EllipticExp

Notations

Traditional name

Elliptic exponential

Traditional notation

$\operatorname{eexp}(z;\, a,\, b)$

Mathematica StandardForm notation

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EllipticExp[z, {a, b}]
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Primary definition

09.55.02.0001.01

$$\operatorname{eexp}(z;\, a,\, b) = \{x,\, y\} \,/;\, \left(z = \operatorname{elog}(x,\, y;\, a,\, b) \,/;\, y^2 - x\left(x^2 + a\,x + b\right) = 0\right)$$

Specific values

Specialized values

09.55.03.0001.01

$$\operatorname{eexp}\left(-\frac{\sqrt{y^2}}{\sqrt{x}\;y} F_1\left(\frac{1}{2};\, \frac{1}{2},\, \frac{1}{2};\, \frac{3}{2};\, \frac{-a-\sqrt{a^2-4\,b}}{2\,x},\, \frac{\sqrt{a^2-4\,b}-a}{2\,x}\right);\, a,\, b\right) = \{x,\, y\} \,/;\, y^2 - x\left(x^2 + a\,x + b\right) = 0$$

General characteristics

Domain and analyticity

$\operatorname{eexp}(z;\, a,\, b)$ is an vector-valued function of z, a and b, that is analytic in each component and it is defined over $\mathbb{C}^3$.

09.55.04.0001.01

$$(z * \{a * b\}) \longrightarrow \operatorname{eexp}(z;\, a,\, b) :: (\mathbb{C} \otimes \{\mathbb{C} \otimes \mathbb{C}\}) \longrightarrow \{\mathbb{C} \otimes \mathbb{C}\}$$

Symmetries and periodicities

Mirror symmetry

09.55.04.0002.01

$$\operatorname{eexp}\left(\bar{z};\, \bar{a},\, \bar{b}\right) = \overline{\operatorname{eexp}(z;\, a,\, b)}$$

Periodicity

No periodicity

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

Differential equations

Ordinary nonlinear differential equations

09.55.13.0001.01

$$4\,w(z)^3 + 4\,a\,w(z)^2 + 4\,b\,w(z) - w'(z)^2 = 0 \;/;\; \{w(z),\, v(z)\} = \text{eexp}(z;\, a,\, b)$$

Identities

Functional identities

09.55.17.0001.01

$$\text{eexp}(z;\, a,\, b) = \text{eexp}\left(z - n\sqrt{8}\,\sqrt{\frac{a - \sqrt{a^2 - 4\,b}}{b}}\;K\!\left(\frac{2\sqrt{a^2 - 4\,b}}{a + \sqrt{a^2 - 4\,b}}\right);\, a,\, b\right) /;\; n \in \mathbb{Z}$$

09.55.17.0002.01

$$\text{eexp}(z;\, a,\, b) = \text{eexp}\left(z - i\,n\sqrt{8}\,\sqrt{\frac{a - \sqrt{a^2 - 4\,b}}{b}}\;K\!\left(1 - \frac{2\sqrt{a^2 - 4\,b}}{a + \sqrt{a^2 - 4\,b}}\right);\, a,\, b\right) /;\; n \in \mathbb{Z}$$

Differentiation

Low-order differentiation

With respect to *z*

09.55.20.0001.01

$$\frac{\partial\,\text{eexp}(z;\, a,\, b)}{\partial z} = \text{eexp}_z'(z;\, a,\, b)$$

Representations through equivalent functions

With inverse function

09.55.27.0001.01

$$\operatorname{eexp}(\operatorname{elog}(z_1, z_2; a, b); a, b) == \{z_1, z_2\} /; z_1^3 + a z_1^2 + b z_1 - z_2^2 == 0$$

With related functions

Involving Jacobi functions

09.55.27.0002.01

$$x == \frac{1}{2}\left(a + \sqrt{a^2 - 4 b}\right) \cot^2\left(\operatorname{am}\left(-\frac{\sqrt{2}\, z}{\sqrt{\frac{a-\sqrt{a^2-4 b}}{b}}} \,\middle|\, \frac{2 \sqrt{a^2 - 4 b}}{a + \sqrt{a^2 - 4 b}}\right)\right) /; \{x, y\} == \operatorname{eexp}(z; a, b)$$

Involving Weierstrass functions

09.55.27.0003.01

$$x == \sqrt[3]{4}\, \wp\left(\sqrt[3]{2}\, z; \sqrt[3]{4} \left(\frac{a^2}{3} - b\right), \frac{a b}{3} - \frac{2 a^3}{27}\right) - \frac{a}{3} /; \{x, y\} == \operatorname{eexp}(z; a, b)$$

History

– D. Masser (1975)

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