Catalan

Notations

Traditional name

Catalan constant

Traditional notation

C

Mathematica StandardForm notation

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Catalan
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Primary definition

02.07.02.0001.01

$$C == \sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k+1)^2}$$

$C ==$ Catalan is the Catalan constant.

Specific values

02.07.03.0001.01

$$C == 0.915965594177219015054603514932384110774149374281672134266498119621763019776254769479356512\ldots$$

Above approximate numerical value of C shows 90 decimal digits.

General characteristics

The Catalan's number C is a constant. It is a positive real number. Whether C is irrational is not known.

Series representations

Generalized power series

02.07.06.0001.01

$$C == 2 \sum_{k=0}^{\infty} \frac{1}{(4\,k+1)^2} - \frac{\pi^2}{8}$$

02.07.06.0002.01

$$C = \frac{\pi^2}{8} - 2\sum_{k=0}^{\infty} \frac{1}{(4k+3)^2}$$

02.07.06.0003.01

$$C = 3\sum_{k=0}^{\infty} \frac{1}{2^{4k}} \left(-\frac{1}{2(8k+2)^2} + \frac{1}{2^2(8k+3)^2} - \frac{1}{2^3(8k+5)^2} + \frac{1}{2^3(8k+6)^2} - \frac{1}{2^4(8k+7)^2} + \frac{1}{2(8k+1)^2} \right) - 2\sum_{k=0}^{\infty} \frac{1}{2^{12k}} \left(\frac{1}{2^4(8k+2)^2} + \frac{1}{2^6(8k+3)^2} - \frac{1}{2^9(8k+5)^2} - \frac{1}{2^{10}(8k+6)^2} - \frac{1}{2^{12}(8k+7)^2} + \frac{1}{2^3(8k+1)^2} \right)$$

02.07.06.0004.01

$$C = \frac{1}{2}\sum_{k=0}^{\infty} \frac{4^k k!^2}{(2k)!\,(2k+1)^2}$$

02.07.06.0005.01

$$C = \sqrt{2}\sum_{k=0}^{\infty} \frac{(2k)!}{8^k k!^2 (2k+1)^2} - \frac{\pi}{4}\log(2)$$

02.07.06.0006.01

$$C = \frac{\pi}{8}\log\left(\sqrt{3}+2\right) + \frac{3}{8}\sum_{k=0}^{\infty} \frac{k!^2}{(2k)!\,(2k+1)^2}$$

The above formula is used for the numerical computation of Catalan's constant in *Mathematica*.

02.07.06.0015.01

$$C = \frac{3}{4}\sum_{k=0}^{\infty} \frac{(-1)^k}{4^k} \left(-\frac{2}{(4k+2)^2} + \frac{1}{(4k+3)^2} + \frac{2}{(4k+1)^2} \right) - \frac{1}{32}\sum_{k=0}^{\infty} \frac{(-1)^k}{64^k} \left(\frac{4}{(4k+2)^2} + \frac{1}{(4k+3)^2} + \frac{8}{(4k+1)^2} \right)$$

G.Huvent (2006)

02.07.06.0007.01

$$C = \frac{\pi}{2}\log(2) - \frac{1}{32}\pi\sum_{k=0}^{\infty} \frac{(2k+1)!^2}{16^k k!^4 (k+1)^3}$$

02.07.06.0008.01

$$C = \frac{\pi}{2}\log(2) + \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}(\psi(k+1)+\gamma)$$

02.07.06.0009.01

$$C = \frac{\pi}{4}\log(2) + \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\left(\psi\left(k+\frac{3}{2}\right)+\gamma\right)$$

02.07.06.0010.01

$$C = \frac{\pi}{4}\log(2) + \frac{1}{4}\sum_{k=0}^{\infty} \frac{2^k k!^2}{(2k+1)!}\psi\left(k+\frac{3}{2}\right) + \frac{\gamma\pi}{8}$$

02.07.06.0011.01

$$C = 1 - \sum_{k=1}^{\infty} \frac{k\,\zeta(2k+1)}{4^{2k}}$$

02.07.06.0012.01

$$C = \frac{1}{16} \sum_{k=1}^{\infty} \frac{(3^k - 1)(k+1)\zeta(k+2)}{4^k}$$

02.07.06.0013.01

$$C = \frac{1}{8} \sum_{k=2}^{\infty} \frac{k}{2^k} \zeta\left(k+1, \frac{3}{4}\right)$$

02.07.06.0014.01

$$C = 1 - \frac{1}{8} \sum_{k=2}^{\infty} \frac{k}{2^k} \zeta\left(k+1, \frac{5}{4}\right)$$

Integral representations

On the real axis

Of the direct function

02.07.07.0001.01

$$C = \frac{1}{4} \int_0^{\infty} \frac{e^{t/2}\, t}{e^t + 1} \, dt$$

02.07.07.0002.01

$$C = \frac{1}{2} \int_0^{\infty} t \operatorname{sech}(t) \, dt$$

02.07.07.0003.01

$$C = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{t}{\sin(t)} \, dt$$

02.07.07.0004.01

$$C = -\int_0^1 \frac{\log(t)}{t^2 + 1} \, dt$$

02.07.07.0005.01

$$C = \frac{1}{4} \int_0^1 \frac{1}{\sqrt{1 - t^2}} \log\left(\frac{1+t}{1-t}\right) dt$$

02.07.07.0006.01

$$C = -\int_0^1 \frac{1}{t^2 + 1} \log\left(\frac{1-t}{\sqrt{2}}\right) dt$$

02.07.07.0007.01

$$C = -\int_0^1 \frac{1}{t^2 + 1} \log\left(\frac{1}{2}(1 - t^2)\right) dt$$

02.07.07.0008.01

$$C = \frac{1}{2} \int_0^{\infty} \frac{1}{t^2 + 1} \log\left(t + \sqrt{t^2 + 1}\right) dt$$

02.07.07.0009.01

$$C == -2\int_0^{\frac{\pi}{4}} \log(2\sin(t))\,dt$$

02.07.07.0010.01

$$C == 2\int_0^{\frac{\pi}{4}} \log(2\cos(t))\,dt$$

02.07.07.0011.01

$$C == -\int_0^{\frac{\pi}{4}} \log(\tan(t))\,dt$$

02.07.07.0012.01

$$C == \int_0^{\frac{\pi}{4}} \log(\cot(t))\,dt$$

02.07.07.0013.01

$$C == \int_0^1 \frac{\tan^{-1}(t)}{t}\,dt$$

02.07.07.0014.01

$$C == \int_0^{\frac{\pi}{2}} \sinh^{-1}(\sin(t))\,dt$$

02.07.07.0015.01

$$C == \int_0^{\frac{\pi}{2}} \sinh^{-1}(\cos(t))\,dt$$

02.07.07.0016.01

$$C == \int_0^{\frac{\pi}{2}} \operatorname{csch}^{-1}(\csc(t))\,dt$$

02.07.07.0017.01

$$C == \int_0^{\frac{\pi}{2}} \operatorname{csch}^{-1}(\sec(t))\,dt$$

02.07.07.0018.01

$$C == \frac{1}{4}\int_0^1 \frac{K(t)}{\sqrt{t}}\,dt$$

02.07.07.0019.01

$$C == \frac{1}{2}\int_0^1 K\left(t^2\right)dt$$

02.07.07.0020.01

$$C == \int_0^1 E\left(t^2\right)dt - \frac{1}{2}$$

Involving the direct function

02.07.07.0021.01

$$C == \frac{3}{4}\int_0^{\frac{\pi}{6}} \frac{t}{\sin(t)}\,dt + \frac{\pi}{8}\log\left(2+\sqrt{3}\right)$$

02.07.07.0022.01

$$C = \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} \frac{t^2}{\sin(t)} \, dt + \frac{7\zeta(3)}{4\pi}$$

02.07.07.0023.01

$$C = -\frac{\pi^2}{4} \int_0^1 \left(t - \frac{1}{2}\right) \sec(\pi t) \, dt$$

02.07.07.0024.01

$$C = \int_0^{\frac{\pi}{2}} \frac{t \csc(t)}{\cos(t) + \sin(t)} \, dt - \frac{\pi}{4} \log(2)$$

02.07.07.0025.01

$$C = \frac{\pi}{4} \log(2) - \int_0^{\frac{\pi}{2}} \frac{t \csc(t)}{\cos(t) - \sin(t)} \, dt$$

02.07.07.0026.01

$$C = \frac{\pi}{4} \log(2) - \frac{1}{2} \int_0^1 \frac{\log(1-t)}{\sqrt{t}\,(t+1)} \, dt$$

02.07.07.0027.01

$$C = \frac{\pi}{2} \log(2) - \frac{1}{2} \int_0^1 \frac{\log(t+1)}{\sqrt{t}\,(t+1)} \, dt$$

02.07.07.0028.01

$$C = \frac{\pi}{8} \log(2) - \int_0^1 \frac{\log(1-t)}{t^2+1} \, dt$$

02.07.07.0029.01

$$C = \int_1^\infty \frac{\log(t+1)}{t^2+1} \, dt - \frac{\pi}{8} \log(2)$$

02.07.07.0030.01

$$C = \int_0^\infty \frac{\log(t+1)}{t^2+1} \, dt - \frac{\pi}{4} \log(2)$$

02.07.07.0031.01

$$C = \frac{\pi}{4} \log(2) - \int_0^\infty \frac{\log(|1-t|)}{t^2+1} \, dt$$

02.07.07.0032.01

$$C = -\frac{\pi}{8} \log(2) - 2 \int_0^1 \frac{\log(t)}{\sqrt{2-t^2}} \, dt$$

02.07.07.0033.01

$$C = \frac{1}{2} \int_0^1 \frac{\log(t+1)}{\sqrt{1-t^2}} \, dt + \frac{\pi}{4} \log(2)$$

02.07.07.0034.01

$$C = -\frac{1}{2} \int_0^1 \frac{\log(1-t)}{\sqrt{1-t^2}} \, dt - \frac{\pi}{4} \log(2)$$

02.07.07.0035.01

$$C = 4\int_0^1 \frac{t}{t^4+1}\log\left(t+\frac{1}{t}\right)dt - \frac{\pi}{4}\log(2)$$

02.07.07.0036.01

$$C = \frac{\pi}{4}\log(2) - 4\int_0^1 \frac{t}{t^4+1}\log\left(\frac{1}{t}-t\right)dt$$

02.07.07.0037.01

$$C = 4\int_1^\infty \frac{t}{t^4+1}\log\left(t+\frac{1}{t}\right)dt - \frac{\pi}{4}\log(2)$$

02.07.07.0038.01

$$C = \frac{\pi}{4}\log(2) - 4\int_1^\infty \frac{t}{t^4+1}\log\left(t-\frac{1}{t}\right)dt$$

02.07.07.0039.01

$$C = 2\int_0^\infty \log(\cosh(t))\,\operatorname{sech}(2\,t)\,dt + \frac{\pi}{4}\log(2)$$

02.07.07.0040.01

$$C = -2\int_0^{\frac{\pi}{4}} \log(\sin(t))\,dt - \frac{\pi}{2}\log(2)$$

02.07.07.0041.01

$$C = \frac{1}{2}\int_0^{\frac{\pi}{2}} \log(1+\sin(t))\,dt + \frac{\pi}{4}\log(2)$$

02.07.07.0042.01

$$C = -\frac{1}{2}\int_0^{\frac{\pi}{2}} \log(1-\sin(t))\,dt - \frac{\pi}{4}\log(2)$$

02.07.07.0043.01

$$C = \frac{1}{2}\int_0^{\frac{\pi}{2}} \log(1+\cos(t))\,dt + \frac{\pi}{4}\log(2)$$

02.07.07.0044.01

$$C = -\frac{1}{2}\int_0^{\frac{\pi}{2}} \log(1-\cos(t))\,dt - \frac{\pi}{4}\log(2)$$

02.07.07.0045.01

$$C = 2\int_0^{\frac{\pi}{4}} \log(\cos(t)+\sin(t))\,dt + \frac{\pi}{4}\log(2)$$

02.07.07.0046.01

$$C = -2\int_0^{\frac{\pi}{4}} \log(\cos(t)-\sin(t))\,dt - \frac{\pi}{4}\log(2)$$

02.07.07.0047.01

$$C = \int_0^{\frac{\pi}{2}} \log(1+\tan(t))\,dt - \frac{\pi}{4}\log(2)$$

02.07.07.0048.01

$$C = \frac{\pi}{8} \log(2) - \int_0^{\frac{\pi}{4}} \log(1 - \tan(t))\, dt$$

02.07.07.0049.01

$$C = -\int_0^1 \tan^{-1}(t)^2\, dt + \frac{\pi^2}{16} + \frac{\pi}{4} \log(2)$$

02.07.07.0050.01

$$C = \frac{2}{\pi} \int_0^1 \frac{\tan^{-1}(t)^2}{t}\, dt + \frac{7\,\zeta(3)}{4\,\pi}$$

02.07.07.0051.01

$$C = \frac{(-1)^{n-1}}{2} \left(4\,n \int_0^{\frac{\pi}{4}} \log\left(\cos^{\frac{1}{n}}(t) + \sin^{\frac{1}{n}}(t)\right) dt + \pi \log(2) - n \sum_{k=0}^{n-1} (-1)^k \left(1 - \frac{2\,k+1}{2\,n}\right) \pi \log\left(2 \cos\left(\frac{(2\,k+1)\,\pi}{2\,n}\right) + 2\right)\right) /; n \in \mathbb{N}^+$$

Multiple integral representations

02.07.07.0052.01

$$C = \frac{1}{4} \int_0^1 \int_0^1 \frac{1}{\sqrt{1-x}\,\sqrt{1-y}\,(x+y)}\, dx\, dy$$

Continued fraction representations

02.07.10.0001.01

$$C = 1 - \cfrac{1/2}{3 + \cfrac{4}{1 + \cfrac{4}{3 + \cfrac{16}{1 + \cfrac{16}{3 + \cfrac{36}{1 + \cfrac{36}{3 + \ldots}}}}}}}$$

02.07.10.0002.01

$$C = 1 - \frac{1}{2\left(3 + \mathrm{K}_k\left(\left(2\left\lfloor \frac{k+1}{2} \right\rfloor\right)^2, 3^{(k-1) \bmod 2}\right)_1^{\infty}\right)}$$

02.07.10.0003.01

$$C = \frac{1}{2} + \cfrac{1/2}{\frac{1}{2} + \cfrac{1}{\frac{1}{2} + \cfrac{2}{\frac{1}{2} + \cfrac{4}{\frac{1}{2} + \cfrac{6}{\frac{1}{2} + \cfrac{9}{\frac{1}{2} + \cfrac{12}{\frac{1}{2} + \ldots}}}}}}}$$

02.07.10.0004.01

$$C = \frac{1}{2} + \frac{1}{1 + 2\,\mathrm{K}_k\left(\frac{1}{16}\left(\left((-1)^k - 1\right)^2 (k+1)^2 + 2\left(1 + (-1)^k\right) k\,(k+2)\right), \frac{1}{2}\right)_1^\infty}$$

Complex characteristics

Real part

02.07.19.0001.01

$\operatorname{Re}(C) = C$

Imaginary part

02.07.19.0002.01

$\operatorname{Im}(C) = 0$

Absolute value

02.07.19.0003.01

$|C| = C$

Argument

02.07.19.0004.01

$\arg(C) = 0$

Conjugate value

02.07.19.0005.01

$\overline{C} = C$

Signum value

02.07.19.0006.01

$\operatorname{sgn}(C) = 1$

Differentiation

Low-order differentiation

02.07.20.0001.01

$$\frac{\partial C}{\partial z} == 0$$

Fractional integro-differentiation

02.07.20.0002.01

$$\frac{\partial^\alpha C}{\partial z^\alpha} == \frac{z^{-\alpha} C}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

02.07.21.0001.01

$$\int C \, dz == C z$$

02.07.21.0002.01

$$\int z^{\alpha-1} C \, dz == \frac{z^\alpha C}{\alpha}$$

Integral transforms

Fourier exp transforms

02.07.22.0001.01

$$\mathcal{F}_t[C](z) == C \sqrt{2\pi} \; \delta(z)$$

Inverse Fourier exp transforms

02.07.22.0002.01

$$\mathcal{F}_t^{-1}[C](z) == C \sqrt{2\pi} \; \delta(z)$$

Fourier cos transforms

02.07.22.0003.01

$$\mathcal{F}c_t[C](z) == C \sqrt{\frac{\pi}{2}} \; \delta(z)$$

Fourier sin transforms

02.07.22.0004.01

$$\mathcal{F}s_t[C](z) == \sqrt{\frac{2}{\pi}} \; \frac{C}{z}$$

Laplace transforms

02.07.22.0005.01

$$\mathcal{L}_t[C](z) = \frac{C}{z}$$

Inverse Laplace transforms

02.07.22.0006.01

$$\mathcal{L}_t^{-1}[C](z) = C\,\delta(z)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_pF_q$

02.07.26.0001.01

$$C = {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; -1\right)$$

Through Meijer *G*

Classical cases

02.07.26.0002.01

$$C = \frac{1}{4} G_{3,3}^{1,3}\left(1 \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}, 0 \\ 0, -\frac{1}{2}, -\frac{1}{2} \end{matrix}\right)$$

02.07.26.0009.01

$$C = C\,G_{0,1}^{1,0}(z \mid 0) + C\,G_{1,2}^{1,1}\left(z \middle| \begin{matrix} 1 \\ 1, 0 \end{matrix}\right)$$

Through other functions

02.07.26.0003.01

$$C = \frac{1}{8}\left(\psi^{(1)}\left(\frac{1}{4}\right) - \pi^2\right)$$

02.07.26.0004.01

$$C = \frac{1}{4}\Phi\left(-1, 2, \frac{1}{2}\right)$$

02.07.26.0005.01

$$C = \frac{i}{8}\log^2(2) + \frac{\pi}{8}\log(2) + i\,\mathrm{Li}_2\left(\frac{1-i}{2}\right) - \frac{5\,i\,\pi^2}{96}$$

02.07.26.0006.01

$$C = \frac{\pi}{8}\log(2) + \frac{i}{2}\left(\mathrm{Li}_2\left(\frac{1-i}{2}\right) - \mathrm{Li}_2\left(\frac{1+i}{2}\right)\right)$$

02.07.26.0007.01

$$C = \frac{1}{16}\left(\zeta\left(2, \frac{1}{4}\right) - \zeta\left(2, \frac{3}{4}\right)\right)$$

02.07.26.0008.01

$$C = \frac{\pi}{24} - \frac{\pi}{2} \log(\text{Glaisher}) + 4 \pi \zeta^{(1,0)}\left(-1, \frac{1}{4}\right)$$

Inequalities

02.07.29.0001.01

$$\frac{9}{10} < C < 1$$

History

– E. Catalan (1865, 1883)
– M. Leclert (1865)
– M. Bresse (1867)
– J. W. L. Glaisher (1877)

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