BesselY

Notations

Traditional name

Bessel function of the second kind

Traditional notation

$Y_\nu(z)$

Mathematica StandardForm notation

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BesselY[ν, z]
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Primary definition

03.03.02.0001.01

$$Y_\nu(z) == \csc(\pi\, \nu)\, (\cos(\nu\, \pi)\, J_\nu(z) - J_{-\nu}(z)) \;/;\; \nu \notin \mathbb{Z}$$

03.03.02.0002.01

$$Y_\nu(z) == \lim_{\mu \to \nu} Y_\mu(z) \;/;\; \nu \in \mathbb{Z}$$

Specific values

Specialized values

For fixed ν

03.03.03.0001.01

$$Y_\nu(0) == \tilde{\infty} \;/;\; \mathrm{Re}(\nu) \neq 0$$

03.03.03.0002.01

$$Y_\nu(0) == ¿ \;/;\; \mathrm{Re}(\nu) == 0 \land \nu \neq 0$$

For fixed z

Explicit rational ν

03.03.03.0013.01

$$Y_{-\frac{14}{3}}(z) = -\frac{1}{81\, 3^{5/6}\, z^{14/3}} \left(1760 \sqrt[3]{2} \left(9 z^{4/3} \left(1-\frac{9 z^2}{110}\right) \left(\text{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\, \text{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - \frac{\left(\frac{3}{2}\right)^{2/3} \left(81 z^4-4320 z^2+14\,080\right) \left(\text{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\, \text{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)}{1760}\right)\right)$$

03.03.03.0014.01

$$Y_{-\frac{9}{2}}(z) = \sqrt{\frac{2}{\pi}}\, \frac{\left(5 z \left(2 z^2-21\right) \cos(z)+\left(z^4-45 z^2+105\right) \sin(z)\right)}{z^{9/2}}$$

03.03.03.0015.01

$$Y_{-\frac{13}{3}}(z) = \frac{1}{27\, 3^{5/6}\, z^{13/3}} \left(1120 \sqrt[3]{2} \left(\sqrt[3]{2} \sqrt[6]{3} \left(1-\frac{9 z^2}{80}\right) \left(\sqrt{3}\, \text{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-3\, \text{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{2/3} + 2 \left(\frac{81 z^4}{4480}-\frac{27 z^2}{40}+1\right) \left(\text{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\sqrt{3}\, \text{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0016.01

$$Y_{-\frac{11}{3}}(z) = \frac{1}{27\, 3^{5/6}\, z^{11/3}} \left(80 \sqrt[3]{2} \left(\frac{1}{4} \left(9 z^2-32\right) \left(\text{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\, \text{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) \left(\frac{3}{2}\right)^{2/3} + 9 \left(1-\frac{9 z^2}{160}\right) z^{4/3} \left(\text{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\, \text{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0017.01

$$Y_{-\frac{7}{2}}(z) = -\sqrt{\frac{2}{\pi}}\, \frac{z \left(z^2-15\right) \cos(z)+3 \left(5-2 z^2\right) \sin(z)}{z^{7/2}}$$

03.03.03.0018.01

$$Y_{-\frac{10}{3}}(z) = -\frac{1}{9\, 3^{5/6}\, z^{10/3}} \left(56 \sqrt[3]{2} \left(\sqrt[3]{2} \sqrt[6]{3} \left(1-\frac{9 z^2}{112}\right) \left(\sqrt{3}\, \text{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-3\, \text{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{2/3} + 2 \left(1-\frac{9 z^2}{14}\right) \left(\text{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\sqrt{3}\, \text{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0019.01

$$Y_{-\frac{8}{3}}(z) = -\frac{1}{9\, 3^{5/6}\, z^{8/3}} \left(5 \sqrt[3]{2} \left(\frac{1}{5} \left(9 z^2-40\right) \left(\text{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\, \text{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) \left(\frac{3}{2}\right)^{2/3} + 9 z^{4/3} \left(\text{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\, \text{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0020.01

$$Y_{-\frac{5}{2}}(z) = -\sqrt{\frac{2}{\pi}}\, \frac{3 z \cos(z)+\left(z^2-3\right) \sin(z)}{z^{5/2}}$$

03.03.03.0021.01

$$Y_{-\frac{7}{3}}(z) = \frac{1}{3\,3^{5/6}\,z^{7/3}}\,4\sqrt[3]{2}\left(\sqrt[3]{2}\,\sqrt[6]{3}\left(\sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{2/3} + 2\left(1 - \frac{9\,z^2}{16}\right)\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.03.03.0022.01

$$Y_{-\frac{5}{3}}(z) = \frac{1}{3\,2^{2/3}\,3^{5/6}\,z^{5/3}}\left(9\,z^{4/3}\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - 4\sqrt[3]{2}\,3^{2/3}\left(\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.03.03.0023.01

$$Y_{-\frac{3}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{z\cos(z) - \sin(z)}{z^{3/2}}$$

03.03.03.0024.01

$$Y_{-\frac{4}{3}}(z) = -\frac{1}{2^{2/3}\,3^{5/6}\,z^{4/3}}\left(\sqrt[3]{2}\,\sqrt[6]{3}\left(\sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{2/3} + 2\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.03.03.0009.01

$$Y_{-\frac{2}{3}}(z) = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,z^{2/3}}\left(\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.03.03.0003.01

$$Y_{-\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{\sin(z)}{\sqrt{z}}$$

03.03.03.0007.01

$$Y_{-\frac{1}{3}}(z) = \frac{1}{2^{2/3}\,\sqrt[3]{3}\,\sqrt[3]{z}}\left(\sqrt{3}\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.03.03.0006.01

$$Y_{\frac{1}{3}}(z) = -\frac{1}{2^{2/3}\,\sqrt[3]{3}\,\sqrt[3]{z}}\left(\sqrt{3}\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 3\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.03.03.0004.01

$$Y_{\frac{1}{2}}(z) = -\sqrt{\frac{2}{\pi}}\,\frac{\cos(z)}{\sqrt{z}}$$

03.03.03.0008.01

$$Y_{\frac{2}{3}}(z) = \frac{1}{\sqrt[3]{2}\,\sqrt[6]{3}\,z^{2/3}}\left(\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.03.03.0025.01

$$Y_{\frac{4}{3}}(z) = -\frac{1}{2^{2/3}\,3^{5/6}\,z^{4/3}}\left(\sqrt[3]{2}\,\sqrt[6]{3}\left(\sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+3\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)z^{2/3}+2\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.03.03.0026.01

$$Y_{\frac{3}{2}}(z) = -\frac{\sqrt{\frac{2}{\pi}}\,(\cos(z)+z\sin(z))}{z^{3/2}}$$

03.03.03.0027.01

$$Y_{\frac{5}{3}}(z) = -\frac{1}{3\,2^{2/3}\,3^{5/6}\,z^{5/3}}\left(9\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)z^{4/3}+4\sqrt[3]{2}\,3^{2/3}\left(\sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.03.03.0028.01

$$Y_{\frac{7}{3}}(z) = -\frac{1}{3\,3^{5/6}\,z^{7/3}}\left(4\sqrt[3]{2}\left(\sqrt[3]{2}\,\sqrt[6]{3}\left(\sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+3\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)z^{2/3}+2\left(1-\frac{9z^2}{16}\right)\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0029.01

$$Y_{\frac{5}{2}}(z) = \sqrt{\frac{2}{\pi}}\;\frac{\left(\left(z^2-3\right)\cos(z)-3z\sin(z)\right)}{z^{5/2}}$$

03.03.03.0030.01

$$Y_{\frac{8}{3}}(z) = -\frac{1}{9\,3^{5/6}\,z^{8/3}}\left(5\sqrt[3]{2}\left(9z^{4/3}\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)-\frac{1}{5}\left(\frac{3}{2}\right)^{2/3}\left(9z^2-40\right)\left(\sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0031.01

$$Y_{\frac{10}{3}}(z) = -\frac{1}{9\,3^{5/6}\,z^{10/3}}\left(56\sqrt[3]{2}\left(\sqrt[3]{2}\,\sqrt[6]{3}\left(1-\frac{9z^2}{112}\right)\left(\sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+3\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)z^{2/3}+2\left(1-\frac{9z^2}{14}\right)\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0032.01

$$Y_{\frac{7}{2}}(z) = \sqrt{\frac{2}{\pi}}\;\frac{\left(3\left(2z^2-5\right)\cos(z)+z\left(z^2-15\right)\sin(z)\right)}{z^{7/2}}$$

03.03.03.0033.01

$$Y_{\frac{11}{3}}(z) = -\frac{1}{27\,3^{5/6}\,z^{11/3}}\left(80\sqrt[3]{2}\left(9z^{4/3}\left(1-\frac{9z^2}{160}\right)\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)-\frac{1}{4}\left(\frac{3}{2}\right)^{2/3}\left(9z^2-32\right)\left(\sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0034.01

$$Y_{\frac{13}{3}}(z) = -\frac{1}{27\,3^{5/6}\,z^{13/3}}\left(1120\,\sqrt[3]{2}\left(\sqrt[3]{2}\,\sqrt[6]{3}\left(1-\frac{9\,z^2}{80}\right)\left(\sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+3\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)z^{2/3} + 2\left(\frac{81\,z^4}{4480}-\frac{27\,z^2}{40}+1\right)\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

03.03.03.0035.01

$$Y_{\frac{9}{2}}(z) = -\sqrt{\frac{2}{\pi}}\;\frac{\left(\left(z^4-45\,z^2+105\right)\cos(z)+5\,z\left(21-2\,z^2\right)\sin(z)\right)}{z^{9/2}}$$

03.03.03.0036.01

$$Y_{\frac{14}{3}}(z) = -\frac{1}{81\,3^{5/6}\,z^{14/3}}\left(1760\,\sqrt[3]{2}\left(9\left(1-\frac{9\,z^2}{110}\right)z^{4/3}\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)+\left(\frac{3}{2}\right)^{2/3}\frac{\left(81\,z^4-4320\,z^2+14\,080\right)}{1760}\left(\sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)-\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)\right)$$

Symbolic rational ν

03.03.03.0005.01

$$Y_\nu(z) = \sqrt{\frac{2}{\pi}}\;\frac{(-1)^{\nu+\frac{1}{2}}}{\sqrt{z}}\left(\sin\left(\frac{\pi}{2}\left(\nu+\frac{1}{2}\right)+z\right)\sum_{j=0}^{\left\lfloor\frac{2|\nu|-1}{4}\right\rfloor}\frac{(-1)^j\left(|\nu|+2\,j-\frac{1}{2}\right)!\,(2\,z)^{-2\,j}}{(2\,j)!\left(|\nu|-2\,j-\frac{1}{2}\right)!} + \cos\left(\frac{\pi}{2}\left(\nu+\frac{1}{2}\right)+z\right)\sum_{j=0}^{\left\lfloor\frac{2|\nu|-3}{4}\right\rfloor}\frac{(-1)^j\left(|\nu|+2\,j+\frac{1}{2}\right)!\,(2\,z)^{-2\,j-1}}{(2\,j+1)!\left(|\nu|-2\,j-\frac{3}{2}\right)!}\right)\;/;\;\nu-\frac{1}{2}\in\mathbb{Z}$$

03.03.03.0010.01

$$Y_\nu(z) = -\frac{(-1)^{\frac{1}{2}(\mathrm{sgn}(\nu)+1)\left(|\nu|-\frac{4}{3}\right)}\,2^{|\nu|-2}\,z^{-|\nu|}\,\Gamma\left(-\frac{1}{3}\right)}{3^{5/6}\,\Gamma(1-|\nu|)}\left(\sqrt[3]{2}\,\sqrt[6]{3}\left(\sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+3\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\mathrm{sgn}(\nu)\right)\left(\sum_{k=0}^{|\nu|-\frac{4}{3}}\frac{\left(|\nu|-k-\frac{4}{3}\right)!}{k!\left(|\nu|-2\,k-\frac{4}{3}\right)!\left(\frac{4}{3}\right)_k(1-|\nu|)_k}\left(\frac{z^2}{4}\right)^k\right)z^{2/3} + 2\left(\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)+\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\mathrm{sgn}(\nu)\right)\sum_{k=0}^{|\nu|-\frac{1}{3}}\frac{\left(|\nu|-k-\frac{1}{3}\right)!}{k!\left(|\nu|-2\,k-\frac{1}{3}\right)!\left(\frac{1}{3}\right)_k(1-|\nu|)_k}\left(\frac{z^2}{4}\right)^k\right)\;/;\;|\nu|-\frac{1}{3}\in\mathbb{Z}$$

03.03.03.0011.01

$$Y_\nu(z) = \frac{(-1)^{\frac{1}{2}(\operatorname{sgn}(\nu)+1)\left(|\nu|-\frac{2}{3}\right)} 2^{|\nu|-\frac{7}{3}} z^{-|\nu|} \Gamma\left(-\frac{2}{3}\right)}{3\, 3^{5/6}\, \Gamma(1-|\nu|)} \left(9 z^{4/3}\left(\operatorname{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \operatorname{sgn}(\nu)\sqrt{3}\, \operatorname{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) \sum_{k=0}^{|\nu|-\frac{5}{3}} \frac{\left(|\nu|-k-\frac{5}{3}\right)!}{k!\left(|\nu|-2k-\frac{5}{3}\right)!\left(\frac{5}{3}\right)_k (1-|\nu|)_k} \left(\frac{z^2}{4}\right)^k - 4\sqrt[3]{2}\, 3^{2/3} \left(\operatorname{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \operatorname{sgn}(\nu)\sqrt{3}\, \operatorname{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) \sum_{k=0}^{|\nu|-\frac{2}{3}} \frac{\left(|\nu|-k-\frac{2}{3}\right)!}{k!\left(|\nu|-2k-\frac{2}{3}\right)!\left(\frac{2}{3}\right)_k (1-|\nu|)_k} \left(\frac{z^2}{4}\right)^k\right) /; |\nu| - \frac{2}{3} \in \mathbb{Z}$$

Values at fixed points

03.03.03.0012.01

$$Y_0(0) = -\infty$$

Values at infinities

03.03.03.0037.01

$$\lim_{x\to\infty} Y_\nu(x) = 0$$

03.03.03.0038.01

$$\lim_{x\to-\infty} Y_\nu(x) = 0$$

03.03.03.0039.01

$$Y_\nu\left(e^{i\lambda}\infty\right) = \begin{cases} 0 & \lambda = 0 \lor \lambda = \pi \\ \tilde{\infty} & \text{True} \end{cases} /; \operatorname{Im}(\lambda) = 0$$

03.03.03.0040.01

$$Y_\nu(\infty) = 0$$

03.03.03.0041.01

$$Y_\nu(-\infty) = 0$$

General characteristics

Domain and analyticity

$Y_\nu(z)$ is an analytical function of ν and z which is defined on $\mathbb{C}^2$.

03.03.04.0001.01

$$(\nu * z) \longrightarrow Y_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

03.03.04.0002.01

$$Y_{-n}(z) = (-1)^n Y_n(z) /; n \in \mathbb{Z}$$

Mirror symmetry

03.03.04.0003.01

$$Y_{\bar{\nu}}(\bar{z}) == \overline{Y_\nu(z)} \;/;\; z \notin (-\infty, 0)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, the function $Y_\nu(z)$ has an essential singularity at $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is a branch point.

03.03.04.0004.01

$$\mathcal{S}ing_z(Y_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$$

With respect to ν

For fixed z, the function $Y_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

03.03.04.0005.01

$$\mathcal{S}ing_\nu(Y_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$$

Branch points

With respect to z

For fixed ν, the function $Y_\nu(z)$ has two branch points: $z == 0$, $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is an essential singularity.

03.03.04.0006.01

$$\mathcal{BP}_z(Y_\nu(z)) == \{0, \tilde{\infty}\}$$

03.03.04.0007.01

$$\mathcal{R}_z(Y_\nu(z), 0) == \log \;/;\; \nu \notin \mathbb{Q}$$

03.03.04.0008.01

$$\mathcal{R}_z\left(Y_{\frac{p}{q}}(z), 0\right) == q \;/;\; p \in \mathbb{Z} \bigwedge q-1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$$

03.03.04.0009.01

$$\mathcal{R}_z(Y_\nu(z), \tilde{\infty}) == \log \;/;\; \nu \notin \mathbb{Q}$$

03.03.04.0010.01

$$\mathcal{R}_z\left(Y_{\frac{p}{q}}(z), \tilde{\infty}\right) == q \;/;\; p \in \mathbb{Z} \bigwedge q-1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$$

With respect to ν

For fixed z, the function $Y_\nu(z)$ does not have branch points.

03.03.04.0011.01

$$\mathcal{BP}_\nu(Y_\nu(z)) == \{\}$$

Branch cuts

With respect to z

For fixed ν, the function $Y_\nu(z)$ has one infinitely long branch cut. For fixed ν, the function $Y_\nu(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

03.03.04.0012.01

$$\mathcal{BC}_z(Y_\nu(z)) = \{\{(-\infty, 0), -i\}\}$$

03.03.04.0013.01

$$\lim_{\epsilon\to+0} Y_\nu(x + i\,\epsilon) = Y_\nu(x) \;/;\; x \in \mathbb{R} \land x < 0$$

03.03.04.0014.01

$$\lim_{\epsilon\to+0} Y_\nu(x - i\,\epsilon) = \cot(\nu\,\pi)\,e^{-2\pi i\nu}\,J_\nu(x) - \csc(\nu\,\pi)\,e^{2\pi i\nu}\,J_{-\nu}(x) \;/;\; \nu \notin \mathbb{Z} \bigwedge x \in \mathbb{R} \bigwedge x < 0$$

03.03.04.0017.01

$$\lim_{\epsilon\to+0} Y_\nu(x - i\,\epsilon) = e^{2 i\pi\nu}\,Y_\nu(x) - 4\,i\cos^2(\pi\,\nu)\,J_\nu(x) \;/;\; x \in \mathbb{R} \land x < 0$$

03.03.04.0015.01

$$\lim_{\epsilon\to+0} Y_\nu(x - i\,\epsilon) = Y_\nu(x) - 4\,i\,J_\nu(x) \;/;\; \nu \in \mathbb{Z} \land x \in \mathbb{R} \land x < 0$$

With respect to ν

For fixed z, the function $Y_\nu(z)$ is an entire function of ν and does not have branch cuts.

03.03.04.0016.01

$$\mathcal{BC}_\nu(Y_\nu(z)) = \{\}$$

Series representations

Generalized power series

Expansions at $\nu = \pm n$

03.03.06.0019.01

$$Y_\nu(z) \propto Y_n(z) + \left(\frac{1}{2}\,n!\left(\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{1}{(n-k)\,k!}\,Y_k(z)\left(\frac{z}{2}\right)^k - \frac{\pi}{2}\,J_n(z)\right)(\nu - n) + \ldots \;/;\; (\nu \to n) \land n \in \mathbb{N}$$

Expansions at generic point $z = z_0$

For the function itself

03.03.06.0020.01

$$Y_\nu(z) \propto Y_\nu(z_0)\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} + 4\,i\,J_\nu(z_0)\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor + \frac{1}{2}\left((Y_{\nu-1}(z_0)-Y_{\nu+1}(z_0))\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} + 4\,i\,(J_{\nu-1}(z_0)-J_{\nu+1}(z_0))\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor\right)(z-z_0) + \frac{1}{8}\left((Y_{\nu-2}(z_0)-2\,Y_\nu(z_0)+Y_{\nu+2}(z_0))\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} + 4\,i\,(J_{\nu-2}(z_0)-2\,J_\nu(z_0)+J_{\nu+2}(z_0))\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor\right)(z-z_0)^2 + \ldots \;/;\; (z\to z_0)$$

03.03.06.0021.01

$$Y_\nu(z) \propto Y_\nu(z_0)\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} + 4\,i\,J_\nu(z_0)\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor + \frac{1}{2}\left((Y_{\nu-1}(z_0)-Y_{\nu+1}(z_0))\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} + 4\,i\,(J_{\nu-1}(z_0)-J_{\nu+1}(z_0))\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor\right)(z-z_0) + \frac{1}{8}\left((Y_{\nu-2}(z_0)-2\,Y_\nu(z_0)+Y_{\nu+2}(z_0))\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} + 4\,i\,(J_{\nu-2}(z_0)-2\,J_\nu(z_0)+J_{\nu+2}(z_0))\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor\right)(z-z_0)^2 + O\left((z-z_0)^3\right)$$

03.03.06.0022.01

$$Y_\nu(z) = \sum_{k=0}^{\infty}\frac{Y_\nu^{(0,k)}(z_0)}{k!}(z-z_0)^k \;/;\; |\arg(z_0)| < \pi$$

03.03.06.0023.01

$$Y_\nu(z) = \sum_{k=0}^{\infty}\frac{1}{k!}\,G_{3,5}^{2,2}\left(\frac{z_0}{2}, \frac{1}{2} \,\middle|\, \begin{matrix}\frac{1-k}{2}, -\frac{k}{2}, -\frac{k+\nu+1}{2} \\ \frac{\nu-k}{2}, -\frac{k+\nu}{2}, \frac{1}{2}, 0, -\frac{k+\nu+1}{2}\end{matrix}\right)(z-z_0)^k \;/;\; |\arg(z_0)| < \pi$$

03.03.06.0024.01

$$Y_\nu(z) = 2^{-2\nu}\sqrt{\pi}\, z_0^{-\nu}\csc(\pi\nu) \sum_{k=0}^{\infty}\frac{2^k z_0^{-k}}{k!}\left(-16^\nu\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\Gamma(1-\nu)\, {}_2\tilde{F}_3\!\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1}{2}(-k-\nu+1), \frac{1}{2}(-k-\nu+2), 1-\nu; -\frac{z_0^2}{4}\right) + z_0^{2\nu}\left(\frac{1}{z_0}\right)^{\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\cos(\pi\nu)\,\Gamma(\nu+1)\, {}_2\tilde{F}_3\!\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-k+\nu+1), \frac{1}{2}(-k+\nu+2), \nu+1; -\frac{z_0^2}{4}\right)\right)(z-z_0)^k \;/;\; \nu\notin\mathbb{Z}$$

03.03.06.0025.01

$$Y_\nu(z) = \sum_{k=0}^{\infty}\frac{2^{-k}}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}\left(Y_{-2j+k+\nu}(z_0)\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} + 4\,i\,J_{-2j+k+\nu}(z_0)\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor\right)(z-z_0)^k$$

03.03.06.0026.01

$$Y_\nu(z) = \sum_{k=0}^{\infty}\frac{z_0^{-k}}{k!}\sum_{m=0}^{k}(-1)^{k+m}\binom{k}{m}(-\nu)_{k-m}\sum_{i=0}^{m}\frac{(-1)^{i-1}\,2^{2i-m}\,(-m)_{2(m-i)}\,(\nu)_i}{(m-i)!}\left(\frac{z_0}{2}\sum_{j=0}^{i-1}\frac{(i-j-1)!}{j!\,(i-2j-1)!\,(-i-\nu+1)_j\,(\nu)_{j+1}}\left(\frac{z_0^2}{4}\right)^j Y_{\nu-1}(z_0) + \sum_{j=0}^{i}\frac{(i-j)!}{j!\,(i-2j)!\,(-i-\nu+1)_j\,(\nu)_j}\left(\frac{z_0^2}{4}\right)^j Y_\nu(z_0)\right)(z-z_0)^k \;/;\; |\arg(z_0)|<\pi$$

03.03.06.0027.01

$$Y_\nu(z) \propto Y_\nu(z_0)\left(\frac{1}{z_0}\right)^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{-\nu\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} + 4\,i\,J_\nu(z_0)\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor(1+O(z-z_0))$$

Expansions on branch cuts

For the function itself

03.03.06.0028.01

$$Y_\nu(z) \propto e^{-2i\pi\nu\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}Y_\nu(x) + 4\,i\,J_\nu(x)\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor - \frac{1}{2}\left(e^{-2i\pi\nu\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}(Y_{\nu+1}(x)-Y_{\nu-1}(x)) - 4\,i\,(J_{\nu-1}(x)-J_{\nu+1}(x))\cos^2(\pi\nu)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\right)(z-x) + \frac{1}{8}\left(4\,i\,(J_{\nu-2}(x)-2J_\nu(x)+J_{\nu+2}(x))\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\cos^2(\pi\nu) + e^{-2i\pi\nu\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}(Y_{\nu-2}(x)-2Y_\nu(x)+Y_{\nu+2}(x))\right)(z-x)^2 + \ldots \;/;\; (z\to x)\land x\in\mathbb{R}\land x<0$$

03.03.06.0029.01

$$Y_\nu(z) \propto e^{-2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} Y_\nu(x) + 4 i J_\nu(x) \cos^2(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor - \frac{1}{2} \left(e^{-2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (Y_{\nu+1}(x) - Y_{\nu-1}(x)) - 4 i (J_{\nu-1}(x) - J_{\nu+1}(x)) \cos^2(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \right) (z-x) + \frac{1}{8} \left(4 i (J_{\nu-2}(x) - 2 J_\nu(x) + J_{\nu+2}(x)) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \cos^2(\pi \nu) + e^{-2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (Y_{\nu-2}(x) - 2 Y_\nu(x) + Y_{\nu+2}(x)) \right) (z-x)^2 ++ O\left((z-x)^3\right) /; x \in \mathbb{R} \land x < 0$$

03.03.06.0030.01

$$Y_\nu(z) = 2^{-2\nu} \sqrt{\pi}\, x^{-\nu} \csc(\pi \nu) \sum_{k=0}^{\infty} \frac{2^k x^{-k}}{k!} \left(-16^\nu e^{-2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \Gamma(1-\nu)\, {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1 - \frac{\nu}{2}; \frac{1}{2}(-k-\nu+1), \frac{1}{2}(-k-\nu+2), 1-\nu; -\frac{x^2}{4} \right) + x^{2\nu} e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \cos(\pi \nu) \Gamma(\nu+1)\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-k+\nu+1), \frac{1}{2}(-k+\nu+2), \nu+1; -\frac{x^2}{4} \right) \right) (z-x)^k /; \nu \notin \mathbb{Z}$$

03.03.06.0031.01

$$Y_\nu(z) = \sum_{k=0}^{\infty} \frac{2^{-k}}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} \left(4 i J_{-2j+k+\nu}(x) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \cos^2(\pi \nu) + Y_{-2j+k+\nu}(x)\, e^{-2 \nu \pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \right) (z-x)^k /; x \in \mathbb{R} \land x < 0$$

03.03.06.0032.01

$$Y_\nu(z) \propto e^{-2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} Y_\nu(x) + 4 i J_\nu(x) \cos^2(\pi \nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor (1 + O(z-x)) /; x \in \mathbb{R} \land x < 0$$

Expansions at $z == 0$

For the function itself

General case

03.03.06.0001.02

$$Y_\nu(z) == -\frac{\cos(\pi \nu) \Gamma(-\nu)}{\pi} \left(\frac{z}{2}\right)^\nu \left(1 - \frac{z^2}{4(\nu+1)} + \frac{z^4}{32(\nu+1)(\nu+2)} - \ldots\right) - \frac{\Gamma(\nu)}{\pi} \left(\frac{z}{2}\right)^{-\nu} \left(1 - \frac{z^2}{4(1-\nu)} + \frac{z^4}{32(1-\nu)(2-\nu)} - \ldots\right) /; (z \to 0) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0033.01

$$Y_\nu(z) == -\frac{\cos(\pi \nu) \Gamma(-\nu)}{\pi} \left(\frac{z}{2}\right)^\nu \left(1 - \frac{z^2}{4(\nu+1)} + \frac{z^4}{32(\nu+1)(\nu+2)} - O\left(z^6\right)\right) - \frac{\Gamma(\nu)}{\pi} \left(\frac{z}{2}\right)^{-\nu} \left(1 - \frac{z^2}{4(1-\nu)} + \frac{z^4}{32(1-\nu)(2-\nu)} - O\left(z^6\right)\right) /; \nu \notin \mathbb{Z}$$

03.03.06.0034.01

$$Y_\nu(z) == -\frac{\Gamma(\nu)}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{2}\right)^{2k-\nu}}{(1-\nu)_k\, k!} - \frac{\Gamma(-\nu) \cos(\nu \pi)}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{2}\right)^{2k+\nu}}{(\nu+1)_k\, k!} /; \nu \notin \mathbb{Z}$$

03.03.06.0002.01

$$Y_\nu(z) = \csc(\pi\nu)\left(\cos(\nu\pi)\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+\nu+1)\,k!}\left(\frac{z}{2}\right)^{2k+\nu} - \sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k-\nu+1)\,k!}\left(\frac{z}{2}\right)^{2k-\nu}\right) /;\ \nu \notin \mathbb{Z}$$

03.03.06.0035.01

$$Y_\nu(z) = \csc(\pi\nu)\left(\frac{\cos(\nu\pi)}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^{\nu}{}_0F_1\left(;\nu+1;-\frac{z^2}{4}\right) - \frac{1}{\Gamma(1-\nu)}\left(\frac{z}{2}\right)^{-\nu}{}_0F_1\left(;1-\nu;-\frac{z^2}{4}\right)\right) /;\ \nu \notin \mathbb{Z}$$

03.03.06.0003.01

$$Y_\nu(z) = 2^{-\nu} z^{\nu}\cot(\pi\nu)\,{}_0\tilde{F}_1\left(;\nu+1;-\frac{z^2}{4}\right) - 2^{\nu} z^{-\nu}\csc(\pi\nu)\,{}_0\tilde{F}_1\left(;1-\nu;-\frac{z^2}{4}\right) /;\ \nu \notin \mathbb{Z}$$

03.03.06.0006.01

$$Y_\nu(z) \propto -\frac{2^{\nu}\ \Gamma(\nu)}{\pi} z^{-\nu}\left(1+O\left(z^2\right)\right) - \frac{2^{-\nu}\cos(\nu\pi)\,\Gamma(-\nu)}{\pi} z^{\nu}\left(1+O\left(z^2\right)\right) /;\ \nu \notin \mathbb{Z}$$

03.03.06.0036.01

$$Y_\nu(z) = F_\infty(z,\nu) /; \left(\left(F_m(z,\nu) = \csc(\pi\nu)\left(\cos(\nu\pi)\sum_{k=0}^{m}\frac{(-1)^k\left(\frac{z}{2}\right)^{2k+\nu}}{\Gamma(k+\nu+1)\,k!} - \sum_{k=0}^{m}\frac{(-1)^k\left(\frac{z}{2}\right)^{2k-\nu}}{\Gamma(k-\nu+1)\,k!}\right) = Y_\nu(z) - \frac{(-1)^m\csc(\pi\nu)}{(m+1)!}\left(\frac{2^{-2m+\nu-2}z^{2m-\nu+2}}{\Gamma(m-\nu+2)}\,{}_1F_2\left(1;m+2,m-\nu+2;-\frac{z^2}{4}\right) - \frac{2^{-2m-\nu-2}z^{2m+\nu+2}\cos(\pi\nu)}{\Gamma(m+\nu+2)}\,{}_1F_2\left(1;m+2,m+\nu+2;-\frac{z^2}{4}\right)\right)\right) \bigwedge m \in \mathbb{N}\right) /;\ \nu \notin \mathbb{Z}$$

Summed form of the truncated series expansion.

Logarithmic cases

03.03.06.0037.01

$$Y_0(z) = \frac{2}{\pi}\log\left(\frac{z}{2}\right)\left(1-\frac{z^2}{4}+\frac{z^4}{64}+\ldots\right) - \frac{1}{\pi}\left(-2\gamma+\frac{1}{4}(-2+2\gamma)z^2+\frac{1}{64}(3-2\gamma)z^4+\ldots\right) /;\ (z\to 0)$$

03.03.06.0038.01

$$Y_1(z) \propto \frac{z}{\pi}\log\left(\frac{z}{2}\right)\left(1-\frac{z^2}{8}+\frac{z^4}{192}+\ldots\right) - \frac{2}{\pi z} - \frac{z}{2\pi}\left(-2\gamma+1+\frac{1}{8}\left(-\frac{5}{2}+2\gamma\right)z^2+\frac{1}{192}\left(\frac{10}{3}-2\gamma\right)z^4+\ldots\right) /;\ (z\to 0)$$

03.03.06.0039.01

$$Y_2(z) \propto -\frac{z^2}{8\pi}\left(\frac{3}{2}-2\gamma-\frac{1}{12}\left(\frac{17}{6}-2\gamma\right)z^2+\frac{1}{384}\left(\frac{43}{12}-2\gamma\right)z^4+\ldots\right) - \frac{4}{\pi z^2}\left(1+\frac{z^2}{4}\right) + \frac{z^2}{4\pi}\log\left(\frac{z}{2}\right)\left(1-\frac{z^2}{12}+\frac{z^4}{384}+\ldots\right) /;\ (z\to 0)$$

03.03.06.0040.01

$$Y_n(z) \propto -\frac{(n-1)!}{\pi}\left(1+\frac{1}{4(n-1)}z^2+\frac{1}{32(n-1)(n-2)}z^4+\ldots\right)\left(\frac{z}{2}\right)^{-n}-\frac{1}{\pi\, n!}\left(\frac{z}{2}\right)^n\left(\psi(n+1)-\gamma-\frac{(\psi(n+2)-\gamma+1)z^2}{4(n+1)}+\frac{\left(\psi(n+3)+\frac{3}{2}-\gamma\right)z^4}{32(n+1)(n+2)}+\ldots\right)+\frac{2}{\pi\, n!}\log\left(\frac{z}{2}\right)\left(\frac{z}{2}\right)^n\left(1-\frac{z^2}{4(n+1)}+\frac{z^4}{32(n+1)(n+2)}+\ldots\right)\,/;\,(z\to 0)\land n-3\in\mathbb{N}$$

03.03.06.0041.01

$$Y_\nu(z) = \frac{1}{\pi}(-1)^{\frac{1}{2}(|\nu|-\nu)}\left(-\left(\frac{z}{2}\right)^{-|\nu|}\sum_{k=0}^{|\nu|-1}\frac{(|\nu|-k-1)!}{k!}\left(\frac{z}{2}\right)^{2k}+2\log\left(\frac{z}{2}\right)\left(\frac{z}{2}\right)^{|\nu|}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\,(k+|\nu|)!}\left(\frac{z}{2}\right)^{2k}-\left(\frac{z}{2}\right)^{|\nu|}\sum_{k=0}^{\infty}\frac{(-1)^k(\psi(k+1)+\psi(k+|\nu|+1))}{k!\,(k+|\nu|)!}\left(\frac{z}{2}\right)^{2k}\right)/;\,\nu\in\mathbb{Z}$$

03.03.06.0004.01

$$Y_\nu(z) = \frac{(-1)^{\frac{(|\nu|-\nu)}{2}}}{\pi}\left(2\log\left(\frac{z}{2}\right)\left(\frac{z}{2}\right)^{|\nu|}{}_0\tilde{F}_1\left(;|\nu|+1;-\frac{z^2}{4}\right)-\sum_{k=0}^{|\nu|-1}\frac{(|\nu|-k-1)!}{k!}\left(\frac{z}{2}\right)^{2k-|\nu|}-\sum_{k=0}^{\infty}\frac{(-1)^k(\psi(k+1)+\psi(k+|\nu|+1))}{k!\,(k+|\nu|)!}\left(\frac{z}{2}\right)^{2k+|\nu|}\right)/;\,\nu\in\mathbb{Z}$$

03.03.06.0042.01

$$Y_n(z) = -\frac{1}{\pi}\left(\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{(n-k-1)!}{k!}\left(\frac{z}{2}\right)^{2k}+\frac{2}{\pi}\log\left(\frac{z}{2}\right)\left(\frac{z}{2}\right)^n\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\,(k+n)!}\left(\frac{z}{2}\right)^{2k}-\frac{1}{\pi}\left(\frac{z}{2}\right)^n\sum_{k=0}^{\infty}\frac{(-1)^k(\psi(k+1)+\psi(k+n+1))}{k!\,(k+n)!}\left(\frac{z}{2}\right)^{2k}\,/;\,n\in\mathbb{N}$$

03.03.06.0005.01

$$Y_n(z) = \frac{2}{\pi}J_n(z)\log\left(\frac{z}{2}\right)-\frac{1}{\pi}\sum_{k=0}^{n-1}\frac{(n-k-1)!}{k!}\left(\frac{z}{2}\right)^{2k-n}-\frac{1}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(\psi(k+1)+\psi(k+n+1))}{k!\,(k+n)!}\left(\frac{z}{2}\right)^{2k+n}\,/;\,n\in\mathbb{N}$$

03.03.06.0007.01

$$Y_\nu(z) \propto \frac{(-1)^{\frac{(|\nu|-\nu)}{2}}}{\pi}\left(2\log\left(\frac{z}{2}\right)\delta_{|\nu|}\left(1+O\left(z^2\right)\right)-\sum_{k=0}^{|\nu|-1}\frac{(-k+|\nu|-1)!}{k!}\left(\frac{z}{2}\right)^{2k-|\nu|}\left(1+O\left(z^2\right)\right)\right)/;\,\nu\in\mathbb{Z}$$

03.03.06.0043.01

$$Y_0(z) = \frac{2}{\pi}\log\left(\frac{z}{2}\right)\left(1+O\left(z^2\right)\right)+\frac{2\gamma}{\pi}\left(1+O\left(z^2\right)\right)$$

03.03.06.0044.01

$$Y_1(z) \propto \frac{z}{\pi}\log\left(\frac{z}{2}\right)\left(1+O\left(z^2\right)\right)-\frac{2}{\pi z}\left(1+O\left(z^2\right)\right)$$

03.03.06.0045.01

$$Y_2(z) \propto -\frac{4}{\pi z^2}\left(1+O\left(z^2\right)\right)+\frac{z^2}{4\pi}\log\left(\frac{z}{2}\right)\left(1+O\left(z^2\right)\right)\,/;\,(z\to 0)$$

03.03.06.0046.01

$$Y_n(z) \propto -\frac{(n-1)!}{\pi}\left(\frac{z}{2}\right)^{-n}\left(1+O\left(z^2\right)\right) + \frac{2}{\pi\, n!}\log\left(\frac{z}{2}\right)\left(\frac{z}{2}\right)^n\left(1+O\left(z^2\right)\right) \;/;\; n-3\in\mathbb{N}$$

03.03.06.0047.01

$$Y_n(z) = F_\infty(z, n) \;/;$$

$$\left(F_m(z, n) = \frac{2}{\pi}\log\left(\frac{z}{2}\right)J_n(z) - \frac{1}{\pi}\sum_{k=0}^{n-1}\frac{(-k+n-1)!\left(\frac{z}{2}\right)^{2k-n}}{k!} - \frac{1}{\pi}\left(\frac{z}{2}\right)^n\sum_{k=0}^{m}\frac{(-1)^k\left(-2\log\left(\frac{z}{2}\right)+\psi(k+1)+\psi(k+n+1)\right)}{k!\,(k+n)!}\left(\frac{z}{2}\right)^{2k} = \right.$$

$$Y_n(z) - \frac{(-1)^m\, 2^{-2m-n-1}\, z^{2m+n+2}}{\pi\,(m+1)!\,(m+n+1)!}\log\left(\frac{z}{2}\right){}_1F_2\left(1;\, m+2,\, m+n+2;\, -\frac{z^2}{4}\right) -$$

$$\left.(-1)^n\, G_{3,5}^{2,2}\left(\frac{z^2}{4}\,\middle|\, \begin{array}{c} m+\frac{n}{2}+1,\, m+\frac{n}{2}+1,\, -\frac{1}{2}(n+1) \\ m+\frac{n}{2}+1,\, m+\frac{n}{2}+1,\, \frac{n}{2},\, -\frac{n}{2},\, -\frac{1}{2}(n+1) \end{array}\right) \bigwedge n\in\mathbb{N}\right) \bigwedge m\in\mathbb{N}$$

Summed form of the truncated series expansion.

Generic formulas for main term

03.03.06.0048.01

$$Y_\nu(z) \propto \begin{cases} \frac{2}{\pi}\log\left(\frac{z}{2}\right) + \frac{\gamma\, 2}{\pi} & \nu = 0 \\ -\frac{1}{\pi}\left((-1)^{\frac{|\nu|-\nu}{2}}\,(|\nu|-1)!\right)\left(\frac{z}{2}\right)^{-|\nu|} & \nu\in\mathbb{Z}\land\nu\neq 0 \\ -\frac{\Gamma(\nu)}{\pi}\left(\frac{z}{2}\right)^{-\nu} - \frac{\cos(\pi\nu)\,\Gamma(-\nu)}{\pi}\left(\frac{z}{2}\right)^{\nu} & \text{True} \end{cases} \;/;\; (z\to 0)$$

Asymptotic series expansions

Expansions inside Stokes sectors

Expansions containing $z \to \infty$

In exponential form ||| In exponential form

03.03.06.0049.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^{-iz+\frac{1}{4}i\pi(2\nu+3)}\left(1 - \frac{i\left(-1+4\nu^2\right)}{8z} - \frac{9-40\nu^2+16\nu^4}{128z^2} + \ldots\right) + \right.$$

$$\left. e^{iz-\frac{1}{4}i\pi(2\nu+3)}\left(1 + \frac{i\left(-1+4\nu^2\right)}{8z} - \frac{9-40\nu^2+16\nu^4}{128z^2} + \ldots\right)\right) \;/;\; |\arg(z)|<\pi \bigwedge (|z|\to\infty)$$

03.03.06.0050.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{2\pi z}}\left(e^{-iz+\frac{1}{4}i\pi(2\nu+3)}\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(\frac{i}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right)\right) + \right.$$

$$\left. e^{iz-\frac{1}{4}i\pi(2\nu+3)}\left(\sum_{k=0}^{n}\frac{\left(\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\nu\right)_k}{k!}\left(-\frac{i}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right)\right)\right) \;/;\; |\arg(z)|<\pi \bigwedge (|z|\to\infty)$$

03.03.06.0051.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{2\pi z}} \left(e^{-iz+\frac{1}{4}i\pi(2\nu+3)} {}_2F_0\left(\nu+\frac{1}{2}, \frac{1}{2}-\nu; ; \frac{i}{2z}\right) + e^{iz-\frac{1}{4}i\pi(2\nu+3)} {}_2F_0\left(\nu+\frac{1}{2}, \frac{1}{2}-\nu; ; -\frac{i}{2z}\right)\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.03.06.0052.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{2\pi z}} \left(e^{-iz+\frac{1}{4}i\pi(2\nu+3)} \left(1+O\left(\frac{1}{z}\right)\right) + e^{iz-\frac{1}{4}i\pi(2\nu+3)} \left(1+O\left(\frac{1}{z}\right)\right)\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

In trigonometric form ||| In trigonometric form

03.03.06.0053.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{z}} \sqrt{\frac{2}{\pi}} \left(\sin\left(z-\frac{\pi\nu}{2}-\frac{\pi}{4}\right)\left(1-\frac{16\nu^4-40\nu^2+9}{128z^2}+\frac{16\left(2\nu^2\left(8\left(\nu^2-21\right)\nu^2+987\right)-3229\right)\nu^2+11025}{98304z^4}+\ldots\right)+\right.$$
$$\left.\frac{4\nu^2-1}{8z}\cos\left(z-\frac{\pi\nu}{2}-\frac{\pi}{4}\right)\left(1+\frac{16\nu^4-40\nu^2+9}{128z^2}+\frac{16\left(2\nu^2\left(8\left(\nu^2-21\right)\nu^2+987\right)-3229\right)\nu^2+11025}{98304z^4}+\ldots\right)\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.03.06.0054.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{z}} \sqrt{\frac{2}{\pi}}$$
$$\left(\sin\left(z-\frac{\pi\nu}{2}-\frac{\pi}{4}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{4}(1-2\nu)\right)_k\left(\frac{1}{4}(3-2\nu)\right)_k\left(\frac{1}{4}(2\nu+1)\right)_k\left(\frac{1}{4}(2\nu+3)\right)_k}{\left(\frac{1}{2}\right)_k k!}\left(-\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2n+2}}\right)\right)+\frac{4\nu^2-1}{8z}\cos\left(z-\frac{\pi\nu}{2}-\right.\right.$$
$$\left.\left.\frac{\pi}{4}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{4}(3-2\nu)\right)_k\left(\frac{1}{4}(5-2\nu)\right)_k\left(\frac{1}{4}(2\nu+3)\right)_k\left(\frac{1}{4}(2\nu+5)\right)_k}{\left(\frac{3}{2}\right)_k k!}\left(-\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2n+2}}\right)\right)\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.03.06.0008.01

$$Y_\nu(z) \propto \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{z}} \left(\sin\left(z-\frac{(1+2\nu)\pi}{4}\right) {}_4F_1\left(\frac{1}{4}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}; -\frac{1}{z^2}\right)+\right.$$
$$\left.\frac{4\nu^2-1}{8z}\cos\left(z-\frac{(1+2\nu)\pi}{4}\right) {}_4F_1\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}; -\frac{1}{z^2}\right)\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.03.06.0009.01

$$Y_\nu(z) \propto \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{z}} \left(\sin\left(z-\frac{(1+2\nu)\pi}{4}\right)\left(1+O\left(\frac{1}{z^2}\right)\right)+\frac{4\nu^2-1}{8z}\cos\left(z-\frac{(1+2\nu)\pi}{4}\right)\left(1+O\left(\frac{1}{z^2}\right)\right)\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

Expansions containing $z \to -\infty$

In exponential form ||| In exponential form

03.03.06.0055.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{-2\pi z}} \left(e^{-i z + \frac{1}{4}(\pi i)(2\nu+1)} \left(1 - \frac{i(-1+4\nu^2)}{8z} - \frac{9-40\nu^2+16\nu^4}{128 z^2} + \ldots \right) + e^{i z - \frac{1}{4} i \pi (2\nu-3)} \left(2 + e^{2 i \pi \nu}\right) \left(1 + \frac{i(-1+4\nu^2)}{8z} - \frac{9-40\nu^2+16\nu^4}{128 z^2} + \ldots \right) \right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.03.06.0056.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{-2\pi z}} \left(e^{-i z + \frac{1}{4}(\pi i)(2\nu+1)} \left(\sum_{k=0}^{n} \frac{\left(\nu+\frac{1}{2}\right)_k \left(\frac{1}{2}-\nu\right)_k}{k!} \left(\frac{i}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) + e^{i z - \frac{1}{4} i \pi (2\nu-3)} \left(2 + e^{2 i \pi \nu}\right) \left(\sum_{k=0}^{n} \frac{\left(\nu+\frac{1}{2}\right)_k \left(\frac{1}{2}-\nu\right)_k}{k!} \left(-\frac{i}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) \right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.03.06.0057.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{-2\pi z}} \left(e^{-i z + \frac{1}{4}(\pi i)(2\nu+1)} \, {}_2F_0\left(\nu + \frac{1}{2}, \frac{1}{2} - \nu;; \frac{i}{2z}\right) + e^{i z - \frac{1}{4} i \pi (2\nu-3)} \left(2 + e^{2 i \pi \nu}\right) {}_2F_0\left(\nu + \frac{1}{2}, \frac{1}{2} - \nu;; -\frac{i}{2z}\right) \right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.03.06.0058.01

$$Y_\nu(z) \propto \frac{1}{\sqrt{-2\pi z}} \left(e^{-i z + \frac{1}{4}(\pi i)(2\nu+1)} \left(1 + O\left(\frac{1}{z}\right)\right) + e^{i z - \frac{1}{4} i \pi (2\nu-3)} \left(2 + e^{2 i \pi \nu}\right) \left(1 + O\left(\frac{1}{z}\right)\right) \right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

In trigonometric form ||| In trigonometric form

03.03.06.0059.01

$$Y_\nu(z) \propto \frac{\sqrt{2}\csc(\pi\nu)}{\sqrt{-\pi z}} \left(\left((-1)^\nu \cos(\nu\pi) \cos\left(\frac{\pi(2\nu+1)}{4} + z\right) - (-1)^{-\nu} \cos\left(\frac{\pi(1-2\nu)}{4} + z\right) \right) \left(1 - \frac{9-40\nu^2+16\nu^4}{128 z^2} + \frac{11025 - 51664\nu^2 + 31584\nu^4 - 5376\nu^6 + 256\nu^8}{98304 z^4} + \ldots \right) + \frac{1-4\nu^2}{8z} \left((-1)^\nu \cos(\nu\pi) \sin\left(\frac{\pi(2\nu+1)}{4} + z\right) - (-1)^{-\nu} \sin\left(\frac{\pi(1-2\nu)}{4} + z\right) \right) \left(1 - \frac{225 - 136\nu^2 + 16\nu^4}{384 z^2} + \frac{893025 - 656784\nu^2 + 137824\nu^4 - 10496\nu^6 + 256\nu^8}{491520 z^4} + \ldots \right) \right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.03.06.0060.01

$$Y_\nu(z) \propto \frac{\sqrt{2}\,\csc(\pi\nu)}{\sqrt{-\pi z}} \left(\left((-1)^\nu \cos(\nu\pi) \cos\left(\frac{\pi(2\nu+1)}{4}+z\right) - (-1)^{-\nu} \cos\left(\frac{\pi(1-2\nu)}{4}+z\right) \right) \left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(1-2\nu)\right)_k \left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(2\nu+1)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right) \right) + \frac{1-4\nu^2}{8z} \left((-1)^\nu \cos(\nu\pi) \sin\left(\frac{\pi(2\nu+1)}{4}+z\right) - (-1)^{-\nu} \sin\left(\frac{\pi(1-2\nu)}{4}+z\right) \right) \left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(5-2\nu)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k \left(\frac{1}{4}(2\nu+5)\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right) \right) \right) /;\ 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.03.06.0061.01

$$Y_\nu(z) \propto \frac{\sqrt{2}\,\csc(\pi\nu)}{\sqrt{-\pi z}} \left(\left((-1)^\nu \cos(\nu\pi) \cos\left(\frac{\pi(2\nu+1)}{4}+z\right) - (-1)^{-\nu} \cos\left(\frac{\pi(1-2\nu)}{4}+z\right) \right) {}_4F_1\left(\frac{1-2\nu}{4}, \frac{3-2\nu}{4}, \frac{2\nu+1}{4}, \frac{2\nu+3}{4}; \frac{1}{2}; -\frac{1}{z^2}\right) + \frac{1-4\nu^2}{8z} \left((-1)^\nu \cos(\nu\pi) \sin\left(\frac{\pi(2\nu+1)}{4}+z\right) - (-1)^{-\nu} \sin\left(\frac{\pi(1-2\nu)}{4}+z\right) \right) {}_4F_1\left(\frac{3-2\nu}{4}, \frac{5-2\nu}{4}, \frac{2\nu+3}{4}, \frac{2\nu+5}{4}; \frac{3}{2}; -\frac{1}{z^2}\right) \right) /;\ 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.03.06.0062.01

$$Y_\nu(z) \propto \frac{\sqrt{2}\,\csc(\pi\nu)}{\sqrt{-\pi z}} \left(\left((-1)^\nu \cos(\nu\pi) \cos\left(\frac{\pi(2\nu+1)}{4}+z\right) - (-1)^{-\nu} \cos\left(\frac{\pi(1-2\nu)}{4}+z\right) \right) \left(1 + O\left(\frac{1}{z^2}\right)\right) + \frac{1-4\nu^2}{8z} \left((-1)^\nu \cos(\nu\pi) \sin\left(\frac{\pi(2\nu+1)}{4}+z\right) - (-1)^{-\nu} \sin\left(\frac{\pi(1-2\nu)}{4}+z\right) \right) \left(1 + O\left(\frac{1}{z^2}\right)\right) \right) /;\ 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-containing arguments

03.03.06.0063.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{\sqrt{2\pi}\,\sqrt[4]{z^2}} \left(\exp\left(\frac{i\pi}{4} - i\sqrt{z^2}\right) \left(e^{\frac{i\pi\nu}{2}} z^\nu \left(z^2\right)^{-\frac{\nu}{2}} \cos(\pi\nu) - e^{-\frac{\pi i\nu}{2}} z^{-\nu} \left(z^2\right)^{\nu/2} \right) \left(1 + \frac{i\left(1-4\nu^2\right)}{8\sqrt{z^2}} - \frac{9-40\nu^2+16\nu^4}{128z^2} + \ldots \right) + \exp\left(-\frac{i\pi}{4} + i\sqrt{z^2}\right) \left(e^{-\frac{\pi i\nu}{2}} z^\nu \left(z^2\right)^{-\frac{\nu}{2}} \cos(\pi\nu) - e^{\frac{i\pi\nu}{2}} z^{-\nu} \left(z^2\right)^{\nu/2} \right) \left(1 - \frac{i\left(1-4\nu^2\right)}{8\sqrt{z^2}} - \frac{9-40\nu^2+16\nu^4}{128z^2} + \ldots \right) \right) /;\ (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0064.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{\sqrt{2\pi}\,\sqrt[4]{z^2}}\left(\exp\left(\frac{i\pi}{4} - i\sqrt{z^2}\right)\left(e^{\frac{i\pi\nu}{2}} z^\nu \left(z^2\right)^{-\frac{\nu}{2}} \cos(\pi\nu) - e^{-\frac{\pi i\nu}{2}} z^{-\nu} \left(z^2\right)^{\nu/2}\right)\left(\sum_{k=0}^{n} \frac{\left(\nu+\frac{1}{2}\right)_k \left(\frac{1}{2}-\nu\right)_k}{k!}\left(\frac{i}{2\sqrt{z^2}}\right)^k + O\left(\frac{1}{z^{n+1}}\right)\right) + \exp\left(-\frac{i\pi}{4} + i\sqrt{z^2}\right)\left(e^{-\frac{\pi i\nu}{2}} z^\nu \left(z^2\right)^{-\frac{\nu}{2}} \cos(\pi\nu) - e^{\frac{i\pi\nu}{2}} z^{-\nu} \left(z^2\right)^{\nu/2}\right)\left(\sum_{k=0}^{n} \frac{\left(\nu+\frac{1}{2}\right)_k \left(\frac{1}{2}-\nu\right)_k}{k!}\left(-\frac{i}{2\sqrt{z^2}}\right)^k + O\left(\frac{1}{z^{n+1}}\right)\right)\right) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0010.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{\sqrt{2\pi}\,\sqrt[4]{z^2}}\left(\exp\left(\frac{i\pi}{4} - i\sqrt{z^2}\right)\left(e^{\frac{i\pi\nu}{2}} z^\nu \left(z^2\right)^{-\frac{\nu}{2}} \cos(\pi\nu) - e^{-\frac{\pi i\nu}{2}} z^{-\nu} \left(z^2\right)^{\nu/2}\right) {}_2F_0\left(\nu+\frac{1}{2}, \frac{1}{2}-\nu;;\frac{i}{2\sqrt{z^2}}\right) + \exp\left(-\frac{i\pi}{4} + i\sqrt{z^2}\right)\left(e^{-\frac{\pi i\nu}{2}} z^\nu \left(z^2\right)^{-\frac{\nu}{2}} \cos(\pi\nu) - e^{\frac{i\pi\nu}{2}} z^{-\nu} \left(z^2\right)^{\nu/2}\right) {}_2F_0\left(\nu+\frac{1}{2}, \frac{1}{2}-\nu;;-\frac{i}{2\sqrt{z^2}}\right)\right) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0011.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{\sqrt{2\pi}\,\sqrt[4]{z^2}}\left(\exp\left(\frac{i\pi}{4} - i\sqrt{z^2}\right)\left(e^{\frac{i\pi\nu}{2}} z^\nu \left(z^2\right)^{-\frac{\nu}{2}} \cos(\pi\nu) - e^{-\frac{\pi i\nu}{2}} z^{-\nu} \left(z^2\right)^{\nu/2}\right)\left(1 + O\left(\frac{1}{z}\right)\right) + \exp\left(-\frac{i\pi}{4} + i\sqrt{z^2}\right)\left(e^{-\frac{\pi i\nu}{2}} z^\nu \left(z^2\right)^{-\frac{\nu}{2}} \cos(\pi\nu) - e^{\frac{i\pi\nu}{2}} z^{-\nu} \left(z^2\right)^{\nu/2}\right)\left(1 + O\left(\frac{1}{z}\right)\right)\right) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

Using exponential function with branch cut-free arguments

03.03.06.0065.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{2\sqrt{2\pi}} \left(\left(\cos(\nu\pi) \left((-i z)^\nu (i z)^{-\nu} z^{-\frac{1}{2}} e^{\frac{i\pi}{4}(6\nu+1) - i z} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{-\nu-\frac{1}{2}} e^{-i z - \frac{i\pi}{4}(2\nu+1)} z^\nu \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) - \left((-i z)^{-\nu} (i z)^\nu z^{-\frac{1}{2}} e^{\frac{i\pi}{4}(1-6\nu) - i z} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{\nu-\frac{1}{2}} z^{-\nu} e^{-i z - \frac{i\pi}{4}(1-2\nu)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) \right) \left(1 - \frac{i(-1+4\nu^2)}{8z} - \frac{9 - 40\nu^2 + 16\nu^4}{128 z^2} + \ldots \right) + \left(\cos(\nu\pi) \left((-i z)^\nu (i z)^{-\nu} z^{-\frac{1}{2}} e^{i z + \frac{i\pi}{4}(2\nu-1)} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{-\nu-\frac{1}{2}} z^\nu e^{i z + \frac{i\pi}{4}(2\nu+1)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) - \left((-i z)^{-\nu} (i z)^\nu z^{-\frac{1}{2}} e^{i z - \frac{i\pi}{4}(2\nu+1)} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{\nu-\frac{1}{2}} z^{-\nu} e^{i z + \frac{i\pi}{4}(1-2\nu)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) \right) \left(1 + \frac{i(-1+4\nu^2)}{8z} - \frac{9 - 40\nu^2 + 16\nu^4}{128 z^2} + \ldots \right) \right) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0066.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{2\sqrt{2\pi}} \left(\left(\cos(\nu\pi) \left((-i z)^\nu (i z)^{-\nu} z^{-\frac{1}{2}} e^{\frac{i\pi}{4}(6\nu+1) - i z} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{-\nu-\frac{1}{2}} e^{-i z - \frac{i\pi}{4}(2\nu+1)} z^\nu \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) - \left((-i z)^{-\nu} (i z)^\nu z^{-\frac{1}{2}} e^{\frac{i\pi}{4}(1-6\nu) - i z} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{\nu-\frac{1}{2}} z^{-\nu} e^{-i z - \frac{i\pi}{4}(1-2\nu)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) \right) \left(\sum_{k=0}^{n} \frac{\left(\nu + \frac{1}{2}\right)_k \left(\frac{1}{2} - \nu\right)_k}{k!} \left(\frac{i}{2z} \right)^k + O\left(\frac{1}{z^{n+1}} \right) \right) + \left(\cos(\nu\pi) \left((-i z)^\nu (i z)^{-\nu} z^{-\frac{1}{2}} e^{i z + \frac{i\pi}{4}(2\nu-1)} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{-\nu-\frac{1}{2}} z^\nu e^{i z + \frac{i\pi}{4}(2\nu+1)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) - \left((-i z)^{-\nu} (i z)^\nu z^{-\frac{1}{2}} e^{i z - \frac{i\pi}{4}(2\nu+1)} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{\nu-\frac{1}{2}} z^{-\nu} e^{i z + \frac{i\pi}{4}(1-2\nu)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) \right) \left(\sum_{k=0}^{n} \frac{\left(\nu + \frac{1}{2}\right)_k \left(\frac{1}{2} - \nu\right)_k}{k!} \left(-\frac{i}{2z} \right)^k + O\left(\frac{1}{z^{n+1}} \right) \right) \right) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0067.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{2\sqrt{2\pi}} \left(\left(\cos(\nu\pi) \left((-i z)^\nu (i z)^{-\nu} z^{-\frac{1}{2}} e^{\frac{i\pi}{4}(6\nu+1)-i z} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{-\nu-\frac{1}{2}} e^{-i z-\frac{i\pi}{4}(2\nu+1)} z^\nu \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) - \left((-i z)^{-\nu} (i z)^\nu z^{-\frac{1}{2}} e^{\frac{i\pi}{4}(1-6\nu)-i z} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{\nu-\frac{1}{2}} z^{-\nu} e^{-i z-\frac{i\pi}{4}(1-2\nu)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) \right) {}_2F_0\left(\nu + \frac{1}{2}, \frac{1}{2} - \nu;; \frac{i}{2z} \right) + \left(\cos(\nu\pi) \left((-i z)^\nu (i z)^{-\nu} z^{-\frac{1}{2}} e^{i z+\frac{i\pi}{4}(2\nu-1)} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{-\nu-\frac{1}{2}} z^\nu e^{i z+\frac{i\pi}{4}(2\nu+1)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) - \left((-i z)^{-\nu} (i z)^\nu z^{-\frac{1}{2}} e^{i z-\frac{i\pi}{4}(2\nu+1)} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{\nu-\frac{1}{2}} z^{-\nu} e^{i z+\frac{i\pi}{4}(1-2\nu)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) \right) {}_2F_0\left(\nu + \frac{1}{2}, \frac{1}{2} - \nu;; -\frac{i}{2z} \right) \right) /; (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

03.03.06.0068.01

$$Y_\nu(z) \propto \frac{\sqrt[4]{-1}\, z^{\nu-1} \left(z^2\right)^{-\frac{\nu}{2}-\frac{1}{4}}}{2\sqrt{2\pi}} \left(e^{-i(\pi\nu-z)} \left(i e^{\frac{i\pi\nu}{2}} \left(i + e^{i\pi\nu}\right) z - i e^{\frac{i\pi\nu}{2}} \left(-i + e^{i\pi\nu}\right) \sqrt{z^2} - 2 i e^{\frac{i\pi\nu}{2}} \left(-i + e^{i\pi\nu}\right) z \left\lfloor \frac{1}{2} - \frac{\arg(z)}{\pi} \right\rfloor + 2 i e^{\frac{i\pi\nu}{2}} \left(i + e^{i\pi\nu}\right) \sqrt{z^2} \left\lfloor \frac{1}{2} - \frac{\arg(z)}{\pi} \right\rfloor \right) {}_2F_0\left(\nu + \frac{1}{2}, \frac{1}{2} - \nu;; -\frac{i}{2z} \right) - e^{-i(z+\pi\nu)} \left(2 e^{\frac{i\pi\nu}{2}} z \left\lfloor \frac{1}{2} - \frac{\arg(z)}{\pi} \right\rfloor \left(1 + i e^{i\pi\nu}\right) - e^{\frac{i\pi\nu}{2}} \sqrt{z^2} \left(1 + i e^{i\pi\nu}\right) + e^{\frac{i\pi\nu}{2}} \left(1 - i e^{i\pi\nu}\right) z + 2 e^{\frac{i\pi\nu}{2}} \left(i + e^{i\pi\nu}\right) i \sqrt{z^2} \left\lfloor \frac{1}{2} - \frac{\arg(z)}{\pi} \right\rfloor \right) {}_2F_0\left(\nu + \frac{1}{2}, \frac{1}{2} - \nu;; \frac{i}{2z} \right) \right) /; (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

03.03.06.0069.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{2\sqrt{2\pi}} \left(\left(\cos(\nu\pi) \left((-i z)^\nu (i z)^{-\nu} z^{-\frac{1}{2}} e^{\frac{i\pi}{4}(6\nu+1)-i z} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{-\nu-\frac{1}{2}} e^{-i z-\frac{i\pi}{4}(2\nu+1)} z^\nu \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) - \left((-i z)^{-\nu} (i z)^\nu z^{-\frac{1}{2}} e^{\frac{i\pi}{4}(1-6\nu)-i z} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{\nu-\frac{1}{2}} z^{-\nu} e^{-i z-\frac{i\pi}{4}(1-2\nu)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) \right) \left(1 + O\left(\frac{1}{z}\right) \right) + \left(\cos(\nu\pi) \left((-i z)^\nu (i z)^{-\nu} z^{-\frac{1}{2}} e^{i z+\frac{i\pi}{4}(2\nu-1)} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{-\nu-\frac{1}{2}} z^\nu e^{i z+\frac{i\pi}{4}(2\nu+1)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) - \left((-i z)^{-\nu} (i z)^\nu z^{-\frac{1}{2}} e^{i z-\frac{i\pi}{4}(2\nu+1)} \left(\frac{\sqrt{z^2}}{z} + 1 \right) + (-z)^{\nu-\frac{1}{2}} z^{-\nu} e^{i z+\frac{i\pi}{4}(1-2\nu)} \left(1 - \frac{\sqrt{z^2}}{z} \right) \right) \right) \left(1 + O\left(\frac{1}{z}\right) \right) \right) /; (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

03.03.06.0070.01

$$Y_\nu(z) \propto \begin{cases} -\frac{i\, e^{-\frac{i\pi}{4}+i z-\frac{i\pi\nu}{2}}}{\sqrt{2\pi}\,\sqrt{z}} + \frac{i\, e^{\frac{i\pi}{4}-i z+\frac{i\pi\nu}{2}}}{\sqrt{2\pi}\,\sqrt{z}} & \arg(z) \leq 0 \\ -\frac{e^{\frac{i\pi}{4}+i z-\frac{i\pi\nu}{2}}\sqrt{2}}{\sqrt{\pi z}} - \frac{e^{-\frac{i\pi}{4}-i z+\frac{i\pi\nu}{2}}}{\sqrt{2\pi}\,\sqrt{z}} - \frac{e^{\frac{i\pi}{4}+i z+\frac{3 i\pi\nu}{2}}}{\sqrt{2\pi}\,\sqrt{z}} & \text{True} \end{cases} \quad /;\ (|z| \to \infty)$$

Expansions for any *z* in trigonometric form

Using trigonometric functions with branch cut-containing arguments

03.03.06.0071.01

$$Y_\nu(z) \propto \sqrt{\frac{2}{\pi}}\, z^{-\nu} \left(z^2\right)^{-\frac{2\nu+1}{4}} \csc(\pi\nu) \Bigg(\left(z^{2\nu} \cos(\pi\nu) \cos\left(\sqrt{z^2} - \frac{(1+2\nu)\pi}{4}\right) - \left(z^2\right)^{\nu} \cos\left(\sqrt{z^2} - \frac{(1-2\nu)\pi}{4}\right)\right)$$
$$\left(1 - \frac{9 - 40\nu^2 + 16\nu^4}{128 z^2} + \frac{11025 - 51664\nu^2 + 31584\nu^4 - 5376\nu^6 + 256\nu^8}{98304 z^4} + \ldots\right) -$$
$$\frac{4\nu^2 - 1}{8\sqrt{z^2}} \left(z^{2\nu} \cos(\pi\nu) \sin\left(\sqrt{z^2} - \frac{(1+2\nu)\pi}{4}\right) - \left(z^2\right)^{\nu} \sin\left(\sqrt{z^2} - \frac{(1-2\nu)\pi}{4}\right)\right)$$
$$\left(1 - \frac{225 - 136\nu^2 + 16\nu^4}{384 z^2} + \frac{893025 - 656784\nu^2 + 137824\nu^4 - 10496\nu^6 + 256\nu^8}{491520 z^4} + \ldots\right)\Bigg) /;\ (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

03.03.06.0072.01

$$Y_\nu(z) \propto \sqrt{\frac{2}{\pi}}\, z^{-\nu} \left(z^2\right)^{-\frac{2\nu+1}{4}} \csc(\pi\nu) \Bigg(\left(z^{2\nu} \cos(\pi\nu) \cos\left(\sqrt{z^2} - \frac{(1+2\nu)\pi}{4}\right) - \left(z^2\right)^{\nu} \cos\left(\sqrt{z^2} - \frac{(1-2\nu)\pi}{4}\right)\right)$$
$$\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(1-2\nu)\right)_k \left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(2\nu+1)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right)\right) -$$
$$\frac{1}{8\sqrt{z^2}} \Bigg(\left(4\nu^2 - 1\right)\left(z^{2\nu} \cos(\pi\nu) \sin\left(\sqrt{z^2} - \frac{(1+2\nu)\pi}{4}\right) - \left(z^2\right)^{\nu} \sin\left(\sqrt{z^2} - \frac{(1-2\nu)\pi}{4}\right)\right)$$
$$\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(5-2\nu)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k \left(\frac{1}{4}(2\nu+5)\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right)\right)\Bigg)\Bigg) /;\ (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

03.03.06.0012.01

$$Y_\nu(z) \propto \sqrt{\frac{2}{\pi}}\, z^{-\nu} \left(z^2\right)^{-\frac{2\nu+1}{4}} \csc(\pi\nu) \Bigg(\left(z^{2\nu} \cos(\pi\nu) \cos\left(\sqrt{z^2} - \frac{(1+2\nu)\pi}{4}\right) - \left(z^2\right)^\nu \cos\left(\sqrt{z^2} - \frac{(1-2\nu)\pi}{4}\right)\right)$$

$$ {}_4F_1\left(\frac{1}{4}(1-2\nu), \frac{1}{4}(3-2\nu), \frac{1}{4}(2\nu+1), \frac{1}{4}(2\nu+3); \frac{1}{2}; -\frac{1}{z^2}\right) -$$

$$\frac{1}{8\sqrt{z^2}} \left(\left(4\nu^2-1\right)\left(z^{2\nu}\cos(\pi\nu)\sin\left(\sqrt{z^2} - \frac{(1+2\nu)\pi}{4}\right) - \left(z^2\right)^\nu \sin\left(\sqrt{z^2} - \frac{(1-2\nu)\pi}{4}\right)\right)\right.$$

$$\left. {}_4F_1\left(\frac{1}{4}(3-2\nu), \frac{1}{4}(5-2\nu), \frac{1}{4}(2\nu+3), \frac{1}{4}(2\nu+5); \frac{3}{2}; -\frac{1}{z^2}\right)\right)\Bigg) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0013.01

$$Y_\nu(z) \propto \sqrt{\frac{2}{\pi}}\, z^{-\nu} \left(z^2\right)^{-\frac{2\nu+1}{4}} \csc(\pi\nu) \Bigg(\left(z^{2\nu} \cos(\pi\nu) \cos\left(\sqrt{z^2} - \frac{(1+2\nu)\pi}{4}\right) - \left(z^2\right)^\nu \cos\left(\sqrt{z^2} - \frac{(1-2\nu)\pi}{4}\right)\right)\left(1 + O\left(\frac{1}{z^2}\right)\right) -$$

$$\frac{4\nu^2-1}{8\sqrt{z^2}} \left(z^{2\nu}\cos(\pi\nu)\sin\left(\sqrt{z^2} - \frac{(1+2\nu)\pi}{4}\right) - \left(z^2\right)^\nu \sin\left(\sqrt{z^2} - \frac{(1-2\nu)\pi}{4}\right)\right)\left(1 + O\left(\frac{1}{z^2}\right)\right)\Bigg) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

Using trigonometric functions with branch cut-free arguments

03.03.06.0073.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{\sqrt{2\pi}} \left(\left(\cos(\pi\nu) \left(\frac{e^{i\pi\nu} z \left(z + i\sqrt{-z^2}\right)}{(-z)^{5/2}} \cos\left(z + \frac{\pi(2\nu+1)}{4}\right) + \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \cos\left(z - \frac{\pi(2\nu+1)}{4}\right) \right) + \frac{e^{-i\pi\nu}\left(z + i\sqrt{-z^2}\right)}{(-z)^{3/2}} \cos\left(z + \frac{\pi(1-2\nu)}{4}\right) - \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \cos\left(z + \frac{\pi(2\nu-1)}{4}\right) \right) \left(1 - \frac{9 - 40\nu^2 + 16\nu^4}{128 z^2} + \frac{11025 - 51664\nu^2 + 31584\nu^4 - 5376\nu^6 + 256\nu^8}{98304 z^4} + \ldots\right) + \frac{1 - 4\nu^2}{8z} \left(\cos(\pi\nu) \left(\frac{e^{i\pi\nu} z \left(z + i\sqrt{-z^2}\right)}{(-z)^{5/2}} \sin\left(z + \frac{\pi(2\nu+1)}{4}\right) + \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \sin\left(z - \frac{\pi(2\nu+1)}{4}\right) \right) + \frac{e^{-i\pi\nu}\left(z + i\sqrt{-z^2}\right)}{(-z)^{3/2}} \sin\left(z + \frac{\pi(1-2\nu)}{4}\right) - \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \sin\left(z + \frac{\pi(2\nu-1)}{4}\right) \right) \left(1 - \frac{225 - 136\nu^2 + 16\nu^4}{384 z^2} + \frac{893025 - 656784\nu^2 + 137824\nu^4 - 10496\nu^6 + 256\nu^8}{491520 z^4} + \ldots\right) \right) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0074.01

$$Y_\nu(z) \propto \frac{(1-i)\, e^{-\frac{1}{2} i(\pi\nu+2z)} \left(1 - 4\nu^2\right)}{16\sqrt{\pi}\,(-z)^{3/2}} \left(e^{2iz} \left(\frac{2\sqrt{-z^2}}{z} - i \right) + \frac{(-1)^\nu\, i\sqrt{-z^2}}{z} \right) \left(1 - \frac{(3-2\nu)(2\nu+3)(2\nu+5)(5-2\nu)}{384 z^2} + \frac{256\nu^8 - 10496\nu^6 + 137824\nu^4 - 656784\nu^2 + 893025}{491520 z^4} + \ldots\right) - \frac{(1+i)\, e^{-\frac{1}{2} i(\pi\nu+2z)}}{2\sqrt{\pi}\sqrt{-z}} \left(e^{2iz} \left(\frac{2\sqrt{-z^2}}{z} - i \right) - \frac{i\,(-1)^\nu \sqrt{-z^2}}{z} \right) \left(1 - \frac{(1-2\nu)(2\nu+1)(2\nu+3)(3-2\nu)}{128 z^2} + \frac{256\nu^8 - 5376\nu^6 + 31584\nu^4 - 51664\nu^2 + 11025}{98304 z^4} + \ldots\right) /; (|z| \to \infty) \land \nu \in \mathbb{Z}$$

03.03.06.0075.01

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{\sqrt{2\pi}} \left(\left(\cos(\pi\nu) \left(\frac{e^{i\pi\nu} z \left(z + i\sqrt{-z^2}\right)}{(-z)^{5/2}} \cos\left(z + \frac{\pi(2\nu+1)}{4}\right) + \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \cos\left(z - \frac{\pi(2\nu+1)}{4}\right) \right) + \right.\right.$$

$$\left. \frac{e^{-i\pi\nu}\left(z + i\sqrt{-z^2}\right)}{(-z)^{3/2}} \cos\left(z + \frac{\pi(1-2\nu)}{4}\right) - \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \cos\left(z + \frac{\pi(2\nu-1)}{4}\right) \right)$$

$$\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(1-2\nu)\right)_k \left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(2\nu+1)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right) \right) +$$

$$\frac{1-4\nu^2}{8z} \left(\cos(\pi\nu) \left(\frac{e^{i\pi\nu} z \left(z + i\sqrt{-z^2}\right)}{(-z)^{5/2}} \sin\left(z + \frac{\pi(2\nu+1)}{4}\right) + \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \sin\left(z - \frac{\pi(2\nu+1)}{4}\right) \right) + \right.$$

$$\left. \frac{e^{-i\pi\nu}\left(z + i\sqrt{-z^2}\right)}{(-z)^{3/2}} \sin\left(z + \frac{\pi(1-2\nu)}{4}\right) - \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \sin\left(z + \frac{\pi(2\nu-1)}{4}\right) \right)$$

$$\left.\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(5-2\nu)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k \left(\frac{1}{4}(2\nu+5)\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right) \right) \right) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.03.06.0076.01

$$Y_\nu(z) \propto -\frac{(1+i)\, e^{-\frac{1}{2} i (\pi\nu + 2z)}}{2\sqrt{\pi}\,\sqrt{-z}} \left(e^{2iz} \left(\frac{2\sqrt{-z^2}}{z} - i \right) - \frac{i\left((-1)^\nu \sqrt{-z^2}\right)}{z} \right)$$

$$\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(1-2\nu)\right)_k \left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(2\nu+1)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k \left(-\frac{1}{z^2}\right)^k}{\left(\frac{1}{2}\right)_k k!} + O\left(\frac{1}{z^{2n+2}}\right) \right) +$$

$$\frac{(1-i)\, e^{-\frac{1}{2} i (\pi\nu + 2z)} \left(1 - 4\nu^2\right)}{16\sqrt{\pi}\,(-z)^{3/2}} \left(e^{2iz} \left(\frac{2\sqrt{-z^2}}{z} - i \right) + \frac{(-1)^\nu i \sqrt{-z^2}}{z} \right)$$

$$\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{4}(3-2\nu)\right)_k \left(\frac{1}{4}(5-2\nu)\right)_k \left(\frac{1}{4}(2\nu+3)\right)_k \left(\frac{1}{4}(2\nu+5)\right)_k \left(-\frac{1}{z^2}\right)^k}{\left(\frac{3}{2}\right)_k k!} + O\left(\frac{1}{z^{2n+2}}\right) \right) /; (|z| \to \infty) \land \nu \in \mathbb{Z}$$

03.03.06.0014.02

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{\sqrt{2\pi}} \left(\left(\cos(\pi\nu)\left(\frac{e^{i\pi\nu} z \left(z + i\sqrt{-z^2}\right)}{(-z)^{5/2}} \cos\left(z + \frac{\pi(2\nu+1)}{4}\right) + \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \cos\left(z - \frac{\pi(2\nu+1)}{4}\right)\right) + \frac{e^{-i\pi\nu}\left(z + i\sqrt{-z^2}\right)}{(-z)^{3/2}} \cos\left(z + \frac{\pi(1-2\nu)}{4}\right) - \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \cos\left(z + \frac{\pi(2\nu-1)}{4}\right)\right) {}_4F_1\left(\frac{1}{4} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}, \frac{\nu}{2} + \frac{1}{4}, \frac{\nu}{2} + \frac{3}{4}; \frac{1}{2}; -\frac{1}{z^2}\right) + \frac{1-4\nu^2}{8z} \left(\cos(\pi\nu)\left(\frac{e^{i\pi\nu} z\left(z + i\sqrt{-z^2}\right)}{(-z)^{5/2}} \sin\left(z + \frac{\pi(2\nu+1)}{4}\right) + \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \sin\left(z - \frac{\pi(2\nu+1)}{4}\right)\right) + \frac{e^{-i\pi\nu}\left(z + i\sqrt{-z^2}\right)}{(-z)^{3/2}} \sin\left(z + \frac{\pi(1-2\nu)}{4}\right) - \frac{\left(z - i\sqrt{-z^2}\right)}{z^{3/2}} \sin\left(z + \frac{\pi(2\nu-1)}{4}\right)\right) {}_4F_1\left(\frac{3}{4} - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}, \frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{5}{4}; \frac{3}{2}; -\frac{1}{z^2}\right)\right) /; (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

03.03.06.0077.01

$$Y_\nu(z) \propto -\frac{(1+i)\, e^{-\frac{1}{2} i (\pi\nu + 2z)}}{2\sqrt{\pi}\,\sqrt{-z}} \left(e^{2iz}\left(\frac{2\sqrt{-z^2}}{z} - i\right) - \frac{i(-1)^\nu \sqrt{-z^2}}{z}\right) {}_4F_1\left(\frac{1}{4} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}, \frac{\nu}{2} + \frac{1}{4}, \frac{\nu}{2} + \frac{3}{4}; \frac{1}{2}; -\frac{1}{z^2}\right) + \frac{(1-i)\, e^{-\frac{1}{2} i (\pi\nu + 2z)} \left(1 - 4\nu^2\right)}{16\sqrt{\pi}\,(-z)^{3/2}} \left(e^{2iz}\left(\frac{2\sqrt{-z^2}}{z} - i\right) + \frac{(-1)^\nu i \sqrt{-z^2}}{z}\right) {}_4F_1\left(\frac{3}{4} - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}, \frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{5}{4}; \frac{3}{2}; -\frac{1}{z^2}\right) /; (|z| \to \infty) \land \nu \in \mathbb{Z}$$

03.03.06.0015.02

$$Y_\nu(z) \propto \frac{\csc(\pi\nu)}{\sqrt{2\pi}} \left(\left(\frac{e^{i\pi\nu} z \left(z + i\sqrt{-z^2}\right) \cos\left(z + \frac{1}{4}\pi(2\nu+1)\right)}{(-z)^{5/2}} + \frac{\left(z - i\sqrt{-z^2}\right) \cos\left(z - \frac{1}{4}\pi(2\nu+1)\right)}{z^{3/2}}\right) \cos(\pi\nu) + \frac{e^{-i\pi\nu}\left(z + i\sqrt{-z^2}\right) \cos\left(z + \frac{1}{4}\pi(1-2\nu)\right)}{(-z)^{3/2}} - \frac{\left(z - i\sqrt{-z^2}\right) \cos\left(z + \frac{1}{4}\pi(2\nu-1)\right)}{z^{3/2}}\right) \left(1 + O\left(\frac{1}{z}\right)\right) /; (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

03.03.06.0078.01

$$Y_\nu(z) \propto \begin{cases} \frac{\sqrt{2}}{\sqrt{\pi}\sqrt{z}} \sin\left(z - \frac{\pi\nu}{2} - \frac{\pi}{4}\right) & \arg(z) \leq 0 \\ -\frac{\sqrt{2}}{\sqrt{\pi}\sqrt{z}} \left(\cos\left(z - \frac{\pi\nu}{2} + \frac{\pi}{4}\right) + e^{iz + \frac{i\pi\nu}{2} + \frac{i\pi}{4}} \cos(\pi\nu)\right) & \text{True} \end{cases} \;/;\; (|z| \to \infty)$$

Residue representations

03.03.06.0016.02

$$Y_\nu(z) = \csc(\pi\nu) \sum_{j=0}^{\infty} \operatorname{res}_s \left(\Gamma(s) \cos(\pi s) \left(\frac{\cos(\pi\nu) \left(\frac{z}{2}\right)^\nu}{\Gamma(1+\nu-s)} - \frac{\left(\frac{z}{2}\right)^{-\nu}}{\Gamma(1-\nu-s)} \right) \left(-\frac{z^2}{4}\right)^{-s} \right)(-j) \;/;\; \nu \notin \mathbb{Z}$$

03.03.06.0017.01

$$Y_\nu(z) = \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\Gamma\left(s+\frac{\nu}{2}\right) \left(\frac{z}{2}\right)^{-2s}}{\Gamma\left(s-\frac{\nu+1}{2}\right) \Gamma\left(\frac{\nu+3}{2}-s\right)} \Gamma\left(s-\frac{\nu}{2}\right) \right)\left(-j+\frac{\nu}{2}\right) + \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\Gamma\left(s-\frac{\nu}{2}\right) \left(\frac{z}{2}\right)^{-2s}}{\Gamma\left(s-\frac{\nu+1}{2}\right) \Gamma\left(\frac{\nu+3}{2}-s\right)} \Gamma\left(s+\frac{\nu}{2}\right) \right)\left(-j-\frac{\nu}{2}\right) \;/;\; \nu \notin \mathbb{Z}$$

03.03.06.0018.02

$$Y_n(z) = \sum_{j=0}^{|n|-1} \operatorname{res}_s \left(\frac{\Gamma\left(s+\frac{|n|}{2}\right) \left(\frac{z}{2}\right)^{-2s}}{\Gamma\left(s-\frac{n+1}{2}\right) \Gamma\left(\frac{n+3}{2}-s\right)} \Gamma\left(s-\frac{|n|}{2}\right) \right)\left(\frac{|n|}{2}-j\right) + \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(\frac{z}{2}\right)^{-2s}}{\Gamma\left(s-\frac{n+1}{2}\right) \Gamma\left(\frac{n+3}{2}-s\right)} \Gamma\left(s+\frac{|n|}{2}\right) \Gamma\left(s-\frac{|n|}{2}\right) \right)\left(-\frac{|n|}{2}-j\right) \;/;$$

$n \in \mathbb{Z}$

Integral representations

On the real axis

Of the direct function

03.03.07.0001.01

$$Y_\nu(z) = -\frac{2^{\nu+1} z^{-\nu}}{\sqrt{\pi}\, \Gamma\left(\frac{1}{2}-\nu\right)} \int_1^\infty \left(t^2-1\right)^{-\nu-\frac{1}{2}} \cos(z t)\, dt \;/;\; |\operatorname{Re}(\nu)| < \frac{1}{2} \bigwedge z > 0$$

03.03.07.0002.01

$$Y_\nu(z) = \frac{1}{\pi} \int_0^\pi \sin(z \sin(t) - t\nu)\, dt - \frac{1}{\pi} \int_0^\infty e^{-z\sinh(t)} \left(e^{-\nu t} \cos(\nu\pi) + e^{\nu t}\right) dt \;/;\; \arg(z) < \frac{\pi}{2}$$

03.03.07.0003.01

$$Y_0(z) = \frac{4}{\pi^2} \int_0^{\frac{\pi}{2}} \cos(z\cos(t)) \left(\log\left(2 z \sin^2(t)\right) + \gamma\right) dt$$

03.03.07.0004.01

$$Y_0(x) = -\frac{2}{\pi} \int_0^\infty \cos(x\cosh(t))\, dt \;/;\; x > 0$$

Contour integral representations

03.03.07.0005.01

$$Y_\nu(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma\left(s+\frac{\nu}{2}\right) \Gamma\left(s-\frac{\nu}{2}\right)}{\Gamma\left(s-\frac{\nu+1}{2}\right) \Gamma\left(\frac{\nu+3}{2}-s\right)} \left(\frac{x}{2}\right)^{-2s} ds \;/;\; x > 0 \bigwedge \frac{1}{2} |\operatorname{Re}(\nu)| < \gamma < \frac{3}{4}$$

03.03.07.0006.01

$$Y_\nu(z) = z^{-\nu} \left(z^2\right)^{\nu/2} \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}\right)\Gamma\left(s-\frac{\nu}{2}\right)}{\Gamma\left(s-\frac{\nu+1}{2}\right)\Gamma\left(\frac{\nu+3}{2}-s\right)} \left(\frac{z^2}{4}\right)^{-s} ds - z^{-\nu}\left(z^2\right)^{-\nu/2}\left(\left(z^2\right)^{\nu} - z^{2\nu}\right)\cot(\pi\nu) \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}\right)}{\Gamma\left(1+\frac{\nu}{2}-s\right)}\left(\frac{z^2}{4}\right)^{-s} ds \;/;\; \nu \notin \mathbb{Z}$$

Limit representations

03.03.09.0001.01

$$Y_\nu(z) = -\frac{2}{\pi} \lim_{\lambda\to\infty} \lambda^\nu Q_\lambda^{-\nu}\left(\cos\left(\frac{z}{\lambda}\right)\right)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.03.13.0001.01

$$w''(z)\, z^2 + w'(z)\, z + \left(z^2 - \nu^2\right) w(z) = 0 \;/;\; w(z) = c_1 J_\nu(z) + c_2 Y_\nu(z)$$

03.03.13.0002.01

$$W_z(J_\nu(z),\, Y_\nu(z)) = \frac{2}{\pi z}$$

03.03.13.0003.01

$$w''(z) + a\, w(z)\, z^n = 0 \;/;\; w(z) = \sqrt{z}\left(c_1 J_{\frac{1}{n+2}}\left(\frac{2\sqrt{a}}{n+2} z^{\frac{n+2}{2}}\right) + c_2 Y_{\frac{1}{n+2}}\left(\frac{2\sqrt{a}}{n+2} z^{\frac{n+2}{2}}\right)\right)$$

03.03.13.0004.01

$$W_z\left(\sqrt{z}\, J_{\frac{1}{n+2}}\left(\frac{2\sqrt{a}\, z^{\frac{n+2}{2}}}{n+2}\right),\, \sqrt{z}\, Y_{\frac{1}{n+2}}\left(\frac{2\sqrt{a}\, z^{\frac{n+2}{2}}}{n+2}\right)\right) = \frac{n+2}{\pi}$$

03.03.13.0005.01

$$w''(z) + \left(m^2 - \frac{1}{z^2}\left(\nu^2 - \frac{1}{4}\right)\right) w(z) = 0 \;/;\; w(z) = c_1 \sqrt{z}\, J_\nu\left(\sqrt{m^2}\, z\right) + c_2 \sqrt{z}\, Y_\nu\left(\sqrt{m^2}\, z\right)$$

03.03.13.0006.01

$$W_z\left(\sqrt{z}\, J_\nu\left(\sqrt{m^2}\, z\right),\, \sqrt{z}\, Y_\nu\left(\sqrt{m^2}\, z\right)\right) = \frac{2}{\pi}$$

03.03.13.0007.01

$$w''(z) + \left(\frac{m^2}{4z} - \frac{\nu^2 - 1}{4z^2}\right) w(z) = 0 \;/;\; w(z) = c_1 \sqrt{z}\, J_\nu\left(\sqrt{m^2}\sqrt{z}\right) + c_2 \sqrt{z}\, Y_\nu\left(\sqrt{m^2}\sqrt{z}\right)$$

03.03.13.0008.01

$$W_z\left(\sqrt{z}\, J_\nu\left(\sqrt{m^2}\sqrt{z}\right),\, \sqrt{z}\, Y_\nu\left(\sqrt{m^2}\sqrt{z}\right)\right) = \frac{1}{\pi}$$

03.03.13.0009.01

$$w''(z) - \frac{2\nu - 1}{z} w'(z) + m^2 w(z) = 0 \;/;\; w(z) = c_1 z^\nu J_\nu(m z) + c_2 z^\nu Y_\nu(m z)$$

03.03.13.0010.01

$$W_z(z^\nu J_\nu(m z), z^\nu Y_\nu(m z)) = \frac{2}{\pi} z^{2\nu-1}$$

03.03.13.0013.01

$$w''(z) + \left(m^2 e^{2z} - \nu^2\right) w(z) = 0 \,/;\, w(z) = c_1 J_\nu(m e^z) + c_2 Y_\nu(m e^z)$$

03.03.13.0014.01

$$W_z(J_\nu(m e^z), Y_\nu(m e^z)) = \frac{2}{\pi}$$

03.03.13.0015.01

$$\left(z^2 - \nu^2\right) z^2 w''(z) + \left(z^2 - 3\nu^2\right) z\, w'(z) + \left(\left(z^2 - \nu^2\right)^2 - z^2 - \nu^2\right) w(z) = 0 \,/;\, w(z) = c_1 \frac{\partial J_\nu(z)}{\partial z} + c_2 \frac{\partial Y_\nu(z)}{\partial z}$$

03.03.13.0016.01

$$W_z\left(\frac{\partial J_\nu(z)}{\partial z}, \frac{\partial Y_\nu(z)}{\partial z}\right) = \frac{1}{2\pi z}\left(4 - \pi\nu J_{\nu+1}(z) Y_{\nu-1}(z) + \pi\nu J_{\nu-1}(z) Y_{\nu+1}(z)\right)$$

03.03.13.0020.01

$$w''(z) - \left(\frac{g''(z)}{g'(z)} - \frac{g'(z)}{g(z)}\right) w'(z) - \left(\frac{\nu^2}{g(z)^2} - 1\right) g'(z)^2 w(z) = 0 \,/;\, w(z) = c_1 J_\nu(g(z)) + c_2 Y_\nu(g(z))$$

03.03.13.0021.01

$$W_z(J_\nu(g(z)), Y_\nu(g(z))) = \frac{2 g'(z)}{\pi g(z)}$$

03.03.13.0022.01

$$w''(z) - \left(-\frac{g'(z)}{g(z)} + \frac{2h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) - \left(\left(\frac{\nu^2}{g(z)^2} - 1\right) g'(z)^2 + \frac{h'(z) g'(z)}{g(z) h(z)} + \frac{h(z) h''(z) - 2h'(z)^2}{h(z)^2} - \frac{h'(z) g''(z)}{h(z) g'(z)}\right) w(z) = 0 \,/;$$
$$w(z) = c_1 h(z) J_\nu(g(z)) + c_2 h(z) Y_\nu(g(z))$$

03.03.13.0023.01

$$W_z(h(z) J_\nu(g(z)), h(z) Y_\nu(g(z))) = \frac{2 h(z)^2 g'(z)}{\pi g(z)}$$

03.03.13.0011.01

$$z^2 w''(z) + (1 - 2s) z w'(z) + \left(a^2 r^2 z^{2r} - \nu^2 r^2 + s^2\right) w(z) = 0 \,/;\, w(z) = c_1 z^s J_\nu(a z^r) + c_2 z^s Y_\nu(a z^r)$$

03.03.13.0012.01

$$W_z(z^s J_\nu(a z^r), z^s Y_\nu(a z^r)) = \frac{2 r z^{2s-1}}{\pi}$$

03.03.13.0024.01

$$w''(z) - 2\log(s) w'(z) + \left(\left(a^2 r^{2z} - \nu^2\right) \log^2(r) + \log^2(s)\right) w(z) = 0 \,/;\, w(z) = c_1 s^z J_\nu(a r^z) + c_2 s^z Y_\nu(a r^z)$$

03.03.13.0025.01

$$W_z(s^z J_\nu(a r^z), s^z Y_\nu(a r^z)) = \frac{2 s^{2z} \log(r)}{\pi}$$

Involving related functions

03.03.13.0017.01

$$\left(\prod_{k=1}^{4}\left(z\frac{d}{dz}\right)\right)w(z)-2\left(\mu^2+\nu^2\right)\left(\prod_{k=1}^{2}\left(z\frac{d}{dz}\right)\right)w(z)+\left(\nu^2-\mu^2\right)^2 w(z)+4z^2\left(\left(\prod_{k=1}^{2}\left(z\frac{d}{dz}\right)\right)w(z)+2w(z)+3z\,w'(z)\right)=0\;/;$$
$$w(z)=c_1 J_\mu(z) J_\nu(z)+c_2 J_\nu(z) Y_\mu(z)+c_3 J_\mu(z) Y_\nu(z)+c_4 Y_\mu(z) Y_\nu(z)$$

03.03.13.0018.01

$$\left(\prod_{k=1}^{3}\left(z\frac{d}{dz}\right)\right)w(z)+4\left(z^2-\nu^2\right)z\,w'(z)+4z^2 w(z)=0\;/;\;w(z)=c_1 J_\nu(z)^2+c_2 Y_\nu(z) J_\nu(z)+c_3 Y_\nu(z)^2$$

03.03.13.0019.01

$$z^3 w^{(3)}(z)+\left(4z^2-4\nu^2+1\right)z\,w'(z)+\left(4\nu^2-1\right)w(z)=0\;/;\;w(z)=c_1 z J_\nu(z)^2+c_2 z Y_\nu(z) J_\nu(z)+c_3 z Y_\nu(z)^2$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.03.16.0001.01

$$Y_\nu(-z)=z^\nu Y_\nu(z)(-z)^{-\nu}+\left((-z)^\nu z^{-\nu}-(-z)^{-\nu}z^\nu\right)J_\nu(z)\cot(\pi\nu)\;/;\;\nu\notin\mathbb{Z}$$

03.03.16.0002.01

$$Y_\nu(-z)=(-1)^\nu\left(Y_\nu(z)-\frac{2}{\pi}(\log(z)-\log(-z))J_\nu(z)\right)/;\;\nu\in\mathbb{Z}$$

03.03.16.0003.01

$$Y_\nu(i z)=-\frac{2z^\nu}{\pi(i z)^\nu}K_\nu(z)+\left(\frac{(i z)^\nu\cos(\pi\nu)}{z^\nu}-\frac{z^\nu}{(i z)^\nu}\right)\csc(\pi\nu)I_\nu(z)\;/;\;\nu\notin\mathbb{Z}$$

03.03.16.0004.01

$$Y_\nu(i z)=-\frac{2}{\pi i^\nu}K_\nu(z)+\frac{2i^\nu}{\pi}(\log(i z)-\log(z))I_\nu(z)\;/;\;\nu\in\mathbb{Z}$$

03.03.16.0005.01

$$Y_\nu(-i z)=-\frac{2z^\nu}{\pi(-i z)^\nu}K_\nu(z)+\left(\frac{(-i z)^\nu\cos(\pi\nu)}{z^\nu}-\frac{z^\nu}{(-i z)^\nu}\right)\csc(\pi\nu)I_\nu(z)\;/;\;\nu\notin\mathbb{Z}$$

03.03.16.0006.01

$$Y_\nu(-i z)=-\frac{2}{\pi(-i)^\nu}K_\nu(z)+\frac{2(-i)^\nu}{\pi}(\log(-i z)-\log(z))I_\nu(z)\;/;\;\nu\in\mathbb{Z}$$

03.03.16.0007.01

$$Y_\nu\left(c\left(d z^n\right)^m\right)=\frac{\left(c\,d^m z^{m n}\right)^\nu}{\left(c\left(d z^n\right)^m\right)^\nu}Y_\nu\left(c\,d^m z^{m n}\right)+\cot(\pi\nu)\left(\frac{\left(c\left(d z^n\right)^m\right)^\nu}{\left(c\,d^m z^{m n}\right)^\nu}-\frac{\left(c\,d^m z^{m n}\right)^\nu}{\left(c\left(d z^n\right)^m\right)^\nu}\right)J_\nu\left(c\,d^m z^{m n}\right)/;\;2m\in\mathbb{Z}\bigwedge\nu\notin\mathbb{Z}$$

03.03.16.0008.01

$$Y_\nu\left(c\left(d z^n\right)^m\right)=\left(\frac{\left(d z^n\right)^m}{d^m z^{m n}}\right)^\nu\left(Y_\nu\left(c\,d^m z^{m n}\right)+\frac{2}{\pi}\left(\log\left(c\left(d z^n\right)^m\right)-\log\left(c\,d^m z^{m n}\right)\right)J_\nu\left(c\,d^m z^{m n}\right)\right)/;\;2m\in\mathbb{Z}\land\nu\in\mathbb{Z}$$

03.03.16.0013.01

$$Y_\nu\left(\sqrt{z^2}\right)=z^\nu\left(z^2\right)^{-\frac{\nu}{2}}Y_\nu(z)+\cot(\pi\nu)\left(z^{-\nu}\left(z^2\right)^{\nu/2}-z^\nu\left(z^2\right)^{-\frac{\nu}{2}}\right)J_\nu(z)\;/;\;\nu\notin\mathbb{Z}$$

03.03.16.0014.01

$$Y_\nu\left(\sqrt{z^2}\right) = \left(\frac{\sqrt{z^2}}{z}\right)^\nu \left(Y_\nu(z) + \frac{2\left(\log\left(\sqrt{z^2}\right) - \log(z)\right)}{\pi} J_\nu(z)\right) /; \nu \in \mathbb{Z}.$$

Addition formulas

03.03.16.0009.01

$$Y_\nu(z_1 - z_2) = \sum_{k=-\infty}^{\infty} Y_{k+\nu}(z_1)\, J_k(z_2) /; \left|\frac{z_2}{z_1}\right| < 1$$

03.03.16.0010.01

$$Y_\nu(z_1 + z_2) = \sum_{k=-\infty}^{\infty} Y_{\nu-k}(z_1)\, J_k(z_2) /; \left|\frac{z_2}{z_1}\right| < 1$$

Multiple arguments

03.03.16.0011.01

$$Y_\nu(z_1 z_2) = z_1^\nu \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(z_1^2 - 1\right)^k Y_{k+\nu}(z_2) \left(\frac{z_2}{2}\right)^k /; \left|z_1^2 - 1\right| < 1$$

03.03.16.0012.01

$$Y_\nu(z_1 z_2) = z_1^{-\nu} \sum_{k=0}^{\infty} \frac{\left(z_1^2 - 1\right)^k}{k!} Y_{\nu-k}(z_2) \left(\frac{z_2}{2}\right)^k /; \left|z_1^2 - 1\right| < 1$$

Identities

Recurrence identities

Consecutive neighbors

03.03.17.0001.01

$$Y_\nu(z) = \frac{2(\nu+1)}{z} Y_{\nu+1}(z) - Y_{\nu+2}(z)$$

03.03.17.0002.01

$$Y_\nu(z) = \frac{2(\nu-1)}{z} Y_{\nu-1}(z) - Y_{\nu-2}(z)$$

Distant neighbors

Increasing

03.03.17.0003.01

$$Y_\nu(z) = 2^{n-1} z^{-n} (\nu+1)_{n-1} \left(2(n+\nu) \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(n-k)!}{k!\,(n-2k)!\,(-n-\nu)_k\,(\nu+1)_k} \left(\frac{z^2}{4}\right)^k Y_{n+\nu}(z) - z \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{(n-k-1)!}{k!\,(n-2k-1)!\,(-n-\nu+1)_k\,(\nu+1)_k} \left(\frac{z^2}{4}\right)^k Y_{n+\nu+1}(z) \right) /;\ n \in \mathbb{N}$$

03.03.17.0014.01

$$Y_\nu(z) = 2^{n-1} z^{-n} (\nu+1)_{n-1} \left(2(n+\nu)\, {}_3F_4\left(1, \frac{1-n}{2}, -\frac{n}{2}; 1, -n, -n-\nu, \nu+1; -z^2\right) Y_{n+\nu}(z) - z\, {}_3F_4\left(1, \frac{1-n}{2}, 1-\frac{n}{2}; 1, 1-n, -n-\nu+1, \nu+1; -z^2\right) Y_{n+\nu+1}(z) \right) /;\ n \in \mathbb{N}$$

03.03.17.0006.01

$$Y_\nu(z) = \frac{\left(4(\nu+1)(\nu+2) - z^2\right) Y_{\nu+2}(z) - 2z(\nu+1) Y_{\nu+3}(z)}{z^2}$$

03.03.17.0007.01

$$Y_\nu(z) = \frac{4(\nu+2)\left(2(\nu+1)(\nu+3) - z^2\right) Y_{\nu+3}(z) + z\left(z^2 - 4(\nu+1)(\nu+2)\right) Y_{\nu+4}(z)}{z^3}$$

03.03.17.0008.01

$$Y_\nu(z) = \frac{1}{z^4} \left(\left(z^4 - 12(\nu+2)(\nu+3) z^2 + 16(\nu+1)(\nu+2)(\nu+3)(\nu+4)\right) Y_{\nu+4}(z) + 4z(\nu+2)\left(z^2 - 2(\nu+1)(\nu+3)\right) Y_{\nu+5}(z) \right)$$

03.03.17.0009.01

$$Y_\nu(z) = \frac{1}{z^5} \left(2(\nu+3)\left(3z^4 - 16(\nu+2)(\nu+4) z^2 + 16(\nu+1)(\nu+2)(\nu+4)(\nu+5)\right) Y_{\nu+5}(z) - z\left(z^4 - 12(\nu+2)(\nu+3) z^2 + 16(\nu+1)(\nu+2)(\nu+3)(\nu+4)\right) Y_{\nu+6}(z) \right)$$

03.03.17.0015.01

$$Y_\nu(z) = C_n(\nu, z) Y_{\nu+n}(z) - C_{n-1}(\nu, z) Y_{\nu+n+1}(z) /;\ C_0(\nu, z) = 1 \bigwedge C_1(\nu, z) = \frac{2(\nu+1)}{z} \bigwedge C_n(\nu, z) = \frac{2(n+\nu)}{z} C_{n-1}(\nu, z) - C_{n-2}(\nu, z) \bigwedge n \in \mathbb{N}^+$$

03.03.17.0016.01

$$Y_\nu(z) = C_n(\nu, z) Y_{\nu+n}(z) - C_{n-1}(\nu, z) Y_{\nu+n+1}(z) /;\ C_n(\nu, z) = 2^n z^{-n} (\nu+1)_n\, {}_2F_3\left(\frac{1-n}{2}, -\frac{n}{2}; \nu+1, -n, -n-\nu; -z^2\right) \bigwedge n \in \mathbb{N}^+$$

Decreasing

03.03.17.0004.01

$$Y_\nu(z) = (-1)^n 2^{n-1} z^{-n} (1-\nu)_{n-1} \left(z \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{(n-k-1)!}{k!\,(n-2k-1)!\,(1-\nu)_k\,(\nu-n+1)_k} \left(\frac{z^2}{4}\right)^k Y_{\nu-n-1}(z) + 2(n-\nu) \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(n-k)!}{k!\,(n-2k)!\,(1-\nu)_k\,(\nu-n)_k} \left(\frac{z^2}{4}\right)^k Y_{\nu-n}(z) \right) /;\ n \in \mathbb{N}$$

03.03.17.0017.01

$$Y_\nu(z) = (-1)^n 2^{n-1} z^{-n} (1-\nu)_{n-1} \left(2(n-\nu)\, {}_3F_4\left(1, \frac{1-n}{2}, -\frac{n}{2}; 1, -n, 1-\nu, \nu-n; -z^2\right) Y_{\nu-n}(z) + z\, {}_3F_4\left(1, \frac{1-n}{2}, 1-\frac{n}{2}; 1, 1-n, 1-\nu, \nu-n+1; -z^2\right) Y_{\nu-n-1}(z)\right) /; n \in \mathbb{N}$$

03.03.17.0010.01

$$Y_\nu(z) == -\frac{2z(\nu-1)Y_{\nu-3}(z) + \left(z^2 - 4(\nu-2)(\nu-1)\right)Y_{\nu-2}(z)}{z^2}$$

03.03.17.0011.01

$$Y_\nu(z) == \frac{z\left(z^2 - 4(\nu-2)(\nu-1)\right)Y_{\nu-4}(z) - 4\left(z^2 - 2(\nu-3)(\nu-1)\right)(\nu-2)Y_{\nu-3}(z)}{z^3}$$

03.03.17.0012.01

$$Y_\nu(z) == \frac{1}{z^4}\left(4z\left(z^2 - 2(\nu-3)(\nu-1)\right)(\nu-2)Y_{\nu-5}(z) + \left(z^4 - 12(\nu-3)(\nu-2)z^2 + 16(\nu-4)(\nu-3)(\nu-2)(\nu-1)\right)Y_{\nu-4}(z)\right)$$

03.03.17.0013.01

$$Y_\nu(z) == -\frac{1}{z^5}\left(z\left(z^4 - 12(\nu-3)(\nu-2)z^2 + 16(\nu-4)(\nu-3)(\nu-2)(\nu-1)\right)Y_{\nu-6}(z) - 2\left(3z^4 - 16(\nu-4)(\nu-2)z^2 + 16(\nu-5)(\nu-4)(\nu-2)(\nu-1)\right)(\nu-3)Y_{\nu-5}(z)\right)$$

03.03.17.0018.01

$$Y_\nu(z) == C_n(\nu, z)\,Y_{\nu-n}(z) - C_{n-1}(\nu, z)\,Y_{\nu-n-1}(z) \;/; C_0(\nu, z) == 1 \bigwedge C_1(\nu, z) == \frac{2(\nu-1)}{z} \bigwedge C_n(\nu, z) == \frac{2(-n+\nu)}{z} C_{n-1}(\nu, z) - C_{n-2}(\nu, z) \bigwedge n \in \mathbb{N}^+$$

03.03.17.0019.01

$$Y_\nu(z) == C_n(\nu, z)\,Y_{\nu-n}(z) - C_{n-1}(\nu, z)\,Y_{\nu-n-1}(z) \;/; C_n(\nu, z) == (-2)^n z^{-n} (1-\nu)_n\, {}_2F_3\left(\frac{1-n}{2}, -\frac{n}{2}; 1-\nu, -n, \nu-n; -z^2\right) \bigwedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

03.03.17.0005.01

$$Y_\nu(z) == \frac{z}{2\nu}\left(Y_{\nu-1}(z) + Y_{\nu+1}(z)\right)$$

Differentiation

Low-order differentiation

With respect to ν

03.03.20.0001.01

$$Y_\nu^{(1,0)}(z) == \csc(\pi\nu)\left(-\pi\, Y_{-\nu}(z) + \left(J_{-\nu}(z) + J_\nu(z)\cos(\pi\nu)\right)\log\left(\frac{z}{2}\right) - \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\left(\frac{\psi(k-\nu+1)}{\Gamma(k-\nu+1)}\left(\frac{z}{2}\right)^{2k-\nu} + \frac{\cos(\pi\nu)\,\psi(k+\nu+1)}{\Gamma(k+\nu+1)}\left(\frac{z}{2}\right)^{2k+\nu}\right)\right) /; \nu \notin \mathbb{Z}$$

03.03.20.0002.01

$$Y_\nu^{(1,0)}(z) = -\frac{2^{\nu-2} z^{2-\nu} \csc(\pi \nu)}{(\nu-1)\,\Gamma(2-\nu)} F_{2\times0\times1}^{0\times1\times2}\left(\begin{matrix} ;1;1,1-\nu; \\ 2,2-\nu;;2-\nu; \end{matrix} -\frac{z^2}{4}, -\frac{z^2}{4}\right) + \frac{2^{-\nu-2} z^{\nu+2} \cot(\pi \nu)}{(\nu+1)\,\Gamma(\nu+2)} F_{2\times0\times1}^{0\times1\times2}\left(\begin{matrix} ;1;1,1+\nu; \\ 2,2+\nu;;2+\nu; \end{matrix} -\frac{z^2}{4}, -\frac{z^2}{4}\right) - (J_\nu(z)\cot(\pi\nu) + J_{-\nu}(z)\csc(\pi\nu))(\log(2) - \log(z) + \psi(\nu)) - \frac{\pi \nu \csc^2(\pi\nu) + \cot(\pi\nu)}{\nu} J_\nu(z) \;/;\; \nu \notin \mathbb{Z}$$

03.03.20.0003.01

$$Y_n^{(1,0)}(z) = \frac{1}{2} n! \left(\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{1}{(n-k)\,k!} Y_k(z) \left(\frac{z}{2}\right)^k - \frac{\pi}{2} J_n(z) \;/;\; n \in \mathbb{N}$$

03.03.20.0017.01

$$Y_{-n}^{(1,0)}(z) = \frac{(-1)^{n-1}}{2} n! \left(\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{1}{(n-k)\,k!} Y_k(z) \left(\frac{z}{2}\right)^k + \frac{(-1)^{n-1}\pi}{2} J_n(z) \;/;\; n \in \mathbb{N}$$

03.03.20.0018.01

$$Y_{n+\frac{1}{2}}^{(1,0)}(z) = -\pi J_{n+\frac{1}{2}}(z) + \frac{2(2z)^{-n-\frac{1}{2}}}{n!\sqrt{\pi}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} (-1)^k 2^{2k} \binom{n}{2k} (2n-2k)! \left(\cos(z)\left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) + \cos(z)\operatorname{Ci}(2z) + \sin(z)\operatorname{Si}(2z)\right) z^{2k} + \frac{2(2z)^{\frac{1}{2}-n}}{n!\sqrt{\pi}} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} (-1)^k 2^{2k} \binom{n}{2k+1} (-2k+2n-1)! \left(\left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sin(z) + \operatorname{Ci}(2z)\sin(z) - \cos(z)\operatorname{Si}(2z)\right) z^{2k} \;/;\; n \in \mathbb{N}$$

Brychkov Yu.A. (2005)

03.03.20.0019.01

$$Y_{-n-\frac{1}{2}}^{(1,0)}(z) = -\pi J_{-n-\frac{1}{2}}(z) - \frac{(-1)^n 2(2z)^{\frac{1}{2}-n}}{n!\sqrt{\pi}} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} (-1)^k 2^{2k} \binom{n}{2k+1} (-2k+2n-1)! \left(\cos(z)\left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) - \cos(z)\operatorname{Ci}(2z) - \sin(z)\operatorname{Si}(2z)\right) z^{2k} + \frac{(-1)^n 2(2z)^{-n-\frac{1}{2}}}{n!\sqrt{\pi}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} (-1)^k 2^{2k} \binom{n}{2k} (2n-2k)! \left(\left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sin(z) - \operatorname{Ci}(2z)\sin(z) + \cos(z)\operatorname{Si}(2z)\right) z^{2k} \;/;\; n \in \mathbb{N}$$

Brychkov Yu.A. (2005)

With respect to *z*

03.03.20.0004.01

$$\frac{\partial Y_\nu(z)}{\partial z} = Y_{\nu-1}(z) - \frac{\nu}{z} Y_\nu(z)$$

03.03.20.0005.01

$$\frac{\partial Y_\nu(z)}{\partial z} = \frac{\nu}{z} Y_\nu(z) - Y_{\nu+1}(z)$$

03.03.20.0006.01

$$\frac{\partial Y_\nu(z)}{\partial z} == \frac{1}{2}\left(Y_{\nu-1}(z) - Y_{\nu+1}(z)\right)$$

03.03.20.0007.01

$$\frac{\partial Y_0(z)}{\partial z} == -Y_1(z)$$

03.03.20.0008.01

$$\frac{\partial (z^\nu\, Y_\nu(z))}{\partial z} == z^\nu\, Y_{\nu-1}(z)$$

03.03.20.0009.01

$$\frac{\partial (z^{-\nu}\, Y_\nu(z))}{\partial z} == -z^{-\nu}\, Y_{\nu+1}(z)$$

03.03.20.0010.01

$$\frac{\partial^2 Y_\nu(z)}{\partial z^2} == \frac{1}{4}\left(Y_{\nu-2}(z) - 2\, Y_\nu(z) + Y_{\nu+2}(z)\right)$$

Symbolic differentiation

With respect to ν

03.03.20.0011.02

$$Y_\nu^{(m,0)}(z) == \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{z}{2}\right)^{2k} \frac{\partial^m \left(\csc(\pi\,\nu)\left(\frac{\cos(\nu\,\pi)}{\Gamma(k+\nu+1)}\left(\frac{z}{2}\right)^{\nu} - \frac{1}{\Gamma(k-\nu+1)}\left(\frac{z}{2}\right)^{-\nu}\right)\right)}{\partial \nu^m} \,/;\, m \in \mathbb{N} \bigwedge \nu \notin \mathbb{Z}$$

With respect to z

03.03.20.0020.01

$$\frac{\partial^n Y_\nu(z)}{\partial z^n} == z^{-n} \sum_{m=0}^{n} (-1)^{m+n} \binom{n}{m} (-\nu)_{n-m} \sum_{k=0}^{m} \frac{(-1)^{k-1}\, 2^{2k-m}\, (-m)_{2(m-k)}\, (\nu)_k}{(m-k)!}$$
$$\left(\frac{z}{2} \sum_{j=0}^{k-1} \frac{(k-j-1)!}{j!\,(k-2j-1)!\,(1-k-\nu)_j\,(\nu)_{j+1}} \left(\frac{z^2}{4}\right)^j Y_{\nu-1}(z) - \sum_{j=0}^{k} \frac{(k-j)!}{j!\,(k-2j)!\,(1-k-\nu)_j\,(\nu)_j} \left(\frac{z^2}{4}\right)^j Y_\nu(z)\right) /;\, n \in \mathbb{N}$$

03.03.20.0012.02

$$\frac{\partial^n Y_\nu(z)}{\partial z^n} == 2^{n-2\nu} \sqrt{\pi}\, z^{-n-\nu} \csc(\pi\,\nu) \left(z^{2\nu} \cos(\pi\,\nu)\, \Gamma(\nu+1)\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{\nu-n+1}{2}, \frac{\nu-n+2}{2}, \nu+1; -\frac{z^2}{4}\right) -\right.$$
$$\left. 16^\nu\, \Gamma(1-\nu)\, {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, \frac{2-\nu}{2}; \frac{1-n-\nu}{2}, \frac{2-n-\nu}{2}, 1-\nu; -\frac{z^2}{4}\right)\right) /;\, \nu \notin \mathbb{Z} \bigwedge n \in \mathbb{N}$$

03.03.20.0013.02

$$\frac{\partial^n Y_\nu(z)}{\partial z^n} == G_{3,5}^{2,2}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1-n}{2}, -\frac{n}{2}, -\frac{n+\nu+1}{2} \\ \frac{\nu-n}{2}, -\frac{n+\nu}{2}, \frac{1}{2}, 0, -\frac{n+\nu+1}{2} \end{array}\right) /;\, n \in \mathbb{N}$$

03.03.20.0014.02

$$\frac{\partial^n Y_\nu(z)}{\partial z^n} == 2^{-n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} Y_{2k-n+\nu}(z) \,/;\, n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to *z*

03.03.20.0015.01

$$\frac{\partial^\alpha Y_\nu(z)}{\partial z^\alpha} = 2^{\alpha-2\nu}\sqrt{\pi}\, z^{-\alpha-\nu}\csc(\pi\nu)\left(z^{2\nu}\cos(\pi\nu)\,\Gamma(\nu+1)\,{}_2\tilde{F}_3\left(\frac{\nu+1}{2},\frac{\nu+2}{2};\frac{1}{2}(-\alpha+\nu+1),\frac{1}{2}(-\alpha+\nu+2),\nu+1;-\frac{z^2}{4}\right)-16^\nu\,\Gamma(1-\nu)\,{}_2\tilde{F}_3\left(\frac{1-\nu}{2},1-\frac{\nu}{2};1-\nu,\frac{1}{2}(-\alpha-\nu+1),\frac{1}{2}(-\alpha-\nu+2);-\frac{z^2}{4}\right)\right)/;\nu\notin\mathbb{Z}$$

03.03.20.0021.01

$$\frac{\partial^\alpha Y_\nu(z)}{\partial z^\alpha} = \frac{1}{\pi}(-1)^{\frac{|\nu|-\nu}{2}}\left(-2^{|\nu|}z^{-\alpha-|\nu|}\sum_{k=\left\lfloor\frac{|\nu|-1}{2}\right\rfloor+1}^{|\nu|-1}\frac{(|\nu|-k-1)!\,\Gamma(2k-|\nu|+1)}{k!\,\Gamma(2k-\alpha-|\nu|+1)}\left(\frac{z}{2}\right)^{2k}+\right.$$
$$(-1)^{|\nu|}2^{|\nu|}z^{-\alpha-|\nu|}\sum_{k=0}^{\left\lfloor\frac{|\nu|-1}{2}\right\rfloor}\frac{(|\nu|-k-1)!\,(\log(z)-\psi(2k-\alpha-|\nu|+1)+\psi(|\nu|-2k))}{k!\,(|\nu|-2k-1)!\,\Gamma(2k-\alpha-|\nu|+1)}\left(\frac{z}{2}\right)^{2k}+$$
$$2^{1-|\nu|}z^{|\nu|-\alpha}\sum_{k=0}^{\infty}\frac{(-1)^k\,\mathcal{FC}_{\log}^{(\alpha)}(z,2k+|\nu|)}{k!\,(k+|\nu|)!}\left(\frac{z}{2}\right)^{2k}-2^{1-|\nu|}\log(2)\,z^{|\nu|-\alpha}\sum_{k=0}^{\infty}\frac{(-1)^k\,\Gamma(2k+|\nu|+1)}{k!\,(k+|\nu|)!\,\Gamma(2k-\alpha+|\nu|+1)}\left(\frac{z}{2}\right)^{2k}-$$
$$\left.2^{-|\nu|}\,z^{|\nu|-\alpha}\sum_{k=0}^{\infty}\frac{(-1)^k\,\Gamma(2k+|\nu|+1)\,(\psi(k+1)+\psi(k+|\nu|+1))}{k!\,(k+|\nu|)!\,\Gamma(2k-\alpha+|\nu|+1)}\left(\frac{z}{2}\right)^{2k}\right)/;\nu\in\mathbb{Z}$$

03.03.20.0016.01

$$\frac{\partial^\alpha Y_\nu(z)}{\partial z^\alpha} = \frac{2^{\alpha-2\nu+1}\,\Gamma(\nu+1)\,z^{\nu-\alpha}}{\sqrt{\pi}}\log\left(\frac{z}{2}\right){}_2\tilde{F}_3\left(\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1;-\frac{\alpha}{2}+\frac{\nu}{2}+\frac{1}{2},-\frac{\alpha}{2}+\frac{\nu}{2}+1,\nu+1;-\frac{z^2}{4}\right)-$$
$$\frac{z^{-\alpha}}{\pi}\sum_{k=\left\lfloor\frac{\nu+1}{2}\right\rfloor}^{\nu-1}\frac{(\nu-k-1)!\,(2k-\nu)!}{k!\,\Gamma(2k-\alpha-\nu+1)}\left(\frac{z}{2}\right)^{2k-\nu}-\sum_{k=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}\frac{(\nu-k-1)!\,\mathcal{FC}_{\exp}^{(\alpha)}(z,2k-\nu)\,z^{2k-\alpha-\nu}}{\pi\,2^{2k-\nu}\,k!}-$$
$$\frac{z^{-\alpha}}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2k+\nu)!\,(\psi(k+1)+\psi(k+\nu+1)-2\psi(2k+\nu+1)+2\psi(2k-\alpha+\nu+1))}{k!\,(k+\nu)!\,\Gamma(2k-\alpha+\nu+1)}\left(\frac{z}{2}\right)^{2k+\nu}/;\nu\in\mathbb{N}$$

Integration

Indefinite integration

Involving only one direct function

03.03.21.0001.01

$$\int Y_\nu(a z)\,dz = 2^{-\nu-1}z\,(a z)^{-\nu}\csc(\pi\nu)$$
$$\left((a z)^{2\nu}\cos(\pi\nu)\,\Gamma\left(\frac{\nu+1}{2}\right){}_1\tilde{F}_2\left(\frac{\nu+1}{2};\nu+1,\frac{\nu+3}{2};-\frac{1}{4}a^2z^2\right)-4^\nu\,\Gamma\left(\frac{1}{2}-\frac{\nu}{2}\right){}_1\tilde{F}_2\left(\frac{1}{2}-\frac{\nu}{2};1-\nu,\frac{3}{2}-\frac{\nu}{2};-\frac{1}{4}a^2z^2\right)\right)/;\nu\notin\mathbb{Z}$$

03.03.21.0002.01

$$\int Y_\nu(z)\,dz = (-1)^{\frac{|\nu|-\nu-2}{2}} G_{2,4}^{3,0}\left(\frac{z}{2}, \frac{1}{2} \middle| \begin{array}{c} 1, -\frac{|\nu|}{2} \\ 0, \frac{1}{2}(1-|\nu|), \frac{1}{2}(|\nu|+1), -\frac{|\nu|}{2} \end{array}\right) /; \frac{\nu+1}{2} \in \mathbb{Z}$$

03.03.21.0003.01

$$\int Y_\nu(z)\,dz = G_{2,4}^{2,1}\left(\frac{z}{2}, \frac{1}{2} \middle| \begin{array}{c} 1, -\frac{|\nu|}{2} \\ \frac{1}{2}(1-|\nu|), \frac{1}{2}(|\nu|+1), 0, -\frac{|\nu|}{2} \end{array}\right) /; \frac{\nu}{2} \in \mathbb{Z}$$

03.03.21.0004.01

$$\int Y_0(z)\,dz = \frac{1}{2}\pi z\,(Y_0(z)\,\boldsymbol{H}_{-1}(z) + Y_1(z)\,\boldsymbol{H}_0(z))$$

03.03.21.0005.01

$$\int Y_1(z)\,dz = -Y_0(z)$$

03.03.21.0006.01

$$\int Y_2(z)\,dz = G_{2,4}^{2,1}\left(\frac{z}{2}, \frac{1}{2} \middle| \begin{array}{c} 1, -1 \\ -\frac{1}{2}, \frac{3}{2}, -1, 0 \end{array}\right)$$

03.03.21.0131.01

$$\int Y_2(z)\,dz = \frac{1}{2}(-2 z\,Y_2(z) + \pi z\,Y_1(z)\,\boldsymbol{H}_0(z) + (\pi z\,Y_2(z) - 2\pi\,Y_1(z))\,\boldsymbol{H}_1(z))$$

03.03.21.0132.01

$$\int Y_3(z)\,dz = -Y_0(z) - 2\,Y_2(z)$$

03.03.21.0133.01

$$\int Y_4(z)\,dz = \frac{-2\,(z\,Y_3(z) + Y_4(z))\,z^2 - 3\pi\,(z\,Y_4(z) - 6\,Y_3(z))\,\boldsymbol{H}_1(z)\,z + 3\pi\left(\left(z^2 - 24\right)Y_3(z) + 4 z\,Y_4(z)\right)\boldsymbol{H}_2(z)}{6 z}$$

03.03.21.0134.01

$$\int Y_5(z)\,dz = -Y_0(z) - 2\,Y_2(z) - 2\,Y_4(z)$$

03.03.21.0135.01

$$\int Y_{2n}(z)\,dz = \frac{1}{2}\pi z\,(Y_0(z)\,\boldsymbol{H}_{-1}(z) + Y_1(z)\,\boldsymbol{H}_0(z)) - 2\sum_{k=0}^{n-1} Y_{2k+1}(z) /; n \in \mathbb{N}$$

03.03.21.0136.01

$$\int Y_{2n+1}(z)\,dz = -Y_0(z) - 2\sum_{k=1}^{n} Y_{2k}(z) /; n \in \mathbb{N}$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear arguments

03.03.21.0007.01

$$\int z^{\alpha-1} Y_\nu(a z)\, dz = 2^{-\nu-1} z^\alpha (a z)^{-\nu} \csc(\pi \nu) \left((a z)^{2\nu} \cos(\pi \nu)\, \Gamma\!\left(\frac{\alpha+\nu}{2}\right) {}_1\tilde{F}_2\!\left(\frac{\alpha+\nu}{2}; \nu+1, \frac{1}{2}(\alpha+\nu+2); -\frac{1}{4} a^2 z^2\right) - 4^\nu\, \Gamma\!\left(\frac{\alpha-\nu}{2}\right) {}_1\tilde{F}_2\!\left(\frac{\alpha-\nu}{2}; 1-\nu, \frac{1}{2}(\alpha-\nu+2); -\frac{1}{4} a^2 z^2\right)\right) /; \nu \notin \mathbb{Z}$$

03.03.21.0008.01

$$\int z^{\alpha-1} Y_\nu(z)\, dz = \frac{2^\nu z^{\alpha-\nu} \csc(\pi \nu)}{(\nu-\alpha)\, \Gamma(1-\nu)}\, {}_1F_2\!\left(\frac{\alpha-\nu}{2}; 1-\nu, \frac{\alpha-\nu}{2}+1; -\frac{z^2}{4}\right) + \frac{2^{-\nu} z^{\alpha+\nu} \cot(\pi \nu)}{(\alpha+\nu)\, \Gamma(\nu+1)}\, {}_1F_2\!\left(\frac{\alpha+\nu}{2}; \nu+1, \frac{\alpha+\nu}{2}+1; -\frac{z^2}{4}\right) /; \nu \notin \mathbb{Z}$$

03.03.21.0009.01

$$\int z^{\alpha-1} Y_0(z)\, dz = \frac{z^\alpha}{\alpha^2} \left(\alpha\, Y_0(z)\, {}_1F_2\!\left(1; \frac{\alpha}{2}+1, \frac{\alpha}{2}; -\frac{z^2}{4}\right) + z\, Y_1(z)\, {}_1F_2\!\left(1; \frac{\alpha}{2}+1, \frac{\alpha}{2}+1; -\frac{z^2}{4}\right)\right)$$

03.03.21.0010.01

$$\int z^{1-\nu} Y_\nu(z)\, dz = -J_{\nu-1}(z) \cot(\pi \nu)\, z^{1-\nu} - J_{1-\nu}(z) \csc(\pi \nu)\, z^{1-\nu}$$

03.03.21.0011.01

$$\int z^{-\nu} Y_\nu(z)\, dz = \frac{2^\nu \csc(\pi \nu)\, z^{1-2\nu}}{(2\nu-1)\, \Gamma(1-\nu)}\, {}_1F_2\!\left(\frac{1}{2}-\nu; 1-\nu, \frac{3}{2}-\nu; -\frac{z^2}{4}\right) + \frac{2^{-\nu} \cot(\pi \nu)\, z}{\Gamma(\nu+1)}\, {}_1F_2\!\left(\frac{1}{2}; \frac{3}{2}, \nu+1; -\frac{z^2}{4}\right)$$

03.03.21.0012.01

$$\int z^{\nu+3} Y_\nu(z)\, dz = 2(\nu+1)\, J_{\nu+2}(z) \cot(\pi \nu)\, z^{\nu+2} + \frac{\csc(\pi \nu)}{\Gamma(-\nu)} \left(2 J_{-\nu-2}(z)\, \Gamma(-\nu)\, z^{\nu+2} + J_{-\nu-1}(z)\, \Gamma(-\nu)\, z^{\nu+3} - J_{\nu+3}(z) \cos(\pi \nu)\, \Gamma(-\nu)\, z^{\nu+3} + 2^{\nu+3} + 2^{\nu+3} \nu\right)$$

03.03.21.0013.01

$$\int z^{\nu+1} Y_\nu(z)\, dz = J_{\nu+1}(z) \cot(\pi \nu)\, z^{\nu+1} + \left(z^{\nu+1} J_{-\nu-1}(z)\right) \csc(\pi \nu)$$

03.03.21.0014.01

$$\int z^\nu Y_\nu(z)\, dz = 2^{-\nu-1} z \csc(\pi \nu) \left(z^{2\nu} \cos(\pi \nu)\, \Gamma\!\left(\nu+\frac{1}{2}\right) {}_1\tilde{F}_2\!\left(\nu+\frac{1}{2}; \nu+1, \nu+\frac{3}{2}; -\frac{z^2}{4}\right) - 4^\nu \sqrt{\pi}\, {}_1\tilde{F}_2\!\left(\frac{1}{2}; 1-\nu, \frac{3}{2}; -\frac{z^2}{4}\right)\right)$$

03.03.21.0015.01

$$\int z\, Y_0(z)\, dz = z\, Y_1(z)$$

03.03.21.0137.01

$$\int \frac{Y_{2n}(z)}{z}\, dz = -\frac{1}{2n} (Y_0(z) + Y_{2n}(z)) - \frac{1}{n} \sum_{k=1}^{n-1} Y_{2k}(z) /; n \in \mathbb{N}^+$$

03.03.21.0138.01

$$\int \frac{Y_{2n+1}(z)}{z}\, dz = \frac{1}{2n+1} \left(-Y_{2n+1}(z) + \frac{1}{2} (\pi z) (Y_0(z)\, \boldsymbol{H}_{-1}(z) + Y_1(z)\, \boldsymbol{H}_0(z)) - 2 \sum_{k=0}^{n-1} Y_{2k+1}(z)\right) /; n \in \mathbb{N}$$

03.03.21.0139.01

$$\int \frac{Y_{2n}(z)}{z^2}\,dz = -\frac{Y_{2n}(z)}{z} + \frac{1}{2n-1}\left(-\frac{1}{2}Y_{2n-1}(z) + \frac{1}{4}(\pi z)\left(Y_0(z)\,\boldsymbol{H}_{-1}(z) + Y_1(z)\,\boldsymbol{H}_0(z)\right) - \sum_{k=0}^{n-2} Y_{2k+1}(z)\right) - \frac{1}{2n+1}\left(-\frac{1}{2}Y_{2n+1}(z) + \frac{1}{4}(\pi z)\left(Y_0(z)\,\boldsymbol{H}_{-1}(z) + Y_1(z)\,\boldsymbol{H}_0(z)\right) - \sum_{k=0}^{n-1} Y_{2k+1}(z)\right) /;\, n \in \mathbb{N}^+$$

03.03.21.0140.01

$$\int \frac{Y_{2n+1}(z)}{z^2}\,dz = \frac{Y_0(z) + Y_{2n+2}(z)}{4(n+1)} - \frac{Y_0(z) + Y_{2n}(z)}{4n} - \frac{Y_{2n+1}(z)}{z} - \frac{1}{2n}\sum_{k=1}^{n-1} Y_{2k}(z) + \frac{1}{2(n+1)}\sum_{k=1}^{n} Y_{2k}(z) /;\, n \in \mathbb{N}^+$$

03.03.21.0141.01

$$\int \frac{Y_\nu(z)}{z^m}\,dz = \frac{1}{2(m-1)}\int \frac{1}{z^{m-1}}\left(Y_{\nu-1}(z) - Y_{\nu+1}(z)\right)dz - \frac{Y_\nu(z)}{(m-1)\,z^{m-1}} /;\, m \in \mathbb{Z} \land m > 1$$

Power arguments

03.03.21.0016.01

$$\int z^{\alpha-1}\,Y_\nu(a\,z^r)\,dz = \frac{1}{r}\left(2^{-\nu-1} z^\alpha (a\,z^r)^{-\nu} \csc(\pi\nu)\left((a\,z^r)^{2\nu}\cos(\pi\nu)\,\Gamma\!\left(\frac{\alpha+r\nu}{2r}\right){}_1\tilde{F}_2\!\left(\frac{\alpha+r\nu}{2r};\, \nu+1,\, \frac{\alpha+r(\nu+2)}{2r};\, -\frac{1}{4}a^2 z^{2r}\right) - 4^\nu\,\Gamma\!\left(\frac{\alpha-r\nu}{2r}\right){}_1\tilde{F}_2\!\left(\frac{\alpha-r\nu}{2r};\, 1-\nu,\, \frac{1}{2}\left(\frac{\alpha}{r}-\nu+2\right);\, -\frac{1}{4}a^2 z^{2r}\right)\right)\right)$$

Involving exponential function

Involving exp

Linear arguments

03.03.21.0017.01

$$\int e^{-i a z}\,Y_\nu(a\,z)\,dz = \frac{2^{-\nu} z\,(a\,z)^\nu \cot(\pi\nu)}{\Gamma(\nu+2)}\,{}_2F_2\!\left(\nu+\frac{1}{2},\, \nu+1;\, \nu+2,\, 2\nu+1;\, -2\,i\,a\,z\right) - \frac{2^\nu z\,(a\,z)^{-\nu}\csc(\pi\nu)}{\Gamma(2-\nu)}\,{}_2F_2\!\left(\frac{1}{2}-\nu,\, 1-\nu;\, 1-2\nu,\, 2-\nu;\, -2\,i\,a\,z\right)$$

03.03.21.0018.01

$$\int e^{i a z}\,Y_\nu(a\,z)\,dz = \frac{2^{-\nu} z\,(a\,z)^\nu \cot(\pi\nu)}{\Gamma(\nu+2)}\,{}_2F_2\!\left(\nu+\frac{1}{2},\, \nu+1;\, \nu+2,\, 2\nu+1;\, 2\,i\,a\,z\right) - \frac{2^\nu z\,(a\,z)^{-\nu}\csc(\pi\nu)}{\Gamma(2-\nu)}\,{}_2F_2\!\left(\frac{1}{2}-\nu,\, 1-\nu;\, 1-2\nu,\, 2-\nu;\, 2\,i\,a\,z\right)$$

Power arguments

03.03.21.0019.01

$$\int e^{-i a z^r} Y_\nu(a z^r)\, dz = \frac{2^\nu z \csc(\pi \nu) (a z^r)^{-\nu}}{(r \nu - 1) \Gamma(1-\nu)} \, {}_2F_2\left(\frac{1}{2} - \nu, \frac{1}{r} - \nu; 1 - 2\nu, -\nu + \frac{1}{r} + 1; -2 i a z^r\right) + \frac{2^{-\nu} z \cot(\pi \nu) (a z^r)^\nu}{(r \nu + 1) \Gamma(\nu + 1)} \, {}_2F_2\left(\nu + \frac{1}{2}, \nu + \frac{1}{r}; \nu + \frac{1}{r} + 1, 2\nu + 1; -2 i a z^r\right)$$

03.03.21.0020.01

$$\int e^{i a z^r} Y_\nu(a z^r)\, dz = \frac{2^\nu z \csc(\pi \nu) (a z^r)^{-\nu}}{(r \nu - 1) \Gamma(1-\nu)} \, {}_2F_2\left(\frac{1}{2} - \nu, \frac{1}{r} - \nu; 1 - 2\nu, -\nu + \frac{1}{r} + 1; 2 i a z^r\right) + \frac{2^{-\nu} z \cot(\pi \nu) (a z^r)^\nu}{(r \nu + 1) \Gamma(\nu + 1)} \, {}_2F_2\left(\nu + \frac{1}{2}, \nu + \frac{1}{r}; \nu + \frac{1}{r} + 1, 2\nu + 1; 2 i a z^r\right)$$

Involving exponential function and a power function

Involving exp and power

Linear arguments

03.03.21.0021.01

$$\int z^{\alpha-1} e^{-i a z} Y_\nu(a z)\, dz = \frac{2^{-\nu} z^\alpha (a z)^{-\nu}}{\pi} \left(\frac{4^\nu \Gamma(\nu)}{\nu - \alpha} \, {}_2F_2\left(\frac{1}{2} - \nu, \alpha - \nu; 1 - 2\nu, \alpha - \nu + 1; -2 i a z\right) - \frac{(a z)^{2\nu} \cos(\pi \nu) \Gamma(-\nu)}{\alpha + \nu} \, {}_2F_2\left(\nu + \frac{1}{2}, \alpha + \nu; \alpha + \nu + 1, 2\nu + 1; -2 i a z\right)\right)$$

03.03.21.0022.01

$$\int z^{-\nu} e^{-i a z} Y_\nu(a z)\, dz = \frac{1}{a (2\nu - 1) \Gamma(\nu)} \left(2^{-\nu} e^{-i a z} z^{-\nu} \csc(\pi \nu) \left(2^\nu a i z J_{1-\nu}(a z) \Gamma(\nu) + 2^\nu a z J_{-\nu}(a z) \Gamma(\nu) - i \cos(\pi \nu) \left(2 e^{i a z} (a z)^\nu - 2^\nu a z J_{\nu-1}(a z) \Gamma(\nu) - i\, 2^\nu a z J_\nu(a z) \Gamma(\nu)\right)\right)\right)$$

03.03.21.0023.01

$$\int z^\nu e^{-i a z} Y_\nu(a z)\, dz = \frac{1}{a (2\nu + 1) \Gamma(-\nu)} \left(e^{-i a z} z^\nu (a z)^{-\nu} \csc(\pi \nu) \left(a i z J_{-\nu-1}(a z) \Gamma(-\nu) (a z)^\nu - a z J_{-\nu}(a z) \Gamma(-\nu) (a z)^\nu + a z J_\nu(a z) \cos(\pi \nu) \Gamma(-\nu) (a z)^\nu + a i z J_{\nu+1}(a z) \cos(\pi \nu) \Gamma(-\nu) (a z)^\nu + 2^{\nu+1} e^{i a z} (-i)\right)\right)$$

03.03.21.0024.01

$$\int z^{\alpha-1} e^{i a z} Y_\nu(a z)\, dz = \frac{2^{-\nu} z^\alpha (a z)^{-\nu}}{\pi} \left(\frac{4^\nu \Gamma(\nu)}{\nu - \alpha} \, {}_2F_2\left(\frac{1}{2} - \nu, \alpha - \nu; 1 - 2\nu, \alpha - \nu + 1; 2 i a z\right) - \frac{(a z)^{2\nu} \cos(\pi \nu) \Gamma(-\nu)}{\alpha + \nu} \, {}_2F_2\left(\nu + \frac{1}{2}, \alpha + \nu; \alpha + \nu + 1, 2\nu + 1; 2 i a z\right)\right)$$

03.03.21.0025.01

$$\int z^{-\nu} e^{i a z} Y_\nu(a z)\, dz = \frac{1}{a (2\nu - 1) \Gamma(\nu)} \left(2^{-\nu} z^{-\nu} \csc(\pi \nu) \left(2^\nu a e^{i a z} (-i) z J_{1-\nu}(a z) \Gamma(\nu) + 2^\nu a e^{i a z} z J_{-\nu}(a z) \Gamma(\nu) + i \cos(\pi \nu) \left(2 (a z)^\nu - 2^\nu a e^{i a z} z J_{\nu-1}(a z) \Gamma(\nu) + 2^\nu a e^{i a z} i z J_\nu(a z) \Gamma(\nu)\right)\right)\right)$$

03.03.21.0026.01

$$\int z^{\nu}\, e^{i a z}\, Y_{\nu}(a z)\, dz = \frac{1}{a\,(2\nu+1)\,\Gamma(-\nu)} \left(z^{\nu}\,(a z)^{-\nu} \csc(\pi \nu) \left(-i\, a\, e^{i a z}\, z\, J_{-\nu-1}(a z)\, \Gamma(-\nu)\,(a z)^{\nu} - a\, e^{i a z}\, z\, J_{-\nu}(a z)\, \Gamma(-\nu)\,(a z)^{\nu} + a\, e^{i a z}\, z\, J_{\nu}(a z) \cos(\pi \nu)\, \Gamma(-\nu)\,(a z)^{\nu} - i\, a\, e^{i a z}\, z\, J_{\nu+1}(a z) \cos(\pi \nu)\, \Gamma(-\nu)\,(a z)^{\nu} + 2^{\nu+1}\, i\right)\right)$$

Power arguments

03.03.21.0027.01

$$\int z^{\alpha-1}\, e^{-i a z^r}\, Y_{\nu}(a z^r)\, dz = \left(2^{-\nu} z^{\alpha} (a z^r)^{-\nu} \csc(\pi \nu) \left((r \nu - \alpha) \cos(\pi \nu)\, \Gamma(1-\nu)\, {}_2F_2\!\left(\nu + \frac{1}{2}, \frac{\alpha}{r} + \nu; \frac{\alpha}{r} + \nu + 1, 2\nu + 1; -2 i a z^r\right) (a z^r)^{2\nu} + 4^{\nu} (\alpha + r \nu)\, \Gamma(\nu+1)\, {}_2F_2\!\left(\frac{1}{2} - \nu, \frac{\alpha}{r} - \nu; 1 - 2\nu, \frac{\alpha}{r} - \nu + 1; -2 i a z^r\right)\right)\right) \Big/ ((r \nu - \alpha)(\alpha + r \nu)\, \Gamma(1-\nu)\, \Gamma(\nu+1))$$

03.03.21.0028.01

$$\int z^{\alpha-1}\, e^{i a z^r}\, Y_{\nu}(a z^r)\, dz = \left(2^{-\nu} z^{\alpha} (a z^r)^{-\nu} \csc(\pi \nu) \left((r \nu - \alpha) \cos(\pi \nu)\, \Gamma(1-\nu)\, {}_2F_2\!\left(\nu + \frac{1}{2}, \frac{\alpha}{r} + \nu; \frac{\alpha}{r} + \nu + 1, 2\nu + 1; 2 i a z^r\right) (a z^r)^{2\nu} + 4^{\nu} (\alpha + r \nu)\, \Gamma(\nu+1)\, {}_2F_2\!\left(\frac{1}{2} - \nu, \frac{\alpha}{r} - \nu; 1 - 2\nu, \frac{\alpha}{r} - \nu + 1; 2 i a z^r\right)\right)\right) \Big/ ((r \nu - \alpha)(\alpha + r \nu)\, \Gamma(1-\nu)\, \Gamma(\nu+1))$$

Involving trigonometric functions

Involving sin

Linear arguments

03.03.21.0029.01

$$\int \sin(a z)\, Y_{\nu}(a z)\, dz = \frac{2^{-\nu} z\, (a z)^{\nu+1} \cot(\pi \nu)}{(\nu+2)\, \Gamma(\nu+1)}\, {}_3F_4\!\left(\frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + 1, \frac{\nu}{2} + \frac{5}{4}; \frac{3}{2}, \frac{\nu}{2} + 2, \nu + 1, \nu + \frac{3}{2}; -a^2 z^2\right) + \frac{2^{\nu} z\, (a z)^{1-\nu} \csc(\pi \nu)}{(\nu-2)\, \Gamma(1-\nu)}\, {}_3F_4\!\left(\frac{3}{4} - \frac{\nu}{2}, 1 - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}; \frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, 2 - \frac{\nu}{2}; -a^2 z^2\right)$$

03.03.21.0030.01

$$\int \sin(b + a z)\, Y_{\nu}(a z)\, dz = 2^{-\nu} z\, (a z)^{-\nu} \left(\frac{4^{\nu}\, a z \cos(b) \csc(\pi \nu)}{(\nu-2)\, \Gamma(1-\nu)}\, {}_3F_4\!\left(\frac{3}{4} - \frac{\nu}{2}, 1 - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}; \frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, 2 - \frac{\nu}{2}; -a^2 z^2\right) - \frac{4^{\nu} \csc(\pi \nu)\, \sin(b)}{\Gamma(2-\nu)}\, {}_3F_4\!\left(\frac{1}{4} - \frac{\nu}{2}, \frac{1}{2} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}; \frac{1}{2}, \frac{1}{2} - \nu, 1 - \nu, \frac{3}{2} - \frac{\nu}{2}; -a^2 z^2\right) + \frac{1}{\Gamma(\nu+1)\, \Gamma(\nu+3)} \left((a z)^{2\nu} \cot(\pi \nu) \left(a z \cos(b)\, \Gamma(\nu+2)\, {}_3F_4\!\left(\frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + 1, \frac{\nu}{2} + \frac{5}{4}; \frac{3}{2}, \frac{\nu}{2} + 2, \nu + 1, \nu + \frac{3}{2}; -a^2 z^2\right) + (\nu+2)\, \Gamma(\nu+1)\, {}_3F_4\!\left(\frac{\nu}{2} + \frac{1}{4}, \frac{\nu}{2} + \frac{1}{2}, \frac{\nu}{2} + \frac{3}{4}; \frac{1}{2}, \frac{\nu}{2} + \frac{3}{2}, \nu + \frac{1}{2}, \nu + 1; -a^2 z^2\right) \sin(b)\right)\right)\right)$$

Power arguments

03.03.21.0031.01

$$\int \sin(a z^r)\, Y_\nu(a z^r)\, dz = 2^{-\nu} z\, (a z^r)^{1-\nu} \left(\frac{(a z^r)^{2\nu} \cot(\pi \nu)}{(\nu r + r + 1)\, \Gamma(\nu+1)}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}, \frac{\nu}{2}+\frac{1}{2}+\frac{1}{2r}; \frac{3}{2}, \frac{\nu}{2}+\frac{3}{2}+\frac{1}{2r}, \nu+1, \nu+\frac{3}{2}; -a^2 z^{2r}\right) + \frac{4^\nu \csc(\pi\nu)}{(r(\nu-1)-1)\,\Gamma(1-\nu)}\, {}_3F_4\!\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, -\frac{\nu}{2}+\frac{1}{2}+\frac{1}{2r}; \frac{3}{2}, 1-\nu, \frac{3}{2}-\nu, -\frac{\nu}{2}+\frac{3}{2}+\frac{1}{2r}; -a^2 z^{2r}\right)\right)$$

03.03.21.0032.01

$$\int \sin(a z^r + b)\, Y_\nu(a z^r)\, dz = 2^{-\nu} z\, (a z^r)^{-\nu} \left(\frac{4^\nu a z^r \cos(b) \csc(\pi\nu)}{(r(\nu-1)-1)\,\Gamma(1-\nu)}\, {}_3F_4\!\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, -\frac{\nu}{2}+\frac{1}{2}+\frac{1}{2r}; \frac{3}{2}, 1-\nu, \frac{3}{2}-\nu, -\frac{\nu}{2}+\frac{3}{2}+\frac{1}{2r}; -a^2 z^{2r}\right) + \frac{4^\nu \csc(\pi\nu) \sin(b)}{(r\nu-1)\,\Gamma(1-\nu)}\, {}_3F_4\!\left(\frac{1}{4}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}, \frac{1}{2r}-\frac{\nu}{2}; \frac{1}{2}, \frac{1}{2}-\nu, 1-\nu, -\frac{\nu}{2}+\frac{1}{2r}+1; -a^2 z^{2r}\right) + \frac{1}{\Gamma(\nu+1)} \left((a z^r)^{2\nu} \cot(\pi\nu) \left(\frac{a z^r \cos(b)}{\nu r + r + 1}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}, \frac{\nu}{2}+\frac{1}{2}+\frac{1}{2r}; \frac{3}{2}, \frac{\nu}{2}+\frac{3}{2}+\frac{1}{2r}, \nu+1, \nu+\frac{3}{2}; -a^2 z^{2r}\right) + \frac{\sin(b)}{r\nu+1}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{1}{2r}; \frac{1}{2}, \frac{\nu}{2}+\frac{1}{2r}+1, \nu+\frac{1}{2}, \nu+1; -a^2 z^{2r}\right)\right)\right)\right)$$

Involving cos

Linear arguments

03.03.21.0033.01

$$\int \cos(a z)\, Y_\nu(a z)\, dz = \frac{2^{-\nu} z\, (a z)^\nu \cot(\pi\nu)}{\Gamma(\nu+2)}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}, \frac{\nu}{2}+\frac{3}{2}, \nu+\frac{1}{2}, \nu+1; -a^2 z^2\right) - \frac{2^\nu z\, (a z)^{-\nu} \csc(\pi\nu)}{\Gamma(2-\nu)}\, {}_3F_4\!\left(\frac{1}{4}-\frac{\nu}{2}, \frac{1}{2}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}; \frac{1}{2}, \frac{1}{2}-\nu, 1-\nu, \frac{3}{2}-\frac{\nu}{2}; -a^2 z^2\right)$$

03.03.21.0034.01

$$\int \cos(b + a z)\, Y_\nu(a z)\, dz = 2^{-\nu} z\, (a z)^{-\nu} \left(\frac{(a z)^{2\nu} \cos(b) \cot(\pi\nu)}{\Gamma(\nu+2)}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}, \frac{\nu}{2}+\frac{3}{2}, \nu+\frac{1}{2}, \nu+1; -a^2 z^2\right) - \frac{4^\nu \cos(b) \csc(\pi\nu)}{\Gamma(2-\nu)}\, {}_3F_4\!\left(\frac{1}{4}-\frac{\nu}{2}, \frac{1}{2}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}; \frac{1}{2}, \frac{1}{2}-\nu, 1-\nu, \frac{3}{2}-\frac{\nu}{2}; -a^2 z^2\right) + a z \left(\frac{4^\nu \Gamma(\nu)}{2\pi - \pi\nu}\, {}_3F_4\!\left(\frac{3}{4}-\frac{\nu}{2}, 1-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}; \frac{3}{2}, 1-\nu, \frac{3}{2}-\nu, 2-\frac{\nu}{2}; -a^2 z^2\right) - \frac{(a z)^{2\nu} (\nu+1) \cot(\pi\nu)}{\Gamma(\nu+3)}\, {}_3F_4\!\left(\frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}, \frac{\nu}{2}+2, \nu+1, \nu+\frac{3}{2}; -a^2 z^2\right)\right) \sin(b)\right)$$

Power arguments

03.03.21.0035.01

$$\int \cos(a z^r) Y_\nu(a z^r) \, dz = \frac{2^{-\nu} z (a z^r)^\nu \cot(\pi \nu)}{(r \nu + 1) \Gamma(\nu + 1)} {}_3F_4\left(\frac{\nu}{2} + \frac{1}{4}, \frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{1}{2r}; \frac{1}{2}, \frac{\nu}{2} + \frac{1}{2r} + 1, \nu + \frac{1}{2}, \nu + 1; -a^2 z^{2r}\right) + \frac{2^\nu z (a z^r)^{-\nu} \csc(\pi \nu)}{(r \nu - 1) \Gamma(1 - \nu)} {}_3F_4\left(\frac{1}{4} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}, \frac{1}{2r} - \frac{\nu}{2}; \frac{1}{2}, \frac{1}{2} - \nu, 1 - \nu, -\frac{\nu}{2} + \frac{1}{2r} + 1; -a^2 z^{2r}\right)$$

03.03.21.0036.01

$$\int \cos(a z^r + b) Y_\nu(a z^r) \, dz = 2^{-\nu} z (a z^r)^{-\nu} \left(\frac{4^\nu \cos(b) \csc(\pi \nu)}{(r \nu - 1) \Gamma(1 - \nu)} {}_3F_4\left(\frac{1}{4} - \frac{\nu}{2}, \frac{3}{4} - \frac{\nu}{2}, \frac{1}{2r} - \frac{\nu}{2}; \frac{1}{2}, \frac{1}{2} - \nu, 1 - \nu, -\frac{\nu}{2} + \frac{1}{2r} + 1; -a^2 z^{2r}\right) + \frac{1}{\Gamma(\nu + 1)} \left((a z^r)^{2\nu} \cot(\pi \nu) \left(\frac{\cos(b)}{r \nu + 1} {}_3F_4\left(\frac{\nu}{2} + \frac{1}{4}, \frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{1}{2r}; \frac{1}{2}, \frac{\nu}{2} + \frac{1}{2r} + 1, \nu + \frac{1}{2}, \nu + 1; -a^2 z^{2r}\right) - \frac{a z^r \sin(b)}{\nu r + r + 1} {}_3F_4\left(\frac{\nu}{2} + \frac{3}{4}, \frac{\nu}{2} + \frac{5}{4}, \frac{\nu}{2} + \frac{1}{2} + \frac{1}{2r}; \frac{3}{2}, \frac{\nu}{2} + \frac{3}{2} + \frac{1}{2r}, \nu + 1, \nu + \frac{3}{2}; -a^2 z^{2r}\right)\right)\right) - \frac{4^\nu a z^r \csc(\pi \nu) \sin(b)}{(r(\nu - 1) - 1) \Gamma(1 - \nu)} {}_3F_4\left(\frac{3}{4} - \frac{\nu}{2}, \frac{5}{4} - \frac{\nu}{2}, -\frac{\nu}{2} + \frac{1}{2} + \frac{1}{2r}; \frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, -\frac{\nu}{2} + \frac{3}{2} + \frac{1}{2r}; -a^2 z^{2r}\right)\right)$$

Involving trigonometric functions and a power function

Involving sin and power

Linear arguments

03.03.21.0037.01

$$\int z^{\alpha - 1} \sin(a z) Y_\nu(a z) \, dz = 2^{-\nu - 2} \sqrt{\pi} \, z^\alpha (a z)^{1 - \nu} \left((a z)^{2\nu} \cot(\pi \nu) \Gamma\left(\nu + \frac{3}{2}\right) \Gamma\left(\frac{1}{2}(\alpha + \nu + 1)\right) {}_3\tilde{F}_4\left(\frac{1}{4}(2\nu + 3), \frac{1}{4}(2\nu + 5), \frac{1}{2}(\alpha + \nu + 1); \frac{3}{2}, \frac{1}{2}(\alpha + \nu + 3), \nu + 1, \nu + \frac{3}{2}; -a^2 z^2\right) - 4^\nu \csc(\pi \nu) \Gamma\left(\frac{3}{2} - \nu\right) \Gamma\left(\frac{1}{2}(\alpha - \nu + 1)\right) {}_3\tilde{F}_4\left(\frac{1}{4}(3 - 2\nu), \frac{1}{4}(5 - 2\nu), \frac{1}{2}(\alpha - \nu + 1); \frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, \frac{1}{2}(\alpha - \nu + 3); -a^2 z^2\right)\right)$$

03.03.21.0038.01

$$\int z^{\alpha - 1} \sin(b + a z) Y_\nu(a z) \, dz = 2^{-\nu - 2} \sqrt{\pi} \, z^\alpha (a z)^{-\nu} \left((a z)^{2\nu} \cot(\pi \nu) \left(a z \cos(b) \Gamma\left(\nu + \frac{3}{2}\right) \Gamma\left(\frac{1}{2}(\alpha + \nu + 1)\right) {}_3\tilde{F}_4\left(\frac{1}{4}(2\nu + 3), \frac{1}{4}(2\nu + 5), \frac{1}{2}(\alpha + \nu + 1); \frac{3}{2}, \frac{1}{2}(\alpha + \nu + 3), \nu + 1, \nu + \frac{3}{2}; -a^2 z^2\right) + 2 \Gamma\left(\frac{\alpha + \nu}{2}\right) \Gamma\left(\nu + \frac{1}{2}\right) {}_3\tilde{F}_4\left(\frac{1}{4}(2\nu + 1), \frac{1}{4}(2\nu + 3), \frac{\alpha + \nu}{2}; \frac{1}{2}, \frac{1}{2}(\alpha + \nu + 2), \nu + \frac{1}{2}, \nu + 1; -a^2 z^2\right) \sin(b)\right) - 4^\nu \csc(\pi \nu) \left(a z \cos(b) \Gamma\left(\frac{3}{2} - \nu\right) \Gamma\left(\frac{1}{2}(\alpha - \nu + 1)\right) {}_3\tilde{F}_4\left(\frac{1}{4}(3 - 2\nu), \frac{1}{4}(5 - 2\nu), \frac{1}{2}(\alpha - \nu + 1); \frac{3}{2}, 1 - \nu, \frac{3}{2} - \nu, \frac{1}{2}(\alpha - \nu + 3); -a^2 z^2\right) + 2 \Gamma\left(\frac{\alpha - \nu}{2}\right) \Gamma\left(\frac{1}{2} - \nu\right) {}_3\tilde{F}_4\left(\frac{1}{4}(1 - 2\nu), \frac{1}{4}(3 - 2\nu), \frac{\alpha - \nu}{2}; \frac{1}{2}, \frac{1}{2} - \nu, 1 - \nu, \frac{1}{2}(\alpha - \nu + 2); -a^2 z^2\right) \sin(b)\right)\right)$$

Power arguments

03.03.21.0039.01

$$\int z^{\alpha-1}\sin(a\,z^r)\,Y_\nu(a\,z^r)\,dz = -\Big(2^{-\nu} z^\alpha (a\,z^r)^{1-\nu}\csc(\pi\nu)\Big((\nu\,r-r-\alpha)\cos(\pi\nu)\,\Gamma(1-\nu)\,{}_3F_4\Big(\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4},\frac{\alpha}{2r}+\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},\frac{\alpha}{2r}+\frac{\nu}{2}+\frac{3}{2},\nu+1,\nu+\frac{3}{2};-a^2 z^{2r}\Big)(a\,z^r)^{2\nu} + 4^\nu(\nu\,r+r+\alpha)\,\Gamma(\nu+1)\,{}_3F_4\Big(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\alpha}{2r}-\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},1-\nu,\frac{3}{2}-\nu,\frac{\alpha}{2r}-\frac{\nu}{2}+\frac{3}{2};-a^2 z^{2r}\Big)\Big)\Big)\Big/((-\nu\,r+r+\alpha)(\nu\,r+r+\alpha)\,\Gamma(1-\nu)\,\Gamma(\nu+1))$$

03.03.21.0040.01

$$\int z^{\alpha-1}\sin(a\,z^r+b)\,Y_\nu(a\,z^r)\,dz = 2^{-\nu} z^\alpha (a\,z^r)^{-\nu}\Big(\frac{(a\,z^r)^{2\nu+1}\cos(b)\cot(\pi\nu)}{(\nu\,r+r+\alpha)\,\Gamma(\nu+1)}\,{}_3F_4\Big(\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4},\frac{\alpha}{2r}+\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},\frac{\alpha}{2r}+\frac{\nu}{2}+\frac{3}{2},\nu+1,\nu+\frac{3}{2};-a^2 z^{2r}\Big) + \Big(\frac{(a\,z^r)^{2\nu}\cot(\pi\nu)}{(\alpha+r\,\nu)\,\Gamma(\nu+1)}\,{}_3F_4\Big(\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4},\frac{\alpha}{2r}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2r}+\frac{\nu}{2}+1,\nu+\frac{1}{2},\nu+1;-a^2 z^{2r}\Big) - \frac{4^\nu\csc(\pi\nu)}{(\alpha-r\,\nu)\,\Gamma(1-\nu)}\,{}_3F_4\Big(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\alpha}{2r}-\frac{\nu}{2};\frac{1}{2},\frac{1}{2}-\nu,1-\nu,\frac{\alpha}{2r}-\frac{\nu}{2}+1;-a^2 z^{2r}\Big)\Big)\sin(b) - \frac{4^\nu a\,z^r\cos(b)\csc(\pi\nu)}{(-\nu\,r+r+\alpha)\,\Gamma(1-\nu)}\,{}_3F_4\Big(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\alpha}{2r}-\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},1-\nu,\frac{3}{2}-\nu,\frac{\alpha}{2r}-\frac{\nu}{2}+\frac{3}{2};-a^2 z^{2r}\Big)\Big)$$

Involving cos and power

Linear arguments

03.03.21.0041.01

$$\int z^{\alpha-1}\cos(a\,z)\,Y_\nu(a\,z)\,dz = \frac{2^{-\nu} z^\alpha (a\,z)^\nu\cot(\pi\nu)}{(\alpha+\nu)\,\Gamma(\nu+1)}\,{}_3F_4\Big(\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4},\frac{\alpha}{2}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2}+\frac{\nu}{2}+1,\nu+\frac{1}{2},\nu+1;-a^2 z^2\Big) + \frac{2^\nu z^\alpha (a\,z)^{-\nu}\csc(\pi\nu)}{(\nu-\alpha)\,\Gamma(1-\nu)}\,{}_3F_4\Big(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\alpha}{2}-\frac{\nu}{2};\frac{1}{2},\frac{1}{2}-\nu,1-\nu,\frac{\alpha}{2}-\frac{\nu}{2}+1;-a^2 z^2\Big)$$

03.03.21.0042.01

$$\int z^{\alpha-1}\cos(b+a\,z)\,Y_\nu(a\,z)\,dz = 2^{-\nu-2}\sqrt{\pi}\;z^\alpha\,(a\,z)^{-\nu}\left((a\,z)^{2\nu}\cot(\pi\,\nu)\left(2\cos(b)\,\Gamma\left(\nu+\frac{1}{2}\right)\Gamma\left(\frac{\alpha+\nu}{2}\right){}_3\tilde{F}_4\left(\frac{1}{4}(2\,\nu+1),\,\frac{1}{4}(2\,\nu+3),\,\frac{\alpha+\nu}{2};\,\frac{1}{2},\,\frac{1}{2}(\alpha+\nu+2),\,\nu+\frac{1}{2},\,\nu+1;\,-a^2\,z^2\right)-a\,z\,\Gamma\left(\nu+\frac{3}{2}\right)\Gamma\left(\frac{1}{2}(\alpha+\nu+1)\right){}_3\tilde{F}_4\left(\frac{1}{4}(2\,\nu+3),\,\frac{1}{4}(2\,\nu+5),\,\frac{1}{2}(\alpha+\nu+1);\,\frac{3}{2},\,\frac{1}{2}(\alpha+\nu+3),\,\nu+1,\,\nu+\frac{3}{2};\,-a^2\,z^2\right)\sin(b)\right)-4^\nu\csc(\pi\,\nu)\left(2\cos(b)\,\Gamma\left(\frac{1}{2}-\nu\right)\Gamma\left(\frac{\alpha-\nu}{2}\right){}_3\tilde{F}_4\left(\frac{1}{4}(1-2\,\nu),\,\frac{1}{4}(3-2\,\nu),\,\frac{\alpha-\nu}{2};\,\frac{1}{2},\,\frac{1}{2}-\nu,\,1-\nu,\,\frac{1}{2}(\alpha-\nu+2);\,-a^2\,z^2\right)-a\,z\,\Gamma\left(\frac{3}{2}-\nu\right)\Gamma\left(\frac{1}{2}(\alpha-\nu+1)\right){}_3\tilde{F}_4\left(\frac{1}{4}(3-2\,\nu),\,\frac{1}{4}(5-2\,\nu),\,\frac{1}{2}(\alpha-\nu+1);\,\frac{3}{2},\,1-\nu,\,\frac{3}{2}-\nu,\,\frac{1}{2}(\alpha-\nu+3);\,-a^2\,z^2\right)\sin(b)\right)\right)$$

Power arguments

03.03.21.0043.01

$$\int z^{\alpha-1}\cos(a\,z^r)\,Y_\nu(a\,z^r)\,dz = \left(2^{-\nu}\,z^\alpha\,(a\,z^r)^{-\nu}\csc(\pi\,\nu)\left((r\,\nu-\alpha)\cos(\pi\,\nu)\,\Gamma(1-\nu)\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{4},\,\frac{\nu}{2}+\frac{3}{4},\,\frac{\alpha}{2\,r}+\frac{\nu}{2};\,\frac{1}{2},\,\frac{\alpha}{2\,r}+\frac{\nu}{2}+1,\,\nu+\frac{1}{2},\,\nu+1;\,-a^2\,z^{2r}\right)(a\,z^r)^{2\nu}+4^\nu(\alpha+r\,\nu)\,\Gamma(\nu+1)\,{}_3F_4\left(\frac{1}{4}-\frac{\nu}{2},\,\frac{3}{4}-\frac{\nu}{2},\,\frac{\alpha}{2\,r}-\frac{\nu}{2};\,\frac{1}{2},\,\frac{1}{2}-\nu,\,1-\nu,\,\frac{\alpha}{2\,r}-\frac{\nu}{2}+1;\,-a^2\,z^{2r}\right)\right)\right)\Big/((r\,\nu-\alpha)(\alpha+r\,\nu)\,\Gamma(1-\nu)\,\Gamma(\nu+1))$$

03.03.21.0044.01

$$\int z^{\alpha-1}\cos(a\,z^r+b)\,Y_\nu(a\,z^r)\,dz = 2^{-\nu}\,z^\alpha\,(a\,z^r)^{-\nu}\left(a\left(\frac{4^\nu\csc(\pi\,\nu)}{(-\nu\,r+r+\alpha)\,\Gamma(1-\nu)}\,{}_3F_4\left(\frac{3}{4}-\frac{\nu}{2},\,\frac{5}{4}-\frac{\nu}{2},\,\frac{\alpha}{2\,r}-\frac{\nu}{2}+\frac{1}{2};\,\frac{3}{2},\,1-\nu,\,\frac{3}{2}-\nu,\,\frac{\alpha}{2\,r}-\frac{\nu}{2}+\frac{3}{2};\,-a^2\,z^{2r}\right)-\frac{(a\,z^r)^{2\nu}\cot(\pi\,\nu)}{(\nu\,r+r+\alpha)\,\Gamma(\nu+1)}\,{}_3F_4\left(\frac{\nu}{2}+\frac{3}{4},\,\frac{\nu}{2}+\frac{5}{4},\,\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{1}{2};\,\frac{3}{2},\,\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{3}{2},\,\nu+1,\,\nu+\frac{3}{2};\,-a^2\,z^{2r}\right)\right)\sin(b)\,z^r+\frac{(a\,z^r)^{2\nu}\cos(b)\cot(\pi\,\nu)}{(\alpha+r\,\nu)\,\Gamma(\nu+1)}\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{4},\,\frac{\nu}{2}+\frac{3}{4},\,\frac{\alpha}{2\,r}+\frac{\nu}{2};\,\frac{1}{2},\,\frac{\alpha}{2\,r}+\frac{\nu}{2}+1,\,\nu+\frac{1}{2},\,\nu+1;\,-a^2\,z^{2r}\right)-\frac{4^\nu\cos(b)\csc(\pi\,\nu)}{(\alpha-r\,\nu)\,\Gamma(1-\nu)}\,{}_3F_4\left(\frac{1}{4}-\frac{\nu}{2},\,\frac{3}{4}-\frac{\nu}{2},\,\frac{\alpha}{2\,r}-\frac{\nu}{2};\,\frac{1}{2},\,\frac{1}{2}-\nu,\,1-\nu,\,\frac{\alpha}{2\,r}-\frac{\nu}{2}+1;\,-a^2\,z^{2r}\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

Linear arguments

03.03.21.0045.01

$$\int Y_\nu(a\,z)^2\,dz = -\frac{1}{\left(4\,\nu^2-1\right)\Gamma(1-\nu)^2\,\Gamma(\nu+1)^2}\left(4^{-\nu}\,z\,(a\,z)^{-2\,\nu}\csc(\pi\,\nu)\left(2^{2\,\nu+1}\left(4\,\nu^2-1\right)\cot(\pi\,\nu)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\,{}_2F_3\!\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},1-\nu,\nu+1;-a^2\,z^2\right)(a\,z)^{2\,\nu} - (2\,\nu-1)\cos(\pi\,\nu)\cot(\pi\,\nu)\,\Gamma(1-\nu)^2\,{}_2F_3\!\left(\nu+\frac{1}{2},\nu+\frac{1}{2};\nu+1,\nu+\frac{3}{2},2\,\nu+1;-a^2\,z^2\right)(a\,z)^{4\,\nu} + \left(2^{4\,\nu+1}\,\nu+16^\nu\right)\csc(\pi\,\nu)\,\Gamma(\nu+1)^2\,{}_2F_3\!\left(\frac{1}{2}-\nu,\frac{1}{2}-\nu;1-2\,\nu,1-\nu,\frac{3}{2}-\nu;-a^2\,z^2\right)\right)\right)$$

03.03.21.0046.01

$$\int Y_\nu(z)^2\,dz = -\frac{1}{\left(4\,\nu^2-1\right)\Gamma(1-\nu)^2\,\Gamma(\nu+1)^2}\left(4^{-\nu}\,z^{1-2\,\nu}\csc(\pi\,\nu)\left(2^{2\,\nu+1}\left(4\,\nu^2-1\right)\cot(\pi\,\nu)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\,z^{2\,\nu}\,{}_2F_3\!\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},1-\nu,\nu+1;-z^2\right) - (2\,\nu-1)\cos(\pi\,\nu)\cot(\pi\,\nu)\,\Gamma(1-\nu)^2\,z^{4\,\nu}\,{}_2F_3\!\left(\nu+\frac{1}{2},\nu+\frac{1}{2};\nu+1,\nu+\frac{3}{2},2\,\nu+1;-z^2\right) + \left(2^{4\,\nu+1}\,\nu+16^\nu\right)\csc(\pi\,\nu)\,\Gamma(\nu+1)^2\,{}_2F_3\!\left(\frac{1}{2}-\nu,\frac{1}{2}-\nu;1-2\,\nu,1-\nu,\frac{3}{2}-\nu;-z^2\right)\right)\right)$$

Power arguments

03.03.21.0047.01

$$\int Y_\nu(a\,z^r)^2\,dz = -\left(4^{-\nu}\,z\,(a\,z^r)^{-2\,\nu}\csc(\pi\,\nu)\left(2^{2\,\nu+1}\left(4\,r^2\,\nu^2-1\right)\cot(\pi\,\nu)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\,{}_2F_3\!\left(\frac{1}{2},\frac{1}{2\,r};1+\frac{1}{2\,r},1-\nu,\nu+1;-a^2\,z^{2\,r}\right)(a\,z^r)^{2\,\nu} - (2\,r\,\nu-1)\cos(\pi\,\nu)\cot(\pi\,\nu)\,\Gamma(1-\nu)^2\,{}_2F_3\!\left(\nu+\frac{1}{2},\nu+\frac{1}{2\,r};\nu+1,\nu+\frac{1}{2\,r}+1,2\,\nu+1;-a^2\,z^{2\,r}\right)(a\,z^r)^{4\,\nu} + \left(2^{4\,\nu+1}\,r\,\nu+16^\nu\right)\csc(\pi\,\nu)\,\Gamma(\nu+1)^2\,{}_2F_3\!\left(\frac{1}{2}-\nu,\frac{1}{2\,r}-\nu;1-2\,\nu,1-\nu,-\nu+\frac{1}{2\,r}+1;-a^2\,z^{2\,r}\right)\right)\right)\Big/\left(\left(4\,r^2\,\nu^2-1\right)\Gamma(1-\nu)^2\,\Gamma(\nu+1)^2\right)$$

Involving products of the direct function

Linear arguments

03.03.21.0048.01

$$\int Y_\mu(a z)\, Y_\nu(a z)\, dz = 2^{-\mu-\nu} z\, (a z)^{-\mu-\nu} \csc(\pi \mu) \left(\frac{1}{\Gamma(\nu+1)} \left((a z)^{2\nu} \cot(\pi \nu) \left(\frac{(a z)^{2\mu} \cos(\pi \mu)}{(\mu+\nu+1)\, \Gamma(\mu+1)}\, {}_3F_4\!\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1; \mu+1, \frac{\mu}{2}+\frac{\nu}{2}+\frac{3}{2}, \nu+1, \mu+\nu+1; -a^2 z^2 \right) + \frac{4^\mu}{(\mu-\nu-1)\, \Gamma(1-\mu)}\, {}_3F_4\!\left(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}+\frac{\nu}{2}+1; 1-\mu, -\frac{\mu}{2}+\frac{\nu}{2}+\frac{3}{2}, \nu+1, -\mu+\nu+1; -a^2 z^2 \right) \right) \right) + \left(4^\nu (a z)^{2\mu} \cos(\pi \mu) \csc(\pi \nu)\, {}_3F_4\!\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1; \mu+1, 1-\nu, \mu-\nu+1, \frac{\mu}{2}-\frac{\nu}{2}+\frac{3}{2}; -a^2 z^2 \right) \right) \Big/ ((-\mu+\nu-1)\, \Gamma(\mu+1)\, \Gamma(1-\nu)) - \frac{4^{\mu+\nu} \csc(\pi \nu)}{(\mu+\nu-1)\, \Gamma(1-\mu)\, \Gamma(1-\nu)}\, {}_3F_4\!\left(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}-\frac{\nu}{2}+1; 1-\mu, 1-\nu, -\mu-\nu+1, -\frac{\mu}{2}-\frac{\nu}{2}+\frac{3}{2}; -a^2 z^2 \right) \right)$$

03.03.21.0049.01

$$\int Y_\nu(a z)\, Y_{\nu+1}(a z)\, dz = -\Bigg(4^{-\nu-1} (a z)^{-2\nu} \csc(\pi \nu) \csc(\pi (\nu+1)) \Bigg(\cos(\pi \nu)\, \Gamma(-\nu) \left(\pi (a z)^{2(\nu+1)} \nu \cos(\pi \nu)\, \Gamma(1-\nu)\, {}_2F_3\!\left(\nu+1, \nu+\frac{3}{2}; \nu+2, \nu+2, 2\nu+2; -a^2 z^2 \right) - \Gamma(\nu+1) \left(4^\nu a^2\, {}_3F_4\!\left(1, 1, \frac{3}{2}; 2, 2, 1-\nu, \nu+2; -a^2 z^2 \right) (\pi \nu (\nu+1) + \Gamma(1-\nu)\, \Gamma(\nu+2) \sin(\pi \nu))\, z^2 + 2^{2\nu+3} \nu (\nu+1)\, \Gamma(1-\nu)\, \Gamma(\nu+2) \log(a z) \sin(\pi \nu) \right) \right) (a z)^{2\nu} + 4^{2\nu+1} \pi (\nu+1)\, \Gamma(\nu+1)\, \Gamma(\nu+2)\, {}_2F_3\!\left(\frac{1}{2}-\nu, -\nu; 1-\nu, 1-\nu, -2\nu; -a^2 z^2 \right) \Bigg) \Bigg) \Big/ (a \pi \nu (\nu+1)\, \Gamma(1-\nu)\, \Gamma(-\nu)\, \Gamma(\nu+1)\, \Gamma(\nu+2))$$

03.03.21.0050.01

$$\int Y_0(a z)\, Y_1(a z)\, dz = -\frac{Y_0(a z)^2}{2 a}$$

Power arguments

03.03.21.0051.01

$$\int Y_\mu(a z^r)\, Y_\nu(a z^r)\, dz = 2^{-\mu-\nu} z\, (a z^r)^{-\mu-\nu} \csc(\pi\mu) \left(\frac{1}{\Gamma(\nu+1)} \left((a z^r)^{2\nu} \cot(\pi\nu) \left(\frac{1}{(r(\mu+\nu)+1)\,\Gamma(\mu+1)} \left((a z^r)^{2\mu} \cos(\pi\mu)\, {}_3F_4\!\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1, \frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2r}; \mu+1, \frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2r}+1, \nu+1, \mu+\nu+1; -a^2 z^{2r}\right)\right) + \frac{1}{(r\mu-r\nu-1)\,\Gamma(1-\mu)} \left(4^\mu\, {}_3F_4\!\left(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}+\frac{\nu}{2}+1, -\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2r}; 1-\mu, -\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2r}+1, \nu+1, -\mu+\nu+1; -a^2 z^{2r}\right)\right)\right)\right) + \left(4^\nu (a z^r)^{2\mu} \cos(\pi\mu) \csc(\pi\nu)\, {}_3F_4\!\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1, \frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2r}; \mu+1, 1-\nu, \mu-\nu+1, \frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2r}+1; -a^2 z^{2r}\right)\right) \Big/ \left((-r\mu+r\nu-1)\,\Gamma(\mu+1)\,\Gamma(1-\nu)\right) - \left(4^{\mu+\nu} \csc(\pi\nu)\, {}_3F_4\!\left(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}-\frac{\nu}{2}+1, -\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2r}; 1-\mu, 1-\nu, -\mu-\nu+1, -\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2r}+1; -a^2 z^{2r}\right)\right) \Big/ \left((r(\mu+\nu)-1)\,\Gamma(1-\mu)\,\Gamma(1-\nu)\right)\right)$$

03.03.21.0052.01

$$\int Y_\nu\!\left(a\sqrt{z}\right) Y_\nu\!\left(b\sqrt{z}\right) dz = \frac{2\sqrt{z}}{a^2-b^2}\left(b\, Y_{\nu-1}\!\left(b\sqrt{z}\right) Y_\nu\!\left(a\sqrt{z}\right) - a\, Y_{\nu-1}\!\left(a\sqrt{z}\right) Y_\nu\!\left(b\sqrt{z}\right)\right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

Linear arguments

03.03.21.0053.01

$$\int z^{\alpha-1}\, Y_\nu(a z)^2\, dz = \left(4^{-\nu} z^\alpha (a z)^{-2\nu} \csc(\pi\nu) \left(2^{2\nu+1}\left(4\nu^2-\alpha^2\right) \cot(\pi\nu)\, \Gamma(1-\nu)\, \Gamma(\nu+1)\, {}_2F_3\!\left(\frac{1}{2}, \frac{\alpha}{2}; \frac{\alpha}{2}+1, 1-\nu, \nu+1; -a^2 z^2\right) (a z)^{2\nu} + \alpha\left(\left(16^\nu \alpha + 2^{4\nu+1}\nu\right) \csc(\pi\nu)\, \Gamma(\nu+1)^2\, {}_2F_3\!\left(\frac{1}{2}-\nu, \frac{\alpha}{2}-\nu; 1-2\nu, 1-\nu, \frac{\alpha}{2}-\nu+1; -a^2 z^2\right) - (a z)^{4\nu} (2\nu-\alpha) \cos(\pi\nu) \cot(\pi\nu)\, \Gamma(1-\nu)^2\, {}_2F_3\!\left(\nu+\frac{1}{2}, \frac{\alpha}{2}+\nu; \nu+1, \frac{\alpha}{2}+\nu+1, 2\nu+1; -a^2 z^2\right)\right)\right)\right) \Big/ \left(\left(\alpha^3-4\alpha\nu^2\right) \Gamma(1-\nu)^2\, \Gamma(\nu+1)^2\right)$$

03.03.21.0054.01

$$\int z^{1-2\nu} Y_\nu(a z)^2 \, dz = \frac{1}{a^2 (2\nu - 1) \Gamma(\nu)^2} \Big(2^{-2\nu-1} z^{-2\nu} \Big(4 \cot^2(\pi \nu) (a z)^{2\nu} - 4^\nu a^2 z^2 J_{\nu-1}(a z)^2 \cot^2(\pi \nu) \Gamma(\nu)^2 - 4^\nu a^2 z^2 J_\nu(a z)^2 \cot^2(\pi \nu) \Gamma(\nu)^2 - 4^\nu a^2 z^2 J_{1-\nu}(a z)^2 \csc^2(\pi \nu) \Gamma(\nu)^2 - 4^\nu a^2 z^2 J_{-\nu}(a z)^2 \csc^2(\pi \nu) \Gamma(\nu)^2 - 4^\nu a^2 z^2 J_{\nu-2}(a z) J_{-\nu}(a z) \cot(\pi \nu) \csc(\pi \nu) \Gamma(\nu)^2 - 2^{2\nu+1} a z J_{\nu-1}(a z) (a z J_{1-\nu}(a z) - (\nu - 1) J_{-\nu}(a z)) \cot(\pi \nu) \csc(\pi \nu) \Gamma(\nu)^2 - a z \big(2^{2\nu+1} (\nu - 1) J_{1-\nu}(a z) + 4^\nu a z J_{2-\nu}(a z)\big) J_\nu(a z) \cot(\pi \nu) \csc(\pi \nu) \Gamma(\nu)^2\Big)\Big)$$

03.03.21.0055.01

$$\int z^{2\nu+1} Y_\nu(a z)^2 \, dz = \frac{1}{2 a^2 (2\nu + 1) \Gamma(-\nu)^2} \Big(z^{2\nu} (a z)^{-2\nu} \Big(-2 a z (a z J_{-\nu}(a z) - \nu J_{-\nu-1}(a z)) J_\nu(a z) \cot(\pi \nu) \csc(\pi \nu) \Gamma(-\nu)^2 (a z)^{2\nu} - a z (a z J_{1-\nu}(a z) + 2\nu J_{-\nu}(a z)) J_{\nu+1}(a z) \cot(\pi \nu) \csc(\pi \nu) \Gamma(-\nu)^2 (a z)^{2\nu} + J_\nu(a z)^2 \cot^2(\pi \nu) \Gamma(-\nu)^2 (a z)^{2(\nu+1)} + J_{\nu+1}(a z)^2 \cot^2(\pi \nu) \Gamma(-\nu)^2 (a z)^{2(\nu+1)} - \csc^2(\pi \nu) \big(-J_{-\nu-1}(a z)^2 \Gamma(-\nu)^2 (a z)^{2(\nu+1)} - J_{-\nu}(a z)^2 \Gamma(-\nu)^2 (a z)^{2(\nu+1)} + J_{-\nu-1}(a z) J_{\nu-1}(a z) \cos(\pi \nu) \Gamma(-\nu)^2 (a z)^{2(\nu+1)} + 4^{\nu+1}\big)\Big)\Big)$$

03.03.21.0056.01

$$\int z Y_\nu(a z)^2 \, dz = \frac{1}{2} z^2 \left(Y_\nu(a z)^2 - Y_{\nu-1}(a z) Y_{\nu+1}(a z)\right)$$

03.03.21.0057.01

$$\int z Y_0(a z)^2 \, dz = \frac{1}{2} z^2 \left(Y_0(a z)^2 + Y_1(a z)^2\right)$$

03.03.21.0058.01

$$\int \frac{1}{z Y_\nu(a z)^2} \, dz = -\frac{\pi J_\nu(a z)}{2 Y_\nu(a z)}$$

03.03.21.0059.01

$$\int \frac{Y_\nu(a z)^2}{z} \, dz = \frac{1}{4} \left(\cot(\pi \nu) \left(\frac{2^{1-2\nu} \cot(\pi \nu) \, (a z)^{2\nu}}{\nu \Gamma(\nu + 1)^2} \, {}_2F_3\!\left(\nu, \nu + \frac{1}{2}; \nu + 1, \nu + 1, 2\nu + 1; -a^2 z^2\right) + \csc(\pi \nu) \left(a^2 \, {}_3\tilde{F}_4\!\left(1, 1, \frac{3}{2}; 2 - \nu, \nu + 2, 2, 2; -a^2 z^2\right) z^2 + a^2 \, {}_3\tilde{F}_4\!\left(1, 1, \frac{3}{2}; \nu + 2, 2 - \nu, 2, 2; -a^2 z^2\right) z^2 - \frac{8 \log(z)}{\Gamma(1 - \nu) \Gamma(\nu + 1)}\right)\right) - \frac{2^{2\nu+1} (a z)^{-2\nu} \csc^2(\pi \nu)}{\nu \Gamma(1 - \nu)^2} \, {}_2F_3\!\left(\frac{1}{2} - \nu, -\nu; 1 - 2\nu, 1 - \nu, 1 - \nu; -a^2 z^2\right)\right)$$

03.03.21.0060.01

$$\int \frac{Y_\nu(a z)^2}{z^2}\, dz = \frac{1}{z\left(4\nu^2-1\right)}\Big(2a^2 z^2 J_{\nu-1}(a z)^2 \cot^2(\pi\nu) + 2 a z J_{\nu-1}(a z)\,(2 a z J_{1-\nu}(a z) + J_{-\nu}(a z) - J_\nu(a z)\cos(\pi\nu))\csc(\pi\nu)\cot(\pi\nu) + \frac{1}{2}\csc(\pi\nu)\big(4a^2 J_{\nu-2}(a z)\,(J_{-\nu}(a z) - J_\nu(a z)\cos(\pi\nu))\cot(\pi\nu) z^2 + \big(4a^2 J_{-\nu-1}(a z)^2 z^2 - 4a^2 J_{-\nu-2}(a z) J_{-\nu}(a z) z^2 + 4a^2 J_{2-\nu}(a z) J_\nu(a z)\cos(\pi\nu) z^2 - 4a J_{-\nu-1}(a z) J_{-\nu}(a z) z - 4 a J_{1-\nu}(a z) J_\nu(a z)\cos(\pi\nu) z + 4\nu J_{-\nu}(a z)^2 + 2 J_{-\nu}(a z)^2 - 2\nu J_\nu(a z)^2 + J_\nu(a z)^2 + 8\nu J_{-\nu}(a z) J_\nu(a z)\cos(\pi\nu) - 4 J_{-\nu}(a z) J_\nu(a z)\cos(\pi\nu) - 2\nu J_\nu(a z)^2\cos(2\pi\nu) + J_\nu(a z)^2\cos(2\pi\nu)\big)\csc(\pi\nu)\big)\Big)$$

Power arguments

03.03.21.0061.01

$$\int z^{\alpha-1} Y_\nu(a z^r)^2\, dz = \Big(4^{-\nu} z^\alpha (a z^r)^{-2\nu}\csc(\pi\nu)\Big(2^{2\nu+1}\left(4r^2\nu^2-\alpha^2\right)\cot(\pi\nu)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\,{}_2F_3\Big(\frac{1}{2}, \frac{\alpha}{2r}; \frac{\alpha}{2r}+1, 1-\nu, \nu+1; -a^2 z^{2r}\Big)(a z^r)^{2\nu} + \alpha\Big(\left(16^\nu\alpha + 2^{4\nu+1} r\nu\right)\csc(\pi\nu)\,\Gamma(\nu+1)^2\,{}_2F_3\Big(\frac{1}{2}-\nu, \frac{\alpha}{2r}-\nu; 1-2\nu, 1-\nu, \frac{\alpha}{2r}-\nu+1; -a^2 z^{2r}\Big) - (a z^r)^{4\nu}(2r\nu-\alpha)\cos(\pi\nu)\cot(\pi\nu)\,\Gamma(1-\nu)^2\,{}_2F_3\Big(\nu+\frac{1}{2}, \frac{\alpha}{2r}+\nu; \nu+1, \frac{\alpha}{2r}+\nu+1, 2\nu+1; -a^2 z^{2r}\Big)\Big)\Big)\Big)\Big/\left(\left(\alpha^3-4r^2\alpha\nu^2\right)\Gamma(1-\nu)^2\,\Gamma(\nu+1)^2\right)$$

Involving products of the direct function and a power function

Linear arguments

03.03.21.0062.01

$$\int z^{\alpha-1} Y_\mu(a z)\, Y_\nu(a z)\, dz = 2^{-\mu-\nu} z^\alpha (a z)^{-\mu-\nu}\Bigg(\frac{1}{\Gamma(\nu+1)}\Bigg((a z)^{2\nu}\cot(\pi\nu)\Bigg(\frac{1}{(\alpha+\mu+\nu)\,\Gamma(\mu+1)}\Big((a z)^{2\mu}\cot(\pi\mu)\,{}_3F_4\Big(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1, \frac{\alpha}{2}+\frac{\mu}{2}+\frac{\nu}{2}; \mu+1, \frac{\alpha}{2}+\frac{\mu}{2}+\frac{\nu}{2}+1, \nu+1, \mu+\nu+1; -a^2 z^2\Big)\Big) - \frac{1}{(\alpha-\mu+\nu)\,\Gamma(1-\mu)}\Big(4^\mu\csc(\pi\mu)\,{}_3F_4\Big(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}+\frac{\nu}{2}+1, \frac{\alpha}{2}+\frac{\nu}{2}-\frac{\mu}{2}; 1-\mu, \frac{\alpha}{2}+\frac{\nu}{2}-\frac{\mu}{2}+1, \nu+1, -\mu+\nu+1; -a^2 z^2\Big)\Big)\Bigg)\Bigg) - \Big(4^{\mu+\nu}\csc(\pi\mu)\csc(\pi\nu)\,{}_3F_4\Big(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}-\frac{\nu}{2}+1, \frac{\alpha}{2}-\frac{\mu}{2}-\frac{\nu}{2}; 1-\mu, 1-\nu, -\mu-\nu+1, \frac{\alpha}{2}-\frac{\mu}{2}-\frac{\nu}{2}+1; -a^2 z^2\Big)\Big)\Big/ ((-\alpha+\mu+\nu)\,\Gamma(1-\mu)\,\Gamma(1-\nu)) - \Big(4^\nu (a z)^{2\mu}\cot(\pi\mu)\csc(\pi\nu)\,{}_3F_4\Big(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1, \frac{\alpha}{2}+\frac{\mu}{2}-\frac{\nu}{2}; \mu+1, 1-\nu, \mu-\nu+1, \frac{\alpha}{2}+\frac{\mu}{2}-\frac{\nu}{2}+1; -a^2 z^2\Big)\Big)\Big/ ((\alpha+\mu-\nu)\,\Gamma(\mu+1)\,\Gamma(1-\nu))\Bigg)$$

03.03.21.0063.01

$$\int z^{1-\mu-\nu}\, Y_\mu(a\,z)\, Y_\nu(a\,z)\, dz = \Big(2^{-\mu-\nu-1} z^{-\mu-\nu} (a\,z)^{-\mu-\nu} \Big(-4^{\mu+\nu} a^2 \csc(\pi\,\mu)\csc(\pi\,\nu)\,\Gamma(2-\mu)\,\Gamma(\mu)\,\Gamma(\mu+1)\,\Gamma(2-\nu)\,\Gamma(\nu)\,\Gamma(\nu+1)\,{}_2F_3\Big(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}-\frac{\nu}{2}+1; 1-\mu, 1-\nu, -\mu-\nu+2; -a^2 z^2\Big) z^2 - \Gamma(1-\mu)\,\Gamma(1-\nu)\Big(4^\nu (\mu+\nu-1)\cot(\pi\,\mu)\csc(\pi\,\nu)\,\Gamma(2-\mu)\,\Gamma(\mu)\,\Gamma(\nu)\,\Gamma(\nu+1)\,{}_2F_3\Big(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1; \mu+1, 2-\nu, \mu-\nu+1; -a^2 z^2\Big)(a\,z)^{2(\mu+1)} + \cot(\pi\,\nu)\csc(\pi\,\mu)\,\Gamma(\mu+1)\,\Gamma(2-\nu)\Big(4\cos(\pi\,\mu)\,\Gamma(2-\mu)\,\Gamma(\nu+1)\Big({}_2F_3\Big(\frac{\mu}{2}+\frac{\nu}{2}-\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}; \mu, \nu, \mu+\nu; -a^2 z^2\Big)-1\Big)(a\,z)^{2\mu} + 4^\mu a^2 z^2 (\mu+\nu-1)\,\Gamma(\mu)\,\Gamma(\nu)\,{}_2F_3\Big(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}+\frac{\nu}{2}+1; 2-\mu, \nu+1, -\mu+\nu+1; -a^2 z^2\Big)\Big)(a\,z)^{2\nu}\Big)\Big)\Big)\Big/ \left(a^2 (\mu+\nu-1)\,\Gamma(1-\mu)\,\Gamma(2-\mu)\,\Gamma(\mu)\,\Gamma(\mu+1)\,\Gamma(1-\nu)\,\Gamma(2-\nu)\,\Gamma(\nu)\,\Gamma(\nu+1)\right)$$

03.03.21.0064.01

$$\int z^{\mu+\nu+1}\, Y_\mu(a\,z)\, Y_\nu(a\,z)\, dz = \Big(2^{-\mu-\nu-1} z^{\mu+\nu} (a\,z)^{-\mu-\nu}\Big(-4^\nu (\mu+\nu+1)\cot(\pi\,\mu)\csc(\pi\,\nu)\,\Gamma(1-\mu)\,\Gamma(-\mu)\,\Gamma(\mu+1)\,\Gamma(-\nu)\,\Gamma(\nu+1)\,\Gamma(\nu+2)\,{}_2F_3\Big(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1; \mu+2, 1-\nu, \mu-\nu+1; -a^2 z^2\Big)(a\,z)^{2(\mu+1)} - \Gamma(\mu+2)\,\Gamma(1-\nu)\Big(4^\mu (\mu+\nu+1)\cot(\pi\,\nu)\csc(\pi\,\mu)\,\Gamma(-\mu)\,\Gamma(\mu+1)\,\Gamma(-\nu)\,\Gamma(\nu+1)\,{}_2F_3\Big(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}+\frac{\nu}{2}+1; 1-\mu, \nu+2, -\mu+\nu+1; -a^2 z^2\Big)(a\,z)^{2(\nu+1)} + \Gamma(1-\mu)\,\Gamma(\nu+2)\Big(4^{\mu+\nu+1}\csc(\pi\,\mu)\csc(\pi\,\nu)\,\Gamma(\mu+1)\,\Gamma(\nu+1) - (a\,z)^{2(\mu+\nu+1)}\cot(\pi\,\mu)\cot(\pi\,\nu)\,\Gamma(-\mu)\,\Gamma(-\nu)\,{}_2F_3\Big(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1; \mu+1, \nu+1, \mu+\nu+2; -a^2 z^2\Big)\Big)\Big) + 4^{\mu+\nu+1}\csc(\pi\,\mu)\csc(\pi\,\nu)\,\Gamma(1-\mu)\,\Gamma(\mu+1)\,\Gamma(\mu+2)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\,\Gamma(\nu+2)\,{}_2F_3\Big(-\frac{\mu}{2}-\frac{\nu}{2}-\frac{1}{2}, -\frac{\mu}{2}-\frac{\nu}{2}; -\mu, -\mu-\nu, -\nu; -a^2 z^2\Big)\Big)\Big)\Big/ \left(a^2 (\mu+\nu+1)\,\Gamma(1-\mu)\,\Gamma(-\mu)\,\Gamma(\mu+1)\,\Gamma(\mu+2)\,\Gamma(1-\nu)\,\Gamma(-\nu)\,\Gamma(\nu+1)\,\Gamma(\nu+2)\right)$$

03.03.21.0065.01

$$\int z\, Y_\nu(a\,z)\, Y_\nu(b\,z)\, dz = \frac{z}{a^2-b^2}\left(b\, Y_{\nu-1}(b\,z)\, Y_\nu(a\,z) - a\, Y_{\nu-1}(a\,z)\, Y_\nu(b\,z)\right)$$

03.03.21.0066.01

$$\int \frac{Y_\mu(a\,z)\, Y_\nu(a\,z)}{z}\, dz = \frac{1}{\mu^2-\nu^2}\left(a\,z\, Y_{\mu-1}(a\,z)\, Y_\nu(a\,z) - Y_\mu(a\,z)\left(a\,z\, Y_{\nu-1}(a\,z) + (\mu-\nu)\, Y_\nu(a\,z)\right)\right)$$

03.03.21.0067.01

$$\int \frac{\left(\left(a^2-b^2\right)z^2-\mu^2+\nu^2\right) Y_\mu(a\,z)\, Y_\nu(b\,z)}{z}\, dz = Y_\mu(a\,z)\left(b\,z\, Y_{\nu-1}(b\,z) + (\mu-\nu)\, Y_\nu(b\,z)\right) - a\,z\, Y_{\mu-1}(a\,z)\, Y_\nu(b\,z)$$

03.03.21.0068.01

$$\int \frac{Y_\mu(a z)\, Y_\nu(a z)}{z^2}\, dz = 2^{-\mu-\nu}\, a\, (a z)^{-\mu-\nu-1} \left(\frac{1}{\Gamma(\nu+1)} \left((a z)^{2\nu} \cot(\pi \nu) \left(\frac{(a z)^{2\mu} \cot(\pi \mu)}{(\mu+\nu-1)\, \Gamma(\mu+1)}\, {}_2F_3\!\left(\frac{\mu}{2}+\frac{\nu}{2}-\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1; \mu+1, \nu+1, \mu+\nu+1; -a^2 z^2\right) + \frac{4^\mu \csc(\pi \mu)}{(\mu-\nu+1)\, \Gamma(1-\mu)}\, {}_2F_3\!\left(-\frac{\mu}{2}+\frac{\nu}{2}-\frac{1}{2}, -\frac{\mu}{2}+\frac{\nu}{2}+1; 1-\mu, \nu+1, -\mu+\nu+1; -a^2 z^2\right)\right)\right) + \frac{4^\nu (a z)^{2\mu} \cot(\pi \mu) \csc(\pi \nu)}{(-\mu+\nu+1)\, \Gamma(\mu+1)\, \Gamma(1-\nu)}\, {}_2F_3\!\left(\frac{\mu}{2}-\frac{\nu}{2}-\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1; \mu+1, 1-\nu, \mu-\nu+1; -a^2 z^2\right) - \frac{4^{\mu+\nu} \csc(\pi \mu) \csc(\pi \nu)}{(\mu+\nu+1)\, \Gamma(1-\mu)\, \Gamma(1-\nu)}\, {}_2F_3\!\left(-\frac{\mu}{2}-\frac{\nu}{2}-\frac{1}{2}, -\frac{\mu}{2}-\frac{\nu}{2}+1; 1-\mu, 1-\nu, -\mu-\nu+1; -a^2 z^2\right)\right)$$

Power arguments

03.03.21.0069.01

$$\int z^{\alpha-1}\, Y_\mu(a z^r)\, Y_\nu(a z^r)\, dz = 2^{-\mu-\nu} z^\alpha (a z^r)^{-\mu-\nu} \csc(\pi \mu) \left(\frac{1}{\Gamma(\nu+1)} \left((a z^r)^{2\nu} \cot(\pi \nu) \left(\frac{1}{(\alpha + r(\mu+\nu))\, \Gamma(\mu+1)} \left((a z^r)^{2\mu} \cos(\pi \mu)\, {}_3F_4\!\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1, \frac{\alpha}{2r}+\frac{\mu}{2}+\frac{\nu}{2}; \mu+1, \frac{\alpha}{2r}+\frac{\mu}{2}+\frac{\nu}{2}+1, \nu+1, \mu+\nu+1; -a^2 z^{2r}\right)\right) - \frac{1}{(\alpha - r\mu + r\nu)\, \Gamma(1-\mu)} \left(4^\mu\, {}_3F_4\!\left(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}+\frac{\nu}{2}+1, \frac{\alpha}{2r}+\frac{\nu}{2}-\frac{\mu}{2}; 1-\mu, \frac{\alpha}{2r}+\frac{\nu}{2}-\frac{\mu}{2}+1, \nu+1, -\mu+\nu+1; -a^2 z^{2r}\right)\right)\right)\right) + \left(4^{\mu+\nu} \csc(\pi \nu)\, {}_3F_4\!\left(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, -\frac{\mu}{2}-\frac{\nu}{2}+1, \frac{\alpha}{2r}-\frac{\mu}{2}-\frac{\nu}{2}; 1-\mu, 1-\nu, -\mu-\nu+1, \frac{\alpha}{2r}-\frac{\mu}{2}-\frac{\nu}{2}+1; -a^2 z^{2r}\right)\right) \Big/ \left((\alpha - r(\mu+\nu))\, \Gamma(1-\mu)\, \Gamma(1-\nu)\right) - \left(4^\nu (a z^r)^{2\mu} \cos(\pi \mu) \csc(\pi \nu)\, {}_3F_4\!\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1, \frac{\alpha}{2r}+\frac{\mu}{2}-\frac{\nu}{2}; \mu+1, 1-\nu, \mu-\nu+1, \frac{\alpha}{2r}+\frac{\mu}{2}-\frac{\nu}{2}+1; -a^2 z^{2r}\right)\right) \Big/ \left((\alpha + r\mu - r\nu)\, \Gamma(\mu+1)\, \Gamma(1-\nu)\right)\right)$$

03.03.21.0070.01

$$\int z^{\alpha-1}\, Y_{\nu-1}(a z^r)\, Y_\nu(a z^r)\, dz = -\frac{1}{2} z^\alpha \csc^2(\pi \nu) \left(-\frac{1}{a} \left(2 z^{-r} \cos(\pi \nu) \left(\frac{2^{1-2\nu} (a z^r)^{2\nu} \cos(\pi \nu)}{(\alpha + r(2\nu-1))\, \Gamma(\nu)\, \Gamma(\nu+1)}\, {}_2F_3\!\left(\nu+\frac{1}{2}, \frac{\alpha}{2r}+\nu-\frac{1}{2}; 2\nu, \nu+1, \frac{\alpha}{2r}+\nu+\frac{1}{2}; -a^2 z^{2r}\right) + \frac{\sin(\pi \nu)}{\pi (r-\alpha)}\, {}_2F_3\!\left(\frac{1}{2}, \frac{\alpha}{2r}-\frac{1}{2}; \frac{\alpha}{2r}+\frac{1}{2}, 1-\nu, \nu; -a^2 z^{2r}\right)\right)\right) + \frac{4^\nu (a z^r)^{1-2\nu}}{(-2\nu r + r + \alpha)\, \Gamma(1-\nu)\, \Gamma(2-\nu)}\, {}_2F_3\!\left(\frac{3}{2}-\nu, \frac{\alpha}{2r}-\nu+\frac{1}{2}; 2-2\nu, 2-\nu, \frac{\alpha}{2r}-\nu+\frac{3}{2}; -a^2 z^{2r}\right) + \frac{2\cot(\pi \nu)}{(\alpha - r)(r+\alpha)\, \Gamma(2-\nu)\, \Gamma(\nu+1)} \left(a\, r\, {}_2F_3\!\left(\frac{3}{2}, \frac{\alpha}{2r}+\frac{1}{2}; \frac{\alpha}{2r}+\frac{3}{2}, 2-\nu, \nu+1; -a^2 z^{2r}\right) \sin(\pi \nu)\, z^r + \pi (r+\alpha)(\nu-1)\, \nu\, J_{1-\nu}(a z^r)\, J_\nu(a z^r)\right)\right)$$

03.03.21.0071.01

$$\int z^{\alpha-1} Y_{-\nu}(a z^r) Y_\nu(a z^r) \, dz = \frac{1}{2} z^\alpha \csc(\pi \nu) \left(2^{1-2\nu} (a z^r)^{-2\nu} \cot(\pi \nu) \left(\frac{(a z^r)^{4\nu}}{(\alpha + 2 r \nu) \Gamma(\nu+1)^2} {}_2F_3\left(\nu + \frac{1}{2}, \frac{\alpha}{2r} + \nu; \nu + 1, \frac{\alpha}{2r} + \nu + 1, 2\nu + 1; -a^2 z^{2r}\right) + \frac{16^\nu}{(\alpha - 2 r \nu) \Gamma(1-\nu)^2} {}_2F_3\left(\frac{1}{2} - \nu, \frac{\alpha}{2r} - \nu; 1 - 2\nu, 1 - \nu, \frac{\alpha}{2r} - \nu + 1; -a^2 z^{2r}\right)\right) - \frac{(\cos(2\pi\nu) + 3)}{\pi \alpha \nu} {}_2F_3\left(\frac{1}{2}, \frac{\alpha}{2r}; \frac{\alpha}{2r} + 1, 1 - \nu, \nu + 1; -a^2 z^{2r}\right)\right)$$

Involving direct function and Bessel-type functions

Involving Bessel functions

Involving Bessel J

Linear arguments

03.03.21.0072.01

$$\int J_\mu(a z) Y_\nu(a z) \, dz = \frac{1}{\Gamma(\mu+1)} \left(2^{-\mu-\nu} z (a z)^{\mu-\nu} \left(\frac{(a z)^{2\nu} \cot(\pi \nu)}{(\mu + \nu + 1) \Gamma(\nu+1)} {}_3F_4\left(\frac{\mu}{2} + \frac{\nu}{2} + \frac{1}{2}, \frac{\mu}{2} + \frac{\nu}{2} + \frac{1}{2}, \frac{\mu}{2} + \frac{\nu}{2} + 1; \mu + 1, \frac{\mu}{2} + \frac{\nu}{2} + \frac{3}{2}, \nu + 1, \mu + \nu + 1; -a^2 z^2\right) + \frac{4^\nu \csc(\pi \nu)}{(-\mu + \nu - 1) \Gamma(1-\nu)} {}_3F_4\left(\frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2}, \frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2}, \frac{\mu}{2} - \frac{\nu}{2} + 1; \mu + 1, 1 - \nu, \mu - \nu + 1, \frac{\mu}{2} - \frac{\nu}{2} + \frac{3}{2}; -a^2 z^2\right)\right)\right)$$

03.03.21.0073.01

$$\int J_\nu(a z) Y_\nu(a z) \, dz = \frac{z}{2 \Gamma(\nu+1)^2} \left(\frac{4^{-\nu} (a z)^{2\nu} \cot(\pi \nu)}{\nu + \frac{1}{2}} {}_2F_3\left(\nu + \frac{1}{2}, \nu + \frac{1}{2}; \nu + 1, \nu + \frac{3}{2}, 2\nu + 1; -a^2 z^2\right) - \frac{2 \csc(\pi \nu) \Gamma(\nu+1)}{\Gamma(1-\nu)} {}_2F_3\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, 1 - \nu, \nu + 1; -a^2 z^2\right)\right)$$

03.03.21.0074.01

$$\int J_{-\nu}(a z) Y_\nu(a z) \, dz = \frac{z}{\Gamma(1-\nu)^2} \left(\frac{4^\nu \csc(\pi \nu) (a z)^{-2\nu}}{2\nu - 1} {}_2F_3\left(\frac{1}{2} - \nu, \frac{1}{2} - \nu; 1 - 2\nu, 1 - \nu, \frac{3}{2} - \nu; -a^2 z^2\right) + \frac{\cot(\pi \nu) \Gamma(1-\nu)}{\Gamma(\nu+1)} {}_2F_3\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, 1 - \nu, \nu + 1; -a^2 z^2\right)\right)$$

Power arguments

03.03.21.0075.01

$$\int J_\mu(a z^r)\, Y_\nu(a z^r)\, dz == \frac{1}{\Gamma(\mu+1)}\left(2^{-\mu-\nu} z\, (a z^r)^{\mu-\nu}\left(\frac{1}{(r(\mu+\nu)+1)\,\Gamma(\nu+1)}\left((a z^r)^{2\nu}\cot(\pi\nu)\,{}_3F_4\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1, \frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2r}; \mu+1, \frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2r}+1, \nu+1, \mu+\nu+1; -a^2 z^{2r}\right)\right)+\frac{1}{(r(\nu-\mu)-1)\,\Gamma(1-\nu)}\left(4^\nu \csc(\pi\nu)\,{}_3F_4\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1, \frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2r}; \mu+1, 1-\nu, \mu-\nu+1, \frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2r}+1; -a^2 z^{2r}\right)\right)\right)\right)$$

03.03.21.0076.01

$$\int J_\nu(a z^r)\, Y_\nu(a z^r)\, dz == \frac{z}{\Gamma(\nu+1)^2}\left(\frac{4^{-\nu}(a z^r)^{2\nu}\cot(\pi\nu)}{2r\nu+1}\,{}_2F_3\left(\nu+\frac{1}{2}, \nu+\frac{1}{2r}; \nu+1, \nu+\frac{1}{2r}+1, 2\nu+1; -a^2 z^{2r}\right)-\frac{\csc(\pi\nu)\,\Gamma(\nu+1)}{\Gamma(1-\nu)}\,{}_2F_3\left(\frac{1}{2}, \frac{1}{2r}; 1+\frac{1}{2r}, 1-\nu, \nu+1; -a^2 z^{2r}\right)\right)$$

03.03.21.0077.01

$$\int J_{-\nu}(a z^r)\, Y_\nu(a z^r)\, dz == \frac{1}{(2r\nu-1)\,\Gamma(1-\nu)^2\,\Gamma(\nu+1)}\left(z\,(a z^r)^{-2\nu}\csc(\pi\nu)\left((2r\nu-1)\cos(\pi\nu)\,\Gamma(1-\nu)\,{}_2F_3\left(\frac{1}{2}, \frac{1}{2r}; 1+\frac{1}{2r}, 1-\nu, \nu+1; -a^2 z^{2r}\right)(a z^r)^{2\nu}+4^\nu\,\Gamma(\nu+1)\,{}_2F_3\left(\frac{1}{2}-\nu, \frac{1}{2r}-\nu; 1-2\nu, 1-\nu, -\nu+\frac{1}{2r}+1; -a^2 z^{2r}\right)\right)\right)$$

03.03.21.0078.01

$$\int J_\nu\left(a\sqrt{z}\right) Y_\nu\left(b\sqrt{z}\right) dz == \frac{2\sqrt{z}}{a^2-b^2}\left(b\, J_\nu\left(a\sqrt{z}\right) Y_{\nu-1}\left(b\sqrt{z}\right)-a\, J_{\nu-1}\left(a\sqrt{z}\right) Y_\nu\left(b\sqrt{z}\right)\right)$$

03.03.21.0079.01

$$\int J_{-\nu}\left(a\sqrt{z}\right) Y_\nu\left(b\sqrt{z}\right) dz == \frac{2\sqrt{z}}{a^2-b^2}\left(a\, J_{1-\nu}\left(a\sqrt{z}\right) Y_\nu\left(b\sqrt{z}\right)+b\, J_{-\nu}\left(a\sqrt{z}\right)\left(J_{1-\nu}\left(b\sqrt{z}\right)+J_{\nu-1}\left(b\sqrt{z}\right)\cos(\pi\nu)\right)\csc(\pi\nu)\right)$$

Involving Bessel J and power

Linear arguments

03.03.21.0080.01

$$\int z^{\alpha-1} J_\mu(a z)\, Y_\nu(a z)\, dz == \frac{1}{\Gamma(\mu+1)}\left(2^{-\mu-\nu} z^\alpha (a z)^{\mu-\nu}\left(\frac{(a z)^{2\nu}\cot(\pi\nu)}{(\alpha+\mu+\nu)\,\Gamma(\nu+1)}\,{}_3F_4\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1, \frac{\alpha}{2}+\frac{\mu}{2}+\frac{\nu}{2}; \mu+1, \frac{\alpha}{2}+\frac{\mu}{2}+\frac{\nu}{2}+1, \nu+1, \mu+\nu+1; -a^2 z^2\right)-\frac{4^\nu\csc(\pi\nu)}{(\alpha+\mu-\nu)\,\Gamma(1-\nu)}\,{}_3F_4\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1, \frac{\alpha}{2}+\frac{\mu}{2}-\frac{\nu}{2}; \mu+1, 1-\nu, \mu-\nu+1, \frac{\alpha}{2}+\frac{\mu}{2}-\frac{\nu}{2}+1; -a^2 z^2\right)\right)\right)$$

03.03.21.0081.01

$$\int z^{\alpha-1} J_\nu(a z) Y_\nu(a z)\, dz = \frac{z^\alpha}{2\,\Gamma(\nu+1)^2}\left(\frac{2^{1-2\nu} (a z)^{2\nu} \cot(\pi \nu)}{\alpha+2\nu}\, {}_2F_3\left(\nu+\frac{1}{2}, \frac{\alpha}{2}+\nu; \nu+1, \frac{\alpha}{2}+\nu+1, 2\nu+1; -a^2 z^2\right) - \frac{2 \csc(\pi \nu)\,\Gamma(\nu+1)}{\alpha\,\Gamma(1-\nu)}\, {}_2F_3\left(\frac{1}{2}, \frac{\alpha}{2}; \frac{\alpha}{2}+1, 1-\nu, \nu+1; -a^2 z^2\right)\right)$$

03.03.21.0082.01

$$\int z^{\alpha-1} J_{-\nu}(a z) Y_\nu(a z)\, dz = \frac{z^\alpha}{\Gamma(1-\nu)^2}\left(\frac{\cot(\pi \nu)\,\Gamma(1-\nu)}{\alpha\,\Gamma(\nu+1)}\, {}_2F_3\left(\frac{1}{2}, \frac{\alpha}{2}; \frac{\alpha}{2}+1, 1-\nu, \nu+1; -a^2 z^2\right) + \frac{4^\nu (a z)^{-2\nu} \csc(\pi \nu)}{2\nu-\alpha}\, {}_2F_3\left(\frac{1}{2}-\nu, \frac{\alpha}{2}-\nu; 1-2\nu, 1-\nu, \frac{\alpha}{2}-\nu+1; -a^2 z^2\right)\right)$$

03.03.21.0083.01

$$\int z^{\mu+\nu+1} J_\mu(a z) Y_\nu(a z)\, dz = \Bigg(2^{-\mu-\nu-1} z^{\mu+\nu+2} (a z)^{\mu-\nu} \csc(\pi \nu)\bigg((a z)^{2\nu} (\mu+1) \cos(\pi \nu)\,\Gamma(1-\nu)\, {}_2F_3\left(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}+1; \mu+1, \nu+1, \mu+\nu+2; -a^2 z^2\right) - 4^\nu (\mu+\nu+1)\,\Gamma(\nu+1)\, {}_2F_3\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1; \mu+2, 1-\nu, \mu-\nu+1; -a^2 z^2\right)\bigg)\Bigg) \Big/ \left((\mu+1)(\mu+\nu+1)\,\Gamma(\mu+1)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\right)$$

03.03.21.0084.01

$$\int z^{1-\mu-\nu} J_\mu(a z) Y_\nu(a z)\, dz = \Bigg(2^{-\mu-\nu-1} z^{-\mu-\nu} (a z)^{\mu-\nu} \csc(\pi \nu)\bigg(4^\nu a^2 z^2 (\mu+\nu-1)\,\Gamma(\nu+1)\, {}_2F_3\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1; \mu+1, 2-\nu, \mu-\nu+1; -a^2 z^2\right) - 4 (a z)^{2\nu} \mu (\nu-1) \nu \cos(\pi \nu)\,\Gamma(1-\nu)\left({}_2F_3\left(\frac{\mu}{2}+\frac{\nu}{2}-\frac{1}{2}, \frac{\mu}{2}+\frac{\nu}{2}; \mu, \nu, \mu+\nu; -a^2 z^2\right)-1\right)\bigg)\Bigg) \Big/ \left(a^2 (\nu-1)(\mu+\nu-1)\,\Gamma(\mu+1)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\right)$$

03.03.21.0085.01

$$\int z J_\nu(a z) Y_\nu(b z)\, dz = \frac{z}{a^2-b^2}\left(b J_\nu(a z) Y_{\nu-1}(b z) - a J_{\nu-1}(a z) Y_\nu(b z)\right)$$

03.03.21.0086.01

$$\int z J_\nu(a z) Y_\nu(a z)\, dz = -\frac{1}{4} z^2 \left(-2 \cos(\pi \nu) J_\nu(a z)^2 + 2 J_{-\nu}(a z) J_\nu(a z) + J_{-\nu-1}(a z) J_{\nu-1}(a z) + J_{1-\nu}(a z) J_{\nu+1}(a z) + 2 J_{\nu-1}(a z) J_{\nu+1}(a z) \cos(\pi \nu)\right) \csc(\pi \nu)$$

03.03.21.0087.01

$$\int z J_{-\nu}(a z) Y_\nu(b z)\, dz = \frac{z}{a^2-b^2}\left(a J_{1-\nu}(a z) Y_\nu(b z) + b J_{-\nu}(a z) (J_{1-\nu}(b z) + J_{\nu-1}(b z) \cos(\pi \nu)) \csc(\pi \nu)\right)$$

03.03.21.0088.01

$$\int z J_0(a z) Y_0(b z)\, dz = \frac{z}{a^2-b^2}\left(a J_1(a z) Y_0(b z) - b J_0(a z) Y_1(b z)\right)$$

03.03.21.0089.01

$$\int \frac{J_\mu(a z) Y_\nu(a z)}{z}\, dz = \frac{1}{\mu^2-\nu^2}\left(a z J_{\mu-1}(a z) Y_\nu(a z) - J_\mu(a z) (a z Y_{\nu-1}(a z) + (\mu-\nu) Y_\nu(a z))\right)$$

03.03.21.0090.01

$$\int \frac{J_\nu(a z)\, Y_\nu(a z)}{z}\, dz = \frac{1}{\pi (\nu-1)\nu^3 (\nu+1)\Gamma(\nu)^2}\left(4^{-\nu-1}\left(2\pi (a z)^{2\nu}\left(\nu^2-1\right)\cot(\pi\nu)\, {}_2F_3\!\left(\nu, \nu+\frac{1}{2}; \nu+1, \nu+1, 2\nu+1; -a^2 z^2\right) - 4^\nu \nu^2\, \Gamma(\nu)^2 \left(a^2\, {}_3F_4\!\left(1, 1, \frac{3}{2}; 2, 2, 2-\nu, \nu+2; -a^2 z^2\right) z^2 + 4\left(\nu^2-1\right)\log(z)\right)\right)\right)$$

03.03.21.0091.01

$$\int \frac{\left(\left(a^2-b^2\right) z^2 + \mu^2 - \nu^2\right) J_\mu(b z)\, Y_\nu(a z)}{z}\, dz = b z\, J_{\mu-1}(b z)\, Y_\nu(a z) - J_\mu(b z)\left(a z\, Y_{\nu-1}(a z) + (\mu-\nu)\, Y_\nu(a z)\right)$$

03.03.21.0092.01

$$\int (a J_\nu(z) + b Y_\nu(z))\, dz = 2^{-\nu-1}\left(z^{\nu+1}(a + b\cot(\pi\nu))\,\Gamma\!\left(\frac{\nu+1}{2}\right) {}_1\tilde{F}_2\!\left(\frac{\nu+1}{2}; \nu+1, \frac{\nu+3}{2}; -\frac{z^2}{4}\right) - 4^\nu b z^{1-\nu} \csc(\pi\nu)\,\Gamma\!\left(\frac{1}{2}-\frac{\nu}{2}\right) {}_1\tilde{F}_2\!\left(\frac{1}{2}-\frac{\nu}{2}; 1-\nu, \frac{3}{2}-\frac{\nu}{2}; -\frac{z^2}{4}\right)\right)$$

03.03.21.0093.01

$$\int z^{\alpha-1}(a J_\nu(z) + b Y_\nu(z))\, dz = 2^{-\nu-1}\left(z^{\alpha+\nu}(a + b\cot(\pi\nu))\,\Gamma\!\left(\frac{\alpha+\nu}{2}\right) {}_1\tilde{F}_2\!\left(\frac{\alpha+\nu}{2}; \nu+1, \frac{1}{2}(\alpha+\nu+2); -\frac{z^2}{4}\right) - 4^\nu b z^{\alpha-\nu}\csc(\pi\nu)\,\Gamma\!\left(\frac{\alpha-\nu}{2}\right) {}_1\tilde{F}_2\!\left(\frac{\alpha-\nu}{2}; 1-\nu, \frac{1}{2}(\alpha-\nu+2); -\frac{z^2}{4}\right)\right)$$

03.03.21.0094.01

$$\int z^{\alpha-1}(a J_\nu(z) + b Y_\nu(z))^2\, dz = -\left(4^{-\nu} z^{\alpha-2\nu}\left(2^{2\nu+1} b\left(4\nu^2-\alpha^2\right)(a + b\cot(\pi\nu))\csc(\pi\nu)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\, {}_2F_3\!\left(\frac{1}{2}, \frac{\alpha}{2}; \frac{\alpha}{2}+1, 1-\nu, \nu+1; -z^2\right) z^{2\nu} + \alpha\left(b^2\left(16^\nu \alpha + 2^{4\nu+1}\nu\right)\csc^2(\pi\nu)\,\Gamma(\nu+1)^2\, {}_2F_3\!\left(\frac{1}{2}-\nu, \frac{\alpha}{2}-\nu; 1-2\nu, 1-\nu, \frac{\alpha}{2}-\nu+1; -z^2\right) - z^{4\nu}(2\nu-\alpha)(a + b\cot(\pi\nu))^2\,\Gamma(1-\nu)^2\, {}_2F_3\!\left(\nu+\frac{1}{2}, \frac{\alpha}{2}+\nu; \nu+1, \frac{\alpha}{2}+\nu+1, 2\nu+1; -z^2\right)\right)\right)\right) \Big/ \left(\left(4\alpha\nu^2-\alpha^3\right)\Gamma(1-\nu)^2\,\Gamma(\nu+1)^2\right)$$

03.03.21.0095.01

$$\int z^{1-2\nu}(a J_\nu(z) + b Y_\nu(z))^2\, dz = \frac{1}{(2\nu-1)\Gamma(\nu)^2}\left(2^{-2\nu-1} z^{-2\nu}\left(4a^2 z^{2\nu} + 4b^2\cot^2(\pi\nu) z^{2\nu} + 8ab\cot(\pi\nu) z^{2\nu} - 4^\nu b^2 J_{1-\nu}(z)^2\csc^2(\pi\nu)\,\Gamma(\nu)^2 z^2 - 4^\nu b^2 J_{-\nu}(z)^2\csc^2(\pi\nu)\,\Gamma(\nu)^2 z^2 - J_{\nu-1}(z)^2\left(4^\nu a^2 + 2^{2\nu+1} b\cot(\pi\nu)\, a + 4^\nu b^2\cot^2(\pi\nu)\right)\Gamma(\nu)^2 z^2 - J_\nu(z)^2\left(4^\nu a^2 + 2^{2\nu+1} b\cot(\pi\nu)\, a + 4^\nu b^2\cot^2(\pi\nu)\right)\Gamma(\nu)^2 z^2 - 4^\nu a b\, J_{\nu-2}(z)\, J_{-\nu}(z)\csc(\pi\nu)\,\Gamma(\nu)^2 z^2 - 4^\nu b^2 J_{\nu-2}(z)\, J_{-\nu}(z)\cot(\pi\nu)\csc(\pi\nu)\,\Gamma(\nu)^2 z^2 - 2^{2\nu+1} b\, J_{\nu-1}(z)\left(z J_{1-\nu}(z) - (\nu-1) J_{-\nu}(z)\right)(a + b\cot(\pi\nu))\csc(\pi\nu)\,\Gamma(\nu)^2 z - b\left(2^{2\nu+1}(\nu-1) J_{1-\nu}(z) + 4^\nu z J_{2-\nu}(z)\right) J_\nu(z)(a + b\cot(\pi\nu))\csc(\pi\nu)\,\Gamma(\nu)^2 z\right)\right)$$

03.03.21.0096.01

$$\int z^{2\nu+1}\,(a\,J_\nu(z)+b\,Y_\nu(z))^2\,dz = \frac{1}{2\,(2\,\nu+1)\,\Gamma(-\nu)^2}$$
$$\Big(J_\nu(z)^2\,(a+b\cot(\pi\,\nu))^2\,\Gamma(-\nu)^2\,z^{2\,(\nu+1)}+J_{\nu+1}(z)^2\,(a+b\cot(\pi\,\nu))^2\,\Gamma(-\nu)^2\,z^{2\,(\nu+1)}-2\,b\,(z\,J_{-\nu}(z)-\nu\,J_{-\nu-1}(z))\,J_\nu(z)$$
$$(a+b\cot(\pi\,\nu))\csc(\pi\,\nu)\,\Gamma(-\nu)^2\,z^{2\,\nu+1}-b\,(z\,J_{1-\nu}(z)+2\,\nu\,J_{-\nu}(z))\,J_{\nu+1}(z)\,(a+b\cot(\pi\,\nu))\csc(\pi\,\nu)\,\Gamma(-\nu)^2\,z^{2\,\nu+1}-b\csc^2(\pi\,\nu)$$
$$\Big(-b\,J_{-\nu-1}(z)^2\,\Gamma(-\nu)^2\,z^{2\,(\nu+1)}+J_{-\nu-1}(z)\,J_{\nu-1}(z)\,\Gamma(-\nu)^2\,(b\cos(\pi\,\nu)+a\sin(\pi\,\nu))\,z^{2\,(\nu+1)}+b\left(4^{\nu+1}-z^{2\,(\nu+1)}\,J_{-\nu}(z)^2\,\Gamma(-\nu)^2\right)\Big)\Big)$$

03.03.21.0097.01

$$\int \frac{(a\,J_\nu(z)+b\,Y_\nu(z))^2}{z}\,dz =$$
$$\Big(2^{-2\,\nu-1}\,z^{-2\,\nu}\Big(z^{2\,\nu}\,(a+b\cot(\pi\,\nu))\,\Gamma(-\nu)^2\Big(\pi\,z^{2\,\nu}\left(\nu^2-1\right)(a+b\cot(\pi\,\nu))\,{}_2F_3\!\left(\nu,\,\nu+\frac{1}{2};\,\nu+1,\,\nu+1,\,2\,\nu+1;\,-z^2\right)-$$
$$4^\nu\,b\,\nu^2\,\Gamma(\nu)^2\left({}_3F_4\!\left(1,\,1,\,\frac{3}{2};\,2,\,2,\,2-\nu,\,\nu+2;\,-z^2\right)z^2+4\left(\nu^2-1\right)\log(z)\right)\Big)-$$
$$16^\nu\,b^2\,\pi\left(\nu^2-1\right)\csc^2(\pi\,\nu)\,\Gamma(\nu)^2\,{}_2F_3\!\left(\frac{1}{2}-\nu,\,-\nu;\,1-2\,\nu,\,1-\nu,\,1-\nu;\,-z^2\right)\Big)\Big)\Big/\left(\pi\,(\nu-1)\,\nu^3\,(\nu+1)\,\Gamma(-\nu)^2\,\Gamma(\nu)^2\right)$$

03.03.21.0098.01

$$\int \frac{(a\,J_\nu(z)+b\,Y_\nu(z))^2}{z^2}\,dz = \frac{1}{\left(4\,\nu^2-1\right)\Gamma(1-\nu)^2\,\Gamma(\nu+1)^2}$$
$$\Big(4^{-\nu}\,z^{-2\,\nu-1}\Big(2^{2\,\nu+1}\,b\left(4\,\nu^2-1\right)(a+b\cot(\pi\,\nu))\csc(\pi\,\nu)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\,{}_1F_2\!\left(-\frac{1}{2};\,1-\nu,\,\nu+1;\,-z^2\right)z^{2\,\nu}+$$
$$(2\,\nu+1)\,(a+b\cot(\pi\,\nu))^2\,\Gamma(1-\nu)^2\,{}_1F_2\!\left(\nu-\frac{1}{2};\,\nu+1,\,2\,\nu+1;\,-z^2\right)z^{4\,\nu}+$$
$$b^2\left(16^\nu-2^{4\,\nu+1}\,\nu\right)\csc^2(\pi\,\nu)\,\Gamma(\nu+1)^2\,{}_1F_2\!\left(-\nu-\frac{1}{2};\,1-2\,\nu,\,1-\nu;\,-z^2\right)\Big)\Big)$$

03.03.21.0099.01

$$\int z^{\alpha-1}\left(a\,J_\mu(z)+b\,Y_\mu(z)\right)(a_1\,J_\nu(z)+b_1\,Y_\nu(z))\,dz =$$
$$2^{-\mu-\nu}\,z^{\alpha-\mu-\nu}\Bigg(\frac{1}{\Gamma(1-\nu)}\Bigg(4^\nu\csc(\pi\,\nu)\Bigg(-\frac{1}{(-\alpha+\mu+\nu)\,\Gamma(1-\mu)}\Bigg(4^\mu\,b\csc(\pi\,\mu)\,{}_3F_4\!\Bigg(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2},\,-\frac{\mu}{2}-\frac{\nu}{2}+1,\,\frac{\alpha}{2}-\frac{\mu}{2}-\frac{\nu}{2};\,1-$$
$$\mu,\,1-\nu,\,-\mu-\nu+1,\,\frac{\alpha}{2}-\frac{\mu}{2}-\frac{\nu}{2}+1;\,-z^2\Bigg)\Bigg)-\frac{1}{(\alpha+\mu-\nu)\,\Gamma(\mu+1)}\Bigg(z^{2\,\mu}\,(a+b\cot(\pi\,\mu))$$
$${}_3F_4\!\Bigg(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2},\,\frac{\mu}{2}-\frac{\nu}{2}+1,\,\frac{\alpha}{2}+\frac{\mu}{2}-\frac{\nu}{2};\,\mu+1,\,1-\nu,\,\mu-\nu+1,\,\frac{\alpha}{2}+\frac{\mu}{2}-\frac{\nu}{2}+1;\,-z^2\Bigg)\Bigg)\Bigg)b_1\Bigg)+$$
$$\frac{1}{(\alpha+\mu+\nu)\,\Gamma(\mu+1)\,\Gamma(\nu+1)}\Bigg(z^{2\,(\mu+\nu)}\,(a+b\cot(\pi\,\mu))\,{}_3F_4\!\Bigg(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\,\frac{\mu}{2}+\frac{\nu}{2}+1,\,\frac{\alpha}{2}+\frac{\mu}{2}+\frac{\nu}{2};$$
$$\mu+1,\,\frac{\alpha}{2}+\frac{\mu}{2}+\frac{\nu}{2}+1,\,\nu+1,\,\mu+\nu+1;\,-z^2\Bigg)(a_1+\cot(\pi\,\nu)\,b_1)\Bigg)-$$
$$\frac{1}{(\alpha-\mu+\nu)\,\Gamma(1-\mu)\,\Gamma(\nu+1)}\Bigg(4^\mu\,b\,z^{2\,\nu}\csc(\pi\,\mu)\,{}_3F_4\!\Bigg(-\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\,-\frac{\mu}{2}+\frac{\nu}{2}+1,\,\frac{\alpha}{2}+\frac{\nu}{2}-\frac{\mu}{2};$$
$$1-\mu,\,\frac{\alpha}{2}+\frac{\nu}{2}-\frac{\mu}{2}+1,\,\nu+1,\,-\mu+\nu+1;\,-z^2\Bigg)(a_1+\cot(\pi\,\nu)\,b_1)\Bigg)\Bigg)$$

03.03.21.0100.01

$$\int z\,(a\,J_\nu(z)+b\,Y_\nu(z))\,(J_\nu(z)\,a_1+Y_\nu(z)\,b_1)\,dz = -\frac{1}{4\pi}\Big(\csc(\pi\nu)\Big(\big(b\,\pi\,J_{-\nu-1}(z)\,J_{\nu-1}(z)\,z^2+2\,b\,\pi\,J_{-\nu}(z)\,J_\nu(z)\,z^2+b\,\pi\,J_{1-\nu}(z)\,J_{\nu+1}(z)\,z^2-2\,b\,\pi\,J_\nu(z)^2\cos(\pi\nu)\,z^2+2\,b\,\pi\,J_{\nu-1}(z)\,J_{\nu+1}(z)\cos(\pi\nu)\,z^2-2\,a\,\pi\,J_\nu(z)^2\sin(\pi\nu)\,z^2+2\,a\,\pi\,J_{\nu-1}(z)\,J_{\nu+1}(z)\sin(\pi\nu)\,z^2+4\,b\,\nu\sin(\pi\nu)\big)\,a_1+\big(a\,\pi\,J_{-\nu-1}(z)\,J_{\nu-1}(z)\,z^2+2\,a\,\pi\,J_{-\nu}(z)\,J_\nu(z)\,z^2+a\,\pi\,J_{1-\nu}(z)\,J_{\nu+1}(z)\,z^2-2\,a\,\pi\,J_\nu(z)^2\cos(\pi\nu)\,z^2+2\,a\,\pi\,J_{\nu-1}(z)\,J_{\nu+1}(z)\cos(\pi\nu)\,z^2-2\,b\,\pi\,Y_\nu(z)^2\sin(\pi\nu)\,z^2+2\,b\,\pi\,Y_{\nu-1}(z)\,Y_{\nu+1}(z)\sin(\pi\nu)\,z^2+4\,a\,\nu\sin(\pi\nu)\big)\,b_1\Big)\Big)$$

03.03.21.0101.01

$$\int \frac{\big(a\,J_\mu(z)+b\,Y_\mu(z)\big)\,(a_1\,J_\nu(z)+b_1\,Y_\nu(z))}{z}\,dz = \frac{1}{(\mu-\nu)\,(\mu+\nu)}\Big(\big(-a\,z\,J_{\mu+1}(z)\,J_\nu(z)+a\,J_\mu(z)\,((\mu-\nu)\,J_\nu(z)+z\,J_{\nu+1}(z))+b\,\big(z\,J_{\nu+1}(z)\,Y_\mu(z)+J_\nu(z)\,\big((\mu-\nu)\,Y_\mu(z)-z\,Y_{\mu+1}(z)\big)\big)\big)\,a_1+\big(-a\,z\,J_{\mu+1}(z)\,Y_\nu(z)+a\,J_\mu(z)\,((\mu-\nu)\,Y_\nu(z)+z\,Y_{\nu+1}(z))+b\,\big(Y_\mu(z)\,((\mu-\nu)\,Y_\nu(z)+z\,Y_{\nu+1}(z))-z\,Y_{\mu+1}(z)\,Y_\nu(z)\big)\big)\,b_1\Big)$$

03.03.21.0102.01

$$\int \frac{(a\,J_\nu(a\,z)+b\,Y_\nu(a\,z))\,(a_1\,J_\nu(a\,z)+b_1\,Y_\nu(a\,z))}{z}\,dz = \frac{1}{\nu\,\Gamma(1-\nu)^2\,\Gamma(\nu+1)^2}\Big(4^{-\nu-1}\,(a\,z)^{-2\nu}\Big(2\,(a+b\cot(\pi\nu))\,\Gamma(1-\nu)^2\,{}_2F_3\Big(\nu,\,\nu+\frac{1}{2};\,\nu+1,\,\nu+1,\,2\,\nu+1;\,-a^2\,z^2\Big)\,(a_1+\cot(\pi\nu)\,b_1)\,(a\,z)^{4\nu}+4^\nu\csc^2(\pi\nu)\,\Gamma(\nu+1)\Big(-4\,b\,\nu\,\Gamma(1-\nu)\log(z)\sin(\pi\nu)\,a_1\,(a\,z)^{2\nu}+\nu\,\Gamma(1-\nu)^2\,\Gamma(\nu+1)\,{}_3\tilde{F}_4\Big(1,\,1,\,\frac{3}{2};\,\nu+2,\,2-\nu,\,2,\,2;\,-a^2\,z^2\Big)\,(b\sin(\pi\nu)\,a_1+(b\cos(\pi\nu)+a\sin(\pi\nu))\,b_1)\,(a\,z)^{2\,(\nu+1)}-\Big(2^{2\nu+1}\,b\,\Gamma(\nu+1)\,{}_2F_3\Big(\frac{1}{2}-\nu,\,-\nu;\,1-2\,\nu,\,1-\nu,\,1-\nu;\,-a^2\,z^2\Big)-(a\,z)^{2\nu}\,\nu\,\Gamma(1-\nu)\Big(a^2\,b\,z^2\cos(\pi\nu)\,\Gamma(1-\nu)\,\Gamma(\nu+1)\,{}_3\tilde{F}_4\Big(1,\,1,\,\frac{3}{2};\,2-\nu,\,\nu+2,\,2,\,2;\,-a^2\,z^2\Big)-4\log(z)\,(2\,b\cos(\pi\nu)+a\sin(\pi\nu))\Big)\Big)\,b_1\Big)\Big)\Big)$$

03.03.21.0103.01

$$\int \frac{1}{z\,J_\nu(z)\,Y_\nu(z)}\,dz = \frac{1}{2}\,\pi\log\Big(\frac{Y_\nu(z)}{J_\nu(z)}\Big)$$

Power arguments

03.03.21.0104.01

$$\int z^{\alpha-1}\,J_\mu(a\,z^r)\,Y_\nu(a\,z^r)\,dz = \frac{1}{\Gamma(\mu+1)}\Big(2^{-\mu-\nu}\,z^\alpha\,(a\,z^r)^{\mu-\nu}\Big(\frac{1}{(\alpha+r\,(\mu+\nu))\,\Gamma(\nu+1)}\Big((a\,z^r)^{2\nu}\cot(\pi\nu)\,{}_3F_4\Big(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2},\,\frac{\mu}{2}+\frac{\nu}{2}+1,\,\frac{\alpha}{2\,r}+\frac{\mu}{2}+\frac{\nu}{2};\,\mu+1,\,\frac{\alpha}{2\,r}+\frac{\mu}{2}+\frac{\nu}{2}+1,\,\nu+1,\,\mu+\nu+1;\,-a^2\,z^{2r}\Big)\Big)-\frac{1}{(\alpha+r\,(\mu-\nu))\,\Gamma(1-\nu)}\Big(4^\nu\csc(\pi\nu)\,{}_3F_4\Big(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2},\,\frac{\mu}{2}-\frac{\nu}{2}+1,\,\frac{\alpha}{2\,r}+\frac{\mu}{2}-\frac{\nu}{2};\,\mu+1,\,1-\nu,\,\mu-\nu+1,\,\frac{\alpha}{2\,r}+\frac{\mu}{2}-\frac{\nu}{2}+1;\,-a^2\,z^{2r}\Big)\Big)\Big)\Big)$$

03.03.21.0105.01

$$\int z^{\alpha-1} J_\nu(a z^r) Y_\nu(a z^r) \, dz = \frac{z^\alpha}{2\,\Gamma(\nu+1)^2} \left(\frac{2^{1-2\nu} (a z^r)^{2\nu} \cot(\pi \nu)}{\alpha + 2 r \nu} \, {}_2F_3\!\left(\nu + \frac{1}{2}, \frac{\alpha}{2r} + \nu; \nu+1, \frac{\alpha}{2r} + \nu + 1, 2\nu + 1; -a^2 z^{2r}\right) - \frac{2 \csc(\pi \nu) \Gamma(\nu+1)}{\alpha\, \Gamma(1-\nu)} \, {}_2F_3\!\left(\frac{1}{2}, \frac{\alpha}{2r}; \frac{\alpha}{2r} + 1, 1-\nu, \nu+1; -a^2 z^{2r}\right)\right)$$

03.03.21.0106.01

$$\int z^{\alpha-1} J_{\nu+1}(a z^r) Y_\nu(a z^r) \, dz = \frac{1}{4 r \Gamma(\nu+2)} \left(z^\alpha \csc(\pi \nu) \left(\frac{2^{1-2\nu} r (a z^r)^{2\nu+1} \cos(\pi \nu)}{(2 \nu r + r + \alpha) \Gamma(\nu+1)} \, {}_2F_3\!\left(\nu + \frac{3}{2}, \frac{\alpha}{2r} + \nu + \frac{1}{2}; \nu+2, \frac{\alpha}{2r} + \nu + \frac{3}{2}, 2\nu+2; -a^2 z^{2r}\right) + \frac{1}{(r-\alpha)(r+\alpha)\Gamma(1-\nu)} \left(4 r \csc(\pi \nu) \left(\pi (r+\alpha) \nu (\nu+1) J_{-\nu}(a z^r) J_{\nu+1}(a z^r) - a r z^r \, {}_2F_3\!\left(\frac{3}{2}, \frac{\alpha}{2r} + \frac{1}{2}; \frac{\alpha}{2r} + \frac{3}{2}, 1-\nu, \nu+2; -a^2 z^{2r}\right) \sin(\pi \nu)\right)\right)\right)\right)$$

03.03.21.0107.01

$$\int z^{\alpha-1} J_{-\nu}(a z^r) Y_\nu(a z^r) \, dz = \frac{z^\alpha}{\Gamma(1-\nu)^2} \left(\frac{\cot(\pi \nu) \Gamma(1-\nu)}{\alpha\, \Gamma(\nu+1)} \, {}_2F_3\!\left(\frac{1}{2}, \frac{\alpha}{2r}; \frac{\alpha}{2r} + 1, 1-\nu, \nu+1; -a^2 z^{2r}\right) + \frac{4^\nu (a z^r)^{-2\nu} \csc(\pi \nu)}{2 r \nu - \alpha} \, {}_2F_3\!\left(\frac{1}{2} - \nu, \frac{\alpha}{2r} - \nu; 1 - 2\nu, 1-\nu, \frac{\alpha}{2r} - \nu + 1; -a^2 z^{2r}\right)\right)$$

03.03.21.0108.01

$$\int z^{\alpha-1} J_0(a z^r) Y_0(a z^r) \, dz = \frac{z^\alpha}{2 \sqrt{\pi}\, r} G_{4,6}^{2,3}\!\left(a z^r, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, 1 - \frac{\alpha}{2r}, -\frac{1}{2} \\ 0, 0, -\frac{1}{2}, 0, 0, -\frac{\alpha}{2r} \end{matrix}\right)$$

Involving Bessel *I*

Linear arguments

03.03.21.0109.01

$$\int I_\nu(a z) Y_\nu(a z) \, dz = -\frac{1}{4} \sqrt{\pi}\, z\, G_{2,6}^{3,0}\!\left(\frac{a z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{1}{4}, \frac{1}{4}, -\frac{\nu}{2} \end{matrix}\right)$$

03.03.21.0110.01

$$\int I_{-\nu}(a z) Y_\nu(a z) \, dz = \frac{1}{4} \sqrt{\pi}\, z\, G_{3,7}^{3,1}\!\left(\frac{a z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{3}{4}, \frac{1}{4}(-2\nu - 1), \frac{1}{4}(1 - 2\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{1}{4}, \frac{1}{4}(-2\nu-1), \frac{1}{4}(1-2\nu), \frac{\nu}{2} \end{matrix}\right)$$

03.03.21.0111.01

$$\int I_0(a z) Y_0(a z) \, dz = -\frac{1}{4} \sqrt{\pi}\, z\, G_{2,6}^{3,0}\!\left(\frac{a z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4} \end{matrix}\right)$$

Power arguments

03.03.21.0112.01

$$\int I_\nu(a z^r)\, Y_\nu(a z^r)\, dz = -\frac{\sqrt{\pi}\, z}{4 r} G_{3,7}^{3,1}\left(\frac{a z^r}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{1}{4r}, \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{1}{4}, \frac{3}{4}, -\frac{1}{4r}, -\frac{\nu}{2} \end{array}\right)$$

03.03.21.0113.01

$$\int I_{-\nu}(a z^r)\, Y_\nu(a z^r)\, dz = \frac{\sqrt{\pi}\, z}{4 r} G_{3,7}^{3,1}\left(\frac{a z^r}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{1}{4r}, \frac{1}{4}(-2\nu-1), \frac{1}{4}(1-2\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{1}{4r}, \frac{1}{4}(-2\nu-1), \frac{1}{4}(1-2\nu), \frac{\nu}{2} \end{array}\right)$$

03.03.21.0114.01

$$\int I_0(a z^r)\, Y_0(a z^r)\, dz = -\frac{\sqrt{\pi}\, z}{4 r} G_{3,7}^{3,1}\left(\frac{a z^r}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{1}{4r}, \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, 0, \frac{1}{4}, \frac{3}{4}, -\frac{1}{4r} \end{array}\right)$$

03.03.21.0115.01

$$\int I_\nu\left(a\sqrt{z}\right) Y_\nu\left(b\sqrt{z}\right) dz = \frac{2\sqrt{z}}{a^2+b^2}\left(a\, I_{\nu+1}\left(a\sqrt{z}\right) Y_\nu\left(b\sqrt{z}\right) + b\, I_\nu\left(a\sqrt{z}\right) Y_{\nu+1}\left(b\sqrt{z}\right)\right)$$

03.03.21.0116.01

$$\int I_{-\nu}\left(a\sqrt{z}\right) Y_\nu\left(b\sqrt{z}\right) dz = \frac{2\sqrt{z}}{a^2+b^2}\left(a\, I_{1-\nu}\left(a\sqrt{z}\right) Y_\nu\left(b\sqrt{z}\right) - b\, I_{-\nu}\left(a\sqrt{z}\right)\left(J_{1-\nu}\left(b\sqrt{z}\right) + J_{\nu-1}\left(b\sqrt{z}\right)\cos(\pi\nu)\right)\csc(\pi\nu)\right)$$

Involving Bessel *I* and power

Linear arguments

03.03.21.0117.01

$$\int z^{\alpha-1} I_\nu(a z)\, Y_\nu(a z)\, dz = -\frac{1}{4}\sqrt{\pi}\, z^\alpha G_{3,7}^{3,1}\left(\frac{a z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{\alpha}{4}, \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{1}{4}, \frac{3}{4}, -\frac{\alpha}{4}, -\frac{\nu}{2} \end{array}\right)$$

03.03.21.0118.01

$$\int z^{\alpha-1} I_{-\nu}(a z)\, Y_\nu(a z)\, dz = \frac{1}{4}\sqrt{\pi}\, z^\alpha G_{3,7}^{3,1}\left(\frac{a z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{\alpha}{4}, \frac{1}{4}(-2\nu-1), \frac{1}{4}(1-2\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{\alpha}{4}, \frac{1}{4}(-2\nu-1), \frac{1}{4}(1-2\nu), \frac{\nu}{2} \end{array}\right)$$

03.03.21.0119.01

$$\int z^{\alpha-1} I_0(a z)\, Y_0(a z)\, dz = -\frac{1}{4}\sqrt{\pi}\, z^\alpha G_{3,7}^{3,1}\left(\frac{a z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{\alpha}{4}, \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, 0, \frac{1}{4}, \frac{3}{4}, -\frac{\alpha}{4} \end{array}\right)$$

03.03.21.0120.01

$$\int z\, I_\nu(b z)\, Y_\nu(a z)\, dz = \frac{z}{a^2+b^2}\left(b\, I_{\nu+1}(b z)\, Y_\nu(a z) + a\, I_\nu(b z)\, Y_{\nu+1}(a z)\right)$$

03.03.21.0121.01

$$\int z\, I_{-\nu}(b z)\, Y_\nu(a z)\, dz = \frac{z}{a^2+b^2}\left(b\, I_{1-\nu}(b z)\, Y_\nu(a z) - a\, I_{-\nu}(b z)\left(J_{1-\nu}(a z) + J_{\nu-1}(a z)\cos(\pi\nu)\right)\csc(\pi\nu)\right)$$

Power arguments

03.03.21.0122.01

$$\int z^{\alpha-1} I_\nu(a z^r)\, Y_\nu(a z^r)\, dz = -\frac{\sqrt{\pi}\, z^\alpha}{4 r} G_{3,7}^{3,1}\left(\frac{a z^r}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{\alpha}{4r}, \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{1}{4}, \frac{3}{4}, -\frac{\alpha}{4r}, -\frac{\nu}{2} \end{array}\right)$$

03.03.21.0123.01

$$\int z^{\alpha-1} I_{-\nu}(a z^r)\, Y_\nu(a z^r)\, dz = \frac{\sqrt{\pi}\, z^\alpha}{4 r} G_{3,7}^{3,1}\left(\frac{a z^r}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{\alpha}{4r}, \frac{1}{4}(-2\nu-1), \frac{1}{4}(1-2\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{\alpha}{4r}, \frac{1}{4}(-2\nu-1), \frac{1}{4}(1-2\nu), \frac{\nu}{2} \end{array}\right)$$

03.03.21.0124.01

$$\int z^{\alpha-1} I_0(a z^r)\, Y_0(a z^r)\, dz = -\frac{\sqrt{\pi}\, z^\alpha}{4 r} G_{3,7}^{3,1}\left(\frac{a z^r}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} 1-\frac{\alpha}{4r}, \frac{1}{4}, \frac{3}{4} \\ 0, 0, \frac{1}{2}, 0, \frac{1}{4}, \frac{3}{4}, -\frac{\alpha}{4r} \end{array}\right)$$

Definite integration

For the direct function itself

03.03.21.0125.01

$$\int_0^\infty Y_\nu(t)\, dt = -\tan\left(\frac{\pi\nu}{2}\right) /; |\mathrm{Re}(\nu)| < 1$$

03.03.21.0126.01

$$\int_0^\infty t^{\alpha-1} Y_\nu(t)\, dt = -\frac{2^{\alpha-1}}{\pi} \cos\left(\frac{1}{2}\pi(\alpha-\nu)\right) \Gamma\left(\frac{\alpha-\nu}{2}\right) \Gamma\left(\frac{\alpha+\nu}{2}\right) /; \mathrm{Re}(\alpha) > |\mathrm{Re}(\nu)| \bigwedge \mathrm{Re}(\alpha) < \frac{3}{2}$$

Involving the direct function

03.03.21.0127.01

$$\int_0^\infty Y_\nu(t)^2\, dt = \frac{\log(4)}{2\pi} + \tan(\pi\nu) - \frac{1}{\pi}\psi\left(\nu + \frac{1}{2}\right) /; |\mathrm{Re}(\nu)| < \frac{1}{2}$$

03.03.21.0128.01

$$\int_0^\infty t^{\alpha-1} Y_\nu(t)^2\, dt = \frac{1}{\pi^{3/2}} \Gamma\left(\frac{\alpha}{2}+\nu\right) \left(\frac{1}{\Gamma\left(\frac{\alpha+1}{2}\right)} \cos\left(\frac{1}{2}\pi(\alpha-2\nu)\right) \Gamma\left(\frac{\alpha}{2}\right) \Gamma\left(\frac{\alpha}{2}-\nu\right) - \frac{\pi}{\alpha\, \Gamma\left(-\frac{\alpha}{2}\right) \Gamma\left(-\frac{\alpha}{2}+\nu+1\right)} \Gamma\left(\frac{1}{2}-\frac{\alpha}{2}\right)\right) /;$$

$$\mathrm{Re}(\alpha) > 2|\mathrm{Re}(\nu)| \land \mathrm{Re}(\alpha) < 2$$

03.03.21.0129.01

$$\int_\beta^\infty Z_\nu(a t, a\beta)\, Z_\nu(b t, b\beta)\, t\, dt = \frac{\delta(a-b)}{a} /;$$

$$Z_\nu(x, y) = \frac{J_\nu(y)\, Y_\nu(x) - J_\nu(x)\, Y_\nu(y)}{\sqrt{J_\nu(y)^2 + Y_\nu(y)^2}} \bigwedge \nu \in \mathbb{R} \bigwedge \beta \in \mathbb{R} \bigwedge \beta \geq 0 \bigwedge a \in \mathbb{R} \bigwedge b \in \mathbb{R}$$

A. Grosberg

03.03.21.0130.01

$$\int_0^\infty Z_\nu(t r, t\beta)\, Z_\nu(t s, t\beta)\, t\, dt = \frac{\delta(r-s)}{r} /; Z_\nu(x, y) = \frac{J_\nu(y)\, Y_\nu(x) - J_\nu(x)\, Y_\nu(y)}{\sqrt{J_\nu(y)^2 + Y_\nu(y)^2}} \bigwedge \nu \in \mathbb{R} \bigwedge \beta \in \mathbb{R} \bigwedge r \in \mathbb{R} \bigwedge s \in \mathbb{R}$$

A. Grosberg

Integral transforms

Fourier cos transforms

03.03.22.0001.01

$$\mathcal{F}c_t[Y_\nu(t)](z) = -\frac{2^{-\nu-\frac{1}{2}}}{\sqrt{\pi}} \sec\left(\frac{\pi\nu}{2}\right) z^{-\nu-1} \left(4^\nu z^{2\nu} {}_2F_1\left(\frac{1}{2}-\frac{\nu}{2}, 1-\frac{\nu}{2}; 1-\nu; \frac{1}{z^2}\right) + \cos(\pi\nu)\, {}_2F_1\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \nu+1; \frac{1}{z^2}\right)\right) /;$$
$$|\mathrm{Re}(\nu)| < 1$$

Fourier sin transforms

03.03.22.0002.01

$$\mathcal{F}s_t[Y_\nu(t)](z) = \frac{2^{-\nu-\frac{1}{2}}}{\sqrt{\pi}} z^{-\nu-1} \csc\left(\frac{\pi\nu}{2}\right) \left(\cos(\pi\nu)\, {}_2F_1\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \nu+1; \frac{1}{z^2}\right) - 4^\nu z^{2\nu} {}_2F_1\left(\frac{1}{2}-\frac{\nu}{2}, 1-\frac{\nu}{2}; 1-\nu; \frac{1}{z^2}\right)\right) /;$$
$$|\mathrm{Re}(\nu)| < 2$$

Laplace transforms

03.03.22.0003.01

$$\mathcal{L}_t[Y_\nu(t)](z) = 2^{-\nu} z^{-\nu-1} \left(\cot(\pi\nu)\, {}_2F_1\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \nu+1; -\frac{1}{z^2}\right) - 4^\nu z^{2\nu} \csc(\pi\nu)\, {}_2F_1\left(\frac{1}{2}-\frac{\nu}{2}, 1-\frac{\nu}{2}; 1-\nu; -\frac{1}{z^2}\right)\right) /; |\mathrm{Re}(\nu)| < 1$$

Mellin transforms

03.03.22.0004.01

$$\mathcal{M}_t[Y_\nu(t)](z) = -\frac{2^{z-1}}{\pi} \cos\left(\frac{\pi(z-\nu)}{2}\right) \Gamma\left(\frac{z-\nu}{2}\right) \Gamma\left(\frac{z+\nu}{2}\right) /; \mathrm{Re}(z) > |\mathrm{Re}(\nu)| \bigwedge \mathrm{Re}(z) < \frac{3}{2}$$

Hankel transforms

03.03.22.0005.01

$$\mathcal{H}_{t;\mu}[Y_\nu(t)](z) = -\frac{\sqrt{2}\, z^{-\nu-1}}{\pi}$$
$$\left(\frac{z^{2\nu}\,\Gamma(\nu)}{\Gamma\left(\frac{1}{4}(2\mu+2\nu+1)\right)} \Gamma\left(\frac{1}{4}(2\mu-2\nu+3)\right) {}_2F_1\left(\frac{1}{4}(-2\mu-2\nu+3), \frac{1}{4}(2\mu-2\nu+3); 1-\nu; \frac{1}{z^2}\right) + \frac{\cos(\pi\nu)\,\Gamma(-\nu)}{\Gamma\left(\frac{1}{4}(2\mu-2\nu+1)\right)}\right.$$
$$\left.\Gamma\left(\frac{1}{4}(2\mu+2\nu+3)\right) {}_2F_1\left(\frac{1}{4}(-2\mu+2\nu+3), \frac{1}{4}(2\mu+2\nu+3); \nu+1; \frac{1}{z^2}\right)\right) /; \mathrm{Re}(\mu-\nu) > -\frac{3}{2} \bigwedge \mathrm{Re}(\mu+\nu) > -\frac{3}{2}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_0\tilde{F}_1$

03.03.26.0001.02

$$Y_\nu(z) = 2^{-\nu} z^\nu \cot(\nu\pi)\, {}_0\tilde{F}_1\left(; \nu+1; -\frac{z^2}{4}\right) - 2^\nu z^{-\nu} \csc(\nu\pi)\, {}_0\tilde{F}_1\left(; 1-\nu; -\frac{z^2}{4}\right) /; \nu \notin \mathbb{Z}$$

Involving ${}_0F_1$

03.03.26.0002.01

$$Y_\nu(z) = -\frac{2^\nu z^{-\nu}\Gamma(\nu)}{\pi}\, {}_0F_1\left(; 1-\nu; -\frac{z^2}{4}\right) - \frac{2^{-\nu} z^\nu \cos(\nu\pi)\Gamma(-\nu)}{\pi}\, {}_0F_1\left(; \nu+1; -\frac{z^2}{4}\right) /; \nu \notin \mathbb{Z}$$

Involving hypergeometric U

03.03.26.0089.01

$$Y_\nu(z) = \frac{z^{-\nu}}{\pi}\left(-2^{\nu+1} e^{-iz}\sqrt{\pi}\, U\left(\nu+\frac{1}{2}, 2\nu+1, 2iz\right)(iz)^{2\nu} - 2^{-\nu}\left(z^{2\nu}\cos(\pi\nu) - (iz)^{2\nu}\right)\Gamma(-\nu)\, {}_0F_1\left(; \nu+1; -\frac{z^2}{4}\right)\right) /; \nu \notin \mathbb{Z}$$

03.03.26.0090.01

$$Y_\nu(z) = -\frac{2z^\nu}{\pi}\left((-1)^\nu 2^\nu e^{-iz}\sqrt{\pi}\, U\left(\nu+\frac{1}{2}, 2\nu+1, 2iz\right) + 2^{-\nu}\, {}_0\tilde{F}_1\left(; \nu+1; -\frac{z^2}{4}\right)(\log(iz) - \log(z))\right) /; \nu \in \mathbb{Z}$$

Involving ${}_1F_1$

03.03.26.0091.01

$$Y_\nu(z) = -\frac{2^{-\nu}\cos(\nu\pi)\Gamma(-\nu) e^{-iz} z^\nu}{\pi}\, {}_1F_1\left(\nu+\frac{1}{2}; 2\nu+1; 2iz\right) - \frac{2^\nu\Gamma(\nu) z^{-\nu} e^{-iz}}{\pi}\, {}_1F_1\left(\frac{1}{2}-\nu; 1-2\nu; 2iz\right) /; \nu \notin \mathbb{Z}$$

Through Meijer G

Classical cases for the direct function itself

03.03.26.0003.01

$$Y_\nu\left(\sqrt{z^2}\right) = G_{1,3}^{2,0}\left(\frac{z^2}{4} \middle| \begin{array}{c} -\frac{1}{2}(\nu+1) \\ \frac{\nu}{2}, -\frac{\nu}{2}, -\frac{1}{2}(\nu+1) \end{array}\right)$$

03.03.26.0004.01

$$Y_\nu(z) = z^{-\nu}\left(z^2\right)^{\nu/2} G_{1,3}^{2,0}\left(\frac{z^2}{4} \middle| \begin{array}{c} -\frac{1}{2}(\nu+1) \\ \frac{\nu}{2}, -\frac{\nu}{2}, -\frac{1}{2}(\nu+1) \end{array}\right) - z^{-\nu}\left(z^2\right)^{-\frac{\nu}{2}}\left(\left(z^2\right)^\nu - z^{2\nu}\right)\cot(\pi\nu)\, G_{0,2}^{1,0}\left(\frac{z^2}{4} \middle| \frac{\nu}{2}, -\frac{\nu}{2}\right) /; \nu \notin \mathbb{Z}$$

03.03.26.0005.01

$$Y_\nu(z) = G_{1,3}^{2,0}\left(\frac{z^2}{4} \middle| \begin{array}{c} -\frac{1}{2}(\nu+1) \\ \frac{\nu}{2}, -\frac{\nu}{2}, -\frac{1}{2}(\nu+1) \end{array}\right) /; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

03.03.26.0006.01

$$Y_\nu\left(\sqrt{z}\right) = G_{1,3}^{2,0}\left(\frac{z}{4} \middle| \begin{array}{c} -\frac{\nu+1}{2} \\ \frac{\nu}{2}, -\frac{\nu}{2}, -\frac{\nu+1}{2} \end{array}\right)$$

03.03.26.0092.01

$$Y_{-\nu}\left(\sqrt{z}\right) + Y_\nu\left(\sqrt{z}\right) = -2\cos\left(\frac{\pi\nu}{2}\right) G_{1,3}^{2,0}\left(\frac{z}{4} \middle| \begin{array}{c} \frac{1}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1}{2} \end{array}\right)$$

03.03.26.0093.01

$$Y_\nu\left(\sqrt{z}\right) - Y_{-\nu}\left(\sqrt{z}\right) = -2\sin\left(\frac{\pi\nu}{2}\right) G_{1,3}^{2,0}\left(\frac{z}{4} \middle| \begin{matrix} 0 \\ -\frac{\nu}{2}, \frac{\nu}{2}, 0 \end{matrix}\right)$$

Classical cases involving cos

03.03.26.0007.01

$$\cos\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{1}{\sqrt{2}} G_{3,5}^{2,2}\left(z \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4}, -\frac{\nu+1}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, -\frac{\nu+1}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

Classical cases involving sin

03.03.26.0008.01

$$\sin\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{1}{\sqrt{2}} G_{3,5}^{2,2}\left(z \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4}, -\frac{\nu}{2} \\ \frac{\nu+1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu}{2} \end{matrix}\right)$$

Classical cases for powers of *Y*

03.03.26.0009.01

$$Y_\nu\left(\sqrt{z}\right)^2 = \frac{2}{\sqrt{\pi}} G_{2,4}^{3,0}\left(z \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}-\nu \\ 0, -\nu, \nu, \frac{1}{2}-\nu \end{matrix}\right) + \frac{1}{\sqrt{\pi}} G_{1,3}^{1,1}\left(z \middle| \begin{matrix} \frac{1}{2} \\ \nu, 0, -\nu \end{matrix}\right)$$

03.03.26.0094.01

$$Y_{-\nu}\left(\sqrt{z}\right)^2 + Y_\nu\left(\sqrt{z}\right)^2 = \frac{4\cos(\pi\nu)}{\sqrt{\pi}} G_{2,4}^{3,0}\left(z \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, -\nu, \nu, \frac{1}{2} \end{matrix}\right) + \frac{2\cos(\pi\nu)}{\sqrt{\pi}} G_{2,4}^{2,1}\left(z \middle| \begin{matrix} \frac{1}{2}, 0 \\ \nu, -\nu, 0, 0 \end{matrix}\right)$$

03.03.26.0095.01

$$Y_{-\nu}\left(\sqrt{z}\right)^2 - Y_\nu\left(\sqrt{z}\right)^2 = \frac{2\sin(\pi\nu)}{\sqrt{\pi}} G_{1,3}^{2,0}\left(z \middle| \begin{matrix} \frac{1}{2} \\ \nu, -\nu, 0 \end{matrix}\right) - \frac{4\sin(\pi\nu)}{\sqrt{\pi}} G_{1,3}^{2,0}\left(z \middle| \begin{matrix} \frac{1}{2} \\ -\nu, \nu, 0 \end{matrix}\right)$$

Classical cases for products of *Y*

03.03.26.0010.01

$$Y_{-\nu}\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{2}{\sqrt{\pi}} G_{2,4}^{3,0}\left(z \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, -\nu, \nu, \frac{1}{2} \end{matrix}\right) + \frac{1}{\sqrt{\pi}} G_{1,3}^{1,1}\left(z \middle| \begin{matrix} \frac{1}{2} \\ 0, -\nu, \nu \end{matrix}\right)$$

03.03.26.0096.01

$$Y_{\nu-1}\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{2}{\sqrt{\pi}} G_{2,4}^{3,0}\left(z \middle| \begin{matrix} 0, 1-\nu \\ \frac{1}{2}(2\nu-1), -\frac{1}{2}, \frac{1}{2}(1-2\nu), 1-\nu \end{matrix}\right) + \frac{1}{\sqrt{\pi}} G_{1,3}^{1,1}\left(z \middle| \begin{matrix} 0 \\ \frac{1}{2}(2\nu-1), -\frac{1}{2}, \frac{1}{2}(1-2\nu) \end{matrix}\right)$$

03.03.26.0011.01

$$Y_{\frac{1}{2}-\nu}\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{2}{\sqrt{\pi}} G_{2,4}^{3,0}\left(z \middle| \begin{matrix} 0, \frac{1}{2} \\ -\frac{1}{4}, \frac{1}{4}-\nu, \nu-\frac{1}{4}, \frac{1}{4} \end{matrix}\right) + \frac{1}{\sqrt{\pi}} G_{2,4}^{1,2}\left(z \middle| \begin{matrix} 0, \frac{1}{2} \\ \frac{1}{4}, -\frac{1}{4}, \nu-\frac{1}{4}, \frac{1}{4}-\nu \end{matrix}\right)$$

03.03.26.0012.01

$$Y_\mu\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{2}{\sqrt{\pi}} G_{3,5}^{4,0}\left(z \middle| \begin{matrix} 0, \frac{1}{2}, \frac{1}{2}(-\mu-\nu+1) \\ \frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2}, -\frac{\mu+\nu}{2}, \frac{1-\mu-\nu}{2} \end{matrix}\right) + \frac{1}{\sqrt{\pi}} G_{2,4}^{1,2}\left(z \middle| \begin{matrix} 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2}, -\frac{\mu+\nu}{2} \end{matrix}\right) /; -\mu-\nu-1 \notin \mathbb{N}$$

03.03.26.0097.01

$$Y_{-n-\nu-1}\left(\sqrt{z}\right)Y_\nu\left(\sqrt{z}\right)=\frac{\csc(\nu\pi)}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{n}{2}+1\\-\frac{1}{2}(n+1)-\nu,\frac{n+1}{2},\frac{n}{2}+1,\frac{n+1}{2}+\nu,-\frac{1}{2}(n+1)\end{matrix}\right)+$$
$$\frac{\cot(\nu\pi)(-1)^n}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{n}{2}+\nu+1\\\frac{n+1}{2},\frac{n+1}{2}+\nu,-\frac{1}{2}(n+1)-\nu,-\frac{1}{2}(n+1),\frac{n}{2}+\nu+1\end{matrix}\right)-$$
$$\frac{2\cot^2(\nu\pi)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{\frac{1}{2}(2k-n-1)}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)};\,n\in\mathbb{N}$$

03.03.26.0098.01

$$Y_{-\nu-1}\left(\sqrt{z}\right)Y_\nu\left(\sqrt{z}\right)=$$
$$\frac{2\cos(\pi\nu)\cot(\pi\nu)}{\pi z}+\frac{\csc(\nu\pi)}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},1\\-\nu-\frac{1}{2},\frac{1}{2},1,\nu+\frac{1}{2},-\frac{1}{2}\end{matrix}\right)+\frac{\cot(\pi\nu)}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\nu+1\\\frac{1}{2},\nu+\frac{1}{2},-\nu-\frac{1}{2},-\frac{1}{2},\nu+1\end{matrix}\right)$$

03.03.26.0099.01

$$Y_{-\nu-2}\left(\sqrt{z}\right)Y_\nu\left(\sqrt{z}\right)=$$
$$-\frac{4(\nu+1)\cos(\pi\nu)\cot(\pi\nu)}{\pi z^2}+\frac{\csc(\nu\pi)}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{3}{2}\\-\nu-1,1,\frac{3}{2},\nu+1,-1\end{matrix}\right)-\frac{\cot(\pi\nu)}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\nu+\frac{3}{2}\\1,\nu+1,-\nu-1,-1,\nu+\frac{3}{2}\end{matrix}\right)$$

03.03.26.0100.01

$$Y_{-\mu}\left(\sqrt{z}\right)Y_{-\nu}\left(\sqrt{z}\right)+Y_\mu\left(\sqrt{z}\right)Y_\nu\left(\sqrt{z}\right)=$$
$$\frac{4\cos\left(\frac{1}{2}\pi(\mu+\nu)\right)}{\sqrt{\pi}}G_{3,5}^{4,0}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{1}{2}\\\frac{\mu+\nu}{2},-\frac{1}{2}(\mu+\nu),\frac{\nu-\mu}{2},\frac{\mu-\nu}{2},\frac{1}{2}\end{matrix}\right)+\frac{2\cos\left(\frac{1}{2}\pi(\mu+\nu)\right)}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},0\\\frac{\mu+\nu}{2},-\frac{1}{2}(\mu+\nu),\frac{\nu-\mu}{2},\frac{\mu-\nu}{2}\end{matrix}\right)$$

03.03.26.0101.01

$$Y_\mu\left(\sqrt{z}\right)Y_\nu\left(\sqrt{z}\right)-Y_{-\mu}\left(\sqrt{z}\right)Y_{-\nu}\left(\sqrt{z}\right)=$$
$$\frac{4\sin\left(\frac{1}{2}\pi(\mu+\nu)\right)}{\sqrt{\pi}}G_{3,5}^{4,0}\left(z\,\middle|\,\begin{matrix}0,0,\frac{1}{2}\\\frac{\mu+\nu}{2},-\frac{1}{2}(\mu+\nu),\frac{\nu-\mu}{2},\frac{\mu-\nu}{2},0\end{matrix}\right)-\frac{2\sin\left(\frac{1}{2}\pi(\mu+\nu)\right)}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2}\\\frac{\mu+\nu}{2},-\frac{1}{2}(\mu+\nu),\frac{\mu-\nu}{2},\frac{\nu-\mu}{2}\end{matrix}\right)$$

Classical cases involving Bessel *J*

03.03.26.0013.01

$$\cos(a\pi)J_\nu\left(\sqrt{z}\right)+\sin(a\pi)Y_\nu\left(\sqrt{z}\right)=G_{1,3}^{2,0}\left(\frac{z}{4}\,\middle|\,\begin{matrix}-a-\frac{\nu}{2}\\-\frac{\nu}{2},\frac{\nu}{2},-a-\frac{\nu}{2}\end{matrix}\right)$$

03.03.26.0102.01

$$\cos(a\pi)J_\nu\left(\sqrt{z}\right)-\sin(a\pi)Y_\nu\left(\sqrt{z}\right)=G_{1,3}^{2,0}\left(\frac{z}{4}\,\middle|\,\begin{matrix}a-\frac{\nu}{2}\\-\frac{\nu}{2},\frac{\nu}{2},a-\frac{\nu}{2}\end{matrix}\right)$$

03.03.26.0019.01

$$J_\nu\left(\sqrt{z}\right)Y_\nu\left(\sqrt{z}\right)=-\frac{1}{\sqrt{\pi}}G_{1,3}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2}\\0,\nu,-\nu\end{matrix}\right)$$

03.03.26.0020.01

$$J_{-\nu}\left(\sqrt{z}\right)Y_{\nu}\left(\sqrt{z}\right)=\frac{1}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},-\nu-\frac{1}{2}\\0,-\nu,-\nu-\frac{1}{2},\nu\end{matrix}\right)$$

03.03.26.0021.01

$$J_{\nu+1}\left(\sqrt{z}\right)Y_{\nu}\left(\sqrt{z}\right)=\frac{1}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},0\\\frac{1}{2},\nu+\frac{1}{2},-\frac{1}{2},-\frac{1}{2}-\nu\end{matrix}\right)$$

03.03.26.0022.01

$$J_{\nu+2}\left(\sqrt{z}\right)Y_{\nu}\left(\sqrt{z}\right)=\frac{1}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2}\\1,\nu+1,-1,-\nu-1\end{matrix}\right)$$

03.03.26.0023.01

$$J_{\mu}\left(\sqrt{z}\right)Y_{\nu}\left(\sqrt{z}\right)=-\frac{1}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{\mu-\nu+1}{2}\\\frac{\mu+\nu}{2},\frac{\mu-\nu}{2},\frac{\mu-\nu+1}{2},-\frac{\mu+\nu}{2},\frac{\nu-\mu}{2}\end{matrix}\right)\text{ /; }-\mu-\nu-1\notin\mathbb{N}$$

03.03.26.0103.01

$$Y_{-n-\nu-1}\left(\sqrt{z}\right)J_{\nu}\left(\sqrt{z}\right)=\frac{(-1)^n}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{n}{2}+\nu+1\\\frac{n+1}{2},\frac{n+1}{2}+\nu,-\frac{1}{2}(n+1)-\nu,-\frac{1}{2}(n+1),\frac{n}{2}+\nu+1\end{matrix}\right)-$$
$$\frac{2\cot(\nu\pi)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{\frac{2k-n-1}{2}}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\text{ /; }n\in\mathbb{N}$$

03.03.26.0104.01

$$Y_{-\nu-1}\left(\sqrt{z}\right)J_{\nu}\left(\sqrt{z}\right)=\frac{2\cos(\pi\nu)}{\pi\sqrt{z}}+\frac{1}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\nu+1\\\frac{1}{2},\nu+\frac{1}{2},-\frac{1}{2},-\nu-\frac{1}{2},\nu+1\end{matrix}\right)$$

03.03.26.0105.01

$$Y_{-\nu-2}\left(\sqrt{z}\right)J_{\nu}\left(\sqrt{z}\right)=-\frac{4(\nu+1)\cos(\pi\nu)}{\pi z}-\frac{1}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2},\nu+\frac{3}{2}\\1,\nu+1,-1,-\nu-1,\nu+\frac{3}{2}\end{matrix}\right)$$

03.03.26.0024.01

$$J_{\nu}\left(\sqrt{z}\right)Y_{-\nu}\left(\sqrt{z}\right)+J_{-\nu}\left(\sqrt{z}\right)Y_{\nu}\left(\sqrt{z}\right)=-\frac{2}{\sqrt{\pi}}G_{1,3}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2}\\\nu,-\nu,0\end{matrix}\right)$$

03.03.26.0025.01

$$J_{\nu}\left(\sqrt{z}\right)Y_{-\nu}\left(\sqrt{z}\right)-J_{-\nu}\left(\sqrt{z}\right)Y_{\nu}\left(\sqrt{z}\right)=\frac{\sin(2\pi\nu)}{\pi^{5/2}}G_{1,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2}\\0,\nu,-\nu\end{matrix}\right)$$

03.03.26.0026.01

$$J_{\nu}\left(\sqrt{z}\right)Y_{\mu}\left(\sqrt{z}\right)+J_{\mu}\left(\sqrt{z}\right)Y_{\nu}\left(\sqrt{z}\right)=-\frac{2}{\sqrt{\pi}}G_{2,4}^{3,0}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2}\\\frac{\mu+\nu}{2},\frac{\mu-\nu}{2},\frac{\nu-\mu}{2},-\frac{\mu+\nu}{2}\end{matrix}\right)$$

03.03.26.0027.01

$$J_{\nu}\left(\sqrt{z}\right)Y_{\mu}\left(\sqrt{z}\right)-J_{\mu}\left(\sqrt{z}\right)Y_{\nu}\left(\sqrt{z}\right)=\frac{\sin(\pi(\nu-\mu))}{\pi^{5/2}}G_{2,4}^{3,2}\left(z\,\middle|\,\begin{matrix}0,\frac{1}{2}\\\frac{\mu+\nu}{2},\frac{\mu-\nu}{2},\frac{\nu-\mu}{2},-\frac{\mu+\nu}{2}\end{matrix}\right)\text{ /; }-\mu-\nu-1\notin\mathbb{N}$$

03.03.26.0014.01

$$J_\nu\left(\sqrt{z}\right)^2 + Y_\nu\left(\sqrt{z}\right)^2 = \frac{2\cos(\pi\nu)}{\pi^{5/2}} G_{1,3}^{3,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, -\nu, \nu \end{matrix}\right)$$

03.03.26.0015.01

$$J_\nu\left(\sqrt{z}\right)^2 - Y_\nu\left(\sqrt{z}\right)^2 = -\frac{2}{\sqrt{\pi}} G_{2,4}^{3,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2}-\nu \\ 0, -\nu, \nu, \frac{1}{2}-\nu \end{matrix}\right)$$

03.03.26.0016.01

$$J_\nu\left(\sqrt{z}\right) J_{-\nu}\left(\sqrt{z}\right) - Y_\nu\left(\sqrt{z}\right) Y_{-\nu}\left(\sqrt{z}\right) = -\frac{2}{\sqrt{\pi}} G_{2,4}^{3,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, -\nu, \nu, \frac{1}{2} \end{matrix}\right)$$

03.03.26.0017.01

$$J_\mu\left(\sqrt{z}\right) J_\nu\left(\sqrt{z}\right) - Y_\mu\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = -\frac{2}{\sqrt{\pi}} G_{3,5}^{4,0}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2}, \frac{1-\mu-\nu}{2} \\ \frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2}, -\frac{\mu+\nu}{2}, \frac{1-\mu-\nu}{2} \end{matrix}\right)$$

03.03.26.0018.01

$$J_\mu\left(\sqrt{z}\right) J_\nu\left(\sqrt{z}\right) + Y_\mu\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{\cos(\pi\mu)}{\pi^{5/2}} G_{2,4}^{3,2}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}, -\frac{\mu+\nu}{2}, \frac{\nu-\mu}{2} \end{matrix}\right) + \frac{\cos(\pi\nu)}{\pi^{5/2}} G_{2,4}^{3,2}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, -\frac{\mu+\nu}{2}, \frac{\nu-\mu}{2}, \frac{\mu-\nu}{2} \end{matrix}\right) /; \mu+\nu \notin \mathbb{Z}$$

Classical cases involving cos, sin, *J*

03.03.26.0028.01

$$\sin\left(\sqrt{z}\right) J_\nu\left(\sqrt{z}\right) + \cos\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = -\sqrt{2}\, G_{2,4}^{3,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

03.03.26.0029.01

$$\cos\left(\sqrt{z}\right) J_\nu\left(\sqrt{z}\right) - \sin\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \sqrt{2}\, G_{2,4}^{3,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1-\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2} \end{matrix}\right)$$

03.03.26.0030.01

$$\cos\left(\sqrt{z}\right) J_\nu\left(\sqrt{z}\right) + \sin\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{\cos(\nu\pi)}{\pi^2\sqrt{2}} G_{2,4}^{3,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1-\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2} \end{matrix}\right)$$

03.03.26.0031.01

$$\sin\left(\sqrt{z}\right) J_\nu\left(\sqrt{z}\right) - \cos\left(\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{\cos(\nu\pi)}{\pi^2\sqrt{2}} G_{2,4}^{3,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

03.03.26.0032.01

$$\sin\left(a+\sqrt{z}\right) J_\nu\left(\sqrt{z}\right) - \cos\left(a+\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{\cos(\pi\nu)}{\pi^2\sqrt{2}} G_{3,5}^{4,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1-\nu}{2}-\frac{a}{\pi} \\ -\frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1-\nu}{2}-\frac{a}{\pi} \end{matrix}\right)$$

03.03.26.0106.01

$$\cos\left(a+\sqrt{z}\right) J_\nu\left(\sqrt{z}\right) + \sin\left(a+\sqrt{z}\right) Y_\nu\left(\sqrt{z}\right) = \frac{\cos(\pi\nu)}{\sqrt{2}\,\pi^2} G_{3,5}^{4,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, -\frac{a}{\pi}-\frac{\nu}{2} \\ \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{a}{\pi}-\frac{\nu}{2} \end{matrix}\right)$$

Classical cases involving Bessel *I*

03.03.26.0033.01

$$I_\nu\left(\sqrt[4]{z}\right) Y_\nu\left(\sqrt[4]{z}\right) = -\sqrt{\pi}\; G_{2,6}^{3,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{1}{4}, \frac{3}{4}, -\frac{\nu}{2} \end{matrix}\right)$$

03.03.26.0034.01

$$I_{-\nu}\left(\sqrt[4]{z}\right) Y_\nu\left(\sqrt[4]{z}\right) = \sqrt{\pi}\; G_{2,6}^{3,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} -\frac{2\nu+1}{4}, \frac{1-2\nu}{4} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-2\nu}{4}, -\frac{1+2\nu}{4} \end{matrix}\right)$$

03.03.26.0107.01

$$I_{-\nu}\left(\sqrt[4]{z}\right) Y_\nu\left(\sqrt[4]{z}\right) = \sqrt{\pi}\; G_{2,6}^{3,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} -\frac{1}{4}(2\nu+1), \frac{1}{4}(1-2\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1}{4}(1-2\nu), -\frac{1}{4}(2\nu+1) \end{matrix}\right)$$

Classical cases involving Bessel *K*

03.03.26.0038.01

$$K_\nu\left(\sqrt[4]{z}\right) Y_\nu\left(\sqrt[4]{z}\right) = -\frac{1}{4\sqrt{\pi}}\; G_{1,5}^{4,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

03.03.26.0108.01

$$K_\nu\left(\sqrt[4]{z}\right) Y_{-\nu}\left(\sqrt[4]{z}\right) = -\frac{1}{4\sqrt{\pi}}\; G_{1,5}^{4,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

03.03.26.0035.01

$$K_0\left(\sqrt[4]{z}\right) - \frac{\pi}{2} Y_0\left(\sqrt[4]{z}\right) = \frac{\pi}{2}\; G_{0,4}^{2,0}\left(\frac{z}{256} \,\middle|\, 0, 0, \frac{1}{2}, \frac{1}{2}\right)$$

03.03.26.0036.01

$$K_n\left(\sqrt[4]{z}\right) + \frac{\pi}{2} Y_n\left(\sqrt[4]{z}\right) = \frac{\pi}{2}\; G_{1,5}^{3,0}\left(\frac{z}{256} \,\middle|\, \begin{matrix} -\frac{n}{4} \\ \frac{2-n}{4}, \frac{n}{4}, \frac{n+2}{4}, -\frac{n}{4}, -\frac{n}{4} \end{matrix}\right) /; n \in \mathbb{N}$$

03.03.26.0037.01

$$K_n\left(\sqrt[4]{z}\right) - \frac{\pi}{2} Y_n\left(\sqrt[4]{z}\right) = \frac{\pi}{2}\; G_{1,5}^{3,0}\left(\frac{z}{256} \,\middle|\, \begin{matrix} \frac{2-n}{4} \\ -\frac{n}{4}, \frac{n}{4}, \frac{n+2}{4}, \frac{2-n}{4}, \frac{2-n}{4} \end{matrix}\right) /; n \in \mathbb{N}$$

Classical cases involving Struve *H*

03.03.26.0039.01

$$Y_\nu\left(\sqrt{z}\right) - \boldsymbol{H}_\nu\left(\sqrt{z}\right) = -\frac{\cos(\nu\pi)}{\pi^2}\; G_{1,3}^{3,1}\left(\frac{z}{4} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{\nu}{2} \end{matrix}\right)$$

Classical cases involving ${}_0F_1$

03.03.26.0040.01

$$Y_\nu(z)\, {}_0F_1\left(; b; -\frac{z^2}{4}\right) = -\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\; G_{3,5}^{2,2}\left(z^2 \,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1-\nu}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, -b-\frac{\nu}{2}+1, -b+\frac{\nu}{2}+1 \end{matrix}\right) /;$$

$$-b-\nu \notin \mathbb{N} \bigwedge \nu - b \notin \mathbb{N} \bigwedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

03.03.26.0109.01

$$Y_\nu(z)\,{}_0F_1\!\left(;\nu-n;-\frac{z^2}{4}\right)=\frac{2^{-n+\nu-1}\,z^{-\nu}\,\Gamma(\nu-n)}{\sqrt{\pi}}\left(z^\nu\,G_{3,5}^{2,2}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{1}{2}(n-\nu+1),\ \frac{1}{2}(n-\nu+2),\ n-\frac{\nu}{2}+\frac{1}{2}\\ n-\frac{\nu}{2}+1,\ \frac{\nu}{2},\ n-\frac{3\nu}{2}+1,\ -\frac{\nu}{2},\ n-\frac{\nu}{2}+\frac{1}{2}\end{array}\right)-\right.$$
$$\left.2\csc(\nu\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,z^{2k}\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-\nu+1)\,\Gamma(k-n+\nu)}\right)/;\ n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0110.01

$$Y_\nu(z)\,{}_0F_1\!\left(;-n-\nu;-\frac{z^2}{4}\right)=\frac{2^{-n-\nu-1}\,\Gamma(-n-\nu)}{\sqrt{\pi}}\left(2\cot(\nu\pi)\,z^\nu\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,z^{2k}\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}+\right.$$
$$\left.(-1)^n\,G_{3,5}^{2,2}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{1}{2}(n+\nu+1),\ \frac{1}{2}(n+\nu+2),\ \frac{1-\nu}{2}\\ n+\frac{\nu}{2}+1,\ -\frac{\nu}{2},\ n+\frac{3\nu}{2}+1,\ \frac{\nu}{2},\ \frac{1-\nu}{2}\end{array}\right)\right)/;\ n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0111.01

$$Y_\nu(z)\,{}_0F_1\!\left(;\nu;-\frac{z^2}{4}\right)=\frac{2^{\nu-1}\,\Gamma(\nu)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(z^2\,\middle|\,\begin{array}{c}1-\frac{\nu}{2},\ \frac{1-\nu}{2}\\ 1-\frac{\nu}{2},\ \frac{\nu}{2},\ 1-\frac{3\nu}{2},\ -\frac{\nu}{2}\end{array}\right)-\frac{2^\nu\,z^{-\nu}\csc(\nu\pi)}{\Gamma(1-\nu)}\ /;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0112.01

$$Y_\nu(z)\,{}_0F_1\!\left(;\nu-1;-\frac{z^2}{4}\right)=2^{\nu-2}\left(\frac{\Gamma(\nu-1)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(z^2\,\middle|\,\begin{array}{c}1-\frac{\nu}{2},\ \frac{3-\nu}{2}\\ 2-\frac{\nu}{2},\ \frac{\nu}{2},\ 2-\frac{3\nu}{2},\ -\frac{\nu}{2}\end{array}\right)-\frac{4\,z^{-\nu}\csc(\pi\nu)}{\Gamma(1-\nu)}\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0113.01

$$Y_\nu(z)\,{}_0F_1\!\left(;-\nu;-\frac{z^2}{4}\right)=-\frac{2^{-\nu-1}\,\Gamma(-\nu)}{\pi}\left(\cos(\pi\nu)\,z^\nu+\sqrt{\pi}\,G_{2,4}^{2,1}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{\nu+1}{2},\ \frac{1-\nu}{2}\\ -\frac{\nu}{2},\ \frac{\nu}{2},\ \frac{1-\nu}{2},\ \frac{3\nu}{2}+1\end{array}\right)\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0114.01

$$Y_\nu(z)\,{}_0F_1\!\left(;-\nu;-\frac{z^2}{4}\right)=2^{-\nu-1}\left(\frac{2\cot(\pi\nu)\,z^\nu}{\Gamma(\nu+1)}+\frac{\Gamma(-\nu)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{\nu+1}{2},\ \frac{\nu+2}{2},\ \frac{1-\nu}{2}\\ -\frac{\nu}{2},\ \frac{\nu+2}{2},\ \frac{1-\nu}{2},\ \frac{\nu}{2},\ \frac{3\nu}{2}+1\end{array}\right)\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0115.01

$$Y_\nu(z)\,{}_0F_1\!\left(;-\nu-1;-\frac{z^2}{4}\right)=2^{-\nu-2}\left(\frac{4\,z^\nu\cot(\pi\nu)}{\Gamma(\nu+1)}-\frac{\Gamma(-\nu-1)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(z^2\,\middle|\,\begin{array}{c}\frac{\nu+2}{2},\ \frac{\nu+3}{2},\ \frac{1-\nu}{2}\\ -\frac{\nu}{2},\ \frac{\nu+4}{2},\ \frac{1-\nu}{2},\ \frac{\nu}{2},\ \frac{3\nu}{2}+2\end{array}\right)\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0041.01

$$Y_\nu(z)\,{}_0F_1\!\left(;\nu+1;\frac{z^2}{4}\right)==02^{-\frac{\nu}{2}}\sqrt{\pi}\,\Gamma(\nu+1)\,G_{2,6}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}\frac{1-\nu}{4},\ \frac{3-\nu}{4}\\ -\frac{\nu}{4},\ \frac{2-\nu}{4},\ \frac{\nu}{4},\ \frac{1-\nu}{4},\ \frac{3-\nu}{4},\ -\frac{1}{4}(3\nu)\end{array}\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0042.01

$$Y_\nu(z)\,{}_0F_1\!\left(;1-\nu;\frac{z^2}{4}\right)==2^{\nu/2}\sqrt{\pi}\,\Gamma(1-\nu)\,G_{2,6}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}-\frac{1}{4}(\nu+1),\ \frac{1-\nu}{4}\\ -\frac{\nu}{4},\ \frac{\nu}{4},\ \frac{\nu+2}{4},\ \frac{1-\nu}{4},\ \frac{3\nu}{4},\ -\frac{1}{4}(\nu+1)\end{array}\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0116.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;b;-z) = -\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1-\nu}{2}\\ -\frac{\nu}{2},\,\frac{\nu}{2},\,\frac{1-\nu}{2},\,1-b-\frac{\nu}{2},\,1-b+\frac{\nu}{2}\end{matrix}\right)/;\,-b-\nu\notin\mathbb{N}\bigwedge -b+\nu\notin\mathbb{N}$$

03.03.26.0117.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;\nu-n;-z) = \frac{2^{-n-1}\,z^{-\frac{\nu}{2}}\,\Gamma(\nu-n)}{\sqrt{\pi}}\left(2^\nu\,z^{\nu/2}\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{1}{2}(n-\nu+1),\,\frac{1}{2}(n-\nu+2),\,n-\frac{\nu}{2}+\frac{1}{2}\\ n-\frac{\nu}{2}+1,\,\frac{\nu}{2},\,n-\frac{3\nu}{2}+1,\,-\frac{\nu}{2},\,n-\frac{\nu}{2}+\frac{1}{2}\end{matrix}\right)-\right.$$
$$\left.2\csc(\nu\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-\nu+1)\,\Gamma(k-n+\nu)}\right)/;\,n\in\mathbb{N}$$

03.03.26.0118.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;-n-\nu;-z) = \frac{2^{-n-\nu-1}\,\Gamma(-n-\nu)}{\sqrt{\pi}}\left(2^{\nu+1}\cot(\nu\pi)\,z^{\nu/2}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}+\right.$$
$$\left.(-1)^n\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{1}{2}(n+\nu+1),\,\frac{1}{2}(n+\nu+2),\,\frac{1-\nu}{2}\\ n+\frac{\nu}{2}+1,\,-\frac{\nu}{2},\,n+\frac{3\nu}{2}+1,\,\frac{\nu}{2},\,\frac{1-\nu}{2}\end{matrix}\right)\right)/;\,n\in\mathbb{N}$$

03.03.26.0119.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;\nu;-z) = \frac{2^{\nu-1}\,\Gamma(\nu)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(4z\,\middle|\,\begin{matrix}1-\frac{\nu}{2},\,\frac{1-\nu}{2}\\ 1-\frac{\nu}{2},\,\frac{\nu}{2},\,1-\frac{3\nu}{2},\,-\frac{\nu}{2}\end{matrix}\right)-\frac{z^{-\frac{\nu}{2}}\csc(\nu\pi)}{\Gamma(1-\nu)}$$

03.03.26.0120.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;\nu-1;-z) = \frac{2^{\nu-2}\,\Gamma(\nu-1)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(4z\,\middle|\,\begin{matrix}1-\frac{\nu}{2},\,\frac{3}{2}-\frac{\nu}{2}\\ 2-\frac{\nu}{2},\,\frac{\nu}{2},\,2-\frac{3\nu}{2},\,-\frac{\nu}{2}\end{matrix}\right)-\frac{z^{-\frac{\nu}{2}}\csc(\pi\nu)}{\Gamma(1-\nu)}$$

03.03.26.0121.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;-\nu;-z) = -\frac{2^{-\nu-1}\,\Gamma(-\nu)}{\pi}\left(2^\nu\cos(\pi\nu)\,z^{\nu/2}+\sqrt{\pi}\,G_{2,4}^{2,1}\!\left(4z\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\,\frac{1-\nu}{2}\\ -\frac{\nu}{2},\,\frac{\nu}{2},\,\frac{1-\nu}{2},\,\frac{3\nu}{2}+1\end{matrix}\right)\right)$$

03.03.26.0122.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;-\nu;-z) = \frac{\cot(\pi\nu)\,z^{\nu/2}}{\Gamma(\nu+1)}+\frac{2^{-\nu-1}\,\Gamma(-\nu)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\,\frac{\nu+2}{2},\,\frac{1-\nu}{2}\\ -\frac{\nu}{2},\,\frac{\nu+2}{2},\,\frac{1-\nu}{2},\,\frac{\nu}{2},\,\frac{3\nu}{2}+1\end{matrix}\right)$$

03.03.26.0123.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;-\nu-1;-z) = \frac{z^{\nu/2}\cot(\pi\nu)}{\Gamma(\nu+1)}-\frac{2^{-\nu-2}\,\Gamma(-\nu-1)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{\nu+2}{2},\,\frac{\nu+3}{2},\,\frac{1-\nu}{2}\\ \frac{\nu+4}{2},\,-\frac{\nu}{2},\,\frac{3\nu}{2}+2,\,\frac{\nu}{2},\,\frac{1-\nu}{2}\end{matrix}\right)$$

03.03.26.0124.01

$$Y_\nu\left(2\sqrt{z}\right)\,_0F_1(;\nu+1;z) = -2^{-\frac{\nu}{2}}\sqrt{\pi}\,\Gamma(\nu+1)\,G_{2,6}^{3,0}\!\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{3}{4}-\frac{\nu}{4},\,\frac{1}{4}-\frac{\nu}{4}\\ \frac{1}{2}-\frac{\nu}{4},\,-\frac{\nu}{4},\,\frac{\nu}{4},\,\frac{3}{4}-\frac{\nu}{4},\,\frac{1}{4}-\frac{\nu}{4},\,-\frac{1}{4}(3\nu)\end{matrix}\right)/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0125.01

$$Y_\nu\left(2\sqrt{z}\right)\,{}_0F_1(;1-\nu;z)=2^{\nu/2}\sqrt{\pi}\,\Gamma(1-\nu)\,G_{2,6}^{3,0}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(-\nu-1),\ \frac{1-\nu}{4}\\ \frac{\nu+2}{4},\ \frac{\nu}{4},\ -\frac{\nu}{4},\ \frac{1}{4}(-\nu-1),\ \frac{3\nu}{4},\ \frac{1-\nu}{4}\end{matrix}\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

Classical cases involving ${}_0\tilde{F}_1$

03.03.26.0043.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)==-\frac{2^{b-1}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\ 1-\frac{b}{2},\ \frac{1-\nu}{2}\\ -\frac{\nu}{2},\ \frac{\nu}{2},\ \frac{1-\nu}{2},\ 1-b-\frac{\nu}{2},\ 1-b+\frac{\nu}{2}\end{matrix}\right)/;\ -b-\nu\notin\mathbb{N}\bigwedge\nu-b\notin\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0126.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;\nu-n;-\frac{z^2}{4}\right)=\frac{2^{-n+\nu-1}z^{-\nu}}{\sqrt{\pi}}\left(z^\nu G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1}{2}(n-\nu+1),\ \frac{1}{2}(n-\nu+2),\ n-\frac{\nu}{2}+\frac{1}{2}\\ n-\frac{\nu}{2}+1,\ \frac{\nu}{2},\ n-\frac{3\nu}{2}+1,\ -\frac{\nu}{2},\ n-\frac{\nu}{2}+\frac{1}{2}\end{matrix}\right)-\right.$$

$$\left.2\csc(\nu\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-\nu+1)\,\Gamma(k-n+\nu)}\right)/;\ n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0127.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;-n-\nu;-\frac{z^2}{4}\right)=\frac{2^{-n-\nu-1}}{\sqrt{\pi}}\left(2\cot(\nu\pi)\,z^\nu\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}+\right.$$

$$\left.(-1)^n G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1}{2}(n+\nu+1),\ \frac{1}{2}(n+\nu+2),\ \frac{1-\nu}{2}\\ n+\frac{\nu}{2}+1,\ -\frac{\nu}{2},\ n+\frac{3\nu}{2}+1,\ \frac{\nu}{2},\ \frac{1-\nu}{2}\end{matrix}\right)\right)/;\ n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0128.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;\nu;-\frac{z^2}{4}\right)=\frac{2^{\nu-1}}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{matrix}1-\frac{\nu}{2},\ \frac{1-\nu}{2}\\ 1-\frac{\nu}{2},\ \frac{\nu}{2},\ 1-\frac{3\nu}{2},\ -\frac{\nu}{2}\end{matrix}\right)-\frac{2^\nu z^{-\nu}}{\pi}/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0129.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;\nu-1;-\frac{z^2}{4}\right)=2^{\nu-2}\left(\frac{1}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{matrix}1-\frac{\nu}{2},\ \frac{3-\nu}{2}\\ 2-\frac{\nu}{2},\ \frac{\nu}{2},\ 2-\frac{3\nu}{2},\ -\frac{\nu}{2}\end{matrix}\right)-\frac{4z^{-\nu}(\nu-1)}{\pi}\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0130.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;-\nu;-\frac{z^2}{4}\right)=-\frac{2^{-\nu-1}}{\pi}\left(\cos(\pi\nu)\,z^\nu+\sqrt{\pi}\,G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\ \frac{1-\nu}{2}\\ -\frac{\nu}{2},\ \frac{\nu}{2},\ \frac{1-\nu}{2},\ \frac{3\nu}{2}+1\end{matrix}\right)\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0131.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;-\nu;-\frac{z^2}{4}\right)=2^{-\nu-1}\left(\frac{1}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\ \frac{\nu+2}{2},\ \frac{1-\nu}{2}\\ -\frac{\nu}{2},\ \frac{\nu+2}{2},\ \frac{1-\nu}{2},\ \frac{\nu}{2},\ \frac{3\nu}{2}+1\end{matrix}\right)-\frac{2z^\nu\cos(\pi\nu)}{\pi}\right)/;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0132.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;-\nu-1;-\frac{z^2}{4}\right)=2^{-\nu-2}\left(\frac{4\,z^\nu\,(\nu+1)\cos(\pi\,\nu)}{\pi}-\frac{1}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{\nu+2}{2},\frac{\nu+3}{2},\frac{1-\nu}{2}\\ -\frac{\nu}{2},\frac{\nu+4}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+2\end{array}\right)\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.03.26.0044.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;1+\nu;\frac{z^2}{4}\right)=2^{-\frac{\nu}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}\frac{1-\nu}{4},\frac{3-\nu}{4}\\ -\frac{\nu}{4},\frac{2-\nu}{4},\frac{\nu}{4},\frac{1-\nu}{4},\frac{3-\nu}{4},-\frac{3\nu}{4}\end{array}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.03.26.0045.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;1-\nu;\frac{z^2}{4}\right)=2^{\nu/2}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}-\frac{\nu+1}{4},\frac{1-\nu}{4}\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1-\nu}{4},\frac{3\nu}{4},-\frac{\nu+1}{4}\end{array}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

03.03.26.0133.01

$$Y_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;b;-z)=-\frac{2^{b-1}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1-\nu}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},1-b-\frac{\nu}{2},1-b+\frac{\nu}{2}\end{array}\right)\;/;\;-b-\nu\notin\mathbb{N}\bigwedge-b+\nu\notin\mathbb{N}$$

03.03.26.0134.01

$$Y_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;\nu-n;-z)=\frac{2^{-n-1}\,z^{-\frac{\nu}{2}}}{\sqrt{\pi}}\left(2^\nu\,z^{\nu/2}\,G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1}{2}(n-\nu+1),\frac{1}{2}(n-\nu+2),n-\frac{\nu}{2}+\frac{1}{2}\\ n-\frac{\nu}{2}+1,\frac{\nu}{2},n-\frac{3\nu}{2}+1,-\frac{\nu}{2},n-\frac{\nu}{2}+\frac{1}{2}\end{array}\right)-\right.$$

$$\left.2\csc(\nu\,\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-\nu+1)\,\Gamma(k-n+\nu)}\right)\;/;\;n\in\mathbb{N}$$

03.03.26.0135.01

$$Y_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;-n-\nu;-z)=\frac{2^{-n-\nu-1}}{\sqrt{\pi}}\left(2^{\nu+1}\cot(\nu\,\pi)\,z^{\nu/2}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}+\right.$$

$$\left.(-1)^n\,G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1}{2}(n+\nu+1),\frac{1}{2}(n+\nu+2),\frac{1-\nu}{2}\\ n+\frac{\nu}{2}+1,-\frac{\nu}{2},n+\frac{3\nu}{2}+1,\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)\right)\;/;\;n\in\mathbb{N}$$

03.03.26.0136.01

$$Y_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;\nu;-z)=\frac{2^{\nu-1}}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(4\,z\,\middle|\,\begin{array}{c}1-\frac{\nu}{2},\frac{1-\nu}{2}\\ 1-\frac{\nu}{2},\frac{\nu}{2},1-\frac{3\nu}{2},-\frac{\nu}{2}\end{array}\right)-\frac{z^{-\frac{\nu}{2}}}{\pi}$$

03.03.26.0137.01

$$Y_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;\nu-1;-z)=\frac{2^{\nu-2}}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(4\,z\,\middle|\,\begin{array}{c}1-\frac{\nu}{2},\frac{3}{2}-\frac{\nu}{2}\\ 2-\frac{\nu}{2},\frac{\nu}{2},2-\frac{3\nu}{2},-\frac{\nu}{2}\end{array}\right)-\frac{z^{-\frac{\nu}{2}}(\nu-1)}{\pi}$$

03.03.26.0138.01

$$Y_\nu\left(2\sqrt{z}\right){}_0\tilde{F}_1(;-\nu;-z)=-\frac{2^{-\nu-1}}{\pi}\left(2^\nu\cos(\pi\,\nu)\,z^{\nu/2}+\sqrt{\pi}\,G_{2,4}^{2,1}\left(4\,z\,\middle|\,\begin{array}{c}\frac{\nu+1}{2},\frac{1-\nu}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{3\nu}{2}+1\end{array}\right)\right)$$

03.03.26.0139.01

$$Y_\nu\left(2\sqrt{z}\right)\,{}_0\tilde{F}_1(;-\nu;-z) = \frac{2^{-\nu-1}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4\,z\;\middle|\;\begin{matrix}\frac{\nu+1}{2},\frac{\nu+2}{2},\frac{1-\nu}{2}\\ -\frac{\nu}{2},\frac{\nu+2}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{3\nu}{2}+1\end{matrix}\right) - \frac{z^{\nu/2}\cos(\pi\,\nu)}{\pi}$$

03.03.26.0140.01

$$Y_\nu\left(2\sqrt{z}\right)\,{}_0\tilde{F}_1(;-\nu-1;z) = \frac{z^{\nu/2}\,(\nu+1)\cos(\pi\,\nu)}{\pi} - \frac{2^{-\nu-2}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4\,z\;\middle|\;\begin{matrix}\frac{\nu+2}{2},\frac{\nu+3}{2},\frac{1-\nu}{2}\\ \frac{\nu+4}{2},-\frac{\nu}{2},\frac{3\nu}{2}+2,\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)$$

03.03.26.0141.01

$$Y_\nu\left(2\sqrt{z}\right)\,{}_0\tilde{F}_1(;\nu+1;z) = -2^{-\frac{\nu}{2}}\sqrt{\pi}\;G_{2,6}^{3,0}\!\left(\frac{z^2}{4}\;\middle|\;\begin{matrix}\frac{3}{4}-\frac{\nu}{4},\frac{1}{4}-\frac{\nu}{4}\\ \frac{1}{2}-\frac{\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{3}{4}-\frac{\nu}{4},\frac{1}{4}-\frac{\nu}{4},-\frac{1}{4}(3\,\nu)\end{matrix}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

03.03.26.0142.01

$$Y_\nu\left(2\sqrt{z}\right)\,{}_0\tilde{F}_1(;1-\nu;z) = 2^{\nu/2}\sqrt{\pi}\;G_{2,6}^{3,0}\!\left(\frac{z^2}{4}\;\middle|\;\begin{matrix}\frac{1}{4}(-\nu-1),\frac{1-\nu}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},-\frac{\nu}{4},\frac{1}{4}(-\nu-1),\frac{3\nu}{4},\frac{1-\nu}{4}\end{matrix}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

Generalized cases for the direct function itself

03.03.26.0085.01

$$Y_\nu(z) = G_{1,3}^{2,0}\!\left(\frac{z}{2},\frac{1}{2}\;\middle|\;\begin{matrix}-\frac{1}{2}(\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(\nu+1)\end{matrix}\right)$$

03.03.26.0143.01

$$Y_{-\nu}(z)+Y_\nu(z) = -2\cos\left(\frac{\pi\,\nu}{2}\right)G_{1,3}^{2,0}\!\left(\frac{z}{2},\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},\frac{1}{2}\end{matrix}\right)$$

03.03.26.0144.01

$$Y_\nu(z)-Y_{-\nu}(z) = -2\sin\left(\frac{\pi\,\nu}{2}\right)G_{1,3}^{2,0}\!\left(\frac{z}{2},\frac{1}{2}\;\middle|\;\begin{matrix}0\\ -\frac{\nu}{2},\frac{\nu}{2},0\end{matrix}\right)$$

Generalized cases involving cos

03.03.26.0046.01

$$\cos(z)\,Y_\nu(z) = \frac{1}{\sqrt{2}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{4},\frac{3}{4},-\frac{1}{2}(\nu+1)\\ -\frac{\nu}{2},\frac{\nu}{2},-\frac{1}{2}(\nu+1),\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

03.03.26.0086.01

$$\cos(a+z)\,Y_\nu(z) = \frac{1}{\sqrt{2}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{a}{\pi}+\frac{\nu}{2}\\ \frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{a}{\pi}+\frac{\nu}{2}\end{matrix}\right) - \frac{\cos(\pi\,\nu)}{\pi^2\sqrt{2}}\,G_{3,5}^{4,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1-\nu}{2}-\frac{a}{\pi}\\ -\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\frac{1-\nu}{2}-\frac{a}{\pi}\end{matrix}\right)$$

Generalized cases involving sin

03.03.26.0047.01

$$\sin(z)\,Y_\nu(z) = \frac{1}{\sqrt{2}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{4},\frac{3}{4},-\frac{\nu}{2}\\ \frac{\nu+1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},-\frac{\nu}{2},\frac{\nu}{2}\end{matrix}\right)$$

03.03.26.0087.01

$$\sin(a+z)\,Y_\nu(z) = \frac{\cos(\pi\nu)}{\sqrt{2}\,\pi^2}\,G_{3,5}^{4,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},-\frac{a}{\pi}-\frac{\nu}{2}\\ \frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{a}{\pi}-\frac{\nu}{2}\end{matrix}\right)-\frac{1}{\sqrt{2}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{2}\left(\frac{2a}{\pi}+\nu+1\right)\\ \frac{\nu}{2},\frac{\nu+1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2}\left(\frac{2a}{\pi}+\nu+1\right)\end{matrix}\right)$$

Generalized cases for powers of *Y*

03.03.26.0048.01

$$Y_\nu(z)^2 = \frac{2}{\sqrt{\pi}}\,G_{2,4}^{3,0}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}-\nu\\ 0,-\nu,\nu,\frac{1}{2}-\nu\end{matrix}\right)+\frac{1}{\sqrt{\pi}}\,G_{1,3}^{1,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}\\ \nu,0,-\nu\end{matrix}\right)$$

03.03.26.0145.01

$$Y_{-\nu}(z)^2+Y_\nu(z)^2 = \frac{4\cos(\pi\nu)}{\sqrt{\pi}}\,G_{2,4}^{3,0}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\ 0,-\nu,\nu,\frac{1}{2}\end{matrix}\right)+\frac{2\cos(\pi\nu)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},0\\ \nu,-\nu,0,0\end{matrix}\right)$$

03.03.26.0146.01

$$Y_{-\nu}(z)^2-Y_\nu(z)^2 = \frac{2\sin(\pi\nu)}{\sqrt{\pi}}\,G_{1,3}^{2,0}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}\\ \nu,-\nu,0\end{matrix}\right)-\frac{4\sin(\pi\nu)}{\sqrt{\pi}}\,G_{1,3}^{2,0}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}\\ -\nu,\nu,0\end{matrix}\right)$$

Generalized cases for products of *Y*

03.03.26.0049.01

$$Y_{-\nu}(z)\,Y_\nu(z) = \frac{2}{\sqrt{\pi}}\,G_{2,4}^{3,0}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\ 0,-\nu,\nu,\frac{1}{2}\end{matrix}\right)+\frac{1}{\sqrt{\pi}}\,G_{1,3}^{1,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,-\nu,\nu\end{matrix}\right)$$

03.03.26.0147.01

$$Y_{\nu-1}(z)\,Y_\nu(z) = \frac{2}{\sqrt{\pi}}\,G_{2,4}^{3,0}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}0,1-\nu\\ \frac{1}{2}(2\nu-1),-\frac{1}{2},\frac{1}{2}(1-2\nu),1-\nu\end{matrix}\right)+\frac{1}{\sqrt{\pi}}\,G_{1,3}^{1,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}0\\ \frac{1}{2}(2\nu-1),-\frac{1}{2},\frac{1}{2}(1-2\nu)\end{matrix}\right)$$

03.03.26.0050.01

$$Y_{\frac{1}{2}-\nu}(z)\,Y_\nu(z) = \frac{2}{\sqrt{\pi}}\,G_{2,4}^{3,0}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}0,\frac{1}{2}\\ -\frac{1}{4},\frac{1}{4}-\nu,\nu-\frac{1}{4},\frac{1}{4}\end{matrix}\right)+\frac{1}{\sqrt{\pi}}\,G_{2,4}^{1,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}0,\frac{1}{2}\\ \frac{1}{4},-\frac{1}{4},\nu-\frac{1}{4},\frac{1}{4}-\nu\end{matrix}\right)$$

03.03.26.0051.01

$$Y_\mu(z)\,Y_\nu(z) = \frac{2}{\sqrt{\pi}}\,G_{3,5}^{4,0}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{1-\mu-\nu}{2}\\ \frac{\mu+\nu}{2},\frac{\mu-\nu}{2},\frac{\nu-\mu}{2},-\frac{\mu+\nu}{2},\frac{1-\mu-\nu}{2}\end{matrix}\right)+\frac{1}{\sqrt{\pi}}\,G_{2,4}^{1,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}0,\frac{1}{2}\\ \frac{\mu+\nu}{2},\frac{\mu-\nu}{2},\frac{\nu-\mu}{2},-\frac{\mu+\nu}{2}\end{matrix}\right)\;/;\;-\mu-\nu-1\notin\mathbb{N}$$

03.03.26.0148.01

$$Y_{-n-\nu-1}(z)\,Y_\nu(z) = \frac{\csc(\pi\nu)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{n}{2}+1\\ -\frac{1}{2}(n+1)-\nu,\frac{n+1}{2},\frac{n}{2}+1,\frac{n+1}{2}+\nu,-\frac{1}{2}(n+1)\end{matrix}\right)+$$

$$\frac{\cot(\nu\pi)(-1)^n}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}0,\frac{1}{2},\frac{n}{2}+\nu+1\\ \frac{n+1}{2},\frac{n+1}{2}+\nu,-\frac{1}{2}(n+1)-\nu,-\frac{1}{2}(n+1),\frac{n}{2}+\nu+1\end{matrix}\right)-$$

$$\frac{2\cot^2(\nu\pi)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k-n-1}\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\;/;\;n\in\mathbb{N}$$

03.03.26.0149.01

$$Y_{-\nu-1}(z)\,Y_\nu(z) = \frac{2\cos(\pi\nu)\cot(\pi\nu)}{\pi z} + \frac{\csc(\pi\nu)}{\sqrt{\pi}} G_{3,5}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, 1 \\ -\nu-\frac{1}{2}, \frac{1}{2}, 1, \nu+\frac{1}{2}, -\frac{1}{2} \end{matrix}\right) + \frac{\cot(\pi\nu)}{\sqrt{\pi}} G_{3,5}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, \nu+1 \\ \frac{1}{2}, \nu+\frac{1}{2}, -\nu-\frac{1}{2}, -\frac{1}{2}, \nu+1 \end{matrix}\right)$$

03.03.26.0150.01

$$Y_{-\nu-2}(z)\,Y_\nu(z) = -\frac{4(\nu+1)\cos(\pi\nu)\cot(\pi\nu)}{\pi z^2} + \frac{\csc(\pi\nu)}{\sqrt{\pi}} G_{3,5}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, \frac{3}{2} \\ -\nu-1, 1, \frac{3}{2}, \nu+1, -1 \end{matrix}\right) - \frac{\cot(\pi\nu)}{\sqrt{\pi}} G_{3,5}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, \nu+\frac{3}{2} \\ 1, \nu+1, -\nu-1, -1, \nu+\frac{3}{2} \end{matrix}\right)$$

03.03.26.0151.01

$$Y_{-\mu}(z)\,Y_{-\nu}(z) + Y_\mu(z)\,Y_\nu(z) = \frac{4\cos\left(\frac{1}{2}\pi(\mu+\nu)\right)}{\sqrt{\pi}} G_{3,5}^{4,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, \frac{1}{2} \\ \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu), \frac{\nu-\mu}{2}, \frac{\mu-\nu}{2}, \frac{1}{2} \end{matrix}\right) + \frac{2\cos\left(\frac{1}{2}\pi(\mu+\nu)\right)}{\sqrt{\pi}} G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 0 \\ \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu), \frac{\nu-\mu}{2}, \frac{\mu-\nu}{2} \end{matrix}\right)$$

03.03.26.0152.01

$$Y_\mu(z)\,Y_\nu(z) - Y_{-\mu}(z)\,Y_{-\nu}(z) = \frac{4\sin\left(\frac{1}{2}\pi(\mu+\nu)\right)}{\sqrt{\pi}} G_{3,5}^{4,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu), \frac{\nu-\mu}{2}, \frac{\mu-\nu}{2}, 0 \end{matrix}\right) - \frac{2\sin\left(\frac{1}{2}\pi(\mu+\nu)\right)}{\sqrt{\pi}} G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu), \frac{\mu-\nu}{2}, \frac{\nu-\mu}{2} \end{matrix}\right)$$

Generalized cases involving Bessel J

03.03.26.0052.01

$$J_\nu(z)\cos(a\pi) + Y_\nu(z)\sin(a\pi) = G_{1,3}^{2,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} -a-\frac{\nu}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, -a-\frac{\nu}{2} \end{matrix}\right)$$

03.03.26.0153.01

$$\cos(a\pi)\,J_\nu(z) - \sin(a\pi)\,Y_\nu(z) = G_{1,3}^{2,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} a-\frac{\nu}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, a-\frac{\nu}{2} \end{matrix}\right)$$

03.03.26.0053.01

$$J_\nu(z)\,Y_\nu(z) = -\frac{1}{\sqrt{\pi}} G_{1,3}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \nu, -\nu \end{matrix}\right)$$

03.03.26.0054.01

$$J_{-\nu}(z)\,Y_\nu(z) = \frac{1}{\sqrt{\pi}} G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, -\nu-\frac{1}{2} \\ 0, -\nu, -\nu-\frac{1}{2}, \nu \end{matrix}\right)$$

03.03.26.0055.01

$$J_{\nu+1}(z)\,Y_\nu(z) = \frac{1}{\sqrt{\pi}} G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 0 \\ \frac{1}{2}, \nu+\frac{1}{2}, -\frac{1}{2}, -\nu-\frac{1}{2} \end{matrix}\right)$$

03.03.26.0056.01

$$J_{\nu+2}(z)\,Y_\nu(z) = \frac{1}{\sqrt{\pi}} G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ 1, \nu+1, -1, -\nu-1 \end{matrix}\right)$$

03.03.26.0057.01

$$J_\mu(z)\,Y_\nu(z) = -\frac{1}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}0,\ \frac{1}{2},\ \frac{\mu-\nu+1}{2}\\ \frac{\mu+\nu}{2},\ \frac{\mu-\nu}{2},\ \frac{\mu-\nu+1}{2},\ -\frac{\mu+\nu}{2},\ \frac{\nu-\mu}{2}\end{matrix}\right)\,/;\ -\mu-\nu-1\notin\mathbb{N}$$

03.03.26.0154.01

$$J_\nu(z)\,Y_{-n-\nu-1}(z) = \frac{(-1)^n}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}0,\ \frac{1}{2},\ \frac{n}{2}+\nu+1\\ \frac{n+1}{2},\ \frac{n+1}{2}+\nu,\ -\frac{1}{2}(n+1)-\nu,\ -\frac{1}{2}(n+1),\ \frac{n}{2}+\nu+1\end{matrix}\right) - \frac{2\cot(\nu\pi)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,z^{2k-n-1}\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}\,/;\ n\in\mathbb{N}$$

03.03.26.0155.01

$$J_\nu(z)\,Y_{-\nu-1}(z) = \frac{2\cos(\pi\nu)}{\pi z} + \frac{1}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}0,\ \frac{1}{2},\ \nu+1\\ \frac{1}{2},\ \nu+\frac{1}{2},\ -\frac{1}{2},\ -\nu-\frac{1}{2},\ \nu+1\end{matrix}\right)$$

03.03.26.0156.01

$$J_\nu(z)\,Y_{-\nu-2}(z) = -\frac{4(\nu+1)\cos(\pi\nu)}{\pi z^2} - \frac{1}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}0,\ \frac{1}{2},\ \nu+\frac{3}{2}\\ 1,\ \nu+1,\ -1,\ -\nu-1,\ \nu+\frac{3}{2}\end{matrix}\right)$$

03.03.26.0058.01

$$J_\nu(z)\,Y_{-\nu}(z) + J_{-\nu}(z)\,Y_\nu(z) = -\frac{2}{\sqrt{\pi}}\,G_{1,3}^{2,0}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{2}\\ \nu,\ -\nu,\ 0\end{matrix}\right)$$

03.03.26.0059.01

$$J_\nu(z)\,Y_{-\nu}(z) - J_{-\nu}(z)\,Y_\nu(z) = \frac{\sin(2\pi\nu)}{\pi^{5/2}}\,G_{1,3}^{3,1}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{2}\\ 0,\ \nu,\ -\nu\end{matrix}\right)$$

03.03.26.0060.01

$$J_\nu(z)\,Y_\mu(z) + J_\mu(z)\,Y_\nu(z) = -\frac{2}{\sqrt{\pi}}\,G_{2,4}^{3,0}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}0,\ \frac{1}{2}\\ \frac{\mu+\nu}{2},\ \frac{\mu-\nu}{2},\ \frac{\nu-\mu}{2},\ -\frac{\mu+\nu}{2}\end{matrix}\right)$$

03.03.26.0061.01

$$J_\nu(z)\,Y_\mu(z) - J_\mu(z)\,Y_\nu(z) = \frac{\sin(\pi(\nu-\mu))}{\pi^{5/2}}\,G_{2,4}^{3,2}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}0,\ \frac{1}{2}\\ \frac{\mu-\nu}{2},\ \frac{\nu-\mu}{2},\ \frac{\mu+\nu}{2},\ -\frac{\mu+\nu}{2}\end{matrix}\right)\,/;\ -\mu-\nu-1\notin\mathbb{N}$$

03.03.26.0062.01

$$J_\nu(z)^2 + Y_\nu(z)^2 = \frac{2\cos(\pi\nu)}{\pi^{5/2}}\,G_{1,3}^{3,1}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{2}\\ 0,\ -\nu,\ \nu\end{matrix}\right)$$

03.03.26.0063.01

$$J_\nu(z)^2 - Y_\nu(z)^2 = -\frac{2}{\sqrt{\pi}}\,G_{2,4}^{3,0}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{2},\ \frac{1}{2}-\nu\\ 0,\ -\nu,\ \nu,\ \frac{1}{2}-\nu\end{matrix}\right)$$

03.03.26.0088.01

$$J_{-\nu}(z)\,J_\nu(z) + Y_{-\nu}(z)\,Y_\nu(z) = \frac{2\cos^2(\pi\nu)}{\pi^{5/2}}\,G_{1,3}^{3,1}\!\left(z,\ \frac{1}{2}\;\middle|\;\begin{matrix}\frac{1}{2}\\ 0,\ -\nu,\ \nu\end{matrix}\right)$$

03.03.26.0064.01

$$J_\nu(z)\,J_{-\nu}(z) - Y_\nu(z)\,Y_{-\nu}(z) = -\frac{2}{\sqrt{\pi}}\,G_{2,4}^{3,0}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\ \frac{1}{2}\\ 0,\ -\nu,\ \nu,\ \frac{1}{2}\end{matrix}\right)$$

03.03.26.0065.01

$$J_\mu(z)\,J_\nu(z) - Y_\mu(z)\,Y_\nu(z) = -\frac{2}{\sqrt{\pi}}\,G_{3,5}^{4,0}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}0,\ \frac{1}{2},\ \frac{1-\mu-\nu}{2}\\ \frac{\mu+\nu}{2},\ \frac{\mu-\nu}{2},\ \frac{\nu-\mu}{2},\ -\frac{\mu+\nu}{2},\ \frac{1-\mu-\nu}{2}\end{matrix}\right)$$

03.03.26.0066.01

$$J_\mu(z)\,J_\nu(z) + Y_\mu(z)\,Y_\nu(z) = \frac{\cos(\pi\,\mu)}{\pi^{5/2}}\,G_{2,4}^{3,2}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}0,\ \frac{1}{2}\\ \frac{\mu+\nu}{2},\ \frac{\mu-\nu}{2},\ -\frac{\mu+\nu}{2},\ \frac{\nu-\mu}{2}\end{matrix}\right) + \frac{\cos(\pi\,\nu)}{\pi^{5/2}}\,G_{2,4}^{3,2}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}0,\ \frac{1}{2}\\ \frac{\mu+\nu}{2},\ -\frac{\mu+\nu}{2},\ \frac{\nu-\mu}{2},\ \frac{\mu-\nu}{2}\end{matrix}\right) /;\ \mu+\nu \notin \mathbb{Z}$$

Generalized cases involving cos, sin, J

03.03.26.0067.01

$$\sin(z)\,J_\nu(z) + \cos(z)\,Y_\nu(z) = -\sqrt{2}\,G_{2,4}^{3,0}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\ \frac{3}{4}\\ -\frac{\nu}{2},\ \frac{\nu}{2},\ \frac{\nu+1}{2},\ \frac{1-\nu}{2}\end{matrix}\right)$$

03.03.26.0068.01

$$\cos(z)\,J_\nu(z) - \sin(z)\,Y_\nu(z) = \sqrt{2}\,G_{2,4}^{3,0}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\ \frac{3}{4}\\ \frac{1-\nu}{2},\ \frac{\nu}{2},\ \frac{\nu+1}{2},\ -\frac{\nu}{2}\end{matrix}\right)$$

03.03.26.0069.01

$$\sin(z)\,J_\nu(z) - \cos(z)\,Y_\nu(z) = \frac{\cos(\nu\,\pi)}{\pi^2\,\sqrt{2}}\,G_{2,4}^{3,2}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\ \frac{3}{4}\\ -\frac{\nu}{2},\ \frac{\nu}{2},\ \frac{\nu+1}{2},\ \frac{1-\nu}{2}\end{matrix}\right)$$

03.03.26.0070.01

$$\cos(z)\,J_\nu(z) + \sin(z)\,Y_\nu(z) = \frac{\cos(\nu\,\pi)}{\pi^2\,\sqrt{2}}\,G_{2,4}^{3,2}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\ \frac{3}{4}\\ \frac{1-\nu}{2},\ \frac{\nu}{2},\ \frac{\nu+1}{2},\ -\frac{\nu}{2}\end{matrix}\right)$$

03.03.26.0071.01

$$\sin(a+z)\,J_\nu(z) - \cos(a+z)\,Y_\nu(z) = \frac{\cos(\pi\,\nu)}{\pi^2\,\sqrt{2}}\,G_{3,5}^{4,2}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\ \frac{3}{4},\ \frac{1-\nu}{2}-\frac{a}{\pi}\\ -\frac{\nu}{2},\ \frac{1-\nu}{2},\ \frac{\nu}{2},\ \frac{\nu+1}{2},\ \frac{1-\nu}{2}-\frac{a}{\pi}\end{matrix}\right)$$

03.03.26.0157.01

$$\cos(a+z)\,J_\nu(z) + \sin(a+z)\,Y_\nu(z) = \frac{\cos(\pi\,\nu)}{\sqrt{2}\,\pi^2}\,G_{3,5}^{4,2}\!\left(z,\ \frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\ \frac{3}{4},\ -\frac{a}{\pi}-\frac{\nu}{2}\\ \frac{1-\nu}{2},\ -\frac{\nu}{2},\ \frac{\nu}{2},\ \frac{\nu+1}{2},\ -\frac{a}{\pi}-\frac{\nu}{2}\end{matrix}\right)$$

Generalized cases involving Bessel I

03.03.26.0072.01

$$I_\nu(z)\,Y_\nu(z) = -\sqrt{\pi}\,G_{2,6}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\ \frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\ \frac{3}{4}\\ 0,\ \frac{1}{2},\ \frac{\nu}{2},\ \frac{1}{4},\ \frac{3}{4},\ -\frac{\nu}{2}\end{matrix}\right)$$

03.03.26.0073.01

$$I_{-\nu}(z)\,Y_\nu(z) = \sqrt{\pi}\, G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{1+2\nu}{4}, \frac{1-2\nu}{4} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-2\nu}{4}, -\frac{1+2\nu}{4} \end{matrix}\right)$$

03.03.26.0158.01

$$I_{-\nu}(z)\,Y_\nu(z) = \sqrt{\pi}\, G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{1}{4}(2\nu+1), \frac{1}{4}(1-2\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1}{4}(1-2\nu), -\frac{1}{4}(2\nu+1) \end{matrix}\right)$$

Generalized cases involving Bessel *K*

03.03.26.0074.01

$$K_\nu(z)\,Y_\nu(z) = -\frac{1}{4\sqrt{\pi}}\, G_{1,5}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

03.03.26.0159.01

$$K_\nu(z)\,Y_{-\nu}(z) = -\frac{1}{4\sqrt{\pi}}\, G_{1,5}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

03.03.26.0075.01

$$K_0(z) - \frac{\pi}{2} Y_0(z) = \frac{\pi}{2} G_{0,4}^{2,0}\left(\frac{z}{4}, \frac{1}{4} \,\middle|\, 0, 0, \frac{1}{2}, \frac{1}{2}\right)$$

03.03.26.0076.01

$$K_n(z) + \frac{\pi}{2} Y_n(z) = \frac{\pi}{2} G_{1,5}^{3,0}\left(\frac{z}{4}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{n}{4} \\ \frac{2-n}{4}, \frac{n}{4}, \frac{n+2}{4}, -\frac{n}{4}, -\frac{n}{4} \end{matrix}\right) /; n \in \mathbb{N}$$

03.03.26.0077.01

$$K_n(z) - \frac{\pi}{2} Y_n(z) = \frac{\pi}{2} G_{1,5}^{3,0}\left(\frac{z}{4}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{2-n}{4} \\ -\frac{n}{4}, \frac{n}{4}, \frac{n+2}{4}, \frac{2-n}{4}, \frac{2-n}{4} \end{matrix}\right) /; n \in \mathbb{N}$$

Generalized cases involving Struve *H*

03.03.26.0078.01

$$Y_\nu(z) - \boldsymbol{H}_\nu(z) = -\frac{\cos(\nu\pi)}{\pi^2} G_{1,3}^{3,1}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{\nu}{2} \end{matrix}\right)$$

Generalized cases involving ${}_0F_1$

03.03.26.0080.01

$$Y_\nu(z)\,{}_0F_1\left(; \nu+1; \frac{z^2}{4}\right) = -2^{-\frac{\nu}{2}} \sqrt{\pi}\, \Gamma(\nu+1)\, G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{4}, \frac{3-\nu}{4} \\ -\frac{\nu}{4}, \frac{2-\nu}{4}, \frac{\nu}{4}, \frac{1-\nu}{4}, \frac{3-\nu}{4}, -\frac{3\nu}{4} \end{matrix}\right)$$

03.03.26.0081.01

$$Y_\nu(z)\,{}_0F_1\left(; 1-\nu; \frac{z^2}{4}\right) = 2^{\nu/2} \sqrt{\pi}\, \Gamma(1-\nu)\, G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{1+\nu}{4}, \frac{1-\nu}{4} \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1-\nu}{4}, \frac{3\nu}{4}, -\frac{1+\nu}{4} \end{matrix}\right)$$

03.03.26.0079.01

$$Y_\nu(z)\,{}_0F_1\left(;b;-\frac{z^2}{4}\right)=-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1-\nu}{2}\\-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},1-b-\frac{\nu}{2},1-b+\frac{\nu}{2}\end{array}\right)/;-b-\nu\notin\mathbb{N}\bigwedge\nu-b\notin\mathbb{N}$$

03.03.26.0160.01

$$Y_\nu(z)\,{}_0F_1\left(;\nu-n;-\frac{z^2}{4}\right)=\frac{2^{-n+\nu-1}\,z^{-\nu}\,\Gamma(\nu-n)}{\sqrt{\pi}}\left(z^\nu\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{2}(n-\nu+1),\frac{1}{2}(n-\nu+2),n-\frac{\nu}{2}+\frac{1}{2}\\n-\frac{\nu}{2}+1,\frac{\nu}{2},n-\frac{3\nu}{2}+1,-\frac{\nu}{2},n-\frac{\nu}{2}+\frac{1}{2}\end{array}\right)-\right.$$

$$\left.2\csc(\nu\pi)\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{(-1)^{k+\lfloor\frac{n+1}{2}\rfloor}z^{2k}\,\Gamma\left(k-n+\lfloor\frac{n}{2}\rfloor+\frac{1}{2}\right)\left(1-k+\lfloor\frac{n}{2}\rfloor\right)_{n-\lfloor\frac{n}{2}\rfloor}}{k!\,\Gamma(k-\nu+1)\,\Gamma(k-n+\nu)}\right)/;n\in\mathbb{N}$$

03.03.26.0161.01

$$Y_\nu(z)\,{}_0F_1\left(;-n-\nu;-\frac{z^2}{4}\right)=\frac{2^{-n-\nu-1}\,\Gamma(-n-\nu)}{\sqrt{\pi}}\left(2\cot(\nu\pi)\,z^\nu\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{(-1)^{k+\lfloor\frac{n+1}{2}\rfloor}z^{2k}\,\Gamma\left(k-n+\lfloor\frac{n}{2}\rfloor+\frac{1}{2}\right)\left(1-k+\lfloor\frac{n}{2}\rfloor\right)_{n-\lfloor\frac{n}{2}\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}+\right.$$

$$\left.(-1)^n\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{2}(n+\nu+1),\frac{1}{2}(n+\nu+2),\frac{1-\nu}{2}\\n+\frac{\nu}{2}+1,-\frac{\nu}{2},n+\frac{3\nu}{2}+1,\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)\right)/;n\in\mathbb{N}$$

03.03.26.0162.01

$$Y_\nu(z)\,{}_0F_1\left(;\nu;-\frac{z^2}{4}\right)=\frac{2^{\nu-1}\,\Gamma(\nu)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\frac{\nu}{2},\frac{1-\nu}{2}\\1-\frac{\nu}{2},\frac{\nu}{2},1-\frac{3\nu}{2},-\frac{\nu}{2}\end{array}\right)-\frac{2^\nu\,z^{-\nu}\csc(\nu\pi)}{\Gamma(1-\nu)}$$

03.03.26.0163.01

$$Y_\nu(z)\,{}_0F_1\left(;\nu-1;-\frac{z^2}{4}\right)=2^{\nu-2}\left(\frac{\Gamma(\nu-1)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\frac{\nu}{2},\frac{3}{2}-\frac{\nu}{2}\\2-\frac{\nu}{2},\frac{\nu}{2},2-\frac{3\nu}{2},-\frac{\nu}{2}\end{array}\right)-\frac{4\,z^{-\nu}\csc(\pi\nu)}{\Gamma(1-\nu)}\right)$$

03.03.26.0164.01

$$Y_\nu(z)\,{}_0F_1\left(;-\nu;-\frac{z^2}{4}\right)=-\frac{2^{-\nu-1}\,\Gamma(-\nu)}{\pi}\left(\cos(\pi\nu)\,z^\nu+\sqrt{\pi}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{\nu+1}{2},\frac{1-\nu}{2}\\\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}+1,\frac{1-\nu}{2}\end{array}\right)\right)$$

03.03.26.0165.01

$$Y_\nu(z)\,{}_0F_1\left(;-\nu;-\frac{z^2}{4}\right)=2^{-\nu-1}\left(\frac{2\cot(\pi\nu)\,z^\nu}{\Gamma(\nu+1)}+\frac{\Gamma(-\nu)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{\nu+1}{2},\frac{\nu+2}{2},\frac{1-\nu}{2}\\\frac{\nu+2}{2},-\frac{\nu}{2},\frac{3\nu}{2}+1,\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)\right)$$

03.03.26.0166.01

$$Y_\nu(z)\,{}_0F_1\left(;-\nu-1;-\frac{z^2}{4}\right)=2^{-\nu-2}\left(\frac{4\,z^\nu\cot(\pi\nu)}{\Gamma(\nu+1)}-\frac{\Gamma(-\nu-1)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{\nu+2}{2},\frac{\nu+3}{2},\frac{1-\nu}{2}\\\frac{\nu+4}{2},-\frac{\nu}{2},\frac{3\nu}{2}+2,\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)\right)$$

Generalized cases involving ${}_0\tilde{F}_1$

03.03.26.0083.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;\nu+1;\frac{z^2}{4}\right)=-2^{-\frac{\nu}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1-\nu}{4},\frac{3-\nu}{4}\\-\frac{\nu}{4},\frac{2-\nu}{4},\frac{\nu}{4},\frac{1-\nu}{4},\frac{3-\nu}{4},-\frac{3\nu}{4}\end{matrix}\right)$$

03.03.26.0084.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;1-\nu;\frac{z^2}{4}\right)=2^{\nu/2}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}-\frac{1+\nu}{4},\frac{1-\nu}{4}\\-\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1-\nu}{4},\frac{3\nu}{4},-\frac{1+\nu}{4}\end{matrix}\right)$$

03.03.26.0082.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)=-\frac{2^{b-1}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1-\nu}{2}\\-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},1-b-\frac{\nu}{2},1-b+\frac{\nu}{2}\end{matrix}\right)/;-b-\nu\notin\mathbb{N}\land\nu-b\notin\mathbb{N}$$

03.03.26.0167.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)=\frac{2^{-n+\nu-1}z^{-\nu}}{\sqrt{\pi}}\left(z^\nu G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}(n-\nu+1),\frac{1}{2}(n-\nu+2),n-\frac{\nu}{2}+\frac{1}{2}\\n-\frac{\nu}{2}+1,\frac{\nu}{2},n-\frac{3\nu}{2}+1,-\frac{\nu}{2},n-\frac{\nu}{2}+\frac{1}{2}\end{matrix}\right)-\right.$$

$$\left.2\csc(\nu\pi)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-\nu+1)\,\Gamma(k-n+\nu)}\right)/;n\in\mathbb{N}$$

03.03.26.0168.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;-\nu-n;-\frac{z^2}{4}\right)=\frac{2^{-n-\nu-1}}{\sqrt{\pi}}\left(2\cot(\nu\pi)\,z^\nu\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(k-n-\nu)\,\Gamma(k+\nu+1)}+\right.$$

$$\left.(-1)^n G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}(n+\nu+1),\frac{1}{2}(n+\nu+2),\frac{1-\nu}{2}\\n+\frac{\nu}{2}+1,-\frac{\nu}{2},n+\frac{3\nu}{2}+1,\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)\right)/;n\in\mathbb{N}$$

03.03.26.0169.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;\nu;-\frac{z^2}{4}\right)=\frac{2^{\nu-1}}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{\nu}{2},\frac{1-\nu}{2}\\1-\frac{\nu}{2},\frac{\nu}{2},1-\frac{3\nu}{2},-\frac{\nu}{2}\end{matrix}\right)-\frac{2^\nu z^{-\nu}}{\pi}$$

03.03.26.0170.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;\nu-1;-\frac{z^2}{4}\right)=2^{\nu-2}\left(\frac{1}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{\nu}{2},\frac{3}{2}-\frac{\nu}{2}\\2-\frac{\nu}{2},\frac{\nu}{2},2-\frac{3\nu}{2},-\frac{\nu}{2}\end{matrix}\right)-\frac{4z^{-\nu}(\nu-1)}{\pi}\right)$$

03.03.26.0171.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;-\nu;-\frac{z^2}{4}\right)=-\frac{2^{-\nu-1}}{\pi}\left(\cos(\pi\nu)\,z^\nu+\sqrt{\pi}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{1-\nu}{2}\\\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}+1,\frac{1-\nu}{2}\end{matrix}\right)\right)$$

03.03.26.0172.01

$$Y_\nu(z)\,{}_0\tilde{F}_1\left(;-\nu;-\frac{z^2}{4}\right)=2^{-\nu-1}\left(\frac{1}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{\nu+2}{2},\frac{1-\nu}{2}\\\frac{\nu+2}{2},-\frac{\nu}{2},\frac{3\nu}{2}+1,\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)-\frac{2z^\nu\cos(\pi\nu)}{\pi}\right)$$

03.03.26.0173.01

$$Y_\nu(z)\, {}_0\tilde{F}_1\left(; -\nu-1; -\frac{z^2}{4}\right) = 2^{-\nu-2}\left(\frac{4\, z^\nu\, (\nu+1)\cos(\pi\,\nu)}{\pi} - \frac{1}{\sqrt{\pi}}\, G_{3,5}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{\nu+2}{2}, \frac{\nu+3}{2}, \frac{1-\nu}{2} \\ \frac{\nu+4}{2}, -\frac{\nu}{2}, \frac{3\,\nu}{2}+2, \frac{\nu}{2}, \frac{1-\nu}{2} \end{array}\right)\right)$$

Representations through equivalent functions

With related functions

03.03.27.0001.01

$$Y_\nu(z) = \csc(\pi\,\nu)\,(\cos(\nu\,\pi)\, J_\nu(z) - J_{-\nu}(z))\ /;\ \nu \notin \mathbb{Z}$$

03.03.27.0002.01

$$J_\nu(z)\, Y_{\nu+1}(z) - J_{\nu+1}(z)\, Y_\nu(z) = -\frac{2}{\pi\, z}$$

03.03.27.0003.02

$$Y_\nu(z) = \frac{(i\, z)^{-\nu}}{\pi}\, z^{-\nu}\left(\pi\csc(\pi\,\nu)\left(z^{2\,\nu}\cos(\pi\,\nu) - (i\, z)^{2\,\nu}\right) I_\nu(i\, z) - 2\,(i\, z)^{2\,\nu} K_\nu(i\, z)\right)\ /;\ \nu \notin \mathbb{Z}$$

03.03.27.0007.01

$$Y_\nu(z) = -\frac{2}{\pi}\,(i^\nu\, K_\nu(i\, z) + (\log(i\, z) - \log(z))\, i^{-\nu}\, I_\nu(i\, z))\ /;\ \nu \in \mathbb{Z}$$

03.03.27.0004.01

$$Y_\nu(z) = \frac{z^{-2\,\nu}}{\pi}\left(-2\, z^\nu\, K_\nu(-i\, z)\,(-i\, z)^\nu - \pi\, J_\nu(z)\csc(\pi\,\nu)\,(-i\, z)^{2\,\nu} + \pi\, z^{2\,\nu}\, J_\nu(z)\cot(\pi\,\nu)\right)\ /;\ \nu \notin \mathbb{Z}$$

03.03.27.0008.01

$$Y_\nu(z) = -\frac{2}{\pi}\,(i^\nu\, K_\nu(i\, z) + (\log(i\, z) - \log(z))\, J_\nu(z))\ /;\ \nu \in \mathbb{Z}$$

03.03.27.0005.01

$$Y_\nu(z) = 2^{-\nu}\, z^\nu\cot(\pi\,\nu)\, {}_0\tilde{F}_1\left(; \nu+1; -\frac{z^2}{4}\right) - 2^\nu\, z^{-\nu}\csc(\pi\,\nu)\, {}_0\tilde{F}_1\left(; 1-\nu; -\frac{z^2}{4}\right)\ /;\ \nu \notin \mathbb{Z}$$

03.03.27.0006.01

$$Y_\nu(z) = -\frac{2^{-\nu}\, z^\nu\cos(\nu\,\pi)\,\Gamma(-\nu)}{\pi}\, {}_0F_1\left(; \nu+1; -\frac{z^2}{4}\right) - \frac{2^\nu\, z^{-\nu}\,\Gamma(\nu)}{\pi}\, {}_0F_1\left(; 1-\nu; -\frac{z^2}{4}\right)\ /;\ \nu \notin \mathbb{Z}$$

03.03.27.0009.01

$$Y_\nu(z) = \csc(\pi\,\nu)\left(e^{\frac{1}{4}(-3)\, i\,\pi\,\nu}\left(-(-1)^{3/4}\, z\right)^\nu\left(i\operatorname{bei}_{-\nu}\left(-(-1)^{3/4}\, z\right) - \operatorname{ber}_{-\nu}\left(-(-1)^{3/4}\, z\right)\right) z^{-\nu} + e^{\frac{3\, i\,\pi\,\nu}{4}}\left(-(-1)^{3/4}\, z\right)^{-\nu}\cos(\pi\,\nu)\left(\operatorname{ber}_\nu\left(-(-1)^{3/4}\, z\right) - i\operatorname{bei}_\nu\left(-(-1)^{3/4}\, z\right)\right) z^\nu\right)\ /;\ \nu \notin \mathbb{Z}$$

03.03.27.0010.01

$$Y_\nu\left(\sqrt[4]{-1}\, z\right) = \csc(\pi\,\nu)\left(e^{\frac{3\, i\,\pi\,\nu}{4}}\left(\sqrt[4]{-1}\, z\right)^\nu\cos(\pi\,\nu)\,(\operatorname{ber}_\nu(z) - i\operatorname{bei}_\nu(z))\, z^{-\nu} + e^{\frac{1}{4}(-3)\, i\,\pi\,\nu}\left(\sqrt[4]{-1}\, z\right)^{-\nu}(i\operatorname{bei}_{-\nu}(z) - \operatorname{ber}_{-\nu}(z))\, z^\nu\right)\ /;\ \nu \notin \mathbb{Z}$$

03.03.27.0011.01

$$Y_\nu(z) = \frac{2\,(-1)^\nu}{\pi}\left(i\,\mathrm{kei}_\nu\left(-(-1)^{3/4}\,z\right) - \mathrm{ker}_\nu\left(-(-1)^{3/4}\,z\right)\right) + \frac{(-1)^\nu\,\left(-4\,i\log(z) + 4\,i\log\left(-(-1)^{3/4}\,z\right) + \pi\right)}{2\,\pi}\left(\mathrm{bei}_\nu\left(-(-1)^{3/4}\,z\right) + i\,\mathrm{ber}_\nu\left(-(-1)^{3/4}\,z\right)\right) /;\ \nu \in \mathbb{Z}$$

03.03.27.0012.01

$$Y_\nu\left(\sqrt[4]{-1}\,z\right) = \frac{(-1)^\nu}{2\,\pi}\,(2\,i\,\pi\,\mathrm{ber}_\nu(z) + 4\,i\,\mathrm{kei}_\nu(z) - 4\,\mathrm{ker}_\nu(z) + \mathrm{bei}_\nu(z)\,(i\log(4) + 4\,i\log(z) - 4\,i\log((1+i)\,z) + \pi)) /;\ \nu \in \mathbb{Z}$$

Zeros

When ν is real, the functions $Y_\nu(z)$ and $\left(\frac{\partial Y_\nu(z)}{\partial z}\right)$ each have an infinite number of real zeros, all of which are simple with the possible exception of $z = 0$.

03.03.30.0001.01

$$Y_\nu(z) = 0 /;\ z = z_k \land k \in \mathbb{N} \land \nu \in \mathbb{R} \land \mathrm{Re}(z_k) = z_k$$

03.03.30.0002.02

$$\frac{\partial Y_\nu(z)}{\partial z} = 0 /;\ z = z_k \land k \in \mathbb{N} \land \nu \in \mathbb{R} \land \mathrm{Re}\,(z_k) = z_k$$

Theorems

Y-transformation

$$\hat{f}_\nu\,(y) = \int_0^\infty f\,(x)\,\sqrt{x\,y}\;Y_\nu\,(x\,y)\,dx \Leftrightarrow\ f(x) = \int_0^\infty \hat{f}_\nu\,(y)\,\sqrt{x\,y}\;\mathbf{H}_\nu(x\,y)\,dy /;\ \mathrm{Re}(\nu) \geq -\frac{1}{2}$$

History

– C. G. Neumann (1867)
– J. Watson (1867) introduced the notation Y
– H. Hankel (1869)
– H. Weber (1873)
– L. Schläfli (1875)

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