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Csc

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Notations

Traditional name

Cosecant

Traditional notation

$\csc(z)$

Mathematica StandardForm notation

`Csc[z]`

Primary definition

01.10.02.0001.01

$$\csc(z) = \frac{1}{\sin(z)} = \frac{2\,i}{e^{i\,z} - e^{-i\,z}}$$

Specific values

Specialized values

01.10.03.0001.01

$$\csc(\pi\, m) = \tilde{\infty} \;/;\; m \in \mathbb{Z}$$

01.10.03.0002.01

$$\csc\left(\pi\left(\frac{1}{2} + m\right)\right) = (-1)^m \;/;\; m \in \mathbb{Z}$$

Values at fixed points

01.10.03.0003.01

$$\csc(0) = \tilde{\infty}$$

01.10.03.0004.01

$$\csc\left(\frac{\pi}{12}\right) = \sqrt{2} + \sqrt{6}$$

01.10.03.0005.01

$$\csc\left(\frac{\pi}{12}\right) = \left(z;\, z^4 - 16\, z^2 + 16\right)_4^{-1}$$

01.10.03.0006.01

$$\csc\left(\frac{\pi}{10}\right) == 1+\sqrt{5}$$

01.10.03.0007.01

$$\csc\left(\frac{\pi}{9}\right) == \frac{4\sqrt[3]{2}}{\left(i+\sqrt{3}\right)\sqrt[3]{-1+i\sqrt{3}}+\left(-i+\sqrt{3}\right)\sqrt[3]{-1-i\sqrt{3}}}$$

01.10.03.0008.01

$$\csc\left(\frac{\pi}{9}\right) == \left(z;\, 3z^6-36z^4+96z^2-64\right)_6^{-1}$$

01.10.03.0009.01

$$\csc\left(\frac{\pi}{9}\right) == \frac{2(-1)^{11/18}}{-1+(-1)^{2/9}}$$

01.10.03.0010.01

$$\csc\left(\frac{\pi}{8}\right) == \sqrt{2\left(2+\sqrt{2}\right)}$$

01.10.03.0011.01

$$\csc\left(\frac{\pi}{8}\right) == \left(z;\, z^4-8z^2+8\right)_4^{-1}$$

01.10.03.0012.01

$$\csc\left(\frac{\pi}{8}\right) == \frac{2(-1)^{5/8}}{-1+\sqrt[4]{-1}}$$

01.10.03.0013.01

$$\csc\left(\frac{\pi}{7}\right) == \left(24\sqrt[3]{14-42\,i\sqrt{3}}\right) \Big/ \left(\sqrt[3]{2}\left(\left(i+\sqrt{3}\right)\left(\sqrt[3]{28-84\,i\sqrt{3}}-2\,i\sqrt{7}\right)\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}+2\left(\sqrt{7}+i\sqrt{21}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}+\left(-i+\sqrt{3}\right)\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right)-4\,7^{5/6}\sqrt[3]{2-6\,i\sqrt{3}}\right)$$

01.10.03.0014.01

$$\csc\left(\frac{\pi}{7}\right) == \left(z;\, 7z^6-56z^4+112z^2-64\right)_6^{-1}$$

01.10.03.0015.01

$$\csc\left(\frac{\pi}{7}\right) == \frac{2(-1)^{9/14}}{-1+(-1)^{2/7}}$$

01.10.03.0016.01

$$\csc\left(\frac{\pi}{6}\right) == 2$$

01.10.03.0017.01

$$\csc\left(\frac{\pi}{5}\right) == \sqrt{2+\frac{2}{\sqrt{5}}}$$

01.10.03.0018.01

$$\csc\left(\frac{\pi}{5}\right) = \left(z;\, 5 z^4 - 20 z^2 + 16\right)_4^{-1}$$

01.10.03.0019.01

$$\csc\left(\frac{2\pi}{9}\right) = -\frac{2 i}{\sqrt[3]{-\frac{1}{2} i \left(-i + \sqrt{3}\right)} - \sqrt[3]{\frac{1}{2} i \left(i + \sqrt{3}\right)}}$$

01.10.03.0020.01

$$\csc\left(\frac{2\pi}{9}\right) = \left(z;\, 3 z^6 - 36 z^4 + 96 z^2 - 64\right)_5^{-1}$$

01.10.03.0021.01

$$\csc\left(\frac{2\pi}{9}\right) = \frac{2 (-1)^{13/18}}{-1 + (-1)^{4/9}}$$

01.10.03.0022.01

$$\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$$

01.10.03.0023.01

$$\csc\left(\frac{2\pi}{7}\right) = \left(24 \sqrt[3]{14 - 42 i \sqrt{3}}\right) \Big/ \left(4\, 7^{5/6} \sqrt[3]{2 - 6 i \sqrt{3}} + \sqrt[3]{2} \left(2 \left(\sqrt[3]{28 - 84 i \sqrt{3}} + \sqrt{7}\, i - \sqrt{21}\right) i \sqrt[3]{14 + i \sqrt{7} + 3 \sqrt{21}} + 4 \sqrt{7} \sqrt[3]{14 - i \sqrt{7} - 3 \sqrt{21}} + \left(i + \sqrt{3}\right) \sqrt[3]{28 + 2 i \sqrt{7} + 6 \sqrt{21}} \left(14 - i \sqrt{7} - 3 \sqrt{21}\right)^{2/3}\right)\right)$$

01.10.03.0024.01

$$\csc\left(\frac{2\pi}{7}\right) = \left(z;\, 7 z^6 - 56 z^4 + 112 z^2 - 64\right)_5^{-1}$$

01.10.03.0025.01

$$\csc\left(\frac{2\pi}{7}\right) = \frac{2 (-1)^{11/14}}{-1 + (-1)^{4/7}}$$

01.10.03.0026.01

$$\csc\left(\frac{3\pi}{10}\right) = \sqrt{5} - 1$$

01.10.03.0027.01

$$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$$

01.10.03.0028.01

$$\csc\left(\frac{3\pi}{8}\right) = \frac{2}{\sqrt{2 + \sqrt{2}}}$$

01.10.03.0029.01

$$\csc\left(\frac{3\pi}{8}\right) = \left(z;\, z^4 - 8 z^2 + 8\right)_3^{-1}$$

01.10.03.0030.01

$$\csc\left(\frac{3\pi}{8}\right) = \frac{2(-1)^{7/8}}{-1+(-1)^{3/4}}$$

01.10.03.0031.01

$$\csc\left(\frac{2\pi}{5}\right) = \sqrt{2-\frac{2}{\sqrt{5}}}$$

01.10.03.0032.01

$$\csc\left(\frac{2\pi}{5}\right) = \left(z;\, 5z^4-20z^2+16\right)_3^{-1}$$

01.10.03.0033.01

$$\csc\left(\frac{5\pi}{12}\right) = \sqrt{6}-\sqrt{2}$$

01.10.03.0034.01

$$\csc\left(\frac{5\pi}{12}\right) = \left(z;\, z^4-16z^2+16\right)_3^{-1}$$

01.10.03.0035.01

$$\csc\left(\frac{3\pi}{7}\right) = \left(24\sqrt[3]{7-21i\sqrt{3}}\right)\Big/\left(4\,7^{5/6}\sqrt[3]{1-3i\sqrt{3}}+4\sqrt{7}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}-\right.$$
$$2i\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{28+2i\sqrt{7}+6\sqrt{21}}+\sqrt{3}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\sqrt[3]{28-2i\sqrt{7}-6\sqrt{21}}-$$
$$\left.i\sqrt[3]{28-2i\sqrt{7}-6\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}+2\sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\;i\right)$$

01.10.03.0036.01

$$\csc\left(\frac{3\pi}{7}\right) = \left(z;\, 7z^6-56z^4+112z^2-64\right)_4^{-1}$$

01.10.03.0037.01

$$\csc\left(\frac{3\pi}{7}\right) = \frac{2(-1)^{13/14}}{-1+(-1)^{6/7}}$$

01.10.03.0038.01

$$\csc\left(\frac{4\pi}{9}\right) = \frac{4}{\left(i+\sqrt{3}\right)\sqrt[3]{-\frac{1}{2}i\left(-i+\sqrt{3}\right)}+\left(-i+\sqrt{3}\right)\sqrt[3]{\frac{1}{2}i\left(i+\sqrt{3}\right)}}$$

01.10.03.0039.01

$$\csc\left(\frac{4\pi}{9}\right) = \left(z;\, 3z^6-36z^4+96z^2-64\right)_4^{-1}$$

01.10.03.0040.01

$$\csc\left(\frac{4\pi}{9}\right) = \frac{2(-1)^{17/18}}{-1+(-1)^{8/9}}$$

01.10.03.0041.01

$$\csc\left(\frac{\pi}{2}\right) = 1$$

01.10.03.0042.01

$$\csc\left(\frac{5\pi}{9}\right) == \frac{4}{\left(i+\sqrt{3}\right)\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)}+\left(-i+\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}}$$

01.10.03.0043.01

$$\csc\left(\frac{5\pi}{9}\right) == \left(z;\,3\,z^6-36\,z^4+96\,z^2-64\right)_4^{-1}$$

01.10.03.0044.01

$$\csc\left(\frac{5\pi}{9}\right) == \frac{2\sqrt[18]{-1}}{1+\sqrt[9]{-1}}$$

01.10.03.0045.01

$$\csc\left(\frac{4\pi}{7}\right) == \left(24\sqrt[3]{7-21\,i\sqrt{3}}\right)\Big/\Big(4\,7^{5/6}\sqrt[3]{1-3\,i\sqrt{3}}+4\sqrt{7}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}-$$
$$2\,i\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}+\sqrt{3}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\sqrt[3]{28-2\,i\sqrt{7}-6\sqrt{21}}-$$
$$i\sqrt[3]{28-2\,i\sqrt{7}-6\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}+2\sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\;i\Big)$$

01.10.03.0046.01

$$\csc\left(\frac{4\pi}{7}\right) == \left(z;\,7\,z^6-56\,z^4+112\,z^2-64\right)_4^{-1}$$

01.10.03.0047.01

$$\csc\left(\frac{4\pi}{7}\right) == \frac{2\sqrt[14]{-1}}{1+\sqrt[7]{-1}}$$

01.10.03.0048.01

$$\csc\left(\frac{7\pi}{12}\right) == \sqrt{6}-\sqrt{2}$$

01.10.03.0049.01

$$\csc\left(\frac{7\pi}{12}\right) == \left(z;\,z^4-16\,z^2+16\right)_3^{-1}$$

01.10.03.0050.01

$$\csc\left(\frac{3\pi}{5}\right) == \sqrt{2-\frac{2}{\sqrt{5}}}$$

01.10.03.0051.01

$$\csc\left(\frac{3\pi}{5}\right) == \left(z;\,5\,z^4-20\,z^2+16\right)_3^{-1}$$

01.10.03.0052.01

$$\csc\left(\frac{5\pi}{8}\right) == \frac{2}{\sqrt{2+\sqrt{2}}}$$

01.10.03.0053.01

$$\csc\left(\frac{5\pi}{8}\right) == \left(z;\,z^4-8\,z^2+8\right)_3^{-1}$$

01.10.03.0054.01

$$\csc\left(\frac{5\pi}{8}\right) == \frac{2\sqrt[8]{-1}}{1+\sqrt[4]{-1}}$$

01.10.03.0055.01

$$\csc\left(\frac{2\pi}{3}\right) == \frac{2}{\sqrt{3}}$$

01.10.03.0056.01

$$\csc\left(\frac{7\pi}{10}\right) == \sqrt{5} - 1$$

01.10.03.0057.01

$$\csc\left(\frac{5\pi}{7}\right) == \left(24\sqrt[3]{14-42\,i\sqrt{3}}\right) \Big/ \left(4\,7^{5/6}\sqrt[3]{2-6\,i\sqrt{3}} + \sqrt[3]{2}\left(2\left(\sqrt[3]{28-84\,i\sqrt{3}} + \sqrt{7}\,i - \sqrt{21}\right)i\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + 4\sqrt{7}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + \left(i+\sqrt{3}\right)\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right)\right)$$

01.10.03.0058.01

$$\csc\left(\frac{5\pi}{7}\right) == \left(z;\,7z^6 - 56z^4 + 112z^2 - 64\right)_5^{-1}$$

01.10.03.0059.01

$$\csc\left(\frac{5\pi}{7}\right) == \frac{2(-1)^{3/14}}{1+(-1)^{3/7}}$$

01.10.03.0060.01

$$\csc\left(\frac{3\pi}{4}\right) == \sqrt{2}$$

01.10.03.0061.01

$$\csc\left(\frac{7\pi}{9}\right) == -\frac{2\,i}{\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)} - \sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}}$$

01.10.03.0062.01

$$\csc\left(\frac{7\pi}{9}\right) == \left(z;\,3z^6 - 36z^4 + 96z^2 - 64\right)_5^{-1}$$

01.10.03.0063.01

$$\csc\left(\frac{7\pi}{9}\right) == \frac{2(-1)^{5/18}}{1+(-1)^{5/9}}$$

01.10.03.0064.01

$$\csc\left(\frac{4\pi}{5}\right) == \sqrt{2+\frac{2}{\sqrt{5}}}$$

01.10.03.0065.01

$$\csc\left(\frac{4\pi}{5}\right) == \left(z;\,5z^4 - 20z^2 + 16\right)_4^{-1}$$

01.10.03.0066.01

$$\csc\left(\frac{5\pi}{6}\right) = 2$$

01.10.03.0067.01

$$\csc\left(\frac{6\pi}{7}\right) = \left(24\sqrt[3]{14 - 42\,i\sqrt{3}}\right) \Big/ \left(\sqrt[3]{2}\left(\left(i+\sqrt{3}\right)\left(\sqrt[3]{28-84\,i\sqrt{3}} - 2\,i\sqrt{7}\right)\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + 2\left(\sqrt{7}+i\sqrt{21}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + \left(-i+\sqrt{3}\right)\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right) - 4\,7^{5/6}\sqrt[3]{2-6\,i\sqrt{3}}\right)$$

01.10.03.0068.01

$$\csc\left(\frac{6\pi}{7}\right) = \left(z;\, 7z^6 - 56z^4 + 112z^2 - 64\right)_6^{-1}$$

01.10.03.0069.01

$$\csc\left(\frac{6\pi}{7}\right) = \frac{2(-1)^{5/14}}{1+(-1)^{5/7}}$$

01.10.03.0070.01

$$\csc\left(\frac{7\pi}{8}\right) = \sqrt{2\left(2+\sqrt{2}\right)}$$

01.10.03.0071.01

$$\csc\left(\frac{7\pi}{8}\right) = \left(z;\, z^4 - 8z^2 + 8\right)_4^{-1}$$

01.10.03.0072.01

$$\csc\left(\frac{7\pi}{8}\right) = \frac{2(-1)^{3/8}}{1+(-1)^{3/4}}$$

01.10.03.0073.01

$$\csc\left(\frac{8\pi}{9}\right) = \frac{4\sqrt[3]{2}}{\left(i+\sqrt{3}\right)\sqrt[3]{-1+i\sqrt{3}} + \left(-i+\sqrt{3}\right)\sqrt[3]{-1-i\sqrt{3}}}$$

01.10.03.0074.01

$$\csc\left(\frac{8\pi}{9}\right) = \left(z;\, 3z^6 - 36z^4 + 96z^2 - 64\right)_6^{-1}$$

01.10.03.0075.01

$$\csc\left(\frac{8\pi}{9}\right) = \frac{2(-1)^{7/18}}{1+(-1)^{7/9}}$$

01.10.03.0076.01

$$\csc\left(\frac{9\pi}{10}\right) = 1+\sqrt{5}$$

01.10.03.0077.01

$$\csc\left(\frac{11\pi}{12}\right) = \sqrt{6}+\sqrt{2}$$

01.10.03.0078.01

$$\csc\left(\frac{11\pi}{12}\right) = \left(z;\, z^4 - 16 z^2 + 16\right)_4^{-1}$$

01.10.03.0079.01

$$\csc(\pi) = \tilde{\infty}$$

01.10.03.0080.01

$$\csc\left(\frac{13\pi}{12}\right) = -2\sqrt{2+\sqrt{3}}$$

01.10.03.0081.01

$$\csc\left(\frac{13\pi}{12}\right) = \left(z;\, z^4 - 16 z^2 + 16\right)_1^{-1}$$

01.10.03.0082.01

$$\csc\left(\frac{11\pi}{10}\right) = -1-\sqrt{5}$$

01.10.03.0083.01

$$\csc\left(\frac{10\pi}{9}\right) = -\frac{4\sqrt[3]{2}}{\left(i+\sqrt{3}\right)\sqrt[3]{-1+i\sqrt{3}} + \left(-i+\sqrt{3}\right)\sqrt[3]{-1-i\sqrt{3}}}$$

01.10.03.0084.01

$$\csc\left(\frac{10\pi}{9}\right) = \left(z;\, 3 z^6 - 36 z^4 + 96 z^2 - 64\right)_1^{-1}$$

01.10.03.0085.01

$$\csc\left(\frac{10\pi}{9}\right) = -\frac{2(-1)^{7/18}}{1+(-1)^{7/9}}$$

01.10.03.0086.01

$$\csc\left(\frac{9\pi}{8}\right) = -\sqrt{2\left(2+\sqrt{2}\right)}$$

01.10.03.0087.01

$$\csc\left(\frac{9\pi}{8}\right) = \left(z;\, z^4 - 8 z^2 + 8\right)_1^{-1}$$

01.10.03.0088.01

$$\csc\left(\frac{9\pi}{8}\right) = -\frac{2(-1)^{3/8}}{1+(-1)^{3/4}}$$

01.10.03.0089.01

$$\csc\left(\frac{8\pi}{7}\right) = -\left(24\sqrt[3]{14-42 i\sqrt{3}}\right) \Big/ \left(\sqrt[3]{2}\left(\left(i+\sqrt{3}\right)\left(\sqrt[3]{28-84 i\sqrt{3}} - 2 i\sqrt{7}\right)\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + 2\left(\sqrt{7}+i\sqrt{21}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + \left(-i+\sqrt{3}\right)\sqrt[3]{28+2 i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right) - 4\, 7^{5/6}\sqrt[3]{2-6 i\sqrt{3}}\right)$$

01.10.03.0090.01

$$\csc\left(\frac{8\pi}{7}\right) = \left(z;\, 7 z^6 - 56 z^4 + 112 z^2 - 64\right)_1^{-1}$$

01.10.03.0091.01

$$\csc\left(\frac{8\pi}{7}\right) = -\frac{2(-1)^{5/14}}{1+(-1)^{5/7}}$$

01.10.03.0092.01

$$\csc\left(\frac{7\pi}{6}\right) = -2$$

01.10.03.0093.01

$$\csc\left(\frac{6\pi}{5}\right) = -\sqrt{2+\frac{2}{\sqrt{5}}}$$

01.10.03.0094.01

$$\csc\left(\frac{6\pi}{5}\right) = \left(z;\, 5z^4-20z^2+16\right)_1^{-1}$$

01.10.03.0095.01

$$\csc\left(\frac{11\pi}{9}\right) = \frac{2i}{\sqrt[3]{-\frac{1}{2}i\left(-i+\sqrt{3}\right)}-\sqrt[3]{\frac{1}{2}i\left(i+\sqrt{3}\right)}}$$

01.10.03.0096.01

$$\csc\left(\frac{11\pi}{9}\right) = \left(z;\, 3z^6-36z^4+96z^2-64\right)_2^{-1}$$

01.10.03.0097.01

$$\csc\left(\frac{11\pi}{9}\right) = -\frac{2(-1)^{5/18}}{1+(-1)^{5/9}}$$

01.10.03.0098.01

$$\csc\left(\frac{5\pi}{4}\right) = -\sqrt{2}$$

01.10.03.0099.01

$$\csc\left(\frac{9\pi}{7}\right) = -\left(24\sqrt[3]{14-42i\sqrt{3}}\right)\Big/\left(4\,7^{5/6}\sqrt[3]{2-6i\sqrt{3}}+\sqrt[3]{2}\left(2\left(\sqrt[3]{28-84i\sqrt{3}}+\sqrt{7}\,i-\sqrt{21}\right)i\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}+4\sqrt{7}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}+\left(i+\sqrt{3}\right)\sqrt[3]{28+2i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right)\right)$$

01.10.03.0100.01

$$\csc\left(\frac{9\pi}{7}\right) = \left(z;\, 7z^6-56z^4+112z^2-64\right)_2^{-1}$$

01.10.03.0101.01

$$\csc\left(\frac{9\pi}{7}\right) = -\frac{2(-1)^{3/14}}{1+(-1)^{3/7}}$$

01.10.03.0102.01

$$\csc\left(\frac{13\pi}{10}\right) = 1-\sqrt{5}$$

01.10.03.0103.01

$$\csc\left(\frac{4\pi}{3}\right) == -\frac{2}{\sqrt{3}}$$

01.10.03.0104.01

$$\csc\left(\frac{11\pi}{8}\right) == -\frac{2}{\sqrt{2+\sqrt{2}}}$$

01.10.03.0105.01

$$\csc\left(\frac{11\pi}{8}\right) == \left(z;\, z^4 - 8\,z^2 + 8\right)_2^{-1}$$

01.10.03.0106.01

$$\csc\left(\frac{11\pi}{8}\right) == -\frac{2\sqrt[8]{-1}}{1+\sqrt[4]{-1}}$$

01.10.03.0107.01

$$\csc\left(\frac{7\pi}{5}\right) == -\sqrt{2-\frac{2}{\sqrt{5}}}$$

01.10.03.0108.01

$$\csc\left(\frac{7\pi}{5}\right) == \left(z;\, 5\,z^4 - 20\,z^2 + 16\right)_2^{-1}$$

01.10.03.0109.01

$$\csc\left(\frac{17\pi}{12}\right) == \sqrt{2} - \sqrt{6}$$

01.10.03.0110.01

$$\csc\left(\frac{17\pi}{12}\right) == \left(z;\, z^4 - 16\,z^2 + 16\right)_2^{-1}$$

01.10.03.0111.01

$$\csc\left(\frac{10\pi}{7}\right) == -\left(24\sqrt[3]{7-21\,i\sqrt{3}}\right)\Big/\Big(4\,7^{5/6}\sqrt[3]{1-3\,i\sqrt{3}} + 4\sqrt{7}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - 2\,i\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}} + \sqrt{3}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\sqrt[3]{28-2\,i\sqrt{7}-6\sqrt{21}} - i\sqrt[3]{28-2\,i\sqrt{7}-6\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3} + 2\sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\; i\Big)$$

01.10.03.0112.01

$$\csc\left(\frac{10\pi}{7}\right) == \left(z;\, 7\,z^6 - 56\,z^4 + 112\,z^2 - 64\right)_3^{-1}$$

01.10.03.0113.01

$$\csc\left(\frac{10\pi}{7}\right) == -\frac{2\sqrt[14]{-1}}{1+\sqrt[7]{-1}}$$

01.10.03.0114.01

$$\csc\left(\frac{13\pi}{9}\right) == -\frac{4}{\left(i+\sqrt{3}\right)\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)} + \left(-i+\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}}$$

01.10.03.0115.01

$$\csc\left(\frac{13\pi}{9}\right) == \left(z;\, 3z^6 - 36z^4 + 96z^2 - 64\right)_3^{-1}$$

01.10.03.0116.01

$$\csc\left(\frac{13\pi}{9}\right) == -\frac{2\sqrt[18]{-1}}{1+\sqrt[9]{-1}}$$

01.10.03.0117.01

$$\csc\left(\frac{3\pi}{2}\right) == -1$$

01.10.03.0118.01

$$\csc\left(\frac{14\pi}{9}\right) == -\frac{4}{\left(i+\sqrt{3}\right)\sqrt[3]{-\frac{1}{2}i\left(-i+\sqrt{3}\right)} + \left(-i+\sqrt{3}\right)\sqrt[3]{\frac{1}{2}i\left(i+\sqrt{3}\right)}}$$

01.10.03.0119.01

$$\csc\left(\frac{14\pi}{9}\right) == \left(z;\, 3z^6 - 36z^4 + 96z^2 - 64\right)_3^{-1}$$

01.10.03.0120.01

$$\csc\left(\frac{14\pi}{9}\right) == -\frac{2(-1)^{17/18}}{-1+(-1)^{8/9}}$$

01.10.03.0121.01

$$\csc\left(\frac{11\pi}{7}\right) == -\left(24\sqrt[3]{7-21i\sqrt{3}}\right)\Big/\left(4\,7^{5/6}\sqrt[3]{1-3i\sqrt{3}} + 4\sqrt{7}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - 2i\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{28+2i\sqrt{7}+6\sqrt{21}} + \sqrt{3}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\sqrt[3]{28-2i\sqrt{7}-6\sqrt{21}} - i\sqrt[3]{28-2i\sqrt{7}-6\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3} + 2\sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\; i\right)$$

01.10.03.0122.01

$$\csc\left(\frac{11\pi}{7}\right) == \left(z;\, 7z^6 - 56z^4 + 112z^2 - 64\right)_3^{-1}$$

01.10.03.0123.01

$$\csc\left(\frac{11\pi}{7}\right) == -\frac{2(-1)^{13/14}}{-1+(-1)^{6/7}}$$

01.10.03.0124.01

$$\csc\left(\frac{19\pi}{12}\right) == \sqrt{2} - \sqrt{6}$$

01.10.03.0125.01

$$\csc\left(\frac{8\pi}{5}\right) == -\sqrt{2-\frac{2}{\sqrt{5}}}$$

01.10.03.0126.01

$$\csc\left(\frac{8\pi}{5}\right) == \left(z;\, 5z^4 - 20z^2 + 16\right)_2^{-1}$$

01.10.03.0127.01

$$\csc\left(\frac{13\pi}{8}\right) == -\frac{2}{\sqrt{2+\sqrt{2}}}$$

01.10.03.0128.01

$$\csc\left(\frac{13\pi}{8}\right) == \left(z;\, z^4 - 8z^2 + 8\right)_2^{-1}$$

01.10.03.0129.01

$$\csc\left(\frac{13\pi}{8}\right) == -\frac{2(-1)^{7/8}}{-1+(-1)^{3/4}}$$

01.10.03.0130.01

$$\csc\left(\frac{5\pi}{3}\right) == -\frac{2}{\sqrt{3}}$$

01.10.03.0131.01

$$\csc\left(\frac{17\pi}{10}\right) == 1-\sqrt{5}$$

01.10.03.0132.01

$$\csc\left(\frac{12\pi}{7}\right) == -\left(24\sqrt[3]{14-42\,i\sqrt{3}}\right) \Big/ \left(4\,7^{5/6}\sqrt[3]{2-6\,i\sqrt{3}} + \sqrt[3]{2}\left(2\left(\sqrt[3]{28-84\,i\sqrt{3}} + \sqrt{7}\,i - \sqrt{21}\right) i\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + 4\sqrt{7}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + \left(i+\sqrt{3}\right)\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right)\right)$$

01.10.03.0133.01

$$\csc\left(\frac{12\pi}{7}\right) == \left(z;\, 7z^6 - 56z^4 + 112z^2 - 64\right)_2^{-1}$$

01.10.03.0134.01

$$\csc\left(\frac{12\pi}{7}\right) == -\frac{2(-1)^{11/14}}{-1+(-1)^{4/7}}$$

01.10.03.0135.01

$$\csc\left(\frac{7\pi}{4}\right) == -\sqrt{2}$$

01.10.03.0136.01

$$\csc\left(\frac{16\pi}{9}\right) == \frac{2\,i}{\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)} - \sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}}$$

01.10.03.0137.01

$$\csc\left(\frac{16\pi}{9}\right) == \left(z;\, 3z^6 - 36z^4 + 96z^2 - 64\right)_2^{-1}$$

01.10.03.0138.01

$$\csc\left(\frac{16\pi}{9}\right) == -\frac{2(-1)^{13/18}}{-1+(-1)^{4/9}}$$

01.10.03.0139.01

$$\csc\left(\frac{9\pi}{5}\right) = -\sqrt{2+\frac{2}{\sqrt{5}}}$$

01.10.03.0140.01

$$\csc\left(\frac{9\pi}{5}\right) = \left(z;\, 5z^4-20z^2+16\right)_1^{-1}$$

01.10.03.0141.01

$$\csc\left(\frac{11\pi}{6}\right) = -2$$

01.10.03.0142.01

$$\csc\left(\frac{13\pi}{7}\right) = -\left(24\sqrt[3]{14-42\,i\sqrt{3}}\right) \Big/ \left(\sqrt[3]{2}\left(\left(i+\sqrt{3}\right)\left(\sqrt[3]{28-84\,i\sqrt{3}}-2\,i\sqrt{7}\right)\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}+2\left(\sqrt{7}+i\sqrt{21}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}+\left(-i+\sqrt{3}\right)\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right)-4\,7^{5/6}\sqrt[3]{2-6\,i\sqrt{3}}\right)$$

01.10.03.0143.01

$$\csc\left(\frac{13\pi}{7}\right) = \left(z;\, 7z^6-56z^4+112z^2-64\right)_1^{-1}$$

01.10.03.0144.01

$$\csc\left(\frac{13\pi}{7}\right) = -\frac{2(-1)^{9/14}}{-1+(-1)^{2/7}}$$

01.10.03.0145.01

$$\csc\left(\frac{15\pi}{8}\right) = -\sqrt{2\left(2+\sqrt{2}\right)}$$

01.10.03.0146.01

$$\csc\left(\frac{15\pi}{8}\right) = \left(z;\, z^4-8z^2+8\right)_1^{-1}$$

01.10.03.0147.01

$$\csc\left(\frac{15\pi}{8}\right) = -\frac{2(-1)^{5/8}}{-1+\sqrt[4]{-1}}$$

01.10.03.0148.01

$$\csc\left(\frac{17\pi}{9}\right) = -\frac{4\sqrt[3]{2}}{\left(i+\sqrt{3}\right)\sqrt[3]{-1+i\sqrt{3}}+\left(-i+\sqrt{3}\right)\sqrt[3]{-1-i\sqrt{3}}}$$

01.10.03.0149.01

$$\csc\left(\frac{17\pi}{9}\right) = \left(z;\, 3z^6-36z^4+96z^2-64\right)_1^{-1}$$

01.10.03.0150.01

$$\csc\left(\frac{17\pi}{9}\right) = -\frac{2(-1)^{11/18}}{-1+(-1)^{2/9}}$$

01.10.03.0151.01

$$\csc\left(\frac{19\pi}{10}\right) = -1 - \sqrt{5}$$

01.10.03.0152.01

$$\csc\left(\frac{23\pi}{12}\right) = -2\sqrt{2+\sqrt{3}}$$

01.10.03.0153.01

$$\csc\left(\frac{23\pi}{12}\right) = \left(z;\, z^4 - 16 z^2 + 16\right)_1^{-1}$$

01.10.03.0154.01

$$\csc(2\pi) = \tilde{\infty}$$

01.10.03.0155.01

$$\csc\left(\frac{\pi}{17}\right) = 4 \Bigg/ \left(\sqrt{\left(8 - \sqrt{\left(2\left(\sqrt{\left(2\left(\sqrt{2\left(17-\sqrt{17}\right)} + 6\sqrt{17} - \sqrt{34\left(17-\sqrt{17}\right)} + 8\sqrt{2\left(17+\sqrt{17}\right)} + 34\right)\right)} + \sqrt{17} - \sqrt{2\left(17-\sqrt{17}\right)} + 15\right)\right)}\right)}\right)$$

01.10.03.0156.01

$$\csc\left(\frac{\pi}{30}\right) = 2 + \sqrt{5} + \sqrt{15 + 6\sqrt{5}}$$

$\csc\left(\frac{n\pi}{m}\right)$ can be expressed using only square roots if $n \in \mathbb{Z}$ and m is a product of a power of 2 and distinct Fermat primes $\{3, 5, 17, 257, \ldots\}$.

Values at infinities

01.10.03.0157.01

$$\csc(i\,\infty) = 0$$

01.10.03.0158.01

$$\csc(-i\,\infty) = 0$$

01.10.03.0159.01

$$\csc(\tilde{\infty}) = ¿$$

General characteristics

Domain and analyticity

$\csc(z)$ is an analytical function of z which is defined in the whole complex z-plane with the exception of countably many points $z = k\pi \,/;\, k \in \mathbb{Z}$.

01.10.04.0001.01

$$z \longrightarrow \csc(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\csc(z)$ is an odd function.

01.10.04.0002.01

$$\csc(-z) == -\csc(z)$$

Mirror symmetry

01.10.04.0003.01

$$\csc(\bar{z}) == \overline{\csc(z)}$$

Periodicity

$\csc(z)$ is a periodic function with period 2π.

01.10.04.0010.01

$$\csc(z + 2\pi) == \csc(z)$$

01.10.04.0004.01

$$\csc(z + 2\pi m) == \csc(z) /; m \in \mathbb{Z}$$

01.10.04.0005.01

$$\csc(z + \pi m) == (-1)^m \csc(z) /; m \in \mathbb{Z}$$

Poles and essential singularities

The function $\csc(z)$ has an infinite set of singular points:

a) $z == \pi k /; k \in \mathbb{Z}$ are the simple poles with residues $(-1)^k$;

b) $z == \tilde{\infty}$ is an essential singular point.

01.10.04.0006.01

$$\mathcal{S}ing_z(\csc(z)) == \{\{\{\pi k, 1\} /; k \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}\}$$

01.10.04.0007.01

$$\operatorname{res}_z(\csc(z))(\pi k) == (-1)^k /; k \in \mathbb{Z}$$

Branch points

The function $\csc(z)$ does not have branch points.

01.10.04.0008.01

$$\mathcal{BP}_z(\csc(z)) == \{\}$$

Branch cuts

The function $\csc(z)$ does not have branch cuts.

01.10.04.0009.01

$$\mathcal{BC}_z(\csc(z)) == \{\}$$

Series representations

Generalized power series

Expansions at $z == z_0$

For the function itself

01.10.06.0023.01

$$\csc(z) \propto \csc(z_0) - \cot(z_0)\csc(z_0)(z - z_0) + 3\csc(z_0)\left(\frac{1}{3}\cos(2 z_0)\csc^2(z_0) + \frac{1}{2}\right)(z - z_0)^2 + \ldots /; (z \to z_0)$$

01.10.06.0024.01

$$\csc(z) \propto \csc(z_0) - \cot(z_0)\csc(z_0)(z - z_0) + 3\csc(z_0)\left(\frac{1}{3}\cos(2 z_0)\csc^2(z_0) + \frac{1}{2}\right)(z - z_0)^2 + O\left((z - z_0)^3\right)$$

01.10.06.0025.01

$$\sum_{j=0}^{\infty} \frac{(-i)^{k+1}}{k!} \sum_{j=0}^{k} \frac{(-1)^j\, j!}{2^j} \mathcal{S}_k^{(j)} \left(\left(i\cot\left(\frac{z_0}{2}\right) + 1\right)^j \left(i\cot\left(\frac{z_0}{2}\right) - 1\right) - 2^k\,(i\cot(z_0) + 1)^j\,(i\cot(z_0) - 1)\right)(z - z_0)^k$$

01.10.06.0026.01

$$\csc(z) = \csc(z_0) \sum_{k=0}^{\infty} \left(\delta_k + (k+1) \sum_{m=0}^{k} \sum_{j=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \frac{(-1)^{j+k}\, 2^{1-m}\,(m - 2j)^k \csc^m(z_0)}{(m+1)\, j!\,(m-j)!\,(k-m)!} \cos\left(\frac{\pi(m-k)}{2} + (m - 2j) z_0\right)\right)(z - z_0)^k$$

01.10.06.0027.01

$$\csc(z) \propto \csc(z_0)\,(1 + O(z - z_0))$$

Expansions at $z = 0$

For the function itself

01.10.06.0001.02

$$\csc(z) \propto \frac{1}{z} + \frac{z}{6} + \frac{7 z^3}{360} + \ldots /; (z \to 0)$$

01.10.06.0028.01

$$\csc(z) \propto \frac{1}{z} + \frac{z}{6} + \frac{7 z^3}{360} + O\left(z^5\right)$$

01.10.06.0002.01

$$\csc(z) = \frac{1}{z} + \sum_{k=1}^{\infty} \frac{(-1)^{k-1}\, 2\left(2^{2k-1} - 1\right) B_{2k}\, z^{2k-1}}{(2k)!} /; |z| < \pi$$

01.10.06.0003.02

$$\csc(z) \propto \frac{1}{z} + \frac{z}{6} + O\left(z^3\right)$$

Expansions at $z = \frac{\pi}{2}$

For the function itself

01.10.06.0004.02

$$\csc(z) \propto 1 + \frac{1}{2}\left(z - \frac{\pi}{2}\right)^2 + \frac{5}{24}\left(z - \frac{\pi}{2}\right)^4 + \ldots /; \left(z \to \frac{\pi}{2}\right)$$

01.10.06.0029.01

$$\csc(z) \propto 1 + \frac{1}{2}\left(z - \frac{\pi}{2}\right)^2 + \frac{5}{24}\left(z - \frac{\pi}{2}\right)^4 + O\left(\left(z - \frac{\pi}{2}\right)^6\right)$$

01.10.06.0005.01

$$\csc(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} E_{2k} \left(z - \frac{\pi}{2}\right)^{2k} /; \left|z - \frac{\pi}{2}\right| < \frac{\pi}{2}$$

01.10.06.0006.02

$$\csc(z) \propto 1 + O\left(\left(z - \frac{\pi}{2}\right)^2\right)$$

q-series

01.10.06.0007.01

$$\csc(z) = -2\,i \sum_{k=1}^{\infty} q^{2k-1} /; q = e^{iz}$$

Dirichlet series

01.10.06.0008.01

$$\csc(z) = -2\,i\,e^{iz} \sum_{k=0}^{\infty} e^{2izk} /; \operatorname{Im}(z) > 0$$

01.10.06.0009.01

$$\csc(z) = 2\,i\,e^{-iz} \sum_{k=0}^{\infty} e^{-2izk} /; \operatorname{Im}(z) < 0$$

Asymptotic series expansions

01.10.06.0010.01

$$\csc(z) \propto -2\,i\,e^{iz}\, {}_1F_0\left(1;\,;\, e^{2iz}\right) /; \operatorname{Im}(z) > 0 \land (|z| \to \infty)$$

01.10.06.0011.01

$$\csc(z) \propto -2\,i\,e^{iz} \left(1 + O\left(e^{2iz}\right)\right) /; \operatorname{Im}(z) > 0 \land (|z| \to \infty)$$

01.10.06.0012.01

$$\csc(z) \propto 2\,i\,e^{-iz}\, {}_1F_0\left(1;\,;\, e^{-2iz}\right) /; \operatorname{Im}(z) < 0 \land (|z| \to \infty)$$

01.10.06.0013.01

$$\csc(z) \propto 2\,i\,e^{-iz} \left(1 + O\left(e^{-2iz}\right)\right) /; \operatorname{Im}(z) < 0 \land (|z| \to \infty)$$

01.10.06.0014.01

$$\csc(z) \propto \csc(z) /; \operatorname{Im}(z) = 0 \land (|z| \to \infty)$$

01.10.06.0015.01

$$\csc(z) \propto -2\,i\,e^{iz} /; \left(z \to e^{i\phi}\,\infty\right) \bigwedge 0 < \phi < \pi$$

01.10.06.0016.01

$$\csc(z) \propto 2\,i\,e^{-iz} /; \left(z \to e^{i\phi}\,\infty\right) \bigwedge -\pi < \phi < 0$$

01.10.06.0030.01

$$\csc(z) \propto \begin{cases} 2\,i\,e^{-i z} & -\pi < \arg(z) < 0 \\ -2\,i\,e^{i z} & 0 < \arg(z) < \pi \\ \csc(z) & \text{True} \end{cases} \;/;\; (|z| \to \infty)$$

Other series representations

01.10.06.0017.01

$$\csc(z) = \frac{1}{z} + 2\,z \sum_{k=1}^{\infty} \frac{(-1)^k}{z^2 - \pi^2 k^2} \;/;\; \frac{z}{\pi} \notin \mathbb{Z}$$

01.10.06.0018.01

$$\csc(z) = \frac{1}{z} + \frac{z}{\pi} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \frac{(-1)^k}{k\,(z - \pi\,k)} \;/;\; \frac{z}{\pi} \notin \mathbb{Z}$$

01.10.06.0019.01

$$\csc(z) = \sum_{k=-\infty}^{\infty} \frac{(-1)^k\,z}{z^2 - \pi^2 k^2} \;/;\; \frac{z}{\pi} \notin \mathbb{Z}$$

01.10.06.0020.01

$$\log(z \csc(z)) = \sum_{k=1}^{\infty} \frac{\zeta(2\,k)}{k} \left(\frac{z}{\pi}\right)^{2k} \;/;\; |z| < \pi$$

01.10.06.0021.01

$$\csc^2(z) = \sum_{k=-\infty}^{\infty} \frac{1}{(z - \pi\,k)^2} \;/;\; \frac{z}{\pi} \notin \mathbb{Z}$$

01.10.06.0022.01

$$\csc^2(z) = \sum_{k=-\infty}^{\infty} \frac{1}{(z + \pi\,k)^2} \;/;\; \frac{z}{\pi} \notin \mathbb{Z}$$

Integral representations

On the real axis

Of the direct function

01.10.07.0001.01

$$\csc(z) = \frac{1}{\pi} \int_0^{\infty} \frac{1}{t^2 + t}\, t^{z/\pi}\, dt \;/;\; 0 < \operatorname{Re}(z) < \pi$$

Product representations

01.10.08.0001.01

$$\csc(z) = \frac{1}{z} \prod_{k=1}^{\infty} \frac{\pi^2 k^2}{\pi^2 k^2 - z^2}$$

01.10.08.0002.01

$$\csc(z) == \frac{1}{z} \prod_{k=1}^{\infty} \sec\left(\frac{z}{2^k}\right) /; |z| < 1$$

Limit representations

01.10.09.0001.01

$$\csc(z) == \lim_{n\to\infty} \sum_{k=-n}^{n} \frac{(-1)^k}{z - \pi k} /; \frac{z}{\pi} \notin \mathbb{Z}$$

Differential equations

Ordinary nonlinear differential equations

01.10.13.0001.01

$$w'(z)^2 - w(z)^4 + w(z)^2 == 0 /; w(z) == \csc(z)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

01.10.16.0001.01

$$\csc(-z) == -\csc(z)$$

01.10.16.0002.01

$$\csc(a\,(b\,z^c)^m) == \frac{(b\,z^c)^m \csc(a\,b^m\,z^{m\,c})}{b^m\,z^{m\,c}} /; 2\,m \in \mathbb{Z}$$

01.10.16.0003.01

$$\csc\left(\sqrt{z^2}\right) == \frac{\sqrt{z^2}\,\csc(z)}{z}$$

Argument involving inverse trigonometric and hyperbolic functions

Involving $\sin^{-1}$

01.10.16.0004.01

$$\csc\left(\sin^{-1}(z)\right) == \frac{1}{z}$$

01.10.16.0016.01

$$\csc\left(\frac{1}{2}\sin^{-1}(z)\right) == \frac{\sqrt{2}\,\sqrt{z^2}}{z\sqrt{1-\sqrt{1-z^2}}}$$

01.10.16.0060.01

$$\csc\left(i \sin^{-1}(z)\right) = \frac{2\, i \left(i\, z + \sqrt{1 - z^2}\right)^i}{\left(i\, z + \sqrt{1 - z^2}\right)^{2\, i} - 1}$$

01.10.16.0061.01

$$\csc\left(a \sin^{-1}(z)\right) = \frac{2\, i \left(i\, z + \sqrt{1 - z^2}\right)^a}{\left(i\, z + \sqrt{1 - z^2}\right)^{2\, a} - 1}$$

Involving $\cos^{-1}$

01.10.16.0005.01

$$\csc\left(\cos^{-1}(z)\right) = \frac{1}{\sqrt{1 - z^2}}$$

01.10.16.0017.01

$$\csc\left(\frac{1}{2} \cos^{-1}(z)\right) = \frac{\sqrt{2}}{\sqrt{1 - z}}$$

01.10.16.0062.01

$$\csc\left(i \cos^{-1}(z)\right) = \frac{2\, i\, e^{\pi/2} \left(i\, z + \sqrt{1 - z^2}\right)^i}{1 - e^{\pi} \left(i\, z + \sqrt{1 - z^2}\right)^{2\, i}}$$

01.10.16.0063.01

$$\csc\left(a \cos^{-1}(z)\right) = \frac{2\, i\, e^{\frac{i\, a\, \pi}{2}} \left(i\, z + \sqrt{1 - z^2}\right)^a}{e^{i\, a\, \pi} - \left(i\, z + \sqrt{1 - z^2}\right)^{2\, a}}$$

Involving $\tan^{-1}$

01.10.16.0006.01

$$\csc\left(\tan^{-1}(z)\right) = \frac{\sqrt{1 + z^2}}{z}$$

01.10.16.0064.01

$$\csc\left(\tan^{-1}(x, y)\right) = \frac{\sqrt{x^2 + y^2}}{y}$$

01.10.16.0018.01

$$\csc\left(\frac{1}{2} \tan^{-1}(z)\right) = \frac{\sqrt{2}\, \sqrt{z^2}}{z \sqrt{1 - \frac{1}{\sqrt{z^2 + 1}}}}$$

01.10.16.0065.01

$$\csc\left(\frac{1}{2}\tan^{-1}(x, y)\right) = \frac{2(i x + y)\sqrt{\frac{x+i y}{\sqrt{x^2+y^2}}}}{-x + i y + \sqrt{x^2 + y^2}}$$

01.10.16.0066.01

$$\csc\left(i \tan^{-1}(z)\right) = -\frac{2 i \left(z^2 + 1\right)^{i/2}}{(1 - i z)^i - (i z + 1)^i}$$

01.10.16.0067.01

$$\csc\left(i \tan^{-1}(x, y)\right) = \frac{2 i \left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^i}{\left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^{2 i} - 1}$$

01.10.16.0068.01

$$\csc\left(a \tan^{-1}(z)\right) = -\frac{2 i \left(z^2 + 1\right)^{a/2}}{(1 - i z)^a - (i z + 1)^a}$$

01.10.16.0069.01

$$\csc\left(a \tan^{-1}(x, y)\right) = \frac{2 i \left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^a}{\left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^{2 a} - 1}$$

Involving $\cot^{-1}$

01.10.16.0007.01

$$\csc\left(\cot^{-1}(z)\right) = \sqrt{1 + \frac{1}{z^2}}\ z$$

01.10.16.0019.01

$$\csc\left(\frac{1}{2}\cot^{-1}(z)\right) = \sqrt{\frac{1}{z^2}}\ \frac{\sqrt{2}\ z}{\sqrt{1 - \frac{1}{\sqrt{1+\frac{1}{z^2}}}}}$$

01.10.16.0070.01

$$\csc\left(i \cot^{-1}(z)\right) = \frac{2 i \left(1 + \frac{1}{z^2}\right)^{i/2}}{\left(\frac{i+z}{z}\right)^i - \left(\frac{-i+z}{z}\right)^i}$$

01.10.16.0071.01

$$\csc\left(a \cot^{-1}(z)\right) = \frac{2 i \left(1 + \frac{1}{z^2}\right)^{a/2}}{\left(\frac{i+z}{z}\right)^a - \left(\frac{-i+z}{z}\right)^a}$$

Involving $\csc^{-1}$

01.10.16.0008.01

$$\csc\left(\csc^{-1}(z)\right) = z$$

01.10.16.0020.01

$$\csc\left(\frac{1}{2}\csc^{-1}(z)\right) = \sqrt{\frac{1}{z^2}}\;\frac{\sqrt{2}\;z}{\sqrt{1-\sqrt{1-\frac{1}{z^2}}}}$$

01.10.16.0072.01

$$\csc\left(i\csc^{-1}(z)\right) = \frac{2\,i\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{i}}{\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2\,i}-1}$$

01.10.16.0073.01

$$\csc\left(a\csc^{-1}(z)\right) = \frac{2\,i\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{a}}{\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2\,a}-1}$$

Involving $\sec^{-1}$

01.10.16.0009.01

$$\csc\left(\sec^{-1}(z)\right) = \frac{\sqrt{z^2}}{\sqrt{z^2-1}}$$

01.10.16.0021.01

$$\csc\left(\frac{1}{2}\sec^{-1}(z)\right) = \frac{\sqrt{2\,z}}{\sqrt{z-1}}$$

01.10.16.0074.01

$$\csc\left(i\sec^{-1}(z)\right) = -\frac{2\,i\,e^{\pi/2}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{i}}{e^{\pi}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2\,i}-1}$$

01.10.16.0075.01

$$\csc\left(a\sec^{-1}(z)\right) = -\frac{2\,i\,e^{\frac{1}{2}(-\pi)\,i\,a}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{a}}{e^{-\pi\,i\,a}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2\,a}-1}$$

Involving $\sinh^{-1}$

01.10.16.0076.01

$$\csc\left(\sinh^{-1}(z)\right) = \frac{2\,i\left(z+\sqrt{z^2+1}\right)^{i}}{\left(z+\sqrt{z^2+1}\right)^{2\,i}-1}$$

01.10.16.0010.01

$$\csc\left(i\sinh^{-1}(z)\right) == -\frac{i}{z}$$

01.10.16.0022.01

$$\csc\left(\frac{i}{2}\sinh^{-1}(z)\right) == -\frac{i\sqrt{2}\sqrt{z^2}}{z\sqrt{\sqrt{z^2+1}-1}}$$

01.10.16.0077.01

$$\csc\left(a\sinh^{-1}(z)\right) = \frac{2\,i\left(z+\sqrt{z^2+1}\right)^{i\,a}}{\left(z+\sqrt{z^2+1}\right)^{2\,i\,a}-1}$$

Involving $\cosh^{-1}$

01.10.16.0078.01

$$\csc\left(\cosh^{-1}(z)\right) = \frac{2\,i\left(z+\sqrt{z-1}\sqrt{z+1}\right)^{i}}{\left(z+\sqrt{z-1}\sqrt{z+1}\right)^{2\,i}-1}$$

01.10.16.0011.01

$$\csc\left(i\cosh^{-1}(z)\right) == -\frac{i}{\sqrt{z-1}\sqrt{z+1}}$$

01.10.16.0023.01

$$\csc\left(\frac{i}{2}\cosh^{-1}(z)\right) == -\frac{i\sqrt{2}}{\sqrt{z-1}}$$

01.10.16.0079.01

$$\csc\left(a\cosh^{-1}(z)\right) = \frac{2\,i\left(z+\sqrt{z-1}\sqrt{z+1}\right)^{i\,a}}{\left(z+\sqrt{z-1}\sqrt{z+1}\right)^{2\,i\,a}-1}$$

Involving $\tanh^{-1}$

01.10.16.0080.01

$$\csc(\tanh^{-1}(z)) = -\frac{2\,i\,(1-z^2)^{i/2}}{(1-z)^i-(z+1)^i}$$

01.10.16.0012.01

$$\csc(i\tanh^{-1}(z)) = -\frac{i\sqrt{1-z^2}}{z}$$

01.10.16.0024.01

$$\csc\left(\frac{i}{2}\tanh^{-1}(z)\right) = -\frac{i\sqrt{2}\sqrt{z^2}}{z\sqrt{\frac{1}{\sqrt{1-z^2}}-1}}$$

01.10.16.0081.01

$$\csc(a\tanh^{-1}(z)) = -\frac{2\,i\,(1-z^2)^{\frac{i a}{2}}}{(1-z)^{i a}-(z+1)^{i a}}$$

Involving $\coth^{-1}$

01.10.16.0082.01

$$\csc(\coth^{-1}(z)) = \frac{2\,i\left(1-\frac{1}{z^2}\right)^{i/2}}{\left(1+\frac{1}{z}\right)^i-\left(1-\frac{1}{z}\right)^i}$$

01.10.16.0013.01

$$\csc(i\coth^{-1}(z)) = -i\sqrt{1-\frac{1}{z^2}}\;z$$

01.10.16.0025.01

$$\csc\left(\frac{i}{2}\coth^{-1}(z)\right) = -\sqrt{\frac{1}{z^2}}\;\frac{i\sqrt{2}\;z}{\sqrt{\frac{1}{\sqrt{1-\frac{1}{z^2}}}-1}}$$

01.10.16.0083.01

$$\csc(a\coth^{-1}(z)) = \frac{2\,i\left(1-\frac{1}{z^2}\right)^{\frac{i a}{2}}}{\left(1+\frac{1}{z}\right)^{i a}-\left(1-\frac{1}{z}\right)^{i a}}$$

Involving csch^{-1}

01.10.16.0084.01

$$\csc\left(\operatorname{csch}^{-1}(z)\right) = \frac{2\,i\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{i}}{\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2i}-1}$$

01.10.16.0014.01

$$\csc\left(i\operatorname{csch}^{-1}(z)\right) = -i\,z$$

01.10.16.0026.01

$$\csc\left(\frac{i}{2}\operatorname{csch}^{-1}(z)\right) = -\sqrt{\frac{1}{z^2}}\;\frac{i\sqrt{2}\;z}{\sqrt{\sqrt{1+\frac{1}{z^2}}-1}}$$

01.10.16.0085.01

$$\csc\left(a\operatorname{csch}^{-1}(z)\right) = \frac{2\,i\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{i\,a}}{\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2\,i\,a}-1}$$

Involving sech^{-1}

01.10.16.0086.01

$$\csc\left(\operatorname{sech}^{-1}(z)\right) = \frac{2\,i\left(\sqrt{\frac{1}{z}-1}\;\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{i}}{\left(\sqrt{\frac{1}{z}-1}\;\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{2i}-1}$$

01.10.16.0015.01

$$\csc\left(i\operatorname{sech}^{-1}(z)\right) = -\frac{i\,z}{1-z}\sqrt{\frac{1-z}{1+z}}$$

01.10.16.0027.01

$$\csc\left(\frac{i}{2}\operatorname{sech}^{-1}(z)\right) = -\sqrt{\frac{1}{z}}\;\frac{i\sqrt{2}\;z}{\sqrt{1-z}}$$

01.10.16.0087.01

$$\csc\left(a\operatorname{sech}^{-1}(z)\right) = \frac{2\,i\left(\sqrt{\frac{1}{z}-1}\;\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{i\,a}}{\left(\sqrt{\frac{1}{z}-1}\;\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{2\,i\,a}-1}$$

Addition formulas

01.10.16.0028.01

$$\csc(a+b) = \frac{1}{\cos(b)\sin(a)+\cos(a)\sin(b)}$$

01.10.16.0029.01

$$\csc(a-b) = \frac{1}{\cos(b)\sin(a)-\cos(a)\sin(b)}$$

01.10.16.0030.01

$$\csc(a+b\,i) = \frac{2\cosh(b)\sin(a)-2\,i\cos(a)\sinh(b)}{\cosh(2\,b)-\cos(2\,a)}$$

01.10.16.0031.01

$$\csc(a-i\,b) = \frac{2\cosh(b)\sin(a)+2\,i\cos(a)\sinh(b)}{\cosh(2\,b)-\cos(2\,a)}$$

Half-angle formulas

01.10.16.0032.02

$$\csc\left(\frac{z}{2}\right) = \frac{\sqrt{2}}{\sqrt{1-\cos(z)}} \;/;\; 0<\operatorname{Re}(z)<2\pi \lor \operatorname{Re}(z)=0 \land \operatorname{Im}(z)>0 \lor \operatorname{Re}(z)=2\pi \land \operatorname{Im}(z)<0$$

01.10.16.0033.01

$$\csc\left(\frac{z}{2}\right) = \frac{\sqrt{2\,z^2}}{z\sqrt{1-\cos(z)}} \;/;\; |\operatorname{Re}(z)|<2\pi$$

01.10.16.0034.01

$$\csc\left(\frac{z}{2}\right) = \frac{\sqrt{2}\,(-1)^{\left\lfloor\frac{\operatorname{Re}(z)}{2\pi}\right\rfloor}}{\sqrt{1-\cos(z)}}\left(1-\left(1+(-1)^{\left\lfloor\frac{\operatorname{Re}(z)}{2\pi}\right\rfloor+\left\lfloor-\frac{\operatorname{Re}(z)}{2\pi}\right\rfloor}\right)\theta(-\operatorname{Im}(z))\right)$$

Multiple arguments

Argument involving numeric multiples of variable

01.10.16.0035.01

$$\csc(2\,z) = \frac{1}{2}\csc(z)\sec(z)$$

01.10.16.0043.02

$$\csc(3\,z) = \frac{\csc^3(z)}{3\csc^2(z)-4}$$

Argument involving symbolic multiples of variable

01.10.16.0044.01

$$\csc(n\,z) = \frac{\csc(z)}{\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(-1)^k\binom{-k+n-1}{k}2^{-2k+n-1}\cos^{-2k+n-1}(z)} \;/;\; n\in\mathbb{N}^+$$

01.10.16.0036.01

$$\csc(n\,z) = 2^{1-n}\prod_{k=0}^{n-1}\csc\left(\frac{\pi\,k}{n}+z\right) /;\; n\in\mathbb{N}$$

01.10.16.0037.01

$$\csc(n\,z) = \frac{\csc(z)}{U_{n-1}(\cos(z))}$$

Products, sums, and powers of the direct function

Products of the direct function

01.10.16.0045.01

$$\csc(a)\csc(b) = \frac{2}{\cos(a-b)-\cos(a+b)}$$

Products involving the direct function

01.10.16.0046.01

$$\csc(a)\sec(b) = \frac{2}{\sin(a-b)+\sin(a+b)}$$

Sums of the direct function

01.10.16.0038.01

$$\csc(a)+\csc(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)\csc(a)\csc(b)$$

01.10.16.0039.01

$$\csc(a)-\csc(b) = -2\sin\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)\csc(a)\csc(b)$$

Sums involving the direct function

Involving other trigonometric functions

Involving sec

01.10.16.0047.01

$$\csc(z)+\sec(z) = \sqrt{2}\,\cos\left(z-\frac{\pi}{4}\right)\csc(z)\sec(z)$$

01.10.16.0048.01

$$\csc(z)-\sec(z) = \sqrt{2}\,\cos\left(z+\frac{\pi}{4}\right)\csc(z)\sec(z)$$

01.10.16.0049.01

$$\csc(a)+\sec(b) = 2\cos\left(\frac{a-b}{2}-\frac{\pi}{4}\right)\cos\left(\frac{a+b}{2}-\frac{\pi}{4}\right)\csc(a)\sec(b)$$

01.10.16.0050.01

$$\csc(a)-\sec(b) = 2\cos\left(\frac{a+b}{2}+\frac{\pi}{4}\right)\cos\left(\frac{a-b}{2}+\frac{\pi}{4}\right)\csc(a)\sec(b)$$

01.10.16.0051.01

$$a\csc(z)+b\sec(z) = 2\,b\sqrt{1+\frac{a^2}{b^2}}\,\sin\left(z+\tan^{-1}\left(\frac{a}{b}\right)\right)\csc(2\,z)$$

Involving hyperbolic functions

Involving csch

01.10.16.0052.01

$$\csc(z) + i \operatorname{csch}(z) = 2\, i \sin\left(\frac{z\, e^{-\frac{1}{4}(i\pi)}}{\sqrt{2}}\right) \cos\left(\frac{z\, e^{\frac{i\pi}{4}}}{\sqrt{2}}\right) \csc(z) \operatorname{csch}(z)$$

01.10.16.0053.01

$$\csc(z) - i \operatorname{csch}(z) = -2\, i \sin\left(\frac{z\, e^{\frac{i\pi}{4}}}{\sqrt{2}}\right) \cos\left(\frac{z\, e^{-\frac{1}{4}(i\pi)}}{\sqrt{2}}\right) \csc(z) \operatorname{csch}(z)$$

01.10.16.0054.01

$$\csc(a) + i \operatorname{csch}(b) = 2\, i \sin\left(\frac{1}{2}(a - i\, b)\right) \cos\left(\frac{1}{2}(a + b\, i)\right) \csc(a) \operatorname{csch}(b)$$

01.10.16.0055.01

$$\csc(a) - i \operatorname{csch}(b) = -2\, i \sin\left(\frac{1}{2}(a + b\, i)\right) \cos\left(\frac{1}{2}(a - i\, b)\right) \csc(a) \operatorname{csch}(b)$$

Involving sech

01.10.16.0056.01

$$\csc(z) + \operatorname{sech}(z) = 2 \cos\left(\frac{z\, e^{-\frac{1}{4}(i\pi)}}{\sqrt{2}} - \frac{\pi}{4}\right) \cos\left(\frac{z\, e^{\frac{i\pi}{4}}}{\sqrt{2}} - \frac{\pi}{4}\right) \csc(z) \operatorname{sech}(z)$$

01.10.16.0057.01

$$\csc(z) - \operatorname{sech}(z) = 2 \cos\left(\frac{e^{\frac{i\pi}{4}} z}{\sqrt{2}} + \frac{\pi}{4}\right) \cos\left(\frac{e^{-\frac{1}{4}(i\pi)} z}{\sqrt{2}} + \frac{\pi}{4}\right) \csc(z) \operatorname{sech}(z)$$

01.10.16.0058.01

$$\csc(a) + \operatorname{sech}(b) = 2 \cos\left(\frac{1}{2}(a - i\, b) - \frac{\pi}{4}\right) \cos\left(\frac{1}{2}(a + b\, i) - \frac{\pi}{4}\right) \csc(a) \operatorname{sech}(b)$$

01.10.16.0059.01

$$\csc(a) - \operatorname{sech}(b) = 2 \cos\left(\frac{1}{2}(a + b\, i) + \frac{\pi}{4}\right) \cos\left(\frac{1}{2}(a - i\, b) + \frac{\pi}{4}\right) \csc(a) \operatorname{sech}(b)$$

Powers of the direct function

01.10.16.0040.01

$$\csc^2(z) = \frac{2 \sec(2\, z)}{\sec(2\, z) - 1}$$

Powers involving the direct function

01.10.16.0041.01

$$\csc^2(a) - \csc^2(b) = -\csc^2(a) \csc^2(b) \sin(a - b) \sin(a + b)$$

01.10.16.0042.01

$$\csc^2(a) - \sec^2(b) = \cos(a-b)\cos(a+b)\csc^2(a)\sec^2(b)$$

Identities

Functional identities

01.10.17.0001.01

$$4\csc^2(2z)\left(\csc^2(z) - 1\right) = \csc^4(z)$$

01.10.17.0002.01

$$\csc^4(z_1)\csc^4(z_2) - 2\csc^2(z_1)\left(\csc^2(z_1) + \csc^2(z_2) - 2\right)\csc^2(z_1+z_2)\csc^2(z_2) + \left(\csc^2(z_1) - \csc^2(z_2)\right)^2\csc^4(z_1+z_2) = 0$$

Complex characteristics

Real part

01.10.19.0001.01

$$\operatorname{Re}(\csc(x+i\,y)) = -\frac{2\cosh(y)\sin(x)}{\cos(2x) - \cosh(2y)}$$

01.10.19.0007.01

$$\operatorname{Re}(\csc(z)) = -\frac{2\cosh(\operatorname{Im}(z))\sin(\operatorname{Re}(z))}{\cos(2\operatorname{Re}(z)) - \cosh(2\operatorname{Im}(z))}$$

Imaginary part

01.10.19.0002.01

$$\operatorname{Im}(\csc(x+i\,y)) = \frac{2\cos(x)\sinh(y)}{\cos(2x) - \cosh(2y)}$$

01.10.19.0008.01

$$\operatorname{Im}(\csc(z)) = \frac{2\cos(\operatorname{Re}(z))\sinh(\operatorname{Im}(z))}{\cos(2\operatorname{Re}(z)) - \cosh(2\operatorname{Im}(z))}$$

Absolute value

01.10.19.0003.01

$$|\csc(x+i\,y)| = \frac{\sqrt{2}}{\sqrt{\cosh(2y) - \cos(2x)}}$$

01.10.19.0009.01

$$|\csc(z)| = \frac{\sqrt{2}}{\sqrt{\cosh(2\operatorname{Im}(z)) - \cos(2\operatorname{Re}(z))}}$$

Argument

01.10.19.0004.01

$$\arg(\csc(x+i\,y)) = \tan^{-1}\left(-\frac{2\cosh(y)\sin(x)}{\cos(2x) - \cosh(2y)}, \frac{2\cos(x)\sinh(y)}{\cos(2x) - \cosh(2y)}\right)$$

01.10.19.0005.01

$$\arg(\csc(x+i\,y)) == \frac{\pi}{2}\,\operatorname{sgn}\left(\frac{\operatorname{sgn}(\cos(x)\sinh(y))}{\operatorname{sgn}(\cos(2\,x)-\cosh(2\,y))}+\frac{1}{2}\right)\left(\frac{\operatorname{sgn}(\cosh(y)\sin(x))}{\operatorname{sgn}(\cos(2\,x)-\cosh(2\,y))}+1\right)-\tan^{-1}(\cot(x)\tanh(y))$$

01.10.19.0010.01

$$\arg(\csc(z)) == \tan^{-1}\left(-\frac{2\cosh(\operatorname{Im}(z))\sin(\operatorname{Re}(z))}{\cos(2\operatorname{Re}(z))-\cosh(2\operatorname{Im}(z))},\ \frac{2\cos(\operatorname{Re}(z))\sinh(\operatorname{Im}(z))}{\cos(2\operatorname{Re}(z))-\cosh(2\operatorname{Im}(z))}\right)$$

01.10.19.0011.01

$$\arg(\csc(z)) == \frac{1}{2}\,\pi\,\operatorname{sgn}\left(\frac{\operatorname{sgn}(\cos(\operatorname{Re}(z))\sinh(\operatorname{Im}(z)))}{\operatorname{sgn}(\cos(2\operatorname{Re}(z))-\cosh(2\operatorname{Im}(z)))}+\frac{1}{2}\right)\left(\frac{\operatorname{sgn}(\cosh(\operatorname{Im}(z))\sin(\operatorname{Re}(z)))}{\operatorname{sgn}(\cos(2\operatorname{Re}(z))-\cosh(2\operatorname{Im}(z)))}+1\right)-\tan^{-1}(\cot(\operatorname{Re}(z))\tanh(\operatorname{Im}(z)))$$

Conjugate value

01.10.19.0006.01

$$\overline{\csc(x+i\,y)} == \frac{1}{\cosh(y)\sin(x)-i\cos(x)\sinh(y)}$$

01.10.19.0012.01

$$\overline{\csc(z)} == \frac{1}{\cosh(\operatorname{Im}(z))\sin(\operatorname{Re}(z))-i\cos(\operatorname{Re}(z))\sinh(\operatorname{Im}(z))}$$

Signum value

01.10.19.0013.01

$$\operatorname{sgn}(\csc(x+i\,y)) == \frac{\sqrt{\cosh(2\,y)-\cos(2\,x)}}{\sqrt{2}\,(\cosh(y)\sin(x)+i\cos(x)\sinh(y))}$$

01.10.19.0014.01

$$\operatorname{sgn}(\csc(z)) == \frac{\sqrt{\cosh(2\operatorname{Im}(z))-\cos(2\operatorname{Re}(z))}}{\sqrt{2}\,(\cosh(\operatorname{Im}(z))\sin(\operatorname{Re}(z))+i\cos(\operatorname{Re}(z))\sinh(\operatorname{Im}(z)))}$$

Differentiation

Low-order differentiation

01.10.20.0001.01

$$\frac{\partial\csc(z)}{\partial z} == -\cot(z)\csc(z)$$

01.10.20.0002.01

$$\frac{\partial^2\csc(z)}{\partial z^2} == \csc(z)\left(\cot^2(z)+\csc^2(z)\right)$$

Symbolic differentiation

01.10.20.0003.01

$$\frac{\partial^n\csc(z)}{\partial z^n} == (-1)^n\,n!\,z^{-n-1}+\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k\,(2\,k-n-1)!}\left(2^{2\,k-1}-1\right)B_{2\,k}\,z^{2\,k-n-1}\ /;\ |z|<\pi\bigwedge n\in\mathbb{N}^+$$

01.10.20.0006.01

$$\frac{\partial^n \csc(z)}{\partial z^n} = (-i)^{n+1} \sum_{k=0}^{n} \frac{(-1)^k\, k!\, \mathcal{S}_n^{(k)}}{2^k} \left(\left(i \cot\left(\frac{z}{2}\right) + 1\right)^k \left(i \cot\left(\frac{z}{2}\right) - 1\right) - 2^n \left(i \cot(z) + 1\right)^k \left(i \cot(z) - 1\right) \right) /;\ n \in \mathbb{N}$$

01.10.20.0004.01

$$\frac{\partial^n \csc(z)}{\partial z^n} = \csc(z) \left(\delta_n + (n+1)! \sum_{k=0}^{n} \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{(-1)^{j+n}\, 2^{1-k}\, (k-2j)^n \csc^k(z)}{(k+1)\, j!\, (k-j)!\, (n-k)!} \cos\left(\frac{\pi (k-n)}{2} + (k-2j) z \right) \right) /;\ n \in \mathbb{N}$$

01.10.20.0007.01

$$\frac{\partial^n \csc(z)}{\partial z^n} = i^n \csc(z) \sum_{j=0}^{n} \sum_{k=0}^{j} (-1)^k \binom{n}{j} 2^{j-k}\, k!\, \mathcal{S}_j^{(k)} \left(1 - \cot(z)\, i\right)^k /;\ n \in \mathbb{N}$$

Victor Adamchik (2005)

Fractional integro-differentiation

01.10.20.0005.02

$$\frac{\partial^\alpha \csc(z)}{\partial z^\alpha} = \mathcal{FC}_{\exp}^{(\alpha)}(z, -1)\, z^{-\alpha-1} + \sum_{k=1}^{\infty} \frac{(-1)^{k-1} \left(2^{2k-1} - 1\right) B_{2k}\, z^{2k-\alpha-1}}{\Gamma(2k-\alpha)\, k} /;\ |z| < \pi$$

01.10.20.0008.01

$$\csc^{(\alpha)}(c z) = \lim_{\nu \to \alpha} \frac{(c z)^{-\nu-1}}{\Gamma(-\nu)} \left(4 \log(2) + 2 \log(\pi) - \log(-c z) + \psi(-\nu) + \gamma\right) - \pi^{-\alpha-1} \left((-c z)^\alpha (c z)^{-\alpha} \psi^{(\alpha)}\left(-\frac{c z}{\pi}\right) - \psi^{(\alpha)}\left(\frac{c z}{\pi}\right) \right) + 2^{-\alpha} \pi^{-\alpha-1} \left((-c z)^\alpha (c z)^{-\alpha} \psi^{(\alpha)}\left(-\frac{c z}{2\pi}\right) - \psi^{(\alpha)}\left(\frac{c z}{2\pi}\right) \right)$$

Integration

Indefinite integration

Involving only one direct function

01.10.21.0019.01

$$\int \csc(b + a z)\, dz = \frac{\log\left(\tan\left(\frac{1}{2}(b + a z)\right)\right)}{a}$$

01.10.21.0020.01

$$\int \csc(a z)\, dz = \frac{\log\left(\tan\left(\frac{a z}{2}\right)\right)}{a}$$

01.10.21.0021.01

$$\int \csc(z)\, dz = \log\left(\tan\left(\frac{z}{2}\right)\right)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Involving z^n and linear arguments

01.10.21.0022.01

$$\int z\csc(a z+b)\,dz = \frac{1}{a^2}\left((b+a z)\left(\log\left(1-e^{i(b+a z)}\right)-\log\left(1+e^{i(b+a z)}\right)\right)-b\log\left(\tan\left(\frac{1}{2}(b+a z)\right)\right)+i\left(\text{Li}_2\left(-e^{i(b+a z)}\right)-\text{Li}_2\left(e^{i(b+a z)}\right)\right)\right)$$

01.10.21.0023.01

$$\int z^n\csc(a z)\,dz = -2\,i\,e^{i a z}\,n!\sum_{j=0}^{n}\frac{1}{(n-j)!}(-1)^j z^{n-j}(i a)^{-j-1}\,{}_{j+2}F_{j+1}\left(\frac{1}{2},\ldots,\frac{1}{2},1;\frac{3}{2},\ldots,\frac{3}{2};e^{2 i a z}\right)/;n\in\mathbb{N}$$

01.10.21.0024.01

$$\int z\csc(a z)\,dz = \frac{a z\left(\log\left(1-e^{i a z}\right)-\log\left(1+e^{i a z}\right)\right)+i\left(\text{Li}_2\left(-e^{i a z}\right)-\text{Li}_2\left(e^{i a z}\right)\right)}{a^2}$$

01.10.21.0025.01

$$\int z^2\csc(a z)\,dz = \frac{a^2\left(\log\left(1-e^{i a z}\right)-\log\left(1+e^{i a z}\right)\right)z^2+2\,a\,i\left(\text{Li}_2\left(-e^{i a z}\right)-\text{Li}_2\left(e^{i a z}\right)\right)z+2\left(\text{Li}_3\left(e^{i a z}\right)-\text{Li}_3\left(-e^{i a z}\right)\right)}{a^3}$$

01.10.21.0026.01

$$\int z^3\csc(a z)\,dz = \frac{1}{8a^4}\left(i\left(2a^4z^4-8\,i\,a^3\log\left(1-e^{-i a z}\right)z^3+8a^3\,i\log\left(1+e^{i a z}\right)z^3+24a^2\,\text{Li}_2\left(e^{-i a z}\right)z^2+24a^2\,\text{Li}_2\left(-e^{i a z}\right)z^2-48\,i\,a\,\text{Li}_3\left(e^{-i a z}\right)z+48\,a\,i\,\text{Li}_3\left(-e^{i a z}\right)z-\pi^4-48\,\text{Li}_4\left(e^{-i a z}\right)-48\,\text{Li}_4\left(-e^{i a z}\right)\right)\right)$$

01.10.21.0027.01

$$\int z^4\csc(a z)\,dz = \frac{1}{10a^5}\left(2a^5\,i\,z^5+10a^4\log\left(1-e^{-i a z}\right)z^4-10a^4\log\left(1+e^{i a z}\right)z^4+40a^3\,i\,\text{Li}_2\left(e^{-i a z}\right)z^3+40a^3\,i\,\text{Li}_2\left(-e^{i a z}\right)z^3+120a^2\,\text{Li}_3\left(e^{-i a z}\right)z^2-120a^2\,\text{Li}_3\left(-e^{i a z}\right)z^2-240\,i\,a\,\text{Li}_4\left(e^{-i a z}\right)z-240\,i\,a\,\text{Li}_4\left(-e^{i a z}\right)z-i\pi^5-240\,\text{Li}_5\left(e^{-i a z}\right)+240\,\text{Li}_5\left(-e^{i a z}\right)\right)$$

Involving exponential function

Involving exp

Involving $a^{b z}$

01.10.21.0028.01

$$\int a^{b z}\csc(c z)\,dz = -\frac{2a^{b z}e^{i c z}}{c-i b\log(a)}\,{}_2F_1\left(\frac{c-i b\log(a)}{2c},1;\frac{3}{2}-\frac{i b\log(a)}{2c};e^{2 i c z}\right)$$

01.10.21.0029.01

$$\int e^{b z}\csc(a z)\,dz = -\frac{2e^{(b+i a)z}}{a-i b}\,{}_2F_1\left(\frac{a-i b}{2a},1;\frac{3}{2}-\frac{i b}{2a};e^{2 i a z}\right)$$

01.10.21.0030.01

$$\int e^{-i a z} \csc(a z) \, dz == \frac{\log\left(-1 + e^{-2 i a z}\right)}{a}$$

01.10.21.0031.01

$$\int e^{i a z} \csc(a z) \, dz == \frac{\log\left(-1 + e^{2 i a z}\right)}{a}$$

Involving exponential function and a power function

Involving exp and power

Involving $z^n e^{b z}$

01.10.21.0032.01

$$\int z^n e^{b z} \csc(c z) \, dz == -2 i \; n! \, e^{(b+i c) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (b + i c)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{-i b + c}{2 c}, \ldots, \frac{-i b + c}{2 c}, 1; \frac{-i b + c}{2 c} + 1, \ldots, \frac{-i b + c}{2 c} + 1; e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.10.21.0033.01

$$\int z^n e^{-i c z} \csc(c z) \, dz == -\frac{2 i z^{1+n}}{1+n} - 2 i e^{2 i c z} n! \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (i c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(1, \ldots, 1, 1; 2, \ldots, 2; e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.10.21.0034.01

$$\int z^n e^{-i c z (2 q+1)} \csc(c z) \, dz == 2 i \; n! \left(-\frac{z^{n+1}}{(n+1)!} + e^{2 i c z} \sum_{j=0}^{n} \frac{(-2 i c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(1, \ldots, 1, 1; 2, \ldots, 2; e^{2 i c z}\right) + \sum_{j=0}^{n} \sum_{k=0}^{q-1} \frac{e^{2 i c (k-q) z} (2 i c (q-k))^{-j-1} z^{n-j}}{(n-j)!}\right) /; n \in \mathbb{N} \land q \in \mathbb{N}^+$$

Arguments involving inverse trigonometric functions

Involving $\sin^{-1}$

01.10.21.0035.01

$$\int \csc\left(\sin^{-1}(z)\right) dz == \log(z)$$

01.10.21.0036.01

$$\int \csc\left(a \sin^{-1}(z)\right) dz == -\frac{1}{a^2 - 1} \left(e^{i (a-1) \sin^{-1}(z)} \left((a+1) \, {}_2F_1\left(\frac{a-1}{2 a}, 1; \frac{3}{2} - \frac{1}{2 a}; e^{2 i a \sin^{-1}(z)}\right) + (a-1) \, e^{2 i \sin^{-1}(z)} \, {}_2F_1\left(\frac{a+1}{2 a}, 1; \frac{1}{2}\left(3 + \frac{1}{a}\right); e^{2 i a \sin^{-1}(z)}\right)\right)\right)$$

Involving $\cos^{-1}$

01.10.21.0037.01

$$\int \csc\left(\cos^{-1}(z)\right) dz = \sin^{-1}(z)$$

01.10.21.0038.01

$$\int \csc\left(a \cos^{-1}(z)\right) dz = \frac{1}{a^2 - 1} \left(i\, e^{-i \cos^{-1}(z)} \left((a+1)\, e^{i a \cos^{-1}(z)}\, {}_2F_1\left(\frac{a-1}{2a}, 1; \frac{3}{2} - \frac{1}{2a}; e^{2 i a \cos^{-1}(z)}\right) - (a-1)\, e^{i (a+2) \cos^{-1}(z)}\, {}_2F_1\left(\frac{a+1}{2a}, 1; \frac{1}{2}\left(3 + \frac{1}{a}\right); e^{2 i a \cos^{-1}(z)}\right)\right)\right)$$

Involving $\tan^{-1}$

01.10.21.0039.01

$$\int \csc\left(\tan^{-1}(z)\right) dz = \log(z) - \log\left(\sqrt{z^2+1} + 1\right) + \sqrt{z^2+1}$$

Involving $\cot^{-1}$

01.10.21.0040.01

$$\int \csc\left(\cot^{-1}(z)\right) dz = \frac{\sqrt{1 + \frac{1}{z^2}}\, z \left(\sqrt{z^2+1}\, z + \sinh^{-1}(z)\right)}{2\sqrt{z^2+1}}$$

Involving $\csc^{-1}$

01.10.21.0041.01

$$\int \csc\left(\csc^{-1}(z)\right) dz = \frac{z^2}{2}$$

Involving $\sec^{-1}$

01.10.21.0042.01

$$\int \csc\left(\sec^{-1}(z)\right) dz = \sqrt{1 - \frac{1}{z^2}}\, z$$

Arguments involving inverse hyperbolic functions

Involving $\sinh^{-1}$

01.10.21.0043.01

$$\int \csc\left(\sinh^{-1}(z)\right) dz = \frac{1}{2} e^{(-1+i)\sinh^{-1}(z)} \left((-1-i)\, e^{2\sinh^{-1}(z)}\, {}_2F_1\left(\frac{1}{2} - \frac{i}{2}, 1; \frac{3}{2} - \frac{i}{2}; e^{2 i \sinh^{-1}(z)}\right) - (1-i)\, {}_2F_1\left(\frac{1}{2} + \frac{i}{2}, 1; \frac{3}{2} + \frac{i}{2}; e^{2 i \sinh^{-1}(z)}\right)\right)$$

01.10.21.0044.01

$$\int \csc(a \sinh^{-1}(z))\, dz = -\frac{1}{a^2+1}\left(e^{i(a+i)\sinh^{-1}(z)}\left((a+i)\, e^{2\sinh^{-1}(z)}\, {}_2F_1\left(\frac{a-i}{2a}, 1; \frac{3}{2}-\frac{i}{2a}; e^{2ia\sinh^{-1}(z)}\right)+(a-i)\, {}_2F_1\left(\frac{a+i}{2a}, 1; \frac{3}{2}+\frac{i}{2a}; e^{2ia\sinh^{-1}(z)}\right)\right)\right)$$

Involving $\cosh^{-1}$

01.10.21.0045.01

$$\int \csc(\cosh^{-1}(z))\, dz = \left(-\frac{1}{2}-\frac{i}{2}\right) e^{(-1+i)\cosh^{-1}(z)}\left(i\, {}_2F_1\left(\frac{1}{2}+\frac{i}{2}, 1; \frac{3}{2}+\frac{i}{2}; e^{2i\cosh^{-1}(z)}\right)+e^{2\cosh^{-1}(z)}\, {}_2F_1\left(\frac{1}{2}-\frac{i}{2}, 1; \frac{3}{2}-\frac{i}{2}; e^{2i\cosh^{-1}(z)}\right)\right)$$

01.10.21.0046.01

$$\int \csc(a \cosh^{-1}(z))\, dz = \frac{1}{a^2+1}\left(e^{i(a+i)\cosh^{-1}(z)}\left((a-i)\, {}_2F_1\left(\frac{a+i}{2a}, 1; \frac{3}{2}+\frac{i}{2a}; e^{2ia\cosh^{-1}(z)}\right)-(a+i)\, e^{2\cosh^{-1}(z)}\, {}_2F_1\left(\frac{a-i}{2a}, 1; \frac{3}{2}-\frac{i}{2a}; e^{2ia\cosh^{-1}(z)}\right)\right)\right)$$

Involving trigonometric functions

Involving sin

Involving $\sin(b\, z)$

01.10.21.0047.01

$$\int \sin(b\, z)\csc(c\, z)\, dz = \frac{i\, e^{i(c-b)z}}{(b-c)(b+c)}\left((b+c)\, {}_2F_1\left(\frac{c-b}{2c}, 1; \frac{3}{2}-\frac{b}{2c}; e^{2icz}\right)+(b-c)\, e^{2ibz}\, {}_2F_1\left(\frac{b+c}{2c}, 1; \frac{b+3c}{2c}; e^{2icz}\right)\right)$$

Involving power of sin

Involving $\sin^m(b\, z)$

01.10.21.0048.01

$$\int \sin^m(b\, z)\csc(c\, z)\, dz = \frac{2^{1-m}\tanh^{-1}\left(e^{icz}\right)(m \bmod 2 - 1)}{c}\binom{m}{\frac{m}{2}} - 2^{1-m}\, e^{icz}\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^s\binom{m}{s}\left(\frac{e^{\frac{i m\pi}{2}-i b(m-2s)z}\, {}_2F_1\left(\frac{c-b(m-2s)}{2c}, 1; \frac{c-b(m-2s)}{2c}+1; e^{2icz}\right)}{c-b(m-2s)}+\frac{e^{i b(m-2s)z-\frac{i m\pi}{2}}\, {}_2F_1\left(\frac{c+b(m-2s)}{2c}, 1; \frac{c+b(m-2s)}{2c}+1; e^{2icz}\right)}{c+b(m-2s)}\right) /; m\in\mathbb{N}^+$$

Involving cos

Involving cos(b z)

01.10.21.0049.01

$$\int \cos(b\,z)\csc(c\,z)\,dz = \frac{e^{i(c-b)z}}{(b-c)(b+c)}\left((b+c)\,{}_2F_1\!\left(\frac{c-b}{2c},1;\frac{3}{2}-\frac{b}{2c};e^{2icz}\right)-(b-c)\,e^{2ibz}\,{}_2F_1\!\left(\frac{b+c}{2c},1;\frac{b+3c}{2c};e^{2icz}\right)\right)$$

Involving power of cos

Involving cos^m(b z)

01.10.21.0050.01

$$\int \cos^m(b\,z)\csc(c\,z)\,dz = \frac{2^{1-m}\tanh^{-1}\!\left(e^{icz}\right)(m \bmod 2-1)}{c}\binom{m}{\frac{m}{2}}-2^{1-m}\,e^{icz}\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{s}\left(\frac{e^{-ib(m-2s)z}\,{}_2F_1\!\left(\frac{c-b(m-2s)}{2c},1;\frac{c-b(m-2s)}{2c}+1;e^{2icz}\right)}{c-b(m-2s)}+\frac{e^{ib(m-2s)z}\,{}_2F_1\!\left(\frac{c+b(m-2s)}{2c},1;\frac{c+b(m-2s)}{2c}+1;e^{2icz}\right)}{c+b(m-2s)}\right)/;\,m\in\mathbb{N}^+$$

Involving trigonometric and a power functions

Involving sin and power

Involving z^n sin(a + b z)

01.10.21.0051.01

$$\int z^n\sin(a+b\,z)\csc(c\,z)\,dz = e^{-ia+i(c-b)z}\,n!\sum_{j=0}^{n}\frac{1}{(n-j)!}\left((-1)^j(-ib+ic)^{-j-1}z^{n-j}\,{}_{j+2}F_{j+1}\!\left(\frac{c-b}{2c},\ldots,\frac{c-b}{2c},1;\frac{c-b}{2c}+1,\ldots,\frac{c-b}{2c}+1;e^{2icz}\right)\right)-e^{ia+i(c+b)z}\,n!\sum_{j=0}^{n}\frac{1}{(n-j)!}\left((-1)^j(ib+ic)^{-j-1}z^{n-j}\,{}_{j+2}F_{j+1}\!\left(\frac{b+c}{2c},\ldots,\frac{b+c}{2c},1;\frac{b+c}{2c}+1,\ldots,\frac{b+c}{2c}+1;e^{2icz}\right)\right)/;\,n\in\mathbb{N}$$

01.10.21.0052.01

$$\int z^n\sin(b\,z)\csc(c\,z)\,dz = -i\,e^{icz}\,n!\left(e^{-\frac{1}{2}(i\pi)+ibz}\sum_{j=0}^{n}\frac{(-1)^j(ib+ic)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b+c}{2c},\ldots,\frac{b+c}{2c},1;\frac{b+c}{2c}+1,\ldots,\frac{b+c}{2c}+1;e^{2icz}\right)+e^{\frac{i\pi}{2}-ibz}\sum_{j=0}^{n}\frac{(-1)^j(-ib+ic)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{-b+c}{2c},\ldots,\frac{-b+c}{2c},1;\frac{-b+c}{2c}+1,\ldots,\frac{-b+c}{2c}+1;e^{2icz}\right)\right)/;\,n\in\mathbb{N}$$

Involving powers of sin and power

Involving $z^n \sin^m(b z)$

01.10.21.0053.01

$$\int z^n \sin^m(b z) \csc(c z) \, dz = i\, 2^{1-m} e^{i c z} \binom{m}{\frac{m}{2}} n! \,(m \bmod 2 - 1) \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (i c)^{-j-1} {}_{j+2}F_{j+1}\!\left(\frac{1}{2}, \ldots, \frac{1}{2}, 1; \frac{3}{2}, \ldots, \frac{3}{2}; e^{2 i c z}\right) - i\, 2^{1-m} e^{i c z} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(e^{b i (m-2k) z - \frac{i m \pi}{2}} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (b i (m-2k) + i c)^{-j-1} {}_{j+2}F_{j+1}\!\left(\frac{c + b(-2k+m)}{2c}, \ldots, \frac{c + b(-2k+m)}{2c}, 1; \frac{c + b(-2k+m)}{2c} + 1, \ldots, \frac{c + b(-2k+m)}{2c} + 1; e^{2 i c z}\right) + e^{\frac{i \pi m}{2} - i b (m-2k) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (-i b (m-2k) + i c)^{-j-1} {}_{j+2}F_{j+1}\!\left(\frac{c - b(-2k+m)}{2c}, \ldots, \frac{c - b(-2k+m)}{2c}, 1; \frac{c - b(-2k+m)}{2c} + 1, \ldots, \frac{c - b(-2k+m)}{2c} + 1; e^{2 i c z}\right) \right) /;\; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving cos and power

Involving $z^n \cos(a + b z)$

01.10.21.0054.01

$$\int z^n \cos(a + b z) \csc(c z) \, dz = -i\, e^{i a + i (c+b) z} n! \sum_{j=0}^{n} \frac{(-1)^j (i c + i b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\!\left(\frac{b+c}{2c}, \ldots, \frac{b+c}{2c}, 1; \frac{b+c}{2c} + 1, \ldots, \frac{b+c}{2c} + 1; e^{2 i c z}\right) - i\, e^{-i a + i (c-b) z} n! \sum_{j=0}^{n} \frac{(-1)^j (i c - i b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\!\left(\frac{-b+c}{2c}, \ldots, \frac{-b+c}{2c}, 1; \frac{-b+c}{2c} + 1, \ldots, \frac{-b+c}{2c} + 1; e^{2 i c z}\right) /;\; n \in \mathbb{N}$$

01.10.21.0055.01

$$\int z^n \cos(b z) \csc(c z) \, dz = -i\, e^{i c z} n! \left(e^{-i b z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (-i b + i c)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\!\left(\frac{-b+c}{2c}, \ldots, \frac{-b+c}{2c}, 1; \frac{-b+c}{2c} + 1, \ldots, \frac{-b+c}{2c} + 1; e^{2 i c z}\right) + e^{i b z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i b + i c)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\!\left(\frac{b+c}{2c}, \ldots, \frac{b+c}{2c}, 1; \frac{b+c}{2c} + 1, \ldots, \frac{b+c}{2c} + 1; e^{2 i c z}\right) \right) /;\; n \in \mathbb{N}$$

Involving powers of cos and power

Involving $z^n \cos^m(b z)$

01.10.21.0056.01

$$\int z^n \cos^m(b z) \csc(c z)\, dz = i\, 2^{1-m} e^{i c z} \binom{m}{\frac{m}{2}} n!\, (m \bmod 2 - 1) \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (i c)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{1}{2}, \ldots, \frac{1}{2}, 1; \frac{3}{2}, \ldots, \frac{3}{2}; e^{2 i c z}\right) - i\, 2^{1-m} e^{i c z} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(e^{b i (m-2k) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (b i (m-2k) + i c)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{c + b(-2k+m)}{2c}, \ldots, \frac{c + b(-2k+m)}{2c}, 1; \frac{c + b(-2k+m)}{2c} + 1, \ldots, \frac{c + b(-2k+m)}{2c} + 1; e^{2 i c z}\right) + e^{-i b (m-2k) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (-i b (m-2k) + i c)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{c - b(-2k+m)}{2c}, \ldots, \frac{c - b(-2k+m)}{2c}, 1; \frac{c - b(-2k+m)}{2c} + 1, \ldots, \frac{c - b(-2k+m)}{2c} + 1; e^{2 i c z}\right)\right) /;\, n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving trigonometric and exponential functions

Involving sin and exp

Involving $e^{p z} \sin(b z)$

01.10.21.0057.01

$$\int e^{p z} \sin(b z)\, \csc(c z)\, dz = e^{i c z} \left(\frac{e^{(-i b + p) z} {}_2F_1\left(1, -\frac{b}{2c} + \frac{1}{2} - \frac{i p}{2c}; -\frac{b}{2c} + \frac{3}{2} - \frac{i p}{2c}; e^{2 i c z}\right)}{-i b + i c + p} - \frac{e^{(i b + p) z} {}_2F_1\left(1, \frac{b}{2c} + \frac{1}{2} - \frac{i p}{2c}; \frac{b}{2c} + \frac{3}{2} - \frac{i p}{2c}; e^{2 i c z}\right)}{i b + i c + p}\right)$$

01.10.21.0058.01

$$\int e^{i (a-c) z} \sin(a z) \csc(c z)\, dz = z + \frac{i e^{2 i a z} {}_2F_1\left(1, \frac{a}{c}; \frac{a+c}{c}; e^{2 i c z}\right)}{2a} + \frac{i \log\left(1 - e^{2 i c z}\right)}{2c}$$

01.10.21.0059.01

$$\int e^{-i (a+c) z} \sin(a z) \csc(c z)\, dz = -z + \frac{e^{-2 i a z} i}{2a} {}_2F_1\left(1, -\frac{a}{c}; 1 - \frac{a}{c}; e^{2 i c z}\right) - \frac{i \log\left(1 - e^{2 i c z}\right)}{2c}$$

Involving powers of sin and exp

Involving $e^{p z} \sin^m(b z)$

01.10.21.0060.01

$$\int e^{p z}\sin^m(b z)\csc(c z)\,dz = i\,\frac{2^{1-m}\,e^{(i c+p)z}\,(m \bmod 2-1)}{i c+p}\binom{m}{\frac{m}{2}}\,{}_2F_1\!\left(\frac{-i p+c}{2c},1;\frac{-i p+c}{2c}+1;e^{2 i c z}\right)-$$
$$i\,2^{1-m}\,e^{i c z}\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^s\binom{m}{s}\left(\frac{e^{(p+b i (m-2s))z-\frac{i m \pi}{2}}\,{}_2F_1\!\left(\frac{b(m-2s)-i p+c}{2c},1;\frac{b(m-2s)-i p+c}{2c}+1;e^{2 i c z}\right)}{i c+p+b i(m-2s)}+\right.$$
$$\left.\frac{e^{\frac{i\pi m}{2}+(p-i b(m-2s))z}\,{}_2F_1\!\left(\frac{-i p-b(m-2s)+c}{2c},1;\frac{-i p-b(m-2s)+c}{2c}+1;e^{2 i c z}\right)}{i c+p-i b(m-2s)}\right)/;\,m\in\mathbb{N}^+$$

Involving cos and exp

Involving $e^{p z}\cos(b z)$

01.10.21.0061.01

$$\int e^{p z}\cos(b z)\csc(c z)\,dz = \frac{1}{2}\left(1-e^{2 i c z}\right)\csc(c z)$$
$$\left(\frac{e^{(i b+p)z}\,{}_2F_1\!\left(1,\frac{b}{2c}+\frac{1}{2}-\frac{i p}{2c};\frac{b}{2c}+\frac{3}{2}-\frac{i p}{2c};e^{2 i c z}\right)}{i b+i c+p}+\frac{e^{(-i b+p)z}\,{}_2F_1\!\left(1,-\frac{b}{2c}+\frac{1}{2}-\frac{i p}{2c};-\frac{b}{2c}+\frac{3}{2}-\frac{i p}{2c};e^{2 i c z}\right)}{-i b+i c+p}\right)$$

01.10.21.0062.01

$$\int e^{i(a-c)z}\cos(a z)\csc(c z)\,dz = -i z-\frac{e^{2 i a z}}{2a}\,{}_2F_1\!\left(1,\frac{a}{c};1+\frac{a}{c};e^{2 i c z}\right)+\frac{\log\left(1-e^{2 i c z}\right)}{2c}$$

01.10.21.0063.01

$$\int e^{-i(a+c)z}\cos(a z)\csc(c z)\,dz = -i z+\frac{e^{-2 i a z}}{2a}\,{}_2F_1\!\left(1,-\frac{a}{c};1-\frac{a}{c};e^{2 i c z}\right)+\frac{\log\left(1-e^{2 i c z}\right)}{2c}$$

Involving powers of cos and exp

Involving $e^{p z}\cos^m(b z)$

01.10.21.0064.01

$$\int e^{p z}\cos^m(b z)\csc(c z)\,dz = \frac{i\,2^{1-m}\,e^{(i c+p)z}}{p+i c}\binom{m}{\frac{m}{2}}(m \bmod 2-1)\,{}_2F_1\!\left(\frac{c-i p}{2c},1;\frac{c-i p}{2c}+1;e^{2 i c z}\right)-$$
$$i\,2^{1-m}\,e^{i c z}\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{s}\left(\frac{e^{(p+b i(m-2s))z}\,{}_2F_1\!\left(\frac{c-i p+b(m-2s)}{2c},1;\frac{c-i p+b(m-2s)}{2c}+1;e^{2 i c z}\right)}{p+i b(m-2s)+i c}+\right.$$
$$\left.\frac{e^{(p-i b(m-2s))z}\,{}_2F_1\!\left(\frac{c-i p-b(m-2s)}{2c},1;\frac{c-i p-b(m-2s)}{2c}+1;e^{2 i c z}\right)}{p-i b(m-2s)+i c}\right)/;\,m\in\mathbb{N}^+$$

Involving trigonometric, exponential and a power functions

Involving sin, exp and power

Involving $z^n e^{pz} \sin(a+bz)\csc(cz)$

01.10.21.0065.01

$$\int z^n e^{pz}\sin(a+bz)\csc(cz)\,dz = e^{-ia+(ic-ib+p)z}\,n!\sum_{j=0}^{n}\frac{(-1)^j (ic-ib+p)^{-j-1} z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\!\left(\frac{c-b-ip}{2c},\ldots,\frac{c-b-ip}{2c},1;\frac{c-b-ip}{2c}+1,\ldots,\frac{c-b-ip}{2c}+1;e^{2icz}\right) - e^{ia+(ic+ib+p)z}\,n!\sum_{j=0}^{n}\frac{(-1)^j (ic+ib+p)^{-j-1} z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\!\left(\frac{c+b-ip}{2c},\ldots,\frac{c+b-ip}{2c},1;\frac{c+b-ip}{2c}+1,\ldots,\frac{c+b-ip}{2c}+1;e^{2icz}\right)\,/;\,n\in\mathbb{N}$$

01.10.21.0066.01

$$\int z^n e^{pz}\sin(bz)\csc(cz)\,dz = e^{icz}\,n!\left(e^{(-ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(-ib+p+ic)^{-j-1}}{(n-j)!}\;{}_{j+2}F_{j+1}\!\left(\frac{-b-ip+c}{2c},\ldots,\frac{-b-ip+c}{2c},1;\frac{-b-ip+c}{2c}+1,\ldots,\frac{-b-ip+c}{2c}+1;e^{2icz}\right) - e^{(ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(ib+p+ic)^{-j-1}}{(n-j)!}\;{}_{j+2}F_{j+1}\!\left(\frac{b-ip+c}{2c},\ldots,\frac{b-ip+c}{2c},1;\frac{b-ip+c}{2c}+1,\ldots,\frac{b-ip+c}{2c}+1;e^{2icz}\right)\right)\,/;\,n\in\mathbb{N}\land p+ia\neq -ic\land p-ia\neq -ic$$

01.10.21.0067.01

$$\int z^n e^{i(b-c)z}\sin(bz)\csc(cz)\,dz = \frac{z^{n+1}}{n+1} + e^{2icz}\,n!\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(2ic)^{-j-1}}{(n-j)!}\;{}_{j+2}F_{j+1}\!\left(1,\ldots,1,1;2,\ldots,2;e^{2icz}\right) - e^{2ibz}\,n!\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(2ib)^{-j-1}}{(n-j)!}\;{}_{j+2}F_{j+1}\!\left(\frac{b}{c},\ldots,\frac{b}{c},1;\frac{b}{c}+1,\ldots,\frac{b}{c}+1;e^{2icz}\right)\,/;\,n\in\mathbb{N}$$

01.10.21.0068.01

$$\int z^n e^{-i(b+c)z}\sin(bz)\csc(cz)\,dz = -\frac{z^{n+1}}{n+1} - n!\,e^{2icz}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(2ic)^{-j-1}}{(n-j)!}\;{}_{j+2}F_{j+1}\!\left(1,\ldots,1,1;2,\ldots,2;e^{2icz}\right) - n!\,e^{-2ibz}\sum_{j=0}^{n}\frac{z^{n-j}(2ib)^{-j-1}}{(n-j)!}\;{}_{j+2}F_{j+1}\!\left(-\frac{b}{c},\ldots,-\frac{b}{c},1;1-\frac{b}{c},\ldots,1-\frac{b}{c};e^{2icz}\right)\,/;\,n\in\mathbb{N}$$

Involving powers of sin, exp and power

Involving $z^n e^{pz} \sin^m(bz)\csc(cz)$

01.10.21.0069.01

$$\int z^n e^{pz}\sin^m(bz)\csc(cz)\,dz = i\,2^{1-m}\,e^{(ic+p)z}\binom{m}{\frac{m}{2}}n!\,(m \bmod 2-1)\sum_{j=0}^{n}\frac{1}{(n-j)!}(-1)^j z^{n-j}(p+ic)^{-j-1}\,{}_{j+2}F_{j+1}\left(\frac{c-ip}{2c},\ldots,\frac{c-ip}{2c},1;\frac{c-ip}{2c}+1,\ldots,\frac{c-ip}{2c}+1;e^{2icz}\right)-$$
$$i\,2^{1-m}\,e^{icz}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(e^{(bi(m-2k)+p)z-\frac{im\pi}{2}}\sum_{j=0}^{n}\frac{1}{(n-j)!}(-1)^j z^{n-j}(bi(m-2k)+p+ic)^{-j-1}\,{}_{j+2}F_{j+1}\left(\frac{c-ip+b(-2k+m)}{2c},\ldots,\frac{c-ip+b(-2k+m)}{2c},1;\frac{c-ip+b(-2k+m)}{2c}+1,\ldots,\frac{c-ip+b(-2k+m)}{2c}+1;e^{2icz}\right)+e^{\frac{i\pi m}{2}+(-ib(m-2k)+p)z}\sum_{j=0}^{n}\frac{1}{(n-j)!}(-1)^j z^{n-j}(-ib(m-2k)+p+ic)^{-j-1}\,{}_{j+2}F_{j+1}\left(\frac{c-ip-b(-2k+m)}{2c},\ldots,\frac{c-ip-b(-2k+m)}{2c},1;\frac{c-ip-b(-2k+m)}{2c}+1,\ldots,\frac{c-ip-b(-2k+m)}{2c}+1;e^{2icz}\right)\right)\;/;\;n\in\mathbb{N}\land m\in\mathbb{N}^+$$

Involving cos, exp and power

Involving $z^n e^{pz}\cos(a+bz)\csc(cz)$

01.10.21.0070.01

$$\int z^n e^{pz}\cos(a+bz)\csc(cz)\,dz = -i\,e^{-ia+(ic-ib+p)z}\,n!\sum_{j=0}^{n}\frac{(-1)^j(ic-ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c-b-ip}{2c},\ldots,\frac{c-b-ip}{2c},1;\frac{c-b-ip}{2c}+1,\ldots,\frac{c-b-ip}{2c}+1;e^{2icz}\right)-i\,e^{ia+(ic+ib+p)z}\,n!\sum_{j=0}^{n}\frac{(-1)^j(ic+ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c+b-ip}{2c},\ldots,\frac{c+b-ip}{2c},1;\frac{c+b-ip}{2c}+1,\ldots,\frac{c+b-ip}{2c}+1;e^{2icz}\right)\;/;\;n\in\mathbb{N}$$

01.10.21.0071.01

$$\int z^n e^{pz}\cos(bz)\csc(cz)\,dz = -i\,e^{icz}\,n!\left(e^{(-ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(-ib+p+ic)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c-b-ip}{2c},\ldots,\frac{c-b-ip}{2c},1;\frac{c-b-ip}{2c}+1,\ldots,\frac{c-b-ip}{2c}+1;e^{2icz}\right)+e^{(ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(ib+p+ic)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c+b-ip}{2c},\ldots,\frac{c+b-ip}{2c},1;\frac{c+b-ip}{2c}+1,\ldots,\frac{c+b-ip}{2c}+1;e^{2icz}\right)\right)\;/;\;n\in\mathbb{N}$$

01.10.21.0072.01

$$\int z^n e^{i(b-c)z}\cos(b z)\csc(c z)\,dz = -\frac{i z^{n+1}}{n+1} - i e^{2 i c z} n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}(1, \ldots, 1, 1; 2, \ldots, 2; e^{2 i c z}) - i e^{2 i b z} n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i b)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b}{c}, \ldots, \frac{b}{c}, 1; \frac{b}{c}+1, \ldots, \frac{b}{c}+1; e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.10.21.0073.01

$$\int z^n e^{-i(b+c)z}\cos(b z)\csc(c z)\,dz = -\frac{i z^{n+1}}{n+1} - i n! e^{2 i c z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}(1, \ldots, 1, 1; 2, \ldots, 2; e^{2 i c z}) + i n! e^{-2 i b z} \sum_{j=0}^{n} \frac{z^{n-j} (2 i b)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(-\frac{b}{c}, \ldots, -\frac{b}{c}, 1; 1-\frac{b}{c}, \ldots, 1-\frac{b}{c}; e^{2 i c z}\right) /; n \in \mathbb{N}$$

Involving powers of cos, exp and power

Involving $z^n e^{p z} \cos^m(b z) \csc(c z)$

01.10.21.0074.01

$$\int z^n e^{p z}\cos^m(b z)\csc(c z)\,dz = i\, 2^{1-m} e^{(i c+p) z} \binom{m}{\frac{m}{2}} n! (m \bmod 2 - 1) \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (p + i c)^{-j-1} \, {}_{j+2}F_{j+1}\left(\frac{c - i p}{2 c}, \ldots, \frac{c - i p}{2 c}, 1; \frac{c - i p}{2 c}+1, \ldots, \frac{c - i p}{2 c}+1; e^{2 i c z}\right) - i\, 2^{1-m} e^{i c z} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(e^{(b i (m-2k)+p) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (b i (m-2k) + p + i c)^{-j-1} \, {}_{j+2}F_{j+1}\left(\frac{c - i p + b(-2k+m)}{2 c}, \ldots, \frac{c - i p + b(-2k+m)}{2 c}, 1; \frac{c - i p + b(-2k+m)}{2 c}+1, \ldots, \frac{c - i p + b(-2k+m)}{2 c}+1; e^{2 i c z}\right) + e^{(-i b (m-2k)+p) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (-i b (m-2k) + p + i c)^{-j-1} \, {}_{j+2}F_{j+1}\left(\frac{c - i p - b(-2k+m)}{2 c}, \ldots, \frac{c - i p - b(-2k+m)}{2 c}, 1; \frac{c - i p - b(-2k+m)}{2 c}+1, \ldots, \frac{c - i p - b(-2k+m)}{2 c}+1; e^{2 i c z}\right)\right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving functions of the direct function

Involving powers of the direct function

Involving powers of csc

Linear argument

01.10.21.0075.01

$$\int \csc^{\nu}(c\,z)\,dz = -\frac{\cos(c\,z)\csc^{\nu-1}(c\,z)\,\sin^2(c\,z)^{\frac{\nu-1}{2}}}{c}\,{}_2F_1\left(\frac{1}{2},\frac{\nu+1}{2};\frac{3}{2};\cos^2(c\,z)\right)$$

01.10.21.0076.01

$$\int \csc^2(c\,z)\,dz = -\frac{\cot(c\,z)}{c}$$

01.10.21.0077.01

$$\int \csc^3(c\,z)\,dz = -\frac{\cot(c\,z)\csc(c\,z)+\log\left(\cos\left(\frac{c\,z}{2}\right)\right)-\log\left(\sin\left(\frac{c\,z}{2}\right)\right)}{2\,c}$$

01.10.21.0078.01

$$\int \csc^4(c\,z)\,dz = -\frac{\cot(c\,z)\left(\csc^2(c\,z)+2\right)}{3\,c}$$

01.10.21.0079.01

$$\int \csc^5(c\,z)\,dz = -\frac{\cot(c\,z)\csc(c\,z)\left(2\csc^2(c\,z)+3\right)+3\log\left(\cos\left(\frac{c\,z}{2}\right)\right)-3\log\left(\sin\left(\frac{c\,z}{2}\right)\right)}{8\,c}$$

01.10.21.0080.01

$$\int \csc^6(c\,z)\,dz = -\frac{\cot(c\,z)\left(3\csc^4(c\,z)+4\csc^2(c\,z)+8\right)}{15\,c}$$

01.10.21.0081.01

$$\int \csc^7(c\,z)\,dz = \frac{15\left(\log\left(\sin\left(\frac{c\,z}{2}\right)\right)-\log\left(\cos\left(\frac{c\,z}{2}\right)\right)\right)-\cot(c\,z)\csc(c\,z)\left(8\csc^4(c\,z)+10\csc^2(c\,z)+15\right)}{48\,c}$$

01.10.21.0082.01

$$\int \csc^8(c\,z)\,dz = -\frac{\cot(c\,z)\left(5\csc^6(c\,z)+6\csc^4(c\,z)+8\csc^2(c\,z)+16\right)}{35\,c}$$

01.10.21.0159.01

$$\int \csc^{2n}(c\,z)\,dz = -\frac{\cos(c\,z)\csc^{2n-1}(c\,z)}{c\,(2\,n-1)}\sum_{k=0}^{n-1}\frac{(1-n)_k\sin^{2k}(c\,z)}{\left(\frac{3}{2}-n\right)_k}\;/;\;n\in\mathbb{N}^+$$

01.10.21.0160.01

$$\int \csc^{2n+1}(c\,z)\,dz = \frac{4^{-n}}{c}\binom{2\,n}{n}\left(\log\left(\sin\left(\frac{c\,z}{2}\right)\right)-\log\left(\cos\left(\frac{c\,z}{2}\right)\right)\right)-\frac{\cos(c\,z)\left(\frac{1}{2}\right)_n}{c\,(2\,n!)}\sum_{k=1}^{n}\frac{\csc^{2k}(c\,z)\,(k-1)!}{\left(\frac{1}{2}\right)_k}\;/;\;n\in\mathbb{N}$$

01.10.21.0161.01

$$\int \csc^{2n}(c\,z)\,dz = -\frac{\cos(c\,z)\csc^{2n-1}(c\,z)}{c\,(2\,n-1)}\,{}_2F_1\left(1,1-n;\frac{3}{2}-n;\sin^2(c\,z)\right)/;\;n\in\mathbb{N}^+$$

01.10.21.0162.01

$$\int \csc^{2n+1}(c\,z)\,dz = \frac{\binom{2\,n}{n}}{4^n\,c}\left(\tanh^{-1}(\cos(c\,z))-\log\left(\cos\left(\frac{c\,z}{2}\right)\right)+\log\left(\sin\left(\frac{c\,z}{2}\right)\right)\right)-\frac{\cos(c\,z)}{c}\,{}_2F_1\left(\frac{1}{2},n+1;\frac{3}{2};\cos^2(c\,z)\right)/;\;n\in\mathbb{N}$$

01.10.21.0083.01

$$\int \csc^{\frac{1}{2}}(c\,z)\,dz = -\frac{2\csc^{\frac{1}{2}}(c\,z)\sin^{\frac{1}{2}}(c\,z)}{c} F\!\left(\frac{1}{4}(\pi-2\,c\,z)\,\middle|\,2\right)$$

01.10.21.0084.01

$$\int \frac{1}{\csc^{\frac{1}{2}}(c\,z)}\,dz = -\frac{2}{c\csc^{\frac{1}{2}}(c\,z)\sin^{\frac{1}{2}}(c\,z)} E\!\left(\frac{1}{4}(\pi-2\,c\,z)\,\middle|\,2\right)$$

Involving products of the direct functions

01.10.21.0085.01

$$\int \csc(b+a\,z)\csc(a\,z)\,dz = \frac{\csc(b)\left(\log(\sin(a\,z))-\log(\sin(b+a\,z))\right)}{a}$$

01.10.21.0086.01

$$\int \csc(b-a\,z)\csc(a\,z)\,dz = \frac{\csc(b)\left(\log(-\sin(a\,z))-\log(\sin(b-a\,z))\right)}{a}$$

Involving powers of products of the direct function

01.10.21.0087.01

$$\int \sqrt{\csc(c\,z)\csc(2\,c\,z)}\,dz = -\frac{2^{3/4}\left(-\cot^2(c\,z)\right)^{3/4}\sqrt{\csc(c\,z)\csc(2\,c\,z)}\tan(c\,z)}{3\,c\sqrt{\frac{\csc(c\,z)}{\sqrt{\cos(2\,c\,z)+1}}}}\sqrt{\frac{\csc(c\,z)}{\sqrt{\cos^2(c\,z)}}}\;{}_2F_1\!\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\csc^2(c\,z)\right)$$

Involving rational functions of the direct function

Involving $(a+b\csc(z))^{-n}$

01.10.21.0088.01

$$\int \frac{1}{a+b\csc(z)}\,dz = \frac{1}{a}\left(z-\frac{2\,b}{\sqrt{b^2-a^2}}\tan^{-1}\!\left(\frac{a+b\tan\left(\frac{z}{2}\right)}{\sqrt{b^2-a^2}}\right)\right)$$

01.10.21.0089.01

$$\int \frac{1}{(a+b\csc(z))^2}\,dz =$$

$$\frac{\csc(z)\,(b+a\sin(z))}{a^2\,(a+b\csc(z))^2}\left(\frac{a\cot(z)\,b^2}{b^2-a^2}-\frac{2\left(b^2-2\,a^2\right)\csc(z)\,(b+a\sin(z))\,b}{\left(b^2-a^2\right)^{3/2}}\tan^{-1}\!\left(\frac{a+b\tan\left(\frac{z}{2}\right)}{\sqrt{b^2-a^2}}\right)+z\,(a+b\csc(z))\right)$$

Involving $\left(a+b\csc^2(z)\right)^{-n}$

01.10.21.0090.01

$$\int \frac{1}{a+b\csc^2(z)}\,dz = \frac{1}{a}\left(z-\frac{\sqrt{b}}{\sqrt{a+b}}\tan^{-1}\!\left(\frac{\sqrt{a+b}\,\tan(z)}{\sqrt{b}}\right)\right)$$

01.10.21.0091.01

$$\int \frac{1}{\left(a+b\csc^2(z)\right)^2}\,dz = \frac{1}{8a^2\left(b\csc^2(z)+a\right)^2}\left((\cos(2z)a-a-2b)\csc^4(z)\left(2z(\cos(2z)a-a-2b)+\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(z)}{\sqrt{b}}\right)(-\cos(2z)a+a+2b)}{(a+b)^{3/2}}-\frac{a\,b\sin(2z)}{a+b}\right)\right)$$

Involving algebraic functions of the direct function

Involving $(a+b\csc(cz))^\beta$

01.10.21.0092.01

$$\int \csc(cz)\,(a+b\csc(cz))^\beta\,dz = -\frac{(a+b\csc(cz))^{\beta+1}\tan(cz)}{b\,c\,(\beta+1)}\sqrt{\frac{b(\csc(cz)+1)}{b-a}}\sqrt{\frac{b-b\csc(cz)}{a+b}}\;F_1\left(\beta+1;\frac{1}{2},\frac{1}{2};\beta+2;\frac{a+b\csc(cz)}{a+b},\frac{a+b\csc(cz)}{a-b}\right)$$

01.10.21.0093.01

$$\int \csc(cz)\sqrt{a+b\csc(cz)}\,dz = \left(2i(\csc(cz)+1)\sqrt{\frac{b-b\csc(cz)}{a+b}}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\csc(cz)}\right)\middle|\frac{a+b}{a-b}\right)-F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\csc(cz)}\right)\middle|\frac{a+b}{a-b}\right)\right)\tan(cz)\right)\Big/\left(\sqrt{-\frac{1}{a+b}}\;c\sqrt{\frac{b(\csc(cz)+1)}{b-a}}\right)$$

01.10.21.0094.01

$$\int \frac{\csc(cz)}{\sqrt{a+b\csc(cz)}}\,dz = -\frac{i}{\sqrt{b-a}\;c\sqrt{\frac{b(\sin(cz)+1)}{b+a\sin(cz)}}}\left(2\sqrt{\frac{b(\csc(cz)-1)}{a+b\csc(cz)}}\;F\left(i\sinh^{-1}\left(\frac{\sqrt{b-a}}{\sqrt{a+b\csc(cz)}}\right)\middle|\frac{a+b}{a-b}\right)\sec(cz)(\sin(cz)+1)\right)$$

Involving $((a+b\csc(cz))^n)^\beta$

01.10.21.0095.01

$$\int \csc(c z)\left((a+b\csc(c z))^n\right)^\beta \, dz = -\frac{1}{b c (n\beta+1)} F_1\left(n\beta+1; \frac{1}{2}, \frac{1}{2}; n\beta+2; \frac{a+b\csc(c z)}{a+b}, \frac{a+b\csc(c z)}{a-b}\right) \sqrt{\frac{b(\csc(c z)+1)}{b-a}} \sqrt{\frac{b-b\csc(c z)}{a+b}} \left((a+b\csc(c z))^n\right)^\beta \sec(c z)(b+a\sin(c z))$$

01.10.21.0096.01

$$\int \csc(c z)\sqrt{(a+b\csc(c z))^3}\, dz = \left(\sqrt{2}\cos(c z)(\csc(c z)+1)\sqrt{(a+b\csc(c z))^3}\sin^3(c z)\left(-2b(b+a\sin(c z))\cot^2(c z) - \frac{1}{\sqrt{\frac{b-b\csc(c z)}{a+b}}} \left((\csc(c z)-1)\sqrt{\frac{a+b\csc(c z)}{a+b}}\left(-8a\sqrt{\frac{b(\csc(c z)+1)}{b-a}}(b-a)E\left(\sin^{-1}\left(\sqrt{\frac{a+b\csc(c z)}{a+b}}\right)\middle|\frac{a+b}{a-b}\right) + 8ab\sqrt{\frac{b(\csc(c z)+1)}{b-a}} F\left(\sin^{-1}\left(\sqrt{\frac{a+b\csc(c z)}{a+b}}\right)\middle|\frac{a+b}{a-b}\right) + \sqrt{2}\left(3a^2+b^2\right)\sqrt{\csc(c z)+1}\, F\left(\sin^{-1}\left(\sqrt{\frac{b-b\csc(c z)}{a+b}}\right)\middle|\frac{a+b}{2b}\right)\right)\sin(c z)\right)\right)\right) \Big/ \left(3c\sqrt{\cos^2(c z)}\sqrt{\cos(2 c z)+1}(\sin(c z)+1)(b+a\sin(c z))^2\right)$$

01.10.21.0097.01

$$\int \frac{\csc(c z)}{\sqrt{(a+b\csc(c z))^3}}\, dz = -\left(2(a+b\csc(c z))^{3/2}\left(\frac{1}{\sqrt{-\frac{1}{a+b}}}\left(i(a-b)\sqrt{\frac{b(\csc(c z)+1)}{b-a}}\sqrt{\frac{b-b\csc(c z)}{a+b}} \left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\csc(c z)}\right)\middle|\frac{a+b}{a-b}\right) - F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\csc(c z)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right) - \frac{b^2\cot^2(c z)}{\sqrt{a+b\csc(c z)}}\right)\tan(c z)\right) \Big/ \left(b\left(a^2-b^2\right)c\sqrt{(a+b\csc(c z))^3}\right)$$

Involving $\left(a+b\csc^2(c z)\right)^\beta$

01.10.21.0098.01

$$\int \left(a+b\csc^2(c\,z)\right)^{\beta}\,dz = -\frac{\sqrt{\cos^2(c\,z)}\,\left(b\csc^2(c\,z)+a\right)^{\beta}\tan(c\,z)}{2\,c\,\beta-c}\left(\frac{a\sin^2(c\,z)}{b}+1\right)^{-\beta} F_1\!\left(\frac{1}{2}-\beta;\,\frac{1}{2},\,-\beta;\,\frac{3}{2}-\beta;\,\sin^2(c\,z),\,-\frac{a\sin^2(c\,z)}{b}\right)$$

01.10.21.0099.01

$$\int \sqrt{a+b\csc^2(c\,z)}\,dz = \frac{1}{c\sqrt{\cos(2\,c\,z)\,a-a-2\,b}}\left(\sqrt{2}\sqrt{b\csc^2(c\,z)+a}\left(\sqrt{a}\,\log\!\left(\sqrt{2}\sqrt{a}\cos(c\,z)+\sqrt{\cos(2\,c\,z)\,a-a-2\,b}\right)-\sqrt{-b}\,\tanh^{-1}\!\left(\frac{\sqrt{2}\sqrt{-b}\cos(c\,z)}{\sqrt{\cos(2\,c\,z)\,a-a-2\,b}}\right)\right)\sin(c\,z)\right)$$

01.10.21.0100.01

$$\int \frac{1}{\sqrt{a+b\csc^2(c\,z)}}\,dz = -\frac{\sqrt{\cos(2\,c\,z)\,a-a-2\,b}\,\csc(c\,z)\log\!\left(\sqrt{2}\sqrt{a}\cos(c\,z)+\sqrt{\cos(2\,c\,z)\,a-a-2\,b}\right)}{\sqrt{2}\sqrt{a}\,c\sqrt{b\csc^2(c\,z)+a}}$$

01.10.21.0101.01

$$\int \csc(c\,z)\left(a+b\csc^2(c\,z)\right)^{\beta}\,dz = -\frac{\sqrt{-\cot^2(c\,z)}\,\left(b\csc^2(c\,z)+a\right)^{\beta}\left(\frac{b\csc^2(c\,z)}{a}+1\right)^{-\beta}\sec(c\,z)}{c}F_1\!\left(\frac{1}{2};\,\frac{1}{2},\,-\beta;\,\frac{3}{2};\,\csc^2(c\,z),\,-\frac{b\csc^2(c\,z)}{a}\right)$$

01.10.21.0102.01

$$\int \csc(c\,z)\sqrt{a+b\csc^2(c\,z)}\,dz = -\frac{\sqrt{-\cot^2(c\,z)}\sqrt{b\csc^2(c\,z)+a}\,\tan(c\,z)}{c\sqrt{\frac{b\csc^2(c\,z)}{a}+1}}E\!\left(\sin^{-1}(\csc(c\,z))\,\middle|\,-\frac{b}{a}\right)$$

01.10.21.0103.01

$$\int \frac{\csc(c\,z)}{\sqrt{a+b\csc^2(c\,z)}}\,dz = \frac{\csc(c\,z)}{\sqrt{2}\,c\sqrt{a+b\csc^2(c\,z)}}\sqrt{\frac{-\cos(2\,c\,z)\,a+a+2\,b}{b}}\,F\!\left(c\,z\,\middle|\,-\frac{a}{b}\right)$$

Involving $\left(\left(a+b\csc^2(c\,z)\right)^n\right)^{\beta}$

01.10.21.0104.01

$$\int \left(\left(a+b\csc^2(c\,z)\right)^n\right)^{\beta}\,dz = -\frac{1}{2\,c\,n\,\beta-c}\left(F_1\!\left(\frac{1}{2}-n\,\beta;\,\frac{1}{2},\,-n\,\beta;\,\frac{3}{2}-n\,\beta;\,\sin^2(c\,z),\,-\frac{a\sin^2(c\,z)}{b}\right)\sqrt{\cos^2(c\,z)}\,\left(\left(b\csc^2(c\,z)+a\right)^n\right)^{\beta}\left(\frac{a\sin^2(c\,z)}{b}+1\right)^{-n\,\beta}\tan(c\,z)\right)$$

01.10.21.0105.01

$$\int \sqrt{\left(a+b\csc^2(c\,z)\right)^3}\,dz = \left(\sqrt{\left(b\csc^2(c\,z)+a\right)^3}\,\sin(c\,z)\left(\sqrt{-b}\left(b\cos(c\,z)\sqrt{\cos(2\,c\,z)\,a-a-2\,b}-2\sqrt{2}\,a^{3/2}\log\left(\sqrt{2}\,\sqrt{a}\,\cos(c\,z)+\sqrt{\cos(2\,c\,z)\,a-a-2\,b}\right)\sin^2(c\,z)\right)-\sqrt{2}\,b\,(3\,a+b)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{-b}\,\cos(c\,z)}{\sqrt{\cos(2\,c\,z)\,a-a-2\,b}}\right)\sin^2(c\,z)\right)\right)\Big/\left(\sqrt{-b}\,c\,(\cos(2\,c\,z)\,a-a-2\,b)^{3/2}\right)$$

01.10.21.0106.01

$$\int \frac{1}{\sqrt{\left(a+b\csc^2(c\,z)\right)^3}}\,dz = \left(\csc^2(c\,z)\left(\sqrt{2}\,(\cos(2\,c\,z)\,a-a-2\,b)^{3/2}\csc(c\,z)\log\left(\sqrt{2}\,\sqrt{a}\,\cos(c\,z)+\sqrt{\cos(2\,c\,z)\,a-a-2\,b}\right)-\frac{2\sqrt{a}\,b\,(\cos(2\,c\,z)\,a-a-2\,b)\cot(c\,z)}{a+b}\right)\right)\Big/\left(4\,a^{3/2}\,c\sqrt{\left(b\csc^2(c\,z)+a\right)^3}\right)$$

01.10.21.0107.01

$$\int \csc(c\,z)\left(\left(a+b\csc^2(c\,z)\right)^n\right)^\beta\,dz = -\frac{\sqrt{-\cot^2(c\,z)}\,\left(\left(b\csc^2(c\,z)+a\right)^n\right)^\beta\left(\frac{b\csc^2(c\,z)}{a}+1\right)^{-n\,\beta}\sec(c\,z)}{c}\,F_1\left(\frac{1}{2};\frac{1}{2},-n\,\beta;\frac{3}{2};\csc^2(c\,z),-\frac{b\csc^2(c\,z)}{a}\right)$$

01.10.21.0108.01

$$\int \csc(c\,z)\sqrt{\left(a+b\csc^2(c\,z)\right)^3}\,dz = \left(\sqrt{\left(b\csc^2(c\,z)+a\right)^3}\left(-4\sqrt{2}\,b\,(2\,a+b)\sqrt{\frac{-\cos(2\,c\,z)\,a+a+2\,b}{b}}\,E\left(c\,z\,\middle|\,-\frac{a}{b}\right)\sin^3(c\,z)+2\sqrt{2}\sqrt{\frac{-\cos(2\,c\,z)\,a+a+2\,b}{b}}\left(3\,a^2+5\,b\,a+2\,b^2\right)F\left(c\,z\,\middle|\,-\frac{a}{b}\right)\sin^3(c\,z)-\cos(c\,z)\left(6\,a^2+13\,b\,a+(2\,a+b)\cos(4\,c\,z)\,a+8\,b^2-2\left(4\,a^2+7\,b\,a+2\,b^2\right)\cos(2\,c\,z)\right)\right)\right)\Big/\left(3\,c\,(-\cos(2\,c\,z)\,a+a+2\,b)^2\right)$$

01.10.21.0109.01

$$\int \frac{\csc(c\,z)}{\sqrt{\left(a+b\csc^2(c\,z)\right)^3}}\,dz = \left((\cos(2\,c\,z)\,a-a-2\,b)\csc^3(c\,z)\left(\sqrt{2}\,\sqrt{\frac{-\cos(2\,c\,z)\,a+a+2\,b}{b}}\;b\,E\!\left(c\,z\,\middle|\,-\frac{a}{b}\right)-\sqrt{2}\,(a+b)\sqrt{\frac{-\cos(2\,c\,z)\,a+a+2\,b}{b}}\;F\!\left(c\,z\,\middle|\,-\frac{a}{b}\right)+a\sin(2\,c\,z)\right)\right)\Big/\left(4\,a\,(a+b)\,c\sqrt{\left(b\csc^2(c\,z)+a\right)^3}\right)$$

Involving functions of the direct function and a power function

Involving powers of the direct function and a power function

Involving powers of csch and power

Involving z^n and linear arguments

01.10.21.0110.01

$$\int z^n \csc^\nu(c\,z)\,dz = n!\,\csc^\nu(c\,z)\left(1-e^{2\,i\,c\,z}\right)^\nu \sum_{j=0}^{n} \frac{(-1)^j z^{n-j}}{(n-j)!\,(i\,c\,\nu)^{j+1}}\;{}_{j+2}F_{j+1}\!\left(\frac{\nu}{2},\ldots,\frac{\nu}{2},\nu;\frac{\nu}{2}+1,\ldots,\frac{\nu}{2}+1;e^{2\,i\,c\,z}\right)/;\;n\in\mathbb{N}^+$$

01.10.21.0111.01

$$\int z\csc^\nu(c\,z)\,dz = \frac{\left(1-e^{2\,i\,c\,z}\right)^\nu\csc^\nu(c\,z)}{c^2\,\nu^2}\left({}_3F_2\!\left(\frac{\nu}{2},\frac{\nu}{2},\nu;\frac{\nu}{2}+1,\frac{\nu}{2}+1;e^{2\,i\,c\,z}\right)-i\,c\,z\,\nu\;{}_2F_1\!\left(\frac{\nu}{2},\nu;\frac{\nu}{2}+1;e^{2\,i\,c\,z}\right)\right)$$

01.10.21.0112.01

$$\int z\csc^2(c\,z)\,dz = \frac{\log(\sin(c\,z))-c\,z\cot(c\,z)}{c^2}$$

01.10.21.0113.01

$$\int z\csc^3(c\,z)\,dz = -\frac{c\,z\cot(c\,z)\csc(c\,z)+\csc(c\,z)-c\,z\log\left(1-e^{i\,c\,z}\right)+c\,z\log\left(1+e^{i\,c\,z}\right)-i\,\mathrm{Li}_2\left(-e^{i\,c\,z}\right)+i\,\mathrm{Li}_2\left(e^{i\,c\,z}\right)}{2\,c^2}$$

01.10.21.0114.01

$$\int z\csc^4(c\,z)\,dz = -\frac{\csc^2(c\,z)+2\,c\,z\cot(c\,z)\left(\csc^2(c\,z)+2\right)-4\log(\sin(c\,z))}{6\,c^2}$$

01.10.21.0115.01

$$\int z\csc^5(c\,z)\,dz = -\frac{1}{24\,c^2}\left(6\,c\,z\cot(c\,z)\csc^3(c\,z)+2\csc^3(c\,z)+9\,c\,z\cot(c\,z)\csc(c\,z)+9\csc(c\,z)-9\,c\,z\log\left(1-e^{i\,c\,z}\right)+9\,c\,z\log\left(1+e^{i\,c\,z}\right)-9\,i\,\mathrm{Li}_2\left(-e^{i\,c\,z}\right)+9\,i\,\mathrm{Li}_2\left(e^{i\,c\,z}\right)\right)$$

01.10.21.0116.01

$$\int z^2\csc^2(c\,z)\,dz = -\frac{c\,z\left(i\,c\,z+c\cot(c\,z)\,z-2\log\left(1-e^{2\,i\,c\,z}\right)\right)+i\,\mathrm{Li}_2\left(e^{2\,i\,c\,z}\right)}{c^3}$$

01.10.21.0117.01

$$\int z^3 \csc^3(c z)\, dz = -\frac{1}{16 c^4} \left(-2 i c^4 z^4 + 8 c^3 \cot(c z) \csc(c z) z^3 - 8 c^3 \log\left(1 - e^{-i c z}\right) z^3 + 8 c^3 \log\left(1 + e^{i c z}\right) z^3 + 24 c^2 \csc(c z) z^2 - 24 i c^2 \operatorname{Li}_2\left(e^{-i c z}\right) z^2 - 48 c \log\left(1 - e^{i c z}\right) z + 48 c \log\left(1 + e^{i c z}\right) z - 48 c \operatorname{Li}_3\left(e^{-i c z}\right) z + 48 c \operatorname{Li}_3\left(-e^{i c z}\right) z + i \pi^4 - 24 i \left(c^2 z^2 + 2\right) \operatorname{Li}_2\left(-e^{i c z}\right) + 48 i \operatorname{Li}_2\left(e^{i c z}\right) + 48 i \operatorname{Li}_4\left(e^{-i c z}\right) + 48 i \operatorname{Li}_4\left(-e^{i c z}\right)\right)$$

Involving functions of the direct function and exponential function

Involving powers of the direct function and exponential function

Involving exp

Involving $e^{b z}$

01.10.21.0118.01

$$\int e^{b z} \csc^{\nu}(c z)\, dz = \frac{e^{b z} \left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z)}{b + i c \nu} \, {}_2F_1\left(\frac{-i b + c \nu}{2 c}, \nu; \frac{1}{2}\left(2 - \frac{i b}{c} + \nu\right); e^{2 i c z}\right)$$

01.10.21.0119.01

$$\int e^{-i c \nu z} \csc^{\nu}(c z)\, dz = \frac{i e^{-i c z \nu} \left(1 - e^{-2 i c z}\right)^{\nu} \csc^{\nu}(c z)}{2 c \nu} \, {}_2F_1\left(\nu, \nu; \nu + 1; e^{-2 i c z}\right)$$

01.10.21.0120.01

$$\int e^{i c z} \csc^2(c z)\, dz = -\frac{i}{c} \left(-\log\left(-1 + e^{i c z}\right) + \log\left(1 + e^{i c z}\right) + \frac{2 e^{i c z}}{-1 + e^{2 i c z}}\right)$$

01.10.21.0121.01

$$\int e^{2 i c z} \csc^2(c z)\, dz = -\frac{2 i \left(\frac{1}{-1 + e^{2 i c z}} - \log\left(-1 + e^{2 i c z}\right)\right)}{c}$$

01.10.21.0122.01

$$\int e^{2 i c z} \csc^4(c z)\, dz = \frac{8 i \left(1 - 3 e^{2 i c z} + 3 e^{4 i c z}\right)}{3 c \left(-1 + e^{2 i c z}\right)^3}$$

01.10.21.0123.01

$$\int e^{-2 i c z} \csc^4(c z)\, dz = \frac{8 i e^{2 i c z} \left(3 - 3 e^{2 i c z} + e^{4 i c z}\right)}{3 c \left(-1 + e^{2 i c z}\right)^3}$$

Involving functions of the direct function, exponential and a power functions

Involving powers of the direct function, exponential and a power functions

Involving exp and power

Involving $z^n e^{b z}$

01.10.21.0124.01

$$\int z^n e^{bz} \csc^\nu(cz)\,dz = n!\csc^\nu(cz)\left(1-e^{2icz}\right)^\nu e^{bz} \sum_{j=0}^n \frac{(-1)^j z^{n-j}}{(n-j)!\,(b+ic\nu)^{j+1}}\; {}_{j+2}F_{j+1}\left(\frac{c\nu-ib}{2c}, \ldots, \frac{c\nu-ib}{2c}, \nu; \frac{c\nu-ib}{2c}+1, \ldots, \frac{c\nu-ib}{2c}+1; e^{2icz}\right) /; n\in\mathbb{N} \bigwedge b \neq -ic\nu$$

01.10.21.0125.01

$$\int z^n e^{-ic\nu z} \csc^\nu(cz)\,dz = \left(1-e^{2icz}\right)^\nu e^{-ic\nu z}\csc^\nu(cz)\left(\frac{z^{n+1}}{n+1} + e^{2icz}\,\nu\, n! \sum_{j=0}^n \frac{(-1)^j (2ic)^{-j-1} z^{n-j}}{(n-j)!}\; {}_{j+3}F_{j+2}(1, \ldots, 1, \nu+1; 2, \ldots, 2; e^{2icz})\right) /; n\in\mathbb{N}$$

01.10.21.0126.01

$$\int z^n e^{-icz(2q+\nu)} \csc^\nu(cz)\,dz = n!\left(1-e^{2icz}\right)^\nu \csc^\nu(cz) \left(\frac{e^{-icz\nu}\,\Gamma(q+\nu)\, z^{n+1}}{(n+1)!\, q!\,\Gamma(\nu)} - \frac{(\nu)_{q+1} e^{-icz(\nu-2)}}{(q+1)!} \sum_{j=0}^n \frac{z^{n-j}}{(-2ic)^{j+1}(n-j)!}\; {}_{j+3}F_{j+2}(1, \ldots, 1, q+\nu+1; 2, \ldots, 2, q+2; e^{2icz}) - \sum_{j=0}^n \sum_{k=0}^{q-1} \frac{(\nu)_k\, e^{icz(2k-2q-\nu)} z^{n-j}}{(2ic(q-k))^{j+1}\, k!\,(n-j)!}\right) /; n\in\mathbb{N} \bigwedge q\in\mathbb{N}^+$$

Involving functions of the direct function and trigonometric functions

Involving powers of the direct function and trigonometric functions

Involving sin

Involving sin($b\,z$)

01.10.21.0127.01

$$\int \sin(bz)\csc^\nu(cz)\,dz = -\frac{1}{2\left(b^2-c^2\nu^2\right)} e^{-ibz}\left(1-e^{2icz}\right)^\nu \csc^\nu(cz) \left(e^{2ibz}(b-c\nu)\,{}_2F_1\left(\frac{b+c\nu}{2c}, \nu; \frac{1}{2}\left(\frac{b}{c}+\nu+2\right); e^{2icz}\right) + (b+c\nu)\,{}_2F_1\left(-\frac{b-c\nu}{2c}, \nu; \frac{1}{2}\left(-\frac{b}{c}+\nu+2\right); e^{2icz}\right)\right)$$

Involving powers of sin

Involving sin^m($b\,z$)

01.10.21.0128.01

$$\int \sin^m(b\,z)\csc^\nu(c\,z)\,dz = -\frac{i\,2^{-m}\left(1-e^{2\,i\,c\,z}\right)^\nu (1-m \bmod 2)\csc^\nu(c\,z)}{c\,\nu}\binom{m}{\frac{m}{2}}\,{}_2F_1\!\left(\frac{\nu}{2},\,\nu;\,\frac{\nu}{2}+1;\,e^{2\,i\,c\,z}\right) - i\,2^{-m}\left(1-e^{2\,i\,c\,z}\right)^\nu \csc^\nu(c\,z)\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^s\binom{m}{s}\left(\frac{e^{\frac{i\,m\,\pi}{2}-i\,b\,(m-2\,s)\,z}\,{}_2F_1\!\left(\frac{c\,\nu-b\,(m-2\,s)}{2\,c},\,\nu;\,\frac{c\,\nu-b\,(m-2\,s)}{2\,c}+1;\,e^{2\,i\,c\,z}\right)}{c\,\nu-b\,(m-2\,s)} + \frac{e^{i\,b\,(m-2\,s)\,z-\frac{i\,m\,\pi}{2}}\,{}_2F_1\!\left(\frac{b\,(m-2\,s)+c\,\nu}{2\,c},\,\nu;\,\frac{b\,(m-2\,s)+c\,\nu}{2\,c}+1;\,e^{2\,i\,c\,z}\right)}{b\,(m-2\,s)+c\,\nu}\right)\,/;\,m\in\mathbb{N}^+$$

Involving cos

Involving $\cos(b\,z)$

01.10.21.0129.01

$$\int \cos(b\,z)\csc^\nu(c\,z)\,dz = \frac{1}{2\,(b-c\,\nu)\,(b+c\,\nu)}\left(i\,e^{-i\,b\,z}\left(1-e^{2\,i\,c\,z}\right)^\nu\csc^\nu(c\,z)\left((b+c\,\nu)\,{}_2F_1\!\left(-\frac{b-c\,\nu}{2\,c},\,\nu;\,\frac{1}{2}\left(-\frac{b}{c}+\nu+2\right);\,e^{2\,i\,c\,z}\right) - e^{2\,i\,b\,z}\,(b-c\,\nu)\,{}_2F_1\!\left(\frac{b+c\,\nu}{2\,c},\,\nu;\,\frac{1}{2}\left(\frac{b}{c}+\nu+2\right);\,e^{2\,i\,c\,z}\right)\right)\right)$$

Involving powers of cos

Involving $\cos^m(b\,z)$

01.10.21.0130.01

$$\int \cos^m(b\,z)\csc^\nu(c\,z)\,dz = \frac{-i\,2^{-m}\left(1-e^{2\,i\,c\,z}\right)^\nu\,(1-m \bmod 2)\csc^\nu(c\,z)}{c\,\nu}\binom{m}{\frac{m}{2}}\,{}_2F_1\!\left(\frac{\nu}{2},\,\nu;\,\frac{\nu}{2}+1;\,e^{2\,i\,c\,z}\right) - i\,2^{-m}\left(1-e^{2\,i\,c\,z}\right)^\nu\csc^\nu(c\,z)\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{s}\left(\frac{e^{-i\,b\,(m-2\,s)\,z}}{c\,\nu-b\,(m-2\,s)}\,{}_2F_1\!\left(\frac{c\,\nu-b\,(m-2\,s)}{2\,c},\,\nu;\,\frac{c\,\nu-b\,(m-2\,s)}{2\,c}+1;\,e^{2\,i\,c\,z}\right) + \frac{e^{i\,b\,(m-2\,s)\,z}}{b\,(m-2\,s)+c\,\nu}\,{}_2F_1\!\left(\frac{b\,(m-2\,s)+c\,\nu}{2\,c},\,\nu;\,\frac{b\,(m-2\,s)+c\,\nu}{2\,c}+1;\,e^{2\,i\,c\,z}\right)\right)\,/;\,m\in\mathbb{N}^+$$

Involving functions of the direct function, trigonometric and a power functions

Involving powers of the direct function, trigonometric and a power functions

Involving sin and power

Involving $z^n\sin(a+b\,z)\csc^\nu(c\,z)$

01.10.21.0131.01

$$\int z^n \sin(a+b\,z)\csc^\nu(c\,z)\,dz = -\frac{i}{2}\left(1-e^{2icz}\right)^\nu \csc^\nu(c\,z)\,n!\left(e^{ia+ibz}\sum_{j=0}^n \frac{(-1)^j z^{n-j}(i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+c\,\nu}{2\,c},\ldots,\frac{b+c\,\nu}{2\,c},\nu;\frac{b+c\,\nu}{2\,c}+1,\ldots,\frac{b+c\,\nu}{2\,c}+1;e^{2icz}\right)-e^{-ia-ibz}\sum_{j=0}^n \frac{(-1)^j z^{n-j}(-i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+c\,\nu}{2\,c},\ldots,\frac{-b+c\,\nu}{2\,c},\nu;\frac{-b+c\,\nu}{2\,c}+1,\ldots,\frac{-b+c\,\nu}{2\,c}+1;e^{2icz}\right)\right)\,/;\,n\in\mathbb{N}\bigwedge b\neq -c\,\nu\bigwedge b\neq c\,\nu$$

01.10.21.0132.01

$$\int z^n \sin(b\,z)\csc^\nu(c\,z)\,dz = \frac{i}{2}\left(1-e^{2icz}\right)^\nu \csc^\nu(c\,z)\,n!\left(e^{-ibz}\sum_{j=0}^n \frac{(-1)^j z^{n-j}(-i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+c\,\nu}{2\,c},\ldots,\frac{-b+c\,\nu}{2\,c},\nu;\frac{-b+c\,\nu}{2\,c}+1,\ldots,\frac{-b+c\,\nu}{2\,c}+1;e^{2icz}\right)-e^{ibz}\sum_{j=0}^n \frac{(-1)^j z^{n-j}(i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+c\,\nu}{2\,c},\ldots,\frac{b+c\,\nu}{2\,c},\nu;\frac{b+c\,\nu}{2\,c}+1,\ldots,\frac{b+c\,\nu}{2\,c}+1;e^{2icz}\right)\right)\,/;\,n\in\mathbb{N}$$

01.10.21.0133.01

$$\int z^n \sin(c\,\nu\,z)\csc^\nu(c\,z)\,dz = \frac{1}{2}\,i\left(1-e^{2icz}\right)^\nu \csc^\nu(c\,z)\left(\frac{e^{-ic\nu z}z^{n+1}}{n+1}+e^{-ic(\nu-2)z}\,\nu\,n!\sum_{j=0}^n \frac{\left((-1)^j(2\,i\,c)^{-j-1}z^{n-j}\right)}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1,\ldots,1,\nu+1;2,\ldots,2;e^{2icz}\right)-n!\,e^{ic\nu z}\sum_{j=0}^n \frac{\left((-1)^j z^{n-j}\right)}{(n-j)!\,(2\,i\,c\,\nu)^{j+1}}\,{}_{j+2}F_{j+1}\left(\nu,\ldots,\nu,\nu;\nu+1,\ldots,\nu+1;e^{2icz}\right)\right)\,/;\,n\in\mathbb{N}$$

01.10.21.0134.01

$$\int z^n \sin(q\,\nu\,c\,z)\csc^\nu(c\,z)\,dz = -\frac{i}{2}\,n!\csc^\nu(c\,z)\left(1-e^{2icz}\right)^\nu\left(-\frac{e^{-ic\nu z}\,\Gamma\left(\frac{\nu(q+1)}{2}\right)z^{n+1}}{\Gamma\left(\frac{\nu(q-1)}{2}+1\right)\Gamma(\nu)\,(n+1)!}+e^{iq\nu cz}\sum_{j=0}^n \frac{(-1)^j z^{n-j}}{(n-j)!\,(i\,c\,\nu\,(q+1))^{j+1}}\,{}_{j+2}F_{j+1}\left(\frac{\nu(q+1)}{2},\ldots,\frac{\nu(q+1)}{2},\nu;\frac{\nu(q+1)}{2}+1,\ldots,\frac{\nu(q+1)}{2}+1;e^{2icz}\right)+\sum_{j=0}^n \frac{e^{icz(2-\nu)}(\nu)_{\frac{(q-1)\nu}{2}+1}\,z^{n-j}}{(n-j)!\,(-2\,i\,c)^{j+1}\left(\frac{(q-1)\nu}{2}+1\right)!}\,{}_{j+3}F_{j+2}\left(1,\ldots,1,\frac{(q+1)\,\nu}{2}+1;2,\ldots,2,\frac{(q-1)\,\nu}{2}+2;e^{2icz}\right)+\sum_{j=0}^n\sum_{k=0}^{\frac{(q-1)\nu}{2}-1} \frac{(\nu)_k\,z^{n-j}\,e^{icz(2k-q\nu)}}{(i\,c\,(-2\,k+q\,\nu-\nu))^{j+1}\,(n-j)!\,k!}\right)\,/;\,n\in\mathbb{N}\bigwedge \frac{(q-1)\,\nu}{2}\in\mathbb{N}^+$$

Involving powers of sin and power

Involving $z^n \sin^m(b z) \csc^\nu(c z)$

01.10.21.0135.01

$$\int z^n \sin^m(b z) \csc^\nu(c z)\, dz = 2^{-m} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \left(1 - e^{2 i c z}\right)^\nu n! \csc^\nu(c z) \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (i c \nu)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{\nu}{2}, \ldots, \frac{\nu}{2}, \nu; \frac{\nu}{2}+1, \ldots, \frac{\nu}{2}+1; e^{2 i c z}\right) + 2^{-m} \left(1 - e^{2 i c z}\right)^\nu n! \csc^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(e^{(b i (m-2k)) z - \frac{i m \pi}{2}} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (b i (m-2k) + i c \nu)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{c \nu + b(-2k+m)}{2c}, \ldots, \frac{c \nu + b(-2k+m)}{2c}, \nu; \frac{c \nu + b(-2k+m)}{2c} + 1, \ldots, \frac{c \nu + b(-2k+m)}{2c} + 1; e^{2 i c z}\right) + e^{\frac{i \pi m}{2} + (-i b (m-2k)) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (-i b (m-2k) + i c \nu)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{c \nu - b(-2k+m)}{2c}, \ldots, \boldsymbol{\frac{c \nu - b(-2k+m)}{2c}}, \nu; \frac{c \nu - b(-2k+m)}{2c} + 1, \ldots, \frac{c \nu - b(-2k+m)}{2c} + 1; e^{2 i c z}\right) \right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving cos and power

Involving $z^n \cos(a + b z) \csc^\nu(c z)$

01.10.21.0136.01

$$\int z^n \cos(a + b z) \csc^\nu(c z)\, dz = \frac{1}{2} \left(1 - e^{2 i c z}\right)^\nu \csc^\nu(c z)\, n! \left(e^{i a + i b z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i b + i c \nu)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + c \nu}{2c}, \ldots, \frac{b + c \nu}{2c}, \nu; \frac{b + c \nu}{2c} + 1, \ldots, \frac{b + c \nu}{2c} + 1; e^{2 i c z}\right) + e^{-i a - i b z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (-i b + i c \nu)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b + c \nu}{2c}, \ldots, \frac{-b + c \nu}{2c}, \nu; \frac{-b + c \nu}{2c} + 1, \ldots, \frac{-b + c \nu}{2c} + 1; e^{2 i c z}\right) \right) /; n \in \mathbb{N} \land b \neq c \nu \land b \neq -c \nu$$

01.10.21.0137.01

$$\int z^n \cos(b z) \csc^\nu(c z)\, dz = \frac{1}{2} \left(1 - e^{2 i c z}\right)^\nu \csc^\nu(c z)\, n! \left(e^{-i b z} \sum_{j=0}^{n} \frac{\left((-1)^j z^{n-j} (-i b + i c \nu)^{-j-1}\right)}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c \nu - b}{2c}, \ldots, \frac{c \nu - b}{2c}, \nu; \frac{c \nu - b}{2c} + 1, \ldots, \frac{c \nu - b}{2c} + 1; e^{2 i c z}\right) + e^{i b z} \sum_{j=0}^{n} \frac{\left((-1)^j z^{n-j} (i b + i c \nu)^{-j-1}\right)}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + c \nu}{2c}, \ldots, \frac{b + c \nu}{2c}, \nu; \frac{b + c \nu}{2c} + 1, \ldots, \frac{b + c \nu}{2c} + 1; e^{2 i c z}\right) \right) /; n \in \mathbb{N}$$

01.10.21.0138.01

$$\int z^n \cos(c\,\nu\,z)\csc^\nu(c\,z)\,dz = \frac{1}{2}\left(1-e^{2 i c z}\right)^\nu \csc^\nu(c\,z)\left(\frac{e^{-i c \nu z} z^{n+1}}{n+1} + e^{-i c(\nu-2) z}\,\nu\, n! \sum_{j=0}^{n} \frac{\left((-1)^j (2\,i\,c)^{-j-1} z^{n-j}\right)}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu+1; 2, \ldots, 2; e^{2 i c z}\right) + e^{i c \nu z}\, n! \sum_{j=0}^{n} \frac{\left((-1)^j z^{n-j}\right)}{(n-j)!\,(2\,i\,c\,\nu)^{j+1}}\,{}_{j+2}F_{j+1}\left(\nu, \ldots, \nu, \nu; \nu+1, \ldots, \nu+1; e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

01.10.21.0139.01

$$\int z^n \cos(q\,\nu\,c\,z)\csc^\nu(c\,z)\,dz = \frac{1}{2}\, n!\csc^\nu(c\,z)\left(1-e^{2 i c z}\right)^\nu \left(\frac{e^{-i c \nu z}\,\Gamma\left(\frac{1}{2}\nu(q+1)\right) z^{n+1}}{\Gamma\left(\frac{1}{2}(q-1)\nu+1\right)\Gamma(\nu)\,(n+1)!} - \sum_{j=0}^{n} \frac{\left(e^{i c z(2-\nu)}\,(\nu)_{\frac{1}{2}(q-1)\nu+1}\, z^{n-j}\right)}{(n-j)!\,(-2\,i\,c)^{j+1}\left(\frac{1}{2}(q-1)\nu+1\right)!}\,{}_{j+3}F_{j+2}\left(1, \ldots, 1, \frac{1}{2}(q+1)\nu+1; 2, \ldots, 2, \frac{1}{2}(q-1)\nu+2; e^{2 i c z}\right) + e^{i q \nu c z} \sum_{j=0}^{n} \frac{\left((-1)^j z^{n-j}\right)}{(n-j)!\,(i\,c\,\nu(q+1))^{j+1}}\,{}_{j+2}F_{j+1}\left(\frac{1}{2}(q+1)\nu, \ldots, \frac{1}{2}(q+1)\nu, \nu; \frac{1}{2}(q+1)\nu+1, \ldots, \frac{1}{2}(q+1)\nu+1; e^{2 i c z}\right) - \sum_{j=0}^{n} \sum_{k=0}^{\frac{1}{2}\nu(q-1)-1} \frac{\left((\nu)_k\, z^{n-j}\right) e^{i c z(2k-q\nu)}}{(i\,c(-2k+q\,\nu-\nu))^{j+1}\,(n-j)!\,k!}\right) /; n \in \mathbb{N} \bigwedge \frac{(q-1)\nu}{2} \in \mathbb{N}^+$$

Involving powers of cos and power

Involving $z^n \cos^m(b\,z)\csc^\nu(c\,z)$

01.10.21.0140.01

$$\int z^n \cos^m(b\,z)\csc^\nu(c\,z)\,dz = 2^{-m}\left(1-e^{2 i c z}\right)^\nu \binom{m}{\frac{m}{2}} n!\,(1-m \bmod 2)\left(\sum_{j=0}^{n} \frac{(-1)^j z^{n-j}(i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{\nu}{2}, \ldots, \frac{\nu}{2}, \nu; \frac{\nu}{2}+1, \ldots, \frac{\nu}{2}+1; e^{2 i c z}\right)\right)\csc^\nu(c\,z) + 2^{-m}\left(1-e^{2 i c z}\right)^\nu n!\csc^\nu(c\,z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2}\right\rfloor} \binom{m}{k}\left(e^{-i b(m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j}(i\,c\,\nu - i\,b(m-2k))^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c\,\nu-b(m-2k)}{2c}, \ldots, \frac{c\,\nu-b(m-2k)}{2c}, \nu; \frac{c\,\nu-b(m-2k)}{2c}+1, \ldots, \frac{c\,\nu-b(m-2k)}{2c}+1; e^{2 i c z}\right) + e^{i b(m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j}(b\,i(m-2k)+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b(m-2k)+c\,\nu}{2c}, \ldots, \frac{b(m-2k)+c\,\nu}{2c}, \nu; \frac{b(m-2k)+c\,\nu}{2c}+1, \ldots, \frac{b(m-2k)+c\,\nu}{2c}+1; e^{2 i c z}\right)\right) /; n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+$$

Involving functions of the direct function, trigonometric and exponential functions

Involving powers of the direct function, trigonometric and exponential functions

Involving sin and exp

Involving $e^{p z} \sin(a z) \csc^{\nu}(c z)$

01.10.21.0141.01

$$\int e^{p z} \sin(a z) \csc^{\nu}(c z)\, dz = \frac{1}{2} i \left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z) \left(\frac{i\, e^{(i a+p) z}\, {}_2F_1\left(\frac{a}{2 c} + \frac{\nu}{2} - \frac{i p}{2 c}, \nu; \frac{a}{2 c} + \frac{\nu}{2} - \frac{i p}{2 c} + 1; e^{2 i c z}\right)}{a - i p + c \nu} + \frac{e^{(-i a+p) z}\, {}_2F_1\left(-\frac{a}{2 c} + \frac{\nu}{2} - \frac{i p}{2 c}, \nu; -\frac{a}{2 c} + \frac{\nu}{2} - \frac{i p}{2 c} + 1; e^{2 i c z}\right)}{-i a + p + i c \nu}\right) /; p \neq i a - i c \nu \bigwedge p \neq -i a - i c \nu$$

01.10.21.0142.01

$$\int e^{i (a - c \nu) z} \sin(a z) \csc^{\nu}(c z)\, dz = \frac{1}{4 a c} \left(e^{-i c z \nu} \left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z) \left(2 i a c z - c\, e^{2 i a z}\, {}_2F_1\left(\frac{a}{c}, \nu; \frac{a}{c} + 1; e^{2 i c z}\right) + a\, e^{2 i c z} \nu\, {}_3F_2\left(1, 1, \nu + 1; 2, 2; e^{2 i c z}\right)\right)\right)$$

01.10.21.0143.01

$$\int e^{-i (a + c \nu) z} \sin(a z) \csc^{\nu}(c z)\, dz = -\frac{1}{4 a c} \left(e^{-i z (2 a + c \nu)} \left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z) \left(c\, {}_2F_1\left(-\frac{a}{c}, \nu; 1 - \frac{a}{c}; e^{2 i c z}\right) + a\, e^{2 i a z} \left(2 i c z + e^{2 i c z} \nu\, {}_3F_2\left(1, 1, \nu + 1; 2, 2; e^{2 i c z}\right)\right)\right)\right)$$

Involving powers of sin and exp

Involving $e^{p z} \sin^{m}(a z) \csc^{\nu}(c z)$

01.10.21.0144.01

$$\int e^{p z} \sin^{m}(a z) \csc^{\nu}(c z)\, dz = \frac{2^{-m} e^{p z} \left(1 - e^{2 i c z}\right)^{\nu} (1 - m \bmod 2) \csc^{\nu}(c z)}{p + i c \nu} \binom{m}{\frac{m}{2}}\, {}_2F_1\left(\frac{c \nu - i p}{2 c}, \nu; \frac{c \nu - i p}{2 c} + 1; e^{2 i c z}\right) + 2^{-m} \left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z) \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^{s} \binom{m}{s} \left(\frac{e^{(p + a i (m - 2 s)) z - \frac{i m \pi}{2}}\, {}_2F_1\left(\frac{-i p + a (m - 2 s) + c \nu}{2 c}, \nu; \frac{-i p + a (m - 2 s) + c \nu}{2 c} + 1; e^{2 i c z}\right)}{p + a i (m - 2 s) + i c \nu} + \frac{e^{\frac{i \pi m}{2} + (p - i a (m - 2 s)) z}\, {}_2F_1\left(\frac{-i p - a (m - 2 s) + c \nu}{2 c}, \nu; \frac{-i p - a (m - 2 s) + c \nu}{2 c} + 1; e^{2 i c z}\right)}{p - i a (m - 2 s) + i c \nu}\right) /; m \in \mathbb{N}^{+}$$

Involving cos and exp

Involving $e^{p z} \cos(a z) \csc^{\nu}(c z)$

01.10.21.0145.01

$$\int e^{p z} \cos(a z) \csc^{\nu}(c z)\, dz = \frac{1}{2}\left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z) \left(\frac{e^{(i a + p) z}\, {}_2F_1\left(\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c}, \nu; \frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c} + 1; e^{2 i c z}\right)}{i a + p + i c \nu} + \frac{e^{(-i a + p) z}\, {}_2F_1\left(-\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c}, \nu; -\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c} + 1; e^{2 i c z}\right)}{-i a + p + i c \nu}\right) /; p \neq i a - i c \nu \bigwedge p \neq -i a - i c \nu$$

01.10.21.0146.01

$$\int e^{i (a - c \nu) z} \cos(a z) \csc^{\nu}(c z)\, dz = \frac{1}{4 a c}\left(e^{-i c z \nu} \left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z) \left(2 a c z - i c e^{2 i a z}\, {}_2F_1\left(\frac{a}{c}, \nu; \frac{a}{c} + 1; e^{2 i c z}\right) - i a e^{2 i c z} \nu\, {}_3F_2\left(1, 1, \nu + 1; 2, 2; e^{2 i c z}\right)\right)\right)$$

01.10.21.0147.01

$$\int e^{-i (a + c \nu) z} \cos(a z) \csc^{\nu}(c z)\, dz = \frac{1}{4 a c}\left(e^{-i z (2 a + c \nu)} \left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z) \left(c i\, {}_2F_1\left(-\frac{a}{c}, \nu; 1 - \frac{a}{c}; e^{2 i c z}\right) + a e^{2 i a z} \left(2 c z - i e^{2 i c z} \nu\, {}_3F_2\left(1, 1, \nu + 1; 2, 2; e^{2 i c z}\right)\right)\right)\right)$$

Involving powers of cos and exp

Involving $e^{p z} \cos^{m}(a z) \csc^{\nu}(c z)$

01.10.21.0148.01

$$\int e^{p z} \cos^{m}(a z) \csc^{\nu}(c z)\, dz = \frac{2^{-m} e^{p z} \left(1 - e^{2 i c z}\right)^{\nu} (1 - m \bmod 2) \csc^{\nu}(c z)}{p + i c \nu} \binom{m}{\frac{m}{2}}\, {}_2F_1\left(\frac{c \nu - i p}{2 c}, \nu; \frac{c \nu - i p}{2 c} + 1; e^{2 i c z}\right) + 2^{-m} \left(1 - e^{2 i c z}\right)^{\nu} \csc^{\nu}(c z) \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{s} \left(\frac{e^{(p - i a (m - 2 s)) z}\, {}_2F_1\left(\frac{-i p - a (m - 2 s) + c \nu}{2 c}, \nu; \frac{-i p - a (m - 2 s) + c \nu}{2 c} + 1; e^{2 i c z}\right)}{p - i a (m - 2 s) + i c \nu} + \frac{e^{(p + a i (m - 2 s)) z}\, {}_2F_1\left(\frac{-i p + a (m - 2 s) + c \nu}{2 c}, \nu; \frac{-i p + a (m - 2 s) + c \nu}{2 c} + 1; e^{2 i c z}\right)}{p + a i (m - 2 s) + i c \nu}\right) /; m \in \mathbb{N}^{+}$$

Involving functions of the direct function, trigonometric, exponential and a power functions

Involving powers of the direct function, trigonometric, exponential and a power functions

Involving sin, exp and power

Involving $z^{n} e^{p z} \sin(a + b z) \csc^{\nu}(c z)$

01.10.21.0149.01

$$\int z^n e^{pz}\sin(a+bz)\csc^\nu(cz)\,dz = -\frac{i}{2}\left(1-e^{2icz}\right)^\nu \csc^\nu(cz)\, n!\left(e^{ia+(p+ib)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(ib+p+ic\nu)^{-j-1}}{(n-j)!}\right.$$
$$_{j+2}F_{j+1}\left(\frac{b-ip+c\nu}{2c},\ldots,\frac{b-ip+c\nu}{2c},\nu;\frac{b-ip+c\nu}{2c}+1,\ldots,\frac{b-ip+c\nu}{2c}+1;e^{2icz}\right)-$$
$$e^{-ia+(p-ib)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(-ib+p+ic\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b-ip+c\nu}{2c},\ldots,\frac{-b-ip+c\nu}{2c},\nu;\right.$$
$$\left.\left.\frac{-b-ip+c\nu}{2c}+1,\ldots,\frac{-b-ip+c\nu}{2c}+1;e^{2icz}\right)\right)\,/;\,n\in\mathbb{N}\bigwedge p+ib\neq -ic\nu\bigwedge p-ib\neq -ic\nu$$

01.10.21.0150.01

$$\int z^n e^{pz}\sin(bz)\csc^\nu(cz)\,dz = \frac{i}{2}\left(1-e^{2icz}\right)^\nu \csc^\nu(cz)\, n!\left(e^{(-ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(-ib+p+ic\nu)^{-j-1}}{(n-j)!}\right.$$
$$_{j+2}F_{j+1}\left(\frac{-b-ip+c\nu}{2c},\ldots,\frac{-b-ip+c\nu}{2c},\nu;\frac{-b-ip+c\nu}{2c}+1,\ldots,\frac{-b-ip+c\nu}{2c}+1;e^{2icz}\right)-$$
$$e^{(ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(ib+p+ic\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b-ip+c\nu}{2c},\ldots,\frac{b-ip+c\nu}{2c},\nu;\frac{b-ip+c\nu}{2c}+1,\right.$$
$$\left.\left.\ldots,\frac{b-ip+c\nu}{2c}+1;e^{2icz}\right)\right)\,/;\,n\in\mathbb{N}\bigwedge p+ib\neq -ic\nu\bigwedge p-ib\neq -ic\nu$$

01.10.21.0151.01

$$\int z^n e^{i(b-c\nu)z}\sin(bz)\csc^\nu(cz)\,dz =$$
$$\frac{i}{2}\left(1-e^{2icz}\right)^\nu\csc^\nu(cz)\left(\frac{e^{-ic\nu z}z^{n+1}}{n+1}+e^{ic(2-\nu)z}\,\nu\, n!\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(2ic)^{-j-1}}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1,\ldots,1,\nu+1;2,\ldots,2;e^{2icz}\right)-\right.$$
$$\left.e^{i(2b-c\nu)z}\,n!\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(2ib)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b}{c},\ldots,\frac{b}{c},\nu;\frac{b}{c}+1,\ldots,\frac{b}{c}+1;e^{2icz}\right)\right)\,/;\,n\in\mathbb{N}$$

01.10.21.0152.01

$$\int z^n e^{-i(b+c\nu)z}\sin(bz)\csc^\nu(cz)\,dz =$$
$$-\frac{i}{2}\left(1-e^{2icz}\right)^\nu\csc^\nu(cz)\left(\frac{e^{-ic\nu z}z^{n+1}}{n+1}+e^{ic(2-\nu)z}\,\nu\, n!\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(2ic)^{-j-1}}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1,\ldots,1,\nu+1;2,\ldots,2;e^{2icz}\right)+\right.$$
$$\left.e^{-i(2b+c\nu)z}\,n!\sum_{j=0}^{n}\frac{z^{n-j}(2ib)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{b}{c},\ldots,-\frac{b}{c},\nu;1-\frac{b}{c},\ldots,1-\frac{b}{c};e^{2icz}\right)\right)\,/;\,n\in\mathbb{N}$$

Involving powers of sin, exp and power

Involving $z^n e^{pz}\sin^m(bz)\csc^\nu(cz)$

01.10.21.0153.01

$$\int z^n e^{pz}\sin^m(bz)\csc^\nu(cz)\,dz = 2^{-m}\binom{m}{\frac{m}{2}}(1-m \bmod 2)\left(1-e^{2icz}\right)^\nu n!\csc^\nu(cz)\,e^{pz}$$
$$\sum_{j=0}^{n}\frac{1}{(n-j)!}(-1)^j z^{n-j}(p+icv)^{-j-1}\,{}_{j+2}F_{j+1}\left(\frac{c\nu-ip}{2c},\ldots,\frac{c\nu-ip}{2c},\nu;\frac{c\nu-ip}{2c}+1,\ldots,\frac{c\nu-ip}{2c}+1;e^{2icz}\right)+$$
$$2^{-m}\left(1-e^{2icz}\right)^\nu n!\csc^\nu(cz)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}$$
$$\left(e^{(bi(m-2k)+p)z-\frac{im\pi}{2}}\sum_{j=0}^{n}\frac{1}{(n-j)!}(-1)^j z^{n-j}(bi(m-2k)+p+ic\nu)^{-j-1}\,{}_{j+2}F_{j+1}\left(\frac{c\nu-ip+b(-2k+m)}{2c},\ldots,\right.\right.$$
$$\left.\frac{c\nu-ip+b(-2k+m)}{2c},\nu;\frac{c\nu-ip+b(-2k+m)}{2c}+1,\ldots,\frac{c\nu-ip+b(-2k+m)}{2c}+1;e^{2icz}\right)+$$
$$e^{\frac{i\pi m}{2}+(-ib(m-2k)+p)z}\sum_{j=0}^{n}\frac{1}{(n-j)!}(-1)^j z^{n-j}(-ib(m-2k)+p+ic\nu)^{-j-1}$$
$${}_{j+2}F_{j+1}\left(\frac{c\nu-ip-b(-2k+m)}{2c},\ldots,\frac{c\nu-ip-b(-2k+m)}{2c},\nu;\right.$$
$$\left.\left.\frac{c\nu-ip-b(-2k+m)}{2c}+1,\ldots,\frac{c\nu-ip-b(-2k+m)}{2c}+1;e^{2icz}\right)\right)/;\ n\in\mathbb{N}\land m\in\mathbb{N}^+$$

Involving cos, exp and power

Involving $z^n e^{pz}\cos(a+bz)\csc^\nu(cz)$

01.10.21.0154.01

$$\int z^n e^{pz}\cos(a+bz)\csc^\nu(cz)\,dz = \frac{1}{2}\left(1-e^{2icz}\right)^\nu\csc^\nu(cz)\,n!\left(e^{ia+(p+ib)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(ib+p+ic\nu)^{-j-1}}{(n-j)!}\right.$$
$${}_{j+2}F_{j+1}\left(\frac{b-ip+c\nu}{2c},\ldots,\frac{b-ip+c\nu}{2c},\nu;\frac{b-ip+c\nu}{2c}+1,\ldots,\frac{b-ip+c\nu}{2c}+1;e^{2icz}\right)+$$
$$e^{-ia+(p-ib)z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(-ib+p+ic\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b-ip+c\nu}{2c},\ldots,\frac{-b-ip+c\nu}{2c},\nu;\right.$$
$$\left.\left.\frac{-b-ip+c\nu}{2c}+1,\ldots,\frac{-b-ip+c\nu}{2c}+1;e^{2icz}\right)\right)/;\ n\in\mathbb{N}\land p+ib\neq -ic\nu\land p-ib\neq -ic\nu$$

01.10.21.0155.01

$$\int z^n e^{p z} \cos(b z) \csc^\nu(c z) \, dz = \frac{1}{2} \left(1 - e^{2 i c z}\right)^\nu \csc^\nu(c z) \, n! \left(e^{(-i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (-i b + p + i c \nu)^{-j-1}}{(n-j)!} \right.$$
$$ {}_{j+2}F_{j+1}\left(\frac{-b - i p + c \nu}{2 c}, \ldots, \frac{-b - i p + c \nu}{2 c}, \nu; \frac{-b - i p + c \nu}{2 c} + 1, \ldots, \frac{-b - i p + c \nu}{2 c} + 1; e^{2 i c z} \right) +$$
$$ e^{(i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i b + p + i c \nu)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b - i p + c \nu}{2 c}, \ldots, \frac{b - i p + c \nu}{2 c}, \nu; \frac{b - i p + c \nu}{2 c} + 1, \right.$$
$$\left. \left. \ldots, \frac{b - i p + c \nu}{2 c} + 1; e^{2 i c z} \right) \right) /; n \in \mathbb{N} \land p + i b \neq -i c \nu \land p - i b \neq -i c \nu$$

01.10.21.0156.01

$$\int z^n e^{i (b - c \nu) z} \cos(b z) \csc^\nu(c z) \, dz =$$
$$\frac{1}{2} \left(1 - e^{2 i c z}\right)^\nu \csc^\nu(c z) \left(\frac{e^{-i c \nu z} z^{n+1}}{n+1} + e^{i c (2 - \nu) z} \nu \, n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; e^{2 i c z}\right) + \right.$$
$$\left. e^{i (2 b - c \nu) z} \, n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i b)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b}{c}, \ldots, \frac{b}{c}, \nu; \frac{b}{c} + 1, \ldots, \frac{b}{c} + 1; e^{2 i c z} \right) \right) /; n \in \mathbb{N}$$

01.10.21.0157.01

$$\int z^n e^{-i (b + c \nu) z} \cos(b z) \csc^\nu(c z) \, dz =$$
$$\frac{1}{2} \left(1 - e^{2 i c z}\right)^\nu \csc^\nu(c z) \left(\frac{e^{-i c \nu z} z^{n+1}}{n+1} + e^{i c (2 - \nu) z} \nu \, n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; e^{2 i c z}\right) - \right.$$
$$\left. n! \, e^{-i (2 b + c \nu) z} \sum_{j=0}^{n} \frac{z^{n-j} (2 i b)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(-\frac{b}{c}, \ldots, -\frac{b}{c}, \nu; 1 - \frac{b}{c}, \ldots, 1 - \frac{b}{c}; e^{2 i c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of cos, exp and power

Involving $z^n e^{p z} \cos^m(b z) \csc^\nu(c z)$

01.10.21.0158.01

$$\int z^n e^{p z} \cos^m(b z) \csc^\nu(c z)\, dz = 2^{-m} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \left(1 - e^{2 i c z}\right)^\nu \csc^\nu(c z)\, n!\, e^{p z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (p + i c \nu)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{c \nu - i p}{2 c}, \ldots, \frac{c \nu - i p}{2 c}, \nu; \frac{c \nu - i p}{2 c} + 1, \ldots, \frac{c \nu - i p}{2 c} + 1; e^{2 i c z}\right) + 2^{-m} \left(1 - e^{2 i c z}\right)^\nu n!\, \csc^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(e^{(b i (m - 2 k) + p) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (b i (m - 2 k) + p + i c \nu)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{c \nu - i p + b(-2 k + m)}{2 c}, \ldots, \frac{c \nu - i p + b(-2 k + m)}{2 c}, \nu; \frac{c \nu - i p + b(-2 k + m)}{2 c} + 1, \ldots, \frac{c \nu - i p + b(-2 k + m)}{2 c} + 1; e^{2 i c z}\right) + e^{(-i b (m - 2 k) + p) z} \sum_{j=0}^{n} \frac{1}{(n-j)!} (-1)^j z^{n-j} (-i b (m - 2 k) + p + i c \nu)^{-j-1} {}_{j+2}F_{j+1}\left(\frac{c \nu - i p - b(-2 k + m)}{2 c}, \ldots, \frac{c \nu - i p - b(-2 k + m)}{2 c}, \nu; \frac{c \nu - i p - b(-2 k + m)}{2 c} + 1, \ldots, \frac{c \nu - i p - b(-2 k + m)}{2 c} + 1; e^{2 i c z}\right) \right) /;\ n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Definite integration

For the direct function itself

01.10.21.0163.01

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc(t)\, dt = \frac{1}{2} \left(\log\left(\sqrt{2} + 2\right) - \log\left(2 - \sqrt{2}\right)\right)$$

01.10.21.0164.01

$$\int_0^{\frac{\pi}{2}} t \csc(t)\, dt = 2 C$$

01.10.21.0165.01

$$\int_0^{\frac{\pi}{2}} t^2 \csc(t)\, dt = 2 C \pi - \frac{7 \zeta(3)}{2}$$

Summation

Finite summation

01.10.23.0002.01

$$\sum_{k=1}^{n-1} \csc^2\left(\frac{k \pi}{n}\right) = \frac{1}{3} \left(n^2 - 1\right) /;\ n \in \mathbb{N}^+$$

01.10.23.0001.01

$$\sum_{k=0}^{n-1} \csc^2\left(\frac{\pi (2 k + 1)}{2 n}\right) = n^2 /;\ n \in \mathbb{N}$$

01.10.23.0003.01

$$\sum_{k=1}^{n} \csc^2\left(\frac{k\pi}{2n+1}\right) == \frac{2}{3} n (n+1) /; n \in \mathbb{N}$$

01.10.23.0004.01

$$\sum_{k=0}^{n-1} \csc^2\left(\frac{\pi k}{n} + z\right) == n^2 \csc^2(n z) /; n \in \mathbb{N}^+$$

01.10.23.0005.01

$$\sum_{k=0}^{n} \csc\left(\frac{z}{2^k}\right) == \cot\left(\frac{z}{2^{n+1}}\right) - \cot(z) /; n \in \mathbb{N}$$

01.10.23.0006.01

$$\sum_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \csc^2\left(\frac{k\pi}{n}\right) == \frac{1}{12} \left(2 n^2 - 3 (-1)^n - 5\right) /; n \in \mathbb{N}^+$$

01.10.23.0007.01

$$\sum_{k=0}^{\left\lfloor \frac{n-2}{2} \right\rfloor} \csc^2\left(\frac{(2k+1)\pi}{2n}\right) == \frac{1}{4} \left(2 n^2 + (-1)^n - 1\right) /; n \in \mathbb{N}$$

Infinite summation

01.10.23.0008.01

$$\sum_{k=1}^{\infty} \frac{\csc\left(k \pi \sqrt{2}\right)}{k^3} == -\frac{13 \pi^3}{360 \sqrt{2}}$$

Products

Finite products

01.10.24.0001.01

$$\prod_{k=1}^{n-1} \csc\left(\frac{k\pi}{n}\right) == \frac{2^{n-1}}{n} /; n \in \mathbb{N}^+$$

01.10.24.0002.01

$$\prod_{k=0}^{n-1} \csc\left(\frac{\pi k}{n} + z\right) == 2^{n-1} \csc(n z) /; n \in \mathbb{N}^+$$

Representations through more general functions

Through hypergeometric functions

01.10.26.0029.01

$$\csc(z) = \frac{2}{z}\, {}_3F_2\left(1, -\frac{z}{\pi}, \frac{z}{\pi}; 1 - \frac{z}{\pi}, \frac{z}{\pi} + 1; -1\right) - \frac{1}{z}$$

Brychkov Yu.A. (2005)

01.10.26.0001.01

$$\csc(z) = \frac{1}{z\, {}_0F_1\left(;\frac{3}{2};-\frac{z^2}{4}\right)}$$

Through Meijer *G*

Classical cases for the direct function itself

01.10.26.0002.01

$$\csc(z) = \frac{\sqrt{z^2}}{z\sqrt{\pi}\, G_{0,2}^{1,0}\left(\frac{z^2}{4} \,\middle|\, \frac{1}{2}, 0\right)}$$

Generalized cases for the direct function itself

01.10.26.0003.01

$$\csc(z) = \frac{1}{\sqrt{\pi}\, G_{0,2}^{1,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \frac{1}{2}, 0\right)}$$

Through other functions

Involving Bessel functions

01.10.26.0004.01

$$\csc(z) = \sqrt{\frac{2}{\pi}}\, \frac{1}{\sqrt{z}\, J_{\frac{1}{2}}(z)}$$

01.10.26.0005.01

$$\csc(z) = \sqrt{\frac{2}{\pi}}\, \frac{i}{\sqrt{i z}\, I_{\frac{1}{2}}(i z)}$$

01.10.26.0006.01

$$\csc(z) = \sqrt{\frac{2}{\pi}}\, \frac{1}{\sqrt{z}\, Y_{-\frac{1}{2}}(z)}$$

Involving Jacobi functions

01.10.26.0007.01

$$\csc(z) = \frac{1}{\operatorname{cd}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)}$$

01.10.26.0008.01

$$\csc(z) = \frac{1}{\operatorname{cn}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)}$$

01.10.26.0009.01

$$\csc(z) = \operatorname{cn}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)$$

01.10.26.0010.01

$$\csc(z) == i \operatorname{cs}(i z \mid 1)$$

01.10.26.0011.01

$$\csc(z) == \operatorname{dc}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)$$

01.10.26.0012.01

$$\csc(z) == \operatorname{dn}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)$$

01.10.26.0013.01

$$\csc(z) == \operatorname{ds}(z \mid 0)$$

01.10.26.0014.01

$$\csc(z) == i \operatorname{ds}(i z \mid 1)$$

01.10.26.0015.01

$$\csc(z) == \operatorname{nc}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)$$

01.10.26.0016.01

$$\csc(z) == \frac{1}{\operatorname{nc}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)}$$

01.10.26.0017.01

$$\csc(z) == \frac{1}{\operatorname{nd}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)}$$

01.10.26.0018.01

$$\csc(z) == \operatorname{ns}(z \mid 0)$$

01.10.26.0019.01

$$\csc(z) == \frac{i}{\operatorname{sc}(i z \mid 1)}$$

01.10.26.0020.01

$$\csc(z) == \frac{i}{\operatorname{sd}(i z \mid 1)}$$

01.10.26.0021.01

$$\csc(z) == \frac{1}{\operatorname{sd}(z \mid 0)}$$

01.10.26.0022.01

$$\csc(z) == \frac{1}{\operatorname{sn}(z \mid 0)}$$

Involving Mathieu functions

01.10.26.0023.01

$$\csc\left(\sqrt{a}\, z\right) == \frac{1}{\operatorname{Se}(a, 0, z)}$$

01.10.26.0024.01

$$\csc\left(\sqrt{a}\, z\right) = -\frac{\sqrt{a}}{\mathrm{Ce}_z(a, 0, z)}$$

Involving some hypergeometric-type functions

01.10.26.0025.01

$$\csc(\pi z) = \frac{\Gamma(1-z)\,\Gamma(z)}{\pi}$$

01.10.26.0026.01

$$\csc(z) = \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{z}\, \boldsymbol{H}_{-\frac{1}{2}}(z)}$$

01.10.26.0027.01

$$\csc(z) = \sqrt{\frac{2}{\pi}}\, \frac{i}{\sqrt{i z}\, \boldsymbol{L}_{-\frac{1}{2}}(i z)}$$

01.10.26.0028.01

$$\csc(n z) = \frac{\csc(z)}{U_{n-1}(\cos(z))}$$

Representations through equivalent functions

With inverse function

01.10.27.0001.01

$$\csc\left(\csc^{-1}(z)\right) = z$$

01.10.27.0002.02

$$\csc^{-1}(\csc(z)) = z \;/;\; -\frac{\pi}{2} < \mathrm{Re}(z) < \frac{\pi}{2} \bigvee \left(\mathrm{Re}(z) = -\frac{\pi}{2} \bigwedge \mathrm{Im}(z) \geq 0\right) \bigvee \left(\mathrm{Re}(z) = \frac{\pi}{2} \bigwedge \mathrm{Im}(z) \leq 0\right)$$

01.10.27.0072.01

$$\csc^{-1}(\csc(z)) = -z - \pi \;/;\; -\frac{3\pi}{2} < \mathrm{Re}(z) < -\frac{\pi}{2} \bigvee \mathrm{Re}(z) = -\frac{3\pi}{2} \bigwedge \mathrm{Im}(z) \geq 0 \bigvee \mathrm{Re}(z) = -\frac{\pi}{2} \bigwedge \mathrm{Im}(z) \leq 0$$

01.10.27.0073.01

$$\csc^{-1}(\csc(z)) = \pi - z \;/;\; \frac{\pi}{2} < \mathrm{Re}(z) < \frac{3\pi}{2} \bigvee \mathrm{Re}(z) = \frac{\pi}{2} \bigwedge \mathrm{Im}(z) \geq 0 \bigvee \mathrm{Re}(z) = \frac{3\pi}{2} \bigwedge \mathrm{Im}(z) \leq 0$$

01.10.27.0074.01

$$\csc^{-1}(\csc(z)) = (-1)^k (z - \pi k) \;/;$$
$$\left(k\pi - \frac{\pi}{2} < \mathrm{Re}(z) < \pi k + \frac{\pi}{2} \bigvee \mathrm{Re}(z) = k\pi - \frac{\pi}{2} \bigwedge \mathrm{Im}(z) \geq 0 \bigvee \mathrm{Re}(z) = \pi k + \frac{\pi}{2} \bigwedge \mathrm{Im}(z) \leq 0\right) \bigwedge k \in \mathbb{Z}$$

01.10.27.0003.01

$$\csc^{-1}(\csc(z)) =$$
$$(-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor} \left(\left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(-\mathrm{Im}(z)) - 1\right) \left(z - \pi \left\lfloor \frac{\mathrm{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \frac{\pi}{2} \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(-\mathrm{Im}(z))\right)$$

01.10.27.0075.01

$$\csc^{-1}(\csc(z)) = \begin{cases} (-1)^{\left\lfloor \frac{2\,\mathrm{Re}(z)+\pi}{2\pi} \right\rfloor} \left(\pi \left\lfloor \frac{2\,\mathrm{Re}(z)-\pi}{2\pi} \right\rfloor - z\right) & \frac{2\,\mathrm{Re}(z)+\pi}{2\pi} \in \mathbb{Z} \bigwedge \mathrm{Im}(z) \leq 0 \\ (-1)^{\left\lfloor \frac{2\,\mathrm{Re}(z)+\pi}{2\pi} \right\rfloor} \left(z - \pi \left\lfloor \frac{2\,\mathrm{Re}(z)+\pi}{2\pi} \right\rfloor\right) & \mathrm{True} \end{cases}$$

With related functions

Involving exp

01.10.27.0004.01

$$\csc(z) = \frac{2\,i}{e^{i z} - e^{-i z}}$$

01.10.27.0005.01

$$\csc(z) = \frac{2\,i\,e^{i z}}{e^{2 i z} - 1}$$

Involving sin

01.10.27.0006.01

$$\csc(z) = \frac{1}{\sin(z)}$$

Involving cos

01.10.27.0007.01

$$\csc(z) = \frac{1}{\cos\left(\frac{\pi}{2} - z\right)}$$

01.10.27.0008.01

$$\csc(z) = -\frac{1}{\cos\left(\frac{\pi}{2} + z\right)}$$

01.10.27.0009.01

$$\csc(z) = \frac{\sqrt{2}}{\sqrt{1 - \cos(2 z)}} \;/;\; 0 < \mathrm{Re}(z) < \pi$$

01.10.27.0010.01

$$\csc(z) = \frac{\sqrt{2}}{\sqrt{1 - \cos(2 z)}} (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\mathrm{Im}(z))\right)$$

01.10.27.0011.01

$$\csc(z) = \frac{\sqrt{z^2}}{z} \frac{1}{\sqrt{1 - \cos^2(z)}} \;/;\; |\mathrm{Re}(z)| < \pi$$

01.10.27.0012.01

$$\csc(z) = \frac{1}{\sqrt{1 - \cos^2(z)}} \;/;\; 0 < \mathrm{Re}(z) < \pi$$

01.10.27.0013.01

$$\csc(z) == \frac{1}{\sqrt{1-\cos^2(z)}} (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \left(1 - \left((-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor} + 1\right) \theta(-\operatorname{Im}(z))\right)$$

01.10.27.0014.01

$$\csc^2(z) == \frac{1}{1-\cos^2(z)}$$

01.10.27.0015.01

$$\csc\left(\frac{\pi}{2} + z\right) == \frac{1}{\cos(z)}$$

01.10.27.0016.01

$$\csc\left(\frac{\pi}{2} - z\right) == \frac{1}{\cos(z)}$$

Involving tan

01.10.27.0017.01

$$\csc(z) == \frac{\tan^2\left(\frac{z}{2}\right) + 1}{2\tan\left(\frac{z}{2}\right)}$$

01.10.27.0018.01

$$\csc(z) == \frac{\sqrt{\tan^2(z) + 1}}{\tan(z)} \;/;\; |\operatorname{Re}(z)| < \frac{\pi}{2}$$

01.10.27.0019.01

$$\csc(z) == \frac{\sqrt{\tan^2(z) + 1}}{\tan(z)} (-1)^{\left\lfloor \frac{1}{2} - \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\operatorname{Im}(z))\right)$$

01.10.27.0020.01

$$\csc^2(z) == \frac{\tan^2(z) + 1}{\tan^2(z)}$$

Involving cot

01.10.27.0021.01

$$\csc(z) == \frac{\cot^2\left(\frac{z}{2}\right) + 1}{2\cot\left(\frac{z}{2}\right)}$$

01.10.27.0022.01

$$\csc(z) == z\sqrt{\frac{1}{z^2}} \sqrt{\cot^2(z) + 1} \;/;\; |\operatorname{Re}(z)| < \pi$$

01.10.27.0023.01

$$\csc(z) == \sqrt{1 + \cot^2(z)} \;/;\; 0 < \operatorname{Re}(z) < \pi$$

01.10.27.0024.01

$$\csc(z) == \sqrt{\cot^2(z) + 1} \,(-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \left(1 - \left((-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor} + 1\right) \theta(\operatorname{Im}(z))\right)$$

01.10.27.0025.01

$$\csc^2(z) = 1 + \cot^2(z)$$

Involving sec

01.10.27.0026.01

$$\csc(z) = \sec\left(\frac{\pi}{2} - z\right)$$

01.10.27.0027.01

$$\csc(z) = -\sec\left(\frac{\pi}{2} + z\right)$$

01.10.27.0028.01

$$\csc(z) = \frac{\sqrt{z^2}}{z} \frac{\sec(z)}{\sqrt{\sec^2(z) - 1}} \;/;\; |\mathrm{Re}(z)| < \frac{\pi}{2}$$

01.10.27.0029.01

$$\csc(z) = \frac{\sec(z)}{\sqrt{\sec^2(z) - 1}} \;/;\; 0 < \mathrm{Re}(z) < \frac{\pi}{2}$$

01.10.27.0030.01

$$\csc(z) = \frac{\sec(z)}{\sqrt{\sec^2(z) - 1}} (-1)^{\left\lfloor \frac{2\,\mathrm{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\mathrm{Im}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(\mathrm{Im}(z))\right)$$

01.10.27.0031.01

$$\csc^2(z) = \frac{\sec^2(z)}{\sec^2(z) - 1}$$

01.10.27.0032.01

$$\csc^2(z) + \sec^2(z) = 4 \csc^2(2 z)$$

01.10.27.0033.01

$$\csc^2(z) - \sec^2(z) = \frac{\csc^2(z) \left(\csc^2(z) - 2\right)}{\csc^2(z) - 1}$$

01.10.27.0034.01

$$\csc(z) + i \sec(z) = 2 e^{i z} \csc(2 z)$$

01.10.27.0035.01

$$\csc(z) - i \sec(z) = 2 e^{-i z} \csc(2 z)$$

01.10.27.0036.01

$$\csc(z) + \sec(z) = \frac{2 \sqrt{2} \csc(2 z)}{\csc\left(z + \frac{\pi}{4}\right)}$$

01.10.27.0037.01

$$\csc(z) - \sec(z) = -\frac{2 \sqrt{2} \csc(2 z)}{\csc\left(z - \frac{\pi}{4}\right)}$$

01.10.27.0038.01

$$a\csc(z)+b\sec(z)=2\sqrt{\frac{b^2}{a^2}+1}\;a\cos\left(z-\tan^{-1}\left(\frac{b}{a}\right)\right)\csc(2z)$$

01.10.27.0039.01

$$\csc\left(\frac{\pi}{2}+z\right)=\sec(z)$$

01.10.27.0040.01

$$\csc\left(\frac{\pi}{2}-z\right)=\sec(z)$$

Involving sinh

01.10.27.0041.01

$$\csc(z)=\frac{i}{\sinh(i\,z)}$$

01.10.27.0042.01

$$\csc(i\,z)=-\frac{i}{\sinh(z)}$$

Involving cosh

01.10.27.0043.01

$$\csc(z)=\frac{1}{\cosh\left(\frac{\pi i}{2}-i\,z\right)}$$

01.10.27.0044.01

$$\csc(z)=-\frac{1}{\cosh\left(\frac{\pi i}{2}+i\,z\right)}$$

01.10.27.0045.01

$$\csc(z)=\frac{\sqrt{z^2}}{z}\,\frac{\sqrt{2}}{\sqrt{1-\cosh(2\,i\,z)}}\;/;\;|\mathrm{Re}(z)|<\pi$$

01.10.27.0046.01

$$\csc(z)=\frac{\sqrt{2}}{\sqrt{1-\cosh(2\,i\,z)}}\;/;\;0<\mathrm{Re}(z)<\pi$$

01.10.27.0047.01

$$\csc(z)=\frac{\sqrt{2}}{\sqrt{1-\cosh(2\,i\,z)}}\,(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rfloor}\left(1-\left(1+(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rfloor+\left\lfloor-\frac{\mathrm{Re}(z)}{\pi}\right\rfloor}\right)\theta(-\mathrm{Im}(z))\right)$$

01.10.27.0048.01

$$\csc(z)=\frac{\sqrt{z^2}}{z}\,\frac{1}{\sqrt{1-\cosh^2(i\,z)}}\;/;\;|\mathrm{Re}(z)|<\pi$$

01.10.27.0049.01

$$\csc(z)=-\frac{1}{\sqrt{1-\cosh^2(i\,z)}}\,(-1)^{\left\lfloor-\frac{\mathrm{Re}(z)}{\pi}\right\rfloor}\left(1-\left(1+(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rfloor+\left\lfloor-\frac{\mathrm{Re}(z)}{\pi}\right\rfloor}\right)\theta(\mathrm{Im}(z))\right)$$

01.10.27.0050.01

$$\csc(z) = \frac{\sqrt{-z^2}}{z} \frac{1}{\sqrt{\cosh^2(i z) - 1}} \,/;\, |\mathrm{Re}(z)| < \frac{\pi}{2}$$

01.10.27.0051.01

$$\csc(z) = \frac{\sqrt{-z^2}}{z} \frac{1}{\sqrt{\cosh^2(i z) - 1}} (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(-\mathrm{Im}(z))\right) /;\, \frac{2 z - \pi}{2 \pi} \notin \mathbb{Z}$$

01.10.27.0052.01

$$\csc^2(z) = \frac{2}{1 - \cosh(2 i z)}$$

01.10.27.0053.01

$$\csc^2(z) = \frac{1}{1 - \cosh^2(i z)}$$

Involving tanh

01.10.27.0054.01

$$\csc(z) = \frac{i \left(1 - \tanh^2\left(\frac{z i}{2}\right)\right)}{2 \tanh\left(\frac{z i}{2}\right)}$$

01.10.27.0055.01

$$\csc(z) = \frac{i \sqrt{1 - \tanh^2(i z)}}{\tanh(i z)} \,/;\, |\mathrm{Re}(z)| < \frac{\pi}{2}$$

01.10.27.0056.01

$$\csc(z) = -\frac{i \sqrt{1 - \tanh^2(i z)}}{\tanh(i z)} (-1)^{\left\lfloor -\frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\mathrm{Im}(z))\right)$$

01.10.27.0057.01

$$\csc^2(z) = \frac{\tanh^2(i z) - 1}{\tanh^2(i z)}$$

Involving coth

01.10.27.0058.01

$$\csc(z) = \frac{i \left(\coth^2\left(\frac{i z}{2}\right) + 1\right)}{2 \coth\left(\frac{i z}{2}\right)}$$

01.10.27.0059.01

$$\csc(z) = \sqrt{1 - \coth^2(i z)} \,/;\, 0 < \mathrm{Re}(z) < \pi$$

01.10.27.0060.01

$$\csc(z) = \sqrt{1 - \coth^2(i z)} \, (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(\mathrm{Im}(z))\right)$$

01.10.27.0061.01

$$\csc^2(z) = 1 - \coth^2(i z)$$

Involving csch

01.10.27.0062.01

$$\csc(z) = i\,\text{csch}(i\,z)$$

01.10.27.0063.01

$$\csc(i\,z) = -i\,\text{csch}(z)$$

Involving sech

01.10.27.0064.01

$$\csc(z) = \text{sech}\left(\frac{\pi\,i}{2} - i\,z\right)$$

01.10.27.0065.01

$$\csc(z) = -\text{sech}\left(\frac{\pi\,i}{2} + i\,z\right)$$

01.10.27.0066.01

$$\csc(z) = \frac{\sqrt{z^2}}{z} \frac{\text{sech}(i\,z)}{\sqrt{\text{sech}^2(i\,z) - 1}} \;/;\; |\text{Re}(z)| < \frac{\pi}{2}$$

01.10.27.0067.01

$$\csc(z) = \frac{\text{sech}(i\,z)}{\sqrt{\text{sech}^2(i\,z) - 1}} (-1)^{\left\lfloor \frac{2\,\text{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\text{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\text{Im}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\text{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(\text{Im}(z))\right)$$

01.10.27.0068.01

$$\csc(z) = -z \sqrt{-\frac{1}{z^2}} \frac{\text{sech}(i\,z)}{\sqrt{1 - \text{sech}^2(i\,z)}} \;/;\; \text{Im}(z) \neq 0$$

01.10.27.0069.01

$$\csc(z) = -\frac{i\,\text{sech}(i\,z)}{\sqrt{1 - \text{sech}^2(i\,z)}} \;/;\; \text{Im}(z) > 0$$

01.10.27.0070.01

$$\csc^2(z) = \frac{\text{sech}^2(i\,z)}{\text{sech}^2(i\,z) - 1}$$

Involving trigonometric and hyperbolic functions

01.10.27.0071.01

$$\csc^2(z) - \sec^2(z) = 4\cot(2\,z)\csc(2\,z)$$

Inequalities

01.10.29.0001.01

$$x\csc(x) + y\csc(x) < 2\sec\left(\frac{x+y}{2}\right) \;/;\; 0 < x < \frac{\pi}{2} \bigwedge 0 < y < \frac{\pi}{2}$$

01.10.29.0002.01

$$x\csc(x) < \frac{\pi}{2} \;/;\; -\frac{\pi}{2} < x < \frac{\pi}{2}$$

01.10.29.0003.01

$$\csc(x) > \frac{1}{x} \;/;\; x > 0$$

01.10.29.0004.01

$$|\csc(x)| > 1 \;/;\; x \in \mathbb{R}$$

Other information

Value properties

01.10.33.0001.01

$$(x \in \mathbb{Q} \bigwedge \csc(x^\circ) \in \mathbb{Q}) \Rightarrow \csc(x) == 1 \vee \csc(x) == -1 \vee \csc(x) == 2 \vee \csc(x) == -2$$

History

– L. Euler (1748)
– T. Olivier, Wait and Jones (1881)

The function csc is encountered often in mathematics and the natural sciences.

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