EllipticE2

Notations

Traditional name

Incomplete elliptic integral of the second kind

Traditional notation

$E(z \mid m)$

Mathematica StandardForm notation

`EllipticE[z, m]`

Primary definition

08.04.02.0001.01

$$E(z \mid m) = \int_0^z \sqrt{1 - m \sin^2(t)}\, dt$$

Specific values

Specialized values

For fixed *z*

08.04.03.0001.01

$$E(z \mid 0) = z$$

08.04.03.0002.01

$$E(z \mid 1) = \sin(z) \;/;\; |\mathrm{Re}(z)| \leq \frac{\pi}{2}$$

08.04.03.0010.01

$$E(z \mid 1) = (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rceil} \sin(z) + 2 \left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rceil$$

For fixed *m*

08.04.03.0003.01

$$E(0 \mid m) = 0$$

08.04.03.0004.01

$$E\left(\frac{\pi}{2} \,\middle|\, m\right) = E(m)$$

08.04.03.0005.01

$$E\left(\frac{k\pi}{2} \,\middle|\, m\right) = k\,E(m) \;/;\; k \in \mathbb{Z}$$

08.04.03.0011.01

$$E\left(\csc^{-1}\left(\sqrt{m}\right) \,\middle|\, m\right) = \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)$$

08.04.03.0012.01

$$E\left(\csc^{-1}\left(\sqrt{m}\right) + \pi k \,\middle|\, m\right) = 2\,k\,E(m) + \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right) /;\; k \in \mathbb{Z}$$

08.04.03.0006.01

$$E(\operatorname{am}(z \mid m) \mid m) = \int_0^z \operatorname{dn}(t \mid m)^2 \, dt$$

08.04.03.0013.01

$$E(\operatorname{am}(z \mid m) \mid m) = \frac{E(m)}{K(m)}\, z + Z(\operatorname{am}(z \mid m) \mid m)$$

08.04.03.0007.01

$$E\left(\sin^{-1}(\operatorname{sn}(z \mid m)) \,\middle|\, m\right) = \int_0^z \operatorname{dn}(t \mid m)^2 \, dt$$

Values at infinities

08.04.03.0008.01

$$E(z \mid \infty) = \tilde{\infty}$$

08.04.03.0009.01

$$E(z \mid -\infty) = \tilde{\infty}$$

General characteristics

Domain and analyticity

$E(z \mid m)$ is an analytical function of z and m which is defined over $\mathbb{C}^2$.

08.04.04.0001.01

$$(z * m) \longrightarrow E(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$E(z \mid m)$ is an odd function with respect to z.

08.04.04.0002.01

$$E(-z \mid m) = -E(z \mid m)$$

Mirror symmetry

08.04.04.0003.01

$$E(\bar{z} \mid \bar{m}) = \overline{E(z \mid m)} \;/;\; \neg\,(m \in \mathbb{R} \land m > 1)$$

Periodicity

$E(z \mid m)$ is a quasi-periodic function with respect to z.

08.04.04.0004.01

$$E(z + \pi k \mid m) == E(z \mid m) + 2 k E(m) /; k \in \mathbb{Z}$$

08.04.04.0010.01

$$E(z \mid m) = E\left(z - \pi \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil \middle| m\right) + 2 \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil E(m)$$

Quasi-symmetry

08.04.04.0011.01

$$E(x + i y \mid m) == E\left(\pi \operatorname{frac}\left(\frac{x}{\pi}\right) + i y \middle| m\right) + 2 \operatorname{sgn}(x) \left\lfloor \frac{|x|}{\pi} \right\rfloor E(m) /; x \in \mathbb{R} \land y \in \mathbb{R}$$

Poles and essential singularities

With respect to *z*

The function $E(z \mid m)$ does not have poles and essential singularities with respect to z.

08.04.04.0005.01

$$\mathcal{Sing}_z(E(z \mid m)) == \{\}$$

With respect to *m*

The function $E(z \mid m)$ does not have poles and essential singularities with respect to m.

08.04.04.0006.01

$$\mathcal{Sing}_m(E(z \mid m)) == \{\}$$

Branch points

With respect to *z*

For fixed m, the function $E(z \mid m)$ has an infinite number of branch points at $z == \pm \sin^{-1}\left(\frac{1}{\sqrt{m}}\right) + \pi k /; k \in \mathbb{Z}$, $z == \frac{\pi}{2} + \pi k /; k \in \mathbb{Z} \land m \notin (0, 1)$, and $z == \tilde{\infty}$. All these are square-root-type branch points.

08.04.04.0007.01

$$\mathcal{BP}_a(E(z \mid m)) == \left\{\left\{\sin^{-1}\left(\frac{1}{\sqrt{m}}\right) + \pi k /; k \in \mathbb{Z}\right\}, \left\{-\sin^{-1}\left(\frac{1}{\sqrt{m}}\right) + \pi k /; k \in \mathbb{Z}\right\}, \left\{\frac{\pi}{2} + \pi k /; k \in \mathbb{Z} \land m \notin (0, 1)\right\}, \tilde{\infty}\right\}$$

08.04.04.0008.01

$$\mathcal{R}_z\left(E(z \mid m), \sin^{-1}\left(\frac{1}{\sqrt{m}}\right) + \pi k\right) == 2 /; k \in \mathbb{Z}$$

08.04.04.0009.01

$$\mathcal{R}_z\left(E(z \mid m), -\sin^{-1}\left(\frac{1}{\sqrt{m}}\right) + \pi k\right) == 2 /; k \in \mathbb{Z}$$

08.04.04.0012.01

$$\mathcal{R}_z\left(E(z \mid m), \frac{\pi}{2} + \pi k\right) == 2 /; k \in \mathbb{Z} \land m \notin (0, 1)$$

With respect to *m*

For fixed z, the function $E(z \mid m)$ has two branch points at $m == \csc^2(z)$and $m == \tilde{\infty}$.

08.04.04.0013.01

$$\mathcal{BP}_m(E(z \mid m)) == \{\csc^2(z), \tilde{\infty}\}$$

Branch cuts

With respect to *z*

General description

For fixed m, the function $E(z \mid m)$ can have up to six infinite sets of branch cuts (it has at least four), which form very complicated curves in the case of generic m.

For fixed real $m < 1$, the function $E(z \mid m)$ does not have branch cuts on the real axis and on the vertical intervals $\{\csc^{-1}(\sqrt{m}) + \pi k, \pi - \csc^{-1}(\sqrt{m}) + \pi k\} /; k \in \mathbb{Z} \land m \in (-\infty, 1)$.
For fixed real $m < 1$, the function $E(z \mid m)$ has four infinite sets of branch cuts located on vertical intervals starting at the points $z == \pi k \pm \csc^{-1}(\sqrt{m}) /; k \in \mathbb{Z}$ and extending to imaginary infinity.

For fixed generic m, the function $E(z \mid m)$ has the following six infinite sets of branch cuts:

1) real intervals $\{\pi k + \csc^{-1}(\sqrt{m}), \pi k + \frac{\pi}{2}\} /; k \in \mathbb{Z} \land m > 1$, where $E(z \mid m)$ is continuous from below (for generic complex m, these branch cuts deform into complicated curves); in the case $m < 1$ these real intervals vanish

2) real intervals $\{\pi k + \frac{\pi}{2}, \pi (k + 1) - \csc^{-1}(\sqrt{m})\} /; k \in \mathbb{Z} \land m > 1$, where $E(z \mid m)$ is continuous from above (for generic complex m, these branch cuts deform into complicated curves); in the case $m < 1$ these real intervals vanish

3) vertical intervals $\{\frac{\pi}{2} + 2 \pi k, \frac{\pi}{2} + 2 \pi k + i \infty\} /; k \in \mathbb{Z} \land m \notin (0, 1)$, or $\{\pi - \csc^{-1}(\sqrt{m}) + 2 \pi k, \frac{\pi}{2} + 2 \pi k + i \infty\} /; k \in \mathbb{Z} \land m \in (0, 1)$, where $E(z \mid m)$ is continuous from the left

4) vertical intervals $\{\frac{3\pi}{2} + 2 \pi k, \frac{3\pi}{2} + 2 \pi k + i \infty\} /; k \in \mathbb{Z} \land m \notin (0, 1)$, or $\{2\pi - \csc^{-1}(\sqrt{m}) + 2 \pi k, \frac{3\pi}{2} + 2 \pi k + i \infty\} /; k \in \mathbb{Z} \land m \in (0, 1)$, where $E(z \mid m)$ is continuous from the right

5) vertical intervals $\{\frac{\pi}{2} + 2 \pi k - i \infty, \frac{\pi}{2} + 2 \pi k\} /; k \in \mathbb{Z} \land m \notin (0, 1)$, or $\{\frac{\pi}{2} + 2 \pi k - i \infty, 2 \pi k + \csc^{-1}(\sqrt{m})\} /; k \in \mathbb{Z} \land m \in (0, 1)$, where $E(z \mid m)$ is continuous from the left

6) vertical intervals $\{\frac{3\pi}{2} + 2 \pi k - i \infty, \frac{3\pi}{2} + 2 \pi k\} /; k \in \mathbb{Z} \land m \notin (0, 1)$, or $\{\frac{3\pi}{2} + 2 \pi k - i \infty, 2 \pi k + \pi + \csc^{-1}(\sqrt{m})\} /; k \in \mathbb{Z} \land m \in (0, 1)$, where $E(z \mid m)$ is continuous from the right.

08.04.04.0014.01

$$\mathcal{BC}_z(E(z\mid m)) = \Big\{\Big\{\Big\{\Big(\pi k + \csc^{-1}(\sqrt{m}),\, \pi k + \frac{\pi}{2}\Big),\, i\Big\} /;\, k\in\mathbb{Z} \land m\in\mathbb{R} \land m>1\Big\},$$
$$\Big\{\Big\{\Big(\pi k + \frac{\pi}{2},\, \pi(k+1) - \csc^{-1}(\sqrt{m})\Big),\, -i\Big\} /;\, k\in\mathbb{Z} \land m\in\mathbb{R} \land m>1\Big\},$$
$$\Big\{\Big\{\Big(2\pi k + \frac{\pi}{2},\, 2k\pi + \frac{\pi}{2} + i\infty\Big),\, 1\Big\} /;\, k\in\mathbb{Z} \land m\notin(0,1)\Big\} \bigvee$$
$$\Big\{\Big\{\Big(2\pi k + \pi - \csc^{-1}(\sqrt{m}),\, 2k\pi + \frac{\pi}{2} + i\infty\Big),\, 1\Big\} /;\, k\in\mathbb{Z} \land m\in(0,1)\Big\},$$
$$\Big\{\Big\{\Big(2\pi k + \frac{3\pi}{2},\, 2k\pi + \frac{3\pi}{2} + i\infty\Big),\, -1\Big\} /;\, k\in\mathbb{Z} \land m\notin(0,1)\Big\} \bigvee$$
$$\Big\{\Big\{\Big(2\pi k + 2\pi - \csc^{-1}(\sqrt{m}),\, 2k\pi + \frac{3\pi}{2} + i\infty\Big),\, -1\Big\} /;\, k\in\mathbb{Z} \land m\in(0,1)\Big\},$$
$$\Big\{\Big\{\Big(2\pi k + \frac{\pi}{2} - i\infty,\, 2k\pi + \frac{\pi}{2}\Big),\, 1\Big\} /;\, k\in\mathbb{Z} \land m\notin(0,1)\Big\} \bigvee$$
$$\Big\{\Big\{\Big(2\pi k + \frac{\pi}{2} - i\infty,\, 2\pi k + \csc^{-1}(\sqrt{m})\Big),\, 1\Big\} /;\, k\in\mathbb{Z} \land m\in(0,1)\Big\},$$
$$\Big\{\Big\{\Big(2\pi k + \frac{\pi}{2} - i\infty,\, 2k\pi + \frac{\pi}{2}\Big),\, -1\Big\} /;\, k\in\mathbb{Z} \land m\notin(0,1)\Big\} \bigvee$$
$$\Big\{\Big\{\Big(2\pi k + \frac{\pi}{2} - i\infty,\, 2\pi k + \pi + \csc^{-1}(\sqrt{m})\Big),\, -1\Big\} /;\, k\in\mathbb{Z} \land m\in(0,1)\Big\}\Big\}$$

Formulas on real axis for real *m*

For $m < 1$

For fixed real $m < 1$, the function $E(z \mid m)$ does not have branch cuts on the real axis.

For $m > 1$

08.04.04.0015.01

$$\lim_{\epsilon\to+0} E(x + i\epsilon \mid m) = -E(x\mid m) + 4\left(\left\lfloor \frac{x}{\pi} - \frac{1}{2}\right\rfloor + 1\right) E(m) + 2\sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right) /;$$
$$x\in\mathbb{R} \bigwedge m\in\mathbb{R} \bigwedge m>1 \bigwedge \pi k + \csc^{-1}(\sqrt{m}) < x < \pi k + \frac{\pi}{2} \bigwedge k\in\mathbb{Z}$$

08.04.04.0016.01

$$\lim_{\epsilon\to+0} E(x - i\epsilon \mid m) = E(x\mid m) /;\, x\in\mathbb{R} \bigwedge m\in\mathbb{R} \bigwedge m>1 \bigwedge \pi k + \csc^{-1}(\sqrt{m}) < x < \pi k + \frac{\pi}{2} \bigwedge k\in\mathbb{Z}$$

08.04.04.0017.01

$$\lim_{\epsilon\to+0} E(x + i\epsilon \mid m) = E(x\mid m) /;\, x\in\mathbb{R} \bigwedge m\in\mathbb{R} \bigwedge m>1 \bigwedge \frac{\pi}{2} + \pi k < x < \pi(k+1) - \csc^{-1}(\sqrt{m}) \bigwedge k\in\mathbb{Z}$$

08.04.04.0018.01

$$\lim_{\epsilon\to+0} E(x - i\epsilon \mid m) = -E(x\mid m) + 4\left(\left\lfloor \frac{x}{\pi} - \frac{1}{2}\right\rfloor + 1\right) E(m) - 2\sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right) /;$$
$$x\in\mathbb{R} \bigwedge m\in\mathbb{R} \bigwedge m>1 \bigwedge \pi k + \frac{\pi}{2} < x < \pi(k+1) - \csc^{-1}(\sqrt{m}) \bigwedge k\in\mathbb{Z}$$

Formulas for vertical intervals

For $m < 1$

For fixed real $m < 1$, the function $E(z \mid m)$ has branch points $\csc^{-1}\left(\sqrt{m}\right) + \pi k \,/;\, k \in \mathbb{Z}$ and $\pi - \csc^{-1}\left(\sqrt{m}\right) + \pi k \,/;\, k \in \mathbb{Z}$. In this case branch cuts lay at the vertical lines beginning from these points and going to imaginary infinity. By this reason for fixed real $m < 1$, the function $E(z \mid m)$ does not have branch cuts on the vertical intervals $\left\{\csc^{-1}\left(\sqrt{m}\right) + \pi k,\, \pi - \csc^{-1}\left(\sqrt{m}\right) + \pi k\right\} /;\, k \in \mathbb{Z} \land m \in (-\infty, 1)$.

For $m > 0$

08.04.04.0019.01

$$\lim_{\epsilon \to +0} E\left(2\pi k + i x + \frac{\pi}{2} - \epsilon \,\middle|\, m\right) = E\left(2\pi k + i x + \frac{\pi}{2} \,\middle|\, m\right) /;\, x \in \mathbb{R} \land k \in \mathbb{Z}$$

08.04.04.0020.01

$$\lim_{\epsilon \to +0} E\left(2\pi k + i x + \frac{\pi}{2} + \epsilon \,\middle|\, m\right) = 4(k+1) E(m) - E\left(i x + \frac{\pi}{2} \,\middle|\, m\right) - 2\sqrt{m} \left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right) /;$$
$$m \in \mathbb{R} \bigwedge x \in \mathbb{R} \bigwedge \left(0 < m < 1 \bigwedge x > -\operatorname{Im}\left(\csc^{-1}\left(\sqrt{m}\right)\right) \bigvee m > 1 \land x < 0\right) \bigwedge k \in \mathbb{Z}$$

08.04.04.0021.01

$$\lim_{\epsilon \to +0} E\left(2\pi k + i x + \frac{\pi}{2} + \epsilon \,\middle|\, m\right) = 4 k E(m) - E\left(i x + \frac{\pi}{2} \,\middle|\, m\right) + 2\sqrt{m} \left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right) /;$$
$$m \in \mathbb{R} \bigwedge x \in \mathbb{R} \bigwedge \left(0 < m < 1 \bigwedge x < \operatorname{Im}\left(\csc^{-1}\left(\sqrt{m}\right)\right) \bigvee m > 1 \land x > 0\right) \bigwedge k \in \mathbb{Z}$$

08.04.04.0022.01

$$\lim_{\epsilon \to +0} E\left(2\pi k + i x + \frac{3\pi}{2} - \epsilon \,\middle|\, m\right) = -F\left(i x + \frac{3\pi}{2} \,\middle|\, m\right) - 2\sqrt{m} \left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right) + 4(k+2) K(m) /;$$
$$m \in \mathbb{R} \bigwedge x \in \mathbb{R} \bigwedge \left(0 < m < 1 \bigwedge x > -\operatorname{Im}\left(\csc^{-1}\left(\sqrt{m}\right)\right) \bigvee m > 1 \land x < 0\right) \bigwedge k \in \mathbb{Z}$$

08.04.04.0023.01

$$\lim_{\epsilon \to +0} E\left(2\pi k + i x + \frac{3\pi}{2} - \epsilon \,\middle|\, m\right) = 4(k+1) E(m) - E\left(i x + \frac{3\pi}{2} \,\middle|\, m\right) + 2\sqrt{m} \left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right) /;$$
$$m \in \mathbb{R} \bigwedge x \in \mathbb{R} \bigwedge \left(0 < m < 1 \bigwedge x < \operatorname{Im}\left(\csc^{-1}\left(\sqrt{m}\right)\right) \bigvee m > 1 \land x > 0\right) \bigwedge k \in \mathbb{Z}$$

08.04.04.0024.01

$$\lim_{\epsilon \to +0} E\left(2\pi k + i x + \frac{3\pi}{2} + \epsilon \,\middle|\, m\right) = E\left(2\pi k + i x + \frac{3\pi}{2} \,\middle|\, m\right) /;\, x \in \mathbb{R} \land k \in \mathbb{Z}$$

With respect to *m*

Branch cut locations: complicated.

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

08.04.06.0009.01

$$E(z \mid m) \propto E(z_0 \mid m) + \sqrt{1 - m \sin^2(z_0)}\, (z - z_0) - \frac{m \sin(2 z_0)}{4 \sqrt{1 - m \sin^2(z_0)}} (z - z_0)^2 + \ldots /; (z \to z_0)$$

08.04.06.0010.01

$$E(z \mid m) \propto E(z_0 \mid m) + \sqrt{1 - m \sin^2(z_0)}\, (z - z_0) - \frac{m \sin(2 z_0)}{4 \sqrt{1 - m \sin^2(z_0)}} (z - z_0)^2 + O\left((z - z_0)^3\right)$$

08.04.06.0011.01

$$E(z \mid m) == E(z_0 \mid m) + \sqrt{1 - m \sin^2(z_0)}\, (z - z_0) + \sqrt{1 - m \sin^2(z_0)} \sum_{k=2}^{\infty} \frac{i^{k-1}}{k!} \binom{k - \frac{3}{2}}{k - 1} \sum_{q=1}^{k-1} \frac{(-1)^q}{1 - 2 q} \binom{k-1}{q} \left(1 - m \sin^2(z_0)\right)^{-q} \sum_{j=0}^{q} \binom{q}{j} m^j (2 - m)^{q-j} 2^{k-j-q-1} \sum_{i=0}^{j} \binom{j}{i} (2 i - j)^{k-1} e^{2 (2 i - j) i z_0} (z - z_0)^k$$

08.04.06.0012.01

$$E(z \mid m) == E(z_0 \mid m) + \sum_{k=1}^{\infty} \frac{1}{k!} \sum_{j=1}^{k} \frac{1}{j!} \left(\sum_{q=0}^{j-1} \binom{j}{q} \sum_{p=0}^{j-q} (-1)^q 2^{q-j} \sin^q(z_0) (2 p + q - j)^k e^{-\frac{1}{2} i \left(\pi (j + k - 2 p - q) + 2 (2 p + q - j) z_0\right)} \binom{j - q}{p} \right. \left. \sum_{i=0}^{j-1} \frac{(1 - j)_{2 (j-i)-2}}{(j - i - 1)! \, (2 \sin(z_0))^{j-2 i-1}} \sum_{s=0}^{i} \binom{i}{s} \left(\frac{1}{2}\right)_s \left(-\frac{1}{2}\right)_{i-s} m^{i-s} \cos^{-2 s-1}(z_0) \left(1 - m \sin^2(z_0)\right)^{s-i+\frac{1}{2}} \right) (z - z_0)^k$$

08.04.06.0013.01

$$E(z \mid m) = \sum_{k=0}^{\infty} \frac{1}{k!} E^{(k,0)}(z_0 \mid m) (z - z_0)^k$$

08.04.06.0014.01

$$E(z \mid m) \propto E(z_0 \mid m) (1 + O(z - z_0))$$

Expansions on branch cuts

Formulas on real axis for real *m*

For $m > 1, \csc^{-1}\left(\sqrt{m}\right) + \pi u < x < \pi\left(u + \frac{1}{2}\right) /; u \in \mathbb{Z}$

08.04.06.0015.01

$$E(z \mid m) \propto \left(2 \left(\left\lfloor \frac{x}{\pi} - \frac{1}{2} \right\rfloor + 1\right) E(m) + \sqrt{m} \left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right) \left(1 - e^{-\pi i \left\lfloor \frac{\arg(x - z)}{2 \pi} \right\rfloor}\right) + E(x \mid m) e^{-\pi i \left\lfloor \frac{\arg(x - z)}{2 \pi} \right\rfloor} + e^{-\pi i \left\lfloor \frac{\arg(x - z)}{2 \pi} \right\rfloor} \left(\sqrt{1 - m \sin^2(x)}\, (z - x) - \frac{m \sin(2 x)}{4 \sqrt{1 - m \sin^2(x)}} (z - x)^2 + \ldots\right) /; (z \to x) \bigwedge x \in \mathbb{R} \bigwedge m \in \mathbb{R} \bigwedge m > 1 \bigwedge \pi u + \csc^{-1}\left(\sqrt{m}\right) < x < \pi u + \frac{\pi}{2} \bigwedge u \in \mathbb{Z}$$

08.04.06.0016.01

$$E(z \mid m) = \left(2\left(\left\lfloor \frac{x}{\pi} - \frac{1}{2} \right\rfloor + 1\right) E(m) + \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(1 - e^{-\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor}\right) + E(x \mid m)\, e^{-\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} + e^{-\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \sum_{k=1}^{\infty} \frac{1}{k!} \sum_{j=1}^{k} \frac{1}{j!} \left(\sum_{q=0}^{j-1} \binom{j}{q} \sum_{p=0}^{j-q} (-1)^q\, 2^{q-j} \sin^q(x)\, (2p+q-j)^k\, e^{-\frac{1}{2} i (\pi (j+k-2p-q) + 2(2p+q-j)x)} \binom{j-q}{p} \sum_{i=0}^{j-1} \frac{(1-j)_{2(j-i)-2}}{(j-i-1)!\, (2\sin(x))^{j-2i-1}} \sum_{s=0}^{i} \binom{i}{s} \left(\frac{1}{2}\right)_s \left(-\frac{1}{2}\right)_{i-s} m^{i-s} \cos^{-2s-1}(x) \left(1 - m \sin^2(x)\right)^{s-i+\frac{1}{2}} \right) (z-x)^k \;/;$$

$$x \in \mathbb{R} \bigwedge m \in \mathbb{R} \bigwedge m > 1 \bigwedge \pi u + \csc^{-1}\left(\sqrt{m}\right) < x < \pi u + \frac{\pi}{2} \bigwedge u \in \mathbb{Z}$$

08.04.06.0017.01

$$E(z \mid m) \propto \left(\left(2\left(\left\lfloor \frac{x}{\pi} - \frac{1}{2} \right\rfloor + 1\right) E(m) + \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(1 - e^{-\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor}\right) + E(x \mid m)\, e^{-\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor}\right)(1 + O(z-x)) \;/;$$

$$(z \to x) \bigwedge x \in \mathbb{R} \bigwedge m \in \mathbb{R} \bigwedge m > 1 \bigwedge \pi u + \csc^{-1}\left(\sqrt{m}\right) < x < \pi u + \frac{\pi}{2} \bigwedge u \in \mathbb{Z}$$

For $m > 1,\ \pi\left(u + \frac{1}{2}\right) < x < \pi(u+1) - \csc^{-1}\left(\sqrt{m}\right) /;\ u \in \mathbb{Z}$

08.04.06.0018.01

$$E(z \mid m) = \left(2\left(\left\lfloor \frac{x}{\pi} - \frac{1}{2} \right\rfloor + 1\right) E(m) - \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(1 - e^{-\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor}\right) + E(x \mid m)\, e^{-\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} + e^{-\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor}\left(\sqrt{1 - m\sin^2(x)}\, (z-x) - \frac{(m \sin(2x))\,(z-x)^2}{4\sqrt{1 - m \sin^2(x)}} + \ldots\right) /;$$

$$(z \to x) \bigwedge x \in \mathbb{R} \bigwedge m \in \mathbb{R} \bigwedge m > 1 \bigwedge \pi u + \frac{\pi}{2} < x < \pi(u+1) - \csc^{-1}\left(\sqrt{m}\right) \bigwedge u \in \mathbb{Z}$$

08.04.06.0019.01

$$E(z \mid m) = \left(2\left(\left\lfloor \frac{x}{\pi} - \frac{1}{2} \right\rfloor + 1\right) E(m) - \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(1 - e^{-\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor}\right) + E(x \mid m)\, e^{-\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} + e^{-\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=1}^{\infty} \frac{1}{k!} \sum_{j=1}^{k} \frac{1}{j!} \left(\sum_{q=0}^{j-1} \binom{j}{q} \sum_{p=0}^{j-q} (-1)^q\, 2^{q-j} \sin^q(x)\, (2p+q-j)^k\, e^{-\frac{1}{2} i (\pi (j+k-2p-q) + 2(2p+q-j)x)} \binom{j-q}{p} \sum_{i=0}^{j-1} \frac{(1-j)_{2(j-i)-2}}{(j-i-1)!\, (2\sin(x))^{j-2i-1}} \sum_{s=0}^{i} \binom{i}{s} \left(\frac{1}{2}\right)_s \left(-\frac{1}{2}\right)_{i-s} m^{i-s} \cos^{-2s-1}(x) \left(1 - m \sin^2(x)\right)^{s-i+\frac{1}{2}} \right) (z-x)^k \;/;$$

$$x \in \mathbb{R} \bigwedge m \in \mathbb{R} \bigwedge m > 1 \bigwedge \pi u + \frac{\pi}{2} < x < \pi(u+1) - \csc^{-1}\left(\sqrt{m}\right) \bigwedge u \in \mathbb{Z}$$

08.04.06.0020.01

$$E(z \mid m) = \left(\left(2\left(\left\lfloor \frac{x}{\pi} - \frac{1}{2} \right\rfloor + 1\right) E(m) - \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(1 - e^{-\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor}\right) + E(x \mid m)\, e^{-\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor}\right)(1 + O(z-x)) \;/;$$

$$(z \to x) \bigwedge x \in \mathbb{R} \bigwedge m \in \mathbb{R} \bigwedge m > 1 \bigwedge \pi u + \frac{\pi}{2} < x < \pi(u+1) - \csc^{-1}\left(\sqrt{m}\right) \bigwedge u \in \mathbb{Z}$$

Formulas for vertical intervals

For $\mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{1}{4}\right) \in \mathbb{Z}$

08.04.06.0021.01

$$E(z \mid m) \propto -e^{-\pi i \left(\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{3}{4} \right\rfloor + \left\lfloor \frac{3}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor\right)} E(z_0 \mid m) + \left(2\left(2\,\mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{1}{4}\right) + 1\right) E(m) - \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(e^{-\pi i \left(\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{3}{4} \right\rfloor + \left\lfloor \frac{3}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor\right)} + 1\right) + \sqrt{1 - m\sin^2(z_0)}\,(z - z_0) - \frac{m\sin(2 z_0)}{4\sqrt{1 - m\sin^2(z_0)}}(z - z_0)^2 + \ldots \;/;\; (z \to z_0) \bigwedge \mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{1}{4}\right) \in \mathbb{Z}$$

08.04.06.0022.01

$$E(z \mid m) = -E(z_0 \mid m)\, e^{-\pi i \left(\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{3}{4} \right\rfloor + \left\lfloor \frac{3}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor\right)} + \left(2\left(2\,\mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{1}{4}\right) + 1\right) E(m) - \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(e^{-\pi i \left(\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{3}{4} \right\rfloor + \left\lfloor \frac{3}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor\right)} + 1\right) + \sum_{k=1}^{\infty} \frac{1}{k!} \sum_{j=1}^{k} \frac{1}{j!} \left(\sum_{q=0}^{j-1} \binom{j}{q} \sum_{p=0}^{j-q} (-1)^q\, 2^{q-j} \sin^q(z_0)\, (2p + q - j)^k\, e^{-\frac{1}{2} i (\pi (j+k-2p-q) + 2(2p+q-j) z_0)} \binom{j-q}{p} \sum_{i=0}^{j-1} \frac{(1-j)_{2(j-i)-2}}{(j-i-1)!\,(2\sin(z_0))^{j-2i-1}} \sum_{s=0}^{i} \binom{i}{s} \left(\frac{1}{2}\right)_s \left(-\frac{1}{2}\right)_{i-s} m^{i-s} \cos^{-2s-1}(z_0) \left(1 - m\sin^2(z_0)\right)^{s-i+\frac{1}{2}}\right)(z - z_0)^k \;/;\; \mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{1}{4}\right) \in \mathbb{Z}$$

08.04.06.0023.01

$$E(z \mid m) \propto \left(\left(2\left(2\,\mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{1}{4}\right) + 1\right) E(m) - \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(e^{-\pi i \left(\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{3}{4} \right\rfloor + \left\lfloor \frac{3}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor\right)} + 1\right) - e^{-\pi i \left(\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{3}{4} \right\rfloor + \left\lfloor \frac{3}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor\right)} E(z_0 \mid m)\right)(1 + O(z - z_0)) \;/;\; (z \to z_0) \bigwedge \mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{1}{4}\right) \in \mathbb{Z}$$

For $\mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{3}{4}\right) \in \mathbb{Z}$

08.04.06.0024.01

$$E(z \mid m) \propto \left(2\left(2\,\mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{3}{4}\right) + 1\right) E(m) + \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right) K\left(\frac{1}{m}\right)\right)\right)\left(1 - e^{-\pi i \left(\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{1}{4} \right\rfloor + \left\lfloor \frac{1}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor\right)}\right) + E(z_0 \mid m)\, e^{-\pi i \left(\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{1}{4} \right\rfloor + \left\lfloor \frac{1}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor\right)} + \sqrt{1 - m\sin^2(z_0)}\,(z - z_0) - \frac{m\sin(2 z_0)}{4\sqrt{1 - m\sin^2(z_0)}}(z - z_0)^2 + \ldots \;/;\; (z \to z_0) \bigwedge \mathrm{Re}\left(\frac{z_0}{2\pi} - \frac{3}{4}\right) \in \mathbb{Z}$$

08.04.06.0025.01

$$E(z\mid m) = \left(2\left(2\operatorname{Re}\left(\frac{z_0}{2\pi}-\frac{3}{4}\right)+1\right)E(m)+\sqrt{m}\left(E\left(\frac{1}{m}\right)+\left(\frac{1}{m}-1\right)K\left(\frac{1}{m}\right)\right)\right)\left(1-e^{-\pi i\left(\left\lfloor\frac{\arg(z-z_0)}{2\pi}+\frac{1}{4}\right\rfloor+\left\lfloor\frac{1}{4}-\frac{\arg(z-z_0)}{2\pi}\right\rfloor\right)}\right)+ E(z_0\mid m)\,e^{-\pi i\left(\left\lfloor\frac{\arg(z-z_0)}{2\pi}+\frac{1}{4}\right\rfloor+\left\lfloor\frac{1}{4}-\frac{\arg(z-z_0)}{2\pi}\right\rfloor\right)}+\sum_{k=1}^{\infty}\frac{1}{k!}\sum_{j=1}^{k}\frac{1}{j!} \left(\sum_{q=0}^{j-1}\binom{j}{q}\sum_{p=0}^{j-q}(-1)^q\,2^{q-j}\sin^q(z_0)\,(2p+q-j)^k\,e^{-\frac{1}{2}i(\pi(j+k-2p-q)+2(2p+q-j)z_0)}\binom{j-q}{p}\sum_{i=0}^{j-1}\frac{(1-j)_{2(j-i)-2}}{(j-i-1)!\,(2\sin(z_0))^{j-2i-1}} \sum_{s=0}^{i}\binom{i}{s}\left(\frac{1}{2}\right)_s\left(-\frac{1}{2}\right)_{i-s}m^{i-s}\cos^{-2s-1}(z_0)\left(1-m\sin^2(z_0)\right)^{s-i+\frac{1}{2}}\right)(z-z_0)^k\;/;\;\operatorname{Re}\left(\frac{z_0}{2\pi}-\frac{3}{4}\right)\in\mathbb{Z}$$

08.04.06.0026.01

$$E(z\mid m)\propto\left(\left(2\left(2\operatorname{Re}\left(\frac{z_0}{2\pi}-\frac{3}{4}\right)+1\right)E(m)+\sqrt{m}\left(E\left(\frac{1}{m}\right)+\left(\frac{1}{m}-1\right)K\left(\frac{1}{m}\right)\right)\right)\left(1-e^{-\pi i\left(\left\lfloor\frac{\arg(z-z_0)}{2\pi}+\frac{1}{4}\right\rfloor+\left\lfloor\frac{1}{4}-\frac{\arg(z-z_0)}{2\pi}\right\rfloor\right)}\right)+ E(z_0\mid m)\,e^{-\pi i\left(\left\lfloor\frac{\arg(z-z_0)}{2\pi}+\frac{1}{4}\right\rfloor+\left\lfloor\frac{1}{4}-\frac{\arg(z-z_0)}{2\pi}\right\rfloor\right)}\right)(1+O(z-z_0))\;/;\;(z\to z_0)\bigwedge\operatorname{Re}\left(\frac{z_0}{2\pi}-\frac{3}{4}\right)\in\mathbb{Z}$$

Expansions at $z = 0$

08.04.06.0027.01

$$E(z\mid m)\propto z-\frac{m z^3}{6}-\frac{m(3m-4)}{120}z^5-\frac{m\left(16-60m+45m^2\right)}{5040}z^7-\frac{m\left(-64+1008m-2520m^2+1575m^3\right)}{362\,880}z^9+\ldots\;/;\;(z\to 0)$$

08.04.06.0001.02

$$E(z\mid m)\propto z-\frac{m z^3}{6}-\frac{m(3m-4)}{120}z^5-\frac{m\left(16-60m+45m^2\right)}{5040}z^7-\frac{m\left(-64+1008m-2520m^2+1575m^3\right)}{362\,880}z^9+O\left(z^{11}\right)$$

08.04.06.0028.01

$$E(z\mid m) = z+\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{j=0}^{k-1}\frac{\left(-\frac{1}{2}\right)_k\binom{2k}{j}(-1)^{i-j+k}\,2^{2i-2k+1}\,(j-k)^{2i}\,m^k}{k!\,(2i+1)!}z^{2i+1}$$

08.04.06.0029.01

$$E(z\mid m)\propto z+O\left(z^3\right)$$

Expansions at $z = \csc^{-1}\left(\sqrt{m}\right)+\pi u\;/;\;u\in\mathbb{Z}$

08.04.06.0030.01

$$E(z\mid m)\propto\sqrt{m}\left(E\left(\frac{1}{m}\right)+\left(\frac{1}{m}-1\right)K\left(\frac{1}{m}\right)\right)+2u\,E(m)+\frac{2\sqrt{2}}{3}\sqrt{-\sqrt{m-1}\,(z-z_0)}\,(z-z_0)\left(1+\frac{3(m-2)}{20\sqrt{m-1}}(z-z_0)-\frac{3m^2+20m-20}{224(m-1)}(z-z_0)^2+\frac{(m-2)^3}{384(m-1)^{3/2}}(z-z_0)^3+\ldots\right)\;/;\;(z\to z_0)\bigwedge z_0=\csc^{-1}\left(\sqrt{m}\right)+\pi u\bigwedge u\in\mathbb{Z}$$

08.04.06.0031.01

$$E(z \mid m) = 2\,u\,E(m) + \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right)K\left(\frac{1}{m}\right)\right) + 2\sqrt{2}\,(z - z_0)\sqrt{-\sqrt{m-1}\,(z - z_0)}\,\sum_{k=0}^{\infty}\frac{1}{2\,k+3}\binom{k-\frac{1}{2}}{k}\sum_{j=0}^{k}\frac{(-1)^j}{1-2\,j}\binom{k}{j}\,p_{j,k}\,(z - z_0)^k\;/;$$
$$z_0 = \csc^{-1}\left(\sqrt{m}\right) + \pi\,u \bigwedge u \in \mathbb{Z} \bigwedge a_0 = 1 \bigwedge a_{2\,k} = \frac{(-1)^k\,2^{2\,k}}{(2\,k+1)!} \bigwedge a_{2\,k+1} = \frac{(-1)^{k-1}\,2^{2\,k}\,(2-m)}{\sqrt{m-1}\,(2\,k+2)!} \bigwedge k \in \mathbb{N} \bigwedge p_{u,0} = 1 \bigwedge p_{u,v} = \frac{1}{v}\sum_{j=1}^{v}(u\,j + j - v)\,a_j\,p_{u,v-j}$$

08.04.06.0032.01

$$E(z \mid m) \propto \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right)K\left(\frac{1}{m}\right)\right) + 2\,u\,E(m) + \frac{2\sqrt{2}}{3}\sqrt{-\sqrt{m-1}\,(z - z_0)}\,(z - z_0)\,(1 + O(z - z_0))\;/;\; z_0 = \csc^{-1}\left(\sqrt{m}\right) + \pi\,u \bigwedge u \in \mathbb{Z}$$

Expansions at $z == -\csc^{-1}\left(\sqrt{m}\right) + \pi\,u\;/;\;u \in \mathbb{Z}$

08.04.06.0033.01

$$E(z \mid m) \propto -\sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right)K\left(\frac{1}{m}\right)\right) + 2\,u\,E(m) + \frac{2\sqrt{2}}{3}\sqrt{\sqrt{m-1}\,(z - z_0)}\,(z - z_0)\left(1 - \frac{3\,(m-2)}{20\sqrt{m-1}}(z - z_0) - \frac{3\,m^2 + 20\,m - 20}{224\,(m-1)}(z - z_0)^2 - \frac{(m-2)^3}{384\,(m-1)^{3/2}}(z - z_0)^3 + \ldots\right)\;/;\;(z \to z_0) \bigwedge z_0 = -\csc^{-1}\left(\sqrt{m}\right) + \pi\,u \bigwedge u \in \mathbb{Z}$$

08.04.06.0034.01

$$E(z \mid m) = 2\,u\,E(m) - \sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right)K\left(\frac{1}{m}\right)\right) + 2\sqrt{2}\,(z - z_0)\sqrt{\sqrt{m-1}\,(z - z_0)}\,\sum_{k=0}^{\infty}\frac{(-1)^k}{2\,k+3}\binom{k-\frac{1}{2}}{k}\sum_{j=0}^{k}\frac{(-1)^j}{1-2\,j}\binom{k}{j}\,p_{j,k}\,(z - z_0)^k\;/;$$
$$z_0 = -\csc^{-1}\left(\sqrt{m}\right) + \pi\,u \bigwedge u \in \mathbb{Z} \bigwedge a_0 = 1 \bigwedge a_{2\,k} = \frac{(-1)^k\,2^{2\,k}}{(2\,k+1)!} \bigwedge a_{2\,k+1} = \frac{(-1)^{k-1}\,2^{2\,k}\,(2-m)}{\sqrt{m-1}\,(2\,k+2)!} \bigwedge k \in \mathbb{N} \bigwedge p_{u,0} = 1 \bigwedge p_{u,v} = \frac{1}{v}\sum_{j=1}^{v}(u\,j + j - v)\,a_j\,p_{u,v-j}$$

08.04.06.0035.01

$$E(z \mid m) \propto -\sqrt{m}\left(E\left(\frac{1}{m}\right) + \left(\frac{1}{m} - 1\right)K\left(\frac{1}{m}\right)\right) + 2\,u\,E(m) + \frac{2\sqrt{2}}{3}\sqrt{\sqrt{m-1}\,(z - z_0)}\,(z - z_0)\,(1 + O(z - z_0))\;/;\; z_0 = -\csc^{-1}\left(\sqrt{m}\right) + \pi\,u \bigwedge u \in \mathbb{Z}$$

Expansions at $z == \pi/2 + 2\,\pi\,u\;/;\;u \in \mathbb{Z} \bigwedge m > 1$

08.04.06.0036.01

$$E(z \mid m) \propto \left(2 \left\lfloor \frac{\mathrm{Re}(z_0)}{\pi} \right\rfloor + \sqrt{-\frac{i}{z-z_0}}\,\sqrt{i\,(z-z_0)} + \frac{\sqrt{(z-z_0)^2}}{z-z_0} + i\,(z-z_0)\sqrt{-\frac{1}{(z-z_0)^2}}\right) E(m) - \left(i\sqrt{-\frac{1}{(z-z_0)^2}}\,(z-z_0) + \frac{\sqrt{(z-z_0)^2}}{z-z_0}\right) E\!\left(\csc^{-1}\!\left(\sqrt{m}\right) \middle| m\right) + \sqrt{1-m}\,(z-z_0) + \frac{m}{6\sqrt{1-m}}\,(z-z_0)^3 + \frac{(m-4)\,m}{120\,(1-m)^{3/2}}\,(z-z_0)^5 + \ldots \;/;\; (z \to z_0) \bigwedge z_0 = \frac{\pi}{2} + 2\pi u \bigwedge u \in \mathbb{Z} \bigwedge m \in \mathbb{R} \bigwedge m > 1$$

08.04.06.0037.01

$$E(z \mid m) = \left(2 \left\lfloor \frac{\mathrm{Re}(z_0)}{\pi} \right\rfloor + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg(z-z_0)}{\pi} \right\rfloor} - (-1)^{\left\lfloor \frac{\arg(z-z_0)}{\pi} \right\rfloor} + (-1)^{\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{3}{4} \right\rfloor + \left\lfloor \frac{1}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor}\right) E(m) - \left(-(-1)^{\left\lfloor \frac{\arg(z-z_0)}{\pi} \right\rfloor} + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg(z-z_0)}{\pi} \right\rfloor}\right) E\!\left(\csc^{-1}\!\left(\sqrt{m}\right) \middle| m\right) + \sqrt{1-m}\,(z-z_0) + \sqrt{1-m}\sum_{k=2}^{\infty} \frac{i^{k-1}}{k!} \binom{k-\frac{3}{2}}{k-1} \sum_{q=1}^{k-1} \frac{(-1)^q}{1-2q} \binom{k-1}{q} (1-m)^{-q} \sum_{j=0}^{q} \binom{q}{j} (-m)^j (2-m)^{q-j}\, 2^{k-j-q-1} \sum_{i=0}^{j} \binom{j}{i} (2i-j)^{k-1} (z-z_0)^k \;/;\; (z \to z_0) \bigwedge z_0 = \frac{\pi}{2} + 2\pi u \bigwedge u \in \mathbb{Z} \bigwedge m \in \mathbb{R} \bigwedge m > 1$$

08.04.06.0038.01

$$E(z \mid m) = \left(2 \left\lfloor \frac{\mathrm{Re}(z_0)}{\pi} \right\rfloor + \sqrt{-\frac{i}{z-z_0}}\,\sqrt{i\,(z-z_0)} + \frac{\sqrt{(z-z_0)^2}}{z-z_0} + i\,(z-z_0)\sqrt{-\frac{1}{(z-z_0)^2}}\right) E(m) - \left(i\sqrt{-\frac{1}{(z-z_0)^2}}\,(z-z_0) + \frac{\sqrt{(z-z_0)^2}}{z-z_0}\right) E\!\left(\csc^{-1}\!\left(\sqrt{m}\right) \middle| m\right) + \sqrt{1-m}\,(z-z_0) + \sqrt{1-m}\sum_{k=2}^{\infty} \frac{i^{k-1}}{k!} \binom{k-\frac{3}{2}}{k-1} \sum_{q=1}^{k-1} \frac{(-1)^q}{1-2q} \binom{k-1}{q} (1-m)^{-q} \sum_{j=0}^{q} \binom{q}{j} (-m)^j (2-m)^{q-j}\, 2^{k-j-q-1} \sum_{i=0}^{j} \binom{j}{i} (2i-j)^{k-1} (z-z_0)^k \;/;\; (z \to z_0) \bigwedge z_0 = \frac{\pi}{2} + 2\pi u \bigwedge u \in \mathbb{Z} \bigwedge m \in \mathbb{R} \bigwedge m > 1$$

08.04.06.0039.01

$$E(z \mid m) \propto \left(\left(2 \left\lfloor \frac{\mathrm{Re}(z_0)}{\pi} \right\rfloor + \sqrt{-\frac{i}{z-z_0}}\,\sqrt{i\,(z-z_0)} + \frac{\sqrt{(z-z_0)^2}}{z-z_0} + i\,(z-z_0)\sqrt{-\frac{1}{(z-z_0)^2}}\right) E(m) - \left(i\sqrt{-\frac{1}{(z-z_0)^2}}\,(z-z_0) + \frac{\sqrt{(z-z_0)^2}}{z-z_0}\right) E\!\left(\csc^{-1}\!\left(\sqrt{m}\right) \middle| m\right)\right) (1 + O(z-z_0)) \;/;\; z_0 = \frac{\pi}{2} + 2\pi u \bigwedge u \in \mathbb{Z} \bigwedge m \in \mathbb{R} \bigwedge m > 1$$

Expansions at $z = 3\pi/2 + 2\pi u /; u \in \mathbb{Z} \bigwedge m > 1$

08.04.06.0040.01

$$E(z \mid m) \propto \left(2\left\lfloor \frac{\operatorname{Re}(z_0)}{\pi} \right\rfloor - \sqrt{\frac{i}{z-z_0}}\,\sqrt{-i\,(z-z_0)} + \frac{\sqrt{(z-z_0)^2}}{z-z_0} + i\,(z-z_0)\sqrt{-\frac{1}{(z-z_0)^2}}\right) E(m) - \left(i\sqrt{-\frac{1}{(z-z_0)^2}}\,(z-z_0) + \frac{\sqrt{(z-z_0)^2}}{z-z_0}\right) E\!\left(\csc^{-1}\!\left(\sqrt{m}\right) \,\middle|\, m\right) + \sqrt{1-m}\,(z-z_0) + \frac{m}{6\sqrt{1-m}}\,(z-z_0)^3 + \frac{(m-4)\,m}{120\,(1-m)^{3/2}}\,(z-z_0)^5 + \ldots \;/;\; (z \to z_0) \bigwedge z_0 = \frac{3\pi}{2} + 2\pi u \bigwedge u \in \mathbb{Z} \bigwedge m \in \mathbb{R} \bigwedge m > 1$$

08.04.06.0041.01

$$E(z \mid m) == \left(2\left\lfloor \frac{\operatorname{Re}(z_0)}{\pi} \right\rfloor + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg(z-z_0)}{\pi} \right\rfloor} - (-1)^{\left\lfloor \frac{\arg(z-z_0)}{\pi} \right\rfloor} - (-1)^{\left\lfloor \frac{\arg(z-z_0)}{2\pi} + \frac{1}{4} \right\rfloor + \left\lfloor \frac{3}{4} - \frac{\arg(z-z_0)}{2\pi} \right\rfloor}\right) E(m) - \left(-(-1)^{\left\lfloor \frac{\arg(z-z_0)}{\pi} \right\rfloor} + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg(z-z_0)}{\pi} \right\rfloor}\right) E\!\left(\csc^{-1}\!\left(\sqrt{m}\right) \,\middle|\, m\right) + \sqrt{1-m}\,(z-z_0) + \sqrt{1-m}\sum_{k=2}^{\infty} \frac{i^{k-1}}{k!} \binom{k-\frac{3}{2}}{k-1} \sum_{q=1}^{k-1} \frac{(-1)^q}{1-2q} \binom{k-1}{q} (1-m)^{-q} \sum_{j=0}^{q} \binom{q}{j} (-m)^j\,(2-m)^{q-j}\,2^{k-j-q-1} \sum_{i=0}^{j} \binom{j}{i} (2i-j)^{k-1}\,(z-z_0)^k \;/;\; (z \to z_0) \bigwedge z_0 == \frac{3\pi}{2} + 2\pi u \bigwedge u \in \mathbb{Z} \bigwedge m \in \mathbb{R} \bigwedge m > 1$$

08.04.06.0042.01

$$E(z \mid m) == \left(2\left\lfloor \frac{\operatorname{Re}(z_0)}{\pi} \right\rfloor - \sqrt{\frac{i}{z-z_0}}\,\sqrt{-i\,(z-z_0)} + \frac{\sqrt{(z-z_0)^2}}{z-z_0} + i\,(z-z_0)\sqrt{-\frac{1}{(z-z_0)^2}}\right) E(m) - \left(i\sqrt{-\frac{1}{(z-z_0)^2}}\,(z-z_0) + \frac{\sqrt{(z-z_0)^2}}{z-z_0}\right) E\!\left(\csc^{-1}\!\left(\sqrt{m}\right) \,\middle|\, m\right) + \sqrt{1-m}\,(z-z_0) + \sqrt{1-m}\sum_{k=2}^{\infty} \frac{i^{k-1}}{k!} \binom{k-\frac{3}{2}}{k-1} \sum_{q=1}^{k-1} \frac{(-1)^q}{1-2q} \binom{k-1}{q} (1-m)^{-q} \sum_{j=0}^{q} \binom{q}{j} (-m)^j\,(2-m)^{q-j}\,2^{k-j-q-1} \sum_{i=0}^{j} \binom{j}{i} (2i-j)^{k-1}\,(z-z_0)^k \;/;\; (z \to z_0) \bigwedge z_0 == \frac{3\pi}{2} + 2\pi u \bigwedge u \in \mathbb{Z} \bigwedge m \in \mathbb{R} \bigwedge m > 1$$

08.04.06.0043.01

$$E(z \mid m) \propto \left(\left(2\left\lfloor \frac{\operatorname{Re}(z_0)}{\pi} \right\rfloor - \sqrt{\frac{i}{z-z_0}}\,\sqrt{-i\,(z-z_0)} + \frac{\sqrt{(z-z_0)^2}}{z-z_0} + i\,(z-z_0)\sqrt{-\frac{1}{(z-z_0)^2}}\right) E(m) - \left(i\sqrt{-\frac{1}{(z-z_0)^2}}\,(z-z_0) + \frac{\sqrt{(z-z_0)^2}}{z-z_0}\right) E\!\left(\csc^{-1}\!\left(\sqrt{m}\right) \,\middle|\, m\right)\right) (1 + O(z-z_0)) \;/;\; z_0 == \frac{3\pi}{2} + 2\pi u \bigwedge u \in \mathbb{Z} \bigwedge m \in \mathbb{R} \bigwedge m > 1$$

Expansions at $z == \infty$

08.04.06.0044.01

$$E(z\mid m)\propto(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil}\left(\frac{\sqrt{-\sin^2(z)}\,\sqrt{-m}}{2\sin(z)}\left(2E\left(\frac{1}{m}\right)+\left(\frac{i\sqrt{-\sin^2(z)}}{\sqrt{\sin^2(z)}}\left(\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)-\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)E(m)+\right.\right.$$
$$2i\left(1-\sqrt{\frac{m}{m-1}}\,\sqrt{\frac{m-1}{m}}\right)\left(\frac{1}{m}K\left(1-\frac{1}{m}\right)-E\left(1-\frac{1}{m}\right)\right)+\frac{2(1-m)}{m}K\left(\frac{1}{m}\right)\Bigg)-$$
$$\csc(z)\sqrt{-\sin^2(z)}\,\sqrt{-m\sin^2(z)}\left(1-\frac{m+1}{2m}\csc^2(z)-\frac{1}{8}\left(1-\frac{1}{m}\right)^2\csc^4(z)+O\left(\csc^6(z)\right)\right)+$$
$$\left.\frac{\csc(z)\sqrt{-\sin^2(z)}}{\sqrt{-m\sin^2(z)}}\left(1-\frac{m+1}{6m}\csc^2(z)-\frac{3m^2+2m+3}{120m^2}\csc^4(z)+O\left(\csc^6(z)\right)\right)\right)+2\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil E(m)\;/;\;(|z|\to\infty)$$

08.04.06.0045.01

$$E(z\mid m)=$$
$$2\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil E(m)+(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil}\left(\frac{\sqrt{-\sin^2(z)}\,\sqrt{-m}}{2\sin(z)}\left(2E\left(\frac{1}{m}\right)+\left(\frac{i\sqrt{-\sin^2(z)}}{\sqrt{\sin^2(z)}}\left(\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)-\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)E(m)+\right.\right.$$
$$2i\left(1-\sqrt{\frac{m}{m-1}}\,\sqrt{\frac{m-1}{m}}\right)\left(\frac{1}{m}K\left(1-\frac{1}{m}\right)-E\left(1-\frac{1}{m}\right)\right)+\frac{2(1-m)}{m}K\left(\frac{1}{m}\right)\Bigg)-\frac{\sqrt{-m\sin^2(z)}\,\sqrt{-\sin^2(z)}}{\sin(z)}$$
$$\left.\sum_{k=0}^{\infty}\frac{\sin^{-2k}(z)\left(-\frac{1}{2}\right)_k}{k!}\,{}_2F_1\left(-k,-\frac{1}{2};\frac{3}{2}-k;\frac{1}{m}\right)+\frac{\sqrt{-\sin^2(z)}}{\sin(z)\sqrt{-m\sin^2(z)}}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\sin^{-2k}(z)}{(2k+1)\,k!}\,{}_2F_1\left(\frac{1}{2},-k;\frac{1}{2}-k;\frac{1}{m}\right)\right)$$

08.04.06.0046.01

$$E(z\mid m)\propto(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil}\left(\sin(z)\frac{\sqrt{-m\sin^2(z)}}{\sqrt{-\sin^2(z)}}\left(1+O\left(\csc^2(z)\right)\right)+\right.$$
$$\frac{\sqrt{-\sin^2(z)}\,\sqrt{-m}}{2\sin(z)}\left(2E\left(\frac{1}{m}\right)+\left(\frac{i\sqrt{-\sin^2(z)}}{\sqrt{\sin^2(z)}}\left(\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)-\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)E(m)+\right.$$
$$2i\left(1-\sqrt{\frac{m}{m-1}}\,\sqrt{\frac{m-1}{m}}\right)\left(\frac{1}{m}K\left(1-\frac{1}{m}\right)-E\left(1-\frac{1}{m}\right)\right)+\frac{2(1-m)}{m}K\left(\frac{1}{m}\right)\Bigg)+$$
$$\left.\frac{\csc(z)\sqrt{-\sin^2(z)}}{\sqrt{-m\sin^2(z)}}\left(1+O\left(\csc^2(z)\right)\right)\right)+2\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil E(m)\;/;\;(|z|\to\infty)$$

Expansions at generic point $m == m_0$

For the function itself

08.04.06.0047.01

$$E(z \mid m) \propto E(z \mid m_0) + \frac{1}{2\, m_0} (E(z \mid m_0) - F(z \mid m_0))\, (m - m_0) + \frac{1}{16\, (m_0 - 1)\, m_0^2 \sqrt{1 - m_0 \sin^2(z)}} \left(2 \sqrt{1 - m_0 \sin^2(z)}\, (2\, (m_0 - 1)\, F(z \mid m_0) - (m_0 - 2)\, E(z \mid m_0)) - m_0 \sin(2\, z)\right) (m - m_0)^2 + \ldots \;/;\; (m \to m_0)$$

08.04.06.0048.01

$$E(z \mid m) \propto E(z \mid m_0) + \frac{1}{2\, m_0} (E(z \mid m_0) - F(z \mid m_0))\, (m - m_0) + \frac{1}{16\, (m_0 - 1)\, m_0^2 \sqrt{1 - m_0 \sin^2(z)}} \left(2 \sqrt{1 - m_0 \sin^2(z)}\, (2\, (m_0 - 1)\, F(z \mid m_0) - (m_0 - 2)\, E(z \mid m_0)) - m_0 \sin(2\, z)\right) (m - m_0)^2 + O\!\left((m - m_0)^3\right)$$

08.04.06.0049.01

$$E(z \mid m) = \frac{\sqrt{\pi}}{2} \sum_{k=0}^{\infty} \frac{(-1)^k \sin^{2k+1}(z)}{k!\, (2\, k + 1)\, \Gamma\!\left(\frac{3}{2} - k\right)} F_1\!\left(k + \frac{1}{2}; \frac{1}{2}, k - \frac{1}{2}; k + \frac{3}{2}; \sin^2(z), m_0 \sin^2(z)\right) (m - m_0)^k$$

08.04.06.0050.01

$$E(z \mid m) = \sum_{k=0}^{\infty} \frac{1}{k!} E^{(0,k)}(z \mid m_0)\, (m - m_0)^k$$

08.04.06.0051.01

$$E(z \mid m) \propto E(z \mid m_0)\, (1 + O(m - m_0))$$

Expansions at $m == 0$

08.04.06.0002.02

$$E(z \mid m) \propto z - \frac{2\, z - \sin(2\, z)}{8} m - \frac{1}{256} (12\, z - 8 \sin(2\, z) + \sin(4\, z))\, m^2 + \ldots \;/;\; (m \to 0)$$

08.04.06.0052.01

$$E(z \mid m) \propto z - \frac{2\, z - \sin(2\, z)}{8} m - \frac{1}{256} (12\, z - 8 \sin(2\, z) + \sin(4\, z))\, m^2 + O\!\left(m^3\right)$$

08.04.06.0053.01

$$E\,(z \mid m) \propto \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(-\frac{1}{2}\right)_k}{k!^2} \left(z - \frac{\cot(z)}{2} \sum_{j=1}^{k} \frac{(j-1)!\, \sin^{2j}(z)}{\left(\frac{1}{2}\right)_j}\right) m^k \;/;\; |m| < 1$$

08.04.06.0054.01

$$E(z \mid m) \propto \frac{\sin(2\, z)}{2} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(-\frac{1}{2}\right)_k \sin^{2k}(z)}{\left(\frac{3}{2}\right)_k k!} {}_2F_1\!\left(1, k + 1; k + \frac{3}{2}; \sin^2(z)\right) m^k + 2 \left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rceil E(m) \;/;\; |m| < 1$$

08.04.06.0004.02

$$E(z \mid m) = \sum_{k=0}^{\infty} \frac{1}{2^{2k}\, k!} \left(-\frac{1}{2}\right)_k \left(z \binom{2\, k}{k} + \sum_{j=1}^{k} \frac{(-1)^j}{j} \binom{2\, k}{k - j} \sin(2\, j\, z)\right) m^k \;/;\; |m| < 1$$

08.04.06.0007.01

$$E(z\mid m)=E(m)-\cos(z)\left(F_{2\times 0\times 0}^{1\times 2\times 2}\left(\begin{matrix}\frac{1}{2};-\frac{1}{2},\frac{1}{2};\frac{1}{2},1;\\ 1,\frac{3}{2};;;\end{matrix}-m\cos^2(z),\cos^2(z)\right)-\frac{m}{2}F_{1\times 1\times 1}^{2\times 1\times 1}\left(\begin{matrix}\frac{1}{2},\frac{3}{2};\frac{1}{2};1;\\ 2;\frac{3}{2};\frac{3}{2};\end{matrix}-m\cos^2(z),m\right)\right)$$

08.04.06.0005.01

$$E(z\mid m)=E(m)-\cos(z)\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{1}{2}\right)_k {}_2F_1\left(\frac{1}{2},\frac{1}{2}-k;\frac{3}{2};\cos^2(z)\right)m^k\;/;\;|m|<1\bigwedge-\frac{\pi}{2}\leq z\leq\frac{\pi}{2}$$

08.04.06.0055.01

$$E(z\mid m)=z+z\sum_{k=1}^{\infty}\left(\frac{\left(\frac{1}{2}\right)_k\left(-\frac{1}{2}\right)_k}{k!^2}+\frac{(-1)^k}{4^k\,k}\binom{k-\frac{3}{2}}{k}{}_2F_1\left(k-\frac{1}{2},k+\frac{1}{2};2k+1;m\right)\sin(2kz)\right)m^k\;/;\;|m|<1$$

08.04.06.0003.02

$$E(z\mid m)=\frac{2z}{\pi}E(m)+\sum_{k=1}^{\infty}\frac{(-1)^k\sin(2kz)}{2^{2k}\,k}\binom{k-\frac{3}{2}}{k}{}_2F_1\left(k-\frac{1}{2},k+\frac{1}{2};2k+1;m\right)m^k$$

08.04.06.0006.01

$$E(z\mid m)=\frac{\pi}{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\frac{1}{2}\right)_k m^k}{k!^2}-\cos(z)\left(\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_j\left(\frac{1}{2}\right)_j\left(\frac{1}{2}\right)_k(-m)^j\cos^{2j+2k}(z)}{(2j+2k+1)\,(j+k)!\,j!}-\frac{m}{2}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{j+k}\left(\frac{3}{2}\right)_{j+k}(-1)^j m^{j+k}\cos^{2j}(z)}{(j+k+1)!\,j!\,(2j+1)\left(\frac{3}{2}\right)_k}\right)$$

08.04.06.0056.01

$$E(z\mid m)\propto z+O(m)$$

Expansions at $m = 1$

08.04.06.0057.01

$$E(z\mid m)\propto(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil}\left(\sin(z)+\frac{1}{2}\left(\sin(z)-\tanh^{-1}(\sin(z))\right)(m-1)+\frac{1}{32}\sec^2(z)\left(6\tanh^{-1}(\sin(z))\cos^2(z)-3\sin(z)-\sin(3z)\right)(m-1)^2+\ldots\right)+2\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil\left(\left(1+\frac{1}{4}(m-1)\left(1+\frac{3(1-m)}{8}+\frac{15}{64}(1-m)^2+\ldots\right)\log(1-m)+\frac{1}{4}(m-1)\left(1-4\log(2)+\frac{1}{16}(24\log(2)-13)(m-1)-\frac{3}{16}(5\log(2)-3)(m-1)^2+\ldots\right)\right)\right)\;/;\;(m\to 1)\bigwedge\neg\frac{2\,\mathrm{Re}(z)+\pi}{4\pi}\in\mathbb{Z}$$

08.04.06.0058.01

$$E(z\mid m)=(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil}\sin(z)\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^{-j}\left(-\frac{1}{2}\right)_j(j)_i\csc^{2i}(z)}{(1-2i)\,j!\,i!}(m-1)^j+2\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil\left(1+\frac{m-1}{4}\log(1-m)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{k!\,(k+1)!}(1-m)^k+\frac{1}{4}(m-1)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{k!\,(k+1)!}\left(-2\psi(k+1)+2\psi\left(k+\frac{1}{2}\right)+\frac{1}{2k^2+3k+1}\right)(1-m)^k\right)\;/;\;\neg\frac{2\,\mathrm{Re}(z)+\pi}{4\pi}\in\mathbb{Z}$$

08.04.06.0059.01

$$E(z \mid m) = (-1)^{\left\lceil \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \sin(z) \left(1 + \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)_k}{k!\,\left(2k - \sin^2(z) - 1\right)} \left(\cos^2(z) \tan^{2k}(z) + \frac{2\,(-1)^k \left(\frac{1}{2}\right)_k}{(k-1)!} \left(\csc^2(z) \sum_{j=1}^{k-1} \frac{(-1)^j\,(j-1)!\,\left(\sin^2(z) + j - k\right)}{\left(\frac{1}{2}\right)_j} \tan^{2j}(z) - \tanh^{-1}(\sin(z)) \csc(z) \left(2k - \sin^2(z) - 1\right) + 1\right)\right) (m-1)^k\right) + 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil \left(1 + \frac{m-1}{4} \log(1-m) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} (1-m)^k + \frac{1}{4}(m-1) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} \left(-2\,\psi(k+1) + 2\,\psi\left(k + \frac{1}{2}\right) + \frac{1}{2k^2 + 3k + 1}\right) (1-m)^k\right) /;\ \neg \frac{2 \operatorname{Re}(z) + \pi}{4\pi} \in \mathbb{Z}$$

08.04.06.0060.01

$$E(z \mid m) = (-1)^{\left\lceil \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \sum_{j=0}^{\infty} \frac{\left(-\frac{1}{2}\right)_j \sin^{2j+1}(z)}{(2j+1)\,j!} \, {}_2F_1\left(j, j + \frac{1}{2}; j + \frac{3}{2}; \sin^2(z)\right) (m-1)^j + 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil \left(1 + \frac{m-1}{4} \log(1-m) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} (1-m)^k + \frac{1}{4}(m-1) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} \left(-2\,\psi(k+1) + 2\,\psi\left(k + \frac{1}{2}\right) + \frac{1}{2k^2 + 3k + 1}\right) (1-m)^k\right) /;\ \neg \frac{2 \operatorname{Re}(z) + \pi}{4\pi} \in \mathbb{Z}$$

08.04.06.0061.01

$$E(z \mid m) = (-1)^{\left\lceil \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \sum_{j=0}^{\infty} \frac{\left(-\frac{1}{2}\right)_j \sin^{2j+1}(z)}{(2j+1)\,j!} \, {}_2F_1\left(j, j + \frac{1}{2}; j + \frac{3}{2}; \sin^2(z)\right) (m-1)^j + 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil E(m) /;\ \neg \frac{2 \operatorname{Re}(z) + \pi}{4\pi} \in \mathbb{Z}$$

08.04.06.0062.01

$$E(z \mid m) \propto 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil + (-1)^{\left\lceil \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \sin(z) + O(m-1) /;\ \neg \frac{2 \operatorname{Re}(z) + \pi}{4\pi} \in \mathbb{Z}$$

Expansions at $m = \infty$

08.04.06.0063.01

$$E(z \mid m) \propto (-1)^{\left\lceil \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \left(\sin(z) + \frac{1}{2}\left(\sin(z) - \tanh^{-1}(\sin(z))\right)(m-1) + \frac{1}{32} \sec^2(z) \left(6 \tanh^{-1}(\sin(z)) \cos^2(z) - 3\sin(z) - \sin(3z)\right)(m-1)^2 + \ldots\right) + 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil \left(\left(1 + \frac{1}{4}(m-1)\left(1 + \frac{3(1-m)}{8} + \frac{15}{64}(1-m)^2 + \ldots\right) \log(1-m) + \frac{1}{4}(m-1)\left(1 - 4\log(2) + \frac{1}{16}(24\log(2) - 13)(m-1) - \frac{3}{16}(5\log(2) - 3)(m-1)^2 + \ldots\right)\right)\right) /;\ (m \to 1)$$

08.04.06.0064.01

$$E(z \mid m) = (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \sin(z) \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^{-j} \left(-\frac{1}{2}\right)_j (j)_i \csc^{2i}(z)}{(1-2i)\, j!\, i!} (m-1)^j + 2 \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil \left(1 + \frac{m-1}{4} \log(1-m) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} (1-m)^k + \frac{1}{4}(m-1) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} \left(-2\psi(k+1) + 2\psi\left(k+\frac{1}{2}\right) + \frac{1}{2k^2+3k+1}\right)(1-m)^k\right)$$

08.04.06.0065.01

$$E(z \mid m) = (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \sin(z) \left(1 + \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)_k}{k!\left(2k - \sin^2(z) - 1\right)} \left(\cos^2(z) \tan^{2k}(z) + \frac{2(-1)^k \left(\frac{1}{2}\right)_k}{(k-1)!} \left(\csc^2(z) \sum_{j=1}^{k-1} \frac{(-1)^j (j-1)! \left(\sin^2(z) + j - k\right)}{\left(\frac{1}{2}\right)_j} \tan^{2j}(z) - \tanh^{-1}(\sin(z)) \csc(z) \left(2k - \sin^2(z) - 1\right) + 1\right)\right)(m-1)^k\right) + 2 \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil \left(1 + \frac{m-1}{4} \log(1-m) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} (1-m)^k + \frac{1}{4}(m-1) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} \left(-2\psi(k+1) + 2\psi\left(k+\frac{1}{2}\right) + \frac{1}{2k^2+3k+1}\right)(1-m)^k\right)$$

08.04.06.0066.01

$$E(z \mid m) = (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \sum_{j=0}^{\infty} \frac{\left(-\frac{1}{2}\right)_j \sin^{2j+1}(z)}{(2j+1)\, j!} \, {}_2F_1\left(j, j+\frac{1}{2}; j+\frac{3}{2}; \sin^2(z)\right)(m-1)^j + 2 \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil \left(1 + \frac{m-1}{4} \log(1-m) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} (1-m)^k + \frac{1}{4}(m-1) \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \left(\frac{3}{2}\right)_k}{k!\,(k+1)!} \left(-2\psi(k+1) + 2\psi\left(k+\frac{1}{2}\right) + \frac{1}{2k^2+3k+1}\right)(1-m)^k\right)$$

08.04.06.0067.01

$$E(z \mid m) = 4E(m) \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil + \frac{2\sin^{-1}\left(\sqrt{m}\, \sin(z)\right)}{\pi\sqrt{m}} \left(m\, E\left(\frac{1}{m}\right) - (m-1)\, K\left(\frac{1}{m}\right)\right) + \sqrt{1 - m\sin^2(z)}\, \tan\left(\frac{z}{2}\right) - \frac{\sin(z)\sqrt{1-m\sin^2(z)}}{8m} \sum_{j=0}^{\infty} \frac{\left(\frac{3}{2}\right)_j^2}{(j+1)!\,(j+2)!} \, {}_4F_3\left(1, 1, j+\frac{3}{2}, j+\frac{3}{2}; \frac{3}{2}, j+2, j+3; \sin^2(z)\right) m^{-j} \;/;\; (|m| \to \infty)$$

08.04.06.0068.01

$$E(z \mid m) \propto 2 \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \sin(z) + O(m-1)$$

Expansions at $m == \infty$ NEW E

08.04.06.0069.01

$$E(z \mid m) \propto (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rfloor} \left(\sin(z) + \frac{1}{2} \left(\sin(z) - \tanh^{-1}(\sin(z)) \right) (m-1) + \frac{1}{32} \sec^2(z) \left(6 \tanh^{-1}(\sin(z)) \cos^2(z) - 3 \sin(z) - \sin(3z) \right) (m-1)^2 + \ldots \right) + 2 \left\lfloor \frac{\text{Re}(z)}{\pi} \right\rceil \left(1 + \frac{1}{4} (m-1) \left(1 + \frac{3(1-m)}{8} + \frac{15}{64} (1-m)^2 + \ldots \right) \log(1-m) + \frac{1}{4} (m-1) \left(1 - 4 \log(2) + \frac{1}{16} (24 \log(2) - 13) (m-1) - \frac{3}{16} (5 \log(2) - 3) (m-1)^2 + \ldots \right) \right) /; (m \to 1)$$

08.04.06.0070.01

$$E(z \mid m) \propto (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rceil} \left(\sin(z) + \frac{1}{2} \left(\sin(z) - \tanh^{-1}(\sin(z)) \right) (m-1) + \frac{1}{32} \sec^2(z) \left(6 \tanh^{-1}(\sin(z)) \cos^2(z) - 3 \sin(z) - \sin(3z) \right) (m-1)^2 + \ldots \right) + 2 \left\lfloor \frac{\text{Re}(z)}{\pi} \right\rceil \left(1 + \frac{1}{4} (m-1) \left(1 + \frac{3(1-m)}{8} + \frac{15}{64} (1-m)^2 + \ldots \right) \log(1-m) + \frac{1}{4} (m-1) \left(1 - 4 \log(2) + \frac{1}{16} (24 \log(2) - 13) (m-1) - \frac{3}{16} (5 \log(2) - 3) (m-1)^2 + \ldots \right) \right) /; (m \to 1)$$

08.04.06.0071.01

$$E(z \mid m) \propto (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rceil} \frac{\sqrt{-m \sin^2(z)}}{\sin(z)} \left(1 - \cos(z) + \frac{2 \log\left(\cos^2\left(\frac{z}{2}\right)\right) - \log\left(-4 m \sin^2(z)\right) - 1}{4 m} + \frac{4 \cos(z) \csc^2(z) + 4 \log\left(\cos^2\left(\frac{z}{2}\right)\right) - 2 \log\left(-4 m \sin^2(z)\right) + 3}{64 m^2} + \frac{2 \cos(z) \left(2 \csc^2(z) + 3\right) \csc^2(z) + 6 \log\left(\cos^2\left(\frac{z}{2}\right)\right) - 3 \log\left(-4 m \sin^2(z)\right) + 6}{256 m^3} + \ldots \right) + 2 \left\lfloor \frac{\text{Re}(z)}{\pi} \right\rceil \left(\sqrt{-m} + \frac{\log(-m)}{4 \sqrt{-m}} \left(1 + \frac{1}{8 m} + \frac{3}{64 m^2} + \ldots \right) + \frac{1}{\sqrt{-m}} \left(\frac{1}{4} + \log(2) + \frac{8 \log(2) - 3}{64 m} + \frac{6 \log(2) - 3}{128 m^2} + \ldots \right) \right) /; (|m| \to \infty)$$

08.04.06.0072.01

$$E(z \mid m) = 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil E(m) + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \frac{\sqrt{-m\sin^2(z)}}{4\,m\sin(z)} \left(2\sqrt{1-\frac{\csc^2(z)}{m}}\; m\sin^2(z) + 2\,m\sin^2(z)\sum_{k=0}^{\infty} \frac{m^{-k}\left(\frac{1}{2}\right)_k\left(-\frac{1}{2}\right)_k}{(k!)^2}\,{}_3F_2\left(1,1,k+\frac{1}{2};2,k+1;\sin^2(z)\right) + \frac{1}{8\,m}\,{}_4F_3\left(1,1,\frac{3}{2},\frac{3}{2};2,2,3;\frac{1}{m}\right) - 2\log\left(\sqrt{1-\frac{\csc^2(z)}{m}}+1\right) + \frac{1}{\pi}\left(\pi\left(\cos(2z)\,m + 3\,m + \log(4) + 1\right) + 4\,(m-1)\,K\left(\frac{1}{m}\right)\left(\log\left(-4\,m\sin^2(z)\right)+2\right) - 4\,m\,E\left(\frac{1}{m}\right)\left(\log\left(-4\,m\sin^2(z)\right)+4\right)\right) + \frac{3}{8\,m^2\sin^2(z)}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}\right)_k\left(\frac{5}{2}\right)_k m^{-k}}{(k+1)!}\,{}_3\tilde{F}_2\left(1,1,k+\frac{5}{2};k+4,2;\frac{\csc^2(z)}{m}\right) + \frac{9}{8\,m^2}\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}\right)_k^2 m^{-k}}{(k+2)!\,(k+3)!}\sum_{j=0}^{k}\frac{1}{j+k+3}\right)$$

08.04.06.0073.01

$$E(z \mid m) = 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil E(m) + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \frac{\sqrt{-m\sin^2(z)}}{4\,m\sin(z)} \left(2\,m\sin^2(z)\sqrt{1-\frac{\csc^2(z)}{m}} - \log\left(-m\sin^2(z)\right) - 2\log\left(\sqrt{1-\frac{\csc^2(z)}{m}}+1\right) + \sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}\right)_k m^{-k}}{k!}\left(2\,m\sin^2(z)\sum_{i=0}^{k}\frac{\left(-\frac{1}{2}\right)_{k-i} m^i \sin^{2i}(z)}{(i+1)\,(k-i)!} - \frac{\left(\frac{1}{2}\right)_k}{(k+1)!}\left(\log\left(-4\,m\sin^2(z)\right) - \sum_{i=0}^{k-2}\frac{2}{i+k+1} - \frac{1}{k} + \frac{1}{k+1}\right) + \frac{\left(\frac{3}{2}\right)_k}{2\,m\sin^2(z)}\,{}_3\tilde{F}_2\left(1,1,k+\frac{3}{2};k+3,2;\frac{\csc^2(z)}{m}\right)\right)\right)$$

08.04.06.0074.01

$$E(z \mid m) = 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil E(m) + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \frac{\sqrt{-m\sin^2(z)}}{4\,m\sin(z)} \sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k}{k!\,m^k}\left(2\,m\sin^2(z)\sum_{i=0}^{k}\frac{\left(-\frac{1}{2}\right)_{k-i} m^i\sin^{2i}(z)}{(i+1)\,(k-i)!} + \frac{\left(\frac{3}{2}\right)_k}{2\,m\sin^2(z)}\,{}_3\tilde{F}_2\left(1,1,k+\frac{3}{2};k+3,2;\frac{\csc^2(z)}{m}\right) - \frac{\left(\frac{1}{2}\right)_k}{(k+1)!}\left(\log\left(-m\sin^2(z)\right) + \psi(k+2) - \psi\left(k+\frac{1}{2}\right)\right)\right)$$

08.04.06.0075.01

$$E(z \mid m) = 2\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil E(m) + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \sin(z)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k \sin^{2k}(z)}{k!\,(2k+1)}\,{}_2F_1\left(k+\frac{1}{2},-\frac{1}{2};k+\frac{3}{2};m\sin^2(z)\right)$$

08.04.06.0076.01

$$E(z \mid m) \propto 2 \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil \left(\frac{4 \log(2) + 1}{4 \sqrt{-m}} \left(1 + O\left(\frac{1}{m}\right)\right) + \frac{\log(-m)}{4 \sqrt{-m}} \left(1 + O\left(\frac{1}{m}\right)\right) + \sqrt{-m} \right) + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \left(1 + O\left(\frac{1}{m}\right)\right) \sqrt{-m \sin^2(z)} \tan\left(\frac{z}{2}\right) /; (|m| \to \infty)$$

Residue representations

08.04.06.0008.01

$$E(z \mid m) = -\frac{\sin(z)}{4 \pi} \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \operatorname{res}_{s,t}\left(\frac{\Gamma\left(\frac{1}{2} - s - t\right) \Gamma(s) \Gamma\left(\frac{1}{2} - s\right) \Gamma(t) \Gamma\left(-\frac{1}{2} - t\right)}{\Gamma\left(\frac{3}{2} - s - t\right)} \left(-\sin^2(z)\right)^{-s} \left(-m \sin^2(z)\right)^{-t} \right)(-j, -k)$$

Other series representations

Expansions $E\left(\sin^{-1}(z) \mid m\right)$ at $z == 0$

08.04.06.0077.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto z + \frac{1 - m}{6} z^3 + \frac{3 - 2m - m^2}{40} z^5 + \ldots /; (z \to 0)$$

08.04.06.0078.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto z + \frac{1 - m}{6} z^3 + \frac{3 - 2m - m^2}{40} z^5 + O\left(z^7\right)$$

08.04.06.0079.01

$$E\left(\sin^{-1}(z) \mid m\right) = \sum_{k=0}^{\infty} \frac{m^k \left(-\frac{1}{2}\right)_k}{(2k + 1)\, k!} \, {}_2F_1\left(\frac{1}{2}, -k; \frac{1}{2} - k; \frac{1}{m}\right) z^{2k+1} /; |z| < 1$$

08.04.06.0080.01

$$E\left(\sin^{-1}(z) \mid m\right) = z \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{j+k} \left(\frac{1}{2}\right)_k \left(-\frac{1}{2}\right)_j m^j}{\left(\frac{3}{2}\right)_{j+k} k!\, j!} z^{2j+2k} /; |z| < 1$$

08.04.06.0081.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto z + O\left(z^3\right)$$

Expansions $E\left(\sin^{-1}(z) \mid m\right)$ at $z == \infty$

08.04.06.0082.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto \frac{m z \sqrt{-z^2}}{\sqrt{-m z^2}} + \frac{\sqrt{-z^2} \sqrt{-m}}{2z} \left(2 E\left(\frac{1}{m}\right) + \left(\frac{i \sqrt{-z^2}}{\sqrt{z^2}} \left(\sqrt{\frac{1}{m}} - \frac{1}{\sqrt{m}} \right) - \sqrt{\frac{1}{m}} - \frac{1}{\sqrt{m}} \right) E(m) + 2 i \left(1 - \sqrt{\frac{m}{m-1}} \sqrt{\frac{m-1}{m}}\right) \left(\frac{1}{m} K\left(1 - \frac{1}{m}\right) - E\left(1 - \frac{1}{m}\right) \right) + \frac{2(1-m)}{m} K\left(\frac{1}{m}\right) \right) + \frac{(1-m)\sqrt{-z^2}}{2z\sqrt{-m z^2}} \left(1 - \frac{-3m^2 + 2m - 7}{12(m-1) m z^2} + \frac{15 m^3 - 9 m^2 - 11 m + 21}{120 (m-1) m^2 z^4} + \ldots \right) /; (|z| \to \infty)$$

08.04.06.0083.01

$$E(\sin^{-1}(z)\mid m)\propto\frac{m\,z\sqrt{-z^2}}{\sqrt{-m\,z^2}}+\frac{\sqrt{-z^2}\,\sqrt{-m}}{2\,z}\left(2\,E\left(\frac{1}{m}\right)+\left(\frac{i\sqrt{-z^2}}{\sqrt{z^2}}\left(\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)-\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)E(m)+2\,i\left(1-\sqrt{\frac{m}{m-1}}\,\sqrt{\frac{m-1}{m}}\right)\left(\frac{1}{m}K\left(1-\frac{1}{m}\right)-E\left(1-\frac{1}{m}\right)\right)+\frac{2\,(1-m)}{m}K\left(\frac{1}{m}\right)\right)+\frac{(1-m)\sqrt{-z^2}}{2\,z\sqrt{-m\,z^2}}\left(1-\frac{-3\,m^2+2\,m-7}{12\,(m-1)\,m\,z^2}+\frac{15\,m^3-9\,m^2-11\,m+21}{120\,(m-1)\,m^2\,z^4}+O\left(\frac{1}{z^6}\right)\right)$$

08.04.06.0084.01

$$E(\sin^{-1}(z)\mid m)=-\frac{\sqrt{1-z^2}\,\sqrt{1-m\,z^2}}{z}+\frac{\sqrt{-z^2}\,\sqrt{-m}}{2\,z}\left(2\,E\left(\frac{1}{m}\right)+\left(\frac{i\sqrt{-z^2}}{\sqrt{z^2}}\left(\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)-\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)E(m)+2\,i\left(1-\sqrt{\frac{m}{m-1}}\,\sqrt{\frac{m-1}{m}}\right)\left(\frac{1}{m}K\left(1-\frac{1}{m}\right)-E\left(1-\frac{1}{m}\right)\right)+\frac{2\,(1-m)}{m}K\left(\frac{1}{m}\right)\right)+\frac{\sqrt{m}\,\sqrt{1-\frac{1}{m\,z^2}}\,\sqrt{1-z^2}}{\sqrt{1-\frac{1}{z^2}}\,\sqrt{1-m\,z^2}}E\left(\sin^{-1}\left(\frac{1}{\sqrt{m}\,z}\right)\middle|\,m\right)$$

08.04.06.0085.01

$$E(\sin^{-1}(z)\mid m)=\frac{\sqrt{-z^2}\,\sqrt{-m}}{2\,z}\left(2\,E\left(\frac{1}{m}\right)+\left(\frac{i\sqrt{-z^2}}{\sqrt{z^2}}\left(\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)-\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)E(m)+2\,i\left(1-\sqrt{\frac{m}{m-1}}\,\sqrt{\frac{m-1}{m}}\right)\left(\frac{1}{m}K\left(1-\frac{1}{m}\right)-E\left(1-\frac{1}{m}\right)\right)+\frac{2\,(1-m)}{m}K\left(\frac{1}{m}\right)\right)+\frac{\sqrt{-z^2}}{z\sqrt{-m\,z^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k}{(2\,k+1)\,k!}\,{}_2F_1\left(\frac{1}{2},-k;\frac{1}{2}-k;\frac{1}{m}\right)z^{-2\,k}+\frac{m\,z\sqrt{-z^2}}{\sqrt{-m\,z^2}}\sum_{k=0}^{\infty}\frac{z^{-2\,k}\left(-\frac{1}{2}\right)_k}{k!}\,{}_2F_1\left(-k,-\frac{1}{2};\frac{3}{2}-k;\frac{1}{m}\right)\;/;\;|z|>1$$

08.04.06.0086.01

$$E(\sin^{-1}(z)\mid m)\propto\frac{m\,z\sqrt{-z^2}}{\sqrt{-m\,z^2}}+\frac{\sqrt{-z^2}\,\sqrt{-m}}{2\,z}\left(2\,E\left(\frac{1}{m}\right)+\left(\frac{i\sqrt{-z^2}}{\sqrt{z^2}}\left(\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)-\sqrt{\frac{1}{m}}-\frac{1}{\sqrt{m}}\right)E(m)+2\,i\left(1-\sqrt{\frac{m}{m-1}}\,\sqrt{\frac{m-1}{m}}\right)\left(\frac{1}{m}K\left(1-\frac{1}{m}\right)-E\left(1-\frac{1}{m}\right)\right)+\frac{2\,(1-m)}{m}K\left(\frac{1}{m}\right)\right)+\frac{(1-m)\sqrt{-z^2}}{2\,z\sqrt{-m\,z^2}}\left(1+O\left(\frac{1}{z^2}\right)\right)$$

Expansions $E(\sin^{-1}(z)\mid m)$ at $m == 0$

08.04.06.0087.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto \sin^{-1}(z) + \frac{1}{4}\left(z\sqrt{1-z^2} - \sin^{-1}(z)\right)m + \frac{1}{64}\left(z\sqrt{1-z^2}\left(2z^2+3\right) - 3\sin^{-1}(z)\right)m^2 + \ldots /; (m \to 0)$$

08.04.06.0088.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto \sin^{-1}(z) + \frac{1}{4}\left(z\sqrt{1-z^2} - \sin^{-1}(z)\right)m + \frac{1}{64}\left(z\sqrt{1-z^2}\left(2z^2+3\right) - 3\sin^{-1}(z)\right)m^2 + O\left(m^3\right)$$

08.04.06.0089.01

$$E\left(\sin^{-1}(z) \mid m\right) = \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)_k z^{2k+1}}{(2k+1)\,k!}\, {}_2F_1\left(\frac{1}{2}, k+\frac{1}{2}; k+\frac{3}{2}; z^2\right) m^k /; |m| < 1$$

08.04.06.0090.01

$$E\left(\sin^{-1}(z) \mid m\right) = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{z^{2j+2k+1}}{(2j+2k+1)\,j!\,k!}\left(\frac{1}{2}\right)_j \left(-\frac{1}{2}\right)_k m^k$$

08.04.06.0091.01

$$E\left(\sin^{-1}(z) \mid m\right) = z\, F_{1\times 0\times 0}^{1\times 1\times 1}\left(\begin{matrix} \frac{1}{2}; -\frac{1}{2}; \frac{1}{2}; \\ \frac{3}{2};;; \end{matrix}\; m z^2, z^2\right)$$

Expansions $E\left(\sin^{-1}(z) \mid m\right)$ at $m = 1$

08.04.06.0092.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto z + \frac{1}{2}\left(z - \tanh^{-1}(z)\right)(m-1) + \frac{1}{16}\left(\frac{3z - 2z^3}{z^2-1} + 3\tanh^{-1}(z)\right)(m-1)^2 + \ldots /; (m \to 1)$$

08.04.06.0093.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto z + \frac{1}{2}\left(z - \tanh^{-1}(z)\right)(m-1) + \frac{1}{16}\left(\frac{3z - 2z^3}{z^2-1} + 3\tanh^{-1}(z)\right)(m-1)^2 + O\left((m-1)^3\right)$$

08.04.06.0094.01

$$E\left(\sin^{-1}(z) \mid m\right) = z + \sum_{k=0}^{\infty}\left(\frac{\left(2 - z^2 + 2k\right)\left(\frac{1}{2}\right)_k z^{2k+1}}{2(k+1)!\left(1-z^2\right)^{k+1}} - \frac{\left(\frac{1}{2}\right)_k z^{2k+1}}{k!}\, {}_2F_1\left(k+\frac{1}{2}, k+2; k+\frac{3}{2}; z^2\right)\right)(m-1)^{k+1}$$

08.04.06.0095.01

$$E\left(\sin^{-1}(z) \mid m\right) \propto z\,(1 + O(m-1))$$

Other expansions

08.04.06.0096.01

$$E(z \mid m) = E(m) - \frac{\pi}{2} + z - \cos(z)\sum_{j=0}^{\infty}\frac{\left(\frac{1}{2}\right)_j\left(\frac{1}{2}\right)_j}{\left(\frac{3}{2}\right)_j\, j!}\left({}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; \frac{1}{2} - j; m\right) - 1\right)\cos^{2j}(z) /; |\cos(z)| < 1$$

08.04.06.0097.01

$$E(z \mid m) = \frac{2z}{\pi}E(m) + \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\frac{m}{4}\right)^k}{k!}\sum_{j=0}^{k-1}\frac{(-1)^{k-j}}{j-k}\binom{2k}{j}\sin(2(j-k)z)$$

Integral representations

On the real axis

Of the direct function

08.04.07.0001.01

$$E(z \mid m) = \int_0^z \sqrt{1 - m \sin^2(t)}\, dt \;/;\; -\frac{\pi}{2} \leq z \leq \frac{\pi}{2}$$

08.04.07.0004.01

$$E(z \mid m) = (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \int_0^z \frac{\cos(t) \sqrt{1 - m \sin^2(t)}}{\sqrt{\cos^2(t)}}\, dt + 2 \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil E(m)$$

08.04.07.0002.01

$$E(z \mid m) = \int_0^{\sin(z)} \frac{\sqrt{1 - m t^2}}{\sqrt{1 - t^2}}\, dt \;/;\; -\frac{\pi}{2} \leq z \leq \frac{\pi}{2}$$

08.04.07.0005.01

$$E(z \mid m) = (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil} \int_0^{\sin(z)} \frac{\sqrt{1 - m t^2}}{\sqrt{1 - t^2}}\, dt + 2 \left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil E(m)$$

Contour integral representations

08.04.07.0003.01

$$E(z \mid m) = -\frac{\sin(z)}{4 \pi (2 \pi i)^2} \int_{\mathcal{L}^*} \int_{\mathcal{L}} \frac{\Gamma\left(\frac{1}{2} - s - t\right) \Gamma(s)\, \Gamma\left(\frac{1}{2} - s\right) \Gamma(t)\, \Gamma\left(-\frac{1}{2} - t\right)}{\Gamma\left(\frac{3}{2} - s - t\right)} \left(-\sin^2(z)\right)^{-s} \left(-m \sin^2(z)\right)^{-t} ds\, dt$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

08.04.13.0003.01

$$(1 - m)\, m\, w''(m) + (1 - m)\, w'(m) + \frac{1}{4} w(m) = \frac{\sin(2 z)}{8 \sqrt{1 - m \sin^2(z)}} \;/;\; w(m) = c_1 E(m) + c_2 (K(1 - m) - E(1 - m)) + E(z \mid m)$$

08.04.13.0001.01

$$(1 - m)\, m \frac{\partial^2 w(m)}{\partial m^2} + (1 - m) \frac{\partial w(m)}{\partial m} + \frac{w(m)}{4} = \frac{\sin(2 z)}{8 \sqrt{1 - m \sin^2(z)}} \;/;\; w(m) = E(z \mid m)$$

08.04.13.0004.01

$$W_m(E(m), K(1 - m) - E(1 - m)) = -\frac{\pi}{4 m}$$

Ordinary nonlinear differential equations

08.04.13.0002.01

$$\left(\frac{\partial w(z)}{\partial z}\right)^4 + \left(\left(\frac{\partial^2 w(z)}{\partial z^2}\right)^2 + m - 2\right)\left(\frac{\partial w(z)}{\partial z}\right)^2 = m - 1 \;/;\; w(z) = E(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

08.04.16.0001.01

$$E(-z \mid m) = -E(z \mid m)$$

08.04.16.0002.01

$$E(z + \pi k \mid m) = E(z \mid m) + 2 k E(m) \;/;\; k \in \mathbb{Z}$$

Products, sums, and powers of the direct function

Sums of the direct function

08.04.16.0003.01

$$E(z_1 \mid m) + E(z_2 \mid m) = E\left(\sin^{-1}(w) \mid m\right) + m\, w \sin(z_1) \sin(z_2) \;/;$$
$$w = \frac{\cos(z_2)\sqrt{1 - m \sin^2(z_2)}\, \sin(z_1) + \cos(z_1)\sqrt{1 - m \sin^2(z_1)}\, \sin(z_2)}{1 - m \sin^2(z_1) \sin^2(z_2)} \bigwedge 0 \le m < 1 \bigwedge |z_1| < 1 \bigwedge |z_2| < 1$$

Differentiation

Low-order differentiation

With respect to *z*

08.04.20.0001.01

$$\frac{\partial E(z \mid m)}{\partial z} = \sqrt{1 - m \sin^2(z)}$$

08.04.20.0002.01

$$\frac{\partial^2 E(z \mid m)}{\partial z^2} = -\frac{m \sin(2 z)}{2\sqrt{1 - m \sin^2(z)}}$$

With respect to *m*

08.04.20.0003.01

$$\frac{\partial E(z \mid m)}{\partial m} = \frac{E(z \mid m) - F(z \mid m)}{2 m}$$

08.04.20.0004.01

$$\frac{\partial^2 E(z \mid m)}{\partial m^2} = -\frac{1}{4 (m - 1) m^2}\left((m - 2) E(z \mid m) - 2 (m - 1) F(z \mid m) + \frac{\sqrt{2}\, m \cos(z) \sin(z)}{\sqrt{\cos(2 z) m - m + 2}}\right)$$

Symbolic differentiation

With respect to z

08.04.20.0012.01

$$\frac{\partial^n E(z\mid m)}{\partial z^n} = \delta_n E(z\mid m) + \delta_{n-1}\sqrt{1-m\sin^2(z)} - \frac{2\, i^{n-1}}{(n-1)!}\left(-\frac{1}{2}\right)_n \sqrt{1-m\sin^2(z)} \sum_{q=1}^{n-1} \frac{(-1)^q}{(1-2q)\left(1-m\sin^2(z)\right)^q} \binom{n-1}{q} \sum_{j=0}^{q} \binom{q}{j} m^j (2-m)^{q-j} 2^{-j+n-q-1} \sum_{i=0}^{j} \binom{j}{i} (2i-j)^{n-1} e^{2(2i-j) i z} /; n \in \mathbb{N}$$

08.04.20.0013.01

$$\frac{\partial^n E(z\mid m)}{\partial z^n} = E(z\mid m)\,\delta_n + \sum_{j=1}^{n} \frac{1}{j!} \sum_{q=0}^{j-1} \binom{j}{q} \sum_{p=0}^{j-q} (-1)^q 2^{q-j} \sin^q(z) (q+2p-j)^n e^{-\frac{1}{2} i (\pi(j+n-q-2p)+2(-j+q+2p)z)} \binom{j-q}{p} \sum_{i=0}^{j-1} \frac{(1-j)_{2(j-i)-2}}{(j-i-1)!\,(2\sin(z))^{j-2i-1}} \sum_{s=0}^{i} \binom{i}{s} \left(\frac{1}{2}\right)_s \left(-\frac{1}{2}\right)_{i-s} m^{i-s} \cos^{-2s-1}(z) \left(1-m\sin^2(z)\right)^{-i+s+\frac{1}{2}} /; n \in \mathbb{N}$$

08.04.20.0005.02

$$\frac{\partial^n E(z\mid m)}{\partial z^n} = E(z\mid m)\,\delta_n + \frac{2(2i)^{n-1}\sqrt{1-m\sin^2(z)}}{\sqrt{\frac{(1-e^{2iz})m+2\sqrt{1-m}-2}{\sqrt{1-m}}}} \sqrt{\frac{e^{2iz} m}{m-2\left(\sqrt{1-m}+1\right)}} \sum_{k=0}^{n-1} \frac{e^{2ikz} \mathcal{S}_{n-1}^{(k)} \left(-\frac{1}{2}\right)_k (-m)^k}{\left(\left(e^{2iz}-1\right)m+2\sqrt{1-m}+2\right)^k} F_1\left(\frac{3}{2}; \frac{1}{2}, -\frac{1}{2}; \frac{3}{2}-k; \frac{\left(e^{2iz}-1\right)m+2\sqrt{1-m}+2}{-m+2\sqrt{1-m}+2}, \frac{\left(e^{2iz}-1\right)m+2\sqrt{1-m}+2}{4\sqrt{1-m}}\right) /; n \in \mathbb{N}$$

08.04.20.0006.02

$$\frac{\partial^n E(z\mid m)}{\partial z^n} = \delta_n E(z\mid m) + 2\left(-\frac{1}{2}\right)_n \sum_{k=0}^{n-1} \frac{(-1)^k \left(1-m\sin^2(z)\right)^{\frac{1}{2}-k}}{k!\,(2k-1)\,(n-k-1)!} \frac{\partial^{n-1}\left(1-m\sin^2(z)\right)^k}{\partial z^{n-1}} /; n \in \mathbb{N}$$

With respect to m

08.04.20.0007.02

$$\frac{\partial^n E(z\mid m)}{\partial m^n} = \frac{(-1)^n \sqrt{\pi}\, \sin^{2n+1}(z)}{2(2n+1)\,\Gamma\left(\frac{3}{2}-n\right)} F_1\left(n+\frac{1}{2}; \frac{1}{2}, n-\frac{1}{2}; n+\frac{3}{2}; \sin^2(z), m\sin^2(z)\right) /; n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to z

08.04.20.0008.01

$$\frac{\partial^\alpha E(z\mid m)}{\partial z^\alpha} = 2^\alpha \sqrt{\pi}\, z^{1-\alpha} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)_k}{k!}\, {}_2F_1\left(k-\frac{1}{2}, k+\frac{1}{2}; 2k+1; m\right) {}_1\tilde{F}_2\left(1; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -k^2 z^2\right) \left(-\frac{m}{4}\right)^k + \frac{2 z^{1-\alpha}}{\pi\, \Gamma(2-\alpha)} E(m)$$

08.04.20.0009.01

$$\frac{\partial^{\alpha} E(z \mid m)}{\partial z^{\alpha}} = \frac{z^{-\alpha}}{\Gamma(1-\alpha)} E(m) - 2^{\alpha} \sqrt{\pi}\, z^{-\alpha} \left(\sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-m)^j\, 2^{-2j-2n}}{(j+n)!\, j!\, (2j+2n+1)} \left(-\frac{1}{2}\right)_j \left(\frac{1}{2}\right)_j \left(\frac{1}{2}\right)_n \sum_{k=0}^{j+n} \binom{2j+2n+1}{k} {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{1}{4}(2j-2k+2n+1)^2 z^2\right) - \frac{m}{2} \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-1)^j\, m^{j+n}\, 2^{-2j}}{(2j+1)\, j!\, (2)_{j+n} \left(\frac{3}{2}\right)_n} \left(\frac{1}{2}\right)_{j+n} \left(\frac{3}{2}\right)_{j+n} \sum_{k=0}^{j} \binom{2j+1}{k} {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{1}{4}(2j-2k+1)^2 z^2\right) \right)$$

With respect to *m*

08.04.20.0010.01

$$\frac{\partial^{\alpha} E(z \mid m)}{\partial m^{\alpha}} = \frac{\pi\, m^{-\alpha}}{2}\, {}_2\tilde{F}_1\left(-\frac{1}{2}, \frac{1}{2}; 1-\alpha; m\right) - \cos(z) \sum_{k=0}^{\infty} \frac{m^{k-\alpha}}{\Gamma(k-\alpha+1)} \left(-\frac{1}{2}\right)_k {}_2F_1\left(\frac{1}{2}, \frac{1}{2}-k; \frac{3}{2}; \cos^2(z)\right)$$

08.04.20.0011.01

$$\frac{\partial^{\alpha} E(z \mid m)}{\partial m^{\alpha}} = \frac{\pi\, m^{-\alpha}}{2}\, {}_2\tilde{F}_1\left(-\frac{1}{2}, \frac{1}{2}; 1-\alpha; m\right) - \frac{m^{-\alpha} \sqrt{\pi}\, \cos(z)}{2}\, \tilde{F}_{2\times 1\times 0}^{1\times 3\times 2}\left(\begin{matrix} \frac{1}{2}; -\frac{1}{2}, \frac{1}{2}, 1; \frac{1}{2}, 1; \\ 1, \frac{3}{2}; 1-\alpha; ; \end{matrix} -m\cos^2(z), \cos^2(z)\right) + \frac{\pi\, m^{1-\alpha} \cos(z)}{8}\, \tilde{F}_{2\times 1\times 1}^{3\times 1\times 1}\left(\begin{matrix} \frac{1}{2}, \frac{3}{2}, 2; \frac{1}{2}; 1; \\ 2, 2-\alpha; \frac{3}{2}; \frac{3}{2}; \end{matrix} -m\cos^2(z), m\right)$$

Integration

Indefinite integration

Involving only one direct function

08.04.21.0001.01

$$\int E(z \mid m)\, dz = \frac{z^2}{\pi} E(m) + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (1-\cos(2kz))}{k!\, k^2} \left(-\frac{1}{2}\right)_k {}_2F_1\left(k-\frac{1}{2}, k+\frac{1}{2}; 2k+1; m\right) \left(\frac{m}{4}\right)^k$$

Involving one direct function and elementary functions

Involving trigonometric functions

Involving sin

08.04.21.0002.01

$$\int \sin(z)\, E(z \mid m)\, dz = \frac{1}{4} \left(\frac{2}{\sqrt{m}} \tan^{-1}\left(\frac{\sqrt{2}\, \sqrt{m}\, \sin(z)}{\sqrt{\cos(2z)\, m - m + 2}} \right) - 4\cos(z)\, E(z \mid m) + \sqrt{2\cos(2z)\, m - 2m + 4}\, \sin(z) \right)$$

08.04.21.0003.01

$$\int E(z \mid m) \sqrt{1 - m\sin^2(z)}\, dz = \frac{1}{2} E(z \mid m)^2$$

08.04.21.0004.01

$$\int \frac{\sin(2z)\,E(z\mid m)}{\sqrt{1-m\sin^2(z)}}\,dz = \frac{1}{2m}\left(-2(m-2)z - 2\sqrt{2\cos(2z)m-2m+4}\;E(z\mid m) + m\sin(2z)\right)$$

Involving cos

08.04.21.0005.01

$$\int \cos(z)\,E(z\mid m)\,dz = \frac{1}{4}\left(\sqrt{2\cos(2z)m-2m+4}\;\cos(z) + 4E(z\mid m)\sin(z) - \frac{2(m-1)\log\left(\sqrt{2}\,\sqrt{m}\,\cos(z)+\sqrt{\cos(2z)m-m+2}\right)}{\sqrt{m}}\right)$$

Involving only one direct function with respect to *m*

08.04.21.0006.01

$$\int E(z\mid m)\,dm = \frac{2}{3}\left((m+1)E(z\mid m) - (1-m)F(z\mid m) - \cot(z)\left(1-\sqrt{1-m\sin^2(z)}\right)\right)$$

Involving one direct function and elementary functions with respect to *m*

Involving power function

08.04.21.0007.01

$$\int z\,E\left(a\mid z^2\right)dz = \frac{1}{3}\left(\sqrt{1-z^2\sin^2(a)}\;\cot(a) + \left(z^2+1\right)E\left(a\mid z^2\right) + \left(z^2-1\right)F\left(a\mid z^2\right)\right)$$

Representations through more general functions

Through hypergeometric functions of two variables

08.04.26.0001.01

$$E(z\mid m) = (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi}\right\rceil}\sin(z)\;F_{1\times 0\times 0}^{1\times 1\times 1}\left(\begin{matrix}\frac{1}{2};\ \frac{1}{2};\ -\frac{1}{2};\\ \frac{3}{2};;;\end{matrix}\ \sin^2(z),\ m\sin^2(z)\right) + 2\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil E(m)$$

08.04.26.0008.01

$$E(z\mid m) = \sin(z)\,F_1\left(\frac{1}{2};\ \frac{1}{2},\ -\frac{1}{2};\ \frac{3}{2};\ \sin^2(z),\ m\sin^2(z)\right)\ /;\ |\mathrm{Re}(z)| < \frac{\pi}{2}$$

08.04.26.0009.01

$$E(z\mid m) = 2\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rceil E(m) + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi}\right\rceil}\sin(z)\;F_1\left(\frac{1}{2};\ \frac{1}{2},\ -\frac{1}{2};\ \frac{3}{2};\ \sin^2(z),\ m\sin^2(z)\right)$$

Through Meijer *G*

Classical cases involving $\tan^{-1}$ in the arguments

08.04.26.0003.01

$$E\left(\tan^{-1}\left(\sqrt[4]{z}\right)\,\middle|\,1-\frac{1}{z}\right) = \frac{\sqrt{z}-1}{2\sqrt{z}} - \frac{1}{4\pi}G_{2,2}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\ 1\\ -\frac{1}{2},\ \frac{1}{2}\end{matrix}\right)\ /;\ z\notin(-\infty,0)$$

Classical cases involving $\cot^{-1}$ in the arguments

08.04.26.0004.01

$$E\left(\cot^{-1}\left(\sqrt[4]{z}\right) \,\middle|\, 1-z\right) = \frac{1-\sqrt{z}}{2} - \frac{1}{4\pi} G_{2,2}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 1 \end{matrix}\right) /;\ z \notin (-\infty, -1)$$

Through other functions

Involving some elliptic-type functions

08.04.26.0005.01

$$E(z \mid m) = (1-m)\,\Pi(m; z \mid m) + \frac{m \sin(2z)}{2\sqrt{1-m\sin^2(z)}}$$

08.04.26.0006.01

$$E(z \mid m) = \frac{E(m)}{K(m)} F(z \mid m) + \frac{\pi}{2K(m)\,\vartheta_4\left(\frac{\pi F(z|m)}{2K(m)}, q(m)\right)} \vartheta_4'\left(\frac{\pi F(z \mid m)}{2K(m)}, q(m)\right)$$

Involving Weierstrass functions

08.04.26.0007.01

$$E(z \mid m) = \frac{1}{K(m)}\left(E(m) F(z \mid m) + \eta_3\,\omega_1 + \omega_1\,\zeta\left(\frac{\omega_1 F(z \mid m)}{K(m)} - \omega_3; g_2, g_3\right)\right) - \frac{\omega_1\ \eta_1}{K(m)^2} F(z \mid m) /;$$

$$m = q^{-1}\left(\exp\left(\frac{i\pi\omega_3}{\omega_1}\right)\right) \bigwedge \{\omega_1, \omega_3\} = \{\omega_1(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge \{\eta_1, \eta_3\} = \{\zeta(\omega_1; g_2, g_3), \zeta(\omega_3; g_2, g_3)\}$$

Representations through equivalent functions

With related functions

08.04.27.0001.01

$$E(\operatorname{am}(z \mid m) \mid m) = Z(\operatorname{am}(z \mid m) \mid m) + \frac{z\,E(m)}{K(m)}$$

08.04.27.0002.01

$$E(\phi \mid m) = \frac{E(m) F(\phi \mid m) + \sqrt{1-n}\ \cot(\phi)\,(\Pi(n \mid m) - K(m))}{K(m)} /;\ \phi = \sin^{-1}\left(\sqrt{\frac{n}{m}}\right) \bigwedge 0 < n < 1 \bigwedge 0 < m < 1$$

Theorems

The surface area of an ellipsoid

The surface area A of an ellipsoid with semi-axes a, b, c is given by

$$A == 2\,\pi\,a\,c\left(\frac{c}{a} + \frac{b}{c}\sqrt{1-\left(\frac{c}{a}\right)^2}\;E(\phi \mid m) + \frac{b\,c}{a^2}\sqrt{\frac{a^2}{a^2-c^2}}\;F(\phi \mid m)\right)\,/;$$

$$\phi == \arcsin\sqrt{1-\left(\frac{c}{a}\right)^2} \bigwedge m == \frac{a^2\left(b^2-c^2\right)}{b^2\left(a^2-c^2\right)} \bigwedge a \geq b \geq c$$

History

– L. Euler (1733, 1757, 1763, 1766)
– J.-L. Lagrange (1783)
– A. M. Legendre (1793, 1811, 1825–1828)
– K. F. Gauss (1799, 1818)
– C. G. J. Jacobi (1827)
– J. Liouville (1840)

Copyright