Divisors

Notations

Traditional name

Divisors

Traditional notation

$\text{divisors}(n)$

Mathematica StandardForm notation

`Divisors[n]`

Primary definition

13.02.02.0001.01

$$\text{divisors}(n) = \{1, d_2, \ldots, d_{m-1}, n\} /; n \in \mathbb{Z} \bigwedge \frac{n}{d_k} \in \mathbb{Z} \bigwedge d_k \in \mathbb{Z} \bigwedge d_k < d_{k+1} \bigwedge d_1 = 1 \bigwedge d_m = n \bigwedge 1 \leq k \leq m-1 \bigwedge m = \sigma_0(n)$$

13.02.02.0002.01

$$\text{divisors}(n) = \{1, d_2, \ldots, d_{m-1}, d_m\} /; \text{Re}(n) \in \mathbb{Z} \bigwedge \text{Im}(n) \in \mathbb{Z} \bigwedge \text{Re}\left(\frac{n}{d_k}\right) \in \mathbb{Z} \bigwedge \text{Im}\left(\frac{n}{d_k}\right) \in \mathbb{Z} \bigwedge \text{Re}(d_k) \in \mathbb{Z} \bigwedge \text{Im}(d_k) \in \mathbb{Z} \bigwedge \text{Re}(d_k) \leq \text{Re}(d_{k+1}) \bigwedge (\text{Im}(d_k) \leq \text{Im}(d_{k+1}) /; \text{Re}(d_k) = \text{Re}(d_{k+1})) \bigwedge d_1 = 1 \bigwedge 1 \leq k \leq m-1 \bigwedge m = \sigma_0(n)$$

13.02.02.0003.01

$$\text{divisors}(-n) = \text{divisors}(n)$$

For integer n, the function $\text{divisors}(n)$ is the list of the integers that divide n.

Examples: The divisors of 4 are 1, 2, and 4, so $\text{divisors}(4) = \{1, 2, 4\}$; the divisors of 5 are 1, and 5, so $\text{divisors}(5) = \{1, 5\}$; the divisors of 6 are 1, 2, 3, and 6, so $\text{divisors}(6) = \{1, 2, 3, 6\}$.

Specific values

Specialized values

13.02.03.0001.01

$$\text{divisors}(p) = \{1, p\} /; p \in \mathbb{P}$$

13.02.03.0002.01

$$\text{divisors}(p^n) = \left\{1, p, p^2, \ldots, p^n\right\} /; p \in \mathbb{P} \land n \in \mathbb{N}^+$$

Values at fixed points

13.02.03.0004.01

divisors(1) == {1}

13.02.03.0005.01

divisors(2) == {1, 2}

13.02.03.0006.01

divisors(3) == {1, 3}

13.02.03.0007.01

divisors(4) == {1, 2, 4}

13.02.03.0008.01

divisors(5) == {1, 5}

13.02.03.0009.01

divisors(6) == {1, 2, 3, 6}

13.02.03.0010.01

divisors(7) == {1, 7}

13.02.03.0011.01

divisors(8) == {1, 2, 4, 8}

13.02.03.0012.01

divisors(9) == {1, 3, 9}

13.02.03.0013.01

divisors(10) == {1, 2, 5, 10}

13.02.03.0014.01

divisors(11) == {1, 11}

13.02.03.0015.01

divisors(12) == {1, 2, 3, 4, 6, 12}

13.02.03.0016.01

divisors(13) == {1, 13}

13.02.03.0017.01

divisors(14) == {1, 2, 7, 14}

13.02.03.0018.01

divisors(15) == {1, 3, 5, 15}

13.02.03.0019.01

divisors(16) == {1, 2, 4, 8, 16}

13.02.03.0020.01

divisors(17) == {1, 17}

13.02.03.0021.01

divisors(18) == {1, 2, 3, 6, 9, 18}

13.02.03.0022.01

divisors(19) == {1, 19}

13.02.03.0023.01

divisors(20) == {1, 2, 4, 5, 10, 20}

13.02.03.0024.01
divisors(21) == {1, 3, 7, 21}

13.02.03.0025.01
divisors(22) == {1, 2, 11, 22}

13.02.03.0026.01
divisors(23) == {1, 23}

13.02.03.0027.01
divisors(24) == {1, 2, 3, 4, 6, 8, 12, 24}

13.02.03.0028.01
divisors(25) == {1, 5, 25}

13.02.03.0029.01
divisors(26) == {1, 2, 13, 26}

13.02.03.0030.01
divisors(27) == {1, 3, 9, 27}

13.02.03.0031.01
divisors(28) == {1, 2, 4, 7, 14, 28}

13.02.03.0032.01
divisors(29) == {1, 29}

13.02.03.0033.01
divisors(30) == {1, 2, 3, 5, 6, 10, 15, 30}

13.02.03.0034.01
divisors(31) == {1, 31}

13.02.03.0035.01
divisors(32) == {1, 2, 4, 8, 16, 32}

13.02.03.0036.01
divisors(33) == {1, 3, 11, 33}

13.02.03.0037.01
divisors(34) == {1, 2, 17, 34}

13.02.03.0038.01
divisors(35) == {1, 5, 7, 35}

13.02.03.0039.01
divisors(36) == {1, 2, 3, 4, 6, 9, 12, 18, 36}

13.02.03.0040.01
divisors(37) == {1, 37}

13.02.03.0041.01
divisors(38) == {1, 2, 19, 38}

13.02.03.0042.01
divisors(39) == {1, 3, 13, 39}

13.02.03.0043.01
divisors(40) == {1, 2, 4, 5, 8, 10, 20, 40}

13.02.03.0044.01

$\text{divisors}(41) = \{1, 41\}$

13.02.03.0045.01

$\text{divisors}(42) = \{1, 2, 3, 6, 7, 14, 21, 42\}$

13.02.03.0046.01

$\text{divisors}(43) = \{1, 43\}$

13.02.03.0047.01

$\text{divisors}(44) = \{1, 2, 4, 11, 22, 44\}$

13.02.03.0048.01

$\text{divisors}(45) = \{1, 3, 5, 9, 15, 45\}$

13.02.03.0049.01

$\text{divisors}(46) = \{1, 2, 23, 46\}$

13.02.03.0050.01

$\text{divisors}(47) = \{1, 47\}$

13.02.03.0051.01

$\text{divisors}(48) = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

13.02.03.0052.01

$\text{divisors}(49) = \{1, 7, 49\}$

13.02.03.0053.01

$\text{divisors}(50) = \{1, 2, 5, 10, 25, 50\}$

13.02.03.0054.01

$\text{divisors}(100) = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

13.02.03.0057.01

$\text{divisors}(1000) = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, 500, 1000\}$

13.02.03.0058.01

$$\text{divisors}(10\,000) = \{1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 125, 200, 250, 400, 500, 625, 1000, 1250, 2000, 2500, 5000, 10\,000\}$$

13.02.03.0056.01

$\text{divisors}(5 + 20\,i) = \{1, 1 + 2\,i, 1 + 4\,i, 2 + i, 5, 5 + 20\,i, 6 + 7\,i, 9 + 2\,i\}$

13.02.03.0055.01

$\text{divisors}(-100) = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

General characteristics

Domain and analyticity

$\text{divisors}(n)$ is a nonanalytical function which is defined over $\mathbb{Z}$.

13.02.04.0001.01

$n \longrightarrow \text{divisors}(n) :: \mathbb{Z} \longrightarrow \mathbb{Z}^m$

Symmetries and periodicities

Parity

divisors(n) is an even function.

13.02.04.0002.01

$$\text{divisors}(-n) = \text{divisors}(n)$$

Periodicity

No periodicity

Products

Finite products

13.02.24.0001.01

$$\prod_{d_j \mid n} d_j = n^{\frac{1}{2}\sigma_0(d_j)} \;/;\; d_j \in \text{divisors}(n)$$

Operations

13.02.25.0001.01

$$\sum_{k \mid n} k^n = \sigma_k(n)$$

13.02.25.0002.01

$$\sum_{0 \mid n} 1 = \sigma_0(n)$$

Representations through equivalent functions

With related functions

13.02.27.0001.01

$$\text{divisors}(n) = \left\{1, \ldots, p_{k_1}^{n_{k_1}}, \ldots, p_{k_h}^{n_{k_h}}, \ldots, n\right\} \;/;\; \text{factors}(n) = \{\{p_1, n_1\}, \{p_2, n_2\}, \ldots, \{p_m, n_m\}\} \land p_k \in \mathbb{P} \land n > 0$$

13.02.27.0002.01

$$\text{divisors}\,(n) = \left\{1, \ldots, p_{k_1}^{n_{k_1}}, \ldots, p_{k_h}^{n_{k_h}}, \ldots, n\right\} \;/;\; n = \prod_{k=1}^{l} p_k^{n_k} \bigwedge p_k \in \mathbb{P}$$

Theorems

The squares problem

Special cases of the s squares problem to find the number $r_s(n)$ of integer solutions of $x_1^2 + x_2^2 + \ldots + x_s^2 = n$ (order and sign changes are taken into account) can be expressed through the divisors of n:

$$r_4(n) = 8 \sum_{\substack{d|n \\ d/4 \notin \mathbb{N}}} d;$$

$$r_8(n) = 16 \sum_{d|n} (-1)^{n+d} d^3;$$

$$r_{16}(n) = -(-1)^n \frac{32}{3} (\tilde{\sigma}_1(n) + \tilde{\sigma}_3(n) + \tilde{\sigma}_5(n)) + (-1)^n \frac{256}{3} \sum_{k=1}^{n-1} (\tilde{\sigma}_1(k)\, \tilde{\sigma}_5(n-k) - \tilde{\sigma}_3(k)\, \tilde{\sigma}_3(n-k)) /;$$

$$\tilde{\sigma}_r(n) == \sum_{d|n} (-1)^{d+n/d} d^r;$$

$$r_{24}(n) = (-1)^n \frac{16}{9} \left(17\, \hat{\sigma}_3(n) + 8\, \hat{\sigma}_5(n) + 2\, \hat{\sigma}_7(n)\right) +$$

$$(-1)^n \frac{512}{9} \sum_{k=1}^{n-1} \left(\hat{\sigma}_3(k)\, \hat{\sigma}_7(n-k) - \hat{\sigma}_5(k)\, \hat{\sigma}_5(n-k)\right) /; \hat{\sigma}_r(n) == \sum_{d|n} (-1)^d d^r$$

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