CosIntegral

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Notations

Traditional name

Cosine integral

Traditional notation

$\mathrm{Ci}(z)$

Mathematica StandardForm notation

`CosIntegral[z]`

Primary definition

06.38.02.0001.01

$$\mathrm{Ci}(z) = \int_0^z \frac{\cos(t)-1}{t}\,dt + \log(z) + \gamma$$

Specific values

Values at fixed points

06.38.03.0001.01

$$\mathrm{Ci}(0) = -\infty$$

Values at infinities

06.38.03.0002.01

$$\mathrm{Ci}(\infty) = 0$$

06.38.03.0003.01

$$\mathrm{Ci}(-\infty) = i\,\pi$$

06.38.03.0004.01

$$\mathrm{Ci}(i\,\infty) = \infty$$

06.38.03.0005.01

$$\mathrm{Ci}(-i\,\infty) = \infty$$

06.38.03.0006.01

$$\mathrm{Ci}(\tilde{\infty}) = ¿$$

General characteristics

Domain and analyticity

Ci(z) is an analytical function of z which is defined over the whole complex z-plane.

06.38.04.0001.01

$$z \longrightarrow \mathrm{Ci}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

06.38.04.0002.01

$$\mathrm{Ci}(\bar{z}) = \overline{\mathrm{Ci}(z)} \;/;\; z \notin (-\infty, 0)$$

Periodicity

No periodicity

Poles and essential singularities

The function Ci(z) has an essential singularity at $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is a branch point.

06.38.04.0003.01

$$\mathcal{S}ing_z(\mathrm{Ci}(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function Ci(z) has two branch points: $z = 0$, $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is an essential singularity.

06.38.04.0004.01

$$\mathcal{BP}_z(\mathrm{Ci}(z)) = \{0, \tilde{\infty}\}$$

06.38.04.0005.01

$$\mathcal{R}_z(\mathrm{Ci}(z), 0) = \log$$

06.38.04.0006.01

$$\mathcal{R}_z(\mathrm{Ci}(z), \tilde{\infty}) = \log$$

Branch cuts

The function Ci(z) is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$ where it is continuous from above.

06.38.04.0007.01

$$\mathcal{BC}_z(\mathrm{Ci}(z)) = \{\{(-\infty, 0), -i\}\}$$

06.38.04.0008.01

$$\lim_{\epsilon \to +0} \mathrm{Ci}(x + i\,\epsilon) = \mathrm{Ci}(x) \;/;\; x < 0$$

06.38.04.0009.01

$$\lim_{\epsilon \to +0} \mathrm{Ci}(x - i\,\epsilon) = \mathrm{Ci}(x) - 2\,\pi\,i \;/;\; x < 0$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

06.38.06.0009.01

$$\mathrm{Ci}(z) \propto \mathrm{Ci}\,(z_0) + \left\lfloor \frac{\arg(z - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \frac{\cos(z_0)}{z_0}\,(z - z_0) - \frac{\cos(z_0) + \sin(z_0)\, z_0}{2\, z_0^2}\,(z - z_0)^2 + \ldots \;/;\; (z \to z_0)$$

06.38.06.0010.01

$$\mathrm{Ci}(z) \propto \mathrm{Ci}\,(z_0) + \left\lfloor \frac{\arg(z - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \frac{\cos(z_0)}{z_0}\,(z - z_0) - \frac{\cos(z_0) + \sin(z_0)\, z_0}{2\, z_0^2}\,(z - z_0)^2 + O\left((z - z_0)^3\right)$$

06.38.06.0011.01

$$\mathrm{Ci}(z) == \mathrm{Ci}(z_0) + \left\lfloor \frac{\arg(z - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \sum_{k=1}^{\infty} \left(\frac{(-1)^{k-1}\, z_0^{-k}}{k} - \frac{\sqrt{\pi}\; 2^{k-3}\, z_0^{2-k}}{k\,!}\; {}_2\tilde{F}_3\left(1, 1;\, 2,\, \frac{3-k}{2},\, 2 - \frac{k}{2};\, -\frac{z_0^2}{4}\right)\right)(z - z_0)^k$$

06.38.06.0012.01

$$\mathrm{Ci}(z) \propto \mathrm{Ci}\,(z_0) + \left\lfloor \frac{\arg(z - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + O(z - z_0)$$

Expansions on branch cuts

For the function itself

06.38.06.0013.01

$$\mathrm{Ci}(z) \propto \mathrm{Ci}\,(x) + 2\, i\, \pi \left\lfloor \frac{\arg(z - x)}{2\pi} \right\rfloor + \frac{\cos(x)}{x}\,(z - x) - \frac{\cos(x) + x \sin(x)}{2\, x^2}\,(z - x)^2 + \ldots \;/;\; (z \to x) \land x \in \mathbb{R} \land x < 0$$

06.38.06.0014.01

$$\mathrm{Ci}(z) \propto \mathrm{Ci}\,(x) + 2\, i\, \pi \left\lfloor \frac{\arg(z - x)}{2\pi} \right\rfloor + \frac{\cos(x)}{x}\,(z - x) - \frac{\cos(x) + x \sin(x)}{2\, x^2}\,(z - x)^2 + O\left((z - x)^3\right) \;/;\; x \in \mathbb{R} \land x < 0$$

06.38.06.0015.01

$$\mathrm{Ci}(z) == \mathrm{Ci}(x) + 2\, i\, \pi \left\lfloor \frac{\arg(z - x)}{2\pi} \right\rfloor + \sum_{k=1}^{\infty} \left(\frac{(-1)^{k-1}\, x^{-k}}{k} - \frac{\sqrt{\pi}\; x^{2-k}\, 2^{k-3}}{k\,!}\; {}_2\tilde{F}_3\left(1, 1;\, 2,\, \frac{3-k}{2},\, 2 - \frac{k}{2};\, -\frac{x^2}{4}\right)\right)(z - x)^k \;/;\; x \in \mathbb{R} \land x < 0$$

06.38.06.0016.01

$$\mathrm{Ci}(z) \propto \mathrm{Ci}\,(x) + 2\, i\, \pi \left\lfloor \frac{\arg(z - x)}{2\pi} \right\rfloor + O(z - x) \;/;\; x \in \mathbb{R} \land x < 0$$

Expansions at $z == 0$

For the function itself

06.38.06.0001.02

$$\mathrm{Ci}(z) \propto \log(z) + \gamma - \frac{1}{4}\, z^2 \left(1 - \frac{z^2}{24} + \frac{z^4}{1080} - \ldots\right) \;/;\; (z \to 0)$$

06.38.06.0017.01

$$\text{Ci}(z) \propto \log(z) + \gamma - \frac{1}{4} z^2 \left(1 - \frac{z^2}{24} + \frac{z^4}{1080} - O(z^6)\right)$$

06.38.06.0002.01

$$\text{Ci}(z) = \log(z) + \gamma + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k z^{2k}}{k (2k)!}$$

06.38.06.0003.01

$$\text{Ci}(z) = \log(z) + \gamma - \frac{z^2}{4} \, {}_2F_3\left(1, 1; 2, 2, \frac{3}{2}; -\frac{z^2}{4}\right)$$

06.38.06.0004.02

$$\text{Ci}(z) \propto \log(z) + \gamma - \frac{z^2}{4} \left(1 + O(z^2)\right)$$

06.38.06.0018.01

$$\text{Ci}(z) = F_\infty(z) \; /; \left(\left(F_n(z) = \log(z) + \gamma + \frac{1}{2} \sum_{k=1}^{n} \frac{(-1)^k z^{2k}}{k (2k)!} = \text{Ci}(z) + \frac{(-1)^n z^{2n+2}}{4 (n+1)^2 (2n+1)!} \, {}_2F_3\left(1, n+1; n+\frac{3}{2}, n+2, n+2; -\frac{z^2}{4}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

06.38.06.0005.01

$$\text{Ci}(z) \propto \log(z) - \frac{\log(z^2)}{2} + \frac{\sin(z)}{z} \, {}_3F_0\left(\frac{1}{2}, 1, 1; ; -\frac{4}{z^2}\right) - \frac{\cos(z)}{z^2} \, {}_3F_0\left(1, 1, \frac{3}{2}; ; -\frac{4}{z^2}\right) /; (|z| \to \infty)$$

06.38.06.0006.01

$$\text{Ci}(z) \propto \log(z) - \frac{\log(z^2)}{2} + \frac{\sin(z)}{z} \left(1 + O\left(\frac{1}{z^2}\right)\right) - \frac{\cos(z)}{z^2} \left(1 + O\left(\frac{1}{z^2}\right)\right) /; (|z| \to \infty)$$

06.38.06.0019.01

$$\text{Ci}(z) \propto \begin{cases} i\pi & \arg(z) = \pi \\ \frac{\sin(z)}{z} & \text{True} \end{cases} /; (|z| \to \infty)$$

Residue representations

06.38.06.0007.02

$$\text{Ci}(z) = -\frac{1}{2} \left(\log(z^2) - 2\log(z)\right) - \frac{\sqrt{\pi}}{2} \, \text{res}_s\left(\frac{\left(\frac{z^2}{4}\right)^{-s}}{\Gamma\left(\frac{1}{2} - s\right)} \frac{\Gamma(s)}{s}\right)(0) - \frac{\sqrt{\pi}}{2} \sum_{j=1}^{\infty} \text{res}_s\left(\frac{\left(\frac{z^2}{4}\right)^{-s}}{s \, \Gamma\left(\frac{1}{2} - s\right)} \Gamma(s)\right)(-j)$$

06.38.06.0008.02

$$\text{Ci}(z) = -\frac{\sqrt{\pi}}{2} \, \text{res}_s\left(\frac{\left(\frac{z}{2}\right)^{-2s}}{\Gamma\left(\frac{1}{2} - s\right)} \frac{\Gamma(s)}{s}\right)(0) - \frac{\sqrt{\pi}}{2} \sum_{j=1}^{\infty} \text{res}_s\left(\frac{\left(\frac{z}{2}\right)^{-2s}}{s \, \Gamma\left(\frac{1}{2} - s\right)} \Gamma(s)\right)(-j)$$

Integral representations

On the real axis

Of the direct function

06.38.07.0001.01

$$\mathrm{Ci}(z) = \int_0^z \frac{\cos(t)-1}{t}\, dt + \log(z) + \gamma$$

06.38.07.0002.01

$$\mathrm{Ci}(z) = -\int_z^\infty \frac{\cos(t)}{t}\, dt \;/;\; |\arg(z)| < \pi$$

Contour integral representations

06.38.07.0003.01

$$\mathrm{Ci}(z) = -\frac{1}{2}\left(\log\left(z^2\right) - 2\log(z)\right) - \frac{\sqrt{\pi}}{4\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)^2}{\Gamma(s+1)\,\Gamma\left(\frac{1}{2}-s\right)} \left(\frac{z^2}{4}\right)^{-s} ds$$

06.38.07.0004.01

$$\mathrm{Ci}(z) = -\frac{\sqrt{\pi}}{4\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)^2}{\Gamma(s+1)\,\Gamma\left(\frac{1}{2}-s\right)} \left(\frac{z}{2}\right)^{-2s} ds$$

Limit representations

06.38.09.0001.01

$$\mathrm{Ci}(x) = -\lim_{n\to\infty}\left(\sum_{k=n+1}^{\infty} \frac{1}{k}\cos\left(\frac{k x}{n}\right)\right) /;\; x \in \mathbb{R} \land x > 0$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

06.38.13.0001.01

$$z\, w^{(3)}(z) + 2\, w''(z) + z\, w'(z) = 0 \;/;\; w(z) = c_1\, \mathrm{Ci}(z) + c_2\, \mathrm{Si}(z) + c_3$$

06.38.13.0004.01

$$W_z(1, \mathrm{Ci}(z), \mathrm{Si}(z)) = \frac{1}{z^2}$$

06.38.13.0002.01

$$z\, w^{(3)}(z) + 2\, w''(z) + z\, w'(z) = 0 \;/;\; w(z) = c_1\, \mathrm{Ci}(z) + c_2\, \mathrm{Ei}(i z) + c_3$$

06.38.13.0005.01

$$W_z(1, \mathrm{Ci}(z), \mathrm{Ei}(i z)) = \frac{i}{z^2}$$

06.38.13.0003.01

$$z\, w^{(3)}(z) + 2\, w''(z) + z\, w'(z) = 0 \;/;\; w(z) = c_1\, \mathrm{Ci}(z) + c_2\, \mathrm{Ei}(-i z) + c_3$$

06.38.13.0006.01

$$W_z(1, \text{Ci}(z), \text{Ei}(-i z)) = -\frac{i}{z^2}$$

06.38.13.0007.01

$$w^{(3)}(z) + \left(\frac{2 g'(z)}{g(z)} - \frac{3 g''(z)}{g'(z)}\right) w''(z) + \left(g'(z)^2 + \frac{3 g''(z)^2}{g'(z)^2} - \frac{2 g''(z)}{g(z)} - \frac{g^{(3)}(z)}{g'(z)}\right) w'(z) = 0 \;/;\; w(z) = c_1 \text{Ci}(g(z)) + c_2 \text{Si}(g(z)) + c_3$$

06.38.13.0008.01

$$W_z(\text{Ci}(g(z)), \text{Si}(g(z)), 1) = \frac{g'(z)^3}{g(z)^2}$$

06.38.13.0009.01

$$w^{(3)}(z) + \left(\frac{2 g'(z)}{g(z)} - \frac{3 h'(z)}{h(z)} - \frac{3 g''(z)}{g'(z)}\right) w''(z) +$$
$$\left(g'(z)^2 - \frac{4 h'(z) g'(z)}{g(z) h(z)} + \frac{6 h'(z)^2}{h(z)^2} + \frac{3 g''(z)^2}{g'(z)^2} + \frac{6 h'(z) g''(z)}{h(z) g'(z)} - \frac{2 g''(z)}{g(z)} - \frac{3 h''(z)}{h(z)} - \frac{g^{(3)}(z)}{g'(z)}\right) w'(z) +$$
$$\left(-\frac{6 h'(z)^3}{h(z)^3} + \frac{4 g'(z) h'(z)^2}{g(z) h(z)^2} - \frac{6 g''(z) h'(z)^2}{h(z)^2 g'(z)} + \frac{6 h''(z) h'(z)}{h(z)^2} - \frac{3 g''(z)^2 h'(z)}{h(z) g'(z)^2} + \frac{2 h'(z) g''(z) - 2 g'(z) h''(z)}{h(z) g(z)} +\right.$$
$$\left.\frac{3 g''(z) h''(z) + h'(z) g^{(3)}(z)}{h(z) g'(z)} - \frac{h'(z) g'(z)^2 + h^{(3)}(z)}{h(z)}\right) w(z) = 0 \;/;\; w(z) = c_1 h(z) \text{Ci}(g(z)) + c_2 h(z) \text{Si}(g(z)) + c_3 h(z)$$

06.38.13.0010.01

$$W_z(h(z) \text{Ci}(g(z)), h(z) \text{Si}(g(z)), h(z)) = \frac{h(z)^3 g'(z)^3}{g(z)^2}$$

06.38.13.0011.01

$$z^3 w^{(3)}(z) - (r + 3 s - 3) z^2 w''(z) + \left(a^2 r^2 z^{2 r} + 3 (s - 1) s + r (2 s - 1) + 1\right) z w'(z) - s \left(a^2 r^2 z^{2 r} + s (r + s)\right) w(z) = 0 \;/;$$
$$w(z) = c_1 z^s \text{Ci}(a z^r) + c_2 z^s \text{Si}(a z^r) + c_3 z^s$$

06.38.13.0012.01

$$W_z(z^s \text{Ci}(a z^r), z^s \text{Si}(a z^r), z^s) = a r^3 z^{r+3 s-3}$$

06.38.13.0013.01

$$w^{(3)}(z) - (\log(r) + 3 \log(s)) w''(z) + \left(a^2 \log^2(r) r^{2 z} + 3 \log^2(s) + 2 \log(r) \log(s)\right) w'(z) -$$
$$\log(s) \left(a^2 \log^2(r) r^{2 z} + \log(s) (\log(r) + \log(s))\right) w(z) = 0 \;/;\; w(z) = c_1 s^z \text{Ci}(a r^z) + c_2 s^z \text{Si}(a r^z) + c_3 s^z$$

06.38.13.0014.01

$$W_z(s^z \text{Ci}(a r^z), s^z \text{Si}(a r^z), s^z) = a r^z s^{3 z} \log^3(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.38.16.0001.01

$$\text{Ci}(-z) = \text{Ci}(z) + \log(-z) - \log(z)$$

06.38.16.0002.01

$$\text{Ci}(i\,z) = \text{Chi}(z) - \log(z) + \log(i\,z)$$

06.38.16.0003.01

$$\text{Ci}(-i\,z) = \text{Chi}(z) - \log(z) + \log(-i\,z)$$

06.38.16.0004.01

$$\text{Ci}\left(\sqrt{z^2}\right) = \text{Ci}(z) - \log(z) + \log\left(\sqrt{z^2}\right)$$

Complex characteristics

Real part

06.38.19.0001.01

$$\text{Re}\,(\text{Ci}(x + i\,y)) = \text{Ci}(x) - \log(x) + \frac{1}{2}\log\left(x^2 + y^2\right) + \frac{1}{2}\sum_{k=1}^{\infty} \frac{y^{2k}}{k\,(2\,k)!}\,{}_1F_2\left(k;\,\frac{1}{2},\,k+1;\,-\frac{x^2}{4}\right)$$

06.38.19.0002.01

$$\text{Re}(\text{Ci}(x + i\,y)) = \frac{1}{2}\log\left(x^2 + y^2\right) + \frac{1}{2}\sum_{k=1}^{\infty} \frac{(-1)^k \left(x^2 + y^2\right)^k}{k\,(2\,k)!}\cos\left(2\,k\tan^{-1}\left(\frac{y}{x}\right)\right) + \gamma$$

06.38.19.0003.01

$$\text{Re}(\text{Ci}(x + i\,y)) = \frac{1}{2}\log\left(x^2 + y^2\right) + \frac{1}{2}\sum_{k=1}^{\infty}\sum_{j=0}^{k} \frac{(-1)^{j+k}\,y^{2j}\,x^{2k-2j}}{k\,(2\,j)!\,(2\,k-2\,j)!} + \gamma$$

06.38.19.0004.01

$$\text{Re}(\text{Ci}(x + i\,y)) = \frac{1}{2}\left(\text{Ci}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

06.38.19.0005.01

$$\text{Im}(\text{Ci}(x + i\,y)) = \tan^{-1}(x,\,y) - x\sum_{k=0}^{\infty} \frac{y^{2j+1}}{(2\,k+1)!}\,{}_1F_2\left(k+1;\,\frac{3}{2},\,k+2;\,-\frac{x^2}{4}\right)$$

06.38.19.0006.01

$$\text{Im}(\text{Ci}(x + i\,y)) = \tan^{-1}(x,\,y) - \frac{1}{2}\sum_{k=0}^{\infty} \frac{(-1)^k \left(x^2 + y^2\right)^{k+1}}{(k+1)\,(2\,k+2)!}\sin\left(2\,(k+1)\tan^{-1}\left(\frac{y}{x}\right)\right)$$

06.38.19.0007.01

$$\text{Im}(\text{Ci}(x + i\,y)) = \tan^{-1}(x,\,y) - \frac{1}{2}\sum_{k=0}^{\infty}\sum_{j=0}^{k} \frac{(-1)^{j+k}\,x^{2k-2j+1}\,y^{2j+1}}{(k+1)\,(2\,j+1)!\,(2\,k-2\,j+1)!}$$

06.38.19.0008.01

$$\text{Im}(\text{Ci}(x + i\,y)) = \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \text{Ci}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

06.38.19.0009.01

$$|\text{Ci}(x+i\,y)| = \sqrt{\text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\text{Ci}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)}$$

Argument

06.38.19.0010.01

$$\arg(\text{Ci}(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(\text{Ci}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \text{Ci}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

06.38.19.0011.01

$$\overline{\text{Ci}(x+i\,y)} = \frac{1}{2}\left(\text{Ci}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right) - \frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \text{Ci}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

06.38.19.0012.01

$$\text{sgn}(\text{Ci}(x+i\,y)) = \left(\frac{i}{y}\sqrt{-\frac{y^2}{x^2}}\;x\left(\text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \text{Ci}\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right)\right) + \text{Ci}\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right) + \text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right) \Big/ \left(2\sqrt{\text{Ci}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\text{Ci}\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right)}\right)$$

Differentiation

Low-order differentiation

06.38.20.0001.01

$$\frac{\partial\,\text{Ci}(z)}{\partial z} = \frac{\cos(z)}{z}$$

06.38.20.0002.01

$$\frac{\partial^2\,\text{Ci}(z)}{\partial z^2} = -\frac{\cos(z)}{z^2} - \frac{\sin(z)}{z}$$

Symbolic differentiation

06.38.20.0006.01

$$\frac{\partial^n\,\text{Ci}(z)}{\partial z^n} = \delta_n\,\text{Ci}(z) - \sum_{k=0}^{n-1}\frac{(-1)^{n-k}\,z^{k-n}\,(n-1)!}{k!}\cos\left(\frac{\pi\,k}{2} + z\right) /;\, n \in \mathbb{N}$$

06.38.20.0003.01

$$\frac{\partial^n \mathrm{Ci}(z)}{\partial z^n} = \delta_n \mathrm{Ci}(z) + \mathrm{Boole}\left(n \neq 0, (n-1)! \sum_{k=0}^{n-1} \frac{(-1)^{n-k-1} z^{k-n}}{k!} \cos\left(z + \frac{\pi k}{2}\right)\right) /; n \in \mathbb{N}$$

06.38.20.0004.01

$$\frac{\partial^n \mathrm{Ci}(z)}{\partial z^n} = (-1)^{n-1} z^{-n} (n-1)! - 2^{n-3} \sqrt{\pi}\, z^{2-n}\, {}_2\tilde{F}_3\left(1, 1; 2, \frac{3-n}{2}, 2 - \frac{n}{2}; -\frac{z^2}{4}\right) /; n \in \mathbb{N}^+$$

Fractional integro-differentiation

06.38.20.0005.01

$$\frac{\partial^\alpha \mathrm{Ci}(z)}{\partial z^\alpha} = \left(\mathcal{F}C_{\log}^{(\alpha)}(z) + \frac{\gamma}{\Gamma(1-\alpha)}\right) z^{-\alpha} - 2^{\alpha-3} \sqrt{\pi}\, z^{2-\alpha}\, {}_2\tilde{F}_3\left(1, 1; 2, \frac{3-\alpha}{2}, 2 - \frac{\alpha}{2}; -\frac{z^2}{4}\right)$$

Integration

Indefinite integration

Involving only one direct function

06.38.21.0001.01

$$\int \mathrm{Ci}(b + a z)\, dz = \frac{(b + a z)\, \mathrm{Ci}(b + a z) - \sin(b + a z)}{a}$$

06.38.21.0002.01

$$\int \mathrm{Ci}(a z)\, dz = z\, \mathrm{Ci}(a z) - \frac{\sin(a z)}{a}$$

06.38.21.0003.01

$$\int \mathrm{Ci}(z)\, dz = z\, \mathrm{Ci}(z) - \sin(z)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

06.38.21.0004.01

$$\int z^{\alpha-1} \mathrm{Ci}(a z)\, dz = \frac{z^\alpha}{2\alpha} \left(\Gamma(\alpha, -i a z) (-i a z)^{-\alpha} + 2\, \mathrm{Ci}(a z) + (i a z)^{-\alpha}\, \Gamma(\alpha, i a z)\right)$$

06.38.21.0005.01

$$\int z^{\alpha-1} \mathrm{Ci}(z)\, dz = \frac{z^\alpha}{\alpha} \mathrm{Ci}(z) + \frac{z^\alpha}{2\alpha} \left((i z)^{-\alpha}\, \Gamma(\alpha, i z) + (-i z)^{-\alpha}\, \Gamma(\alpha, -i z)\right)$$

06.38.21.0006.01

$$\int z\, \mathrm{Ci}(a z)\, dz = -\frac{-a^2\, \mathrm{Ci}(a z)\, z^2 + a \sin(a z)\, z + \cos(a z)}{2 a^2}$$

06.38.21.0007.01

$$\int \frac{\mathrm{Ci}(a\,z)}{z}\,dz = \frac{1}{2}\left(-i\,a\,z\,{}_3F_3(1,1,1;2,2,2;-i\,a\,z) + i\,a\,z\,{}_3F_3(1,1,1;2,2,2;i\,a\,z) + \log(z)\left(2\,\mathrm{Ci}(a\,z) + \Gamma(0,-i\,a\,z) + \Gamma(0,i\,a\,z) - \log(z) + \log(-i\,a\,z) + \log(i\,a\,z) + 2\,\gamma\right)\right)$$

06.38.21.0008.01

$$\int \frac{\mathrm{Ci}(a\,z)}{z^2}\,dz = -\frac{\cos(a\,z) + \mathrm{Ci}(a\,z) + a\,z\,\mathrm{Si}(a\,z)}{z}$$

06.38.21.0009.01

$$\int \frac{\mathrm{Ci}(b + a\,z)}{z^2}\,dz = \frac{a\,z\cos(b)\,\mathrm{Ci}(a\,z) - (b + a\,z)\,\mathrm{Ci}(b + a\,z) - a\,z\sin(b)\,\mathrm{Si}(a\,z)}{b\,z}$$

Power arguments

06.38.21.0010.01

$$\int z^{\alpha-1}\,\mathrm{Ci}(a\,z^r)\,dz = \frac{z^\alpha}{2\,\alpha}\left(\Gamma\!\left(\frac{\alpha}{r}, -i\,a\,z^r\right)(-i\,a\,z^r)^{-\frac{\alpha}{r}} + 2\,\mathrm{Ci}(a\,z^r) + (i\,a\,z^r)^{-\frac{\alpha}{r}}\,\Gamma\!\left(\frac{\alpha}{r}, i\,a\,z^r\right)\right)$$

Involving exponential function

Involving exp

06.38.21.0011.01

$$\int e^{b\,z}\,\mathrm{Ci}(a\,z)\,dz = -\frac{-2\,e^{b\,z}\,\mathrm{Ci}(a\,z) + \mathrm{Ei}((b - i\,a)\,z) + \mathrm{Ei}((b + a\,i)\,z)}{2\,b}$$

06.38.21.0012.01

$$\int e^{i\,a\,z}\,\mathrm{Ci}(a\,z)\,dz = -\frac{i\left(2\,e^{i\,a\,z}\,\mathrm{Ci}(a\,z) - \mathrm{Ei}(2\,i\,a\,z) - \log(a\,z)\right)}{2\,a}$$

06.38.21.0013.01

$$\int e^{-i\,a\,z}\,\mathrm{Ci}(a\,z)\,dz = \frac{i\left(2\,e^{-i\,a\,z}\,\mathrm{Ci}(a\,z) - \mathrm{Ei}(-2\,i\,a\,z) - \log(a\,z)\right)}{2\,a}$$

Involving exponential function and a power function

Involving exp and power

06.38.21.0014.01

$$\int z^n\,e^{b\,z}\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{2}\,n!\,(-b)^{-n-1}\left(\mathrm{Ei}((b - i\,a)\,z) + \mathrm{Ei}((b + a\,i)\,z) - 2\,e^{b\,z}\,\mathrm{Ci}(a\,z)\sum_{k=0}^{n}\frac{(-b\,z)^k}{k!} - e^{(b+a\,i)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b + a\,i}\right)^m\sum_{k=0}^{m-1}\frac{(-b - i\,a)^k\,z^k}{k!} - e^{(b-i\,a)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b - i\,a}\right)^m\sum_{k=0}^{m-1}\frac{(i\,a - b)^k\,z^k}{k!}\right) /;\ n \in \mathbb{N}$$

06.38.21.0015.01

$$\int z^n e^{i a z} \operatorname{Ci}(a z) \, dz = \frac{(-i a)^{-n-1}}{2} \left(n! \left(\operatorname{Ei}(2 i a z) + \log(z) + 2 \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-i a z)^k}{2k} - 2^{-k-1} \Gamma(k, -2 i a z) \right) \right) - 2 \operatorname{Ci}(a z) \Gamma(n+1, -i a z) \right) /; n \in \mathbb{N}$$

06.38.21.0016.01

$$\int z^n e^{-i a z} \operatorname{Ci}(a z) \, dz = \frac{(i a)^{-n-1}}{2} \left(n! \left(\operatorname{Ei}(-2 i a z) + \log(z) + 2 \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(i a z)^k}{2k} - 2^{-k-1} \Gamma(k, 2 i a z) \right) \right) - 2 \operatorname{Ci}(a z) \Gamma(n+1, i a z) \right) /; n \in \mathbb{N}$$

06.38.21.0017.01

$$\int z e^{b z} \operatorname{Ci}(a z) \, dz = \frac{1}{2 b^2 (a^2 + b^2)} \left(\operatorname{Ei}((b + a i) z) a^2 - 2 b e^{b z} \sin(a z) a - 2 b^2 e^{b z} \cos(a z) + 2 (a^2 + b^2) e^{b z} (b z - 1) \operatorname{Ci}(a z) + (a^2 + b^2) \operatorname{Ei}((b - i a) z) + b^2 \operatorname{Ei}((b + a i) z) \right)$$

06.38.21.0018.01

$$\int z^2 e^{b z} \operatorname{Ci}(a z) \, dz = -\frac{1}{b^3} \left(-e^{b z} (b^2 z^2 - 2 b z + 2) \operatorname{Ci}(a z) + \operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z) + \frac{1}{(a^2 + b^2)^2} \left(b e^{b z} \left(b \left((b z - 1) a^2 + b^2 (b z - 3) \right) \cos(a z) + a \left((b z - 2) a^2 + b^2 (b z - 4) \right) \sin(a z) \right) \right) \right)$$

06.38.21.0019.01

$$\int z^3 e^{b z} \operatorname{Ci}(a z) \, dz = \frac{1}{b^4} \left(e^{b z} (b^3 z^3 - 3 b^2 z^2 + 6 b z - 6) \operatorname{Ci}(a z) + 3 (\operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z)) - \frac{1}{(a^2 + b^2)^3} \left(b e^{b z} \left(b \left((b^2 z^2 - b z + 3) a^4 + 2 b^2 (b^2 z^2 - 3 b z + 3) a^2 + b^4 (b^2 z^2 - 5 b z + 11) \right) \cos(a z) + a \left((b^2 z^2 - 3 b z + 6) a^4 + 2 b^2 (b^2 z^2 - 5 b z + 8) a^2 + b^4 (b^2 z^2 - 7 b z + 18) \right) \sin(a z) \right) \right) \right)$$

Involving trigonometric functions

Involving sin

06.38.21.0020.01

$$\int \sin(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{4 b} (-4 \cos(b z) \operatorname{Ci}(a z) + \operatorname{Ei}(-i (a - b) z) + \operatorname{Ei}(i (a - b) z) + \operatorname{Ei}(-i (a + b) z) + \operatorname{Ei}(i (a + b) z))$$

06.38.21.0021.01

$$\int \sin(a z) \operatorname{Ci}(a z) \, dz = \frac{-2 \cos(a z) \operatorname{Ci}(a z) + \operatorname{Ci}(2 a z) + \log(a z)}{2 a}$$

Involving cos

06.38.21.0022.01

$$\int \cos(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{4\,b}\left(i\left(\mathrm{Ei}(-i\,(a-b)\,z) - \mathrm{Ei}(i\,(a-b)\,z) - \mathrm{Ei}(-i\,(a+b)\,z) + \mathrm{Ei}(i\,(a+b)\,z) - 4\,i\,\mathrm{Ci}(a\,z)\sin(b\,z)\right)\right)$$

06.38.21.0023.01

$$\int \cos(a\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{2\,\mathrm{Ci}(a\,z)\sin(a\,z) - \mathrm{Si}(2\,a\,z)}{2\,a}$$

Involving trigonometric functions and a power function

Involving sin and power

06.38.21.0024.01

$$\int z^n \sin(b\,z)\,\mathrm{Ci}(a\,z)\,dz = -\frac{i}{4}\,(i\,b)^{-n-1}\,n!\Bigg(-(-1)^n\,\mathrm{Ei}(-i\,(a-b)\,z) - \mathrm{Ei}(i\,(a-b)\,z) - \mathrm{Ei}(-i\,(a+b)\,z) - (-1)^n\,\mathrm{Ei}(i\,(a+b)\,z) + \frac{1}{\Gamma(n+2)}\left(2\,(n+1)\,\mathrm{Ci}(a\,z)\left((-1)^n\,\Gamma(n+1,\,-i\,b\,z) + \Gamma(n+1,\,i\,b\,z)\right)\right) + e^{i\,(a-b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-a}\right)^m\sum_{k=0}^{m-1}\frac{(-i\,a+i\,b)^k\,z^k}{k!} + e^{-i\,(a+b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{a+b}\right)^m\sum_{k=0}^{m-1}\frac{(i\,a+i\,b)^k\,z^k}{k!} + (-1)^n\,e^{i\,(a+b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{a+b}\right)^m\sum_{k=0}^{m-1}\frac{(-i\,a-i\,b)^k\,z^k}{k!} + (-1)^n\,e^{-i\,(a-b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-a}\right)^m\sum_{k=0}^{m-1}\frac{(i\,a-i\,b)^k\,z^k}{k!}\Bigg)\,/;\,n\in\mathbb{N}$$

06.38.21.0025.01

$$\int z^n \sin(a\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{i^n\,a^{-n-1}}{4}\Bigg(n!\Bigg((-1)^n\,\mathrm{Ei}(-2\,i\,a\,z) + \mathrm{Ei}(2\,i\,a\,z) + (1+(-1)^n)\log(z) + 2\,(-1)^n\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(i\,a\,z)^k}{2\,k} - 2^{-k-1}\,\Gamma(k,\,2\,i\,a\,z)\right) + 2\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(-i\,a\,z)^k}{2\,k} - 2^{-k-1}\,\Gamma(k,\,-2\,i\,a\,z)\right)\Bigg) - 2\,\mathrm{Ci}(a\,z)\left(\Gamma(n+1,\,-i\,a\,z) + (-1)^n\,\Gamma(n+1,\,i\,a\,z)\right)\Bigg)\,/;\,n\in\mathbb{N}$$

06.38.21.0026.01

$$\int z\sin(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{4\,b^2}\Bigg(i\left(\mathrm{Ei}(-i\,(a-b)\,z) - \mathrm{Ei}(i\,(a-b)\,z) - \mathrm{Ei}(-i\,(a+b)\,z) + \mathrm{Ei}(i\,(a+b)\,z)\right) - 4\,\mathrm{Ci}(a\,z)\,(b\,z\cos(b\,z) - \sin(b\,z)) + \frac{4\,b\,(b\cos(a\,z)\sin(b\,z) - a\cos(b\,z)\sin(a\,z))}{b^2-a^2}\Bigg)$$

06.38.21.0027.01

$$\int z^2 \sin(b\,z)\,\mathrm{Ci}(a\,z)\,dz = -\frac{1}{2\,b^3}\left(\mathrm{Ei}(-i\,(a-b)\,z) + \mathrm{Ei}(i\,(a-b)\,z) + \mathrm{Ei}(-i\,(a+b)\,z) + \mathrm{Ei}(i\,(a+b)\,z)\right) + \frac{\cos(a\,z)\left(b\left(b^2-a^2\right)z\sin(b\,z) - \left(a^2-3\,b^2\right)\cos(b\,z)\right)}{(a-b)^2\,b\,(a+b)^2} + \frac{a\sin(a\,z)\left(b\left(a^2-b^2\right)z\cos(b\,z) - 2\left(a^2-2\,b^2\right)\sin(b\,z)\right)}{(a-b)^2\,b^2\,(a+b)^2} - \frac{\mathrm{Ci}(a\,z)\left(\left(b^2\,z^2-2\right)\cos(b\,z) - 2\,b\,z\sin(b\,z)\right)}{b^3}$$

06.38.21.0028.01

$$\int z^3 \sin(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{2 b^4} \Big(3 i (-\operatorname{Ei}(-i (a-b) z) + \operatorname{Ei}(i (a-b) z) + \operatorname{Ei}(-i (a+b) z) - \operatorname{Ei}(i (a+b) z)) + \frac{1}{(a-b)^3 (a+b)^3} \left(2 a b \sin(a z) \left(\left(\left(b^2 z^2 - 6\right) a^4 - 2 b^2 \left(b^2 z^2 - 8\right) a^2 + b^4 \left(b^2 z^2 - 18\right)\right) \cos(b z) + b \left(-3 a^4 + 10 b^2 a^2 - 7 b^4\right) z \sin(b z)\right)\right) + \frac{1}{(b-a)^3 (a+b)^3} \left(2 b^2 \cos(a z) \left(b \left(a^4 - 6 b^2 a^2 + 5 b^4\right) z \cos(b z) + \left(\left(b^2 z^2 - 3\right) a^4 + \left(6 b^2 - 2 b^4 z^2\right) a^2 + b^4 \left(b^2 z^2 - 11\right)\right) \sin(b z)\right)\right) - 2 \operatorname{Ci}(a z) \left(b z \left(b^2 z^2 - 6\right) \cos(b z) - 3 \left(b^2 z^2 - 2\right) \sin(b z)\right) \Big)$$

Involving cos and power

06.38.21.0029.01

$$\int z^n \cos(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{4} (i b)^{-n-1} n! \Bigg(-(-1)^n \operatorname{Ei}(-i (a-b) z) + \operatorname{Ei}(i (a-b) z) + \operatorname{Ei}(-i (a+b) z) - (-1)^n \operatorname{Ei}(i (a+b) z) + \frac{1}{\Gamma(n+2)} (2 (n+1) \operatorname{Ci}(a z) ((-1)^n \Gamma(n+1, -i b z) - \Gamma(n+1, i b z))) - e^{i (a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(-i a + i b)^k z^k}{k!} - e^{-i (a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(i a + i b)^k z^k}{k!} + (-1)^n e^{i (a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(-i a - i b)^k z^k}{k!} + (-1)^n e^{-i (a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(i a - i b)^k z^k}{k!} \Bigg) /; n \in \mathbb{N}$$

06.38.21.0030.01

$$\int z^n \cos(a z) \operatorname{Ci}(a z) \, dz = -\frac{i^{n+1}}{4} a^{-n-1} \Bigg(2 \operatorname{Ci}(a z) (\Gamma(n+1, -i a z) - (-1)^n \Gamma(n+1, i a z)) + n! \Bigg((-1)^n \operatorname{Ei}(-2 i a z) - \operatorname{Ei}(2 i a z) + ((-1)^n - 1) \log(z) - 2 \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-i a z)^k}{2 k} - 2^{-k-1} \Gamma(k, -2 i a z) \right) + 2 (-1)^n \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(i a z)^k}{2 k} - 2^{-k-1} \Gamma(k, 2 i a z) \right) \Bigg) \Bigg) /; n \in \mathbb{N}$$

06.38.21.0031.01

$$\int z \cos(b z) \operatorname{Ci}(a z) \, dz = -\frac{1}{4 b^2} \left(\operatorname{Ei}(-i (a-b) z) + \operatorname{Ei}(i (a-b) z) + \operatorname{Ei}(-i (a+b) z) + \operatorname{Ei}(i (a+b) z) - 2 \operatorname{Ci}(a z) (\Gamma(2, -i b z) + \Gamma(2, i b z)) + \frac{4 b (b \cos(a z) \cos(b z) + a \sin(a z) \sin(b z))}{(a-b) (a+b)} \right)$$

06.38.21.0032.01

$$\int z^2 \cos(b z)\,\mathrm{Ci}(a z)\,dz = \frac{1}{2 b^3}\left(i\left(-\mathrm{Ei}(-i (a-b) z) + \mathrm{Ci}(a z)\,(\Gamma(3, -i b z) - \Gamma(3, i b z)) + \frac{1}{(a-b)^2 (a+b)^2}\left(2 i \left(a (a-b)(a+b) z \sin(a z) - \left(a^2 - 3 b^2\right)\cos(a z)\right)\sin(b z)\, b^2 + 2 i \cos(b z)\left((a-b)(a+b) z \cos(a z)\, b^2 + 2 a \left(a^2 - 2 b^2\right)\sin(a z)\right) b + \left(a^2 - b^2\right)^2 (\mathrm{Ei}(i (a-b) z) + \mathrm{Ei}(-i (a+b) z) - \mathrm{Ei}(i (a+b) z))\right)\right)\right)$$

06.38.21.0033.01

$$\int z^3 \cos(b z)\,\mathrm{Ci}(a z)\,dz = \frac{1}{4 b^4}\left(\frac{1}{(a-b)^3 (a+b)^3}\left(6 (\mathrm{Ei}(-i (a-b) z) + \mathrm{Ei}(i (a-b) z) + \mathrm{Ei}(-i (a+b) z) + \mathrm{Ei}(i (a+b) z))\left(a^2 - b^2\right)^3 + 4 b^2 \cos(b z)\left(a\left(-3 a^4 + 10 b^2 a^2 - 7 b^4\right) z \sin(a z) - \left(-3 a^4 + 6 b^2 a^2 - 11 b^4 + b^2 \left(a^2 - b^2\right)^2 z^2\right)\cos(a z)\right) + 4 b \left(b^2 \left(a^4 - 6 b^2 a^2 + 5 b^4\right) z \cos(a z) - a \left(b^2 \left(a^2 - b^2\right)^2 z^2 - 2 \left(3 a^4 - 8 b^2 a^2 + 9 b^4\right)\right)\sin(a z)\right)\sin(b z)\right) - 2\,\mathrm{Ci}(a z)\,(\Gamma(4, -i b z) + \Gamma(4, i b z))\right)$$

Involving hyperbolic functions

Involving sinh

06.38.21.0034.01

$$\int \sinh(b z)\,\mathrm{Ci}(a z)\,dz = -\frac{-2 \cosh(b z)\,\mathrm{Ci}(a z) + \mathrm{Ci}((a + b i) z) + \mathrm{Ci}((a - i b) z)}{2 b}$$

Involving cosh

06.38.21.0035.01

$$\int \cosh(b z)\,\mathrm{Ci}(a z)\,dz = \frac{2\,\mathrm{Ci}(a z) \sinh(b z) + i\,\mathrm{Si}((a + b i) z) - i\,\mathrm{Si}((a - i b) z)}{2 b}$$

Involving hyperbolic functions and a power function

Involving sinh and power

06.38.21.0036.01

$$\int z^n \sinh(b z) \operatorname{Ci}(a z) \, dz = \frac{n! \, b^{-n-1}}{4} \left(-\operatorname{Ei}((-b + a(-i)) z) - \operatorname{Ei}((i a - b) z) - (-1)^n \operatorname{Ei}((b - i a) z) - (-1)^n \operatorname{Ei}((b + a i) z) + 2 (-1)^n e^{b z} \operatorname{Ci}(a z) \sum_{k=0}^{n} \frac{(-b z)^k}{k!} + 2 e^{-b z} \operatorname{Ci}(a z) \sum_{k=0}^{n} \frac{(b z)^k}{k!} + (-1)^n e^{(b + a i) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b + a i} \right)^m \sum_{k=0}^{m-1} \frac{(-b - i a)^k z^k}{k!} + (-1)^n e^{(b - i a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b - i a} \right)^m \sum_{k=0}^{m-1} \frac{(i a - b)^k z^k}{k!} + e^{(i a - b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b - i a} \right)^m \sum_{k=0}^{m-1} \frac{(b - i a)^k z^k}{k!} + e^{(-b - i a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b + a i} \right)^m \sum_{k=0}^{m-1} \frac{(b + a i)^k z^k}{k!} \right) /; n \in \mathbb{N}$$

06.38.21.0037.01

$$\int z \sinh(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{4 b^2 (a^2 + b^2)} \left(-4 \cos(a z) \sinh(b z) b^2 - 4 a \cosh(b z) \sin(a z) b + (a^2 + b^2) (\operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z) - \operatorname{Ei}(-b z + a(-i) z) - \operatorname{Ei}(i a z - b z) + 4 \operatorname{Ci}(a z) (b z \cosh(b z) - \sinh(b z))) \right)$$

06.38.21.0038.01

$$\int z^2 \sinh(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{4 b^3} \left(4 \operatorname{Ci}(a z) \left((b^2 z^2 + 2) \cosh(b z) - 2 b z \sinh(b z) \right) + \frac{1}{(a^2 + b^2)^2} \left(4 \cos(a z) \left((a^2 + 3 b^2) \cosh(b z) - b (a^2 + b^2) z \sinh(b z) \right) b^2 + 4 \sin(a z) \left(2 a (a^2 + 2 b^2) \sinh(b z) - a b (a^2 + b^2) z \cosh(b z) \right) b - 2 (a^2 + b^2)^2 (\operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z) + \operatorname{Ei}(-b z + a(-i) z) + \operatorname{Ei}(i a z - b z)) \right) \right)$$

06.38.21.0039.01

$$\int z^3 \sinh(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{4 b^4} \left(4 \operatorname{Ci}(a z) \left(b z (b^2 z^2 + 6) \cosh(b z) - 3 (b^2 z^2 + 2) \sinh(b z) \right) + \frac{1}{(a^2 + b^2)^3} \left(6 (\operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z) - \operatorname{Ei}(-b z + a(-i) z) - \operatorname{Ei}(i a z - b z)) (a^2 + b^2)^3 + 4 b \sin(a z) \left(a b (a^2 + b^2) (3 a^2 + 7 b^2) z \sinh(b z) - a \left(b^2 (a^2 + b^2)^2 z^2 + 2 (3 a^4 + 8 b^2 a^2 + 9 b^4) \right) \cosh(b z) \right) + 4 b^2 \cos(a z) \left(b (a^2 + b^2) (a^2 + 5 b^2) z \cosh(b z) - \left(3 a^4 + 6 b^2 a^2 + 11 b^4 + b^2 (a^2 + b^2)^2 z^2 \right) \sinh(b z) \right) \right) \right)$$

Involving cosh and power

06.38.21.0040.01

$$\int z^n \cosh(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{n!\,b^{-n-1}}{4}\Bigg(\mathrm{Ei}((-b+a\,(-i))\,z)+\mathrm{Ei}((i\,a-b)\,z)-(-1)^n\,\mathrm{Ei}((b-i\,a)\,z)-(-1)^n\,\mathrm{Ei}((b+a\,i)\,z)+2\,(-1)^n\,e^{b\,z}\,\mathrm{Ci}(a\,z)\sum_{k=0}^{n}\frac{(-b\,z)^k}{k!}-2\,e^{-b\,z}\,\mathrm{Ci}(a\,z)\sum_{k=0}^{n}\frac{(b\,z)^k}{k!}+(-1)^n\,e^{(b+a\,i)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b+a\,i}\right)^m\sum_{k=0}^{m-1}\frac{(-b-i\,a)^k\,z^k}{k!}+(-1)^n\,e^{(b-i\,a)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-i\,a}\right)^m\sum_{k=0}^{m-1}\frac{(i\,a-b)^k\,z^k}{k!}-e^{(i\,a-b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-i\,a}\right)^m\sum_{k=0}^{m-1}\frac{(b-i\,a)^k\,z^k}{k!}-e^{(-b-i\,a)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b+a\,i}\right)^m\sum_{k=0}^{m-1}\frac{(b+a\,i)^k\,z^k}{k!}\Bigg)\,/;\,n\in\mathbb{N}$$

06.38.21.0041.01

$$\int z\cosh(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{4\,b^2}\Bigg(\mathrm{Ei}((b-i\,a)\,z)+\mathrm{Ei}((b+a\,i)\,z)+\mathrm{Ei}(-b\,z+a\,(-i)\,z)+4\,\mathrm{Ci}(a\,z)\,(b\,z\sinh(b\,z)-\cosh(b\,z))+\frac{1}{a^2+b^2}\left(\left(a^2+b^2\right)\mathrm{Ei}(i\,a\,z-b\,z)-4\,b\,(b\cos(a\,z)\cosh(b\,z)+a\sin(a\,z)\sinh(b\,z))\right)\Bigg)$$

06.38.21.0042.01

$$\int z^2\cosh(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{4\,b^3}\Bigg(4\,\mathrm{Ci}(a\,z)\left(\left(b^2\,z^2+2\right)\sinh(b\,z)-2\,b\,z\cosh(b\,z)\right)+\frac{1}{\left(a^2+b^2\right)^2}\Big(4\cos(a\,z)\left(\left(a^2+3\,b^2\right)\sinh(b\,z)-b\left(a^2+b^2\right)z\cosh(b\,z)\right)b^2+4\sin(a\,z)\left(2\,a\left(a^2+2\,b^2\right)\cosh(b\,z)-a\,b\left(a^2+b^2\right)z\sinh(b\,z)\right)b-2\left(a^2+b^2\right)^2(\mathrm{Ei}((b-i\,a)\,z)+\mathrm{Ei}((b+a\,i)\,z)-\mathrm{Ei}(-b\,z+a\,(-i)\,z)-\mathrm{Ei}(i\,a\,z-b\,z))\Big)\Bigg)$$

06.38.21.0043.01

$$\int z^3\cosh(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{4\,b^4}\Bigg(4\,\mathrm{Ci}(a\,z)\left(b\,z\left(b^2\,z^2+6\right)\sinh(b\,z)-3\left(b^2\,z^2+2\right)\cosh(b\,z)\right)+\frac{1}{\left(a^2+b^2\right)^3}\Big(6\,(\mathrm{Ei}((b-i\,a)\,z)+\mathrm{Ei}((b+a\,i)\,z)+\mathrm{Ei}(-b\,z+a\,(-i)\,z)+\mathrm{Ei}(i\,a\,z-b\,z))\left(a^2+b^2\right)^3+4\,b^2\cos(a\,z)\left(b\left(a^2+b^2\right)\left(a^2+5\,b^2\right)z\sinh(b\,z)-\left(3\,a^4+6\,b^2\,a^2+11\,b^4+b^2\left(a^2+b^2\right)^2z^2\right)\cosh(b\,z)\right)+4\,b\sin(a\,z)\left(a\,b\left(a^2+b^2\right)\left(3\,a^2+7\,b^2\right)z\cosh(b\,z)-a\left(b^2\left(a^2+b^2\right)^2z^2+2\left(3\,a^4+8\,b^2\,a^2+9\,b^4\right)\right)\sinh(b\,z)\right)\Big)\Bigg)$$

Involving logarithm

Involving log

06.38.21.0044.01

$$\int \log(b\,z)\,\text{Ci}(a\,z)\,dz = \frac{a\,z\,\text{Ci}(a\,z)\,(\log(b\,z)-1)-\sin(a\,z)\,(\log(b\,z)-1)+\text{Si}(a\,z)}{a}$$

Involving logarithm and a power function

Involving log and power

06.38.21.0045.01

$$\int z^{\alpha-1}\log(b\,z)\,\text{Ci}(a\,z)\,dz = \frac{1}{2\,\alpha^3}\Big(z^\alpha\,\left(a^2\,z^2\right)^{-\alpha}\,\Big(-\alpha\,\Gamma(\alpha,\,i\,a\,z)\,(-i\,a\,z)^\alpha-\alpha\,\Gamma(\alpha+1)\log(z)\,(-i\,a\,z)^\alpha+\alpha^2\,\Gamma(\alpha,\,i\,a\,z)\log(b\,z)\,(-i\,a\,z)^\alpha-(i\,a\,z)^\alpha\,\alpha\,\Gamma(\alpha,\,-i\,a\,z)+\left(a^2\,z^2\right)^\alpha\,{}_2F_2(\alpha,\,\alpha;\,\alpha+1,\,\alpha+1;\,-i\,a\,z)+\left(a^2\,z^2\right)^\alpha\,{}_2F_2(\alpha,\,\alpha;\,\alpha+1,\,\alpha+1;\,i\,a\,z)-(i\,a\,z)^\alpha\,\alpha\,\Gamma(\alpha+1)\log(z)+(i\,a\,z)^\alpha\,\alpha^2\,\Gamma(\alpha,\,-i\,a\,z)\log(b\,z)+2\left(a^2\,z^2\right)^\alpha\,\alpha\,\text{Ci}(a\,z)\,(\alpha\log(b\,z)-1)\Big)\Big)$$

06.38.21.0046.01

$$\int z\log(b\,z)\,\text{Ci}(a\,z)\,dz = \frac{1}{4\,a^2}\left(-\cos(a\,z)\,(2\log(b\,z)+1)+\text{Ci}(a\,z)\left(-a^2\,z^2+2\,a^2\log(b\,z)\,z^2+2\right)+a\,z\,(1-2\log(b\,z))\sin(a\,z)\right)$$

06.38.21.0047.01

$$\int z^2\log(b\,z)\,\text{Ci}(a\,z)\,dz = \frac{1}{9\,a^3}\left(a^3\,\text{Ci}(a\,z)\,(3\log(b\,z)-1)\,z^3+a^2\sin(a\,z)\,z^2-3\,a^2\log(b\,z)\sin(a\,z)\,z^2-a\cos(a\,z)\,z-6\,a\cos(a\,z)\log(b\,z)\,z+6\log(b\,z)\sin(a\,z)+7\sin(a\,z)-6\,\text{Si}(a\,z)\right)$$

06.38.21.0048.01

$$\int z^3\log(b\,z)\,\text{Ci}(a\,z)\,dz = \frac{1}{16\,a^4}\left(\text{Ci}(a\,z)\left(-a^4\,z^4+4\,a^4\log(b\,z)\,z^4-24\right)+\cos(a\,z)\left(-a^2\,z^2-12\left(a^2\,z^2-2\right)\log(b\,z)+38\right)+a\,z\left(a^2\,z^2-4\left(a^2\,z^2-6\right)\log(b\,z)+14\right)\sin(a\,z)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

06.38.21.0049.01

$$\int \text{Ci}(a\,z)^2\,dz = \frac{a\,z\,\text{Ci}(a\,z)^2-2\sin(a\,z)\,\text{Ci}(a\,z)+\text{Si}(2\,a\,z)}{a}$$

Involving products of the direct function

06.38.21.0050.01

$$\int \text{Ci}(a\,z)\,\text{Ci}(b\,z)\,dz = \frac{1}{2\,a\,b}\,(2\,a\,b\,z\,\text{Ci}(a\,z)\,\text{Ci}(b\,z)-2\,b\sin(a\,z)\,\text{Ci}(b\,z)-2\,a\,\text{Ci}(a\,z)\sin(b\,z)-a\,\text{Si}((a-b)\,z)+b\,\text{Si}((a-b)\,z)+a\,\text{Si}((a+b)\,z)+b\,\text{Si}((a+b)\,z))$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

06.38.21.0051.01

$$\int z^n \operatorname{Ci}(a z)^2 \, dz = \frac{i^{n+1} a^{-n-1}}{2(n+1)} \Bigg(2 \operatorname{Ci}(a z) (\Gamma(n+1, -i a z) - (-1)^n \Gamma(n+1, i a z)) + n! \Bigg((-1)^n \operatorname{Ei}(-2 i a z) - \operatorname{Ei}(2 i a z) + ((-1)^n - 1) \log(z) - 2 \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-i a z)^k}{2k} - 2^{-k-1} \Gamma(k, -2 i a z)\right) + 2 (-1)^n \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(i a z)^k}{2k} - 2^{-k-1} \Gamma(k, 2 i a z)\right)\Bigg)\Bigg) + \frac{z^{n+1}}{n+1} \operatorname{Ci}(a z)^2 \;/;\; n \in \mathbb{N}$$

06.38.21.0052.01

$$\int z \operatorname{Ci}(a z)^2 \, dz = \frac{1}{4 a^2} \left(2 a^2 z^2 \operatorname{Ci}(a z)^2 - 4 (\cos(a z) + a z \sin(a z)) \operatorname{Ci}(a z) - \cos(2 a z) + 2 \operatorname{Ci}(2 a z) + 2 \log(z)\right)$$

06.38.21.0053.01

$$\int z^2 \operatorname{Ci}(a z)^2 \, dz = \frac{1}{12 a^3} \left(4 a^3 \operatorname{Ci}(a z)^2 z^3 + 8 a z - 2 a \cos(2 a z) z - 8 \operatorname{Ci}(a z) \left(2 a z \cos(a z) + \left(a^2 z^2 - 2\right) \sin(a z)\right) + 5 \sin(2 a z) - 8 \operatorname{Si}(2 a z)\right)$$

06.38.21.0054.01

$$\int z^3 \operatorname{Ci}(a z)^2 \, dz = \frac{1}{8 a^4} \Big(2 a^4 \operatorname{Ci}(a z)^2 z^4 + 3 a^2 z^2 - a^2 \cos(2 a z) z^2 + 4 a \sin(2 a z) z + 8 \cos(2 a z) - 12 \operatorname{Ci}(2 a z) - 12 \log(z) - 4 \operatorname{Ci}(a z) \left(3 \left(a^2 z^2 - 2\right) \cos(a z) + a z \left(a^2 z^2 - 6\right) \sin(a z)\right)\Big)$$

Involving products of the direct function and a power function

06.38.21.0055.01

$$\int z^n \operatorname{Ci}(a z) \operatorname{Ci}(b z) \, dz = \frac{\operatorname{Ci}(b z)}{n+1} \left(z^{n+1} \operatorname{Ci}(a z) + \frac{1}{2} i^{n+1} a^{-n-1} \left(\Gamma(n+1, -i a z) + (-1)^{n+1} \Gamma(n+1, i a z)\right)\right) - \frac{(i b)^{-n-1} n!}{4(n+1)} \Bigg(-(-1)^n \operatorname{Ei}(i (b-a) z) + \operatorname{Ei}(i (a-b) z) + \operatorname{Ei}(-i (a+b) z) - (-1)^n \operatorname{Ei}(i (a+b) z) + \frac{2 \operatorname{Ci}(a z)}{n!} ((-1)^n \Gamma(n+1, -i b z) - \Gamma(n+1, i b z)) - e^{i (a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(-i a + i b)^k z^k}{k!} - e^{-i (a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(i a + i b)^k z^k}{k!} + (-1)^n e^{i (a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(-i a - i b)^k z^k}{k!} + (-1)^n e^{-i (a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(i a - i b)^k z^k}{k!}\Bigg) + \frac{a^{-n-1} i^{n+1} n!}{4(n+1)} \sum_{k=0}^{n} \frac{a^k (a-b)^{-k} (a+b)^{-k}}{k!} \Big(-(-1)^n \Gamma(k, i (a+b) z) (a-b)^k - (-1)^n (a+b)^k \Gamma(k, i (a-b) z) + \left(\Gamma(k, -i (a+b) z) (a-b)^k + (a+b)^k \Gamma(k, i (b-a) z)\right)\Big) \;/;\; n \in \mathbb{N}$$

06.38.21.0056.01

$$\int z\,\mathrm{Chi}(a\,z)\,\mathrm{Chi}(b\,z)\,dz = \frac{1}{8\,a^2\,b^2}\left(4\,\mathrm{Chi}(a\,z)\,(\cosh(b\,z) + b\,z\,(b\,z\,\mathrm{Chi}(b\,z) - \sinh(b\,z)))\,a^2 + 4\,b\,\sinh(a\,z)\,\sinh(b\,z)\,a - \left(a^2 + b^2\right)(\mathrm{Ei}((a-b)\,z) + \mathrm{Ei}((b-a)\,z) + \mathrm{Ei}(-(a+b)\,z) + \mathrm{Ei}((a+b)\,z)) + 4\,b^2\,\mathrm{Chi}(b\,z)\,(\cosh(a\,z) - a\,z\,\sinh(a\,z))\right)$$

06.38.21.0057.01

$$\int z^2\,\mathrm{Ci}(a\,z)\,\mathrm{Ci}(b\,z)\,dz = -\frac{1}{3\,a^3\,(a-b)\,b^3\,(a+b)}\left(\left(\left(a^2 + 2\,b^2\right)\cos(a\,z)\,b^2 + a\,(a-b)\,(a+b)\left(a\left(b^2\,z^2 - 2\right)\mathrm{Ci}(a\,z) - b^2\,z\,\sin(a\,z)\right)\right)\sin(b\,z)\,a - a^2\,b\,\cos(b\,z)\left(\left(2\,a^2 + b^2\right)\sin(a\,z) - 2\,a\,(a-b)\,(a+b)\,z\,\mathrm{Ci}(a\,z)\right) + (a-b)\,(a+b)\left(\mathrm{Ci}(b\,z)\left(-a^3\,\mathrm{Ci}(a\,z)\,z^3 + 2\,a\,\cos(a\,z)\,z + \left(a^2\,z^2 - 2\right)\sin(a\,z)\right)b^3 + \left(b^3 - a^3\right)\mathrm{Si}((a-b)\,z) + \left(a^3 + b^3\right)\mathrm{Si}((a+b)\,z)\right)\right)$$

06.38.21.0058.01

$$\int z^3\,\mathrm{Ci}(a\,z)\,\mathrm{Ci}(b\,z)\,dz = -\left((a-b)^2\,(a+b)^2\,\mathrm{Ci}(a\,z)\left(-b^4\,\mathrm{Ci}(b\,z)\,z^4 + b\left(b^2\,z^2 - 6\right)\sin(b\,z)\,z + 3\left(b^2\,z^2 - 2\right)\cos(b\,z)\right)a^4 + b^2\cos(b\,z)\left(\left(3\,a^4 - 14\,b^2\,a^2 + 3\,b^4\right)\cos(a\,z) + a\left(-3\,a^4 + 2\,b^2\,a^2 + b^4\right)z\,\sin(a\,z)\right)a^2 - b\left(\left(a^2\,b^2\left(a^2 - b^2\right)^2 z^2 - 2\left(a^2 + b^2\right)\left(3\,a^4 - 8\,b^2\,a^2 + 3\,b^4\right)\right)\sin(a\,z) - b^2\left(a^5 + 2\,b^2\,a^3 - 3\,b^4\,a\right)z\,\cos(a\,z)\right)\sin(b\,z)\,a + (a-b)^2\,(a+b)^2\left(3\left(a^2\,z^2 - 2\right)\cos(a\,z)\,\mathrm{Ci}(b\,z)\,b^4 + a\,z\left(a^2\,z^2 - 6\right)\mathrm{Ci}(b\,z)\,\sin(a\,z)\,b^4 + 3\left(a^4 + b^4\right)(\mathrm{Ci}((a-b)\,z) + \mathrm{Ci}((a+b)\,z))\right)\right)\Big/\left(4\,a^4\,(a-b)^2\,b^4\,(a+b)^2\right)$$

Involving direct function and Gamma-, Beta-, Erf-type functions

Involving exponential integral-type functions

Involving Ei

06.38.21.0059.01

$$\int \mathrm{Ei}(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{2\,a\,b}\left(-2\,a\,\mathrm{Ci}(a\,z)\left(e^{b\,z} - b\,z\,\mathrm{Ei}(b\,z)\right) + (a + b\,i)\,\mathrm{Ei}((b - i\,a)\,z) + a\,\mathrm{Ei}((b + a\,i)\,z) - i\,b\,\mathrm{Ei}((b + a\,i)\,z) - 2\,b\,\mathrm{Ei}(b\,z)\,\sin(a\,z)\right)$$

Involving Si

06.38.21.0060.01

$$\int \mathrm{Si}(b\,z)\,\mathrm{Ci}(a\,z)\,dz = -\frac{1}{2\,a\,b}\left((a-b)\,\mathrm{Ci}((a-b)\,z) + a\,\mathrm{Ci}((a+b)\,z) + b\,\mathrm{Ci}((a+b)\,z) + 2\,b\,\sin(a\,z)\,\mathrm{Si}(b\,z) - 2\,a\,\mathrm{Ci}(a\,z)\,(\cos(b\,z) + b\,z\,\mathrm{Si}(b\,z))\right)$$

06.38.21.0061.01

$$\int \left(\mathrm{Ci}(b + a\,z)^2 + \mathrm{Si}(b + a\,z)^2\right)dz = \frac{1}{a}\left((b + a\,z)\,\mathrm{Ci}(b + a\,z)^2 - 2\sin(b + a\,z)\,\mathrm{Ci}(b + a\,z) + \mathrm{Si}(b + a\,z)\,(2\cos(b + a\,z) + (b + a\,z)\,\mathrm{Si}(b + a\,z))\right)$$

Involving exponential integral-type functions and a power function

Involving Ei and power

06.38.21.0062.01

$$\int z^n \operatorname{Ei}(b z) \operatorname{Ci}(a z) \, dz = \frac{\left(\Gamma(n+1, -b z) (-b)^{-n-1} + z^{n+1} \operatorname{Ei}(b z)\right) \operatorname{Ci}(a z)}{n+1} - \frac{1}{n+1} \left(\frac{(-b)^{-n-1} n!}{2} (\operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z)) + \frac{i (i a)^{-n}}{2 a} \left(-n! \operatorname{Ei}((b - i a) z) + n! \sum_{k=1}^{n} \frac{a^k (a + b i)^{-k} \Gamma(k, (i a - b) z)}{k!} + \operatorname{Ei}(b z) \Gamma(n+1, i a z) + (-1)^n \left(n! \operatorname{Ei}((b + a i) z) - n! \sum_{k=1}^{n} \frac{a^k (a - i b)^{-k} \Gamma(k, -(b + a i) z)}{k!} - \operatorname{Ei}(b z) \Gamma(n+1, -i a z) \right) \right) - \frac{(-b)^{-n-1} n!}{2} \sum_{k=1}^{n} \frac{1}{k!} \left(b^k \left(\Gamma(k, (i a - b) z) (b - i a)^{-k} + (b + a i)^{-k} \Gamma(k, -(b + a i) z) \right) \right) \right) /; n \in \mathbb{N}$$

06.38.21.0063.01

$$\int z \operatorname{Ei}(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{4} \left(-\frac{2 e^{b z} (b z - 1) \operatorname{Ci}(a z)}{b^2} - \frac{\left(a^2 - b^2\right) (\operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z))}{a^2 b^2} + \frac{2 e^{b z} \sin(a z)}{a b} - \frac{2 \operatorname{Ei}(b z) \left(-a^2 \operatorname{Ci}(a z) z^2 + a \sin(a z) z + \cos(a z)\right)}{a^2} \right)$$

06.38.21.0064.01

$$\int z^2 \operatorname{Ei}(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{3} \left(-\frac{\left(a^2 - 2 b^2\right) e^{b z} \cos(a z)}{a^2 b \left(a^2 + b^2\right)} + \frac{\left(a^3 - i b^3\right) \operatorname{Ei}((b - i a) z) + \left(a^3 + b^3 i\right) \operatorname{Ei}((b + a i) z)}{a^3 b^3} + \frac{e^{b z} \left((b z - 2) a^2 + b^2 (b z + 1)\right) \sin(a z)}{a b^2 \left(a^2 + b^2\right)} + \frac{1}{a^3} \left(\operatorname{Ei}(b z) \left(a^3 \operatorname{Ci}(a z) z^3 - 2 a \cos(a z) z + \left(2 - a^2 z^2\right) \sin(a z)\right) \right) - \frac{e^{b z} \left(b^2 z^2 - 2 b z + 2\right) \operatorname{Ci}(a z)}{b^3} \right)$$

06.38.21.0065.01

$$\int z^3 \operatorname{Ei}(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{4} \left(\left(e^{b z} \left((3 - b z) a^4 + 2 b^2 (b z + 7) a^2 + 3 b^4 (b z + 1)\right) \cos(a z) \right) \Big/ \left(a^2 b^2 \left(a^2 + b^2\right)^2 \right) - \frac{3 \left(a^4 + b^4\right) (\operatorname{Ei}((b - i a) z) + \operatorname{Ei}((b + a i) z))}{a^4 b^4} + \frac{1}{a^3 b^3 \left(a^2 + b^2\right)^2} \left(e^{b z} \left(\left(b^2 z^2 - 3 b z + 6\right) a^6 + 2 b^2 \left(b^2 z^2 - b z + 5\right) a^4 + b^4 \left(b^2 z^2 + b z - 10\right) a^2 - 6 b^6 \right) \sin(a z) \right) + \frac{1}{a^4} \left(\operatorname{Ei}(b z) \left(a^4 \operatorname{Ci}(a z) z^4 + a \left(6 - a^2 z^2\right) \sin(a z) z + \left(6 - 3 a^2 z^2\right) \cos(a z) \right) \right) - \frac{e^{b z} \left(b^3 z^3 - 3 b^2 z^2 + 6 b z - 6\right) \operatorname{Ci}(a z)}{b^4} \right)$$

Involving Si and power

06.38.21.0066.01

$$\int z^n \operatorname{Si}(b z) \operatorname{Ci}(a z) \, dz = \frac{z^{n+1} \operatorname{Si}(b z) \operatorname{Ci}(a z)}{n+1} - \frac{i (i b)^{-n-1} n!}{4 (n+1)} ((-1)^n (\operatorname{Ei}(i b z - i a z) + \operatorname{Ei}(a i z + b i z)) + \operatorname{Ei}(-i b z + a (-i) z)) + \operatorname{Ei}(i a z - i b z) + \frac{1}{2 (n+1)} \left(i (i b)^{-n-1} \operatorname{Ci}(a z) ((-1)^n \Gamma(n+1, -i b z) + \Gamma(n+1, i b z))\right) + \frac{i (i a)^{-n-1} n!}{4 (n+1)} \Bigg((-1)^n \operatorname{Ei}(-i (b-a) z) + \operatorname{Ei}(i (b-a) z) - \operatorname{Ei}(-i (a+b) z) - (-1)^{-n} \operatorname{Ei}(i (a+b) z) + \frac{2 i \operatorname{Si}(b z)}{n!} ((-1)^n \Gamma(n+1, -i a z) - \Gamma(n+1, i a z)) - e^{i (b-a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{a}{a-b}\right)^m \sum_{k=0}^{m-1} \frac{(-i b + i a)^k z^k}{k!} + e^{-i (a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{a}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(i b + i a)^k z^k}{k!} + (-1)^n e^{i (a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{a}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(-i b - i a)^k z^k}{k!} - (-1)^n e^{-i (b-a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\left(\frac{a}{a-b}\right)^m \sum_{k=0}^{m-1} \frac{(i b - i a)^k z^k}{k!}\right)\Bigg) - \frac{i (i b)^{-n-1} n!}{2 (n+1)} \left(\sum_{k=1}^{n} \frac{1}{2 k!} \left(b^k \left(-\Gamma(k, i (b-a) z) (b-a)^{-k} - (a+b)^{-k} \Gamma(k, i (a+b) z)\right)\right) + (-1)^n \sum_{k=1}^{n} \frac{1}{2 k!} \left(b^k \left(-(b-a)^{-k} \Gamma(k, i (a-b) z) - (a+b)^{-k} \Gamma(k, -i (a+b) z)\right)\right)\right) /; n \in \mathbb{N}$$

06.38.21.0067.01

$$\int z^n \operatorname{Si}(a z) \operatorname{Ci}(a z) \, dz = \frac{z^{n+1} \operatorname{Si}(a z) \operatorname{Ci}(a z)}{n+1} + \frac{1}{2 (n+1)} \left(i (i a)^{-n-1} \operatorname{Ci}(a z) ((-1)^n \Gamma(n+1, -i a z) + \Gamma(n+1, i a z))\right) - \frac{i^n a^{-n-1}}{4 (n+1)} \Bigg(n! \Bigg((-1)^n \operatorname{Ei}(-2 i a z) + \operatorname{Ei}(2 i a z) - (1 + (-1)^n) \log(z) - \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-i a z)^k}{k} + 2^{-k} \Gamma(k, -2 i a z)\right) - (-1)^n \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(i a z)^k}{k} + 2^{-k} \Gamma(k, 2 i a z)\right)\Bigg) - 2 i (\Gamma(n+1, -i a z) - (-1)^n \Gamma(n+1, i a z)) \operatorname{Si}(a z)\Bigg) - \frac{i (i a)^{-n-1} n!}{4 (n+1)} \left((\operatorname{Ei}(-2 i a z) + \log(z) + (-1)^n (\operatorname{Ei}(2 i a z) + \log(z))) + (-1)^n \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-i a z)^k}{k} - 2^{-k} \Gamma(k, -2 i a z)\right) + \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(i a z)^k}{k} - 2^{-k} \Gamma(k, 2 i a z)\right)\right) /; n \in \mathbb{N}$$

06.38.21.0068.01

$$\int z \operatorname{Si}(b z) \operatorname{Ci}(a z) \, dz = \frac{1}{8 a^2 b^2} \Big(e^{-i (a+b) z} \Big(2 a^2 e^{i (a+b) z} \operatorname{Ci}(a z) \left(2 b^2 \operatorname{Si}(b z) z^2 + i (\Gamma(2, -i b z) - \Gamma(2, i b z))\right) - i \Big(-2 i e^{i (a+b) z} (\Gamma(2, -i a z) + \Gamma(2, i a z)) \operatorname{Si}(b z) b^2 - a \left(-1 + e^{2 i a z}\right) \left(1 + e^{2 i b z}\right) b + \left(a^2 + b^2\right) e^{i (a+b) z} (\operatorname{Ei}(-i (a-b) z) - \operatorname{Ei}(i (a-b) z) - \operatorname{Ei}(-i (a+b) z) + \operatorname{Ei}(i (a+b) z))\Big)\Big)\Big)$$

06.38.21.0069.01

$$\int z^2\,\mathrm{Si}(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{6}\Bigg(2\,\mathrm{Ci}(a\,z)\,\mathrm{Si}(b\,z)\,z^3 + \frac{1}{b^3}(\mathrm{Ei}(-i\,(a-b)\,z) + \mathrm{Ei}(i\,(a-b)\,z) + \mathrm{Ei}(-i\,(a+b)\,z) + \mathrm{Ei}(i\,(a+b)\,z)) - \frac{1}{(2\,b^2)\,(a^2-b^2)} \left(b\,(b^2-a^2)\,(E_{-1}(-i\,(a-b)\,z) + E_{-1}(i\,(a-b)\,z) + E_{-1}(-i\,(a+b)\,z) + E_{-1}(i\,(a+b)\,z))\,z^2 - 8\,b\cos(a\,z)\cos(b\,z) - 8\,a\sin(a\,z)\sin(b\,z)\right) - \frac{1}{a^3}\Bigg(\frac{1}{(a^2-b^2)^2}\Big(2\,\big(a\,(a-b)\,(a+b)\,z\cos(a\,z) + (b^2-3\,a^2)\sin(a\,z)\big)\sin(b\,z)\,a^2 + 2\,b\cos(b\,z)\,\big(2\,(b^2-2\,a^2)\cos(a\,z) + a\,(b^2-a^2)\,z\sin(a\,z)\big)\,a + (a^2-b^2)^2\,(\mathrm{Ei}(-i\,(a-b)\,z) + \mathrm{Ei}(i\,(a-b)\,z) - \mathrm{Ei}(-i\,(a+b)\,z) - \mathrm{Ei}(i\,(a+b)\,z))\Big) + i\,(\Gamma(3,-i\,a\,z) - \Gamma(3,i\,a\,z))\,\mathrm{Si}(b\,z)\Bigg)\Bigg) - \frac{\mathrm{Ci}(a\,z)\,(\Gamma(3,-i\,b\,z) + \Gamma(3,i\,b\,z))}{b^3}$$

06.38.21.0070.01

$$\int z^3\,\mathrm{Si}(b\,z)\,\mathrm{Ci}(a\,z)\,dz = \frac{1}{8}\Bigg(2\,\mathrm{Ci}(a\,z)\,\mathrm{Si}(b\,z)\,z^4 + \frac{1}{b^4}(3\,i\,(\mathrm{Ei}(-i\,(a-b)\,z) - \mathrm{Ei}(i\,(a-b)\,z) - \mathrm{Ei}(-i\,(a+b)\,z) + \mathrm{Ei}(i\,(a+b)\,z))) + \frac{i\,\mathrm{Ci}(a\,z)\,(\Gamma(4,i\,b\,z) - \Gamma(4,-i\,b\,z))}{b^4} - \frac{i}{2\,b^3\,(a^2-b^2)}\Big((a+b)\,\big((a-b)\,b\,(b\,i\,z\,(E_{-2}(-i\,(a-b)\,z) + E_{-2}(i\,(a-b)\,z) + E_{-2}(-i\,(a+b)\,z) + E_{-2}(i\,(a+b)\,z)) + 3\,(E_{-1}(-i\,(a-b)\,z) - E_{-1}(i\,(a-b)\,z) - E_{-1}(-i\,(a+b)\,z) + E_{-1}(i\,(a+b)\,z)))\,z^2 + 12\,i\sin((a-b)\,z)\big) + 12\,(a-b)\,i\sin((a+b)\,z)\Big) + \frac{i}{2\,a^4}\Bigg(\frac{1}{(a^2-b^2)^3}\Big(6\,(\mathrm{Ei}(-i\,(a-b)\,z) - \mathrm{Ei}(i\,(a-b)\,z) - \mathrm{Ei}(-i\,(a+b)\,z) + \mathrm{Ei}(i\,(a+b)\,z))\,(a^2-b^2)^3 - 4\,i\,a\cos(b\,z)\,\big(a\,b\,(7\,a^4-10\,b^2\,a^2+3\,b^4)\,z\cos(a\,z) + b\,(a^2\,(a^2-b^2)^2\,z^2 - 2\,(9\,a^4-8\,b^2\,a^2+3\,b^4))\sin(a\,z)\big) + 4\,a^2\,i\,\big((-11\,a^4+6\,b^2\,a^2+(a^2-b^2)^2\,z^2\,a^2-3\,b^4)\cos(a\,z) - a\,(5\,a^4-6\,b^2\,a^2+b^4)\,z\sin(a\,z)\big)\sin(b\,z)\Big) - 2\,i\,(\Gamma(4,-i\,a\,z) + \Gamma(4,i\,a\,z))\,\mathrm{Si}(b\,z)\Bigg)\Bigg)$$

Definite integration

Involving the direct function

06.38.21.0071.01

$$\int_0^\infty e^{-t\,z}\,\mathrm{Ci}(t)\,dt = -\frac{\log(1+z^2)}{2\,z}\ /;\ \mathrm{Re}(z) > 0$$

06.38.21.0072.01

$$\int_0^\infty \cos(t)\,\mathrm{Ci}(t)\,dt = -\frac{\pi}{4}$$

06.38.21.0073.01

$$\int_0^\infty \text{Ci}(t)^2 \, dt = \frac{\pi}{2}$$

Involving related functions

06.38.21.0074.01

$$\int_0^\infty \text{Ci}(t)\left(\text{Si}(t) - \frac{\pi}{2}\right) dt = \log(2)$$

Integral transforms

Laplace transforms

06.38.22.0001.01

$$\mathcal{L}_t[\text{Ci}(t)](z) = -\frac{\log(1+z^2)}{2z} \; /; \; \text{Re}(z) > 0$$

Operations

Limit operation

06.38.25.0001.01

$$\lim_{x\to\infty} \text{Ci}(a + b x) = \begin{cases} \pi i & \arg(b) = \pi \wedge 0 \le \arg(a) \le \pi \\ -\pi i & \arg(b) = \pi \wedge -\pi < \arg(a) < 0 \\ 0 & \arg(b) = 0 \\ \infty & |\arg(a)| = \frac{\pi}{2} \bigwedge |\arg(b)| = \frac{\pi}{2} \\ \tilde{\infty} & \text{True} \end{cases}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_pF_q$

06.38.26.0001.01

$$\text{Ci}(z) = -\frac{z^2}{4} \, {}_2F_3\left(1, 1; 2, 2, \frac{3}{2}; -\frac{z^2}{4}\right) + \log(z) + \gamma$$

Through Meijer *G*

Classical cases for the direct function itself

06.38.26.0002.01

$$\text{Ci}(z) = -\frac{\sqrt{\pi}}{2} \, G_{1,3}^{2,0}\left(\frac{z^2}{4} \middle| \begin{matrix} 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right) - \frac{1}{2}\left(\log(z^2) - 2\log(z)\right)$$

06.38.26.0003.01

$$\text{Ci}(z) = -\frac{\sqrt{\pi}}{2} \, G_{1,3}^{2,0}\left(\frac{z^2}{4} \middle| \begin{matrix} 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right) /; \; \text{Re}(z) > 0$$

06.38.26.0004.01

$$\operatorname{Ci}\left(\sqrt{z}\right) = -\frac{1}{2}\sqrt{\pi}\, G_{1,3}^{2,0}\left(\frac{z}{4} \,\middle|\, \begin{matrix} 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right)$$

Classical cases for powers of Ci and Si

06.38.26.0018.01

$$\operatorname{Ci}\left(\sqrt{z}\right)^2 + \operatorname{Si}\left(\sqrt{z}\right)^2 = \frac{1}{2}\pi^{3/2}\, G_{3,5}^{3,1}\left(\frac{z}{4} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) + \frac{\pi^2}{4}$$

06.38.26.0005.01

$$\left(\operatorname{Si}\left(\sqrt{z}\right) - \frac{\pi}{2}\right)^2 + \operatorname{Ci}\left(\sqrt{z}\right)^2 = \frac{1}{2\sqrt{\pi}}\, G_{2,4}^{4,1}\left(\frac{z}{4} \,\middle|\, \begin{matrix} 0, 1 \\ 0, 0, 0, \frac{1}{2} \end{matrix}\right)$$

Classical cases involving cos, sin, Si

06.38.26.0006.01

$$\cos\left(\sqrt{z}\right)\operatorname{Ci}\left(\sqrt{z}\right) + \sin\left(\sqrt{z}\right)\operatorname{Si}\left(\sqrt{z}\right) = -\frac{1}{2}\pi^{3/2}\, G_{2,4}^{2,1}\left(\frac{z}{4} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

06.38.26.0007.01

$$\cos\left(\sqrt{z}\right)\operatorname{Ci}\left(\sqrt{z}\right) + \sin\left(\sqrt{z}\right)\left(\operatorname{Si}\left(\sqrt{z}\right) - \frac{\pi}{2}\right) = -\frac{1}{2\sqrt{\pi}}\, G_{1,3}^{3,1}\left(\frac{z}{4} \,\middle|\, \begin{matrix} 0 \\ 0, 0, \frac{1}{2} \end{matrix}\right)$$

06.38.26.0008.01

$$\sin\left(\sqrt{z}\right)\operatorname{Ci}\left(\sqrt{z}\right) - \cos\left(\sqrt{z}\right)\operatorname{Si}\left(\sqrt{z}\right) = -\frac{1}{2}\pi^{3/2}\, G_{2,4}^{2,1}\left(\frac{z}{4} \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{2}, \frac{1}{2}, 1, 0 \end{matrix}\right)$$

06.38.26.0009.01

$$\sin\left(\sqrt{z}\right)\operatorname{Ci}\left(\sqrt{z}\right) - \cos\left(\sqrt{z}\right)\left(\operatorname{Si}\left(\sqrt{z}\right) - \frac{\pi}{2}\right) = \frac{1}{2\sqrt{\pi}}\, G_{1,3}^{3,1}\left(\frac{z}{4} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Generalized cases for the direct function itself

06.38.26.0010.01

$$\operatorname{Ci}(z) = -\frac{\sqrt{\pi}}{2}\, G_{1,3}^{2,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right)$$

Generalized cases for powers of Ci and Si

06.38.26.0019.01

$$\operatorname{Ci}(z)^2 + \operatorname{Si}(z)^2 = \frac{1}{2}\pi^{3/2}\, G_{3,5}^{3,1}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) + \frac{\pi^2}{4}$$

06.38.26.0011.01

$$\left(\operatorname{Si}(z) - \frac{\pi}{2}\right)^2 + \operatorname{Ci}(z)^2 = \frac{1}{2\sqrt{\pi}}\, G_{2,4}^{4,1}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 0, 1 \\ 0, 0, 0, \frac{1}{2} \end{matrix}\right)$$

Generalized cases involving cos, sin, Si

06.38.26.0012.01

$$\cos(z)\,\text{Ci}(z) + \sin(z)\,\text{Si}(z) = -\frac{1}{2}\pi^{3/2}\,G_{2,4}^{2,1}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

06.38.26.0013.01

$$\cos(z)\,\text{Ci}(z) + \sin(z)\left(\text{Si}(z) - \frac{\pi}{2}\right) = -\frac{1}{2\sqrt{\pi}}\,G_{1,3}^{3,1}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 0 \\ 0, 0, \frac{1}{2} \end{matrix}\right)$$

06.38.26.0014.01

$$\sin(z)\,\text{Ci}(z) - \cos(z)\,\text{Si}(z) = -\frac{1}{2}\pi^{3/2}\,G_{2,4}^{2,1}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{2}, \frac{1}{2}, 1, 0 \end{matrix}\right)$$

06.38.26.0015.01

$$\sin(z)\,\text{Ci}(z) - \cos(z)\left(\text{Si}(z) - \frac{\pi}{2}\right) = \frac{1}{2\sqrt{\pi}}\,G_{1,3}^{3,1}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Through other functions

06.38.26.0016.01

$$\text{Ci}(z) = \log(z) - \frac{1}{2}\left(\Gamma(0, -i z) + \Gamma(0, i z) + \log(-i z) + \log(i z)\right)$$

06.38.26.0017.01

$$\text{Ci}(z) = \log(z) - \frac{1}{2}\left(E_1(-i z) + E_1(i z) + \log(-i z) + \log(i z)\right)$$

Representations through equivalent functions

With related functions

06.38.27.0001.01

$$\text{Ci}(z) = \text{Chi}(i z) - \log(i z) + \log(z)$$

06.38.27.0002.01

$$\text{Ci}(z) = \log(z) + \frac{1}{4}\left(2\left(\text{Ei}(-i z) + \text{Ei}(i z)\right) + \log\left(\frac{i}{z}\right) + \log\left(-\frac{i}{z}\right) - \log(-i z) - \log(i z)\right)$$

06.38.27.0003.01

$$\text{Ci}(z) = \frac{1}{2}\left(\text{li}\left(e^{-i z}\right) + \text{li}\left(e^{i z}\right)\right) + \frac{\pi i}{2}\,\text{sgn}(\text{Im}(z))\,(1 - \text{sgn}(\text{Re}(z))) \;/;\; |\text{Re}(z)| < \pi$$

Theorems

The averaged radiated intensity from a thin linear antenna

The averaged radiated intensity Π from a thin linear antenna of length l driven with a time-dependent current $i = i_0 \cos\left(\omega t + n \pi \frac{x}{l}\right)$, $n \in \mathbb{N}^+$, where x is the coordinate along the antenna, $-l < x < l$, is given by $\Pi \propto i_0^2 (\gamma + \log(2 \pi n) - \text{Ci}(2 \pi n))$.

History

– L. Mascheroni (1790, 1819)
– F.W. Bessel (1812)
– C.A. Bretschneider (1843)
– O. Schlomilch (1846)
– F. Arndt (1847)
– J. W. L. Glaisher (1870) introduced the notations Si and Ci
– N. Nielsen (1904) used the notations Si and Ci

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