# GammaRegularized



## Notations

### Traditional name

Regularized incomplete gamma function

### Traditional notation

$Q(a, z)$

### *Mathematica* StandardForm notation

`GammaRegularized[a, z]`

## Primary definition

06.08.02.0001.01

$$Q(a, z) = \frac{\Gamma(a, z)}{\Gamma(a)}$$

## Specific values

### Specialized values

#### For fixed *a*

06.08.03.0001.01

$$Q(a, 0) = \tilde{\infty} \;/;\; \mathrm{Re}(a) < 0$$

06.08.03.0002.01

$$Q(a, 0) = 1 \;/;\; \mathrm{Re}(a) > 0$$

06.08.03.0013.01

$$Q(a, -1) = \frac{e}{\Gamma(a)} \,\mathrm{Subfactorial}(a-1)$$

#### For fixed *z*

06.08.03.0003.01

$$Q(0, z) = 0$$

06.08.03.0004.01

$$Q\left(\frac{1}{2}, z\right) = \mathrm{erfc}\left(\sqrt{z}\right)$$

06.08.03.0005.01

$$Q\left(n+\frac{1}{2}, z\right) = \operatorname{erfc}\left(\sqrt{z}\right) + \frac{(-1)^{n-1}}{\Gamma(n+1/2)} e^{-z} \sqrt{z} \sum_{k=0}^{n-1} \left(\frac{1}{2}-n\right)_{n-k-1} (-z)^k \;/;\; n \in \mathbb{N}$$

06.08.03.0006.01

$$Q\left(-\frac{1}{2}, z\right) = \operatorname{erfc}\left(\sqrt{z}\right) - \frac{e^{-z}}{\sqrt{\pi}\,\sqrt{z}}$$

06.08.03.0007.01

$$Q\left(\frac{1}{2}-n, z\right) = \operatorname{erfc}\left(\sqrt{z}\right) - \frac{1}{\Gamma(1/2-n)} z^{\frac{1}{2}-n} e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{(1/2-n)_{k+1}} \;/;\; n \in \mathbb{N}$$

06.08.03.0008.01

$$Q(1, z) = e^{-z}$$

06.08.03.0009.01

$$Q(n, z) = e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{k!} \;/;\; n \in \mathbb{N}^+$$

06.08.03.0010.01

$$Q(-n, z) = 0 \;/;\; n \in \mathbb{N}$$

06.08.03.0012.01

$$Q\left(n+\frac{1}{2}, z\right) = \operatorname{erfc}\left(\sqrt{z}\right) + \frac{1}{\Gamma\left(n+\frac{1}{2}\right)} \left(e^{-z} \sum_{k=0}^{n-1} \frac{z^{k+\frac{1}{2}}}{\left(n+\frac{1}{2}\right)_{k-n+1}} - e^{-z} \sum_{k=n}^{-1} \frac{z^{k+\frac{1}{2}}}{\left(n+\frac{1}{2}\right)_{k-n+1}}\right) \;/;\; n \in \mathbb{Z}$$

### Values at infinities

06.08.03.0011.01

$$Q(a, \infty) = 0$$

## General characteristics

### Domain and analyticity

$Q(a, z)$ is an analytical function of $a$ and $z$ which is defined in $\mathbb{C}^2$. For fixed $z$, it is an entire function of $a$.

06.08.04.0001.01

$$(a * z) \longrightarrow Q(a, z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

### Symmetries and periodicities

#### Mirror symmetry

06.08.04.0002.01

$$Q(\overline{a}, \overline{z}) = \overline{Q(a, z)} \;/;\; z \notin (-\infty, 0)$$

#### Periodicity

No periodicity

## Poles and essential singularities

### With respect to *z*

For fixed $a$, the function $Q(a, z)$ has an essential singularity at $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is a branch point for generic $a$.

06.08.04.0003.01

$$\mathcal{Sing}_z(Q(a, z)) == \{\{\tilde{\infty}, \infty\}\}$$

### With respect to *a*

For fixed $z$, the function $Q(a, z)$ has only one singular point at $a = \tilde{\infty}$. It is an essential singular point.

06.08.04.0004.01

$$\mathcal{Sing}_a(Q(a, z)) == \{\{\tilde{\infty}, \infty\}\}$$

## Branch points

### With respect to *z*

For fixed $a$, not being a positive integer, the function $Q(a, z)$ has two branch points: $z == 0$, $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is an essential singularity.

06.08.04.0005.01

$$\mathcal{BP}_z(Q(a, z)) == \{0, \tilde{\infty}\} /; a \notin \mathbb{N}^+$$

06.08.04.0006.01

$$\mathcal{R}_z(Q(a, z), 0) == \log /; a \notin \mathbb{Q}$$

06.08.04.0007.01

$$\mathcal{R}_z\left(Q\left(\frac{p}{q}, z\right), 0\right) == q /; p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$$

06.08.04.0008.01

$$\mathcal{R}_z(Q(a, z), \tilde{\infty}) == \log /; a \notin \mathbb{Q}$$

06.08.04.0009.01

$$\mathcal{R}_z\left(Q\left(\frac{p}{q}, z\right), \tilde{\infty}\right) == q /; p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$$

### With respect to *a*

For fixed $z$, the function $Q(a, z)$ does not have branch points.

06.08.04.0010.01

$$\mathcal{BP}_a(Q(a, z)) == \{\}$$

## Branch cuts

### With respect to *z*

For fixed $a$, not being a positive integer, the function $Q(a, z)$ has one infinitely long branch cut. It is a single-valued function on the $z$-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

06.08.04.0011.01

$$\mathcal{BC}_z(Q(a, z)) == \{\{(-\infty, 0), -i\}\}$$

06.08.04.0012.01

$$\lim_{\epsilon \to +0} Q(a, x + i\,\epsilon) == Q(a, x) \;/;\; x < 0$$

06.08.04.0013.01

$$\lim_{\epsilon \to +0} Q(a, x - i\,\epsilon) == 1 - e^{-2\,i\,\pi\,a}\,(1 - Q(a, x)) \;/;\; x < 0$$

#### With respect to $a$

For fixed $z$, the function $Q(a, z)$ does not have branch cuts.

06.08.04.0014.01

$$\mathcal{BC}_a(Q(a, z)) == \{\}$$

## Series representations

### Generalized power series

#### Expansions at generic point $a == a_0$

### For the function itself

06.08.06.0011.01

$$Q(a, z) \propto Q(a_0, z) + \left(\frac{z^{a_0}}{a_0\,\Gamma(a_0 + 1)}\,{}_2F_2(a_0, a_0; a_0 + 1, a_0 + 1; -z) + (Q(a_0, z) - 1)\,(\log(z) - \psi(a_0))\right)(a - a_0) - \frac{1}{2}\left((1 - Q(a_0, z))\left((\log(z) - \psi(a_0))^2 - \psi^{(1)}(a_0)\right) + \frac{2\,z^{a_0}}{\Gamma(a_0)\,a_0^3}\,({}_3F_3(a_0, a_0, a_0; a_0 + 1, a_0 + 1, a_0 + 1; -z) + a_0(\psi(a_0) - \log(z))\,{}_2F_2(a_0, a_0; a_0 + 1, a_0 + 1; -z))\right)(a - a_0)^2 + \ldots \;/;\; (a \to a_0)$$

06.08.06.0012.01

$$Q(a, z) \propto Q(a_0, z) + \left(\frac{z^{a_0}}{a_0\,\Gamma(a_0 + 1)}\,{}_2F_2(a_0, a_0; a_0 + 1, a_0 + 1; -z) + (Q(a_0, z) - 1)\,(\log(z) - \psi(a_0))\right)(a - a_0) - \frac{1}{2}\left((1 - Q(a_0, z))\left((\log(z) - \psi(a_0))^2 - \psi^{(1)}(a_0)\right) + \frac{2\,z^{a_0}}{\Gamma(a_0)\,a_0^3}\,({}_3F_3(a_0, a_0, a_0; a_0 + 1, a_0 + 1, a_0 + 1; -z) + a_0(\psi(a_0) - \log(z))\,{}_2F_2(a_0, a_0; a_0 + 1, a_0 + 1; -z))\right)(a - a_0)^2 + O\left((a - a_0)^3\right)$$

06.08.06.0013.01

$$Q(a, z) = \sum_{k=0}^{\infty} \sum_{s=0}^{k} \left( \Gamma^{(k-s)}(a_0) - z^{a_0} \sum_{i=0}^{k-s} (-1)^{k-i-s} \binom{k-s}{i} (k-i-s)! \, \Gamma(a_0)^{k-i-s+1} \log^i(z) \, {}_4\tilde{F}_4(c_1, c_2, \ldots, c_{k-i-s+1}; c_1+1, c_2+1, \ldots, c_{k-i-s+1}+1; -z) \right) \sum_{j=0}^{s} \frac{(-1)^j (s+1) \Gamma(a_0)^{-j-1}}{(j+1)! \, (k-s)! \, (s-j)!} \text{Function}\left[u, \Gamma(u)^j\right]^{(s)}(a_0) \, (a-a_0)^k \, /; \, c_1 = c_2 = \ldots = c_{k+1} = a_0 \land k \in \mathbb{N}$$

06.08.06.0014.01

$$Q(a, z) \propto Q(a_0, z) \, (1 + O(a - a_0))$$

**Expansions of $Q(\epsilon - n, z)$ at $\epsilon = 0$**

## For the function itself

06.08.06.0015.01

$$Q(\epsilon - n, z) \propto (-1)^n \, n! \, \Gamma(-n, z) \, \epsilon + (-1)^n \, n! \left( \Gamma(-n, z) \, (-H_n + \log(z) + \gamma) + G_{2,3}^{3,0}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ 0, 0, -n \end{matrix}\right) \right) \epsilon^2 + \frac{1}{6} (-1)^n \, n! \left( 6 \, (-H_n + \log(z) + \gamma) \, G_{2,3}^{3,0}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ 0, 0, -n \end{matrix}\right) + 6 \, G_{3,4}^{4,0}\left(z \,\middle|\, \begin{matrix} 1, 1, 1 \\ 0, 0, 0, -n \end{matrix}\right) + \Gamma(-n, z) \left(3 \log^2(z) - \pi^2 + 3 \, \psi^{(0)}(n+1) \left(\psi^{(0)}(n+1) - 2 \log(z)\right) + 3 \, \psi^{(1)}(n+1)\right) \right) \epsilon^3 + O\left(\epsilon^4\right) \, /; \, n \in \mathbb{N}$$

06.08.06.0016.01

$$Q(\epsilon - n, z) \propto \epsilon \, (-1)^n \sum_{k=0}^{\infty} \left( \sum_{j=0}^{k} \sum_{i=0}^{j} a_{k-j} \, b_{j-i,n} \, c_{i,n} \right) \epsilon^k \, /;$$

$$a_{2k} = \frac{(-1)^k \pi^{2k}}{(2k+1)!} \bigwedge a_{2k+1} = 0 \bigwedge b_{k,n} = \frac{(-1)^k}{k!} \Gamma^{(k)}(n+1) \bigwedge c_{k,n} = \frac{1}{k!} \Gamma^{(k,0)}(-n, z) \bigwedge k \in \mathbb{N} \bigwedge n \in \mathbb{N}$$

06.08.06.0017.01

$$Q(\epsilon - n, z) \propto (-1)^n \, n! \, \Gamma(-n, z) \, \epsilon \, (1 + O(\epsilon)) \, /; \, n \in \mathbb{N}$$

**Expansions at generic point $z = z_0$**

## For the function itself

06.08.06.0018.01

$$Q(a, z) \propto -2 \, i \, e^{-i a \pi} \sin(\pi a) \left\lfloor \frac{\arg(z - z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2\pi} \right\rfloor + \left(\frac{1}{z_0}\right)^{a \left\lfloor \frac{\arg(z - z_0)}{2\pi} \right\rfloor} z_0^{a \left\lfloor \frac{\arg(z - z_0)}{2\pi} \right\rfloor} \left( Q(a, z_0) - \frac{z_0^{a-1} \, e^{-z_0}}{\Gamma(a)} (z - z_0) + \frac{e^{-z_0} \, (-a + z_0 + 1) \, z_0^{a-2}}{2 \, \Gamma(a)} (z - z_0)^2 + \ldots \right) /; \, (z \to z_0)$$

06.08.06.0019.01

$$Q(a, z) \propto -2\, i\, e^{-i a \pi} \sin(\pi a) \left\lfloor \frac{\arg(z - z_0)}{2 \pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2 \pi} \right\rfloor + \left(\frac{1}{z_0}\right)^{a \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{a \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left( Q(a, z_0) - \frac{z_0^{a-1}\, e^{-z_0}}{\Gamma(a)} (z - z_0) + \frac{e^{-z_0} (-a + z_0 + 1)\, z_0^{a-2}}{2\, \Gamma(a)} (z - z_0)^2 + O\left((z - z_0)^3\right) \right)$$

06.08.06.0020.01

$$Q(a, z) = \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{(-1)^{k-j}}{j!\, (k-j)!} \left( \left(\frac{1}{z_0}\right)^{a \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{a \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} Q(a - j, z_0) - \left(1 - e^{-2 i a \pi}\right) \left\lfloor \frac{\arg(z - z_0)}{2 \pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2 \pi} \right\rfloor \right) (z - z_0)^k$$

06.08.06.0021.01

$$Q(a, z) = \left(\frac{1}{z_0}\right)^{a \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{a \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left( Q(a, z_0) + e^{-z_0} \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{(-1)^{k-j}\, z_0^{a-j-1}}{k\, j!\, (k-j-1)!\, \Gamma(a-j)} (z - z_0)^k \right) - 2\, i\, e^{-i a \pi} \sin(\pi a) \left\lfloor \frac{\arg(z - z_0)}{2 \pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2 \pi} \right\rfloor$$

06.08.06.0022.01

$$Q(a, z) \propto -2\, i\, e^{-i a \pi} \sin(\pi a) \left\lfloor \frac{\arg(z - z_0)}{2 \pi} \right\rfloor \left\lfloor \frac{\arg(z_0) + \pi}{2 \pi} \right\rfloor + \left(\frac{1}{z_0}\right)^{a \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{a \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} (Q(a, z_0) + O(z - z_0))$$

**Expansions on branch cuts**

## For the function itself

06.08.06.0023.01

$$Q(a, z) \propto -2\, i\, e^{-i a \pi} \sin(\pi a) \left\lfloor \frac{\arg(z - x)}{2 \pi} \right\rfloor + e^{2 i a \pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left( Q(a, x) - \frac{e^{-x}\, x^{a-1}}{\Gamma(a)} (z - x) + \frac{x - a + 1}{2\, \Gamma(a)} e^{-x}\, x^{a-2} (z - x)^2 + \ldots \right) /; (z \to x) \land x \in \mathbb{R} \land x < 0$$

06.08.06.0024.01

$$Q(a, z) \propto -2\, i\, e^{-i a \pi} \sin(\pi a) \left\lfloor \frac{\arg(z - x)}{2 \pi} \right\rfloor + e^{2 i a \pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left( Q(a, x) - \frac{e^{-x}\, x^{a-1}}{\Gamma(a)} (z - x) + \frac{x - a + 1}{2\, \Gamma(a)} e^{-x}\, x^{a-2} (z - x)^2 + O\left((z - x)^3\right) \right) /; x \in \mathbb{R} \land x < 0$$

06.08.06.0025.01

$$Q(a, z) = e^{2 \pi i a \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left( Q(a, x) + e^{-x} \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{(-1)^{k-j}\, x^{a-j-1}}{k\, j!\, (k-j-1)!\, \Gamma(a-j)} (z - x)^k \right) - 2\, i\, e^{-i a \pi} \sin(\pi a) \left\lfloor \frac{\arg(z - x)}{2 \pi} \right\rfloor /; x \in \mathbb{R} \land x < 0$$

06.08.06.0026.01

$$Q(a, z) \propto -2\, i\, e^{-i a \pi} \sin(\pi a) \left\lfloor \frac{\arg(z - x)}{2 \pi} \right\rfloor + e^{2 i a \pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (Q(a, x) + O(z - x)) /; x \in \mathbb{R} \land x < 0$$

**Expansions at $z = 0$**

## For the function itself

## General case

06.08.06.0001.02

$$Q(a, z) \propto 1 - \frac{z^a}{\Gamma(a+1)} \left(1 - \frac{a\, z}{a+1} + \frac{a\, z^2}{2\,(a+2)} - \ldots\right) /;\ (z \to 0)$$

06.08.06.0027.01

$$Q(a, z) \propto 1 - \frac{z^a}{\Gamma(a+1)} \left(1 - \frac{a\, z}{a+1} + \frac{a\, z^2}{2\,(a+2)} - O\left(z^3\right)\right)$$

06.08.06.0002.02

$$Q(a, z) == 1 - \frac{z^a}{\Gamma(a+1)} \sum_{k=0}^{\infty} \frac{a\,(-z)^k}{(a+k)\, k!}$$

06.08.06.0003.01

$$Q(a, z) == 1 - z^a\, {}_1\tilde{F}_1(a;\, a+1;\, -z)$$

06.08.06.0004.02

$$Q(a, z) \propto 1 - \frac{1}{\Gamma(a+1)}\, z^a (1 + O(z))$$

06.08.06.0028.01

$$Q(a, z) == F_\infty(z, a) /;$$
$$\left(\left(F_n(z, a) == 1 - \frac{z^a}{\Gamma(a)} \sum_{k=0}^{n} \frac{(-z)^k}{(a+k)\, k!} == Q(a, z) - (-1)^n\, z^{a+n+1}\, (a)_{n+1}\, {}_2\tilde{F}_2(1,\, a+n+1;\, n+2,\, a+n+2;\, -z)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

## Special cases

06.08.06.0029.01

$$Q(1, z) \propto 1 - z + O\left(z^3\right)$$

06.08.06.0030.01

$$Q(2, z) \propto 1 - \frac{z^2}{2} + O\left(z^3\right)$$

06.08.06.0031.01

$$Q(n, z) \propto 1 - \frac{z^n}{n!} + O\left(z^{n+1}\right) /;\ n \in \mathbb{N}^+$$

06.08.06.0005.01

$$Q(n, z) == e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{k!} /;\ n \in \mathbb{N}^+$$

06.08.06.0006.02

$$Q(n, z) \propto 1 + O(z^n) /;\ n \in \mathbb{N}^+$$

# Asymptotic series expansions

### Expansions at $a == \infty$

06.08.06.0032.01

$$Q(a,z) \propto 1 - \frac{a^{-a-\frac{1}{2}} e^{a-z} z^a}{\sqrt{2\pi}} \left(1 + \frac{12z-1}{12a} + \frac{288z^2 - 312z + 1}{288a^2} + O\left(\frac{1}{a^3}\right)\right) /; (|a| \to \infty)$$

06.08.06.0033.01

$$Q(a,z) \propto 1 - \frac{z^a}{\Gamma(a+1)} - \frac{z^a}{\Gamma(a)} \sum_{j=0}^{\infty} (-1)^j a^{-j-1} \sum_{k=1}^{\infty} \frac{k^j (-z)^k}{k!} /; (|a| \to \infty)$$

06.08.06.0034.01

$$Q(a,z) \propto 1 - \frac{a^{-a-\frac{1}{2}} e^{a-z} z^a}{\sqrt{2\pi}} \left(1 + O\left(\frac{1}{a}\right)\right) /; (|a| \to \infty)$$

### Expansions at $z == \infty$

06.08.06.0035.01

$$Q(a,z) \propto \frac{e^{-z} z^{a-1}}{\Gamma(a)} \left(1 - \frac{1-a}{z} + \frac{(2-a)(1-a)}{z^2} + O\left(\frac{1}{z^3}\right)\right) /; (|z| \to \infty)$$

06.08.06.0036.01

$$Q(a,z) \propto \frac{e^{-z} z^{a-1}}{\Gamma(a)} \left(\sum_{k=0}^{n} (-1)^k (1-a)_k z^{-k} + O\left(\frac{1}{z^{n+1}}\right)\right) /; (|z| \to \infty) \land n \in \mathbb{N}$$

06.08.06.0037.01

$$Q(a,z) \propto \frac{e^{-z} z^{a-1}}{\Gamma(a)} \sum_{k=0}^{\infty} (-1)^k (1-a)_k z^{-k} /; (|z| \to \infty)$$

06.08.06.0007.01

$$Q(a,z) \propto \frac{1}{\Gamma(a)} e^{-z} z^{a-1} \, {}_2F_0\left(1, 1-a; ; -\frac{1}{z}\right) /; (|z| \to \infty)$$

06.08.06.0008.01

$$Q(a,z) \propto \frac{e^{-z} z^{a-1}}{\Gamma(a)} \left(1 + O\left(\frac{1}{z}\right)\right) /; (|z| \to \infty)$$

## Residue representations

06.08.06.0009.02

$$Q(a,z) == 1 + \frac{1}{\Gamma(a)} \sum_{j=0}^{\infty} \text{res}_s\left(\frac{z^{-s}}{s} \Gamma(a+s)\right)(-a-j)$$

06.08.06.0010.02

$$Q(a,z) == \frac{1}{\Gamma(a)} \left(\text{res}_s\left(\Gamma(a+s) z^{-s} \frac{1}{s}\right)(0) + \sum_{j=0}^{\infty} \text{res}_s\left(\frac{z^{-s}}{s} \Gamma(a+s)\right)(-a-j)\right)$$

# Integral representations

## On the real axis

### Of the direct function

06.08.07.0001.01

$$Q(a, z) = \frac{1}{\Gamma(a)} \int_z^{\infty} t^{a-1} e^{-t} \, dt$$

### Contour integral representations

06.08.07.0002.01

$$Q(a, z) = 1 - \frac{1}{\Gamma(a)\, 2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s+a)\,\Gamma(-s)}{\Gamma(1-s)} z^{-s} \, ds$$

06.08.07.0003.01

$$Q(a, z) = 1 - \frac{1}{\Gamma(a)\, 2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(s+a)\,\Gamma(-s)}{\Gamma(1-s)} z^{-s} \, ds \;/;\; -\mathrm{Re}(a) < \gamma < 1 \bigwedge |\arg(z)| < \frac{\pi}{2}$$

06.08.07.0004.01

$$Q(a, z) = \frac{1}{\Gamma(a)\, 2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s+a)\,\Gamma(s)\, z^{-s}}{\Gamma(s+1)} \, ds$$

06.08.07.0005.01

$$Q(a, z) = \frac{1}{\Gamma(a)\, 2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(s+a)\,\Gamma(s)\, z^{-s}}{\Gamma(s+1)} \, ds \;/;\; \max(-\mathrm{Re}(a), 0) < \gamma \bigwedge |\arg(z)| < \frac{\pi}{2}$$

## Continued fraction representations

06.08.10.0001.01

$$Q(a, z) = \frac{z^a e^{-z}}{\Gamma(a)} \cfrac{1}{z + \cfrac{1-a}{1 + \cfrac{1}{z + \cfrac{2-a}{1 + \cfrac{2}{z + \cfrac{3-a}{1 + \cfrac{3}{z + \ldots}}}}}}} \;/;\; z \notin (-\infty, 0)$$

06.08.10.0002.01

$$Q(a, z) = \frac{z^a e^{-z}}{\Gamma(a) \left( z + \mathrm{K}_k \left( 2^{\frac{1}{2}(-1-(-1)^k)} k^{\frac{1}{2}(1+(-1)^k)} \left( \frac{k+1}{2} - a \right)^{\frac{1}{2}(1-(-1)^k)}, z^{\frac{1}{2}((-1)^k+1)} \right)_1^{\infty} \right)} \;/;\; z \notin (-\infty, 0)$$

06.08.10.0003.01

$$Q(a, z) = \frac{z^a e^{-z}}{\Gamma(a)} \cfrac{1}{1 - a + z + \cfrac{a-1}{3 - a + z + \cfrac{2(a-2)}{5 - a + z + \cfrac{3(a-3)}{7 - a + z + \cfrac{4(a-4)}{9 - a + z + \cfrac{5(a-5)}{11 - a + z + \ldots}}}}}} \;/;\; z \notin (-\infty, 0)$$

06.08.10.0004.01

$$Q(a,z) = \frac{z^a e^{-z}}{\Gamma(a)\left(z-a+1+\mathrm{K}_k(-k(k-a),\,-a+2k+z+1)_1^\infty\right)} \;/;\; z \notin (-\infty, 0)$$

06.08.10.0005.01

$$Q(a,z) = 1 - \frac{z^a e^{-z}}{\Gamma(a)} \cfrac{1}{a - \cfrac{a z}{a+1+\cfrac{z}{a+2-\cfrac{(a+1)z}{a+3+\cfrac{2z}{a+4-\cfrac{(a+2)z}{a+5+\ldots}}}}}} \;/;\; z \notin (-\infty, 0)$$

06.08.10.0006.01

$$Q(a,z) = 1 - \frac{z^a e^{-z}}{\Gamma(a)\left(a+\mathrm{K}_k\left((-1)^k\left(a^{\frac{1}{2}(1-(-1)^k)}+\left\lfloor\frac{k-1}{2}\right\rfloor\right)z,\, a+k\right)_1^\infty\right)} \;//;\; z \notin (-\infty, 0)$$

06.08.10.0007.01

$$Q(a,z) = 1 - \frac{z^a e^{-z}}{\Gamma(a)} \cfrac{1}{a - \cfrac{a z}{a+z+1-\cfrac{(a+1)z}{a+z+2-\cfrac{(a+2)z}{a+z+3-\cfrac{(a+3)z}{a+z+4-\cfrac{(a+4)z}{a+z+5+\ldots}}}}}}$$

06.08.10.0008.01

$$Q(a,z) = 1 - \frac{z^a e^{-z}}{\Gamma(a)\left(a+\mathrm{K}_k(-(a+k-1)z,\, a+k+z)_1^\infty\right)}$$

06.08.10.0009.01

$$Q(a,z) = 1 - \frac{z^a e^{-z}}{\Gamma(a)} \cfrac{1}{a - \cfrac{a z}{a+1+\cfrac{z}{a+2-\cfrac{(a+1)z}{a+3+\cfrac{2z}{a+4-\cfrac{(a+2)z}{a+5+\ldots}}}}}}$$

06.08.10.0010.01

$$Q(a,z) = 1 - \frac{z^a e^{-z}}{\Gamma(a)\left(a+\mathrm{K}_k\left((-1)^k\left(\frac{k}{2}\right)^{\frac{1}{2}(1+(-1)^k)}\left(\frac{k-1}{2}+a\right)^{\frac{1}{2}(1-(-1)^k)} z,\, a+k\right)_1^\infty\right)}$$

## Differential equations

### Ordinary linear differential equations and wronskians

### For the direct function itself

06.08.13.0001.01

$$z\,w''(z) + (1 - a + z)\,w'(z) = 0\;/;\;w(z) = c_1\,Q(a, z) + c_2$$

06.08.13.0002.01

$$W_z(1, Q(a, z)) = -\frac{e^{-z}\,z^{a-1}}{\Gamma(a)}$$

06.08.13.0003.01

$$w''(z) + \left(\frac{(g(z) - a + 1)\,g'(z)}{g(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) = 0\;/;\;w(z) = c_1\,Q(a, g(z)) + c_2$$

06.08.13.0004.01

$$W_z(Q(a, g(z)), 1) = \frac{e^{-g(z)}\,g(z)^{a-1}\,g'(z)}{\Gamma(a)}$$

06.08.13.0005.01

$$w''(z) + \left(\frac{(g(z) - a + 1)\,g'(z)}{g(z)} - \frac{2\,h'(z)}{h(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) + \left(\frac{2\,h'(z)^2}{h(z)^2} + \frac{(a-1)\,g'(z)\,h'(z)}{g(z)\,h(z)} + \frac{g''(z)\,h'(z)}{h(z)\,g'(z)} - \frac{g'(z)\,h'(z) + h''(z)}{h(z)}\right) w(z) = 0\;/;\;w(z) = c_1\,h(z)\,Q(a, g(z)) + c_2\,h(z)$$

06.08.13.0006.01

$$W_z(h(z)\,Q(a, g(z)), h(z)) = \frac{e^{-g(z)}\,g(z)^{a-1}\,h(z)^2\,g'(z)}{\Gamma(a)}$$

06.08.13.0007.01

$$z^2\,w''(z) + (d\,r\,z^r - a\,r - 2\,s + 1)\,z\,w'(z) + s\,(-d\,r\,z^r + a\,r + s)\,w(z) = 0\;/;\;w(z) = c_1\,z^s\,Q(a, d\,z^r) + c_2\,z^s$$

06.08.13.0008.01

$$W_z(z^s\,Q(a, d\,z^r), z^s) = \frac{e^{-d\,z^r}\,r\,z^{2\,s-1}\,(d\,z^r)^a}{\Gamma(a)}$$

06.08.13.0009.01

$$w''(z) - ((a - d\,r^z)\log(r) + 2\log(s))\,w'(z) + \log(s)\,((a - d\,r^z)\log(r) + \log(s))\,w(z) = 0\;/;\;w(z) = c_1\,s^z\,Q(a, d\,r^z) + c_2\,s^z$$

06.08.13.0010.01

$$W_z(s^z\,Q(a, d\,r^z), s^z) = \frac{e^{-d\,r^z}\,(d\,r^z)^a\,s^{2\,z}\log(r)}{\Gamma(a)}$$

# Transformations

## Transformations and argument simplifications

### Argument involving basic arithmetic operations

06.08.16.0001.01

$$Q(a + 1, z) = Q(a, z) + \frac{e^{-z}\,z^a}{\Gamma(a + 1)}$$

06.08.16.0002.01

$$Q(a - 1, z) = Q(a, z) - \frac{e^{-z}\,z^{a-1}}{\Gamma(a)}$$

06.08.16.0003.01

$$Q(a+n, z) == Q(a, z) + \frac{1}{\Gamma(a)} z^{a-1} e^{-z} \sum_{k=1}^{n} \frac{z^k}{(a)_k} /; n \in \mathbb{N}$$

06.08.16.0004.01

$$Q(a-n, z) == Q(a, z) - \frac{1}{\Gamma(a-n)} e^{-z} z^{a-n-1} \sum_{k=1}^{n} \frac{z^k}{(a-n)_k} /; n \in \mathbb{N}$$

# Identities

## Recurrence identities

### Consecutive neighbors

06.08.17.0001.01

$$Q(a, z) == Q(a+1, z) - \frac{e^{-z} z^a}{\Gamma(a+1)}$$

06.08.17.0002.01

$$Q(a, z) == Q(a-1, z) + \frac{e^{-z} z^{a-1}}{\Gamma(a)}$$

### Distant neighbors

06.08.17.0003.02

$$Q(a, z) == Q(a+n, z) - z^{a-1} e^{-z} \sum_{k=1}^{n} \frac{z^k}{\Gamma(a+k)} /; n \in \mathbb{N}$$

06.08.17.0004.02

$$Q(a, z) == Q(a-n, z) + z^{a-1} e^{-z} \sum_{k=0}^{n-1} \frac{z^{-k}}{\Gamma(a-k)} /; n \in \mathbb{N}$$

# Differentiation

## Low-order differentiation

### With respect to *a*

06.08.20.0011.01

$$\frac{\partial Q(a, z)}{\partial a} = \frac{1}{\Gamma(a)} G_{2,3}^{3,0}\left(z \middle| \begin{matrix} 1, 1 \\ 0, 0, a \end{matrix}\right) + Q(a, z) (\log(z) - \psi(a))$$

06.08.20.0001.01

$$\frac{\partial Q(a, z)}{\partial a} == \Gamma(a) z^a \, {}_2\tilde{F}_2(a, a; a+1, a+1; -z) + Q(a, z, 0) (\log(z) - \psi(a))$$

06.08.20.0012.01

$$\frac{\partial^2 Q(a,z)}{\partial a^2} = \frac{1}{\Gamma(a)}\left(2\,G_{3,4}^{4,0}\!\left(z\,\middle|\,\begin{matrix}1,1,1\\0,0,0,a\end{matrix}\right) + 2\,G_{2,3}^{3,0}\!\left(z\,\middle|\,\begin{matrix}1,1\\0,0,a\end{matrix}\right)(\log(z)-\psi(a)) + \Gamma(a)\,Q(a,z)\left(\log^2(z) - 2\,\psi(a)\log(z) + \psi(a)^2 - \psi^{(1)}(a)\right)\right)$$

06.08.20.0002.01

$$\frac{\partial^2 Q(a,z)}{\partial a^2} = Q(a,z,0)\left(\log^2(z) - 2\,\psi(a)\log(z) + \psi(a)^2 - \psi^{(1)}(a)\right) - 2\,z^a\,\Gamma(a)\left(\Gamma(a)\,{}_3\tilde{F}_3(a,a,a;a+1,a+1,a+1;-z) + (\psi(a)-\log(z))\,{}_2\tilde{F}_2(a,a;a+1,a+1;-z)\right)$$

### With respect to $z$

06.08.20.0003.01

$$\frac{\partial Q(a,z)}{\partial z} = -\frac{e^{-z} z^{a-1}}{\Gamma(a)}$$

06.08.20.0004.01

$$\frac{\partial^2 Q(a,z)}{\partial z^2} = \frac{e^{-z} z^{a-2}(z-a+1)}{\Gamma(a)}$$

## Symbolic differentiation

### With respect to $a$

06.08.20.0005.02

$$\frac{\partial^n Q(a,z)}{\partial a^n} = \frac{\Gamma^{(n)}(a)}{\Gamma(a)} - \frac{1}{\Gamma(a)}\sum_{k=0}^{\infty}\frac{(-1)^{n-k}\,\Gamma(n+1,-(a+k)\log(z))}{(a+k)^{n+1}\,k!} \;/;\; n\in\mathbb{N}$$

06.08.20.0006.02

$$\frac{\partial^n Q(a,z)}{\partial a^n} = n!\sum_{k=0}^{n}\left(\Gamma^{(n)}(a) - z^a\sum_{i=0}^{n-k}(-1)^{n-i-k}\binom{n-k}{i}(n-i-k)!\,\Gamma(a)^{n-i-k+1}\log^i(z)\,{}_{n-k-i+1}\tilde{F}_{n-k-i+1}(a_1,a_2,\ldots,a_{n-k-i+1};a_1+1,a_2+1,\ldots,a_{n-k-i+1}+1;-z)\right)\sum_{j=0}^{k}\frac{(-1)^j(k+1)\,\Gamma(a)^{-j-1}}{(j+1)!\,(n-k)!\,(k-j)!}\frac{\partial^k\Gamma(a)^j}{\partial a^k} \;/;\; a_1 = a_2 = \ldots = a_{n+1} = a \bigwedge n\in\mathbb{N}$$

06.08.20.0013.01

$$Q^{(n,0)}(-m,z) = (-1)^m\,n!\sum_{j=0}^{n-1}\sum_{i=0}^{j}a_{n-j-1}\,b_{j-i,m}\,c_{i,m} \;/;\; a_{2k} = \frac{(-1)^k\pi^{2k}}{(2k+1)!}\bigwedge a_{2k+1} = 0 \bigwedge b_{k,m} = \frac{(-1)^k}{k!}\Gamma^{(k)}(m+1)\bigwedge c_{k,m} = \frac{1}{k!}\Gamma^{(k,0)}(-m,z)\bigwedge k\in\mathbb{N}\bigwedge m\in\mathbb{N}\bigwedge n\in\mathbb{N}^+$$

### With respect to $z$

06.08.20.0014.01

$$\frac{\partial^n Q(a,z)}{\partial z^n} = Q(a,z)\,\delta_n + e^{-z}\sum_{k=0}^{n-1}\frac{(-1)^{n-k}(n-1)!\,z^{a-k-1}}{\Gamma(a-k)\,k!\,(-k+n-1)!} \;/;\; n\in\mathbb{N}$$

06.08.20.0007.02

$$\frac{\partial^n Q(a,z)}{\partial z^n} = -a\,z^{-n}\sum_{k=0}^{n}(-1)^{n-k}\binom{n}{k}(1-a-k)_{n-1}\,Q(a+k,z)\,/;\,n\in\mathbb{N}$$

### Fractional integro-differentiation

#### With respect to *a*

06.08.20.0008.01

$$\frac{\partial^\alpha \Gamma(a,z)}{\partial a^\alpha} = a^{-\alpha}\int_z^\infty t^{a-1}\,(a\log(t))^\alpha\; e^{-t}\,Q(-\alpha,0,a\log(t))\,dt$$

#### With respect to *z*

06.08.20.0009.01

$$\frac{\partial^\alpha Q(a,z)}{\partial z^\alpha} = \frac{z^{-\alpha}}{\Gamma(1-\alpha)} - z^{a-\alpha}\,{}_1\tilde{F}_1(a;a-\alpha+1;-z)\,/;\,-a\notin\mathbb{N}^+$$

06.08.20.0010.01

$$\frac{\partial^\alpha Q(a,z)}{\partial z^\alpha} = \frac{z^{-\alpha}}{\Gamma(1-\alpha)} - \frac{1}{\Gamma(a)}\sum_{k=0}^{\infty}\frac{(-1)^k\,\mathcal{FC}_{\exp}^{(\alpha)}(z,a+k)\,z^{a+k-\alpha}}{(a+k)\,k!}$$

## Integration

### Indefinite integration

#### Involving only one direct function

06.08.21.0001.01

$$\int Q(a,z)\,dz = z\,Q(a,z) - a\,Q(a+1,z)$$

#### Involving one direct function and elementary functions

### Involving power function

06.08.21.0002.01

$$\int z^{\alpha-1}\,Q(a,z)\,dz = \frac{1}{\alpha}\left(z^\alpha\,Q(a,z) - \frac{\Gamma(a+\alpha)}{\Gamma(a)}\,Q(a+\alpha,z)\right)$$

#### Involving only one direct function with respect to *a*

06.08.21.0003.01

$$\int Q(a,z)\,da = \frac{1}{\Gamma(a)}\int_z^\infty \frac{t^{a-1}\,e^{-t}}{\log(t)}\,dt$$

## Integral transforms

### Fourier cos transforms

06.08.22.0001.01

$$\mathcal{F}c_t[Q(a,t)](x) = \sqrt{\frac{2}{\pi}} \frac{(x^2+1)^{-\frac{a}{2}} \sin(a \tan^{-1}(x))}{x} + \sqrt{\frac{\pi}{2}}\, \delta(x) \;/;\; x \in \mathbb{R} \land \mathrm{Re}(a) > -1$$

### Fourier sin transforms

06.08.22.0002.01

$$\mathcal{F}s_t[Q(a,t)](x) = \sqrt{\frac{2}{\pi}} \frac{1 + (1+x^2)^{-\frac{a}{2}} \cos(a \tan^{-1}(x))}{x} \;/;\; x \in \mathbb{R} \land \mathrm{Re}(a) > -2$$

### Laplace transforms

06.08.22.0003.01

$$\mathcal{L}_t[Q(a,t)](z) = \frac{1-(z+1)^{-a}}{z} \;/;\; \mathrm{Re}(z) > 0 \land \mathrm{Re}(a) > -1$$

## Representations through more general functions

### Through hypergeometric functions

#### Involving ${}_1\tilde{F}_1$

06.08.26.0001.01

$$Q(a,z) = 1 - z^a \, {}_1\tilde{F}_1(a; a+1; -z) \;/;\; -a \notin \mathbb{N}^+$$

#### Involving ${}_1F_1$

06.08.26.0002.01

$$Q(a,z) = 1 - \frac{z^a}{\Gamma(a+1)} \, {}_1F_1(a; a+1; -z) \;/;\; -a \notin \mathbb{N}^+$$

#### Involving hypergeometric $U$

06.08.26.0003.01

$$Q(a,z) = \frac{1}{\Gamma(a)} e^{-z} \, U(1-a, 1-a, z)$$

### Through Meijer $G$

#### Classical cases for the direct function itself

06.08.26.0004.01

$$Q(a,z) = 1 - \frac{1}{\Gamma(a)} G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1 \\ a, 0 \end{matrix}\right)$$

06.08.26.0005.01

$$Q(a,z) = \frac{1}{\Gamma(a)} G_{1,2}^{2,0}\left(z \,\middle|\, \begin{matrix} 1 \\ 0, a \end{matrix}\right)$$

06.08.26.0006.01

$$Q(a, \sqrt{z}) - Q(a, -\sqrt{z}) = -\frac{2^{a-2}\sqrt{\pi}}{\Gamma(a)} \sqrt{-z^2} \, G_{1,3}^{2,0}\left(-\frac{z}{4} \,\middle|\, \begin{matrix} 0 \\ \frac{a-1}{2}, \frac{a}{2}-1, -1 \end{matrix}\right)$$

### Classical cases involving exp

06.08.26.0007.01

$$e^z\, Q(a, z) = \frac{\sin(\pi a)}{\pi}\, G_{1,2}^{2,1}\!\left(z \,\middle|\, \begin{matrix} a \\ 0, a \end{matrix}\right)$$

06.08.26.0008.01

$$e^{-z}\, Q(a, -z) + e^z\, Q(a, z) = \frac{\sin(\pi a)}{\pi^{3/2}}\, G_{2,4}^{3,2}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{a+1}{2}, \frac{a}{2} \\ \frac{a+1}{2}, \frac{a}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

06.08.26.0009.01

$$e^{-z}\, Q(a, -z) - e^z\, Q(a, z) = -\frac{\sin(\pi a)\sqrt{-z^2}}{\pi^{3/2}\, z}\, G_{2,4}^{3,2}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{a+1}{2}, \frac{a}{2} \\ \frac{a+1}{2}, \frac{a}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

06.08.26.0010.01

$$e^{\frac{\pi i a}{2}-z}\, Q(a, -z) + e^{-\frac{\pi i a}{2}+z}\, Q(a, z) = \frac{\sin(\pi a)}{\pi^{3/2}}\, G_{1,3}^{3,1}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{a}{2} \\ 0, \frac{1}{2}, \frac{a}{2} \end{matrix}\right) /;\, 0 < \arg(z) \le \pi$$

06.08.26.0011.01

$$e^{\frac{\pi i a}{2}-z}\, Q(a, -z) - e^{-\frac{\pi i a}{2}+z}\, Q(a, z) = \frac{i \sin(\pi a)}{\pi^{3/2}}\, G_{1,3}^{3,1}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{a+1}{2} \\ 0, \frac{1}{2}, \frac{a+1}{2} \end{matrix}\right) /;\, 0 < \arg(z) \le \pi$$

### Classical cases for products of incomplete gamma functions ||| Classical cases for products of incomplete gamma functions

06.08.26.0012.01

$$Q(a, -z)\, Q(a, z) = \frac{2^{a-1} \sin(\pi a)}{\pi^{3/2}\, \Gamma(a)}\, G_{2,4}^{4,1}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} a, 1 \\ 0, \frac{a}{2}, \frac{a+1}{2}, a \end{matrix}\right)$$

### Classical cases involving regularized gamma Γ

06.08.26.0017.01

$$\Gamma(a, -z)\, Q(a, z) = \frac{2^{a-1} \sin(\pi a)}{\pi^{3/2}}\, G_{2,4}^{4,1}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} a, 1 \\ 0, \frac{a}{2}, \frac{a+1}{2}, a \end{matrix}\right)$$

### Generalized cases for the direct function itself

06.08.26.0013.01

$$e^{\frac{\pi i a}{2}}\, Q(a, -z) + e^{-\frac{\pi i a}{2}}\, Q(a, z) = \frac{2^a \sqrt{\pi}}{\Gamma(a)}\, G_{1,3}^{2,0}\!\left(-\frac{i z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 1 \\ 0, \frac{a}{2}, \frac{a+1}{2} \end{matrix}\right) /;\, 0 < \arg(z) \le \pi$$

06.08.26.0014.01

$$e^{\frac{\pi i a}{2}}\, Q(a, -z) - e^{-\frac{\pi i a}{2}}\, Q(a, z) = \frac{i\, 2^a \sqrt{\pi}}{\Gamma(a)}\, G_{1,3}^{2,0}\!\left(-\frac{i z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 1 \\ 0, \frac{a+1}{2}, \frac{a}{2} \end{matrix}\right) /;\, 0 < \arg(z) \le \pi$$

### Generalized cases involving exp

06.08.26.0018.01

$$e^{-z}\, Q(a, -z) + e^z\, Q(a, z) = \frac{\sin(a \pi)}{\pi^{3/2}}\, G_{2,4}^{3,2}\!\left(\frac{\sqrt{-z^2}}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{a}{2}, \frac{a+1}{2} \\ \frac{a}{2}, \frac{a+1}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

06.08.26.0019.01

$$e^{-z}\,Q(a,-z)-e^{z}\,Q(a,z)=\frac{z\sin(a\pi)}{\pi^{3/2}\sqrt{-z^2}}\,G_{2,4}^{3,2}\!\left(\frac{\sqrt{-z^2}}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{a}{2},\frac{a+1}{2}\\ \frac{a}{2},\frac{a+1}{2},\frac{1}{2},0\end{matrix}\right)$$

06.08.26.0020.01

$$e^{\frac{\pi i a}{2}-z}\,Q(a,-z)+e^{-\frac{1}{2}(\pi i a)+z}\,Q(a,z)=\frac{\sin(a\pi)}{\pi^{3/2}}\,G_{1,3}^{3,1}\!\left(\frac{\sqrt{-z^2}}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{a}{2}\\ 0,\frac{1}{2},\frac{a}{2}\end{matrix}\right)/;\ 0<\arg(z)\leq\pi$$

06.08.26.0021.01

$$e^{\frac{\pi i a}{2}-z}\,Q(a,-z)-e^{-\frac{1}{2}(\pi i a)+z}\,Q(a,z)=\frac{i\sin(a\pi)}{\pi^{3/2}}\,G_{1,3}^{3,1}\!\left(\frac{\sqrt{-z^2}}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{a+1}{2}\\ 0,\frac{1}{2},\frac{a+1}{2}\end{matrix}\right)/;\ 0<\arg(z)\leq\pi$$

### Through other functions

#### Involving some hypergeometric-type functions

06.08.26.0015.01

$$Q(a,z)==\frac{\Gamma(a,z,0)}{\Gamma(a)}+1\ /;\ \mathrm{Re}(a)>0$$

06.08.26.0016.01

$$Q(a,z)==Q(a,z,0)+1\ /;\ \mathrm{Re}(a)>0$$

## Representations through equivalent functions

### With inverse function

06.08.27.0001.01

$$Q\left(a,Q^{-1}(a,z)\right)==z$$

### With related functions

06.08.27.0002.01

$$Q(a,z)==\frac{\Gamma(a,z)}{\Gamma(a)}$$

06.08.27.0003.01

$$Q(a,z)==\frac{z^a\,E_{1-a}(z)}{\Gamma(a)}$$

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