2} \\ \frac{\sqrt[4]{-1}\sqrt{2}\, e^{\frac{i\pi\nu}{2}}}{\sqrt{\pi}\sqrt{z}} \cosh\left(z - \frac{1}{4} i\pi(2\nu+1)\right) & -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} \\ \frac{(-1)^{3/4}\sqrt{2}\, e^{\frac{3 i\pi\nu}{2}}}{\sqrt{\pi}\sqrt{z}} \cosh\left(z + \frac{1}{4} i\pi(2\nu+1)\right) & \text{True} \end{cases} /; (|z| \to \infty)$$

Residue representations

03.02.06.0017.01

$$I_\nu(z) = \pi\, 2^{-\nu} z^\nu \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(\frac{z^2}{4}\right)^{-s}}{\Gamma\left(s + \frac{1}{2}\right) \Gamma(1 + \nu - s)\, \Gamma\left(\frac{1}{2} - s\right)} \Gamma(s) \right)(-j)$$

03.02.06.0018.01

$$I_\nu(z) = z^\nu \left(-z^2\right)^{-\frac{\nu}{2}} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(-\frac{z^2}{4}\right)^{-s}}{\Gamma\left(1 + \frac{\nu}{2} - s\right)} \Gamma\left(s + \frac{\nu}{2}\right) \right)\left(-j - \frac{\nu}{2}\right)$$

03.02.06.0019.01

$$I_\nu(z) = \pi \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(\frac{z}{2}\right)^{-2s}}{\Gamma\left(\frac{\nu+1}{2} + s\right) \Gamma\left(1 + \frac{\nu}{2} - s\right) \Gamma\left(\frac{1-\nu}{2} - s\right)} \Gamma\left(s + \frac{\nu}{2}\right) \right)\left(-j - \frac{\nu}{2}\right)$$

03.02.06.0020.01

$$I_\nu(z) = z^\nu (i z)^{-\nu} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(\frac{i z}{2}\right)^{-2s}}{\Gamma\left(1 + \frac{\nu}{2} - s\right)} \Gamma\left(s + \frac{\nu}{2}\right) \right)\left(-j - \frac{\nu}{2}\right)$$

Integral representations

On the real axis

Of the direct function

03.02.07.0001.01

$$I_0(z) = \frac{1}{\pi} \int_0^\pi e^{z\cos(t)}\, dt$$

03.02.07.0002.01

$$I_0(z) = \frac{1}{\pi}\int_0^{\pi} e^{-z\cos(t)}\,dt$$

03.02.07.0003.01

$$I_0(z) = \frac{1}{\pi}\int_0^{\pi} \cosh(z\cos(t))\,dt$$

03.02.07.0004.01

$$I_\nu(z) = \frac{2^{1-\nu} z^\nu}{\sqrt{\pi}\,\Gamma\left(\nu+\frac{1}{2}\right)}\int_0^1 \left(1-t^2\right)^{\nu-\frac{1}{2}} \cosh(z\,t)\,dt \,/;\, \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.02.07.0005.01

$$I_\nu(z) = \frac{2^{-\nu}}{\sqrt{\pi}\,\Gamma\left(\nu+\frac{1}{2}\right)} z^\nu \int_{-1}^{-1} \left(1-t^2\right)^{\nu-\frac{1}{2}} e^{-z\,t}\,dt \,/;\, \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.02.07.0006.01

$$I_\nu(z) = \frac{2^{-\nu}}{\sqrt{\pi}\,\Gamma\left(\nu+\frac{1}{2}\right)} z^\nu \int_0^{\pi} e^{-z\cos(t)} \sin^{2\nu}(t)\,dt \,/;\, \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.02.07.0007.01

$$I_\nu(z) = \frac{1}{\pi}\int_0^{\pi} e^{z\cos(t)} \cos(\nu\,t)\,dt \,/;\, \nu \in \mathbb{Z} \land \operatorname{Re}(z) > 0$$

Contour integral representations

03.02.07.0008.01

$$I_\nu(z) = \frac{2^{-\nu-1}}{i} z^\nu \int_{\mathcal{L}} \frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(-s+\nu+1)} \left(\frac{z^2}{4}\right)^{-s} ds$$

03.02.07.0009.01

$$I_\nu(z) = \frac{z^\nu \left(-z^2\right)^{-\frac{\nu}{2}}}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}\right)}{\Gamma\left(1+\frac{\nu}{2}-s\right)} \left(-\frac{z^2}{4}\right)^{-s} ds$$

03.02.07.0010.01

$$I_\nu(z) = \frac{1}{2i} \int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}+s\right)\Gamma\left(1+\frac{\nu}{2}-s\right)\Gamma\left(\frac{1-\nu}{2}-s\right)} \left(\frac{z}{2}\right)^{-2s} ds$$

03.02.07.0011.01

$$I_\nu(z) = \frac{z^\nu (i\,z)^{-\nu}}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}\right)}{\Gamma\left(1+\frac{\nu}{2}-s\right)} \left(\frac{i\,z}{2}\right)^{-2s} ds$$

Limit representations

03.02.09.0001.01

$$I_\nu(z) = \lim_{\lambda\to\infty} \lambda^\nu\, P_\lambda^{-\nu}\left(\cosh\left(\frac{z}{\lambda}\right)\right)$$

03.02.09.0002.01

$$I_\nu(z) == \left(\frac{z}{2}\right)^\nu \lim_{n\to\infty} \frac{1}{n^\nu} L_n^\nu\left(-\frac{z^2}{4\,n}\right)$$

03.02.09.0003.01

$$I_\nu(z) == \frac{z^\nu}{2^\nu\,\Gamma(\nu+1)} \lim_{a\to\infty} {}_1F_1\left(a;\,\nu+1;\,\frac{z^2}{4\,a}\right)$$

03.02.09.0004.01

$$I_\nu(z) == \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu \lim_{n\to\infty} \lim_{m\to\infty} {}_2F_1\left(m,\,n;\,\nu+1;\,\frac{z^2}{4\,m\,n}\right)$$

Generating functions

03.02.11.0001.02

$$\sum_{k=-\infty}^{\infty} I_k(x)\,t^k == \exp\left(\frac{1}{2}\,x\left(t+\frac{1}{t}\right)\right)$$

03.02.11.0002.01

$$\sum_{k=-\infty}^{\infty} I_k(z)\,e^{i\,k\,q} == e^{z\cos(q)}$$

P. Abbott

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.02.13.0001.01

$$z^2\,w''(z)+z\,w'(z)-\left(z^2+\nu^2\right)w(z) == 0\;/;\;w(z) == c_1\,I_\nu(z)+c_2\,K_\nu(z)$$

03.02.13.0002.01

$$W_z(I_\nu(z),\,K_\nu(z)) == -\frac{1}{z}$$

03.02.13.0003.01

$$w''(z)\,z^2+w'(z)\,z-\left(z^2+\nu^2\right)w(z) == 0\;/;\;w(z) == c_1\,I_\nu(z)+c_2\,I_{-\nu}(z)\bigwedge \nu\notin\mathbb{Z}$$

03.02.13.0004.01

$$W_z(I_\nu(z),\,I_{-\nu}(z)) == -\frac{2\sin(\pi\,\nu)}{\pi\,z}$$

03.02.13.0005.01

$$w''(z)-a\,z^n\,w(z) == 0\;/;\;w(z) == \sqrt{z}\left(c_1\,I_{\frac{1}{n+2}}\left(\frac{2\sqrt{a}}{n+2}\,z^{\frac{n+2}{2}}\right)+c_2\,K_{\frac{1}{n+2}}\left(\frac{2\sqrt{a}}{n+2}\,z^{\frac{n+2}{2}}\right)\right)$$

03.02.13.0006.01

$$W_z\left(\sqrt{z}\,I_{\frac{1}{n+2}}\left(\frac{2\sqrt{a}}{n+2}\,z^{\frac{n+2}{2}}\right),\,\sqrt{z}\,K_{\frac{1}{n+2}}\left(\frac{2\sqrt{a}}{n+2}\,z^{\frac{n+2}{2}}\right)\right) == -\frac{n}{2}-1$$

03.02.13.0007.01

$$w''(z) - a\, z^n\, w(z) = 0 \;/;\; w(z) = \sqrt{z} \left(c_1\, I_{\frac{1}{n+2}}\!\left(\frac{2\sqrt{a}}{n+2}\, z^{\frac{n+2}{2}} \right) + c_2\, I_{-\frac{1}{n+2}}\!\left(\frac{2\sqrt{a}}{n+2}\, z^{\frac{n+2}{2}} \right) \right) \bigwedge \frac{1}{n+2} \notin \mathbb{Z}$$

03.02.13.0008.01

$$W_z\!\left(\sqrt{z}\, I_{\frac{1}{n+2}}\!\left(\frac{2\sqrt{a}}{n+2}\, z^{\frac{n+2}{2}} \right),\, \sqrt{z}\, I_{-\frac{1}{n+2}}\!\left(\frac{2\sqrt{a}}{n+2}\, z^{\frac{n+2}{2}} \right) \right) = -\frac{(n+2)}{\pi} \sin\!\left(\frac{\pi}{n+2} \right)$$

03.02.13.0009.01

$$w''(z) - \left(m^2 + \frac{1}{z^2} \left(\nu^2 - \frac{1}{4} \right) \right) w(z) = 0 \;/;\; w(z) = c_1 \sqrt{z}\, I_\nu\!\left(\sqrt{m^2}\; z \right) + c_2 \sqrt{z}\, K_\nu\!\left(\sqrt{m^2}\; z \right)$$

03.02.13.0010.01

$$W_z\!\left(\sqrt{z}\, I_\nu\!\left(\sqrt{m^2}\; z \right),\, \sqrt{z}\, K_\nu\!\left(\sqrt{m^2}\; z \right) \right) = -1$$

03.02.13.0011.01

$$w''(z) - \left(m^2 + \frac{\nu^2 - \frac{1}{4}}{z^2} \right) w(z) = 0 \;/;\; w(z) = c_1 \sqrt{z}\, I_\nu\!\left(\sqrt{m^2}\; z \right) + c_2 \sqrt{z}\, I_{-\nu}\!\left(\sqrt{m^2}\; z \right) \bigwedge \nu \notin \mathbb{Z}$$

03.02.13.0012.01

$$W_z\!\left(\sqrt{z}\, I_\nu\!\left(\sqrt{m^2}\; z \right),\, \sqrt{z}\, I_{-\nu}\!\left(\sqrt{m^2}\; z \right) \right) = -\frac{2 \sin(\pi\, \nu)}{\pi}$$

03.02.13.0013.01

$$w''(z) - \left(\frac{m^2}{4\, z} + \frac{\nu^2 - 1}{4\, z^2} \right) w(z) = 0 \;/;\; w(z) = c_1 \sqrt{z}\, I_\nu\!\left(\sqrt{m^2}\; \sqrt{z} \right) + c_2 \sqrt{z}\, K_\nu\!\left(\sqrt{m^2}\; \sqrt{z} \right)$$

03.02.13.0014.01

$$W_z\!\left(\sqrt{z}\, I_\nu\!\left(\sqrt{m^2}\; \sqrt{z} \right),\, \sqrt{z}\, K_\nu\!\left(\sqrt{m^2}\; \sqrt{z} \right) \right) = -\frac{1}{2}$$

03.02.13.0015.01

$$w''(z) - \left(\frac{m^2}{4\, z} + \frac{\nu^2 - 1}{4\, z^2} \right) w(z) = 0 \;/;\; w(z) = c_1 \sqrt{z}\, I_\nu\!\left(\sqrt{m^2}\; \sqrt{z} \right) + c_2 \sqrt{z}\, I_{-\nu}\!\left(\sqrt{m^2}\; \sqrt{z} \right) \bigwedge \nu \notin \mathbb{Z}$$

03.02.13.0016.01

$$W_z\!\left(\sqrt{z}\, I_\nu\!\left(\sqrt{m^2}\; \sqrt{z} \right),\, \sqrt{z}\, I_{-\nu}\!\left(\sqrt{m^2}\; \sqrt{z} \right) \right) = -\frac{\sin(\pi\, \nu)}{\pi}$$

03.02.13.0017.01

$$w''(z) - \frac{2\, \nu - 1}{z}\, w'(z) - w(z)\, m^2 = 0 \;/;\; w(z) = c_1\, z^\nu\, I_\nu(m\, z) + c_2\, z^\nu\, K_\nu(m\, z)$$

03.02.13.0018.01

$$W_z(z^\nu\, I_\nu(m\, z),\, z^\nu\, K_\nu(m\, z)) = -z^{2\,\nu-1}$$

03.02.13.0019.01

$$w''(z) - \frac{2\, \nu - 1}{z}\, w'(z) - w(z)\, m^2 = 0 \;/;\; w(z) = c_1\, z^\nu\, I_\nu(m\, z) + c_2\, z^\nu\, I_{-\nu}(m\, z) \bigwedge \nu \notin \mathbb{Z}$$

03.02.13.0020.01

$$W_z(z^\nu\, I_\nu(m\, z),\, z^\nu\, I_{-\nu}(m\, z)) = -\frac{2}{\pi}\, z^{2\,\nu-1} \sin(\pi\, \nu)$$

03.02.13.0021.01

$$w''(z)\,z^2+(2\,z+1)\,w'(z)\,z+\left(z-\nu^2\right)w(z)=0\;/;\;w(z)=c_1\,e^{-z}\,I_\nu(z)+c_2\,e^{-z}\,K_\nu(z)$$

03.02.13.0022.01

$$W_z\!\left(e^{-z}\,I_\nu(z),\;e^{-z}\,K_\nu(z)\right)=-\frac{e^{-2\,z}}{z}$$

03.02.13.0023.01

$$w''(z)\,z^2+(2\,z+1)\,w'(z)\,z+\left(z-\nu^2\right)w(z)=0\;/;\;w(z)=c_1\,e^{-z}\,I_\nu(z)+\;c_2\,e^{-z}\,I_{-\nu}(z)\bigwedge\nu\notin\mathbb{Z}$$

03.02.13.0024.01

$$W_z\!\left(e^{-z}\,I_\nu(z),\;e^{-z}\,I_{-\nu}(z)\right)=-\frac{2}{\pi\,z}\,e^{-2\,z}\,\sin(\pi\,\nu)$$

03.02.13.0025.01

$$w''(z)\,z^2+(1-2\,z)\,w'(z)\,z-\left(\nu^2+z\right)w(z)=0\;/;\;w(z)=c_1\,e^{z}\,I_\nu(z)+c_2\,e^{z}\,K_\nu(z)$$

03.02.13.0026.01

$$W_z\!\left(e^{z}\,I_\nu(z),\;e^{z}\,K_\nu(z)\right)=-\frac{e^{2\,z}}{z}$$

03.02.13.0027.01

$$w''(z)\,z^2+(1-2\,z)\,w'(z)\,z-\left(\nu^2+z\right)w(z)=0\;/;\;w(z)=\;c_1\,e^{z}\,I_\nu(z)+\;c_2\,e^{z}\,I_{-\nu}(z)\bigwedge\nu\notin\mathbb{Z}$$

03.02.13.0028.01

$$W_z\!\left(e^{z}\,I_\nu(z),\;e^{z}\,I_{-\nu}(z)\right)=-\frac{2}{\pi\,z}\,e^{2\,z}\,\sin(\pi\,\nu)$$

03.02.13.0029.01

$$w''(z)\,z^2+(1-2\,p)\,w'(z)\,z+\left(-m^2\,q^2\,z^{2\,q}+p^2-\nu^2\,q^2\right)w(z)=0\;/;\;w(z)=c_1\,z^p\,I_\nu\!\left(m\,z^q\right)\;+\;c_2\,z^p\,I_{-\nu}\!\left(m\,z^q\right)$$

03.02.13.0030.01

$$W_z\!\left(z^p\,I_\nu(m\,z^q),\;z^p\,I_{-\nu}(m\,z^q)\right)=-\frac{2\,q}{\pi}\,z^{2\,p-1}\,\sin(\pi\,\nu)$$

03.02.13.0031.01

$$w''(z)-\left(e^{2\,z}\,m^2+\nu^2\right)w(z)=0\;/;\;w(z)=\;c_1\,I_{-\nu}\!\left(m\,e^{z}\right)+\;c_2\,I_\nu\!\left(m\,e^{z}\right)$$

03.02.13.0032.01

$$W_z\!\left(I_\nu(m\,e^{z}),\;I_{-\nu}(m\,e^{z})\right)=-\frac{2\sin(\pi\,\nu)}{\pi}$$

03.02.13.0033.01

$$\left(z^2+\nu^2\right)w''(z)\,z^2+\left(z^2+3\,\nu^2\right)w'(z)\,z-\left(z^2-\nu^2+\left(z^2+\nu^2\right)^2\right)w(z)=0\;/;\;w(z)=c_1\,\frac{\partial I_\nu(z)}{\partial z}+c_2\,\frac{\partial K_\nu(z)}{\partial z}$$

03.02.13.0034.01

$$w^{(4)}(z)-\frac{m^4}{z^2}\,w(z)=0\;/;\;w(z)=c_1\,z\left(I_2\!\left(2\,m\sqrt{z}\right)-J_2\!\left(2\,m\sqrt{z}\right)\right)+$$
$$c_2\,z\left(I_2\!\left(2\,m\sqrt{z}\right)+J_2\!\left(2\,m\sqrt{z}\right)\right)+\;c_3\,G_{0,4}^{2,0}\!\left(\frac{m^4\,z^2}{16}\,\middle|\,0,1,\tfrac{1}{2},\tfrac{3}{2}\right)+c_4\,G_{0,4}^{2,0}\!\left(\frac{m^4\,z^2}{16}\,\middle|\,\tfrac{1}{2},\tfrac{3}{2},0,1\right)$$

03.02.13.0039.01

$$w''(z)-\left(\frac{g''(z)}{g'(z)}-\frac{g'(z)}{g(z)}\right)w'(z)-\left(\frac{\nu^2}{g(z)^2}+1\right)g'(z)^2\,w(z)\;=0\;/;\;w(z)=c_1\,I_\nu(g(z))\;+c_2\,K_\nu(g(z))$$

03.02.13.0040.01

$$W_z(I_\nu(g(z)),\, K_\nu(g(z))) = -\frac{g'(z)}{g(z)}$$

03.02.13.0041.01

$$w''(z) - \left(-\frac{g'(z)}{g(z)} + \frac{2\,h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) - \left(\left(\frac{\nu^2}{g(z)^2} + 1\right) g'(z)^2 + \frac{h'(z)\,g'(z)}{g(z)\,h(z)} + \frac{h(z)\,h''(z) - 2\,h'(z)^2}{h(z)^2} - \frac{h'(z)\,g''(z)}{h(z)\,g'(z)}\right) w(z) = 0 \;/;\; w(z) = c_1\, h(z)\, I_\nu(g(z)) + c_2\, h(z)\, K_\nu(g(z))$$

03.02.13.0042.01

$$W_z(h(z)\, I_\nu(g(z)),\, h(z)\, K_\nu(g(z))) = -\frac{h(z)^2\, g'(z)}{g(z)}$$

03.02.13.0043.01

$$z^2\, w''(z) + z\,(1 - 2\,s)\, w'(z) + \left(s^2 - r^2\left(a^2\, z^{2\,r} + \nu^2\right)\right) w(z) = 0 \;/;\; w(z) = c_1\, z^s\, I_\nu(a\, z^r) + c_2\, z^s\, K_\nu(a\, z^r)$$

03.02.13.0044.01

$$W_z(z^s\, I_\nu(a\, z^r),\, z^s\, K_\nu(a\, z^r)) = -r\, z^{2\,s-1}$$

03.02.13.0045.01

$$w''(z) - 2\log(s)\, w'(z) - \left(\left(a^2\, r^{2\,z} + \nu^2\right)\log^2(r) - \log^2(s)\right) w(z) = 0 \;/;\; w(z) = c_1\, s^z\, I_\nu(a\, r^z) + c_2\, s^z\, K_\nu(a\, r^z)$$

03.02.13.0046.01

$$W_z(s^z\, I_\nu(a\, r^z),\, s^z\, K_\nu(a\, r^z)) = -s^{2\,z}\log(r)$$

Involving related functions

03.02.13.0035.01

$$\left(\prod_{k=1}^{4}\left(z\,\frac{d}{d\,z}\right)\right) w(z) - 2\left(\mu^2 + \nu^2\right)\left(\prod_{k=1}^{2}\left(z\,\frac{d}{d\,z}\right)\right) w(z) + \left(\nu^2 - \mu^2\right)^2 w(z) - 4\,z^2\left(\left(\prod_{k=1}^{2}\left(z\,\frac{d}{d\,z}\right)\right) w(z) + 2\,w(z) + 3\,z\,w'(z)\right) = 0 \;/;\; w(z) = c_1\, I_\mu(z)\, I_\nu(z) + c_2\, I_\nu(z)\, K_\mu(z) + c_3\, I_\mu(z)\, K_\nu(z) + c_4\, K_\mu(z)\, K_\nu(z)$$

03.02.13.0036.01

$$\left(\prod_{k=1}^{4}\left(z\,\frac{d}{d\,z}\right)\right) w(z) - 2\left(\mu^2 + \nu^2\right)\left(\prod_{k=1}^{2}\left(z\,\frac{d}{d\,z}\right)\right) w(z) + \left(\nu^2 - \mu^2\right)^2 w(z) - 4\,z^2\left(\left(\prod_{k=1}^{2}\left(z\,\frac{d}{d\,z}\right)\right) w(z) + 2\,w(z) + 3\,z\,w'(z)\right) = 0 \;/;\; w(z) = c_1\, I_\mu(z)\, I_\nu(z) + c_2\, I_{-\mu}(z)\, I_\nu(z) + c_3\, I_\mu(z)\, I_{-\nu}(z) + c_4\, I_{-\mu}(z)\, I_{-\nu}(z)$$

03.02.13.0037.01

$$\left(\prod_{k=1}^{3}\left(z\,\frac{d}{d\,z}\right)\right) w(z) - 4\left(z^2 + \nu^2\right) z\,\frac{\partial w(z)}{\partial z} - 4\,z^2\, w(z) = 0 \;/;\; w(z) = c_1\, I_\nu(z)^2 + c_2\, K_\nu(z)\, I_\nu(z) + c_3\, K_\nu(z)^2$$

03.02.13.0038.01

$$z^3\, w^{(3)}(z) - z\left(4\,z^2 + 4\,\nu^2 - 1\right) w'(z) + \left(4\,\nu^2 - 1\right) w(z) = 0 \;/;\; w(z) = c_1\, z\, I_\nu(z)^2 + c_2\, z\, K_\nu(z)\, I_\nu(z) + c_3\, z\, K_\nu(z)^2$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.02.16.0001.01

$$I_\nu(-z) == \frac{(-z)^\nu}{z^\nu} I_\nu(z)$$

03.02.16.0002.01

$$I_\nu(i z) == \frac{(i z)^\nu}{z^\nu} J_\nu(z)$$

03.02.16.0003.01

$$I_\nu(-i z) == \frac{(-i z)^\nu}{z^\nu} J_\nu(z)$$

03.02.16.0004.01

$$I_\nu\left(\sqrt{z^2}\right) == z^{-\nu} \left(z^2\right)^{\nu/2} I_\nu(z)$$

03.02.16.0005.01

$$I_\nu(c (d z^n)^m) == \frac{(c (d z^n)^m)^\nu}{(c d^m z^{m n})^\nu} I_\nu(c d^m z^{m n}) /; 2 m \in \mathbb{Z}$$

Addition formulas

03.02.16.0006.01

$$I_\nu(z_1 - z_2) == \sum_{k=-\infty}^{\infty} (-1)^k I_{k+\nu}(z_1) I_k(z_2) /; \left|\frac{z_2}{z_1}\right| < 1 \bigvee \nu \in \mathbb{Z}$$

03.02.16.0007.01

$$I_\nu(z_1 + z_2) == \sum_{k=-\infty}^{\infty} I_{\nu-k}(z_1) I_k(z_2) /; \nu \in \mathbb{Z}$$

Multiple arguments

03.02.16.0008.01

$$I_\nu(z_1 z_2) == z_1^\nu (i z_2)^\nu z_2^{-\nu} (i z_1 z_2)^{-\nu} (z_1 z_2)^\nu \sum_{k=0}^{\infty} \frac{\left(z_1^2 - 1\right)^k}{k!} I_{k+\nu}(z_2) \left(\frac{z_2}{2}\right)^k$$

03.02.16.0009.01

$$I_\nu(z_1 z_2) == z_1^{-\nu} \sum_{k=0}^{\infty} \frac{\left(z_1^2 - 1\right)^k}{k!} I_{\nu-k}(z_2) \left(\frac{z_2}{2}\right)^k /; \left|z_1^2 - 1\right| < 1 \bigvee \nu \in \mathbb{Z}$$

Identities

Recurrence identities

Consecutive neighbors

03.02.17.0001.01

$$I_\nu(z) == \frac{2 (\nu + 1)}{z} I_{\nu+1}(z) + I_{\nu+2}(z)$$

03.02.17.0002.01

$$I_\nu(z) = I_{\nu-2}(z) - \frac{2(\nu-1)}{z} I_{\nu-1}(z)$$

Distant neighbors

Increasing

03.02.17.0003.01

$$I_\nu(z) = 2^{n-1} z^{-n} (\nu+1)_{n-1} \left(2(n+\nu) \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(n-k)!}{k!\,(n-2k)!\,(-n-\nu)_k\,(\nu+1)_k} \left(-\frac{z^2}{4}\right)^k I_{n+\nu}(z) + z \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(n-k-1)!}{k!\,(n-2k-1)!\,(-n-\nu+1)_k\,(\nu+1)_k} \left(-\frac{z^2}{4}\right)^k I_{n+\nu+1}(z) \right) /;\, n \in \mathbb{N}$$

03.02.17.0015.01

$$I_\nu(z) = 2^{n-1} z^{-n} (\nu+1)_{n-1} \left(2(n+\nu)\, {}_3F_4\!\left(1, \frac{1-n}{2}, -\frac{n}{2}; 1, -n, -n-\nu, \nu+1; z^2\right) I_{n+\nu}(z) + z\, {}_3F_4\!\left(1, \frac{1-n}{2}, 1-\frac{n}{2}; 1, 1-n, -n-\nu+1, \nu+1; z^2\right) I_{n+\nu+1}(z) \right) /;\, n \in \mathbb{N}$$

03.02.17.0007.01

$$I_\nu(z) = \frac{\left(z^2 + 4(\nu+1)(\nu+2)\right) I_{\nu+2}(z) + 2z(\nu+1) I_{\nu+3}(z)}{z^2}$$

03.02.17.0008.01

$$I_\nu(z) = \frac{4(\nu+2)\left(z^2 + 2(\nu+1)(\nu+3)\right) I_{\nu+3}(z) + z\left(z^2 + 4(\nu+1)(\nu+2)\right) I_{\nu+4}(z)}{z^3}$$

03.02.17.0009.01

$$I_\nu(z) = \frac{1}{z^4} \left(\left(z^4 + 12(\nu+2)(\nu+3) z^2 + 16(\nu+1)(\nu+2)(\nu+3)(\nu+4)\right) I_{\nu+4}(z) + 4z(\nu+2)\left(z^2 + 2(\nu+1)(\nu+3)\right) I_{\nu+5}(z) \right)$$

03.02.17.0010.01

$$I_\nu(z) = \frac{1}{z^5} \left(2(\nu+3)\left(3z^4 + 16(\nu+2)(\nu+4) z^2 + 16(\nu+1)(\nu+2)(\nu+4)(\nu+5)\right) I_{\nu+5}(z) + z\left(z^4 + 12(\nu+2)(\nu+3) z^2 + 16(\nu+1)(\nu+2)(\nu+3)(\nu+4)\right) I_{\nu+6}(z) \right)$$

03.02.17.0016.01

$$I_\nu(z) = C_n(\nu, z) I_{\nu+n}(z) + C_{n-1}(\nu, z) I_{\nu+n+1}(z) \;/;\; C_0(\nu, z) = 1 \bigwedge C_1(\nu, z) = \frac{2(\nu+1)}{z} \bigwedge C_n(\nu, z) = \frac{2(n+\nu)}{z} C_{n-1}(\nu, z) + C_{n-2}(\nu, z) \bigwedge n \in \mathbb{N}^+$$

03.02.17.0017.01

$$I_\nu(z) = C_n(\nu, z) I_{\nu+n}(z) + C_{n-1}(\nu, z) I_{\nu+n+1}(z) \;/;\, C_n(\nu, z) = 2^n z^{-n} (\nu+1)_n\, {}_2F_3\!\left(\frac{1-n}{2}, -\frac{n}{2}; \nu+1, -n, -n-\nu; z^2\right) \bigwedge n \in \mathbb{N}^+$$

Decreasing

03.02.17.0004.01

$$I_\nu(z) = 2^{n-1} z^{-n} (1-\nu)_{n-1} \left(z \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{(n-k-1)!}{k!\,(n-2k-1)!\,(1-\nu)_k\,(\nu-n+1)_k} \left(-\frac{z^2}{4}\right)^k I_{\nu-n-1}(z) + 2(n-\nu) \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(n-k)!}{k!\,(n-2k)!\,(1-\nu)_k\,(\nu-n)_k} \left(-\frac{z^2}{4}\right)^k I_{\nu-n}(z) \right) /; n \in \mathbb{N}$$

03.02.17.0018.01

$$I_\nu(z) = 2^{n-1} z^{-n} (1-\nu)_{n-1} \left(2(n-\nu)\, {}_3F_4\left(1, \frac{1-n}{2}, -\frac{n}{2}; 1, -n, 1-\nu, \nu-n; z^2\right) I_{\nu-n}(z) + z\, {}_3F_4\left(1, \frac{1-n}{2}, 1-\frac{n}{2}; 1, 1-n, 1-\nu, \nu-n+1; z^2\right) I_{\nu-n-1}(z) \right) /; n \in \mathbb{N}$$

03.02.17.0011.01

$$I_\nu(z) = \frac{\left(z^2 + 4(\nu-2)(\nu-1)\right) I_{\nu-2}(z) - 2z(\nu-1) I_{\nu-3}(z)}{z^2}$$

03.02.17.0012.01

$$I_\nu(z) = \frac{z\left(z^2 + 4(\nu-2)(\nu-1)\right) I_{\nu-4}(z) - 4\left(z^2 + 2(\nu-3)(\nu-1)\right)(\nu-2) I_{\nu-3}(z)}{z^3}$$

03.02.17.0013.01

$$I_\nu(z) = \frac{1}{z^4} \left(\left(z^4 + 12(\nu-3)(\nu-2) z^2 + 16(\nu-4)(\nu-3)(\nu-2)(\nu-1)\right) I_{\nu-4}(z) - 4z\left(z^2 + 2(\nu-3)(\nu-1)\right)(\nu-2) I_{\nu-5}(z) \right)$$

03.02.17.0014.01

$$I_\nu(z) = \frac{1}{z^5} \left(z\left(z^4 + 12(\nu-3)(\nu-2) z^2 + 16(\nu-4)(\nu-3)(\nu-2)(\nu-1)\right) I_{\nu-6}(z) - 2\left(3z^4 + 16(\nu-4)(\nu-2) z^2 + 16(\nu-5)(\nu-4)(\nu-2)(\nu-1)\right)(\nu-3) I_{\nu-5}(z) \right)$$

03.02.17.0019.01

$$I_\nu(z) = C_n(\nu, z) I_{\nu-n}(z) + C_{n-1}(\nu, z) I_{\nu-n-1}(z) \;/;\; C_0(\nu, z) = 1 \bigwedge C_1(\nu, z) = -\frac{2(\nu-1)}{z} \bigwedge C_n(\nu, z) = -\frac{2(-n+\nu)}{z} C_{n-1}(\nu, z) + C_{n-2}(\nu, z) \bigwedge n \in \mathbb{N}^+$$

03.02.17.0020.01

$$I_\nu(z) = C_n(\nu, z) I_{\nu-n}(z) + C_{n-1}(\nu, z) I_{\nu-n-1}(z) \;/;\; C_n(\nu, z) = (-2)^n z^{-n} (1-\nu)_n\, {}_2F_3\left(\frac{1-n}{2}, -\frac{n}{2}; 1-\nu, -n, \nu-n; -z^2\right) \bigwedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

03.02.17.0005.01

$$I_\nu(z) = \frac{z\,(I_{\nu-1}(z) - I_{\nu+1}(z))}{2\nu}$$

Relations of special kind

03.02.17.0006.01

$$I_{\nu+1}(z)\, I_{-\nu}(z) - I_\nu(z)\, I_{-\nu-1}(z) = \frac{2\sin(\pi\nu)}{\pi z}$$

Differentiation

Low-order differentiation

With respect to ν

03.02.20.0001.01

$$I_\nu^{(1,0)}(z) = I_\nu(z)\log\left(\frac{z}{2}\right) - \sum_{k=0}^{\infty}\frac{\psi(k+\nu+1)}{k!\,\Gamma(k+\nu+1)}\left(\frac{z}{2}\right)^{2k+\nu}$$

03.02.20.0002.01

$$I_\nu^{(1,0)}(z) = I_\nu(z)\,(\log(z)-\log(2)-\psi(\nu+1)) - \frac{1}{(\nu+1)\,\Gamma(\nu+2)}\left(\frac{z}{2}\right)^{\nu+2} F_{2\times 0\times 1}^{0\times 1\times 2}\left(\begin{matrix} ;\,1;\,1,\,1+\nu; \\ 2,\,2+\nu;;\,2+\nu; \end{matrix}\ \frac{z^2}{4},\,\frac{z^2}{4}\right)$$

03.02.20.0003.01

$$I_n^{(1,0)}(z) = \frac{(-1)^n}{2}\,n!\left(\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{(-1)^k I_k(z)}{(n-k)\,k!}\left(\frac{z}{2}\right)^k + (-1)^{n-1}K_n(z)\ /;\ n\in\mathbb{N}$$

03.02.20.0018.01

$$I_{-n}^{(1,0)}(z) = \frac{1}{2}\,n!\left(-\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{(-1)^k}{(n-k)\,k!}\,I_k(z)\left(\frac{z}{2}\right)^k - (-1)^n K_n(z) + \sum_{k=0}^{n-1}\frac{(-1)^{n-k-1}(n-k-1)!}{k!}\left(\frac{z}{2}\right)^{2k-n} + \frac{1}{n!}\left(\frac{z}{2}\right)^n\sum_{j=1}^{n}\frac{1}{j}\,{}_1F_2\left(j;\,j+1,\,n+1;\,\frac{z^2}{4}\right)\ /;\ n\in\mathbb{N}^+$$

03.02.20.0019.01

$$I_{n+\frac{1}{2}}^{(1,0)}(z) = \frac{(-1)^n\,2\,(2z)^{\frac{1}{2}-n}}{n!\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}2^{2k}\binom{n}{2k+1}(-2k+2n-1)!\left(\cosh(z)\left(\psi\left(k+\frac{3}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)-\cosh(z)\,\mathrm{Chi}(2z)+\sinh(z)\,\mathrm{Shi}(2z)\right)z^{2k} + \frac{(-1)^n\,2\,(2z)^{-n-\frac{1}{2}}}{n!\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}2^{2k}\binom{n}{2k}(2n-2k)!\left(\left(\psi\left(k-n+\frac{1}{2}\right)-\psi\left(k+\frac{1}{2}\right)\right)\sinh(z)+\mathrm{Chi}(2z)\sinh(z)-\cosh(z)\,\mathrm{Shi}(2z)\right)z^{2k}\ /;\ n\in\mathbb{N}$$

Brychkov Yu.A. (2005)

03.02.20.0020.01

$$I_{-n-\frac{1}{2}}^{(1,0)}(z) = \frac{(-1)^n\,2\,(2z)^{\frac{1}{2}-n}}{n!\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}2^{2k}\binom{n}{2k+1}(-2k+2n-1)!\left(-\left(\psi\left(k+\frac{3}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)\sinh(z)-\mathrm{Chi}(2z)\sinh(z)+\cosh(z)\,\mathrm{Shi}(2z)\right)z^{2k} + \frac{(-1)^n\,2\,(2z)^{-n-\frac{1}{2}}}{n!\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}2^{2k}\binom{n}{2k}(2n-2k)!\left(\cosh(z)\left(\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)+\cosh(z)\,\mathrm{Chi}(2z)-\sinh(z)\,\mathrm{Shi}(2z)\right)z^{2k}\ /;\ n\in\mathbb{N}$$

Brychkov Yu.A. (2005)

With respect to z

03.02.20.0004.01

$$\frac{\partial I_\nu(z)}{\partial z} = I_{\nu-1}(z) - \frac{\nu}{z} I_\nu(z)$$

03.02.20.0005.01

$$\frac{\partial I_\nu(z)}{\partial z} = \frac{\nu}{z} I_\nu(z) + I_{\nu+1}(z)$$

03.02.20.0006.01

$$\frac{\partial I_\nu(z)}{\partial z} = \frac{1}{2} (I_{\nu-1}(z) + I_{\nu+1}(z))$$

03.02.20.0007.01

$$\frac{\partial I_0(z)}{\partial z} = I_1(z)$$

03.02.20.0008.01

$$\frac{\partial (z^\nu I_\nu(z))}{\partial z} = z^\nu I_{\nu-1}(z)$$

03.02.20.0009.01

$$\frac{\partial (z^{-\nu} I_\nu(z))}{\partial z} = z^{-\nu} I_{\nu+1}(z)$$

03.02.20.0010.01

$$\frac{\partial^2 I_\nu(z)}{\partial z^2} = \frac{1}{4} (I_{\nu-2}(z) + 2 I_\nu(z) + I_{\nu+2}(z))$$

Symbolic differentiation

With respect to ν

03.02.20.0011.02

$$I_\nu^{(m,0)}(z) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{z}{2}\right)^{2k} \frac{\partial^m}{\partial \nu^m} \frac{\left(\frac{z}{2}\right)^\nu}{\Gamma(k+\nu+1)} /; m \in \mathbb{N}$$

With respect to z

03.02.20.0012.01

$$I_\nu^{(0,n)}(0) = 0 /; n \in \mathbb{N}^+ \bigwedge \left(\nu \in \mathbb{Z} \land |\nu| > n \bigvee \frac{n-\nu-1}{2} \in \mathbb{N}\right)$$

03.02.20.0013.01

$$I_\nu^{(0,n)}(0) = \frac{2^{-n} n!}{\Gamma\left(\frac{1}{2}(n-\nu+2)\right) \Gamma\left(\frac{1}{2}(n+\nu+2)\right)} /; n \in \mathbb{N}^+ \bigwedge \frac{n-\nu}{2} \in \mathbb{Z} \bigwedge |\nu| \le n$$

03.02.20.0021.01

$$\frac{\partial^n I_\nu(z)}{\partial z^n} = z^{-n} \sum_{m=0}^{n} (-1)^{m+n} \binom{n}{m} (-\nu)_{n-m} \sum_{k=0}^{m} \frac{(-1)^{k-1} 2^{2k-m} (-m)_{2(m-k)} (\nu)_k}{(m-k)!} \left(\frac{z}{2} \sum_{j=0}^{k-1} \frac{(k-j-1)!}{j!\,(k-2j-1)!\,(1-k-\nu)_j\,(\nu)_{j+1}} \left(-\frac{z^2}{4}\right)^j I_{\nu-1}(z) - \sum_{j=0}^{k} \frac{(k-j)!}{j!\,(k-2j)!\,(1-k-\nu)_j\,(\nu)_j} \left(-\frac{z^2}{4}\right)^j I_\nu(z) \right) /;\ n \in \mathbb{N}$$

03.02.20.0014.02

$$\frac{\partial^n I_\nu(z)}{\partial z^n} = 2^{n-2\nu} \sqrt{\pi}\ z^{\nu-n}\ \Gamma(\nu+1)\ {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-n+\nu}{2}, \frac{2-n+\nu}{2}, \nu+1; \frac{z^2}{4}\right) /;\ n \in \mathbb{N}$$

03.02.20.0015.02

$$\frac{\partial^n I_\nu(z)}{\partial z^n} = 2^{-n} \sum_{k=0}^{n} \binom{n}{k} I_{2k-n+\nu}(z) /;\ n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to z

03.02.20.0016.01

$$\frac{\partial^\alpha I_\nu(z)}{\partial z^\alpha} = 2^{\alpha-2\nu} \sqrt{\pi}\ z^{\nu-\alpha}\ \Gamma(\nu+1)\ {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-\alpha+\nu+1), \frac{1}{2}(-\alpha+\nu+2), \nu+1; \frac{z^2}{4}\right) /;\ -\nu \notin \mathbb{N}^+$$

03.02.20.0017.01

$$\frac{\partial^\alpha I_{-n}(z)}{\partial z^\alpha} = 2^{\alpha-2n} \sqrt{\pi}\ z^{n-\alpha}\ \Gamma(n+1)\ {}_2\tilde{F}_3\left(\frac{n+1}{2}, \frac{n+2}{2}; \frac{n-\alpha+1}{2}, \frac{n-\alpha+2}{2}, n+1; \frac{z^2}{4}\right) /;\ n \in \mathbb{N}^+$$

Integration

Indefinite integration

Involving only one direct function

03.02.21.0001.01

$$\int I_\nu(a z)\, dz = 2^{-\nu-1} z (a z)^\nu\ \Gamma\left(\frac{\nu+1}{2}\right) {}_1\tilde{F}_2\left(\frac{\nu+1}{2}; \nu+1, \frac{\nu+3}{2}; \frac{a^2 z^2}{4}\right)$$

03.02.21.0002.01

$$\int I_\nu(z)\, dz = 2^{-\nu-1} z^{\nu+1}\ \Gamma\left(\frac{\nu}{2}+\frac{1}{2}\right) {}_1\tilde{F}_2\left(\frac{\nu}{2}+\frac{1}{2}; \nu+1, \frac{\nu}{2}+\frac{3}{2}; \frac{z^2}{4}\right)$$

03.02.21.0003.01

$$\int I_0(z)\, dz = \frac{1}{2} z \left(I_0(z) \left(\pi\, \boldsymbol{L}_1(z) + 2\right) - \pi\, I_1(z)\, \boldsymbol{L}_0(z)\right)$$

03.02.21.0004.01

$$\int I_1(z)\, dz = I_0(z)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear arguments

03.02.21.0005.01

$$\int z^{\alpha-1} I_\nu(a z)\, dz = 2^{-\nu-1} z^\alpha (a z)^\nu \Gamma\left(\frac{\alpha+\nu}{2}\right) {}_1\tilde{F}_2\left(\frac{\alpha+\nu}{2}; \nu+1, \frac{1}{2}(\alpha+\nu+2); \frac{a^2 z^2}{4}\right)$$

03.02.21.0006.01

$$\int z^{\alpha-1} I_\nu(z)\, dz = 2^{-\nu-1} z^{\alpha+\nu} \Gamma\left(\frac{\alpha}{2}+\frac{\nu}{2}\right) {}_1\tilde{F}_2\left(\frac{\alpha}{2}+\frac{\nu}{2}; \nu+1, \frac{\alpha}{2}+\frac{\nu}{2}+1; \frac{z^2}{4}\right)$$

03.02.21.0007.01

$$\int z^{\alpha-1} I_0(z)\, dz = \frac{z^\alpha}{\alpha} {}_1F_2\left(\frac{\alpha}{2}; 1, \frac{\alpha}{2}+1; \frac{z^2}{4}\right)$$

03.02.21.0008.01

$$\int z^{1-\nu} I_\nu(z)\, dz = z^{1-\nu} I_{\nu-1}(z)$$

03.02.21.0009.01

$$\int z^{-\nu} I_\nu(z)\, dz = \frac{2^{-\nu} z}{\Gamma(\nu+1)} {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \nu+1; \frac{z^2}{4}\right)$$

03.02.21.0010.01

$$\int z^{\nu+3} I_\nu(z)\, dz = \frac{z^{\nu+2}\, \Gamma(\nu+2)}{(\nu+1)\,\Gamma(\nu+1)} (2(\nu+1) I_{\nu+2}(z) + z I_{\nu+3}(z))$$

03.02.21.0011.01

$$\int z^{\nu+1} I_\nu(z)\, dz = z^{\nu+1} I_{\nu+1}(z)$$

03.02.21.0012.01

$$\int z^\nu I_\nu(z)\, dz = \frac{2^{-\nu} z^{2\nu+1}}{(2\nu+1)\,\Gamma(\nu+1)} {}_1F_2\left(\nu+\frac{1}{2}; \nu+1, \nu+\frac{3}{2}; \frac{z^2}{4}\right)$$

03.02.21.0013.01

$$\int z I_0(z)\, dz = z I_1(z)$$

03.02.21.0014.01

$$\int \frac{I_0(z)}{z}\, dz = -\frac{1}{2} G_{1,3}^{2,0}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} 1 \\ 0, 0, 0 \end{matrix}\right)$$

Power arguments

03.02.21.0015.01

$$\int z^{\alpha-1} I_\nu(a z^r)\, dz = \frac{2^{-\nu} z^\alpha (a z^r)^\nu}{(\alpha + r\nu)\,\Gamma(\nu+1)} {}_1F_2\left(\frac{\alpha}{2r}+\frac{\nu}{2}; \frac{\alpha}{2r}+\frac{\nu}{2}+1, \nu+1; \frac{1}{4} a^2 z^{2r}\right)$$

Involving exponential function

Involving exp

Linear arguments

03.02.21.0016.01

$$\int e^{-a z} I_\nu(a z)\, dz = \frac{2^{-\nu} z (a z)^\nu}{(\nu+1)\,\Gamma(\nu+1)}\, {}_2F_2\left(\nu+\frac{1}{2}, \nu+1; \nu+2, 2\nu+1; -2 a z\right)$$

03.02.21.0017.01

$$\int e^{a z} I_\nu(a z)\, dz = \frac{2^{-\nu} z (a z)^\nu}{(\nu+1)\,\Gamma(\nu+1)}\, {}_2F_2\left(\nu+\frac{1}{2}, \nu+1; \nu+2, 2\nu+1; 2 a z\right)$$

Power arguments

03.02.21.0018.01

$$\int e^{-a z^r} I_\nu(a z^r)\, dz = \frac{2^{-\nu} z (a z^r)^\nu}{r \nu\, \Gamma(\nu+1) + \Gamma(\nu+1)}\, {}_2F_2\left(\nu+\frac{1}{2}, \nu+\frac{1}{r}; \nu+\frac{1}{r}+1, 2\nu+1; -2 a z^r\right)$$

03.02.21.0019.01

$$\int e^{a z^r} I_\nu(a z^r)\, dz = \frac{2^{-\nu} z (a z^r)^\nu}{r \nu\, \Gamma(\nu+1) + \Gamma(\nu+1)}\, {}_2F_2\left(\nu+\frac{1}{2}, \nu+\frac{1}{r}; \nu+\frac{1}{r}+1, 2\nu+1; 2 a z^r\right)$$

Involving exponential function and a power function

Involving exp and power

Linear arguments

03.02.21.0020.01

$$\int z^{\alpha-1} e^{-a z} I_\nu(a z)\, dz = \frac{2^{-\nu} z^\alpha (a z)^\nu}{(\alpha+\nu)\,\Gamma(\nu+1)}\, {}_2F_2\left(\nu+\frac{1}{2}, \alpha+\nu; \alpha+\nu+1, 2\nu+1; -2 a z\right)$$

03.02.21.0021.01

$$\int z^{-\nu} e^{-a z} I_\nu(a z)\, dz = \frac{2^{-\nu} e^{-a z} z^{-\nu}}{a (2\nu-1)\,\Gamma(\nu)} \left(2 e^{a z} (a z)^\nu - 2^\nu a z \left(I_{\nu-1}(a z) + I_\nu(a z)\right) \Gamma(\nu)\right)$$

03.02.21.0022.01

$$\int z^\nu e^{-a z} I_\nu(a z)\, dz = \frac{e^{-a z} z^{\nu+1}}{2\nu+1} \left(I_\nu(a z) + I_{\nu+1}(a z)\right)$$

03.02.21.0023.01

$$\int z^{\alpha-1} e^{a z} I_\nu(a z)\, dz = \frac{2^{-\nu} z^\alpha (a z)^\nu}{(\alpha+\nu)\,\Gamma(\nu+1)}\, {}_2F_2\left(\nu+\frac{1}{2}, \alpha+\nu; \alpha+\nu+1, 2\nu+1; 2 a z\right)$$

03.02.21.0024.01

$$\int z^{-\nu} e^{a z} I_\nu(a z)\, dz = \frac{2^{-\nu} z^{-\nu}}{a (2\nu-1)\,\Gamma(\nu)} \left(2^\nu a e^{a z} z \left(I_{\nu-1}(a z) - I_\nu(a z)\right) \Gamma(\nu) - 2 (a z)^\nu\right)$$

03.02.21.0025.01

$$\int z^{\nu} e^{a z} I_{\nu}(a z) \, dz = \frac{e^{a z} z^{\nu+1}}{2 \nu + 1} (I_{\nu}(a z) - I_{\nu+1}(a z))$$

Power arguments

03.02.21.0026.01

$$\int z^{\alpha-1} e^{-a z^r} I_{\nu}(a z^r) \, dz = \frac{2^{-\nu} z^{\alpha} (a z^r)^{\nu}}{(\alpha + r \nu) \Gamma(\nu + 1)} {}_2F_2\left(\nu + \frac{1}{2}, \frac{\alpha}{r} + \nu; \frac{\alpha}{r} + \nu + 1, 2 \nu + 1; -2 a z^r\right)$$

03.02.21.0027.01

$$\int z^{\alpha-1} e^{a z^r} I_{\nu}(a z^r) \, dz = \frac{2^{-\nu} z^{\alpha} (a z^r)^{\nu}}{(\alpha + r \nu) \Gamma(\nu + 1)} {}_2F_2\left(\nu + \frac{1}{2}, \frac{\alpha}{r} + \nu; \frac{\alpha}{r} + \nu + 1, 2 \nu + 1; 2 a z^r\right)$$

Involving hyperbolic functions

Involving sinh

Linear arguments

03.02.21.0028.01

$$\int \sinh(a z) I_{\nu}(a z) \, dz = \frac{2^{-\nu} z (a z)^{\nu+1}}{(\nu + 2) \Gamma(\nu + 1)} {}_3F_4\left(\frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + 1, \frac{\nu}{2} + \frac{5}{4}; \frac{3}{2}, \frac{\nu}{2} + 2, \nu + 1, \nu + \frac{3}{2}; a^2 z^2\right)$$

03.02.21.0029.01

$$\int \sinh(b + a z) I_{\nu}(a z) \, dz = \frac{1}{(\nu + 1)(\nu + 2) \Gamma(\nu + 1)} \left(2^{-\nu} z (a z)^{\nu} \left(a z (\nu + 1) \cosh(b) {}_3F_4\left(\frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + 1, \frac{\nu}{2} + \frac{5}{4}; \frac{3}{2}, \frac{\nu}{2} + 2, \nu + 1, \nu + \frac{3}{2}; a^2 z^2\right) + (\nu + 2) {}_3F_4\left(\frac{\nu}{2} + \frac{1}{4}, \frac{\nu}{2} + \frac{1}{2}, \frac{\nu}{2} + \frac{3}{4}; \frac{1}{2}, \frac{\nu}{2} + \frac{3}{2}, \nu + \frac{1}{2}, \nu + 1; a^2 z^2\right) \sinh(b)\right)\right)$$

Power arguments

03.02.21.0030.01

$$\int \sinh(a z^r) I_{\nu}(a z^r) \, dz = \frac{2^{-\nu} z (a z^r)^{\nu+1}}{(\nu r + r + 1) \Gamma(\nu + 1)} {}_3F_4\left(\frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{5}{4}, \frac{\nu}{2} + \frac{1}{2} + \frac{1}{2 r}; \frac{3}{2}, \frac{\nu}{2} + \frac{3}{2} + \frac{1}{2 r}, \nu + 1, \nu + \frac{3}{2}; a^2 z^{2 r}\right)$$

03.02.21.0031.01

$$\int \sinh(a z^r + b) I_{\nu}(a z^r) \, dz = \frac{1}{(r \nu + 1)(\nu r + r + 1) \Gamma(\nu + 1)} \left(2^{-\nu} z (a z^r)^{\nu} \left(a (r \nu + 1) \cosh(b) {}_3F_4\left(\frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{5}{4}, \frac{\nu}{2} + \frac{1}{2} + \frac{1}{2 r}; \frac{3}{2}, \frac{\nu}{2} + \frac{3}{2} + \frac{1}{2 r}, \nu + 1, \nu + \frac{3}{2}; a^2 z^{2 r}\right) z^r + (\nu r + r + 1) {}_3F_4\left(\frac{\nu}{2} + \frac{1}{4}, \frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{1}{2 r}; \frac{1}{2}, \frac{\nu}{2} + \frac{1}{2 r} + 1, \nu + \frac{1}{2}, \nu + 1; a^2 z^{2 r}\right) \sinh(b)\right)\right)$$

Involving cosh

Linear arguments

03.02.21.0032.01

$$\int \cosh(a z)\, I_\nu(a z)\, dz = \frac{2^{-\nu} z (a z)^\nu}{(\nu+1)\,\Gamma(\nu+1)}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}, \frac{\nu}{2}+\frac{3}{2}, \nu+\frac{1}{2}, \nu+1; a^2 z^2\right)$$

03.02.21.0033.01

$$\int \cosh(b + a z)\, I_\nu(a z)\, dz = \frac{1}{(\nu+1)(\nu+2)\,\Gamma(\nu+1)} \left(2^{-\nu} z (a z)^\nu \left((\nu+2)\cosh(b)\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}, \frac{\nu}{2}+\frac{3}{2}, \nu+\frac{1}{2}, \nu+1; a^2 z^2\right) + a z (\nu+1)\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}, \frac{\nu}{2}+2, \nu+1, \nu+\frac{3}{2}; a^2 z^2\right)\sinh(b)\right)\right)$$

Power arguments

03.02.21.0034.01

$$\int \cosh(a z^r)\, I_\nu(a z^r)\, dz = \frac{2^{-\nu} z (a z^r)^\nu}{r \nu\,\Gamma(\nu+1) + \Gamma(\nu+1)}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{1}{2r}; \frac{1}{2}, \frac{\nu}{2}+\frac{1}{2r}+1, \nu+\frac{1}{2}, \nu+1; a^2 z^{2r}\right)$$

03.02.21.0035.01

$$\int \cosh(a z^r + b)\, I_\nu(a z^r)\, dz = \frac{1}{(r\nu+1)(\nu r + r + 1)\,\Gamma(\nu+1)} \left(2^{-\nu} z (a z^r)^\nu \left(a (r\nu+1)\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}, \frac{\nu}{2}+\frac{1}{2}+\frac{1}{2r}; \frac{3}{2}, \frac{\nu}{2}+\frac{3}{2}+\frac{1}{2r}, \nu+1, \nu+\frac{3}{2}; a^2 z^{2r}\right)\sinh(b)\, z^r + (\nu r + r + 1)\cosh(b)\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{1}{2r}; \frac{1}{2}, \frac{\nu}{2}+\frac{1}{2r}+1, \nu+\frac{1}{2}, \nu+1; a^2 z^{2r}\right)\right)\right)$$

Involving hyperbolic functions and a power function

Involving sinh and power

Linear arguments

03.02.21.0036.01

$$\int z^{\alpha-1} \sinh(a z)\, I_\nu(a z)\, dz = \frac{2^{-\nu} z^\alpha (a z)^{\nu+1}}{(\alpha+\nu+1)\,\Gamma(\nu+1)}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}, \frac{\alpha}{2}+\frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+\frac{3}{2}, \nu+1, \nu+\frac{3}{2}; a^2 z^2\right)$$

03.02.21.0037.01

$$\int z^{\alpha-1} \sinh(b + a z)\, I_\nu(a z)\, dz = \frac{1}{(\alpha+\nu)(\alpha+\nu+1)\,\Gamma(\nu+1)} \left(2^{-\nu} z^\alpha (a z)^\nu \left(a z (\alpha+\nu)\cosh(b)\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}, \frac{\alpha}{2}+\frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+\frac{3}{2}, \nu+1, \nu+\frac{3}{2}; a^2 z^2\right) + (\alpha+\nu+1)\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}, \frac{\alpha}{2}+\frac{\nu}{2}; \frac{1}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+1, \nu+\frac{1}{2}, \nu+1; a^2 z^2\right)\sinh(b)\right)\right)$$

Power arguments

03.02.21.0038.01

$$\int z^{\alpha-1}\sinh(a\,z^r)\,I_\nu(a\,z^r)\,dz = \frac{2^{-\nu}\,z^\alpha\,(a\,z^r)^{\nu+1}}{(\nu\,r+r+\alpha)\,\Gamma(\nu+1)}\,{}_3F_4\left(\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4},\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{3}{2},\nu+1,\nu+\frac{3}{2};a^2\,z^{2\,r}\right)$$

03.02.21.0039.01

$$\int z^{\alpha-1}\sinh(a\,z^r+b)\,I_\nu(a\,z^r)\,dz = \frac{1}{(\alpha+r\,\nu)\,(\nu\,r+r+\alpha)\,\Gamma(\nu+1)}\left(2^{-\nu}\,z^\alpha\,(a\,z^r)^\nu\left(a\,(\alpha+r\,\nu)\cosh(b)\,{}_3F_4\left(\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4},\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{3}{2},\nu+1,\nu+\frac{3}{2};a^2\,z^{2\,r}\right)z^r+\right.\right.$$
$$\left.\left.(\nu\,r+r+\alpha)\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4},\frac{\alpha}{2\,r}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+1,\nu+\frac{1}{2},\nu+1;a^2\,z^{2\,r}\right)\sinh(b)\right)\right)$$

Involving cosh and power

Linear arguments

03.02.21.0040.01

$$\int z^{\alpha-1}\cosh(a\,z)\,I_\nu(a\,z)\,dz = \frac{2^{-\nu}\,z^\alpha\,(a\,z)^\nu}{(\alpha+\nu)\,\Gamma(\nu+1)}\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4},\frac{\alpha}{2}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2}+\frac{\nu}{2}+1,\nu+\frac{1}{2},\nu+1;a^2\,z^2\right)$$

03.02.21.0041.01

$$\int z^{\alpha-1}\cosh(b+a\,z)\,I_\nu(a\,z)\,dz = \frac{1}{(\alpha+\nu)\,(\alpha+\nu+1)\,\Gamma(\nu+1)}\left(2^{-\nu}\,z^\alpha\,(a\,z)^\nu\left((\alpha+\nu+1)\cosh(b)\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4},\frac{\alpha}{2}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2}+\frac{\nu}{2}+1,\nu+\frac{1}{2},\nu+1;a^2\,z^2\right)+\right.\right.$$
$$\left.\left.a\,z\,(\alpha+\nu)\,{}_3F_4\left(\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4},\frac{\alpha}{2}+\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},\frac{\alpha}{2}+\frac{\nu}{2}+\frac{3}{2},\nu+1,\nu+\frac{3}{2};a^2\,z^2\right)\sinh(b)\right)\right)$$

Power arguments

03.02.21.0042.01

$$\int z^{\alpha-1}\cosh(a\,z^r)\,I_\nu(a\,z^r)\,dz = \frac{2^{-\nu}\,z^\alpha\,(a\,z^r)^\nu}{(\alpha+r\,\nu)\,\Gamma(\nu+1)}\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4},\frac{\alpha}{2\,r}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+1,\nu+\frac{1}{2},\nu+1;a^2\,z^{2\,r}\right)$$

03.02.21.0043.01

$$\int z^{\alpha-1}\cosh(a\,z^r+b)\,I_\nu(a\,z^r)\,dz = \frac{1}{(\alpha+r\,\nu)\,(\nu\,r+r+\alpha)\,\Gamma(\nu+1)}\left(2^{-\nu}\,z^\alpha\,(a\,z^r)^\nu\left(a\,(\alpha+r\,\nu)\,{}_3F_4\left(\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4},\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{3}{2},\nu+1,\nu+\frac{3}{2};a^2\,z^{2\,r}\right)\sinh(b)\,z^r+\right.\right.$$
$$\left.\left.(\nu\,r+r+\alpha)\cosh(b)\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4},\frac{\alpha}{2\,r}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+1,\nu+\frac{1}{2},\nu+1;a^2\,z^{2\,r}\right)\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

Linear arguments

03.02.21.0044.01

$$\int I_\nu(a\,z)^2\,dz = \frac{4^{-\nu}\,z\,(a\,z)^{2\,\nu}}{(2\,\nu+1)\,\Gamma(\nu+1)^2}\,{}_2F_3\left(\nu+\frac{1}{2},\,\nu+\frac{1}{2};\,\nu+1,\,\nu+\frac{3}{2},\,2\,\nu+1;\,a^2\,z^2\right)$$

03.02.21.0045.01

$$\int I_\nu(z)^2\,dz = \frac{4^{-\nu}\,z^{2\,\nu+1}}{(2\,\nu+1)\,\Gamma(\nu+1)^2}\,{}_2F_3\left(\nu+\frac{1}{2},\,\nu+\frac{1}{2};\,\nu+1,\,\nu+\frac{3}{2},\,2\,\nu+1;\,z^2\right)$$

03.02.21.0046.01

$$\int \frac{1}{z\,I_{-\nu}(z)\,I_\nu(z)}\,dz = -\frac{1}{2}\,\pi\,\csc(\pi\,\nu)\,\log\left(\frac{I_{-\nu}(z)}{I_\nu(z)}\right)$$

Power arguments

03.02.21.0047.01

$$\int I_\nu(a\,z^r)^2\,dz = \frac{4^{-\nu}\,z\,(a\,z^r)^{2\,\nu}}{(2\,r\,\nu+1)\,\Gamma(\nu+1)^2}\,{}_2F_3\left(\nu+\frac{1}{2},\,\nu+\frac{1}{2\,r};\,\nu+1,\,\nu+\frac{1}{2\,r}+1,\,2\,\nu+1;\,a^2\,z^{2\,r}\right)$$

Involving products of the direct function

Linear arguments

03.02.21.0048.01

$$\int I_\mu(a\,z)\,I_\nu(a\,z)\,dz = \frac{2^{-\mu-\nu}\,z\,(a\,z)^{\mu+\nu}}{(\mu+\nu+1)\,\Gamma(\mu+1)\,\Gamma(\nu+1)}\,{}_3F_4\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\,\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\,\frac{\mu}{2}+\frac{\nu}{2}+1;\,\mu+1,\,\frac{\mu}{2}+\frac{\nu}{2}+\frac{3}{2},\,\nu+1,\,\mu+\nu+1;\,a^2\,z^2\right)$$

03.02.21.0049.01

$$\int I_\nu(a\,z)\,I_{\nu+1}(a\,z)\,dz = \frac{2^{-2\,(\nu+1)}\,z\,(a\,z)^{2\,\nu+1}}{(\nu+1)\,\Gamma(\nu+1)\,\Gamma(\nu+2)}\,{}_2F_3\left(\nu+1,\,\nu+\frac{3}{2};\,\nu+2,\,\nu+2,\,2\,\nu+2;\,a^2\,z^2\right)$$

03.02.21.0050.01

$$\int I_0(a\,z)\,I_1(a\,z)\,dz = \frac{I_0(a\,z)^2}{2\,a}$$

Power arguments

03.02.21.0051.01

$$\int I_\mu(a\,z^r)\,I_\nu(a\,z^r)\,dz = \frac{2^{-\mu-\nu}\,z\,(a\,z^r)^{\mu+\nu}}{(r\,(\mu+\nu)+1)\,\Gamma(\mu+1)\,\Gamma(\nu+1)}\,{}_3F_4\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\,\frac{\mu}{2}+\frac{\nu}{2}+1,\,\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2\,r};\,\mu+1,\,\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2\,r}+1,\,\nu+1,\,\mu+\nu+1;\,a^2\,z^{2\,r}\right)$$

03.02.21.0052.01

$$\int I_\nu\left(a\sqrt{z}\right) I_\nu\left(b\sqrt{z}\right) dz = \frac{2\sqrt{z}}{a^2-b^2}\left(a I_\nu\left(b\sqrt{z}\right) I_{\nu+1}\left(a\sqrt{z}\right) - b I_{\nu+1}\left(b\sqrt{z}\right) I_\nu\left(a\sqrt{z}\right)\right)$$

03.02.21.0053.01

$$\int I_{-\nu}\left(a\sqrt{z}\right) I_\nu\left(b\sqrt{z}\right) dz = \frac{2\sqrt{z}}{b^2-a^2}\left(b I_{\nu+1}\left(b\sqrt{z}\right) I_{-\nu}\left(a\sqrt{z}\right) - a I_\nu\left(b\sqrt{z}\right) I_{-\nu-1}\left(a\sqrt{z}\right)\right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

Linear arguments

03.02.21.0054.01

$$\int z^{\alpha-1} I_\nu(a z)^2 dz = \frac{4^{-\nu} z^\alpha (a z)^{2\nu}}{(\alpha+2\nu)\,\Gamma(\nu+1)^2}\, {}_2F_3\left(\nu+\frac{1}{2}, \frac{\alpha}{2}+\nu; \nu+1, \frac{\alpha}{2}+\nu+1, 2\nu+1; a^2 z^2\right)$$

03.02.21.0055.01

$$\int z^{1-2\nu} I_\nu(a z)^2 dz = \frac{2^{-2\nu-1} z^{-2\nu}}{a^2 (2\nu-1)\,\Gamma(\nu)^2}\left(-4 (a z)^{2\nu} + 4^\nu a^2 z^2 I_{\nu-1}(a z)^2\, \Gamma(\nu)^2 - 4^\nu a^2 z^2 I_\nu(a z)^2\, \Gamma(\nu)^2\right)$$

03.02.21.0056.01

$$\int z^{2\nu+1} I_\nu(a z)^2 dz = \frac{z^{2(\nu+1)}}{4\nu+2}\left(I_\nu(a z)^2 - I_{\nu+1}(a z)^2\right)$$

03.02.21.0057.01

$$\int z I_\nu(a z)^2 dz = \frac{1}{2} z^2 \left(I_\nu(a z)^2 - I_{\nu-1}(a z) I_{\nu+1}(a z)\right)$$

03.02.21.0058.01

$$\int z I_0(a z)^2 dz = \frac{1}{2} z^2 \left(I_0(a z)^2 - I_1(a z)^2\right)$$

03.02.21.0059.01

$$\int \frac{1}{z I_\nu(z)^2} dz = -\frac{K_\nu(z)}{I_\nu(z)}$$

03.02.21.0060.01

$$\int \frac{I_\nu(a z)^2}{z} dz = \frac{2^{-2\nu-1} (a z)^{2\nu}}{\nu^3\, \Gamma(\nu)^2}\, {}_2F_3\left(\nu, \nu+\frac{1}{2}; \nu+1, \nu+1, 2\nu+1; a^2 z^2\right)$$

03.02.21.0061.01

$$\int \frac{I_\nu(a z)^2}{z^2} dz = \frac{1}{z\left(4\nu^2-1\right)}\left(2 a^2 z^2 I_{\nu-1}(a z)^2 - 2 a z I_\nu(a z) I_{\nu-1}(a z) + I_\nu(a z)\left((1-2\nu) I_\nu(a z) - 2 a^2 z^2 I_{\nu-2}(a z)\right)\right)$$

Power arguments

03.02.21.0062.01

$$\int z^{\alpha-1}\, I_\nu(a\, z^r)^2\, dz = \frac{4^{-\nu}\, z^\alpha\, (a\, z^r)^{2\,\nu}}{(\alpha+2\,r\,\nu)\,\Gamma(\nu+1)^2}\, {}_2F_3\!\left(\nu+\frac{1}{2},\, \frac{\alpha}{2\,r}+\nu;\, \nu+1,\, \frac{\alpha}{2\,r}+\nu+1,\, 2\,\nu+1;\, a^2\, z^{2\,r}\right)$$

Involving products of the direct function and a power function

Linear arguments

03.02.21.0063.01

$$\int z^{\alpha-1}\, I_\mu(a\,z)\, I_\nu(a\,z)\, dz = \frac{2^{-\mu-\nu}\, z^\alpha\, (a\,z)^{\mu+\nu}}{(\alpha+\mu+\nu)\,\Gamma(\mu+1)\,\Gamma(\nu+1)}\, {}_3F_4\!\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\, \frac{\mu}{2}+\frac{\nu}{2}+1,\, \frac{\alpha}{2}+\frac{\mu}{2}+\frac{\nu}{2};\, \mu+1,\, \frac{\alpha}{2}+\frac{\mu}{2}+\frac{\nu}{2}+1,\, \nu+1,\, \mu+\nu+1;\, a^2\, z^2\right)$$

03.02.21.0064.01

$$\int z^{1-\mu-\nu}\, I_\mu(a\,z)\, I_\nu(a\,z)\, dz = \frac{2^{-\mu-\nu+1}\, z^{-\mu-\nu}\, (a\,z)^{\mu+\nu}\, \mu\, \nu}{a^2\, (\mu+\nu-1)\,\Gamma(\mu+1)\,\Gamma(\nu+1)} \left({}_2F_3\!\left(\frac{\mu}{2}+\frac{\nu}{2}-\frac{1}{2},\, \frac{\mu}{2}+\frac{\nu}{2};\, \mu,\, \nu,\, \mu+\nu;\, a^2\, z^2\right)-1\right)$$

03.02.21.0065.01

$$\int z^{\mu+\nu+1}\, I_\mu(a\,z)\, I_\nu(a\,z)\, dz = \frac{2^{-\mu-\nu-1}\, z^{\mu+\nu+2}\, (a\,z)^{\mu+\nu}}{(\mu+\nu+1)\,\Gamma(\mu+1)\,\Gamma(\nu+1)}\, {}_2F_3\!\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\, \frac{\mu}{2}+\frac{\nu}{2}+1;\, \mu+1,\, \nu+1,\, \mu+\nu+2;\, a^2\, z^2\right)$$

03.02.21.0066.01

$$\int z\, I_\nu(a\,z)\, I_\nu(b\,z)\, dz = \frac{z}{a^2-b^2}\, (a\, I_\nu(b\,z)\, I_{\nu+1}(a\,z) - b\, I_\nu(a\,z)\, I_{\nu+1}(b\,z))$$

03.02.21.0067.01

$$\int z\, I_{-\nu}(a\,z)\, I_\nu(b\,z)\, dz = \frac{z}{b^2-a^2}\, (b\, I_{\nu+1}(b\,z)\, I_{-\nu}(a\,z) - a\, I_\nu(b\,z)\, I_{-\nu-1}(a\,z))$$

03.02.21.0068.01

$$\int \frac{\left(-\left(a^2-b^2\right) z^2+\mu^2-\nu^2\right) I_\nu(a\,z)\, I_\mu(b\,z)}{z}\, dz = b\, z\, I_{\mu-1}(b\,z)\, I_\nu(a\,z) - I_\mu(b\,z)\, (a\, z\, I_{\nu-1}(a\,z) + (\mu-\nu)\, I_\nu(a\,z))$$

03.02.21.0069.01

$$\int \frac{I_\mu(a\,z)\, I_\nu(a\,z)}{z}\, dz = \frac{1}{\mu^2-\nu^2}\left(a\, z\, I_{\mu-1}(a\,z)\, I_\nu(a\,z) - I_\mu(a\,z)\, (a\, z\, I_{\nu-1}(a\,z) + (\mu-\nu)\, I_\nu(a\,z))\right)$$

03.02.21.0070.01

$$\int \frac{I_\mu(a\,z)\, I_\nu(a\,z)}{z^2}\, dz = \frac{2^{-\mu-\nu}\, a\, (a\,z)^{\mu+\nu-1}}{(\mu+\nu-1)\,\Gamma(\mu+1)\,\Gamma(\nu+1)}\, {}_2F_3\!\left(\frac{\mu}{2}+\frac{\nu}{2}-\frac{1}{2},\, \frac{\mu}{2}+\frac{\nu}{2}+1;\, \mu+1,\, \nu+1,\, \mu+\nu+1;\, a^2\, z^2\right)$$

03.02.21.0071.01

$$\int \frac{I_{\nu-1}(a\,z)\, I_\nu(a\,z)}{z^2}\, dz = \frac{4^{-\nu}\, a\, (a\,z)^{2\,(\nu-1)}}{(\nu-1)\,\Gamma(\nu)\,\Gamma(\nu+1)}\, {}_2F_3\!\left(\nu-1,\, \nu+\frac{1}{2};\, \nu,\, 2\,\nu,\, \nu+1;\, a^2\, z^2\right)$$

Power arguments

03.02.21.0072.01

$$\int z^{\alpha-1} I_\mu(a z^r) I_\nu(a z^r) \, dz = \frac{2^{-\mu-\nu} z^\alpha (a z^r)^{\mu+\nu}}{(\alpha + r(\mu+\nu)) \Gamma(\mu+1) \Gamma(\nu+1)} {}_3F_4\left(\frac{\mu}{2} + \frac{\nu}{2} + \frac{1}{2}, \frac{\mu}{2} + \frac{\nu}{2} + 1, \frac{\alpha}{2r} + \frac{\mu}{2} + \frac{\nu}{2}; \mu+1, \frac{\alpha}{2r} + \frac{\mu}{2} + \frac{\nu}{2} + 1, \nu+1, \mu+\nu+1; a^2 z^{2r}\right)$$

03.02.21.0073.01

$$\int z^{\alpha-1} I_{\nu-1}(a z^r) I_\nu(a z^r) \, dz = \frac{2^{1-2\nu} z^\alpha (a z^r)^{2\nu-1}}{(\alpha + r(2\nu-1)) \Gamma(\nu) \Gamma(\nu+1)} {}_2F_3\left(\nu + \frac{1}{2}, \frac{\alpha}{2r} + \nu - \frac{1}{2}; 2\nu, \nu+1, \frac{\alpha}{2r} + \nu + \frac{1}{2}; a^2 z^{2r}\right)$$

03.02.21.0074.01

$$\int z^{\alpha-1} I_{-\nu}(a z^r) I_\nu(a z^r) \, dz = \frac{z^\alpha \sin(\pi\nu)}{\pi \alpha \nu} {}_2F_3\left(\frac{1}{2}, \frac{\alpha}{2r}; \frac{\alpha}{2r} + 1, 1-\nu, \nu+1; a^2 z^{2r}\right)$$

Involving direct function and Bessel-type functions

Involving Bessel functions

Involving Bessel J

Linear arguments

03.02.21.0075.01

$$\int J_\nu(a z) I_\nu(a z) \, dz = 2^{-3\nu-2} \sqrt{\pi} \, z (a z)^{2\nu} \Gamma\left(\frac{\nu}{2} + \frac{1}{4}\right) {}_1\tilde{F}_4\left(\frac{1}{4}(2\nu+1); \nu+1, \frac{\nu+1}{2}, \frac{\nu+2}{2}, \frac{1}{4}(2\nu+5); -\frac{1}{64} a^4 z^4\right)$$

03.02.21.0076.01

$$\int J_{-\nu}(a z) I_\nu(a z) \, dz = \frac{1}{4} \sqrt{\pi} \, z \, G_{2,6}^{2,1}\left(\frac{a z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{matrix} \frac{3}{4}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, -\frac{1}{4}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

Power arguments

03.02.21.0077.01

$$\int J_\nu(a z^r) I_\nu(a z^r) \, dz = \frac{2^{-3\nu-2} \sqrt{\pi} \, z (a z^r)^{2\nu}}{r} \Gamma\left(\frac{1}{4}\left(2\nu + \frac{1}{r}\right)\right) {}_1\tilde{F}_4\left(\frac{1}{4}\left(2\nu + \frac{1}{r}\right); \nu+1, \frac{\nu+1}{2}, \frac{\nu+2}{2}, \frac{1}{4}\left(2\nu + \frac{1}{r} + 4\right); -\frac{1}{64} a^4 z^{4r}\right)$$

03.02.21.0078.01

$$\int J_{-\nu}(a z^r) I_\nu(a z^r) \, dz = \frac{\sqrt{\pi} \, z}{4r} G_{2,6}^{2,1}\left(\frac{a z^r}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{matrix} 1 - \frac{1}{4r}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, -\frac{1}{4r}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

03.02.21.0079.01

$$\int J_\nu\left(a\sqrt{z}\right) I_\nu\left(b\sqrt{z}\right) dz = \frac{2\sqrt{z}}{a^2+b^2}\left(b \, I_{\nu+1}\left(b\sqrt{z}\right) J_\nu\left(a\sqrt{z}\right) + a \, I_\nu\left(b\sqrt{z}\right) J_{\nu+1}\left(a\sqrt{z}\right)\right)$$

03.02.21.0080.01

$$\int J_{-\nu}\left(a\sqrt{z}\right) I_\nu\left(b\sqrt{z}\right) dz = \frac{2\sqrt{z}}{a^2+b^2}\left(b \, I_{\nu+1}\left(b\sqrt{z}\right) J_{-\nu}\left(a\sqrt{z}\right) - a \, I_\nu\left(b\sqrt{z}\right) J_{-\nu-1}\left(a\sqrt{z}\right)\right)$$

Involving Bessel J and power

Linear arguments

03.02.21.0081.01

$$\int z^{\alpha-1} J_\nu(a z) I_\nu(a z) \, dz = 2^{-3\nu-2} \sqrt{\pi} \, z^\alpha (a z)^{2\nu} \Gamma\left(\frac{1}{4}(\alpha+2\nu)\right) {}_1\tilde{F}_4\left(\frac{1}{4}(\alpha+2\nu); \nu+1, \frac{\nu+1}{2}, \frac{\nu+2}{2}, \frac{1}{4}(\alpha+2\nu+4); -\frac{1}{64} a^4 z^4\right)$$

03.02.21.0082.01

$$\int z^{\alpha-1} J_{-\nu}(a z) I_\nu(a z) \, dz = \frac{1}{4} \sqrt{\pi} \, z^\alpha G_{2,6}^{2,1}\left(\frac{a z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{matrix} 1-\frac{\alpha}{4}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, -\frac{\alpha}{4}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

03.02.21.0083.01

$$\int z J_\nu(a z) I_\nu(b z) \, dz = \frac{z}{a^2+b^2} \left(b I_{\nu+1}(b z) J_\nu(a z) + a I_\nu(b z) J_{\nu+1}(a z)\right)$$

03.02.21.0084.01

$$\int z J_{-\nu}(a z) I_\nu(b z) \, dz = \frac{z}{a^2+b^2} \left(b I_{\nu+1}(b z) J_{-\nu}(a z) - a I_\nu(b z) J_{-\nu-1}(a z)\right)$$

Power arguments

03.02.21.0085.01

$$\int z^{\alpha-1} J_\nu(a z^r) I_\nu(a z^r) \, dz = \frac{2^{-3\nu-2} \sqrt{\pi} \, z^\alpha (a z^r)^{2\nu} \Gamma\left(\frac{\alpha+2 r \nu}{4 r}\right)}{r} {}_1\tilde{F}_4\left(\frac{\alpha+2 r \nu}{4 r}; \nu+1, \frac{\nu+1}{2}, \frac{\nu+2}{2}, \frac{1}{4}\left(\frac{\alpha}{r}+2\nu+4\right); -\frac{1}{64} a^4 z^{4 r}\right)$$

03.02.21.0086.01

$$\int z^{\alpha-1} J_{-\nu}(a z^r) I_\nu(a z^r) \, dz = \frac{\sqrt{\pi} \, z^\alpha}{4 r} G_{2,6}^{2,1}\left(\frac{a z^r}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{matrix} 1-\frac{\alpha}{4 r}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, -\frac{\alpha}{4 r}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

Definite integration

For the direct function itself

03.02.21.0087.01

$$\int_0^\infty t^{\alpha-1} e^{-t} I_\nu(t) \, dt = \frac{1}{2}\left(\frac{2^{-\nu} \Gamma(\alpha+\nu)}{\Gamma(\nu+1)} {}_2F_1\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1; 1\right) + \right.$$

$$\frac{2^\alpha}{\Gamma\left(\frac{1}{2}(-\alpha+\nu+1)\right)} \Gamma\left(\frac{\alpha+\nu+1}{2}\right) \sin\left(\frac{\pi(\alpha+\nu)}{2}\right) {}_2F_1\left(\frac{\alpha-\nu+1}{2}, \frac{\alpha+\nu+1}{2}; \frac{3}{2}; 1\right) +$$

$$\left.\frac{2^{\alpha-1}}{\Gamma\left(\frac{1}{2}(\nu-\alpha+2)\right)} \cos\left(\frac{\pi(\alpha+\nu)}{2}\right) \Gamma\left(\frac{\alpha+\nu}{2}\right) {}_2F_1\left(\frac{\alpha-\nu}{2}, \frac{\alpha+\nu}{2}; \frac{1}{2}; 1\right)\right) /; \operatorname{Re}(\alpha+\nu) > 0 \bigwedge \operatorname{Re}(\alpha) < \frac{1}{2}$$

Integral transforms

Laplace transforms

03.02.22.0001.01

$$\mathcal{L}_t[I_\nu(t)](z) == 2^{-\nu} z^{-\nu-1} {}_2F_1\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \nu+1; \frac{1}{z^2}\right) /; \operatorname{Re}(\nu) > -1$$

Summation

Infinite summation

03.02.23.0001.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k I_{k+\nu}(x) x^k}{k!} == J_\nu(x)$$

03.02.23.0002.01

$$\sum_{k=1}^{\infty} (-1)^k I_k(x)^2 == \frac{1}{2}\left(1 - I_0(x)^2\right)$$

03.02.23.0003.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k (2k+\nu)\Gamma(k+\nu)}{k!} I_{2k+\nu}(x) == \left(\frac{x}{2}\right)^\nu$$

03.02.23.0004.01

$$\sum_{k=1}^{\infty} \frac{I_{2k}(x)}{k} == \frac{1}{2}\left(\log\left(\frac{x}{2}\right) + \gamma\right) I_0(x) + \frac{K_0(x)}{2}$$

03.02.23.0005.01

$$\sum_{k=1}^{\infty} \frac{(2k+\nu)}{k(k+\nu)} I_{2k+\nu}(x) == \frac{(-1)^{\nu-1}\nu!}{2}\left(\sum_{k=0}^{\nu-1} \frac{(-1)^k}{(\nu-k)\,k!}\left(\frac{x}{2}\right)^k I_k(x)\right)\left(\frac{x}{2}\right)^{-\nu} + (-1)^\nu K_\nu(x) + I_\nu(x)\left(\log\left(\frac{x}{2}\right) - \psi(\nu+1)\right) /; \nu \in \mathbb{N}$$

03.02.23.0006.01

$$\sum_{k=-\infty}^{\infty} I_k(x) t^k == \exp\left(\frac{1}{2} x \left(t + \frac{1}{t}\right)\right)$$

03.02.23.0007.01

$$\sum_{k=1}^{\infty} \cos(k t) I_k(x) == \frac{1}{2}\left(e^{x \cos(t)} - I_0(x)\right)$$

03.02.23.0008.01

$$\sum_{k=1}^{\infty} I_k(x) == \frac{1}{2}(e^x - I_0(x))$$

03.02.23.0009.01

$$\sum_{k=1}^{\infty} (-1)^k I_k(x) == \frac{1}{2}(e^{-x} - I_0(x))$$

03.02.23.0010.01

$$\sum_{k=1}^{\infty} (-1)^k \cos(2kt) I_{2k}(x) == \frac{1}{2}(\cosh(x \sin(t)) - I_0(x))$$

03.02.23.0011.01

$$\sum_{k=0}^{\infty}(-1)^k \sin((2k+1)t)\, I_{2k+1}(x) = \frac{1}{2}\sinh(x\sin(t))$$

03.02.23.0012.01

$$\sum_{k=1}^{\infty}\cos(2kt)\, I_{2k}(x) = \frac{1}{2}(\cosh(x\cos(t)) - I_0(x))$$

03.02.23.0013.01

$$\sum_{k=0}^{\infty}\cos((2k+1)t)\, I_{2k+1}(x) = \frac{1}{2}\sinh(x\cos(t))$$

03.02.23.0014.01

$$\sum_{k=1}^{\infty}(-1)^k I_{2k}(x) = \frac{1}{2}(1 - I_0(x))$$

03.02.23.0015.01

$$\sum_{k=1}^{\infty} I_{2k}(x) = \frac{1}{2}(\cosh(x) - I_0(x))$$

03.02.23.0016.01

$$\sum_{k=0}^{\infty} I_{2k+1}(x) = \frac{\sinh(x)}{2}$$

03.02.23.0017.01

$$\sum_{k=0}^{\infty} I_{nk}(z) = \frac{I_0(z)}{2} + \frac{1}{2n}\sum_{k=0}^{n-1} e^{z\cos\left(\frac{2\pi k}{n}\right)} \;/;\; n \in \mathbb{N}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_0\tilde{F}_1$

03.02.26.0001.01

$$I_\nu(z) = \left(\frac{z}{2}\right)^\nu {}_0\tilde{F}_1\left(;\nu+1;\frac{z^2}{4}\right)$$

Involving ${}_0F_1$

03.02.26.0002.01

$$I_\nu(z) = \frac{1}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu {}_0F_1\left(;\nu+1;\frac{z^2}{4}\right) \;/;\; -\nu \notin \mathbb{N}^+$$

Involving ${}_1F_1$

03.02.26.0003.01

$$I_\nu(z) = \frac{z^\nu}{2^\nu e^z\, \Gamma(\nu+1)}\, {}_1F_1\left(\nu+\frac{1}{2}; 2\nu+1; 2z\right)$$

03.02.26.0004.01

$$I_\nu(z) = \frac{z^\nu}{2^\nu\,\Gamma(\nu+1)} \lim_{a\to\infty} {}_1F_1\left(a;\, \nu+1;\, \frac{z^2}{4\,a}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

03.02.26.0005.01

$$I_\nu(z) = \pi\, z^\nu \left(z^2\right)^{-\frac{\nu}{2}} G_{1,3}^{1,0}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu}{2},\, -\frac{\nu}{2},\, \frac{\nu+1}{2} \end{matrix}\right)$$

03.02.26.0006.01

$$I_\nu(z) = \pi\, 2^{-\nu} z^\nu\, G_{1,3}^{1,0}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0,\, -\nu,\, \frac{1}{2} \end{matrix}\right)$$

03.02.26.0007.01

$$I_\nu(z) = z^\nu \left(-z^2\right)^{-\frac{\nu}{2}} G_{0,2}^{1,0}\left(-\frac{z^2}{4} \,\middle|\, \frac{\nu}{2},\, -\frac{\nu}{2}\right)$$

03.02.26.0008.01

$$I_\nu\left(\sqrt{z}\right) = 2^{-\nu} z^{\nu/2}\, G_{0,2}^{1,0}\left(-\frac{z}{4} \,\middle|\, 0,\, -\nu\right)$$

03.02.26.0067.01

$$I_\nu\left(\sqrt{z}\right) = \pi\, G_{1,3}^{1,0}\left(\frac{z}{4} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu}{2},\, -\frac{\nu}{2},\, \frac{\nu+1}{2} \end{matrix}\right)$$

03.02.26.0068.01

$$I_{-\nu}\left(\sqrt{z}\right) + I_\nu\left(\sqrt{z}\right) = 2\,\pi\, G_{2,4}^{2,0}\left(\frac{z}{4} \,\middle|\, \begin{matrix} 0,\, \frac{1}{2} \\ -\frac{\nu}{2},\, \frac{\nu}{2},\, 0,\, \frac{1}{2} \end{matrix}\right)$$

03.02.26.0069.01

$$I_{-\nu}\left(\sqrt{z}\right) - I_\nu\left(\sqrt{z}\right) = \frac{\sin(\pi\,\nu)}{\pi}\, G_{0,2}^{2,0}\left(\frac{z}{4} \,\middle|\, -\frac{\nu}{2},\, \frac{\nu}{2}\right)$$

Classical cases involving exp

03.02.26.0009.01

$$e^{-z}\, I_\nu(z) = \frac{1}{\sqrt{\pi}}\, G_{1,2}^{1,1}\left(2\,z \,\middle|\, \begin{matrix} \frac{1}{2} \\ \nu,\, -\nu \end{matrix}\right)$$

03.02.26.0010.01

$$e^{z}\, I_\nu(z) = -\sqrt{\pi}\, \csc(\pi\,\nu)\, G_{2,3}^{1,1}\left(2\,z \,\middle|\, \begin{matrix} \frac{1}{2},\, 0 \\ \nu,\, -\nu,\, 0 \end{matrix}\right)$$

Classical cases involving cosh

03.02.26.0011.01

$$\cosh\left(\sqrt{z}\right) I_\nu\left(\sqrt{z}\right) = -\frac{\pi}{\sqrt{2}} \csc\left(\frac{\pi\,\nu}{2}\right) G_{3,5}^{1,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4},\, \frac{3}{4},\, 0 \\ \frac{\nu}{2},\, 0,\, -\frac{\nu}{2},\, \frac{1-\nu}{2},\, \frac{\nu+1}{2} \end{matrix}\right)$$

Classical cases involving sinh

03.02.26.0012.01

$$\sinh\left(\sqrt{z}\right) I_{\nu}\left(\sqrt{z}\right) = -\frac{\pi}{\sqrt{2}} \sec\left(\frac{\pi \nu}{2}\right) G_{3,5}^{1,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, 0 \\ \frac{\nu+1}{2}, 0, -\frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu}{2} \end{matrix}\right)$$

Classical cases involving cosh, sinh

03.02.26.0070.01

$$\cosh\left(\sqrt{z}\right) I_{-\nu}\left(\sqrt{z}\right) + \sinh\left(\sqrt{z}\right) I_{\nu}\left(\sqrt{z}\right) = \sqrt{2}\, \pi\, G_{2,4}^{2,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ -\frac{\nu}{2}, \frac{\nu+1}{2}, \frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

03.02.26.0013.01

$$\cosh\left(\sqrt{z}\right) I_{-\nu}\left(\sqrt{z}\right) - \sinh\left(\sqrt{z}\right) I_{\nu}\left(\sqrt{z}\right) = \frac{\cos(\pi \nu)}{\sqrt{2}\, \pi} G_{2,4}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ -\frac{\nu}{2}, \frac{\nu+1}{2}, \frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

03.02.26.0081.01

$$\sinh(z) I_{-\nu}\left(\sqrt{z}\right) - \cosh(z) I_{\nu}\left(\sqrt{z}\right) = -\frac{\cos(\pi \nu)}{\sqrt{2}\, \pi} G_{2,4}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{\nu}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

03.02.26.0082.01

$$\cosh(z) I_{\nu}\left(\sqrt{z}\right) + \sinh(z) I_{-\nu}\left(\sqrt{z}\right) = \sqrt{2}\, \pi\, G_{2,4}^{2,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{\nu}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

Classical cases for powers of Bessel *I*

03.02.26.0014.02

$$I_{\nu}\left(\sqrt{z}\right)^2 = \sqrt{\pi}\, \sec(\pi \nu)\, G_{1,3}^{1,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{2} \\ \nu, 0, -\nu \end{matrix}\right)$$

03.02.26.0071.01

$$I_{-\nu}\left(\sqrt{z}\right)^2 + I_{\nu}\left(\sqrt{z}\right)^2 = 2\sqrt{\pi}\, G_{2,4}^{2,0}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \nu, -\nu, 0, 0 \end{matrix}\right)$$

03.02.26.0015.01

$$I_{-\nu}\left(\sqrt{z}\right)^2 - I_{\nu}\left(\sqrt{z}\right)^2 = \frac{\sin(2\pi \nu)}{\pi^{3/2}} G_{1,3}^{2,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2} \\ -\nu, \nu, 0 \end{matrix}\right)$$

Classical cases for products of Bessel *I*

03.02.26.0016.01

$$I_{-\nu}\left(\sqrt{z}\right) I_{\nu}\left(\sqrt{z}\right) = \sqrt{\pi}\, G_{2,4}^{1,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, \frac{1}{2}, \nu, -\nu \end{matrix}\right)$$

03.02.26.0017.01

$$I_{\nu-1}\left(\sqrt{z}\right) I_{\nu}\left(\sqrt{z}\right) = \sqrt{\pi}\, \csc(\pi \nu)\, G_{2,4}^{1,1}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \nu-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}-\nu, -\frac{1}{2} \end{matrix}\right)$$

03.02.26.0073.01

$$I_\mu\left(\sqrt{z}\right) I_\nu\left(\sqrt{z}\right) == \sqrt{\pi}\; G_{3,5}^{1,2}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{\mu+\nu+1}{2} \\ \frac{\mu+\nu}{2}, -\frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2}, \frac{\mu+\nu+1}{2} \end{array}\right) /; -\mu-\nu-1 \notin \mathbb{N}$$

03.02.26.0018.01

$$I_\mu\left(\sqrt{z}\right) I_\nu\left(\sqrt{z}\right) == \sqrt{\pi}\; \csc\left(\frac{1}{4}\pi\,(2\,\mu+2\,\nu+3)\right) G_{3,5}^{1,2}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{1}{4} \\ \frac{\mu+\nu}{2}, -\frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2}, \frac{1}{4} \end{array}\right) /; -\mu-\nu-1 \notin \mathbb{N}$$

03.02.26.0083.01

$$I_{-n-\nu-1}\left(\sqrt{z}\right) I_\nu\left(\sqrt{z}\right) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} z^{\frac{1}{2}(2k-n-1)}\, \Gamma\left(k-n+\left\lfloor \frac{n}{2} \right\rfloor+\frac{1}{2}\right) \left(1-k+\left\lfloor \frac{n}{2} \right\rfloor\right)_{n-\left\lfloor \frac{n}{2} \right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)} -$$

$$(-1)^{\left\lfloor \frac{n}{2} \right\rfloor} \sqrt{2\pi}\; G_{3,5}^{1,2}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{1}{4} \\ \frac{n+1}{2}, \frac{n+1}{2}+\nu, -\frac{1}{2}(n+1), -\frac{1}{2}(n+1)-\nu, \frac{1}{4} \end{array}\right) /; n \in \mathbb{N}$$

03.02.26.0084.01

$$I_{-\nu-1}\left(\sqrt{z}\right) I_\nu\left(\sqrt{z}\right) = -\sqrt{2\pi}\; G_{3,5}^{1,2}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{1}{4} \\ \frac{1}{2}, \nu+\frac{1}{2}, -\frac{1}{2}, -\nu-\frac{1}{2}, \frac{1}{4} \end{array}\right) - \frac{2\sin(\pi\,\nu)}{\pi\sqrt{z}}$$

03.02.26.0085.01

$$I_{-\nu-2}\left(\sqrt{z}\right) I_\nu\left(\sqrt{z}\right) = \frac{4\,(\nu+1)\sin(\pi\,\nu)}{\pi\,z} - \sqrt{2\pi}\; G_{3,5}^{1,2}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{1}{4} \\ 1, \nu+1, -1, -\nu-1, \frac{1}{4} \end{array}\right)$$

03.02.26.0072.01

$$I_{-\mu}\left(\sqrt{z}\right) I_{-\nu}\left(\sqrt{z}\right) + I_\mu\left(\sqrt{z}\right) I_\nu\left(\sqrt{z}\right) == 2\sqrt{\pi}\; G_{2,4}^{2,0}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu), \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2} \end{array}\right)$$

03.02.26.0019.01

$$I_{-\mu}\left(\sqrt{z}\right) I_{-\nu}\left(\sqrt{z}\right) - I_\mu\left(\sqrt{z}\right) I_\nu\left(\sqrt{z}\right) == \frac{\sin(\pi\,(\mu+\nu))}{\sqrt{\pi^3}}\; G_{2,4}^{2,2}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, -\frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2} \end{array}\right) /; \mu+\nu \notin \mathbb{Z}$$

Classical cases involving Bessel *J*

03.02.26.0020.01

$$J_\nu\left(\sqrt[4]{z}\right) I_\nu\left(\sqrt[4]{z}\right) == \sqrt{\pi}\; G_{0,4}^{1,0}\left(\frac{z}{64} \,\middle|\, \frac{\nu}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}\right)$$

03.02.26.0021.01

$$J_{-\nu}\left(\sqrt[4]{z}\right) I_\nu\left(\sqrt[4]{z}\right) == \sqrt{\pi}\; G_{1,5}^{2,0}\left(\frac{z}{64} \,\middle|\, \begin{array}{c} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{array}\right)$$

Classical cases involving Bessel *K*

03.02.26.0022.01

$$I_\nu\left(\sqrt{z}\right) K_\nu\left(\sqrt{z}\right) == \frac{1}{2\sqrt{\pi}}\; G_{1,3}^{2,1}\left(z \,\middle|\, \begin{array}{c} \frac{1}{2} \\ 0, \nu, -\nu \end{array}\right)$$

03.02.26.0086.01

$$I_{-\nu}\left(\sqrt{z}\right)K_\nu\left(\sqrt{z}\right)=\frac{1}{2\sqrt{\pi}}\,G_{1,3}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2}\\0,-\nu,\nu\end{matrix}\right)$$

03.02.26.0023.01

$$I_\mu\left(\sqrt{z}\right)K_\nu\left(\sqrt{z}\right)\equiv\frac{1}{2\sqrt{\pi}}\,G_{2,4}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2}\\\frac{\mu-\nu}{2},\frac{\mu+\nu}{2},-\frac{\mu+\nu}{2},\frac{\nu-\mu}{2}\end{matrix}\right)/;-\mu-\nu-1\notin\mathbb{N}\bigwedge\nu-\mu-1\notin\mathbb{N}$$

03.02.26.0087.01

$$I_\nu\left(\sqrt{z}\right)K_{n+\nu+1}\left(\sqrt{z}\right)=\frac{(-1)^n\pi^{3/2}\csc\left(\frac{1}{4}\pi\left(4\nu+(-1)^n\right)\right)}{\sqrt{2}}\,G_{4,6}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\nu+\frac{3}{4},\frac{1}{4}\\\frac{n+1}{2},\frac{n+1}{2}+\nu,\nu+\frac{3}{4},\frac{1}{4},-\frac{1}{2}(n+1),-\frac{1}{2}(n+1)-\nu\end{matrix}\right)-$$
$$(-1)^n\sqrt{\pi}\csc(\nu\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{\frac{1}{2}(2k-n-1)}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}/;n\in\mathbb{N}$$

03.02.26.0088.01

$$I_\nu\left(\sqrt{z}\right)K_{-n-\nu-1}\left(\sqrt{z}\right)=\frac{(-1)^n\pi^{3/2}\csc\left(\frac{1}{4}\pi\left(4\nu+(-1)^n\right)\right)}{\sqrt{2}}\,G_{4,6}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\nu+\frac{3}{4},\frac{1}{4}\\\frac{n+1}{2},\frac{n+1}{2}+\nu,\nu+\frac{3}{4},\frac{1}{4},-\frac{1}{2}(n+1),-\frac{1}{2}(n+1)-\nu\end{matrix}\right)-$$
$$(-1)^n\sqrt{\pi}\csc(\nu\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{\frac{1}{2}(2k-n-1)}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}/;n\in\mathbb{N}/;n\in\mathbb{N}$$

03.02.26.0089.01

$$I_\nu\left(\sqrt{z}\right)K_{\nu+1}\left(\sqrt{z}\right)=\frac{\pi^{3/2}}{\cos(\pi\nu)+\sin(\pi\nu)}\,G_{4,6}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\nu+\frac{3}{4},\frac{1}{4}\\\frac{1}{2},\nu+\frac{1}{2},\nu+\frac{3}{4},\frac{1}{4},-\frac{1}{2},-\nu-\frac{1}{2}\end{matrix}\right)+\frac{1}{\sqrt{z}}$$

03.02.26.0090.01

$$I_\nu\left(\sqrt{z}\right)K_{\nu+2}\left(\sqrt{z}\right)=\frac{2(\nu+1)}{z}+\frac{\pi^{3/2}\csc(\pi\nu)}{\cot(\pi\nu)-1}\,G_{4,6}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\nu+\frac{3}{4},\frac{1}{4}\\1,\nu+1,\nu+\frac{3}{4},\frac{1}{4},-1,-\nu-1\end{matrix}\right)$$

03.02.26.0024.01

$$\left(I_{-\nu}\left(\sqrt{z}\right)+I_\nu\left(\sqrt{z}\right)\right)K_\nu\left(\sqrt{z}\right)\equiv\frac{\cos(\pi\nu)}{\sqrt{\pi}}\,G_{1,3}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2}\\-\nu,\nu,0\end{matrix}\right)$$

03.02.26.0091.01

$$\left(I_{-\nu}\left(\sqrt{z}\right)-I_\nu\left(\sqrt{z}\right)\right)K_\nu\left(\sqrt{z}\right)=\frac{\sin(\pi\nu)}{\sqrt{\pi}}\,G_{1,3}^{3,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2}\\0,\nu,-\nu\end{matrix}\right)$$

03.02.26.0025.01

$$I_\nu\left(\sqrt{z}\right)K_\mu\left(\sqrt{z}\right)+I_\mu\left(\sqrt{z}\right)K_\nu\left(\sqrt{z}\right)\equiv\frac{1}{\sqrt{\pi}}\cos\left(\frac{1}{2}\pi(\mu-\nu)\right)G_{2,4}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},0\\\frac{\mu+\nu}{2},\frac{\mu-\nu}{2},\frac{\nu-\mu}{2},-\frac{\mu+\nu}{2}\end{matrix}\right)$$

03.02.26.0026.01

$$I_\nu\left(\sqrt{z}\right)K_\mu\left(\sqrt{z}\right)-I_\mu\left(\sqrt{z}\right)K_\nu\left(\sqrt{z}\right)\equiv\frac{1}{\sqrt{\pi}}\sin\left(\frac{1}{2}\pi(\mu-\nu)\right)G_{2,4}^{3,1}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2}\\\frac{\mu+\nu}{2},\frac{\mu-\nu}{2},\frac{\nu-\mu}{2},-\frac{\mu+\nu}{2}\end{matrix}\right)$$

03.02.26.0092.01

$$I_\nu(\sqrt{z})^2 - \frac{1}{\pi^2} K_\nu(\sqrt{z})^2 = -\sec(\pi\nu)\sqrt{\pi}\, G_{3,5}^{3,0}\left(z \,\middle|\, \begin{array}{c} \frac{1}{2}, \frac{1}{4}(1-2\nu), \frac{1}{4}(3-2\nu) \\ 0, -\nu, \nu, \frac{1}{4}(1-2\nu), \frac{1}{4}(3-2\nu) \end{array}\right)$$

Classical cases involving Bessel Y

03.02.26.0027.01

$$I_\nu\left(\sqrt[4]{z}\right) Y_\nu\left(\sqrt[4]{z}\right) = -\sqrt{\pi}\, G_{2,6}^{3,0}\left(\frac{z}{64} \,\middle|\, \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{1}{4}, \frac{3}{4}, -\frac{\nu}{2} \end{array}\right)$$

03.02.26.0093.01

$$I_\nu\left(\sqrt[4]{z}\right) Y_{-\nu}\left(\sqrt[4]{z}\right) = -\sqrt{\pi}\, G_{2,6}^{3,0}\left(\frac{z}{64} \,\middle|\, \begin{array}{c} \frac{1}{4}(2\nu+1), \frac{1}{4}(2\nu+3) \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{1}{4}(2\nu+3), \frac{1}{4}(2\nu+1), -\frac{\nu}{2} \end{array}\right)$$

03.02.26.0094.01

$$I_\nu\left(\sqrt[4]{z}\right) Y_{-\nu}\left(\sqrt[4]{z}\right) = \sqrt{\pi}\, G_{2,6}^{3,0}\left(\frac{z}{64} \,\middle|\, \begin{array}{c} \frac{1}{4}(2\nu-1), \frac{1}{4}(2\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{4}(2\nu+1), \frac{1}{4}(2\nu-1) \end{array}\right)$$

Classical cases involving Struve L

03.02.26.0028.01

$$I_\nu\left(\sqrt{z}\right) - \boldsymbol{L}_\nu\left(\sqrt{z}\right) = \frac{1}{\pi} G_{1,3}^{2,1}\left(\frac{z}{4} \,\middle|\, \begin{array}{c} \frac{\nu+1}{2} \\ \frac{\nu+1}{2}, \frac{\nu}{2}, -\frac{\nu}{2} \end{array}\right)$$

Classical cases involving ${}_0F_1$

03.02.26.0029.01

$$I_\nu(z)\, {}_0F_1\left(; b; \frac{z^2}{4}\right) = \sqrt{\pi} \csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right)\Gamma(b)\, 2^{b-1} G_{3,5}^{1,2}\left(z^2 \,\middle|\, \begin{array}{c} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{3-2b}{4} \\ \frac{\nu}{2}, -\frac{\nu}{2}, 1-b+\frac{\nu}{2}, 1-b-\frac{\nu}{2}, \frac{3-2b}{4} \end{array}\right) /;$$
$$-b-\nu \notin \mathbb{N} \bigwedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

03.02.26.0095.01

$$I_\nu(z)\, {}_0F_1\left(; -n-\nu; \frac{z^2}{4}\right) = -\frac{2^{-n-\nu-1}\Gamma(-n-\nu)}{\sqrt{\pi}}\left((-1)^{\left\lfloor \frac{n}{2} \right\rfloor}\sqrt{2}\,\pi\, G_{3,5}^{1,2}\left(z^2 \,\middle|\, \begin{array}{c} \frac{1}{2}(n+\nu+1), \frac{1}{2}(n+\nu+2), \frac{1}{4}(2n+2\nu+3) \\ n+\frac{\nu}{2}+1, -\frac{\nu}{2}, \frac{\nu}{2}, n+\frac{3\nu}{2}+1, \frac{1}{4}(2n+2\nu+3) \end{array}\right) -\right.$$
$$\left. 2\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} z^{2k+\nu}\, \Gamma\left(k-n+\left\lfloor \frac{n}{2} \right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor \frac{n}{2} \right\rfloor\right)_{n-\left\lfloor \frac{n}{2} \right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\right) /; n \in \mathbb{N} \bigwedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

03.02.26.0096.01

$$I_\nu(z)\, {}_0F_1\left(; -\nu; \frac{z^2}{4}\right) = \frac{2^{-\nu-1}\Gamma(-\nu)}{\sqrt{\pi}}\left(-\frac{2\sin(\pi\nu)\, z^\nu}{\sqrt{\pi}} - \sqrt{2}\,\pi\, G_{3,5}^{1,2}\left(z^2 \,\middle|\, \begin{array}{c} \frac{\nu+1}{2}, \frac{\nu}{2}+1, \frac{1}{4}(2\nu+3) \\ \frac{\nu}{2}+1, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{3\nu}{2}+1, \frac{1}{4}(2\nu+3) \end{array}\right)\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

03.02.26.0097.01

$$I_\nu(z)\,{}_0F_1\left(;-\nu-1;\frac{z^2}{4}\right)=\frac{2^{-\nu-2}\,\Gamma(-\nu-1)}{\pi}\left(4\,z^\nu\,(\nu+1)\sin(\pi\,\nu)-\sqrt{2}\,\pi^{3/2}\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{\nu+2}{2},\,\frac{\nu+3}{2},\,\frac{1}{4}(2\,\nu+5)\\ \frac{\nu+4}{2},\,-\frac{\nu}{2},\,\frac{\nu}{2},\,\frac{3\,\nu}{2}+2,\,\frac{1}{4}(2\,\nu+5)\end{matrix}\right)\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.02.26.0098.01

$$I_\nu(z)\,{}_0F_1\left(;\nu;\frac{z^2}{4}\right)=2^{\nu-1}\sqrt{\pi}\,\csc\left(\left(\nu+\frac{1}{4}\right)\pi\right)\Gamma(\nu)\,G_{2,4}^{1,1}\left(z^2\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\,\frac{1}{4}(3-2\,\nu)\\ \frac{\nu}{2},\,-\frac{\nu}{2},\,1-\frac{3\,\nu}{2},\,\frac{1}{4}(3-2\,\nu)\end{matrix}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.02.26.0099.01

$$I_\nu(z)\,{}_0F_1\left(;\nu+1;\frac{z^2}{4}\right)=\frac{2^\nu\,e^{-\frac{1}{2}i\pi\nu}\,\Gamma(\nu+1)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-z^2\,\middle|\,\begin{matrix}\frac{1}{2}-\frac{\nu}{2}\\ \frac{\nu}{2},\,-\frac{\nu}{2},\,-\frac{1}{2}(3\,\nu)\end{matrix}\right)/;\,-\pi<\arg(z)\leq 0$$

03.02.26.0100.01

$$I_\nu(z)\,{}_0F_1\left(;1-\nu;\frac{z^2}{4}\right)=\frac{2^{-\nu}\,e^{-\frac{1}{2}i\pi\nu}\,\Gamma(1-\nu)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-z^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu}{2},\,-\frac{\nu}{2},\,\frac{3\,\nu}{2}\end{matrix}\right)/;\,-\pi<\arg(z)\leq 0$$

03.02.26.0030.01

$$I_\nu(z)\,{}_0F_1\left(;\nu+1;-\frac{z^2}{4}\right)=\sqrt{\pi}\,2^{-\frac{\nu}{2}}\,\Gamma(\nu+1)\,z^\nu\left(z^4\right)^{-\frac{\nu}{4}}\,G_{0,4}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\frac{\nu}{4},\,-\frac{\nu}{4},\,\frac{2-\nu}{4},\,-\frac{3\,\nu}{4}\right)$$

03.02.26.0031.01

$$I_\nu(z)\,{}_0F_1\left(;1-\nu;-\frac{z^2}{4}\right)=\sqrt{\pi}\,2^{\nu/2}\,\Gamma(1-\nu)\,G_{1,5}^{2,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\,\nu+2}{4}\\ \frac{\nu}{4},\,\frac{\nu+2}{4},\,-\frac{\nu}{4},\,\frac{3\,\nu}{4},\,\frac{3\,\nu+2}{4}\end{matrix}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.02.26.0101.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;b;z)=\sqrt{\pi}\,\csc\left(\frac{1}{4}\pi\,(2\,b+2\,\nu+1)\right)\Gamma(b)\,2^{b-1}\,G_{3,5}^{1,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}(3-2\,b)\\ \frac{\nu}{2},\,-\frac{\nu}{2},\,-b+\frac{\nu}{2}+1,\,-b-\frac{\nu}{2}+1,\,\frac{1}{4}(3-2\,b)\end{matrix}\right)/;\,-b-\nu\notin\mathbb{N}$$

03.02.26.0102.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;-n-\nu;z)=\frac{2^{-n-\nu-\frac{1}{2}}\,\Gamma(-n-\nu)}{\sqrt{\pi}}\left(2^{\nu+\frac{1}{2}}\,z^{\nu/2}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}-(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\pi\,G_{3,5}^{1,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1}{2}(n+\nu+1),\,\frac{1}{2}(n+\nu+2),\,\frac{1}{4}(2\,n+2\,\nu+3)\\ n+\frac{\nu}{2}+1,\,-\frac{\nu}{2},\,\frac{\nu}{2},\,n+\frac{3\,\nu}{2}+1,\,\frac{1}{4}(2\,n+2\,\nu+3)\end{matrix}\right)\right)/;\,n\in\mathbb{N}$$

03.02.26.0103.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;-\nu;z)=\frac{\Gamma(-\nu)}{\sqrt{\pi}}\left(-\frac{\sin(\pi\,\nu)\,z^{\nu/2}}{\sqrt{\pi}}-2^{-\nu-\frac{1}{2}}\,\pi\,G_{3,5}^{1,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\,\frac{\nu+2}{2},\,\frac{1}{4}(2\,\nu+3)\\ \frac{\nu+2}{2},\,-\frac{\nu}{2},\,\frac{\nu}{2},\,\frac{3\,\nu}{2}+1,\,\frac{1}{4}(2\,\nu+3)\end{matrix}\right)\right)$$

03.02.26.0104.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;-\nu-1;z)=\frac{2^{-\nu-\frac{3}{2}}\,\Gamma(-\nu-1)}{\pi}\left(2^{\nu+\frac{3}{2}}\,z^{\nu/2}\,(\nu+1)\sin(\pi\nu)-\pi^{3/2}\,G_{3,5}^{1,2}\!\left(4z\,\middle|\,\begin{array}{c}\frac{\nu+2}{2},\frac{\nu+3}{2},\frac{1}{4}(2\nu+5)\\ \frac{\nu+4}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+2,\frac{1}{4}(2\nu+5)\end{array}\right)\right)$$

03.02.26.0105.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;\nu;z)=2^{\nu-1}\sqrt{\pi}\,\csc\left(\left(\nu+\frac{1}{4}\right)\pi\right)\Gamma(\nu)\,G_{2,4}^{1,1}\!\left(4z\,\middle|\,\begin{array}{c}\frac{1-\nu}{2},\frac{1}{4}(3-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},1-\frac{3\nu}{2},\frac{1}{4}(3-2\nu)\end{array}\right)$$

03.02.26.0106.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;\nu+1;-z)=2^{-\nu}\sqrt{\pi}\,z^{\nu/2}\,\Gamma(\nu+1)\,G_{0,4}^{1,0}\!\left(\frac{z^2}{4}\,\middle|\,0,-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},-\nu\right)$$

03.02.26.0107.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;1-\nu;-z)=2^{\nu/2}\sqrt{\pi}\,\Gamma(1-\nu)\,G_{1,5}^{2,0}\!\left(\frac{z^2}{4}\,\middle|\,\begin{array}{c}\frac{1}{4}(3\nu+2)\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},-\frac{\nu}{4}\end{array}\right)/;\ -\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

Classical cases involving ${}_0\tilde{F}_1$

03.02.26.0032.01

$$I_\nu(z)\,{}_0\tilde{F}_1\!\left(;b;\frac{z^2}{4}\right)=\sqrt{\pi}\,\csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right)2^{b-1}\,G_{3,5}^{1,2}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{3-2b}{4}\\ \frac{\nu}{2},-\frac{\nu}{2},1-b+\frac{\nu}{2},1-b-\frac{\nu}{2},\frac{3-2b}{4}\end{array}\right)/;$$

$$-b-\nu\notin\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.02.26.0108.01

$$I_\nu(z)\,{}_0\tilde{F}_1\!\left(;-n-\nu;\frac{z^2}{4}\right)=-\frac{2^{-n-\nu-1}}{\sqrt{\pi}}\left((-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\sqrt{2}\,\pi\,G_{3,5}^{1,2}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{1}{2}(n+\nu+1),\frac{1}{2}(n+\nu+2),\frac{1}{4}(2n+2\nu+3)\\ n+\frac{\nu}{2}+1,-\frac{\nu}{2},\frac{\nu}{2},n+\frac{3\nu}{2}+1,\frac{1}{4}(2n+2\nu+3)\end{array}\right)-\right.$$

$$\left.2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k+\nu}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\right)/;\ n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.02.26.0109.01

$$I_\nu(z)\,{}_0\tilde{F}_1\!\left(;-\nu;\frac{z^2}{4}\right)=\frac{2^{-\nu-1}}{\sqrt{\pi}}\left(-\frac{2\sin(\pi\nu)\,z^\nu}{\sqrt{\pi}}-\sqrt{2}\,\pi\,G_{3,5}^{1,2}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{\nu+1}{2},\frac{\nu}{2}+1,\frac{1}{4}(2\nu+3)\\ \frac{\nu}{2}+1,-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+1,\frac{1}{4}(2\nu+3)\end{array}\right)\right)/;\ -\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.02.26.0110.01

$$I_\nu(z)\,{}_0\tilde{F}_1\!\left(;-\nu-1;\frac{z^2}{4}\right)=\frac{2^{-\nu-2}}{\pi}\left(4z^\nu(\nu+1)\sin(\pi\nu)-\sqrt{2}\,\pi^{3/2}\,G_{3,5}^{1,2}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{\nu+2}{2},\frac{\nu+3}{2},\frac{1}{4}(2\nu+5)\\ \frac{\nu+4}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+2,\frac{1}{4}(2\nu+5)\end{array}\right)\right)/;\ -\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.02.26.0111.01

$$I_\nu(z)\,{}_0\tilde{F}_1\!\left(;\nu;\frac{z^2}{4}\right)=2^{\nu-1}\sqrt{\pi}\,\csc\left(\left(\nu+\frac{1}{4}\right)\pi\right)G_{2,4}^{1,1}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{1-\nu}{2},\frac{1}{4}(3-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},1-\frac{3\nu}{2},\frac{1}{4}(3-2\nu)\end{array}\right)/;\ -\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.02.26.0112.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;\nu+1;\frac{z^2}{4}\right)=\frac{2^\nu e^{-\frac{1}{2}i\pi\nu}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-z^2\,\middle|\,\begin{matrix}\frac{1}{2}-\frac{\nu}{2}\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu)\end{matrix}\right)\;/;\;-\pi<\arg(z)\le 0$$

03.02.26.0113.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;1-\nu;\frac{z^2}{4}\right)=\frac{2^{-\nu} e^{-\frac{1}{2}i\pi\nu}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-z^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)\;/;\;-\pi<\arg(z)\le 0$$

03.02.26.0033.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;\nu+1;-\frac{z^2}{4}\right)=\sqrt{\pi}\,2^{-\frac{\nu}{2}}z^\nu\left(z^4\right)^{-\frac{\nu}{4}}G_{0,4}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{3\nu}{4}\right)$$

03.02.26.0034.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;1-\nu;-\frac{z^2}{4}\right)=\sqrt{\pi}\,2^{\nu/2}\,G_{1,5}^{2,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu+2}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{3\nu}{4},\frac{3\nu+2}{4}\end{matrix}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.02.26.0114.01

$$I_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;b;z)=\sqrt{\pi}\,\csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right)2^{b-1}\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{\nu}{2},-\frac{\nu}{2},-b+\frac{\nu}{2}+1,-b-\frac{\nu}{2}+1,\frac{1}{4}(3-2b)\end{matrix}\right)\;/;\;-b-\nu\notin\mathbb{N}$$

03.02.26.0115.01

$$I_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;-n-\nu;z)=\frac{2^{-n-\nu-\frac{1}{2}}}{\sqrt{\pi}}\left(2^{\nu+\frac{1}{2}}z^{\nu/2}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}4^k z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}-\right.$$

$$\left.(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\pi\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{matrix}\frac{1}{2}(n+\nu+1),\frac{1}{2}(n+\nu+2),\frac{1}{4}(2n+2\nu+3)\\ n+\frac{\nu}{2}+1,-\frac{\nu}{2},\frac{\nu}{2},n+\frac{3\nu}{2}+1,\frac{1}{4}(2n+2\nu+3)\end{matrix}\right)\right)\;/;\;n\in\mathbb{N}$$

03.02.26.0116.01

$$I_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;-\nu;-z)=\frac{1}{\sqrt{\pi}}\left(-\frac{\sin(\pi\nu)\,z^{\nu/2}}{\sqrt{\pi}}-2^{-\nu-\frac{1}{2}}\pi\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{\nu+2}{2},\frac{1}{4}(2\nu+3)\\ \frac{\nu+2}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+1,\frac{1}{4}(2\nu+3)\end{matrix}\right)\right)$$

03.02.26.0117.01

$$I_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;-\nu-1;-z)=\frac{2^{-\nu-\frac{3}{2}}}{\pi}\left(2^{\nu+\frac{3}{2}}z^{\nu/2}(\nu+1)\sin(\pi\nu)-\pi^{3/2}\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{matrix}\frac{\nu+2}{2},\frac{\nu+3}{2},\frac{1}{4}(2\nu+5)\\ \frac{\nu+4}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+2,\frac{1}{4}(2\nu+5)\end{matrix}\right)\right)$$

03.02.26.0118.01

$$I_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;\nu;-z)=2^{\nu-1}\sqrt{\pi}\,\csc\left(\left(\nu+\frac{1}{4}\right)\pi\right)G_{2,4}^{1,1}\left(4z\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(3-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},1-\frac{3\nu}{2},\frac{1}{4}(3-2\nu)\end{matrix}\right)$$

03.02.26.0119.01

$$I_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;\nu+1;-z)=2^{-\nu}\sqrt{\pi}\,z^{\nu/2}\,G_{0,4}^{1,0}\left(\frac{z^2}{4}\,\middle|\,0,-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},-\nu\right)$$

03.02.26.0120.01

$$I_\nu\left(2\sqrt{z}\right)\,{}_0\tilde{F}_1(;1-\nu;-z) = 2^{\nu/2}\sqrt{\pi}\,G_{1,5}^{2,0}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(3\nu+2)\\ \frac{\nu+2}{4},\,\frac{\nu}{4},\,\frac{1}{4}(3\nu+2),\,\frac{3\nu}{4},\,-\frac{\nu}{4}\end{matrix}\right) /;\, -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

Generalized cases for the direct function itself

03.02.26.0035.01

$$I_\nu(z) = \pi\,G_{1,3}^{1,0}\left(\frac{z}{2},\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu}{2},\,-\frac{\nu}{2},\,\frac{\nu+1}{2}\end{matrix}\right)$$

03.02.26.0036.01

$$I_\nu(z) = z^\nu\,(i\,z)^{-\nu}\,G_{0,2}^{1,0}\left(\frac{i\,z}{2},\,\frac{1}{2}\,\middle|\,\frac{\nu}{2},\,-\frac{\nu}{2}\right)$$

03.02.26.0037.01

$$I_\nu(z) = 2^{-\nu}\,z^\nu\,G_{0,2}^{1,0}\left(\frac{i\,z}{2},\,\frac{1}{2}\,\middle|\,0,\,-\nu\right)$$

03.02.26.0074.01

$$I_{-\nu}(z) + I_\nu(z) = 2\,\pi\,G_{2,4}^{2,0}\left(\frac{z}{2},\,\frac{1}{2}\,\middle|\,\begin{matrix}0,\,\frac{1}{2}\\ -\frac{\nu}{2},\,\frac{\nu}{2},\,0,\,\frac{1}{2}\end{matrix}\right)$$

03.02.26.0075.01

$$I_{-\nu}(z) - I_\nu(z) = \frac{\sin(\pi\,\nu)}{\pi}\,G_{0,2}^{2,0}\left(\frac{z}{2},\,\frac{1}{2}\,\middle|\,-\frac{\nu}{2},\,\frac{\nu}{2}\right)$$

Generalized cases involving cosh

03.02.26.0038.01

$$\cosh(z)\,I_\nu(z) = -\frac{\pi\csc\left(\frac{\pi\,\nu}{2}\right)}{\sqrt{2}}\,G_{3,5}^{1,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\,\frac{3}{4},\,0\\ \frac{\nu}{2},\,0,\,-\frac{\nu}{2},\,\frac{1-\nu}{2},\,\frac{\nu+1}{2}\end{matrix}\right)$$

03.02.26.0121.01

$$\cosh(a+z)\,I_\nu(z) = \frac{z^\nu}{\sqrt{2}}\,G_{3,5}^{2,2}\left(i\,z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(1-2\,\nu),\,\frac{1}{4}(3-2\,\nu),\,\frac{1}{2}+\frac{i\,a}{\pi}\\ 0,\,\frac{1}{2},\,\frac{1}{2}-\nu,\,-\nu,\,\frac{1}{2}+\frac{i\,a}{\pi}\end{matrix}\right)$$

Generalized cases involving sinh

03.02.26.0039.01

$$\sinh(z)\,I_\nu(z) = -\frac{\pi\sec\left(\frac{\pi\,\nu}{2}\right)}{\sqrt{2}}\,G_{3,5}^{1,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\,\frac{3}{4},\,0\\ \frac{\nu+1}{2},\,0,\,-\frac{\nu}{2},\,\frac{1-\nu}{2},\,\frac{\nu}{2}\end{matrix}\right)$$

03.02.26.0122.01

$$\sinh(a+z)\,I_\nu(z) = -\frac{i\,z^\nu}{\sqrt{2}}\,G_{3,5}^{2,2}\left(i\,z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(1-2\,\nu),\,\frac{1}{4}(3-2\,\nu),\,\frac{i\,a}{\pi}\\ 0,\,\frac{1}{2},\,\frac{i\,a}{\pi},\,\frac{1}{2}-\nu,\,-\nu\end{matrix}\right)$$

Generalized cases involving cosh, sinh

03.02.26.0076.01

$$\cosh(z)\, I_{-\nu}(z) + \sinh(z)\, I_\nu(z) = \sqrt{2}\, \pi\, G_{2,4}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ -\frac{\nu}{2}, \frac{\nu+1}{2}, \frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

03.02.26.0040.01

$$\cosh(z)\, I_{-\nu}(z) - \sinh(z)\, I_\nu(z) = \frac{\cos(\pi \nu)}{\sqrt{2}\, \pi}\, G_{2,4}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ -\frac{\nu}{2}, \frac{\nu+1}{2}, \frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

03.02.26.0123.01

$$\sinh(z)\, I_{-\nu}(z) - \cosh(z)\, I_\nu(z) = -\frac{\cos(\pi \nu)}{\sqrt{2}\, \pi}\, G_{2,4}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{\nu}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

03.02.26.0124.01

$$\cosh(z)\, I_\nu(z) + \sinh(z)\, I_{-\nu}(z) = \sqrt{2}\, \pi\, G_{2,4}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{\nu}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

Generalized cases involving Ai

03.02.26.0041.01

$$\mathrm{Ai}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) I_\nu(z) = \frac{1}{2\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}}\, G_{2,4}^{2,2}\left(z^{2/3}, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}, \frac{2}{3} \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+2), -\frac{\nu}{2}, \frac{1}{6}(2-3\nu) \end{matrix}\right)$$

03.02.26.0125.01

$$\mathrm{Ai}(z)\, I_\nu\left(\frac{2 z^{3/2}}{3}\right) = \frac{z^{-\frac{3\nu}{2}} \left(z^{3/2}\right)^\nu}{2\sqrt[3]{2}\,\sqrt[6]{3}\,\pi^{3/2}}\, G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}, \frac{2}{3} \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+2), -\frac{\nu}{2}, \frac{1}{6}(2-3\nu) \end{matrix}\right)$$

Generalized cases involving Ai′

03.02.26.0042.01

$$\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) I_\nu(z) = -\frac{\sqrt[6]{3}}{2\, 2^{2/3}\, \pi^{3/2}}\, G_{2,4}^{2,2}\left(z^{2/3}, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6} \\ \frac{\nu}{2}, \frac{\nu}{2}+\frac{2}{3}, -\frac{\nu}{2}, \frac{2}{3}-\frac{\nu}{2} \end{matrix}\right)$$

03.02.26.0126.01

$$\mathrm{Ai}'(z)\, I_\nu\left(\frac{2 z^{3/2}}{3}\right) = -\frac{\sqrt[6]{3}\, z^{-\frac{3\nu}{2}} \left(z^{3/2}\right)^\nu}{2\, 2^{2/3}\, \pi^{3/2}}\, G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6} \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+4), -\frac{\nu}{2}, \frac{1}{6}(4-3\nu) \end{matrix}\right)$$

Generalized cases involving Bi

03.02.26.0043.01

$$\mathrm{Bi}\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) I_\nu(z) = \frac{2^{2/3}\sqrt{\pi}}{\sqrt[6]{3}}\, G_{4,6}^{2,2}\left(z^{2/3}, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}, \frac{2}{3}, \frac{1}{6}(3\nu+1), \frac{1}{6}(3\nu+4) \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+2), -\frac{\nu}{2}, \frac{1}{6}(2-3\nu), \frac{1}{6}(3\nu+1), \frac{1}{6}(3\nu+4) \end{matrix}\right)$$

03.02.26.0127.01

$$\mathrm{Bi}(z)\, I_\nu\left(\frac{2 z^{3/2}}{3}\right) = \frac{2^{2/3}\sqrt{\pi}\, z^{-\frac{3\nu}{2}} \left(z^{3/2}\right)^\nu}{\sqrt[6]{3}}\, G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}, \frac{2}{3}, \frac{1}{6}(3\nu+1), \frac{1}{6}(3\nu+4) \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+2), -\frac{\nu}{2}, \frac{1}{6}(2-3\nu), \frac{1}{6}(3\nu+1), \frac{1}{6}(3\nu+4) \end{matrix}\right)$$

Generalized cases involving Bi′

03.02.26.0044.01

$$\operatorname{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) I_\nu(z) = \sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}\,G_{4,6}^{2,2}\left(z^{2/3}, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, \frac{\nu}{2}+\frac{1}{3}, \frac{\nu}{2}+\frac{5}{6} \\ \frac{\nu}{2}, \frac{\nu}{2}+\frac{2}{3}, -\frac{\nu}{2}, \frac{2}{3}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{3}, \frac{\nu}{2}+\frac{5}{6} \end{array}\right)$$

03.02.26.0128.01

$$\operatorname{Bi}'(z)\, I_\nu\left(\frac{2 z^{3/2}}{3}\right) = \sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}\, z^{-\frac{3\nu}{2}} \left(z^{3/2}\right)^\nu G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, \frac{1}{6}(3\nu+2), \frac{\nu}{2}+\frac{5}{6} \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+4), -\frac{\nu}{2}, \frac{1}{6}(4-3\nu), \frac{1}{6}(3\nu+2), \frac{1}{6}(3\nu+5) \end{array}\right)$$

Generalized cases for powers of Bessel *I*

03.02.26.0045.02

$$I_\nu(z)^2 = \sqrt{\pi}\, \sec(\pi\nu)\, G_{1,3}^{1,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{2} \\ \nu, 0, -\nu \end{array}\right)$$

03.02.26.0077.01

$$I_{-\nu}(z)^2 + I_\nu(z)^2 = 2\sqrt{\pi}\, G_{2,4}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ \nu, -\nu, 0, 0 \end{array}\right)$$

03.02.26.0046.01

$$I_{-\nu}(z)^2 - I_\nu(z)^2 = \frac{\sin(2\pi\nu)}{\pi^{3/2}} G_{1,3}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{2} \\ -\nu, \nu, 0 \end{array}\right)$$

Generalized cases for products of Bessel *I*

03.02.26.0047.01

$$I_{-\nu}(z)\, I_\nu(z) = \sqrt{\pi}\, G_{2,4}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{2}, \frac{1}{2} \\ 0, \frac{1}{2}, \nu, -\nu \end{array}\right)$$

03.02.26.0048.01

$$I_{\nu-1}(z)\, I_\nu(z) = \sqrt{\pi}\, \csc(\pi\nu)\, G_{2,4}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ \nu-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}-\nu, -\frac{1}{2} \end{array}\right)$$

03.02.26.0049.01

$$I_\mu(z)\, I_\nu(z) = \sqrt{\pi}\, \csc\left(\frac{1}{4}\pi(2\mu+2\nu+3)\right) G_{3,5}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{1}{4} \\ \frac{\mu+\nu}{2}, -\frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2}, \frac{1}{4} \end{array}\right) /; -\mu-\nu-1 \notin \mathbb{N}$$

03.02.26.0078.01

$$I_\mu(z)\, I_\nu(z) = \sqrt{\pi}\, G_{3,5}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{\mu+\nu+1}{2} \\ \frac{\mu+\nu}{2}, -\frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2}, \frac{\mu+\nu+1}{2} \end{array}\right) /; -\mu-\nu-1 \notin \mathbb{N}$$

03.02.26.0129.01

$$I_{-n-\nu-1}(z)\, I_\nu(z) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} z^{2k-n-1}\, \Gamma\left(k-n+\left\lfloor \frac{n}{2} \right\rfloor + \frac{1}{2}\right) \left(1-k+\left\lfloor \frac{n}{2} \right\rfloor\right)_{n-\left\lfloor \frac{n}{2} \right\rfloor}}{k!\, \Gamma(k-n-\nu)\, \Gamma(k+\nu+1)} -$$

$$(-1)^{\left\lfloor \frac{n}{2} \right\rfloor} \sqrt{2\pi}\, G_{3,5}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{1}{4} \\ \frac{n+1}{2}, \frac{n+1}{2}+\nu, -\frac{1}{2}(n+1), -\frac{1}{2}(n+1)-\nu, \frac{1}{4} \end{array}\right) /; n \in \mathbb{N}$$

03.02.26.0130.01

$$I_{-\nu-1}(z)\, I_{\nu}(z) = -\sqrt{2\pi}\; G_{3,5}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, \frac{1}{4} \\ \frac{1}{2}, \nu+\frac{1}{2}, -\frac{1}{2}, -\nu-\frac{1}{2}, \frac{1}{4} \end{matrix}\right) - \frac{2\sin(\pi\,\nu)}{\pi\, z}$$

03.02.26.0131.01

$$I_{-\nu-2}(z)\, I_{\nu}(z) = \frac{4\,(\nu+1)\sin(\pi\,\nu)}{\pi\, z^2} - \sqrt{2\pi}\; G_{3,5}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, \frac{1}{4} \\ 1, \nu+1, -1, -\nu-1, \frac{1}{4} \end{matrix}\right)$$

03.02.26.0132.01

$$I_{\mu}(z)\, I_{\nu}(z) + I_{-\mu}(z)\, I_{-\nu}(z) = 2\sqrt{\pi}\; G_{2,4}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu), \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2} \end{matrix}\right)$$

03.02.26.0050.01

$$I_{\mu}(z)\, I_{\nu}(z) - I_{-\mu}(z)\, I_{-\nu}(z) == -\frac{\sin(\pi\,(\mu+\nu))}{\pi^{3/2}}\; G_{2,4}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, -\frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2} \end{matrix}\right)$$

Generalized cases involving Bessel *J*

03.02.26.0051.01

$$J_{\nu}(z)\, I_{\nu}(z) == \sqrt{\pi}\; G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \frac{\nu}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}\right)$$

03.02.26.0052.01

$$J_{-\nu}(z)\, I_{\nu}(z) == \sqrt{\pi}\; G_{1,5}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

03.02.26.0079.01

$$(J_{-\nu}(z) + J_{\nu}(z))\, I_{\nu}(z) == 2\cos\left(\frac{\pi\,\nu}{2}\right)\sqrt{\pi}\; G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{\nu}{4}, \frac{\nu+2}{4} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu}{4}, \frac{\nu+2}{4} \end{matrix}\right)$$

03.02.26.0080.01

$$(J_{-\nu}(z) - J_{\nu}(z))\, I_{\nu}(z) == 2\sin\left(\frac{\pi\,\nu}{2}\right)\sqrt{\pi}\; G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu+3}{4}, \frac{\nu+1}{4} \end{matrix}\right)$$

Generalized cases involving Bessel *K*

03.02.26.0053.01

$$I_{\nu}(z)\, K_{\nu}(z) == \frac{1}{2\sqrt{\pi}}\; G_{1,3}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \nu, -\nu \end{matrix}\right)$$

03.02.26.0133.01

$$I_{-\nu}(z)\, K_{\nu}(z) == \frac{1}{2\sqrt{\pi}}\; G_{1,3}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, -\nu, \nu \end{matrix}\right)$$

03.02.26.0054.01

$$I_{\mu}(z)\, K_{\nu}(z) == \frac{1}{2\sqrt{\pi}}\; G_{2,4}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \frac{\mu-\nu}{2}, \frac{\mu+\nu}{2}, \frac{1}{2}(-\mu-\nu), \frac{\nu-\mu}{2} \end{matrix}\right) /; -\mu-\nu-1 \notin \mathbb{N} \bigwedge \nu-\mu-1 \notin \mathbb{N}$$

03.02.26.0134.01

$$I_\nu(z)\,K_{n+\nu+1}(z) = \frac{1}{\sqrt{2}}\left((-1)^n\,\pi^{3/2}\csc\left(\frac{1}{4}\,\pi\,(4\,\nu+(-1)^n)\right)\right)G_{4,6}^{2,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}0,\,\frac{1}{2},\,\nu+\frac{3}{4},\,\frac{1}{4}\\ \frac{n+1}{2},\,\frac{n+1}{2}+\nu,\,\nu+\frac{3}{4},\,\frac{1}{4},\,-\frac{1}{2}\,(n+1),\,-\frac{1}{2}\,(n+1)-\nu\end{array}\right)-$$
$$(-1)^n\sqrt{\pi}\,\csc(\nu\,\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k-n-1}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\;/;\;n\in\mathbb{N}$$

03.02.26.0135.01

$$I_\nu(z)\,K_{-n-\nu-1}(z) = \frac{1}{\sqrt{2}}\left((-1)^n\,\pi^{3/2}\csc\left(\frac{1}{4}\,\pi\,(4\,\nu+(-1)^n)\right)\right)G_{4,6}^{2,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}0,\,\frac{1}{2},\,\nu+\frac{3}{4},\,\frac{1}{4}\\ \frac{n+1}{2},\,\frac{n+1}{2}+\nu,\,\nu+\frac{3}{4},\,\frac{1}{4},\,-\frac{1}{2}\,(n+1),\,-\frac{1}{2}\,(n+1)-\nu\end{array}\right)-$$
$$(-1)^n\sqrt{\pi}\,\csc(\nu\,\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k-n-1}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\;/;\;n\in\mathbb{N}$$

03.02.26.0136.01

$$I_\nu(z)\,K_{\nu+1}(z) = \frac{\pi^{3/2}}{\cos(\pi\,\nu)+\sin(\pi\,\nu)}\,G_{4,6}^{2,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}0,\,\frac{1}{2},\,\nu+\frac{3}{4},\,\frac{1}{4}\\ \frac{1}{2},\,\nu+\frac{1}{2},\,\nu+\frac{3}{4},\,\frac{1}{4},\,-\frac{1}{2},\,-\nu-\frac{1}{2}\end{array}\right)+\frac{1}{z}$$

03.02.26.0137.01

$$I_\nu(z)\,K_{\nu+2}(z) = \frac{2\,(\nu+1)}{z^2}+\frac{\pi^{3/2}\csc(\pi\,\nu)}{\cot(\pi\,\nu)-1}\,G_{4,6}^{2,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}0,\,\frac{1}{2},\,\nu+\frac{3}{4},\,\frac{1}{4}\\ 1,\,\nu+1,\,\nu+\frac{3}{4},\,\frac{1}{4},\,-1,\,-\nu-1\end{array}\right)$$

03.02.26.0055.01

$$(I_{-\nu}(z)+I_\nu(z))\,K_\nu(z) = \frac{\cos(\pi\,\nu)}{\sqrt{\pi}}\,G_{1,3}^{2,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{2}\\ -\nu,\,\nu,\,0\end{array}\right)$$

03.02.26.0138.01

$$(I_{-\nu}(z)-I_\nu(z))\,K_\nu(z) = \frac{\sin(\pi\,\nu)}{\sqrt{\pi}}\,G_{1,3}^{3,0}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{2}\\ 0,\,\nu,\,-\nu\end{array}\right)$$

03.02.26.0056.01

$$I_\nu(z)\,K_\mu(z)+I_\mu(z)\,K_\nu(z) = \frac{\cos\left(\frac{1}{2}\,\pi\,(\mu-\nu)\right)}{\sqrt{\pi}}\,G_{2,4}^{3,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{2},\,0\\ \frac{\mu+\nu}{2},\,\frac{\mu-\nu}{2},\,\frac{\nu-\mu}{2},\,-\frac{\mu+\nu}{2}\end{array}\right)$$

03.02.26.0057.01

$$I_\nu(z)\,K_\mu(z)-I_\mu(z)\,K_\nu(z) = \frac{\sin\left(\frac{1}{2}\,\pi\,(\mu-\nu)\right)}{\sqrt{\pi}}\,G_{2,4}^{3,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}0,\,\frac{1}{2}\\ \frac{\mu+\nu}{2},\,\frac{\mu-\nu}{2},\,\frac{\nu-\mu}{2},\,-\frac{\mu+\nu}{2}\end{array}\right)$$

03.02.26.0139.01

$$I_\nu(z)^2-\frac{1}{\pi^2}\,K_\nu(z)^2 = -\sec(\pi\,\nu)\sqrt{\pi}\,G_{3,5}^{3,0}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{2},\,\frac{1}{4}\,(1-2\,\nu),\,\frac{1}{4}\,(3-2\,\nu)\\ 0,\,-\nu,\,\nu,\,\frac{1}{4}\,(1-2\,\nu),\,\frac{1}{4}\,(3-2\,\nu)\end{array}\right)$$

Generalized cases involving Bessel *Y*

03.02.26.0058.01

$$I_\nu(z)\,Y_\nu(z) = -\sqrt{\pi}\,G_{2,6}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\;\middle|\;\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1}{2},\frac{\nu}{2},\frac{1}{4},\frac{3}{4},-\frac{\nu}{2}\end{matrix}\right)$$

03.02.26.0140.01

$$I_\nu(z)\,Y_{-\nu}(z) = -\sqrt{\pi}\,G_{2,6}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\;\middle|\;\begin{matrix}\frac{1}{4}(2\nu+1),\frac{1}{4}(2\nu+3)\\ 0,\frac{1}{2},\frac{\nu}{2},\frac{1}{4}(2\nu+3),\frac{1}{4}(2\nu+1),-\frac{\nu}{2}\end{matrix}\right)$$

03.02.26.0141.01

$$I_\nu(z)\,Y_{-\nu}(z) = \sqrt{\pi}\,G_{2,6}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\;\middle|\;\begin{matrix}\frac{1}{4}(2\nu-1),\frac{1}{4}(2\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{4}(2\nu+1),\frac{1}{4}(2\nu-1)\end{matrix}\right)$$

Generalized cases involving Struve L

03.02.26.0059.01

$$I_\nu(z) - \boldsymbol{L}_\nu(z) = \frac{1}{\pi}\,G_{1,3}^{2,1}\!\left(\frac{z}{2},\frac{1}{2}\;\middle|\;\begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu+1}{2},\frac{\nu}{2},-\frac{\nu}{2}\end{matrix}\right)$$

Generalized cases involving ${}_0F_1$

03.02.26.0060.01

$$I_\nu(z)\,{}_0F_1\!\left(;b;\frac{z^2}{4}\right) = \sqrt{\pi}\,\csc\!\left(\frac{1}{4}\pi(2b+2\nu+1)\right)\Gamma(b)\,2^{b-1}\,G_{3,5}^{1,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{3-2b}{4}\\ \frac{\nu}{2},-\frac{\nu}{2},1-b+\frac{\nu}{2},1-b-\frac{\nu}{2},\frac{3-2b}{4}\end{matrix}\right)/;\,-b-\nu\notin\mathbb{N}$$

03.02.26.0142.01

$$I_\nu(z)\,{}_0F_1\!\left(;-n-\nu;\frac{z^2}{4}\right) = -\frac{2^{-n-\nu-1}\,\Gamma(-n-\nu)}{\sqrt{\pi}}\left((-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\sqrt{2}\,\pi\,G_{3,5}^{1,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{2}(n+\nu+1),\frac{1}{2}(n+\nu+2),\frac{1}{4}(2n+2\nu+3)\\ n+\frac{\nu}{2}+1,-\frac{\nu}{2},\frac{\nu}{2},n+\frac{3\nu}{2}+1,\frac{1}{4}(2n+2\nu+3)\end{matrix}\right)-\right.$$
$$\left.2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k+\nu}\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\right)/;\,n\in\mathbb{N}$$

03.02.26.0143.01

$$I_\nu(z)\,{}_0F_1\!\left(;-\nu;\frac{z^2}{4}\right) = 2^{-\nu}\left(\frac{z^\nu}{\Gamma(\nu+1)}-\sqrt{\frac{\pi}{2}}\,\Gamma(-\nu)\,G_{4,6}^{2,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{\nu+1}{2},\frac{\nu+2}{2},-\frac{\nu}{2},\frac{1}{4}(2\nu+3)\\ \frac{\nu+2}{2},-\frac{\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+1,\frac{1}{4}(2\nu+3)\end{matrix}\right)\right)$$

03.02.26.0144.01

$$I_\nu(z)\,{}_0F_1\!\left(;-\nu-1;\frac{z^2}{4}\right) = \frac{2^{-\nu-2}\,\Gamma(-\nu-1)}{\pi}\left(4z^\nu(\nu+1)\sin(\pi\nu)-\sqrt{2}\,\pi^{3/2}\,G_{3,5}^{1,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{\nu+2}{2},\frac{\nu+3}{2},\frac{1}{4}(2\nu+5)\\ \frac{\nu+4}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+2,\frac{1}{4}(2\nu+5)\end{matrix}\right)\right)$$

03.02.26.0145.01

$$I_\nu(z)\,{}_0F_1\!\left(;\nu;\frac{z^2}{4}\right) = 2^{\nu-1}\sqrt{\pi}\,\csc\!\left(\left(\nu+\frac{1}{4}\right)\pi\right)\Gamma(\nu)\,G_{2,4}^{1,1}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(3-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},1-\frac{3\nu}{2},\frac{1}{4}(3-2\nu)\end{matrix}\right)$$

03.02.26.0146.01

$$I_\nu(z)\,{}_0F_1\left(;\nu+1;\frac{z^2}{4}\right)=2^\nu\sqrt{\pi}\,\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)\Gamma(\nu+1)\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},\frac{1}{4}-\frac{\nu}{2}\\\frac{\nu}{2},-\frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}-\frac{\nu}{2}\end{array}\right)$$

03.02.26.0147.01

$$I_\nu(z)\,{}_0F_1\left(;1-\nu;\frac{z^2}{4}\right)=2^{\frac{1}{2}-\nu}\sqrt{\pi}\,\Gamma(1-\nu)\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{\nu}{2},\frac{1}{4}(2\nu+1)\end{array}\right)$$

03.02.26.0061.01

$$I_\nu(z)\,{}_0F_1\left(;\nu+1;-\frac{z^2}{4}\right)=\sqrt{\pi}\,2^{-\frac{\nu}{2}}\,\Gamma(\nu+1)\,G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{3\nu}{4}\right)$$

03.02.26.0062.01

$$I_\nu(z)\,{}_0F_1\left(;1-\nu;-\frac{z^2}{4}\right)=2^{\nu/2}\sqrt{\pi}\,\Gamma(1-\nu)\,G_{1,5}^{2,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{3\nu+2}{4}\\\frac{\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{3\nu}{4},\frac{3\nu+2}{4}\end{array}\right)$$

03.02.26.0148.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;\nu+1;-z)=2^{-\frac{\nu}{2}}\sqrt{\pi}\,\Gamma(\nu+1)\,G_{0,4}^{1,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{1}{2}-\frac{\nu}{4},-\frac{1}{4}(3\nu)\right)$$

03.02.26.0149.01

$$I_\nu\left(2\sqrt{z}\right){}_0F_1(;1-\nu;-z)=2^{\nu/2}\sqrt{\pi}\,\Gamma(1-\nu)\,G_{1,5}^{2,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4}(3\nu+2)\\\frac{\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{3\nu}{4},\frac{1}{4}(3\nu+2)\end{array}\right)$$

Generalized cases involving ${}_0\tilde{F}_1$

03.02.26.0063.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)=\sqrt{\pi}\,\csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right)2^{b-1}\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{3-2b}{4}\\\frac{\nu}{2},-\frac{\nu}{2},1-b+\frac{\nu}{2},1-b-\frac{\nu}{2},\frac{3-2b}{4}\end{array}\right)/;-b-\nu\notin\mathbb{N}$$

03.02.26.0150.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;-n-\nu;\frac{z^2}{4}\right)=-\frac{2^{-n-\nu-1}}{\sqrt{\pi}}\left((-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{2}(n+\nu+1),\frac{1}{2}(n+\nu+2),\frac{1}{4}(2n+2\nu+3)\\n+\frac{\nu}{2}+1,-\frac{\nu}{2},\frac{\nu}{2},n+\frac{3\nu}{2}+1,\frac{1}{4}(2n+2\nu+3)\end{array}\right)-\right.$$
$$\left.2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k+\nu}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\right)/;n\in\mathbb{N}$$

03.02.26.0151.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;-\nu;\frac{z^2}{4}\right)=-2^{-\nu}\left(\frac{\sin(\pi\nu)\,z^\nu}{\pi}+\sqrt{\frac{\pi}{2}}\,G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{\nu+1}{2},\frac{\nu+2}{2},-\frac{\nu}{2},\frac{1}{4}(2\nu+3)\\\frac{\nu+2}{2},-\frac{\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+1,\frac{1}{4}(2\nu+3)\end{array}\right)\right)$$

03.02.26.0152.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;-\nu-1;\frac{z^2}{4}\right)=\frac{2^{-\nu-2}}{\pi}\left(4z^\nu(\nu+1)\sin(\pi\nu)-\sqrt{2}\,\pi^{3/2}\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{\nu+2}{2},\frac{\nu+3}{2},\frac{1}{4}(2\nu+5)\\\frac{\nu+4}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+2,\frac{1}{4}(2\nu+5)\end{array}\right)\right)$$

03.02.26.0153.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;\nu;\frac{z^2}{4}\right)=2^{\nu-1}\sqrt{\pi}\,\csc\left(\left(\nu+\frac{1}{4}\right)\pi\right)G_{2,4}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-\nu}{2},\frac{1}{4}(3-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},1-\frac{3\nu}{2},\frac{1}{4}(3-2\nu)\end{array}\right)$$

03.02.26.0154.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;\nu+1;\frac{z^2}{4}\right)=2^{\nu}\sqrt{\pi}\,\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},\frac{1}{4}-\frac{\nu}{2}\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}-\frac{\nu}{2}\end{array}\right)$$

03.02.26.0155.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;1-\nu;\frac{z^2}{4}\right)=2^{\frac{1}{2}-\nu}\sqrt{\pi}\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{\nu}{2},\frac{1}{4}(2\nu+1)\end{array}\right)$$

03.02.26.0064.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;\nu+1;-\frac{z^2}{4}\right)=\sqrt{\pi}\,2^{-\frac{\nu}{2}}\,G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{3\nu}{4}\right)$$

03.02.26.0065.01

$$I_\nu(z)\,{}_0\tilde{F}_1\left(;1-\nu;-\frac{z^2}{4}\right)=2^{\nu/2}\sqrt{\pi}\,G_{1,5}^{2,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{3\nu+2}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{3\nu}{4},\frac{3\nu+2}{4}\end{array}\right)$$

03.02.26.0156.01

$$I_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;\nu+1;-z)=2^{-\frac{\nu}{2}}\sqrt{\pi}\,G_{0,4}^{1,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{1}{2}-\frac{\nu}{4},-\frac{1}{4}(3\nu)\right)$$

03.02.26.0157.01

$$I_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;1-\nu;-z)=2^{\nu/2}\sqrt{\pi}\,G_{1,5}^{2,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{3\nu}{4},\frac{1}{4}(3\nu+2)\end{array}\right)$$

Through other functions

03.02.26.0066.01

$$I_\nu(z)=\boldsymbol{L}_{-\nu}(z)\;/;\;\nu-\frac{1}{2}\in\mathbb{N}$$

Representations through equivalent functions

With related functions

03.02.27.0001.01

$$I_\nu(z)=\frac{z^\nu}{(i\,z)^\nu}\,J_\nu(i\,z)$$

03.02.27.0002.01

$$I_\nu(i\,z)=\frac{(i\,z)^\nu}{z^\nu}\,J_\nu(z)$$

03.02.27.0006.01

$$I_\nu(z)=\frac{z^\nu}{(i\,z)^\nu}\left(\csc(\pi\,\nu)\,Y_{-\nu}(i\,z)-\cot(\pi\,\nu)\,Y_\nu(i\,z)\right)$$

03.02.27.0003.01

$$I_{\nu+1}(z)\,K_\nu(z)+I_\nu(z)\,K_{\nu+1}(z)=\frac{1}{z}$$

03.02.27.0004.01

$$I_\nu(z)=\left(\frac{z}{2}\right)^\nu\,{}_0\tilde{F}_1\!\left(;\nu+1;\frac{z^2}{4}\right)$$

03.02.27.0005.01

$$I_\nu(z)=\frac{1}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu\,{}_0F_1\!\left(;\nu+1;\frac{z^2}{4}\right)/;\ -\nu\notin\mathbb{N}^+$$

03.02.27.0007.01

$$I_\nu(z)=e^{\frac{1}{4}(-3)\,i\,\pi\,\nu}\,z^\nu\left(-(-1)^{3/4}\,z\right)^{-\nu}\left(i\,\mathrm{bei}_\nu\!\left(-(-1)^{3/4}\,z\right)+\mathrm{ber}_\nu\!\left(-(-1)^{3/4}\,z\right)\right)$$

03.02.27.0008.01

$$I_\nu\!\left(\sqrt[4]{-1}\,z\right)=e^{\frac{1}{4}(-3)\,i\,\pi\,\nu}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\left(\mathrm{ber}_\nu(z)+i\,\mathrm{bei}_\nu(z)\right)$$

03.02.27.0009.01

$$I_\nu\!\left(\sqrt[4]{z}\right)J_\nu\!\left(\sqrt[4]{z}\right)=(-z)^{-\frac{\nu}{2}}\,z^{\nu/2}\left(\mathrm{bei}_\nu\!\left(\sqrt[4]{-z}\right)^2+\mathrm{ber}_\nu\!\left(\sqrt[4]{-z}\right)^2\right)$$

Theorems

I-transformation

$$\hat{f}_\nu(y)=\int_0^\infty f(x)\sqrt{x\,y}\,I_\nu(x\,y)\,dx\iff f(x)=\frac{1}{\pi\,i}\int_{\gamma-i\infty}^{\gamma+i\infty}\hat{f}_\nu(y)\sqrt{x\,y}\,K_\nu(x\,y)\,dy\ /;\ \nu\geq-\frac{1}{2}$$

Unitary representations of the real Euclidean group

Bessel functions of integer order appear as the matrix elements of the unitary representations of the real Euclidean group E_3 in the $e^{i\,n\,\phi}$ basis.

The radial current density

The radial current density of a high frequency current with time dependence $i=i_0\cos(\omega\,t)$ flowing through a straight circular conductor of radius R is given by $i(r)\propto i_0\,\mathrm{Re}\left(I_0\!\left(\sqrt{i}\,\frac{r}{\xi}\right)\middle/I_0\!\left(\sqrt{i}\,\frac{R}{\xi}\right)\right)$, where ξ is a function of the conductivity and the frequency ω.

A simple continued fraction

The simple continued fraction with arithmetic progression

$$a+\cfrac{1}{a+d+\cfrac{1}{a+2\,d+\cfrac{1}{a+3\,d+\ddots}}}$$

has the closed form $I_{\frac{a}{d}-1}\left(\frac{2}{d}\right) \Big/ I_{\frac{a}{d}}\left(\frac{2}{d}\right)$.

Random walks

The probability w_t for a random walker who starts at $t = 0$ at the origin of a d dimensional cubic lattice to be at the point $\{n_1, n_2, \ldots, n_d\}$ at time t ($t \in \mathbb{N}$) is :

$$w_t == \frac{\partial^t}{\partial \xi^t} \prod_{k=1}^{d} I_n\left(\frac{\xi}{d}\right) \Big|_{\xi=0}$$

History

– A. B. Basset (1888)

Applications of $I_\nu(z)$ include electrodynamics, solid state physics, celestial mechanics, quantum chromodynamics, radiation theory, and the Aharonov–Bohm effect.

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