GegenbauerC3General

Notations

Traditional name

Gegenbauer function

Traditional notation

$C_\nu^\lambda(z)$

Mathematica StandardForm notation

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GegenbauerC[ν, λ, z]
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Primary definition

07.14.02.0001.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(\nu+2\lambda)}{\nu!\,\Gamma(\lambda)}\,{}_2\tilde{F}_1\left(-\nu,\,\nu+2\lambda;\,\lambda+\frac{1}{2};\,\frac{1-z}{2}\right)$$

Specific values

Specialized values

For fixed ν, λ

07.14.03.0001.01

$$C_\nu^\lambda(0) = \frac{2^\nu\sqrt{\pi}\,\Gamma\left(\lambda+\frac{\nu}{2}\right)}{\Gamma\left(\frac{1-\nu}{2}\right)\Gamma(\nu+1)\,\Gamma(\lambda)}$$

07.14.03.0002.01

$$C_\nu^\lambda(1) = \frac{\Gamma(2\lambda+\nu)}{\Gamma(2\lambda)\,\Gamma(\nu+1)}$$

07.14.03.0003.01

$$C_\nu^\lambda(-1) = \frac{\cos(\pi(\lambda+\nu))\,\Gamma(2\lambda+\nu)\sec(\pi\lambda)}{\Gamma(2\lambda)\,\Gamma(\nu+1)} \;/;\; \mathrm{Re}(\lambda) < \frac{1}{2}$$

07.14.03.0004.01

$$C_\nu^\lambda(-1) = \tilde{\infty} \;/;\; \mathrm{Re}(\lambda) > \frac{1}{2}$$

For fixed ν, z

07.14.03.0005.01

$$C_{\nu}^{0}(z) == 0$$

07.14.03.0006.01

$$C_{\nu}^{-m}(z) == 0 \;/;\; m \in \mathbb{N}$$

07.14.03.0007.01

$$C_{\nu}^{\frac{1}{2}}(z) == P_{\nu}(z)$$

07.14.03.0008.01

$$C_{\nu}^{1}(z) == U_{\nu}(z)$$

07.14.03.0009.01

$$C_{\nu}^{-\frac{k+\nu}{2}}(z) == \tilde{\infty} \;/;\; k \in \mathbb{N}$$

For fixed λ, z

07.14.03.0010.01

$$C_{0}^{\lambda}(z) == 1$$

07.14.03.0011.01

$$C_{1}^{\lambda}(z) == 2\,\lambda\,z$$

07.14.03.0012.01

$$C_{2}^{\lambda}(z) == 2\,\lambda\,(\lambda+1)\,z^2 - \lambda$$

07.14.03.0013.01

$$C_{3}^{\lambda}(z) == \frac{4}{3}\,\lambda\,(\lambda+1)\,(\lambda+2)\,z^3 - 2\,\lambda\,(\lambda+1)\,z$$

07.14.03.0014.01

$$C_{4}^{\lambda}(z) == \frac{2}{3}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,z^4 - 2\,\lambda\,(\lambda+1)\,(\lambda+2)\,z^2 + \frac{1}{2}\,\lambda\,(\lambda+1)$$

07.14.03.0015.01

$$C_{5}^{\lambda}(z) == \frac{4}{15}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,z^5 - \frac{4}{3}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,z^3 + \lambda\,(\lambda+1)\,(\lambda+2)\,z$$

07.14.03.0016.01

$$C_{6}^{\lambda}(z) == \frac{4}{45}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,z^6 - \frac{2}{3}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,z^4 + \lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,z^2 - \frac{1}{6}\,\lambda\,(\lambda+1)\,(\lambda+2)$$

07.14.03.0017.01

$$C_{7}^{\lambda}(z) == \frac{8}{315}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,z^7 - \frac{4}{15}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,z^5 + \frac{2}{3}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,z^3 - \frac{1}{3}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,z$$

07.14.03.0018.01

$$C_{8}^{\lambda}(z) == \frac{2}{315}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,(\lambda+7)\,z^8 - \frac{4}{45}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,z^6 + \frac{1}{3}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,z^4 - \frac{1}{3}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,z^2 + \frac{1}{24}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)$$

07.14.03.0019.01

$$C_9^{\lambda}(z) = \frac{4\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,(\lambda+7)\,(\lambda+8)\,z^9}{2835} - \frac{8}{315}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,(\lambda+7)\,z^7 + \frac{2}{15}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,z^5 - \frac{2}{9}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,z^3 + \frac{1}{12}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,z$$

07.14.03.0020.01

$$C_{10}^{\lambda}(z) = \frac{4\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,(\lambda+7)\,(\lambda+8)\,(\lambda+9)\,z^{10}}{14\,175} - \frac{2}{315}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,(\lambda+7)\,(\lambda+8)\,z^8 + \frac{2}{45}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,(\lambda+7)\,z^6 - \frac{1}{9}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,(\lambda+6)\,z^4 + \frac{1}{12}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)\,(\lambda+5)\,z^2 - \frac{1}{120}\,\lambda\,(\lambda+1)\,(\lambda+2)\,(\lambda+3)\,(\lambda+4)$$

07.14.03.0021.01

$$C_n^{\lambda}(z) = \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k\,(\lambda)_{n-k}\,(2\,z)^{n-2\,k}}{k!\,(n-2\,k)!} \;/;\; n \in \mathbb{N}$$

07.14.03.0022.01

$$C_{-n}^{\lambda}(z) = 0 \;/;\; n \in \mathbb{N}^+$$

07.14.03.0023.01

$$C_{-2\,\lambda-k}^{\lambda}(z) = \tilde{\infty} \;/;\; k \in \mathbb{N}$$

General characteristics

Domain and analyticity

$C_\nu^{\lambda}(z)$ is an analytical function of ν, λ, z which is defined in $\mathbb{C}^3$. For integer ν, $C_\nu^{\lambda}(z)$ degenerates to a polynomial in z.

07.14.04.0001.01

$$(\nu * \lambda * z) \longrightarrow C_\nu^{\lambda}(z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

07.14.04.0002.01

$$C_n^{\lambda}(-z) = (-1)^n\,C_n^{\lambda}(z) \;/;\; n \in \mathbb{N}$$

Mirror symmetry

07.14.04.0003.01

$$C_{\bar{\nu}}^{\bar{\lambda}}(\bar{z}) = \overline{C_\nu^{\lambda}(z)} \;/;\; z \notin (-\infty, -1)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν /; $\nu \notin \mathbb{Z}$, λ, the function $C_\nu^\lambda(z)$ does not have poles and essential singularities.

07.14.04.0004.01

$$\mathcal{Sing}_z\left(C_\nu^\lambda(z)\right) == \{\} /; \nu \notin \mathbb{Z}$$

For positive integer ν, the function $C_\nu^\lambda(z)$ is polynomial and has pole of order ν at $z = \tilde{\infty}$.

07.14.04.0005.01

$$\mathcal{Sing}_z\left(C_\nu^\lambda(z)\right) == \{\{\tilde{\infty}, \nu\}\} /; \nu \in \mathbb{N}^+$$

For nonpositive integer ν, the function $C_\nu^\lambda(z)$ is constant (0 for $\nu < 0$ or 1 for $\nu == 0$).

With respect to λ

For fixed ν, z, the function $C_\nu^\lambda(z)$ has an infinite set of singular points:

a) $\lambda == -\frac{\nu+j}{2}$ /; $j \in \mathbb{N}$, are the simple poles with residues $\frac{(-1)^j 2^{j+\nu} \sqrt{\pi}}{\nu!\, j!\, \Gamma\left(-\frac{\nu+j}{2}\right)} \, {}_2\tilde{F}_1\left(-j, -\nu; \frac{1-\nu-j}{2}; \frac{1-z}{2}\right)$;

b) $\lambda == \tilde{\infty}$ is an essential singular point.

07.14.04.0006.01

$$\mathcal{Sing}_\lambda\left(C_\nu^\lambda(z)\right) == \left\{\left\{\left\{-\frac{\nu+j}{2}, 1\right\} /; j \in \mathbb{N}\right\}, \{\tilde{\infty}, \infty\}\right\}$$

07.14.04.0007.01

$$\operatorname{res}_\lambda\left(C_\nu^\lambda(z)\right)\left(-\frac{\nu+j}{2}\right) == \frac{(-1)^j 2^{j+\nu} \sqrt{\pi}}{\nu!\, j!\, \Gamma\left(-\frac{\nu+j}{2}\right)} \, {}_2\tilde{F}_1\left(-j, -\nu; \frac{1-\nu-j}{2}; \frac{1-z}{2}\right) /; j \in \mathbb{N}$$

With respect to ν

For fixed λ, z, the function $C_\nu^\lambda(z)$ has an infinite set of singular points:

a) $\nu == -j - 2\lambda$ /; $j \in \mathbb{N}$, are the simple poles with residues $\frac{2^{1-2\lambda} \sqrt{\pi}\, (-1)^j}{j!\, \Gamma(1-j-2\lambda)\, \Gamma(\lambda)} \, {}_2\tilde{F}_1\left(-j, j+2\lambda; \lambda+\frac{1}{2}; \frac{1-z}{2}\right)$;

b) $\nu == \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

07.14.04.0008.01

$$\mathcal{Sing}_\nu\left(C_\nu^\lambda(z)\right) == \{\{\{-j - 2\lambda, 1\} /; j \in \mathbb{N}\}, \{\tilde{\infty}, \infty\}\}$$

07.14.04.0009.01

$$\operatorname{res}_\nu\left(C_\nu^\lambda(z)\right)(-j-2\lambda) == \frac{(-1)^j 2^{1-2\lambda} \sqrt{\pi}}{j!\, \Gamma(1-j-2\lambda)\, \Gamma(\lambda)} \, {}_2\tilde{F}_1\left(-j, j+2\lambda; \lambda+\frac{1}{2}; \frac{1-z}{2}\right) /; j \in \mathbb{N}$$

Branch points

With respect to z

For fixed ν /; $\nu \notin \mathbb{Z}$, λ, the function $C_\nu^\lambda(z)$ has two branch points: $z == -1$, $z == \tilde{\infty}$.

For fixed λ and integer ν, the function $C_\nu^\lambda(z)$ does not have branch points.

07.14.04.0010.01

$$\mathcal{BP}_z\left(C_\nu^\lambda(z)\right) == \{-1, \tilde{\infty}\} /; \nu \notin \mathbb{Z}$$

07.14.04.0011.01

$$\mathcal{BP}_z\left(C_\nu^\lambda(z)\right) == \{\} /; \nu \in \mathbb{Z}$$

07.14.04.0012.01

$$\mathcal{R}_z\left(C_\nu^\lambda(z), -1\right) == \log /; \lambda - \frac{1}{2} \in \mathbb{Z} \bigvee \lambda - \frac{1}{2} \notin \mathbb{Q} \bigwedge \neg (\nu \in \mathbb{N} \bigvee -2\lambda - \nu \in \mathbb{N})$$

07.14.04.0013.01

$$\mathcal{R}_z\left(C_\nu^\lambda(z), -1\right) == s /; \lambda - \frac{1}{2} == \frac{r}{s} \bigwedge r \in \mathbb{Z} \bigwedge s - 1 \in \mathbb{N}^+ \bigwedge \gcd(r, s) == 1 \bigwedge \neg (\nu \in \mathbb{N} \bigvee -2\lambda - \nu \in \mathbb{N})$$

07.14.04.0014.01

$$\mathcal{R}_z\left(C_\nu^\lambda(z), \tilde{\infty}\right) == \log /; 2\lambda \in \mathbb{Z} \bigvee \neg (\nu \in \mathbb{Q} \bigwedge 2\lambda + \nu \in \mathbb{Q})$$

07.14.04.0015.01

$$\mathcal{R}_z\left(C_\nu^\lambda(z), \tilde{\infty}\right) == \operatorname{lcm}(s, u) /;$$
$$\nu == \frac{r}{s} \bigwedge 2\lambda + \nu == \frac{t}{u} \bigwedge \{r, s, t, u\} \in \mathbb{Z} \bigwedge s > 1 \bigwedge u > 1 \bigwedge \gcd(r, s) == 1 \bigwedge \gcd(t, u) == 1 \bigwedge \neg (\nu \in \mathbb{N} \bigvee -2\lambda - \nu \in \mathbb{N})$$

With respect to λ

For fixed ν, z, the function $C_\nu^\lambda(z)$ does not have branch points.

07.14.04.0016.01

$$\mathcal{BP}_\lambda\left(C_\nu^\lambda(z)\right) == \{\}$$

With respect to ν

For fixed λ, z, the function $C_\nu^\lambda(z)$ does not have branch points.

07.14.04.0017.01

$$\mathcal{BP}_\nu\left(C_\nu^\lambda(z)\right) == \{\}$$

Branch cuts

With respect to z

For fixed ν /; $\nu \notin \mathbb{Z}$, λ, the function $C_\nu^\lambda(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, -1)$ where it is continuous from above.

For fixed λ and integer ν, the function $C_\nu^\lambda(z)$ does not have branch cuts.

07.14.04.0018.01

$$\mathcal{BC}_z\left(C_\nu^\lambda(z)\right) == \{\{(-\infty, -1), -i\}\} /; \nu \notin \mathbb{Z}$$

07.14.04.0019.01

$$\mathcal{BC}_z\left(C_\nu^\lambda(z)\right) == \{\} \;/;\; \nu \in \mathbb{Z}$$

07.14.04.0020.01

$$\lim_{\epsilon \to +0} C_\nu^\lambda(x - i\,\epsilon) == C_\nu^\lambda(x) \;/;\; x < -1$$

07.14.04.0021.01

$$\lim_{\epsilon \to +0} C_\nu^\lambda(x - i\,\epsilon) == e^{i\,\pi\,\lambda}\left(2\cos(\pi\,(\nu + \lambda))\,C_\nu^\lambda(-x) - e^{i\,\pi\,\lambda}\,C_\nu^\lambda(x)\right) \;/;\; x < -1$$

With respect to λ

For fixed ν, z, the function $C_\nu^\lambda(z)$ does not have branch cuts.

07.14.04.0022.01

$$\mathcal{BC}_\lambda\left(C_\nu^\lambda(z)\right) == \{\}$$

With respect to ν

For fixed λ, z, the function $C_\nu^\lambda(z)$ does not have branch cuts.

07.14.04.0023.01

$$\mathcal{BC}_\nu\left(C_\nu^\lambda(z)\right) == \{\}$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

07.14.06.0037.01

$$C_\nu^\lambda(z) \propto \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\sin(\pi\nu)}{\pi^{3/2}\,\Gamma(\lambda)} \Bigg(-i\,\pi^{3/2}\csc(\pi\nu)\,4^\lambda\,\Gamma(\lambda)\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} \left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor \left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor C_\nu^\lambda(-z_0) - \left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} (z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{1}{2}(z_0+1)\,\middle|\,\begin{matrix}\nu+1,\,-2\lambda-\nu+1\\0,\,\frac{1}{2}-\lambda\end{matrix}\right) + \frac{1}{2}\Bigg(\left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} (z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{1}{2}(z_0+1)\,\middle|\,\begin{matrix}\nu,\,-2\lambda-\nu\\0,\,-\lambda-\frac{1}{2}\end{matrix}\right) + 2\pi i\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} \left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor \left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor \Gamma(1-\nu)\,\Gamma(2\lambda+\nu+1)\,{}_2\tilde{F}_1\!\left(1-\nu,\,2\lambda+\nu+1;\,\lambda+\frac{3}{2};\,\frac{1}{2}(z_0+1)\right)\Bigg)(z-z_0) + \frac{1}{8}\Bigg(2\pi i\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} \left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor \left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor \Gamma(2-\nu)\,\Gamma(2\lambda+\nu+2)\,{}_2\tilde{F}_1\!\left(2-\nu,\,2\lambda+\nu+2;\,\lambda+\frac{5}{2};\,\frac{1}{2}(z_0+1)\right) - \left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} (z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{1}{2}(z_0+1)\,\middle|\,\begin{matrix}\nu-1,\,-2\lambda-\nu-1\\0,\,-\lambda-\frac{3}{2}\end{matrix}\right)\Bigg)(z-z_0)^2 + \ldots \Bigg) /;\ (z\to z_0)$$

07.14.06.0038.01

$$C_\nu^\lambda(z) \propto \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\sin(\pi\nu)}{\pi^{3/2}\,\Gamma(\lambda)} \Bigg(-i\,\pi^{3/2}\csc(\pi\nu)\,4^\lambda\,\Gamma(\lambda)\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor C_\nu^\lambda(-z_0) -$$

$$\left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{1}{2}(z_0+1)\,\middle|\,\begin{matrix}\nu+1,\,-2\lambda-\nu+1\\ 0,\,\frac{1}{2}-\lambda\end{matrix}\right)+$$

$$\frac{1}{2}\Bigg(\left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{1}{2}(z_0+1)\,\middle|\,\begin{matrix}\nu,\,-2\lambda-\nu\\ 0,\,-\lambda-\frac{1}{2}\end{matrix}\right)+2\pi\,i\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor$$

$$\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor\Gamma(1-\nu)\,\Gamma(2\lambda+\nu+1)\,{}_2\tilde{F}_1\!\left(1-\nu,\,2\lambda+\nu+1;\,\lambda+\frac{3}{2};\,\frac{1}{2}(z_0+1)\right)\Bigg)(z-z_0)+$$

$$\frac{1}{8}\Bigg(2\pi\,i\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor\Gamma(2-\nu)\,\Gamma(2\lambda+\nu+2)\,{}_2\tilde{F}_1\!\left(2-\nu,\,2\lambda+\nu+2;\,\lambda+\frac{5}{2};\,\frac{1}{2}(z_0+1)\right)-$$

$$\left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{1}{2}(z_0+1)\,\middle|\,\begin{matrix}\nu-1,\,-2\lambda-\nu-1\\ 0,\,-\lambda-\frac{3}{2}\end{matrix}\right)\Bigg)(z-z_0)^2+O\!\left((z-z_0)^3\right)\Bigg)$$

07.14.06.0039.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\sin(\pi\nu)}{\pi^{3/2}\,\Gamma(\lambda)}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-k}}{k!}\Bigg(2\pi\,i\,(-1)^k\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor\Gamma(k-\nu)\,\Gamma(k+2\lambda+\nu)$$

$${}_2\tilde{F}_1\!\left(k-\nu,\,k+2\lambda+\nu;\,k+\lambda+\frac{1}{2};\,\frac{z_0+1}{2}\right)-$$

$$\left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{z_0+1}{2}\,\middle|\,\begin{matrix}-k+\nu+1,\,-k-2\lambda-\nu+1\\ 0,\,-k-\lambda+\frac{1}{2}\end{matrix}\right)\Bigg)(z-z_0)^k$$

07.14.06.0040.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\sin(\pi\nu)}{\sqrt{\pi}\,\Gamma(\lambda)}\sum_{k=0}^{\infty}\frac{2^{-k}}{k!}\left(2^{k+\lambda-\frac{1}{2}}\pi\sec(\pi\lambda)(z_0+1)^{\frac{1}{2}-k-\lambda+\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}{}_2\tilde{F}_1\left(\lambda+\nu+\frac{1}{2},-\lambda-\nu+\frac{1}{2};-k-\lambda+\frac{3}{2};\frac{z_0+1}{2}\right)+\cos(\pi(\lambda+\nu))\,\Gamma(k-\nu)\,\Gamma(k+2\lambda+\nu)\left(2\,i\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor-\sec(\pi\lambda)\left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\right){}_2\tilde{F}_1\left(k-\nu,k+2\lambda+\nu;k+\lambda+\frac{1}{2};\frac{z_0+1}{2}\right)\right)(z-z_0)^k\ /;\ \frac{1}{2}-\lambda\notin\mathbb{Z}$$

07.14.06.0041.01

$$C_\nu^\lambda(z)\propto-\frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\sin(\pi\nu)}{\pi^{3/2}\,\Gamma(\lambda)}\left(i\,\pi^{3/2}\csc(\pi\nu)\,4^\lambda\,\Gamma(\lambda)\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor C_\nu^\lambda(z_0)+\left(\frac{1}{z_0+1}\right)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(\frac{1}{2}(z_0+1)\,\middle|\,\begin{matrix}\nu+1,-2\lambda-\nu+1\\0,\frac{1}{2}-\lambda\end{matrix}\right)+O(z-z_0)\right)$$

Expansions on branch cuts

For the function itself

07.14.06.0042.01

$$C_\nu^\lambda(z)\propto\frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\sin(\pi\nu)}{\pi^{3/2}\,\Gamma(\lambda)}\left(-i\,\pi^{3/2}\,4^\lambda\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\csc(\pi\nu)\,\Gamma(\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor C_\nu^\lambda(-x)-e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(\frac{x+1}{2}\,\middle|\,\begin{matrix}\nu+1,-2\lambda-\nu+1\\0,\frac{1}{2}-\lambda\end{matrix}\right)+\frac{1}{2}\left(2\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}i\,\pi\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\Gamma(1-\nu)\,\Gamma(2\lambda+\nu+1)\,{}_2\tilde{F}_1\left(1-\nu,2\lambda+\nu+1;\lambda+\frac{3}{2};\frac{x+1}{2}\right)+e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(\frac{x+1}{2}\,\middle|\,\begin{matrix}\nu,-2\lambda-\nu\\0,-\lambda-\frac{1}{2}\end{matrix}\right)\right)(z-x)+\frac{1}{8}\left(2\,\pi\,i\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\Gamma(2-\nu)\,\Gamma(2\lambda+\nu+2)\,{}_2\tilde{F}_1\left(2-\nu,2\lambda+\nu+2;\lambda+\frac{5}{2};\frac{x+1}{2}\right)-e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(\frac{x+1}{2}\,\middle|\,\begin{matrix}\nu-1,-2\lambda-\nu-1\\0,-\lambda-\frac{3}{2}\end{matrix}\right)\right)(z-x)^2+\ldots\right)/;\ (z\to x)\land x\in\mathbb{R}\land x<-1$$

07.14.06.0043.01

$$C_\nu^\lambda(z) \propto \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\sin(\pi\nu)}{\pi^{3/2}\,\Gamma(\lambda)} \left(-i\,\pi^{3/2}\,4^\lambda\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\csc(\pi\nu)\,\Gamma(\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor C_\nu^\lambda(-x) - e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{x+1}{2}\,\middle|\,\begin{matrix}\nu+1,\,-2\lambda-\nu+1\\ 0,\,\frac{1}{2}-\lambda\end{matrix}\right) + \frac{1}{2}\left(2\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor} i\,\pi\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\Gamma(1-\nu)\,\Gamma(2\lambda+\nu+1)\,{}_2\tilde{F}_1\!\left(1-\nu,\,2\lambda+\nu+1;\,\lambda+\frac{3}{2};\,\frac{x+1}{2}\right) + e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{x+1}{2}\,\middle|\,\begin{matrix}\nu,\,-2\lambda-\nu\\ 0,\,-\lambda-\frac{1}{2}\end{matrix}\right)\right)(z-x) + \frac{1}{8}\left(2\pi\,i\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\Gamma(2-\nu)\,\Gamma(2\lambda+\nu+2)\,{}_2\tilde{F}_1\!\left(2-\nu,\,2\lambda+\nu+2;\,\lambda+\frac{5}{2};\,\frac{x+1}{2}\right) - e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{x+1}{2}\,\middle|\,\begin{matrix}\nu-1,\,-2\lambda-\nu-1\\ 0,\,-\lambda-\frac{3}{2}\end{matrix}\right)\right)(z-x)^2 + O\!\left((z-x)^3\right)\right) /;\; x\in\mathbb{R}\land x<-1$$

07.14.06.0044.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\sin(\pi\nu)}{\pi^{3/2}\,\Gamma(\lambda)} \sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-k}}{k!}\left(2\pi\,i\,(-1)^k\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\Gamma(k-\nu)\,\Gamma(k+2\lambda+\nu)\,{}_2\tilde{F}_1\!\left(k-\nu,\,k+2\lambda+\nu;\,k+\lambda+\frac{1}{2};\,\frac{x+1}{2}\right) - e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(\frac{x+1}{2}\,\middle|\,\begin{matrix}-k+\nu+1,\,-k-2\lambda-\nu+1\\ 0,\,-k-\lambda+\frac{1}{2}\end{matrix}\right)\right)(z-x)^k /;\; x\in\mathbb{R}\land x<-1$$

07.14.06.0045.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\sin(\pi\nu)}{\sqrt{\pi}\;\Gamma(\lambda)}\sum_{k=0}^{\infty}\frac{2^{-k}}{k!}\left(2^{k+\lambda-\frac{1}{2}}\,\pi\sec(\pi\lambda)\,(x+1)^{-k-\lambda+\frac{1}{2}}\,e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\,{}_2\tilde{F}_1\!\left(\lambda+\nu+\frac{1}{2},\,-\lambda-\nu+\frac{1}{2};\,-k-\lambda+\frac{3}{2};\,\frac{x+1}{2}\right) + \cos(\pi(\lambda+\nu))\,\Gamma(k-\nu)\,\Gamma(k+2\lambda+\nu)\left(2\,i\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor - \sec(\pi\lambda)\,e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\right){}_2\tilde{F}_1\!\left(k-\nu,\,k+2\lambda+\nu;\,k+\lambda+\frac{1}{2};\,\frac{x+1}{2}\right)\right)(z-x)^k /;\; \frac{1}{2}-\lambda\notin\mathbb{Z}\bigwedge x\in\mathbb{R}\bigwedge x<-1$$

07.14.06.0046.01

$$C_\nu^\lambda(z) = -\frac{2^{\frac{1}{2}-\lambda}\sqrt{\pi}\, e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\sec(\pi\lambda)\sin(\pi\nu)}{\Gamma(\lambda)} \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\left(2\,i\,(1-x)^{\frac{1}{2}-k-\lambda}\cos(\pi(\lambda+\nu))\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor {}_2\tilde{F}_1\left(\lambda+\nu+\frac{1}{2},\frac{1}{2}-\lambda-\nu;\frac{3}{2}-k-\lambda;\frac{1-x}{2}\right)+\frac{\pi(x+1)^{\frac{1}{2}-k-\lambda}}{\Gamma(1-k-2\lambda-\nu)\Gamma(\nu-k+1)}\left(2\,i\csc(\pi\nu)\csc(\pi(2\lambda+\nu))\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor\cos^2(\pi(\lambda+\nu))+e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left(-\csc(\pi\nu)\csc(\pi(2\lambda+\nu))\cos^2(\pi(\lambda+\nu))-1\right)\sec(\pi\lambda)\right) {}_2\tilde{F}_1\left(\lambda+\nu+\frac{1}{2},\frac{1}{2}-\lambda-\nu;k+\lambda+\frac{1}{2};\frac{1-x}{2}\right)\right)(z-x)^k \;/;\; \frac{1}{2}-\lambda\notin\mathbb{Z}\bigwedge x\in\mathbb{R}\bigwedge x<-1$$

07.14.06.0047.01

$$C_\nu^\lambda(z)\propto-\frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\sin(\pi\nu)}{\pi^{3/2}\Gamma(\lambda)}\left(i\,\pi^{3/2}\,4^\lambda\,e^{i\pi\left(\frac{1}{2}-\lambda\right)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\csc(\pi\nu)\,\Gamma(\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor C_\nu^\lambda(-x)+e^{\pi i(1-2\lambda)\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(\frac{x+1}{2}\,\middle|\,\begin{matrix}\nu+1,-2\lambda-\nu+1\\0,\frac{1}{2}-\lambda\end{matrix}\right)+O(z-x)\right)/;\,x\in\mathbb{R}\land x<-1$$

Expansions at $z == 0$

For the function itself

General case

07.14.06.0001.02

$$C_\nu^\lambda(z)\propto\frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda)}\left(\frac{\sqrt{\pi}}{\Gamma\left(\frac{1}{2}-\frac{\nu}{2}\right)\Gamma\left(\lambda+\frac{\nu}{2}+\frac{1}{2}\right)}-\frac{2\sqrt{\pi}\,z}{\Gamma\left(\lambda+\frac{\nu}{2}\right)\Gamma\left(-\frac{\nu}{2}\right)}+\frac{2^{2\lambda-1}\Gamma\left(\lambda+\frac{\nu}{2}+1\right)\Gamma\left(1-\frac{\nu}{2}\right)z^2}{\sqrt{\pi}\,\Gamma(-\nu)\Gamma(2\lambda+\nu)}+\ldots\right)/;\,(z\to 0)$$

07.14.06.0048.01

$$C_\nu^\lambda(z)\propto\frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda)}\left(\frac{\sqrt{\pi}}{\Gamma\left(\frac{1}{2}-\frac{\nu}{2}\right)\Gamma\left(\lambda+\frac{\nu}{2}+\frac{1}{2}\right)}-\frac{2\sqrt{\pi}\,z}{\Gamma\left(\lambda+\frac{\nu}{2}\right)\Gamma\left(-\frac{\nu}{2}\right)}+\frac{2^{2\lambda-1}\Gamma\left(\lambda+\frac{\nu}{2}+1\right)\Gamma\left(1-\frac{\nu}{2}\right)z^2}{\sqrt{\pi}\,\Gamma(-\nu)\Gamma(2\lambda+\nu)}+O\left(z^3\right)\right)$$

07.14.06.0002.01

$$C_\nu^\lambda(z)=\frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda)}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-\nu)_{j+k}(2\lambda+\nu)_{j+k}(-z)^j}{\Gamma\left(j+k+\lambda+\frac{1}{2}\right)j!\,k!\,2^{j+k}}\;/;\;|z|<1$$

07.14.06.0003.01

$$C_\nu^\lambda(z)=\frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda)}\tilde{F}_{1\times0\times0}^{2\times0\times0}\left(\begin{matrix}-\nu,2\lambda+\nu;;;\\\lambda+\frac{1}{2};;;\end{matrix}\;\frac{1}{2},-\frac{z}{2}\right)$$

07.14.06.0004.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)} \sum_{k=0}^{\infty}\sum_{j=0}^{k} \frac{(-\nu)_k\,(2\lambda+\nu)_k\,(-z)^j}{\Gamma\left(k+\lambda+\frac{1}{2}\right) j!\,(k-j)!\,2^k} \;/;\; |z|<1$$

07.14.06.0049.01

$$C_\nu^\lambda(z) = \frac{2^\nu\sqrt{\pi}\,\Gamma\left(\lambda+\frac{\nu}{2}\right)}{\Gamma(\lambda)\,\Gamma\left(\frac{1-\nu}{2}\right)\Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{\left(-\frac{\nu}{2}\right)_k \left(\lambda+\frac{\nu}{2}\right)_k}{\left(\frac{1}{2}\right)_k k!} z^{2k} + \frac{2^\nu\sqrt{\pi}\,z\,\Gamma\left(\lambda+\frac{\nu+1}{2}\right)}{\Gamma(\lambda)\,\Gamma\left(1-\frac{\nu}{2}\right)\Gamma(\nu)} \sum_{k=0}^{\infty} \frac{\left(\frac{1-\nu}{2}\right)_k \left(\frac{\nu+1}{2}+\lambda\right)_k}{\left(\frac{3}{2}\right)_k k!} z^{2k} \;/;\; |z|<1$$

07.14.06.0050.01

$$C_\nu^\lambda(z) = \frac{\cos\left(\frac{\pi\nu}{2}\right)\Gamma\left(\lambda+\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu}{2}+1\right)\Gamma(\lambda)}\, {}_2F_1\left(-\frac{\nu}{2}, \lambda+\frac{\nu}{2}; \frac{1}{2}; z^2\right) + \frac{2^\nu\sqrt{\pi}\,\Gamma\left(\lambda+\frac{\nu+1}{2}\right) z}{\Gamma(\lambda)\,\Gamma\left(1-\frac{\nu}{2}\right)\Gamma(\nu)}\, {}_2F_1\left(\frac{1-\nu}{2}, \frac{\nu+1}{2}+\lambda; \frac{3}{2}; z^2\right)$$

07.14.06.0005.02

$$C_\nu^\lambda(z) \propto \frac{\cos\left(\frac{\pi\nu}{2}\right)\Gamma\left(\lambda+\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu}{2}+1\right)\Gamma(\lambda)} (1+O(z))$$

07.14.06.0051.01

$$C_\nu^\lambda(z) = F_\infty(z, \nu, \lambda) \;/;$$
$$\Bigg(\Bigg(F_m(z,\nu,\lambda) = \frac{2^\nu\sqrt{\pi}\,\Gamma\left(\lambda+\frac{\nu}{2}\right)}{\Gamma(\lambda)\,\Gamma\left(\frac{1-\nu}{2}\right)\Gamma(\nu+1)} \sum_{k=0}^{m} \frac{\left(-\frac{\nu}{2}\right)_k \left(\lambda+\frac{\nu}{2}\right)_k}{\left(\frac{1}{2}\right)_k k!} z^{2k} + \frac{2^\nu\sqrt{\pi}\,z\,\Gamma\left(\lambda+\frac{\nu+1}{2}\right)}{\Gamma(\lambda)\,\Gamma\left(1-\frac{\nu}{2}\right)\Gamma(\nu)} \sum_{k=0}^{m} \frac{\left(\frac{1-\nu}{2}\right)_k \left(\frac{\nu+1}{2}+\lambda\right)_k}{\left(\frac{3}{2}\right)_k k!} z^{2k} =$$
$$C_\nu^{(\lambda)}(z) - \frac{2^\nu\sqrt{\pi}\,z^{2m+2}\,\Gamma\left(\lambda+\frac{\nu}{2}\right)\left(\lambda+\frac{\nu}{2}\right)_{m+1}\left(-\frac{\nu}{2}\right)_{m+1}}{(m+1)!\,\Gamma(\lambda)\,\Gamma\left(\frac{1-\nu}{2}\right)\Gamma(\nu+1)\left(\frac{1}{2}\right)_{m+1}}\, {}_3F_2\left(1, m-\frac{\nu}{2}+1, m+\lambda+\frac{\nu}{2}+1; m+\frac{3}{2}, m+2; z^2\right) -$$
$$\frac{2^\nu\sqrt{\pi}\,z^{2m+3}\,\Gamma\left(\lambda+\frac{\nu+1}{2}\right)\left(\frac{1-\nu}{2}\right)_{m+1}\left(\lambda+\frac{\nu+1}{2}\right)_{m+1}}{(m+1)!\,\Gamma(\lambda)\,\Gamma\left(1-\frac{\nu}{2}\right)\Gamma(\nu)\left(\frac{3}{2}\right)_{m+1}}\, {}_3F_2\left(1, m-\frac{\nu}{2}+\frac{3}{2}, m+\lambda+\frac{\nu}{2}+\frac{3}{2}; m+2, m+\frac{5}{2}; z^2\right)\Bigg) \bigwedge m \in \mathbb{N}\Bigg)$$

Summed form of the truncated series expansion.

Special cases

07.14.06.0006.01

$$C_n^\lambda(z) = \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k\,(\lambda)_{n-k}\,(2z)^{n-2k}}{k!\,(n-2k)!} \;/;\; n \in \mathbb{N}$$

07.14.06.0007.01

$$C_n^\lambda(z) \propto \frac{(-1)^{\left\lfloor \frac{n}{2} \right\rfloor}(\lambda)_{n-\left\lfloor \frac{n}{2} \right\rfloor}(2z)^{n-2\left\lfloor \frac{n}{2} \right\rfloor}}{\left\lfloor \frac{n}{2} \right\rfloor!\left(n-2\left\lfloor \frac{n}{2} \right\rfloor\right)!} \left(1+O\left(z^2\right)\right) \;/;\; (z \to 0) \bigwedge n \in \mathbb{N}^+$$

Generic formulas for main term

07.14.06.0052.01

$$C_\nu^\lambda(z) \propto \begin{cases} 0 & -\nu \in \mathbb{N}^+ \\ \tilde{\infty} & -2\lambda - \nu \in \mathbb{N} \\ \frac{(-1)^{\left\lfloor \frac{\nu}{2} \right\rfloor} (\lambda)_{\nu - \left\lfloor \frac{\nu}{2} \right\rfloor} (2z)^{\nu - 2\left\lfloor \frac{\nu}{2} \right\rfloor}}{\left\lfloor \frac{\nu}{2} \right\rfloor! \left(\nu - 2\left\lfloor \frac{\nu}{2} \right\rfloor\right)!} & \nu \in \mathbb{N} \\ \frac{\cos\left(\frac{\pi\nu}{2}\right) \Gamma\left(\lambda + \frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu}{2} + 1\right) \Gamma(\lambda)} & \text{True} \end{cases} \quad /; \ (z \to 0)$$

Expansions at $z == 1$

For the function itself

General case

07.14.06.0008.02

$$C_\nu^\lambda(z) \propto \frac{2^{1-2\lambda} \sqrt{\pi}\, \Gamma(2\lambda + \nu)}{\Gamma(\nu + 1)\, \Gamma(\lambda)} \left(\frac{1}{\Gamma\left(\lambda + \frac{1}{2}\right)} + \frac{\nu (2\lambda + \nu)(z - 1)}{2\, \Gamma\left(\lambda + \frac{3}{2}\right)} + \frac{-\nu (1 - \nu)(2\lambda + \nu)(2\lambda + \nu + 1)(z - 1)^2}{8\, \Gamma\left(\lambda + \frac{5}{2}\right)} + \ldots \right) /;$$
$$(z \to 1) \bigwedge -\lambda - \frac{1}{2} \notin \mathbb{N}$$

07.14.06.0053.01

$$C_\nu^\lambda(z) \propto \frac{2^{1-2\lambda} \sqrt{\pi}\, \Gamma(2\lambda + \nu)}{\Gamma(\nu + 1)\, \Gamma(\lambda)} \left(\frac{1}{\Gamma\left(\lambda + \frac{1}{2}\right)} + \frac{\nu (2\lambda + \nu)(z - 1)}{2\, \Gamma\left(\lambda + \frac{3}{2}\right)} + \frac{-\nu (1 - \nu)(2\lambda + \nu)(2\lambda + \nu + 1)(z - 1)^2}{8\, \Gamma\left(\lambda + \frac{5}{2}\right)} + O\left((z - 1)^3\right) \right) /;$$
$$-\lambda - \frac{1}{2} \notin \mathbb{N}$$

07.14.06.0009.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda} \sqrt{\pi}\, \Gamma(2\lambda + \nu)}{\Gamma(\nu + 1)\, \Gamma(\lambda)} \sum_{k=0}^{\infty} \frac{(-\nu)_k\, (2\lambda + \nu)_k}{\Gamma\left(k + \lambda + \frac{1}{2}\right) k!} \left(\frac{1 - z}{2} \right)^k /; \left| \frac{1 - z}{2} \right| < 1$$

07.14.06.0054.01

$$C_\nu^\lambda(z) \propto \frac{\Gamma(2\lambda + \nu)}{\Gamma(2\lambda)\, \Gamma(\nu + 1)} \sum_{k=0}^{\infty} \frac{(-\nu)_k\, (2\lambda + \nu)_k}{\left(\lambda + \frac{1}{2}\right)_k k!} \left(\frac{1 - z}{2} \right)^k /; -\lambda - \frac{1}{2} \notin \mathbb{N}$$

07.14.06.0010.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda} \sqrt{\pi}\, \Gamma(2\lambda + \nu)}{\Gamma(\nu + 1)\, \Gamma(\lambda)}\, {}_2\tilde{F}_1\left(-\nu, 2\lambda + \nu; \lambda + \frac{1}{2}; \frac{1 - z}{2} \right)$$

07.14.06.0055.01

$$C_\nu^\lambda(z) \propto \frac{\Gamma(2\lambda + \nu)}{\Gamma(2\lambda)\, \Gamma(\nu + 1)}\, {}_2F_1\left(-\nu, 2\lambda + \nu; \lambda + \frac{1}{2}; \frac{1 - z}{2} \right) /; -\lambda - \frac{1}{2} \notin \mathbb{N}$$

07.14.06.0012.01

$$C_\nu^\lambda(z) \propto \frac{\Gamma(2\lambda + \nu)}{\Gamma(2\lambda)\, \Gamma(\nu + 1)} (1 + O(z - 1)) /; (z \to 1) \bigwedge -\lambda - \frac{1}{2} \notin \mathbb{N}$$

07.14.06.0056.01

$$C_\nu^\lambda(z) = F_\infty(z, \nu, \lambda) /; \left(\left(F_m(z, \nu, \lambda) = \frac{\Gamma(2\lambda+\nu)}{\Gamma(2\lambda)\Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(-\nu)_k (2\lambda+\nu)_k}{\left(\lambda+\frac{1}{2}\right)_k k!} \left(\frac{1-z}{2}\right)^k = C_\nu^\lambda(z) - \frac{2^{-m-1}\Gamma(2\lambda+\nu)(-\nu)_{m+1}(2\lambda+\nu)_{m+1}(1-z)^{m+1}}{(m+1)!\,\Gamma(2\lambda)\Gamma(\nu+1)\left(\lambda+\frac{1}{2}\right)_{m+1}} \, {}_3F_2\left(1, m-\nu+1, m+2\lambda+\nu+1; m+2, m+\lambda+\frac{3}{2}; \frac{1-z}{2}\right)\right) \bigwedge m \in \mathbb{N} \bigwedge -\lambda - \frac{1}{2} \notin \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

07.14.06.0011.01

$$C_n^\lambda(z) = \frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(2\lambda+n)}{n!\,\Gamma(\lambda)} \sum_{k=0}^{n} \frac{(-n)_k (2\lambda+n)_k}{\Gamma\left(k+\lambda+\frac{1}{2}\right) k!} \left(\frac{1-z}{2}\right)^k /; n \in \mathbb{N}$$

07.14.06.0057.01

$$C_\nu^\lambda(z) = -\frac{\sqrt{\pi}\,(2-2z)^{\frac{1}{2}-\lambda}\sec(\pi(\lambda+\nu))\sin(\pi\nu)}{\Gamma(\lambda)\Gamma\left(\frac{3}{2}-\lambda\right)} \left(1 + \frac{(2\lambda+2\nu-1)(2\lambda+2\nu+1)}{4(3-2\lambda)}(z-1) + \frac{(2\lambda+2\nu-3)(2\lambda+2\nu-1)(2\lambda+2\nu+1)(2\lambda+2\nu+3)}{32(3-2\lambda)(5-2\lambda)}(z-1)^2 + \ldots\right) /; (z \to 1) \bigwedge -\lambda - \frac{1}{2} \in \mathbb{N}$$

07.14.06.0058.01

$$C_\nu^\lambda(z) = -\frac{\sqrt{\pi}\,(2-2z)^{\frac{1}{2}-\lambda}\sec(\pi(\lambda+\nu))\sin(\pi\nu)}{\Gamma(\lambda)\Gamma\left(\frac{3}{2}-\lambda\right)} \left(1 + \frac{(2\lambda+2\nu-1)(2\lambda+2\nu+1)}{4(3-2\lambda)}(z-1) + \frac{(2\lambda+2\nu-3)(2\lambda+2\nu-1)(2\lambda+2\nu+1)(2\lambda+2\nu+3)}{32(3-2\lambda)(5-2\lambda)}(z-1)^2 + O\left((z-1)^3\right)\right) /; -\lambda - \frac{1}{2} \in \mathbb{N}$$

07.14.06.0059.01

$$C_\nu^\lambda(z) = -\frac{\sqrt{\pi}\,(2-2z)^{\frac{1}{2}-\lambda}\sec(\pi(\lambda+\nu))\sin(\pi\nu)}{\Gamma(\lambda)} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}-\lambda-\nu\right)_k \left(\lambda+\nu+\frac{1}{2}\right)_k}{\Gamma\left(k-\lambda+\frac{3}{2}\right) k!} \left(\frac{1-z}{2}\right)^k /; -\lambda - \frac{1}{2} \in \mathbb{N}$$

07.14.06.0060.01

$$C_\nu^\lambda(z) = -\frac{\sqrt{\pi}\,(2-2z)^{\frac{1}{2}-\lambda}\sec(\pi(\lambda+\nu))\sin(\pi\nu)}{\Gamma(\lambda)\Gamma\left(\frac{3}{2}-\lambda\right)} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}-\lambda-\nu\right)_k \left(\lambda+\nu+\frac{1}{2}\right)_k}{\left(\frac{3}{2}-\lambda\right)_k k!} \left(\frac{1-z}{2}\right)^k /; -\lambda - \frac{1}{2} \in \mathbb{N}$$

07.14.06.0061.01

$$C_\nu^\lambda(z) = -\frac{\sqrt{\pi}\,(2-2z)^{\frac{1}{2}-\lambda}\sec(\pi(\lambda+\nu))\sin(\pi\nu)}{\Gamma(\lambda)} \, {}_2\tilde{F}_1\left(-\lambda-\nu+\frac{1}{2}, \lambda+\nu+\frac{1}{2}; \frac{3}{2}-\lambda; \frac{1-z}{2}\right) /; -\lambda - \frac{1}{2} \in \mathbb{N}$$

07.14.06.0062.01

$$C_\nu^\lambda(z) = -\frac{\sqrt{\pi}\,(2-2z)^{\frac{1}{2}-\lambda}\sec(\pi(\lambda+\nu))\sin(\pi\nu)}{\Gamma(\lambda)\,\Gamma\left(\frac{3}{2}-\lambda\right)}\,{}_2F_1\left(-\lambda-\nu+\frac{1}{2},\lambda+\nu+\frac{1}{2};\frac{3}{2}-\lambda;\frac{1-z}{2}\right)/;-\lambda-\frac{1}{2}\in\mathbb{N}$$

07.14.06.0063.01

$$C_\nu^\lambda(z) = -\frac{\sqrt{\pi}\,(2-2z)^{\frac{1}{2}-\lambda}\sec(\pi(\lambda+\nu))\sin(\pi\nu)}{\Gamma(\lambda)\,\Gamma\left(\frac{3}{2}-\lambda\right)}\,(1+O(z-1))\;/;(z\to 1)\bigwedge -\lambda-\frac{1}{2}\in\mathbb{N}$$

Generic formulas for main term

07.14.06.0064.01

$$C_\nu^\lambda(z) \propto \begin{cases} 0 & -\nu\in\mathbb{N}^+ \\ \tilde{\infty} & -2\lambda-\nu\in\mathbb{N} \\ -\frac{\sqrt{\pi}\,2^{2^{\frac{1}{2}-\lambda}}\sec(\pi(\lambda+\nu))\sin(\pi\nu)(1-z)^{\frac{1}{2}-\lambda}}{\Gamma\left(\frac{3}{2}-\lambda\right)\Gamma(\lambda)} & -\lambda-\frac{1}{2}\in\mathbb{N} \\ \frac{\Gamma(2\lambda+\nu)}{\Gamma(2\lambda)\Gamma(\nu+1)} & \text{True} \end{cases}\quad /;(z\to 1)$$

Expansions at $z == -1$

For the function itself

General case

07.14.06.0013.02

$$C_\nu^\lambda(z) \propto \frac{\cos(\pi(\lambda+\nu))\sec(\pi\lambda)\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(2\lambda)}\left(1-\frac{\nu(2\lambda+\nu)}{2\lambda+1}(z+1)-\frac{\nu(1-\nu)(2\lambda+\nu)(1+2\lambda+\nu)}{2(2\lambda+1)(2\lambda+3)}(z+1)^2-\ldots\right)-$$
$$\frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)}(z+1)^{\frac{1}{2}-\lambda}\left(1+\frac{(1-2\lambda-2\nu)(1+2\lambda+2\nu)}{4(3-2\lambda)}(z+1)+\right.$$
$$\left.\frac{(1-2\lambda-2\nu)(3-2\lambda-2\nu)(1+2\lambda+2\nu)(3+2\lambda+2\nu)}{32(3-2\lambda)(5-2\lambda)}(z+1)^2+\ldots\right)/;(z\to -1)\bigwedge\lambda+\frac{1}{2}\notin\mathbb{Z}$$

07.14.06.0065.01

$$C_\nu^\lambda(z) \propto \frac{\cos(\pi(\lambda+\nu))\sec(\pi\lambda)\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(2\lambda)}\left(1-\frac{\nu(2\lambda+\nu)}{2\lambda+1}(z+1)-\frac{\nu(1-\nu)(2\lambda+\nu)(1+2\lambda+\nu)}{2(2\lambda+1)(2\lambda+3)}(z+1)^2-O\left((z+1)^3\right)\right)-$$
$$\frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)}(z+1)^{\frac{1}{2}-\lambda}\left(1+\frac{(1-2\lambda-2\nu)(1+2\lambda+2\nu)}{4(3-2\lambda)}(z+1)+\right.$$
$$\left.\frac{(1-2\lambda-2\nu)(3-2\lambda-2\nu)(1+2\lambda+2\nu)(3+2\lambda+2\nu)}{32(3-2\lambda)(5-2\lambda)}(z+1)^2+O\left((z+1)^3\right)\right)/;\lambda+\frac{1}{2}\notin\mathbb{Z}$$

07.14.06.0014.01

$$C_\nu^\lambda(z) = \frac{\cos(\pi(\lambda+\nu))\sec(\pi\lambda)\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(2\lambda)} \sum_{k=0}^{\infty} \frac{(-\nu)_k (2\lambda+\nu)_k}{\left(\lambda+\frac{1}{2}\right)_k k!} \left(\frac{z+1}{2}\right)^k - \frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)} (z+1)^{\frac{1}{2}-\lambda} \sum_{k=0}^{\infty} \frac{\left(\lambda+\nu+\frac{1}{2}\right)_k \left(\frac{1}{2}-\lambda-\nu\right)_k}{\left(\frac{3}{2}-\lambda\right)_k k!} \left(\frac{z+1}{2}\right)^k \;/;\; \left|\frac{z+1}{2}\right| < 1 \bigwedge \lambda+\frac{1}{2} \notin \mathbb{Z}$$

07.14.06.0015.01

$$C_\nu^\lambda(z) = \frac{\cos(\pi(\lambda+\nu))\sec(\pi\lambda)\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(2\lambda)} {}_2F_1\left(-\nu, 2\lambda+\nu; \lambda+\frac{1}{2}; \frac{z+1}{2}\right) - \frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)} (z+1)^{\frac{1}{2}-\lambda} {}_2F_1\left(\lambda+\nu+\frac{1}{2}, \frac{1}{2}-\lambda-\nu; \frac{3}{2}-\lambda; \frac{z+1}{2}\right) \;/;\; \lambda+\frac{1}{2} \notin \mathbb{Z}$$

07.14.06.0016.01

$$C_\nu^\lambda(z) \propto \frac{\cos(\pi(\nu+\lambda))\sec(\pi\lambda)\Gamma(\nu+2\lambda)}{\Gamma(\nu+1)\Gamma(2\lambda)}(1+O(z+1)) - \frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)}(z+1)^{\frac{1}{2}-\lambda}(1+O(z+1)) \;/;\; (z\to -1) \bigwedge \lambda+\frac{1}{2} \notin \mathbb{Z}$$

07.14.06.0066.01

$$C_\nu^\lambda(z) = F_\infty(z,\nu,\lambda) \;/;\; \left(\left(F_m(z,\nu,\lambda) = \frac{\cos(\pi(\lambda+\nu))\sec(\pi\lambda)\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\Gamma(2\lambda)} \sum_{k=0}^{m} \frac{(-\nu)_k(2\lambda+\nu)_k}{\left(\lambda+\frac{1}{2}\right)_k k!}\left(\frac{z+1}{2}\right)^k - \frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)}(z+1)^{\frac{1}{2}-\lambda} \sum_{k=0}^{m} \frac{\left(\lambda+\nu+\frac{1}{2}\right)_k\left(\frac{1}{2}-\lambda-\nu\right)_k}{\left(\frac{3}{2}-\lambda\right)_k k!}\left(\frac{z+1}{2}\right)^k = C_\nu^\lambda(z) - \frac{2^{-m-1}\sec(\pi\lambda)\cos(\pi(\lambda+\nu))\Gamma(2\lambda+\nu)(-\nu)_{m+1}(2\lambda+\nu)_{m+1}}{(m+1)!\,\Gamma(2\lambda)\Gamma(\nu+1)\left(\lambda+\frac{1}{2}\right)_{m+1}}(z+1)^{m+1} {}_3F_2\left(1, m-\nu+1, m+2\lambda+\nu+1; m+2, m+\lambda+\frac{3}{2}; \frac{z+1}{2}\right) + \frac{2^{-m-\lambda-\frac{1}{2}}\sin(\pi\nu)\Gamma\left(\lambda-\frac{1}{2}\right)\left(-\lambda-\nu+\frac{1}{2}\right)_{m+1}\left(\lambda+\nu+\frac{1}{2}\right)_{m+1}}{\sqrt{\pi}\,(m+1)!\,\Gamma(\lambda)\left(\frac{3}{2}-\lambda\right)_{m+1}}(z+1)^{m-\lambda+\frac{3}{2}} {}_3F_2\left(1, m-\lambda-\nu+\frac{3}{2}, m+\lambda+\nu+\frac{3}{2}; m+2, m-\lambda+\frac{5}{2}; \frac{z+1}{2}\right)\right) \bigwedge m \in \mathbb{N}\right) \bigwedge \neg\, \lambda+\frac{1}{2} \in \mathbb{Z}$$

Summed form of the truncated series expansion.

Special cases

07.14.06.0018.01

$$C_\nu^\lambda(z) = \frac{\sin(\nu\pi)\,\Gamma(2\lambda+\nu)}{\pi\,\Gamma(\nu+1)\,\Gamma(2\lambda)}\log\left(\frac{z+1}{2}\right){}_2F_1\left(-\nu,\,2\lambda+\nu;\,\lambda+\frac{1}{2};\,\frac{z+1}{2}\right)-$$
$$\frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\,\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)}(z+1)^{\frac{1}{2}-\lambda}\sum_{k=0}^{\lambda-\frac{3}{2}}\frac{\left(\frac{1}{2}-\lambda-\nu\right)_k\left(\lambda+\nu+\frac{1}{2}\right)_k}{k!\left(\frac{3}{2}-\lambda\right)_k}\left(\frac{z+1}{2}\right)^k-\frac{2^{1-2\lambda}\sin(\nu\pi)\,\Gamma(\nu+2\lambda)}{\sqrt{\pi}\,\Gamma(\nu+1)\,\Gamma(\lambda)}$$
$$\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(2\lambda+\nu)_k}{k!\,\Gamma\left(k+\lambda+\frac{1}{2}\right)}\left(\psi(k+1)+\psi\left(k+\lambda+\frac{1}{2}\right)-\psi(k+2\lambda+\nu)-\psi(k-\nu)\right)\left(\frac{z+1}{2}\right)^k\,/;\,\lambda-\frac{3}{2}\in\mathbb{N}\bigwedge\nu\notin\mathbb{Z}$$

07.14.06.0019.01

$$C_\nu^\lambda(z)\propto\frac{\sin(\nu\pi)\,\Gamma(2\lambda+\nu)}{\pi\,\Gamma(\nu+1)\,\Gamma(2\lambda)}\left(\log\left(\frac{z+1}{2}\right)-\psi\left(\lambda+\frac{1}{2}\right)+\psi(-\nu)+\psi(2\lambda+\nu)+\gamma\right)(1+O(z+1))-$$
$$\frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\,\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)}(z+1)^{\frac{1}{2}-\lambda}(1+O(z+1))\,/;\,(z\to-1)\bigwedge\lambda-\frac{3}{2}\in\mathbb{N}\bigwedge\nu\notin\mathbb{Z}$$

07.14.06.0020.01

$$C_\nu^{\frac{1}{2}}(z) = \frac{\sin(\nu\pi)}{\pi}\left(\log\left(\frac{z+1}{2}\right){}_2F_1\left(-\nu,\,\nu+1;\,1;\,\frac{z+1}{2}\right)-\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{k!^2}(2\,\psi(k+1)-\psi(k+\nu+1)-\psi(k-\nu))\left(\frac{z+1}{2}\right)^k\right)/;$$
$$\nu\notin\mathbb{Z}$$

07.14.06.0021.01

$$C_\nu^{\frac{1}{2}}(z)\propto\frac{\sin(\nu\pi)}{\pi}\left(\log\left(\frac{z+1}{2}\right)+\psi(-\nu)+\psi(\nu+1)+2\gamma\right)(1+O(z+1))\,/;\,(z\to-1)\wedge\nu\notin\mathbb{Z}$$

07.14.06.0022.01

$$C_\nu^\lambda(z) = -\frac{2^{\frac{1}{2}-\lambda}\cos(\pi(\lambda+\nu))}{\left(\frac{1}{2}-\lambda\right)!\,\sqrt{\pi}\,\Gamma(\lambda)}\log\left(\frac{z+1}{2}\right)(z+1)^{\frac{1}{2}-\lambda}{}_2F_1\left(\lambda+\nu+\frac{1}{2},\,\frac{1}{2}-\lambda-\nu;\,\frac{3}{2}-\lambda;\,\frac{z+1}{2}\right)+\frac{2^{\frac{1}{2}-\lambda}\cos(\pi(\lambda+\nu))}{\sqrt{\pi}\,\Gamma(\lambda)}(z+1)^{\frac{1}{2}-\lambda}$$
$$\sum_{k=0}^{\infty}\frac{1}{k!\left(k-\lambda+\frac{1}{2}\right)!}\left(\frac{1}{2}-\lambda-\nu\right)_k\left(\lambda+\nu+\frac{1}{2}\right)_k\left(\psi(k+1)+\psi\left(k-\lambda+\frac{3}{2}\right)-\psi\left(k-\lambda-\nu+\frac{1}{2}\right)-\psi\left(k+\lambda+\nu+\frac{1}{2}\right)\right)\left(\frac{z+1}{2}\right)^k+$$
$$\frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\left(-\lambda-\frac{1}{2}\right)!\,\Gamma(2\lambda+\nu)}{\sqrt{\pi}\,\Gamma(\lambda)\,\Gamma(\nu+1)}\sum_{k=0}^{-\lambda-\frac{1}{2}}\frac{(-\nu)_k\,(2\lambda+\nu)_k}{k!\left(\lambda+\frac{1}{2}\right)_k}\left(\frac{z+1}{2}\right)^k\,/;\,-\lambda-\frac{1}{2}\in\mathbb{N}\bigwedge\nu\notin\mathbb{Z}$$

07.14.06.0023.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\left(-\lambda-\frac{1}{2}\right)!\,\Gamma(2\lambda+\nu)}{\sqrt{\pi}\,\Gamma(\lambda)\,\Gamma(\nu+1)}(1+O(z+1))-\frac{2^{\frac{1}{2}-\lambda}\cos(\pi(\lambda+\nu))}{\left(\frac{1}{2}-\lambda\right)!\,\sqrt{\pi}\,\Gamma(\lambda)}(z+1)^{\frac{1}{2}-\lambda}$$
$$\left(\log\left(\frac{z+1}{2}\right)-\psi\left(\frac{3}{2}-\lambda\right)+\psi\left(\frac{1}{2}-\lambda-\nu\right)+\psi\left(\lambda+\nu+\frac{1}{2}\right)+\gamma\right)(1+O(z+1))\,/;\,-\lambda-\frac{1}{2}\in\mathbb{N}\bigwedge\nu\notin\mathbb{Z}$$

07.14.06.0017.02

$$C_n^\lambda(z)\propto\frac{(-1)^n\,\Gamma(n+2\lambda)}{n!\,\Gamma(2\lambda)}(1+O(z+1))\,/;\,n\in\mathbb{N}$$

Generic formulas for main term

07.14.06.0067.01

$$C_\nu^\lambda(z) \propto \begin{cases} 0 & -\nu \in \mathbb{N}^+ \\ \tilde{\infty} & -2\lambda - \nu \in \mathbb{N} \\ \frac{\sin(\nu\pi)\,\Gamma(2\lambda+\nu)}{\pi\,\Gamma(\nu+1)\,\Gamma(2\lambda)} \left(\log\left(\frac{z+1}{2}\right) - \psi\left(\lambda+\frac{1}{2}\right) + \psi(-\nu) + \psi(2\lambda+\nu) + \gamma\right) - \frac{2^{\frac{1}{2}-\lambda} \sin(\pi\nu)\,\Gamma\left(\lambda-\frac{1}{2}\right)(z+1)^{\frac{1}{2}-\lambda}}{\sqrt{\pi}\,\Gamma(\lambda)} & \lambda - \frac{3}{2} \in \mathbb{N} \bigwedge \nu \\ \frac{\sin(\nu\pi)}{\pi}\left(\log\left(\frac{z+1}{2}\right) + \psi(-\nu) + \psi(\nu+1) + 2\gamma\right) & \lambda = \frac{1}{2} \bigwedge \nu \notin \mathbb{N} \\ \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\left(-\lambda-\frac{1}{2}\right)!\,\Gamma(2\lambda+\nu)}{\sqrt{\pi}\,\Gamma(\lambda)\,\Gamma(\nu+1)} - \frac{2^{\frac{1}{2}-\lambda}\cos(\pi(\lambda+\nu))(z+1)^{\frac{1}{2}-\lambda}}{\left(\frac{1}{2}-\lambda\right)!\,\sqrt{\pi}\,\Gamma(\lambda)} \left(\log\left(\frac{z+1}{2}\right) - \psi\left(\frac{3}{2}-\lambda\right) + \psi\left(-\lambda-\nu+\frac{1}{2}\right) + \psi\left(\lambda+\nu+\frac{1}{2}\right) + \gamma\right) & -\lambda - \frac{1}{2} \in \mathbb{N} \bigwedge \nu \\ \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\,\Gamma\left(\frac{1}{2}-\lambda\right)\Gamma(2\lambda+\nu)}{\sqrt{\pi}\,\Gamma(\lambda)\,\Gamma(\nu+1)} - \frac{2^{\frac{1}{2}-\lambda}\sin(\pi\nu)\,\Gamma\left(\lambda-\frac{1}{2}\right)(z+1)^{\frac{1}{2}-\lambda}}{\sqrt{\pi}\,\Gamma(\lambda)} & \text{True} \end{cases}$$

07.14.06.0068.01

$$C_\nu^\lambda(z) \propto \begin{cases} 0 & -\nu \in \mathbb{N}^+ \\ \tilde{\infty} & -2\lambda - \nu \in \mathbb{N} \\ -\frac{2^{\frac{1}{2}-\lambda}\sin(\pi\nu)\,\Gamma\left(\lambda-\frac{1}{2}\right)(z+1)^{\frac{1}{2}-\lambda}}{\sqrt{\pi}\,\Gamma(\lambda)} & \text{Re}(\lambda) > \frac{1}{2} \bigwedge \nu \notin \mathbb{N} \\ \frac{\sin(\nu\pi)\log(z+1)}{\pi} & \lambda = \frac{1}{2} \bigwedge \nu \notin \mathbb{N} \\ \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\left(-\lambda-\frac{1}{2}\right)!\,\Gamma(2\lambda+\nu)}{\sqrt{\pi}\,\Gamma(\lambda)\,\Gamma(\nu+1)} & \text{Re}(\lambda) < \frac{1}{2} \\ \frac{2^{1-2\lambda}\cos(\pi(\lambda+\nu))\,\Gamma\left(\frac{1}{2}-\lambda\right)\Gamma(2\lambda+\nu)}{\sqrt{\pi}\,\Gamma(\lambda)\,\Gamma(\nu+1)} - \frac{2^{\frac{1}{2}-\lambda}\sin(\pi\nu)\,\Gamma\left(\lambda-\frac{1}{2}\right)(z+1)^{\frac{1}{2}-\lambda}}{\sqrt{\pi}\,\Gamma(\lambda)} & \text{True} \end{cases} \quad /;\ (z \to -1)$$

Expansions at $z == \infty$

For the function itself

Expansions in $1/z$

07.14.06.0069.01

$$C_\nu^\lambda(z) \propto \frac{2^\nu\,\Gamma(\lambda+\nu)\,z^\nu}{\Gamma(\lambda)\,\Gamma(\nu+1)} \left(1 - \frac{(1-\nu)\,\nu}{4\,(1-\lambda-\nu)\,z^2} + \frac{(-3+\nu)(-2+\nu)(-1+\nu)\,\nu}{32\,(-2+\lambda+\nu)(-1+\lambda+\nu)\,z^4} + \ldots\right) - \frac{2^{-2\lambda-\nu}\sin(\pi\nu)\,\Gamma(-\lambda-\nu)\,\Gamma(2\lambda+\nu)\,z^{-2\lambda-\nu}}{\pi\,\Gamma(\lambda)}$$
$$\left(1 + \frac{(2\lambda+\nu)(1+2\lambda+\nu)}{4\,(1+\lambda+\nu)\,z^2} + \frac{(2\lambda+\nu)(1+2\lambda+\nu)(2+2\lambda+\nu)(3+2\lambda+\nu)}{32\,(1+\lambda+\nu)(2+\lambda+\nu)\,z^4} + \ldots\right) /;\ (|z| \to \infty) \bigwedge \lambda + \nu \notin \mathbb{Z}$$

07.14.06.0070.01

$$C_\nu^\lambda(z) \propto \frac{2^\nu\,\Gamma(\lambda+\nu)\,z^\nu}{\Gamma(\lambda)\,\Gamma(\nu+1)}\left(1-\frac{(1-\nu)\,\nu}{4\,(1-\lambda-\nu)\,z^2}+\frac{(-3+\nu)\,(-2+\nu)\,(-1+\nu)\,\nu}{32\,(-2+\lambda+\nu)\,(-1+\lambda+\nu)\,z^4}+O\left(\frac{1}{z^6}\right)\right)- \frac{2^{-2\lambda-\nu}\,\sin(\pi\,\nu)\,\Gamma(-\lambda-\nu)\,\Gamma(2\,\lambda+\nu)\,z^{-2\lambda-\nu}}{\pi\,\Gamma(\lambda)} \left(1+\frac{(2\,\lambda+\nu)\,(1+2\,\lambda+\nu)}{4\,(1+\lambda+\nu)\,z^2}+\frac{(2\,\lambda+\nu)\,(1+2\,\lambda+\nu)\,(2+2\,\lambda+\nu)\,(3+2\,\lambda+\nu)}{32\,(1+\lambda+\nu)\,(2+\lambda+\nu)\,z^4}+O\left(\frac{1}{z^6}\right)\right)/;\ \lambda+\nu\notin\mathbb{Z}$$

07.14.06.0071.01

$$C_\nu^\lambda(z) = \frac{2^\nu\,\Gamma(\lambda+\nu)\,z^\nu}{\Gamma(\lambda)\,\Gamma(\nu+1)}\sum_{k=0}^{\infty}\frac{\left(-\frac{\nu}{2}\right)_k\left(\frac{1-\nu}{2}\right)_k}{(1-\lambda-\nu)_k\,k!}\,z^{-2k}-\frac{2^{-2\lambda-\nu}\,\sin(\pi\,\nu)\,\Gamma(-\lambda-\nu)\,\Gamma(2\,\lambda+\nu)\,z^{-2\lambda-\nu}}{\pi\,\Gamma(\lambda)}\sum_{k=0}^{\infty}\frac{\left(\lambda+\frac{\nu}{2}\right)_k\left(\lambda+\frac{\nu+1}{2}\right)_k}{\lambda+\nu+1}\,z^{-2k}\ /;\ |z|>1\bigwedge\lambda+\nu\notin\mathbb{Z}$$

07.14.06.0072.01

$$C_\nu^\lambda(z) = \frac{2^\nu\,\Gamma(\lambda+\nu)\,z^\nu}{\Gamma(\lambda)\,\Gamma(\nu+1)}\,{}_2F_1\left(-\frac{\nu}{2},\frac{1-\nu}{2};-\lambda-\nu+1;\frac{1}{z^2}\right)- \frac{2^{-2\lambda-\nu}\,\sin(\pi\,\nu)\,\Gamma(-\lambda-\nu)\,\Gamma(2\,\lambda+\nu)\,z^{-2\lambda-\nu}}{\pi\,\Gamma(\lambda)}\,{}_2F_1\left(\lambda+\frac{\nu}{2},\lambda+\frac{\nu+1}{2};\lambda+\nu+1;\frac{1}{z^2}\right)/;\ \lambda+\nu\notin\mathbb{Z}\bigwedge z\notin(-1,0)$$

07.14.06.0073.01

$$C_n^\lambda(z) = \frac{2^n\,z^n\,(\lambda)_n}{n!}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left(-\frac{n}{2}\right)_k\left(\frac{1-n}{2}\right)_k}{(1-n-\lambda)_k\,k!}\,z^{-2k}\ /;\ \lambda+n\notin\mathbb{N}^+$$

07.14.06.0074.01

$$C_n^\lambda(z) = \frac{2^n\,z^n\,(\lambda)_n}{n!}\,{}_2F_1\left(-\frac{n}{2},\frac{1-n}{2};-n-\lambda+1;\frac{1}{z^2}\right)/;\ \lambda+n\notin\mathbb{N}^+$$

07.14.06.0075.01

$$C_\nu^\lambda(z) \propto \frac{2^\nu\,\Gamma(\lambda+\nu)\,z^\nu}{\Gamma(\lambda)\,\Gamma(\nu+1)}\left(1+O\left(\frac{1}{z^2}\right)\right)-\frac{2^{-2\lambda-\nu}\,\sin(\pi\,\nu)\,\Gamma(-\lambda-\nu)\,\Gamma(2\,\lambda+\nu)\,z^{-2\lambda-\nu}}{\pi\,\Gamma(\lambda)}\left(1+O\left(\frac{1}{z^2}\right)\right)/;\ \lambda+\nu\notin\mathbb{Z}$$

07.14.06.0076.01

$$C_\nu^\lambda(z) \propto \frac{2^n\,z^n\,\Gamma(n+\lambda)}{\Gamma(\lambda)\,n!}\left(1+O\left(\frac{1}{z^2}\right)\right)/;\ n\in\mathbb{N}^+$$

07.14.06.0077.01

$$C_\nu^\lambda(z) = F_\infty(z, \nu, \lambda) /; \left(\left(\left(F_m(z, \nu, \lambda) = \frac{2^\nu \Gamma(\lambda+\nu) z^\nu}{\Gamma(\lambda)\Gamma(\nu+1)} \sum_{k=0}^m \frac{\left(-\frac{\nu}{2}\right)_k \left(\frac{1-\nu}{2}\right)_k}{(1-\lambda-\nu)_k\, k!} z^{-2k} - \frac{2^{-2\lambda-\nu} \sin(\pi\nu)\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu) z^{-2\lambda-\nu}}{\pi\Gamma(\lambda)} \sum_{k=0}^m \frac{\left(\lambda+\frac{\nu}{2}\right)_k \left(\lambda+\frac{\nu+1}{2}\right)_k}{(\lambda+\nu+1)_k\, k!} z^{-2k} = C_\nu^\lambda(z) - \frac{2^{-2(m+\lambda+1)-\nu} \csc(\pi(\lambda+\nu))\Gamma(2(m+\lambda+1)+\nu)\sin(\pi\nu) z^{-2(m+\lambda+1)-\nu}}{(m+1)!\,\Gamma(\lambda)\Gamma(m+\lambda+\nu+2)} {}_3F_2\left(1, m+\lambda+\frac{\nu}{2}+1, m+\lambda+\frac{\nu}{2}+\frac{3}{2}; m+2, m+\lambda+\nu+2; \frac{1}{z^2}\right) + \frac{2^{-2m+\nu-2} \csc(\pi(\lambda+\nu))\sin(\pi\nu)\Gamma(2m-\nu+2) z^{-2m+\nu-2}}{(m+1)!\,\Gamma(\lambda)\Gamma(m-\lambda-\nu+2)} {}_3F_2\left(1, m-\frac{\nu}{2}+1, m-\frac{\nu}{2}+\frac{3}{2}; m+2, m-\lambda-\nu+2; \frac{1}{z^2}\right)\right) \bigwedge m \in \mathbb{N}\right) \bigwedge 2\nu \notin \mathbb{Z}\right)$$

Summed form of the truncated series expansion.

Expansions in $1/(1-z)$

07.14.06.0024.02

$$C_\nu^\lambda(z) \propto \frac{2^\nu \Gamma(\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda)} (z-1)^\nu \left(1 - \frac{\nu}{1-z} - \frac{\nu(1-\nu)(3-2\lambda-2\nu)}{4(1-\lambda-\nu)(1-z)^2} - \ldots\right) - \frac{2^{-2\lambda-\nu} \sin(\nu\pi)\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)}{\pi\Gamma(\lambda)} (z-1)^{-2\lambda-\nu} \left(1 + \frac{2\lambda+\nu}{1-z} + \frac{(3+2\lambda+2\nu)(2\lambda+\nu)(1+2\lambda+\nu)}{4(1+\lambda+\nu)(1-z)^2} + \ldots\right) /; (|z| \to \infty) \bigwedge 2\lambda+2\nu \notin \mathbb{Z}$$

07.14.06.0078.01

$$C_\nu^\lambda(z) \propto \frac{2^\nu \Gamma(\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda)} (z-1)^\nu \left(1 - \frac{\nu}{1-z} - \frac{\nu(1-\nu)(3-2\lambda-2\nu)}{4(1-\lambda-\nu)(1-z)^2} - O\left(\frac{1}{z^3}\right)\right) - \frac{2^{-2\lambda-\nu} \sin(\nu\pi)\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)}{\pi\Gamma(\lambda)} (z-1)^{-2\lambda-\nu} \left(1 + \frac{2\lambda+\nu}{1-z} + \frac{(3+2\lambda+2\nu)(2\lambda+\nu)(1+2\lambda+\nu)}{4(1+\lambda+\nu)(1-z)^2} + O\left(\frac{1}{z^3}\right)\right) /; 2\lambda+2\nu \notin \mathbb{Z}$$

07.14.06.0025.01

$$C_\nu^\lambda(z) = \frac{2^\nu \Gamma(\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda)} (z-1)^\nu \sum_{k=0}^\infty \frac{(-\nu)_k \left(\frac{1}{2}-\lambda-\nu\right)_k}{(1-2\lambda-2\nu)_k\, k!} \left(\frac{2}{1-z}\right)^k - \frac{2^{-2\lambda-\nu} \sin(\nu\pi)\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)}{\pi\Gamma(\lambda)} (z-1)^{-2\lambda-\nu} \sum_{k=0}^\infty \frac{(2\lambda+\nu)_k \left(\lambda+\nu+\frac{1}{2}\right)_k}{(2\lambda+2\nu+1)_k\, k!} \left(\frac{2}{1-z}\right)^k /; \left|\frac{1-z}{2}\right| > 1 \bigwedge 2\lambda+2\nu \notin \mathbb{Z}$$

07.14.06.0026.01

$$C_\nu^\lambda(z) = \frac{2^\nu \Gamma(\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda)} (z-1)^\nu {}_2F_1\left(-\nu, -\lambda-\nu+\frac{1}{2}; -2\lambda-2\nu+1; \frac{2}{1-z}\right) - \frac{2^{-2\lambda-\nu} \sin(\nu\pi)\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)}{\pi\Gamma(\lambda)} (z-1)^{-2\lambda-\nu} {}_2F_1\left(2\lambda+\nu, \lambda+\nu+\frac{1}{2}; 2\lambda+2\nu+1; \frac{2}{1-z}\right) /; z \notin (-1, 1) \bigwedge 2\lambda+2\nu \notin \mathbb{Z}$$

07.14.06.0027.01

$$C_\nu^\lambda(z) \propto \frac{2^\nu \Gamma(\lambda+\nu) z^\nu}{\Gamma(\nu+1)\Gamma(\lambda)} \left(1 + O\left(\frac{1}{z}\right)\right) - \frac{2^{-2\lambda-\nu} \sin(\nu\pi)\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu) z^{-2\lambda-\nu}}{\pi\Gamma(\lambda)} \left(1 + O\left(\frac{1}{z}\right)\right) /; (|z| \to \infty) \bigwedge 2\lambda+2\nu \notin \mathbb{Z}$$

07.14.06.0028.01

$$C_\nu^\lambda(z) = \frac{(-1)^{\lambda+\nu-1}\, 2^{1-2\lambda-\nu} \sin(\nu\pi)\, \Gamma(2\lambda+\nu)}{\pi\, \Gamma(\lambda)\, \Gamma(\lambda+\nu+1)} \log\left(\frac{z-1}{2}\right)(z-1)^{-2\lambda-\nu}\, {}_2F_1\left(2\lambda+\nu, \lambda+\nu+\frac{1}{2}; 2\lambda+2\nu+1; \frac{2}{1-z}\right) + \frac{(-1)^{\lambda+\nu-1}\, 2^{1-2\lambda-\nu} \sin(\nu\pi)\, \Gamma(2\lambda+\nu)}{\pi\, \Gamma(\lambda)\, \Gamma(\lambda+\nu+1)} (z-1)^{-2\lambda-\nu} \sum_{k=0}^{\infty} \frac{(2\lambda+\nu)_k}{k!\, (2\lambda+2\nu+1)_k} \left(\lambda+\nu+\frac{1}{2}\right)_k \left(\psi(k+1)+\psi(k+2\lambda+2\nu+1)-\psi\left(\frac{1}{2}-k-\lambda-\nu\right)-\psi(k+2\lambda+\nu)\right)\left(\frac{2}{1-z}\right)^k + \frac{2^{1-2\lambda-\nu}\sqrt{\pi}\, (z-1)^\nu}{\Gamma(\nu+1)\,\Gamma(\lambda)} \sum_{k=0}^{2\lambda+2\nu-1} \frac{(2\lambda+2\nu-k-1)!\, (-\nu)_k}{k!\, \Gamma\left(\frac{1}{2}-k+\lambda+\nu\right)} \left(\frac{2}{1-z}\right)^k /;\ \lambda+\nu \in \mathbb{N} \land \nu \notin \mathbb{Z}$$

07.14.06.0029.01

$$C_\nu^\lambda(z) \propto \frac{(-1)^{\lambda+\nu-1}\, 2^{1-2\lambda-\nu} \sin(\nu\pi)\, \Gamma(2\lambda+\nu)}{\pi\, \Gamma(\lambda)\, \Gamma(\lambda+\nu+1)} z^{-2\lambda-\nu} \left(\log\left(\frac{z-1}{2}\right)-\psi\left(\frac{1}{2}-\lambda-\nu\right)-\psi(2\lambda+\nu)+\psi(2\lambda+2\nu+1)-\gamma\right)\left(1+O\left(\frac{1}{z}\right)\right) + \frac{2^\nu\, \Gamma(\lambda+\nu)\, z^\nu}{\Gamma(\lambda)\, \Gamma(\nu+1)} \left(1+O\left(\frac{1}{z}\right)\right) /;\ (|z| \to \infty) \land \lambda+\nu \in \mathbb{N}^+ \land \nu \notin \mathbb{Z}$$

07.14.06.0030.01

$$C_\nu^{-\nu}(z) \propto \frac{\sin(\pi\nu)\, 2^{\nu+1} z^\nu}{\pi} (\gamma - \log(2) - \log(z-1) + \psi(-\nu)) \left(1+O\left(\frac{1}{z}\right)\right) /;\ (|z| \to \infty) \land \nu \notin \mathbb{Z}$$

07.14.06.0031.01

$$C_\nu^\lambda(z) = -\frac{2^{\nu+1}\, \Gamma(2\lambda+\nu) \sin(\nu\pi)\, (z-1)^{-2\lambda-\nu}}{\sqrt{\pi}\, \Gamma(\lambda)} \sum_{k=0}^{-2\lambda-2\nu-1} \frac{(-2\lambda-2\nu-k-1)!\, (2\lambda+\nu)_k}{k!\, \Gamma\left(\frac{1}{2}-k-\lambda-\nu\right)} \left(\frac{2}{1-z}\right)^k + \frac{(-1)^{\lambda+\nu}\, 2^{\nu+1}}{\Gamma(\nu+1)\, \Gamma(1-\lambda-\nu)\, \Gamma(\lambda)} \log\left(\frac{z-1}{2}\right)(z-1)^\nu\, {}_2F_1\left(-\nu, \frac{1}{2}-\lambda-\nu; 1-2\lambda-2\nu; \frac{2}{1-z}\right) + \frac{(-1)^{\lambda+\nu}\, 2^{\nu+1}}{\Gamma(\nu+1)\, \Gamma(1-\lambda-\nu)\, \Gamma(\lambda)} (z-1)^\nu \sum_{k=0}^{\infty} \frac{(-\nu)_k}{k!\, (1-2\lambda-2\nu)_k} \left(\frac{1}{2}-\lambda-\nu\right)_k \left(\psi(k+1)-\psi\left(\frac{1}{2}-k+\lambda+\nu\right)+\psi(k-2\lambda-2\nu+1)-\psi(k-\nu)\right)\left(\frac{2}{1-z}\right)^k /;\ -\lambda-\nu \in \mathbb{N} \land \nu \notin \mathbb{Z}$$

07.14.06.0032.01

$$C_\nu^\lambda(z) \propto \frac{(-1)^{\lambda+\nu}\, 2^{\nu+1} z^\nu}{\Gamma(\nu+1)\, \Gamma(1-\lambda-\nu)\, \Gamma(\lambda)} \left(\log\left(\frac{z-1}{2}\right)-\psi(-\nu)-\psi\left(\lambda+\nu+\frac{1}{2}\right)+\psi(1-2\lambda-2\nu)-\gamma\right)\left(1+O\left(\frac{1}{z}\right)\right) - \frac{2^{-2\lambda-\nu} \sin(\pi\nu)\, \Gamma(-\lambda-\nu)\, \Gamma(2\lambda+\nu)\, z^{-2\lambda-\nu}}{\pi\, \Gamma(\lambda)} \left(1+O\left(\frac{1}{z}\right)\right) /;\ (|z| \to \infty) \land -\lambda-\nu \in \mathbb{N} \land \nu \notin \mathbb{Z}$$

07.14.06.0033.01

$$C_\nu^\lambda(z) = \frac{2^\nu\, \Gamma(\lambda+\nu)}{\Gamma(\nu+1)\, \Gamma(\lambda)} (z-1)^\nu \sum_{k=0}^{\lambda+\nu-\frac{1}{2}} \frac{(-\nu)_k \left(\frac{1}{2}-\lambda-\nu\right)_k}{k!\, (1-2\lambda-2\nu)_k} \left(\frac{2}{1-z}\right)^k - \frac{(-1)^{\lambda+\nu+\frac{1}{2}}\, 2^{1-2\lambda-\nu} \sin(\nu\pi)}{\Gamma(\lambda)\, \Gamma(\lambda+\nu+1)} \Gamma(2\lambda+\nu)\, (z-1)^{-2\lambda-\nu}\, {}_2F_1\left(2\lambda+\nu, \lambda+\nu+\frac{1}{2}; 2\lambda+2\nu+1; \frac{2}{1-z}\right) /;\ \lambda+\nu-\frac{1}{2} \in \mathbb{N} \bigwedge \nu \notin \mathbb{Z}$$

07.14.06.0034.01

$$C_\nu^\lambda(z) \propto \frac{2^\nu\,\Gamma(\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)}\,z^\nu\left(1+O\left(\frac{1}{z}\right)\right)-\frac{(-1)^{\lambda+\nu+\frac{1}{2}}\,2^{1-2\lambda-\nu}\sin(\nu\pi)}{\Gamma(\lambda)\,\Gamma(\lambda+\nu+1)}\,\Gamma(2\lambda+\nu)\,z^{-\nu-2\lambda}\left(1+O\left(\frac{1}{z}\right)\right)/;\,(|z|\to\infty)\bigwedge\lambda+\nu-\frac{1}{2}\in\mathbb{N}$$

07.14.06.0035.01

$$C_\nu^\lambda(z) = \frac{(-1)^{\frac{1}{2}-\lambda-\nu}\,2^{\nu+1}\,\pi}{\Gamma(\nu+1)\,\Gamma(1-\lambda-\nu)\,\Gamma(\lambda)}\,(z-1)^\nu\,{}_2F_1\left(-\nu,\,\frac{1}{2}-\lambda-\nu;\,1-2\lambda-2\nu;\,\frac{2}{1-z}\right)-$$
$$\frac{2^{-2\lambda-\nu}\,\Gamma(-\lambda-\nu)\,\Gamma(2\lambda+\nu)\sin(\nu\pi)}{\pi\,\Gamma(\lambda)}\,(z-1)^{-2\lambda-\nu}\sum_{k=0}^{-\lambda-\nu-\frac{1}{2}}\frac{\left(\lambda+\nu+\frac{1}{2}\right)_k(2\lambda+\nu)_k}{k!\,(2\lambda+2\nu+1)_k}\left(\frac{2}{1-z}\right)^k/;\,-\lambda-\nu-\frac{1}{2}\in\mathbb{N}\bigwedge\nu\notin\mathbb{Z}$$

07.14.06.0036.01

$$C_\nu^\lambda(z) \propto \frac{(-1)^{\frac{1}{2}-\lambda-\nu}\,2^{\nu+1}\,\pi}{\Gamma(\nu+1)\,\Gamma(1-\lambda-\nu)\,\Gamma(\lambda)}\,z^\nu\left(1+O\left(\frac{1}{z}\right)\right)-\frac{2^{-2\lambda-\nu}\,\Gamma(-\lambda-\nu)\,\Gamma(2\lambda+\nu)\sin(\nu\pi)}{\pi\,\Gamma(\lambda)}\,z^{-2\lambda-\nu}\left(1+O\left(\frac{1}{z}\right)\right)/;$$
$$(|z|\to\infty)\bigwedge-\lambda-\nu-\frac{1}{2}\in\mathbb{N}\bigwedge\nu\notin\mathbb{Z}$$

07.14.06.0079.01

$$C_\nu^\lambda(z) = F_\infty(z,\,\nu,\,\lambda)\,/;$$
$$\Bigg(\Bigg(F_m(z,\,\nu,\,\lambda) = \frac{2^\nu\,\Gamma(\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)}\,(z-1)^\nu\sum_{k=0}^{m}\frac{(-\nu)_k\left(\frac{1}{2}-\lambda-\nu\right)_k}{(1-2\lambda-2\nu)_k\,k!}\left(\frac{2}{1-z}\right)^k-\frac{2^{-2\lambda-\nu}\sin(\nu\pi)\,\Gamma(-\lambda-\nu)\,\Gamma(2\lambda+\nu)}{\pi\,\Gamma(\lambda)}$$
$$(z-1)^{-2\lambda-\nu}\sum_{k=0}^{m}\frac{(2\lambda+\nu)_k\left(\lambda+\nu+\frac{1}{2}\right)_k}{(2\lambda+2\nu+1)_k\,k!}\left(\frac{2}{1-z}\right)^k =$$
$$C_\nu^\lambda(z)-\frac{2^{m-2\lambda-\nu+1}\,(-1)^m\sin(\pi\nu)\,\Gamma(-\lambda-\nu)\,\Gamma(2\lambda+\nu)\left(\lambda+\nu+\frac{1}{2}\right)_{m+1}(2\lambda+\nu)_{m+1}}{\pi\,(m+1)!\,\Gamma(\lambda)\,(2\lambda+2\nu+1)_{m+1}}\,(z-1)^{-m-2\lambda-\nu-1}$$
$${}_3F_2\left(1,\,m+\lambda+\nu+\frac{3}{2},\,m+2\lambda+\nu+1;\,m+2,\,m+2\lambda+2\nu+2;\,\frac{2}{1-z}\right)+$$
$$\frac{2^{m+\nu+1}\,(-1)^m\,\Gamma(\lambda+\nu)\left(-\lambda-\nu+\frac{1}{2}\right)_{m+1}(-\nu)_{m+1}}{(m+1)!\,\Gamma(\lambda)\,\Gamma(\nu+1)\,(-2\lambda-2\nu+1)_{m+1}}\,(z-1)^{-m+\nu-1}$$
$${}_3F_2\left(1,\,m-\nu+1,\,m-\lambda-\nu+\frac{3}{2};\,m+2,\,m-2\lambda-2\nu+2;\,\frac{2}{1-z}\right)\Bigg)\bigwedge m\in\mathbb{N}\Bigg)\bigwedge\neg\,2\lambda+2\nu\in\mathbb{Z}$$

Summed form of the truncated series expansion.

Generic formulas for main term

07.14.06.0080.01

$$C_\nu^\lambda(z) \propto$$

$$\begin{cases} 0 & (-\nu \in \mathbb{N}^+) \bigvee \left(-\lambda - \tfrac{1}{2} \in \mathbb{N} \bigwedge ((-2 \right. \\ \tilde{\infty} & -\lambda - \tfrac{1}{2} \in \mathbb{N} \bigwedge -2\lambda - \nu \in \mathbb{N} \bigwedge \lambda + \\ \frac{2^\nu \Gamma(\lambda+\nu) z^\nu}{\Gamma(\lambda)\Gamma(\nu+1)} + \frac{2^{-2\lambda-\nu+1} \cos(\pi(\lambda+\nu)) \Gamma(2\lambda+\nu) \sin(\pi\nu) z^{-2\lambda-\nu} \left(\log(2)-\log(z)+\psi\left(\frac{1}{2}-\lambda-\nu\right)+\psi(2\lambda+\nu)-\psi(2\lambda+2\nu+1)+\gamma\right)}{\pi\Gamma(\lambda)\Gamma(\lambda+\nu+1)} & 2(\lambda+\nu) \in \mathbb{N}^+ \bigwedge \frac{1}{2} + \lambda + \nu \notin \mathbb{Z} \\ \frac{2^{\nu+1} z^\nu \sin(\pi\nu)(-\log(2)-\log(z)+\psi(-\nu)+\gamma)}{\pi} & \lambda + \nu = 0 \\ -\frac{2^{-2\lambda-\nu} \Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)\sin(\pi\nu) z^{-2\lambda-\nu}}{\pi\Gamma(\lambda)} - \frac{2^{\nu+1}\cos(\pi(\lambda+\nu)) z^\nu \left(\log(2)-\log(z)+\psi(-\nu)+\psi\left(\lambda+\nu+\frac{1}{2}\right)-\psi(1-2\lambda-2\nu)+\gamma\right)}{\Gamma(\lambda)\Gamma(1-\lambda-\nu)\Gamma(\nu+1)} & -2(\lambda+\nu) \in \mathbb{N}^+ \bigwedge \frac{1}{2} - \lambda - \nu \notin \mathbb{Z} \\ \frac{2^\nu \Gamma(\lambda+\nu) z^\nu}{\Gamma(\lambda)\Gamma(\nu+1)} & -\lambda - \frac{1}{2} \in \mathbb{N} \bigwedge \nu \in \mathbb{N} \bigwedge \lambda + \nu > - \\ \frac{2^\nu \Gamma(\lambda+\nu) z^\nu}{\Gamma(\lambda)\Gamma(\nu+1)} - \frac{2^{-2\lambda-\nu}\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)\sin(\pi\nu) z^{-2\lambda-\nu}}{\pi\Gamma(\lambda)} & \text{True} \end{cases}$$

$/;$

$(|z| \to \infty)$

07.14.06.0081.01

$$C_\nu^\lambda(z) \propto$$

$$\begin{cases} 0 & (-\nu \in \mathbb{N}^+) \bigvee \left(-\lambda - \frac{1}{2} \in \mathbb{N} \bigwedge \left(\left(-2\lambda - \nu \in \mathbb{N} \bigwedge \lambda + \nu \geq \frac{1}{2}\right) \bigvee \left(\nu \in \mathbb{N} \bigwedge \lambda + \nu \right.\right.\right. \\ \tilde{\infty} & -\lambda - \frac{1}{2} \in \mathbb{N} \bigwedge -2\lambda - \nu \in \mathbb{N} \bigwedge \lambda + \nu < \frac{1}{2} \\ \frac{2^\nu z^\nu \Gamma(\lambda+\nu)}{\Gamma(\lambda)\Gamma(\nu+1)} & \operatorname{Re}(\lambda+\nu) > 0 \\ -\frac{2^{\nu+1}\sin(\pi\nu)\log(z) z^\nu}{\pi} & \lambda + \nu = 0 \\ -\frac{2^{-2\lambda-\nu}\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)\sin(\pi\nu) z^{-2\lambda-\nu}}{\pi\Gamma(\lambda)} & \operatorname{Re}(\lambda+\nu) < 0 \\ \frac{2^\nu\Gamma(\lambda+\nu) z^\nu}{\Gamma(\lambda)\Gamma(\nu+1)} - \frac{2^{-2\lambda-\nu}\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)\sin(\pi\nu) z^{-2\lambda-\nu}}{\pi\Gamma(\lambda)} & \text{True} \end{cases}$$

$/;$

$(|z| \to \infty)$

Integral representations

On the real axis

Of the direct function

07.14.07.0001.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\Gamma(\nu+2\lambda)}{\nu!\,\Gamma(\lambda)^2} \int_0^\pi \left(z + \sqrt{z^2-1}\cos(t)\right)^\nu \sin^{2\lambda-1}(t)\, dt \;/;\; \operatorname{Re}(\lambda) > 0 \land \operatorname{Re}(z) > 0$$

Integral representations of negative integer order

Rodrigues-type formula.

07.14.07.0002.01

$$C_n^{\lambda}(z) = \frac{(-1)^n \, \Gamma\left(\lambda + \frac{1}{2}\right) \Gamma(n + 2\lambda) \left(1 - z^2\right)^{\frac{1}{2}-\lambda}}{n! \, 2^n \, \Gamma(2\lambda) \, \Gamma\left(n + \lambda + \frac{1}{2}\right)} \frac{\partial^n \left(1 - z^2\right)^{n+\lambda-\frac{1}{2}}}{\partial z^n} \; /; \; n \in \mathbb{N}$$

Generating functions

07.14.11.0001.01

$$C_n^{\lambda}(z) = \left([t^n] \left(t^2 - 2\, z\, t + 1\right)^{-\lambda}\right) /; \; n \in \mathbb{N} \land -1 < z < 1$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

07.14.13.0001.01

$$\left(1 - z^2\right) w''(z) - (2\lambda + 1)\, z\, w'(z) + \nu (\nu + 2\lambda)\, w(z) = 0 \; /; \; w(z) = c_1 \, C_\nu^{\lambda}(z) \; + c_2 \left(1 - z^2\right)^{\frac{1}{4}(1-2\lambda)} Q_{\lambda+\nu-\frac{1}{2}}^{\frac{1}{2}-\lambda}(z)$$

07.14.13.0002.01

$$W_z\left(C_\nu^{\lambda}(z), \left(1 - z^2\right)^{\frac{1}{4}(1-2\lambda)} Q_{\nu+\lambda-\frac{1}{2}}^{\frac{1}{2}-\lambda}(z)\right) = \frac{2^{\frac{1}{2}-\lambda} \sqrt{\pi} \left(1 - z^2\right)^{-\lambda-\frac{1}{2}}}{\Gamma(\lambda)}$$

07.14.13.0003.01

$$w''(z) - \left(\frac{(2\lambda + 1)\, g(z)\, g'(z)}{1 - g(z)^2} + \frac{g''(z)}{g'(z)}\right) w'(z) + \frac{\nu (2\lambda + \nu)\, g'(z)^2}{1 - g(z)^2} w(z) = 0 \; /; \; w(z) = \; c_1 \, C_\nu^{\lambda}(g(z)) + c_2 \left(1 - g(z)^2\right)^{\frac{1-2\lambda}{4}} Q_{\lambda+\nu-\frac{1}{2}}^{\frac{1}{2}-\lambda}(g(z))$$

07.14.13.0004.01

$$W_z\left(C_\nu^{\lambda}(g(z)), \left(1 - g(z)^2\right)^{\frac{1-2\lambda}{4}} Q_{\lambda+\nu-\frac{1}{2}}^{\frac{1}{2}-\lambda}(g(z))\right) = \frac{2^{\frac{1}{2}-\lambda} \sqrt{\pi} \left(1 - g(z)^2\right)^{-\lambda-\frac{1}{2}} g'(z)}{\Gamma(\lambda)}$$

07.14.13.0005.01

$$w''(z) - \left(\frac{(2\lambda + 1)\, g(z)\, g'(z)}{1 - g(z)^2} + \frac{2\, h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) +$$
$$\left(\frac{\nu (2\lambda + \nu)\, g'(z)^2}{1 - g(z)^2} + \frac{(2\lambda + 1)\, g(z)\, h'(z)\, g'(z)}{\left(1 - g(z)^2\right) h(z)} + \frac{2\, h'(z)^2}{h(z)^2} + \frac{h'(z)\, g''(z)}{h(z)\, g'(z)} - \frac{h''(z)}{h(z)}\right) w(z) = 0 \; /;$$
$$w(z) = \; c_1 \, h(z)\, C_\nu^{\lambda}(g(z)) + c_2 \, h(z) \left(1 - g(z)^2\right)^{\frac{1-2\lambda}{4}} Q_{\lambda+\nu-\frac{1}{2}}^{\frac{1}{2}-\lambda}(g(z))$$

07.14.13.0006.01

$$W_z\left(h(z)\, C_\nu^{\lambda}(g(z)), \, h(z) \left(1 - g(z)^2\right)^{\frac{1-2\lambda}{4}} Q_{\lambda+\nu-\frac{1}{2}}^{\frac{1}{2}-\lambda}(g(z))\right) = \frac{2^{\frac{1}{2}-\lambda} \sqrt{\pi} \left(1 - g(z)^2\right)^{-\lambda-\frac{1}{2}} g'(z)\, h(z)^2}{\Gamma(\lambda)}$$

07.14.13.0007.01

$$w''(z) + \frac{-a^2 (2 s - 2 r \lambda - 1) z^{2 r} + r + 2 s - 1}{z \left(a^2 z^{2 r} - 1\right)} w'(z) + \frac{a^2 z^{2 r} (s + r \nu) (s - r (2 \lambda + \nu)) - s (r + s)}{z^2 \left(a^2 z^{2 r} - 1\right)} w(z) = 0 /;$$

$$w(z) = c_1 z^s C_\nu^\lambda(a z^r) + c_2 z^s \left(1 - a^2 z^{2 r}\right)^{\frac{1 - 2 \lambda}{4}} Q_{\lambda + \nu - \frac{1}{2}}^{\frac{1}{2} - \lambda}(a z^r)$$

07.14.13.0008.01

$$W_z\left(z^s C_\nu^\lambda(a z^r), z^s \left(1 - a^2 z^{2 r}\right)^{\frac{1 - 2 \lambda}{4}} Q_{\lambda + \nu - \frac{1}{2}}^{\frac{1}{2} - \lambda}(a z^r)\right) = \frac{2^{\frac{1}{2} - \lambda} a \sqrt{\pi} \, r z^{r + 2 s - 1} \left(1 - a^2 z^{2 r}\right)^{-\lambda - \frac{1}{2}}}{\Gamma(\lambda)}$$

07.14.13.0009.01

$$w''(z) - \left(\frac{a^2 (2 \lambda + 1) \log(r) r^{2 z}}{1 - a^2 r^{2 z}} + \log(r) + 2 \log(s)\right) w'(z) +$$
$$\frac{a^2 r^{2 z} (\nu \log(r) + \log(s)) (\log(s) - (2 \lambda + \nu) \log(r)) - \log(s) (\log(r) + \log(s))}{a^2 r^{2 z} - 1} w(z) = 0 /;$$

$$w(z) = c_1 s^z C_\nu^\lambda(a r^z) + c_2 s^z \left(1 - a^2 r^{2 z}\right)^{\frac{1 - 2 \lambda}{4}} Q_{\lambda + \nu - \frac{1}{2}}^{\frac{1}{2} - \lambda}(a r^z)$$

07.14.13.0010.01

$$W_z\left(s^z C_\nu^\lambda(a r^z), s^z \left(1 - a^2 r^{2 z}\right)^{\frac{1 - 2 \lambda}{4}} Q_{\lambda + \nu - \frac{1}{2}}^{\frac{1}{2} - \lambda}(a r^z)\right) = \frac{2^{\frac{1}{2} - \lambda} a \sqrt{\pi} \, r^z \left(1 - a^2 r^{2 z}\right)^{-\lambda - \frac{1}{2}} s^{2 z} \log(r)}{\Gamma(\lambda)}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.14.16.0001.01

$$C_n^\lambda(-z) = (-1)^n C_n^\lambda(z) /; n \in \mathbb{N}$$

Identities

Recurrence identities

Consecutive neighbors

With respect to ν

07.14.17.0001.01

$$C_\nu^\lambda(z) = \frac{2 (\lambda + \nu + 1) z}{2 \lambda + \nu} C_{\nu + 1}^\lambda(z) - \frac{\nu + 2}{2 \lambda + \nu} C_{\nu + 2}^\lambda(z)$$

07.14.17.0002.01

$$C_\nu^\lambda(z) = \frac{2 (\lambda + \nu - 1) z}{\nu} C_{\nu - 1}^\lambda(z) - \frac{2 \lambda + \nu - 2}{\nu} C_{\nu - 2}^\lambda(z)$$

With respect to λ

07.14.17.0008.01

$$C_\nu^\lambda(z) == \frac{2\lambda\left(2\lambda - 2\left(z^2-1\right)(\lambda+\nu+1)+1\right)}{(2\lambda+\nu)(2\lambda+\nu+1)} C_\nu^{\lambda+1}(z) + \frac{4\lambda(\lambda+1)\left(z^2-1\right)}{(2\lambda+\nu)(2\lambda+\nu+1)} C_\nu^{\lambda+2}(z)$$

07.14.17.0009.01

$$C_\nu^\lambda(z) = \frac{2(\lambda-1)z^2 + 2\nu\left(z^2-1\right) - 4\lambda + 5}{2\left(z^2-1\right)(\lambda-1)} C_\nu^{\lambda-1}(z) + \frac{(\nu+2\lambda-4)(\nu+2\lambda-3)}{4\left(z^2-1\right)((\lambda-3)\lambda+2)} C_\nu^{\lambda-2}(z)$$

Distant neighbors

With respect to ν

07.14.17.0010.01

$$C_\nu^\lambda(z) == \mathcal{C}_n(\nu, \lambda, z)\, C_{\nu+n}^\lambda(z) - \frac{\nu+n+1}{n+2\lambda+\nu-1} \mathcal{C}_{n-1}(\nu, \lambda, z)\, C_{\nu+n+1}^\lambda(z) \,/;\, \mathcal{C}_0(\nu, \lambda, z) == 1 \bigwedge$$
$$\mathcal{C}_1(\nu, \lambda, z) == \frac{2(\lambda+\nu+1)z}{2\lambda+\nu} \bigwedge \mathcal{C}_n(\nu, \lambda, z) == \frac{2z(n+\lambda+\nu)}{n+2\lambda+\nu-1} \mathcal{C}_{n-1}(\nu, \lambda, z) - \frac{\nu+n}{n+2\lambda+\nu-2} \mathcal{C}_{n-2}(\nu, \lambda, z) \bigwedge n \in \mathbb{N}^+$$

07.14.17.0011.01

$$C_\nu^\lambda(z) == \mathcal{C}_n(\nu, \lambda, z)\, C_{\nu-n}^\lambda(z) - \frac{2\lambda+\nu-n-1}{\nu-n+1} \mathcal{C}_{n-1}(\nu, \lambda, z)\, C_{\nu-n-1}^\lambda(z) \,/;\, \mathcal{C}_0(\nu, \lambda, z) == 1 \bigwedge$$
$$\mathcal{C}_1(\nu, \lambda, z) == \frac{2(\lambda+\nu-1)z}{\nu} \bigwedge \mathcal{C}_n(\nu, \lambda, z) == \frac{2z(\lambda+\nu-n)}{\nu-n+1} \mathcal{C}_{n-1}(\nu, \lambda, z) - \frac{2\lambda+\nu-n}{\nu-n+2} \mathcal{C}_{n-2}(\nu, \lambda, z) \bigwedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

Recurrence relations

07.14.17.0003.01

$$(2\lambda+\nu-1)\, C_{\nu-1}^\lambda(z) + (\nu+1)\, C_{\nu+1}^\lambda(z) == 2(\lambda+\nu)\, z\, C_\nu^\lambda(z)$$

07.14.17.0004.01

$$C_\nu^\lambda(z) == \frac{1}{2(\lambda+\nu)z}\left((2\lambda+\nu-1)\, C_{\nu-1}^\lambda(z) + (\nu+1)\, C_{\nu+1}^\lambda(z)\right)$$

07.14.17.0012.01

$$C_\nu^\lambda(z) == \frac{2(\lambda+\nu+1)z}{2\lambda+\nu} C_{\nu+1}^\lambda(z) - \frac{\nu+2}{2\lambda+\nu} C_{\nu+2}^\lambda(z)$$

07.14.17.0013.01

$$C_\nu^\lambda(z) == z\, C_{\nu-1}^\lambda(z) + \frac{2\lambda+\nu-2}{2\lambda-2} C_\nu^{\lambda-1}(z)$$

07.14.17.0014.01

$$C_\nu^\lambda(z) == \frac{(\nu+1)z}{2\lambda+\nu} C_{\nu+1}^\lambda(z) - \frac{2\lambda\left(z^2-1\right)}{2\lambda+\nu} C_\nu^{\lambda+1}(z)$$

Normalized recurrence relation

07.14.17.0005.01

$$z\, p(\nu, z) == \frac{\nu\,(\nu + 2\,\lambda - 1)}{4\,(\nu + \lambda - 1)\,(\nu + \lambda)}\, p(\nu - 1, z) + p(\nu + 1, z)\; /;\; p(\nu, z) == \frac{\Gamma(\nu + 1)}{2^{\nu}\,(\lambda)_{\nu}}\, C_{\nu}^{\lambda}(z)$$

Relations of special kind

07.14.17.0006.01

$$C_{\nu}^{\lambda}(z) == -\frac{\sin(\pi\,\nu)}{\sin(\pi\,(2\,\lambda + \nu))}\, C_{-2\,\lambda-\nu}^{\lambda}(z)$$

07.14.17.0007.01

$$(2\,\lambda + \nu)\,\nu\, C_{\nu}^{\lambda}(z) - 2\,z\,\lambda\,(2\,\lambda + 1)\, C_{\nu-1}^{\lambda+1}(z) - 4\left(z^2 - 1\right)\lambda\,(\lambda + 1)\, C_{\nu-2}^{\lambda+2}(z) == 0$$

Complex characteristics

Real part

07.14.19.0001.01

$$\mathrm{Re}\left(C_n^{\lambda}(x + i\,y)\right) == \sum_{j=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^j\, 2^{2\,j}\,(\lambda)_{2\,j}}{(2\,j)!}\, C_{n-2\,j}^{2\,j+\lambda}(x)\, y^{2\,j}\; /;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R} \bigwedge \lambda \in \mathbb{R} \bigwedge n \in \mathbb{N}$$

Imaginary part

07.14.19.0002.01

$$\mathrm{Im}\left(C_n^{\lambda}(x + i\,y)\right) == \sum_{j=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(-1)^j\, 2^{2\,j+1}\,(\lambda)_{2\,j+1}}{(2\,j+1)!}\, C_{n-2\,j-1}^{2\,j+\lambda+1}(x)\, y^{2\,j+1}\; /;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R} \bigwedge \lambda \in \mathbb{R} \bigwedge n \in \mathbb{N}$$

Differentiation

Low-order differentiation

With respect to ν

07.14.20.0001.01

$$\frac{\partial C_{\nu}^{\lambda}(z)}{\partial \nu} == \frac{(z - 1)\,\Gamma(2\,\lambda + \nu)}{(2\,\lambda + 1)\,\Gamma(\nu + 1)\,\Gamma(2\,\lambda)} \left(\nu\, F_{2\times 0\times 1}^{2\times 1\times 2}\left(\begin{matrix} 1 - \nu,\, 2\,\lambda + \nu + 1;\, 1;\, 1,\, 2\,\lambda + \nu; \\ 2,\, \lambda + \frac{3}{2};;\, 2\,\lambda + \nu + 1; \end{matrix} \frac{1 - z}{2}, \frac{1 - z}{2}\right) + (2\,\lambda + \nu)\, F_{2\times 0\times 1}^{2\times 1\times 2}\left(\begin{matrix} 1 - \nu,\, 2\,\lambda + \nu + 1;\, 1;\, 1,\, -\nu; \\ 2,\, \lambda + \frac{3}{2};;\, 1 - \nu; \end{matrix} \frac{1 - z}{2}, \frac{1 - z}{2}\right)\right) - \frac{2^{1-2\,\lambda}\,\sqrt{\pi}\;\Gamma(2\,\lambda + \nu)}{\Gamma(\nu + 1)\,\Gamma(\lambda)}\,(\psi(\nu + 1) - \psi(2\,\lambda + \nu))\; {}_2\tilde{F}_1\left(-\nu,\, 2\,\lambda + \nu;\, \lambda + \frac{1}{2};\, \frac{1 - z}{2}\right)$$

With respect to λ

07.14.20.0002.01

$$\frac{\partial C_\nu^\lambda(z)}{\partial \lambda} = \frac{2\,\nu\,(z-1)\,\Gamma(2\,\lambda+\nu)}{(2\,\lambda+1)^2\,\Gamma(\nu+1)\,\Gamma(2\,\lambda)}\left((2\,\lambda+1)\,F_{2\times 0\times 1}^{2\times 1\times 2}\left(\begin{matrix}1-\nu,\,2\,\lambda+\nu+1;\,1;\,1,\,2\,\lambda+\nu; \\ 2,\,\lambda+\frac{3}{2};;\,2\,\lambda+\nu+1;\end{matrix}\;\frac{1-z}{2},\,\frac{1-z}{2}\right)-\right.$$
$$\left.(2\,\lambda+\nu)\,F_{2\times 0\times 1}^{2\times 1\times 2}\left(\begin{matrix}1-\nu,\,2\,\lambda+\nu+1;\,1;\,1,\,\lambda+\frac{1}{2}; \\ 2,\,\lambda+\frac{3}{2};;\,\lambda+\frac{3}{2};\end{matrix}\;\frac{1-z}{2},\,\frac{1-z}{2}\right)\right)-$$
$$\frac{2^{2-2\,\lambda}\,\sqrt{\pi}\,\Gamma(2\,\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)}\,(\psi(2\,\lambda)-\psi(2\,\lambda+\nu))\,{}_2\tilde{F}_1\left(-\nu,\,2\,\lambda+\nu;\,\lambda+\frac{1}{2};\,\frac{1-z}{2}\right)$$

With respect to z

Forward shift operator:

07.14.20.0003.01

$$\frac{\partial C_\nu^\lambda(z)}{\partial z} = 2\,\lambda\,C_{\nu-1}^{\lambda+1}(z)$$

07.14.20.0004.01

$$\frac{\partial^2 C_\nu^\lambda(z)}{\partial z^2} = 4\,\lambda\,(\lambda+1)\,C_{\nu-2}^{\lambda+2}(z)$$

Backward shift operator:

07.14.20.0005.01

$$\left(1-z^2\right)\frac{\partial C_\nu^\lambda(z)}{\partial z} + z\,(1-2\,\lambda)\,C_\nu^\lambda(z) = -\frac{(\nu+1)\,(\nu+2\,\lambda-1)}{2\,(\lambda-1)}\,C_{\nu+1}^{\lambda-1}(z)$$

07.14.20.0006.01

$$\frac{\partial\left(\left(1-z^2\right)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda(z)\right)}{\partial z} = -\frac{(\nu+1)\,(\nu+2\,\lambda-1)}{2\,(\lambda-1)}\,\left(1-z^2\right)^{\lambda-\frac{3}{2}}\,C_{\nu+1}^{\lambda-1}(z)$$

Symbolic differentiation

With respect to z

07.14.20.0007.02

$$\frac{\partial^m C_\nu^\lambda(z)}{\partial z^m} = 2^m\,(\lambda)_m\,C_{\nu-m}^{m+\lambda}(z)\;/;\;m\in\mathbb{N}$$

07.14.20.0008.02

$$\frac{\partial^m C_\nu^\lambda(z)}{\partial z^m} = \frac{2^{1-2\,\lambda}\,\sqrt{\pi}\,(z-1)^{-m}\,\Gamma(2\,\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)}\,{}_3\tilde{F}_2\left(1,\,-\nu,\,2\,\lambda+\nu;\,1-m,\,\lambda+\frac{1}{2};\,\frac{1-z}{2}\right)\;/;\;m\in\mathbb{N}$$

Fractional integro-differentiation

With respect to z

07.14.20.0009.01

$$\frac{\partial^\alpha C_\nu^\lambda(z)}{\partial z^\alpha} = \frac{2^{1-2\,\lambda}\,\sqrt{\pi}\,\Gamma(2\,\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)}\,z^{-\alpha}\,\tilde{F}_{1\times 1\times 0}^{2\times 1\times 0}\left(\begin{matrix}-\nu,\,2\,\lambda+\nu;\,1;\,; \\ \lambda+\frac{1}{2};\,1-\alpha;\,;\end{matrix}\;-\frac{z}{2},\,\frac{1}{2}\right)$$

Integration

Indefinite integration

Involving only one direct function

07.14.21.0001.01

$$\int C_\nu^\lambda(z)\,dz = \frac{1}{2(\lambda-1)}\,C_{\nu+1}^{\lambda-1}(z)$$

07.14.21.0002.01

$$\int C_\nu^{\frac{3}{2}}(z)\,dz = P_{\nu+1}(z)$$

07.14.21.0003.01

$$\int C_\nu^2(z)\,dz = \frac{1}{2}\,U_{\nu+1}(z)$$

Involving one direct function and elementary functions

Involving power function

07.14.21.0004.01

$$\int z^{\alpha-1}\,C_\nu^\lambda(z)\,dz = \frac{2^{1-2\lambda}\sqrt{\pi}\;\Gamma(2\lambda+\nu)\,z^\alpha}{\alpha\,\Gamma(\nu+1)\,\Gamma(\lambda)}\,\tilde{F}_{1\times1\times0}^{2\times1\times0}\left(\begin{matrix}-\nu,\,2\lambda+\nu;\,\alpha;\,;\\ \lambda+\frac{1}{2};\,\alpha+1;\,;\end{matrix}\;-\frac{z}{2},\,\frac{1}{2}\right)$$

Involving algebraic functions

07.14.21.0005.01

$$\int \left(1-z^2\right)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda(z)\,dz = -\frac{2\left(1-z^2\right)^{\lambda+\frac{1}{2}}\,\lambda}{\nu\,(\nu+2\lambda)}\,C_{\nu-1}^{\lambda+1}(z)$$

07.14.21.0006.01

$$\int \left(1-z^2\right)^{\frac{1}{2}(-\nu-3)}\,C_\nu^\lambda(z)\,dz = \frac{\left(1-z^2\right)^{\frac{1}{2}(-\nu-1)}}{\nu+2\lambda}\,C_{\nu+1}^\lambda(z)$$

07.14.21.0007.01

$$\int \left(1-z^2\right)^{\frac{1}{2}(\nu+2\lambda-3)}\,C_\nu^\lambda(z)\,dz = -\frac{\left(1-z^2\right)^{\frac{1}{2}(\nu+2\lambda-1)}}{\nu}\,C_{\nu-1}^\lambda(z)$$

Definite integration

Involving the direct function

Orthogonality:

07.14.21.0008.01

$$\int_{-1}^{1}\left(1-t^2\right)^{\lambda-\frac{1}{2}}\,C_m^\lambda(t)\,C_n^\lambda(t)\,dt = \frac{\pi\,2^{1-2\lambda}\,\Gamma(n+2\lambda)}{n!\,(n+\lambda)\,\Gamma(\lambda)^2}\,\delta_{m,n}\;/;\;m\in\mathbb{N}\bigwedge n\in\mathbb{N}\bigwedge \mathrm{Re}(\lambda)>-\frac{1}{2}\bigwedge \lambda\neq 0$$

Summation

Finite summation

07.14.23.0001.01

$$\sum_{k=0}^{n} \frac{(-1)^k 4^k \Gamma(n-k+1) \Gamma(k+\lambda)^2 (2k+2\lambda-1)}{\Gamma(k+n+2\lambda)} \left(z_1^2-1\right)^{k/2} \left(z_2^2-1\right)^{k/2} C_{n-k}^{k+\lambda}(z_1)\, C_{n-k}^{k+\lambda}(z_2)\, C_k^{\lambda-\frac{1}{2}}(\alpha) = \frac{4^{1-\lambda}\sqrt{\pi}\,\Gamma(\lambda)}{\Gamma\left(\lambda-\frac{1}{2}\right)} C_n^{\lambda}\left(z_1 z_2 - \sqrt{z_1^2-1}\sqrt{z_2^2-1}\,\alpha\right)$$

Infinite summation

07.14.23.0002.01

$$\sum_{n=0}^{\infty} C_n^{\lambda}(z)\, w^n = \left(w^2-2zw+1\right)^{-\lambda} /; -1<z<1 \land |w|<1$$

07.14.23.0003.01

$$\sum_{n=0}^{\infty} \frac{\left(\lambda+\frac{1}{2}\right)_n}{(2\lambda)_n} C_n^{\lambda}(z)\, w^n = \frac{2^{\lambda-\frac{1}{2}}}{\sqrt{w^2-2zw+1}} \left(1-wz+\sqrt{w^2-2zw+1}\right)^{\frac{1}{2}-\lambda} /; -1<z<1 \land |w|<1$$

07.14.23.0004.01

$$\sum_{n=0}^{\infty} \frac{C_n^{\lambda}(z)\, w^n}{(2\lambda)_n} = e^{zw}\, {}_0F_1\left(;\lambda+\frac{1}{2};\frac{1}{4}\left(z^2-1\right)w^2\right) /; -1<z<1 \land |w|<1$$

07.14.23.0005.01

$$\sum_{n=0}^{\infty} \frac{1}{(2\lambda)_n \left(\lambda+\frac{1}{2}\right)_n} C_n^{\lambda}(z)\, w^n = {}_0F_1\left(;\lambda+\frac{1}{2};\frac{1}{2}(z-1)w\right) {}_0F_1\left(;\lambda+\frac{1}{2};\frac{1}{2}(z+1)w\right) /; -1<z<1 \land |w|<1$$

07.14.23.0006.01

$$\sum_{n=0}^{\infty} \frac{(\gamma)_n (2\lambda-\gamma)_n}{(2\lambda)_n \left(\lambda+\frac{1}{2}\right)_n} C_n^{\lambda}(z)\, w^n = {}_2F_1\left(\gamma, 2\lambda-\gamma; \lambda+\frac{1}{2}; \frac{1}{2}\left(1-\sqrt{w^2-2zw+1}-w\right)\right) {}_2F_1\left(\gamma, 2\lambda-\gamma; \lambda+\frac{1}{2}; \frac{1}{2}\left(1-\sqrt{w^2-2zw+1}+w\right)\right) /; -1<z<1 \land |w|<1$$

07.14.23.0007.01

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(2\lambda)_n} C_n^{\lambda}(z)\, w^n = (1-wz)^{\lambda}\, {}_2F_1\left(\frac{\lambda}{2}, \frac{\lambda+1}{2}; \lambda+\frac{1}{2}; \frac{\left(z^2-1\right)w^2}{(1-wz)^2}\right) /; -1<z<1 \land |w|<1$$

07.14.23.0008.01

$$\sum_{n=0}^{\infty} \frac{n!\,(n+\lambda)}{\Gamma(n+2\lambda)} C_n^{\lambda}(x)\, C_n^{\lambda}(y) = \frac{\pi\, 2^{1-2\lambda}}{\Gamma(\lambda)^2} \left(1-x^2\right)^{\frac{1-2\lambda}{4}} \left(1-y^2\right)^{\frac{1-2\lambda}{4}} \delta(x-y) /; \mathrm{Re}(\lambda) > -\frac{1}{2} \bigwedge \lambda \neq 0 \bigwedge -1<x<1 \bigwedge -1<y<1$$

Operations

Limit operation

07.14.25.0001.01

$$\lim_{\lambda\to 0} \frac{1}{\lambda} C_\nu^\lambda(z) = C_\nu^{(0)}(z)$$

07.14.25.0002.01

$$\lim_{\lambda\to 0} \frac{1}{\lambda} C_\nu^\lambda(z) = \frac{2}{\nu} T_\nu(z)$$

07.14.25.0003.01

$$\lim_{\lambda\to \infty} \lambda^{-\frac{\nu}{2}} C_\nu^\lambda\left(\frac{z}{\sqrt{\lambda}}\right) = \frac{2}{\nu!} H_\nu(z) /; |z| < 1$$

07.14.25.0004.01

$$\lim_{z\to \infty} (2\,z)^{-\nu} C_\nu^\lambda(z) = \frac{(\lambda)_\nu}{\nu!}$$

Orthogonality, completeness, and Fourier expansions

The set of functions $C_n^\lambda(x)$, $n = 0, 1, \ldots$, forms a complete, orthogonal (with weight $\frac{n!\,(n+\lambda)\,\Gamma(\lambda)^2}{\pi\, 2^{1-2\lambda}\,\Gamma(n+2\lambda)} \left(1-x^2\right)^{\lambda-\frac{1}{2}}$) system on the interval $(-1, 1)$.

07.14.25.0005.01

$$\sum_{n=0}^{\infty} \left(\sqrt{\frac{n!\,(n+\lambda)\,\Gamma(\lambda)^2}{\pi\, 2^{1-2\lambda}\,\Gamma(n+2\lambda)}} \left(1-x^2\right)^{\frac{2\lambda-1}{4}} C_n^\lambda(x)\right)\left(\sqrt{\frac{n!\,(n+\lambda)\,\Gamma(\lambda)^2}{\pi\, 2^{1-2\lambda}\,\Gamma(n+2\lambda)}} \left(1-y^2\right)^{\frac{2\lambda-1}{4}} C_n^\lambda(y)\right) = \delta(x-y) /;$$

$$\mathrm{Re}(\lambda) > -\frac{1}{2} \bigwedge \lambda \neq 0 \bigwedge -1 < x < 1 \bigwedge -1 < y < 1$$

07.14.25.0006.01

$$\int_{-1}^{1} \left(\sqrt{\frac{m!\,(m+\lambda)\,\Gamma(\lambda)^2}{\pi\, 2^{1-2\lambda}\,\Gamma(m+2\lambda)}} \left(1-t^2\right)^{\frac{2\lambda-1}{4}} C_m^\lambda(t)\right)\left(\sqrt{\frac{n!\,(n+\lambda)\,\Gamma(\lambda)^2}{\pi\, 2^{1-2\lambda}\,\Gamma(n+2\lambda)}} \left(1-t^2\right)^{\frac{2\lambda-1}{4}} C_n^\lambda(t)\right) dt = \delta_{m,n} /; \mathrm{Re}(\lambda) > -\frac{1}{2} \bigwedge \lambda \neq 0$$

Any sufficiently smooth function $f(x)$ can be expanded in the system $\left\{C_n^\lambda(x)\right\}_{n=0,1,\ldots}$ as a generalized Fourier series, with its sum converging to $f(x)$ almost everywhere.

07.14.25.0007.01

$$f(x) = \sum_{n=0}^{\infty} c_n\, \psi_n(x) /; c_n = \int_{-1}^{1} \psi_n(t)\, f(t)\, dt \bigwedge \psi_n(x) = \sqrt{\frac{n!\,(n+\lambda)\,\Gamma(\lambda)^2}{\pi\, 2^{1-2\lambda}\,\Gamma(n+2\lambda)}} \left(1-x^2\right)^{\frac{2\lambda-1}{4}} C_n^\lambda(x) \bigwedge -1 < x < 1$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_0\tilde{F}_1$

07.14.26.0001.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)}\,{}_2\tilde{F}_1\left(-\nu, 2\lambda+\nu; \lambda+\frac{1}{2}; \frac{1-z}{2}\right)$$

07.14.26.0044.01

$$C_\nu^\lambda(z) = -\frac{\sqrt{\pi}\,(2-2z)^{\frac{1}{2}-\lambda}\sec(\pi(\lambda+\nu))\sin(\pi\nu)}{\Gamma(\lambda)}\,{}_2\tilde{F}_1\left(-\lambda-\nu+\frac{1}{2}, \lambda+\nu+\frac{1}{2}; \frac{3}{2}-\lambda; \frac{1-z}{2}\right) /; -\lambda-\frac{1}{2} \in \mathbb{N}$$

Involving ${}_2F_1$

07.14.26.0002.01

$$C_\nu^\lambda(z) = \frac{\Gamma(\nu+2\lambda)}{\Gamma(2\lambda)\,\Gamma(\nu+1)}\,{}_2F_1\left(-\nu, \nu+2\lambda; \lambda+\frac{1}{2}; \frac{1-z}{2}\right) /; -\lambda-\frac{1}{2} \notin \mathbb{N}$$

07.14.26.0003.01

$$C_\nu^\lambda(z) = \frac{\cos(\pi(\lambda+\nu))\sec(\pi\lambda)\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(2\lambda)}\,{}_2F_1\left(-\nu, 2\lambda+\nu; \lambda+\frac{1}{2}; \frac{z+1}{2}\right) - \frac{2^{\frac{1}{2}-\lambda}\sin(\nu\pi)\,\Gamma\left(\lambda-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\lambda)}\,(z+1)^{\frac{1}{2}-\lambda}\,{}_2F_1\left(\lambda+\nu+\frac{1}{2}, \frac{1}{2}-\lambda-\nu; \frac{3}{2}-\lambda; \frac{z+1}{2}\right) /; \lambda+\frac{1}{2} \notin \mathbb{Z}$$

07.14.26.0004.01

$$C_\nu^\lambda(z) = \frac{2^\nu\,\Gamma(\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)}\,(z-1)^\nu\,{}_2F_1\left(-\nu, -\lambda-\nu+\frac{1}{2}; -2\lambda-2\nu+1; \frac{2}{1-z}\right) - \frac{2^{-2\lambda-\nu}\sin(\nu\pi)\,\Gamma(-\lambda-\nu)\,\Gamma(2\lambda+\nu)}{\pi\,\Gamma(\lambda)}\,(z-1)^{-2\lambda-\nu}\,{}_2F_1\left(2\lambda+\nu, \lambda+\nu+\frac{1}{2}; 2\lambda+2\nu+1; \frac{2}{1-z}\right) /; 2\lambda+2\nu \notin \mathbb{Z}$$

Through hypergeometric functions of two variables

07.14.26.0005.01

$$C_\nu^\lambda(z) = \frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)}\,\tilde{F}_{1\times0\times0}^{2\times0\times0}\left(\begin{matrix}-\nu, 2\lambda+\nu;;; \\ \lambda+\frac{1}{2};;;\end{matrix}\ \frac{1}{2}, -\frac{z}{2}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

07.14.26.0006.01

$$C_\nu^\lambda(z) = -\frac{2^{1-2\lambda}\sin(\pi\nu)}{\sqrt{\pi}\,\Gamma(\lambda)}\,G_{2,2}^{1,2}\left(\frac{z-1}{2}\,\middle|\,\begin{matrix}\nu+1, -2\lambda-\nu+1 \\ 0, \frac{1}{2}-\lambda\end{matrix}\right) /; \nu \notin \mathbb{Z}$$

07.14.26.0007.01

$$C_n^\lambda(z) = -\frac{2^{1-2\lambda}}{\sqrt{\pi}\,\Gamma(\lambda)}\lim_{m\to n}\sin(\pi m)\,G_{2,2}^{1,2}\left(\frac{z-1}{2}\,\middle|\,\begin{matrix}m+1, -m-2\lambda+1 \\ 0, \frac{1}{2}-\lambda\end{matrix}\right) /; n \in \mathbb{Z}$$

07.14.26.0008.01

$$C_\nu^\lambda(2z+1) = -\frac{2^{1-2\lambda}\sin(\pi\nu)}{\sqrt{\pi}\,\Gamma(\lambda)}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\nu+1, -2\lambda-\nu+1 \\ 0, \frac{1}{2}-\lambda\end{matrix}\right) /; \nu \notin \mathbb{Z}$$

Classical cases involving algebraic functions

07.14.26.0009.01

$$(z+1)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda(2z+1) \;\text{==}\; \frac{\cos((\lambda+\nu)\pi)\,(2\lambda)_\nu}{\cos(\lambda\pi)\,\Gamma\left(\frac{1}{2}-\lambda\right)\Gamma(\nu+1)}\,G_{2,2}^{1,2}\!\left(z\,\middle|\,\begin{matrix}-\lambda-\nu+\frac{1}{2},\,\lambda+\nu+\frac{1}{2}\\ 0,\,\frac{1}{2}-\lambda\end{matrix}\right)$$

07.14.26.0010.01

$$(z+1)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda\left(1+\frac{2}{z}\right) \;\text{==}\; \frac{\cos((\lambda+\nu)\pi)\,(2\lambda)_\nu}{\cos(\lambda\pi)\,\Gamma\left(\frac{1}{2}-\lambda\right)\Gamma(\nu+1)}\,G_{2,2}^{2,1}\!\left(z\,\middle|\,\begin{matrix}\lambda+\frac{1}{2},\,2\lambda\\ -\nu,\,2\lambda+\nu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.14.26.0011.01

$$(z+1)^{-2\lambda-\nu}\,C_\nu^\lambda\left(\frac{1-z}{1+z}\right) \;\text{==}\; \frac{1}{\Gamma(2\lambda)\left(\lambda+\frac{1}{2}\right)_\nu\Gamma(\nu+1)}\,G_{2,2}^{1,2}\!\left(z\,\middle|\,\begin{matrix}1-2\lambda-\nu,\,\frac{1}{2}-\lambda-\nu\\ 0,\,\frac{1}{2}-\lambda\end{matrix}\right)/;\,z\notin(-\infty,-1)$$

07.14.26.0012.01

$$(z+1)^{-2\lambda-\nu}\,C_\nu^\lambda\left(\frac{z-1}{z+1}\right) \;\text{==}\; \frac{1}{\Gamma(2\lambda)\left(\lambda+\frac{1}{2}\right)_\nu\Gamma(\nu+1)}\,G_{2,2}^{2,1}\!\left(z\,\middle|\,\begin{matrix}1-2\lambda-\nu,\,\frac{1}{2}-\lambda-\nu\\ 0,\,\frac{1}{2}-\lambda\end{matrix}\right)/;\,z\notin(-1,0)$$

07.14.26.0013.01

$$(z+1)^{-\lambda-\frac{\nu}{2}}\,C_\nu^\lambda\left(\frac{1}{\sqrt{z+1}}\right) \;\text{==}\; \frac{2^\nu}{\Gamma(\lambda)\,\Gamma(\nu+1)}\,G_{2,2}^{1,2}\!\left(z\,\middle|\,\begin{matrix}\frac{1-\nu}{2}-\lambda,\,1-\lambda-\frac{\nu}{2}\\ 0,\,\frac{1}{2}-\lambda\end{matrix}\right)$$

07.14.26.0014.01

$$(z+1)^{-\lambda-\frac{\nu}{2}}\,C_\nu^\lambda\left(\sqrt{\frac{z}{z+1}}\right) \;\text{==}\; \frac{2^\nu}{\Gamma(\lambda)\,\Gamma(\nu+1)}\,G_{2,2}^{2,1}\!\left(z\,\middle|\,\begin{matrix}1-\lambda-\frac{\nu}{2},\,\frac{1-\nu}{2}\\ 0,\,\frac{1}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.14.26.0015.01

$$(z+1)^{-\lambda-\frac{\nu}{2}}\,C_\nu^\lambda\left(\frac{z+2}{2\sqrt{z+1}}\right) \;\text{==}\; \frac{1}{\Gamma(\lambda)\,\Gamma(\nu+1)}\,G_{2,2}^{1,2}\!\left(z\,\middle|\,\begin{matrix}1-\lambda,\,-2\lambda-\nu+1\\ 0,\,1-2\lambda\end{matrix}\right)$$

07.14.26.0016.01

$$(z+1)^{-\lambda-\frac{\nu}{2}}\,C_\nu^\lambda\left(\frac{2z+1}{2\sqrt{z}\,\sqrt{z+1}}\right) \;\text{==}\; \frac{1}{\Gamma(\lambda)\,\Gamma(\nu+1)}\,G_{2,2}^{2,1}\!\left(z\,\middle|\,\begin{matrix}1-\lambda-\frac{\nu}{2},\,\lambda-\frac{\nu}{2}\\ -\frac{\nu}{2},\,\lambda+\frac{\nu}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

Classical cases involving unit step θ

07.14.26.0017.01

$$\theta(1-|z|)\,(1-z)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda(2z-1) \;\text{==}\; \frac{(2\lambda)_\nu\,\Gamma\left(\lambda+\frac{1}{2}\right)}{\Gamma(\nu+1)}\,G_{2,2}^{2,0}\!\left(z\,\middle|\,\begin{matrix}\lambda+\nu+\frac{1}{2},\,-\lambda-\nu+\frac{1}{2}\\ 0,\,\frac{1}{2}-\lambda\end{matrix}\right)/;\,z\notin(-1,0)$$

07.14.26.0018.01

$$\theta(|z|-1)\,(z-1)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda(2z-1) \;\text{==}\; \frac{(2\lambda)_\nu\,\Gamma\left(\lambda+\frac{1}{2}\right)}{\Gamma(\nu+1)}\,G_{2,2}^{0,2}\!\left(z\,\middle|\,\begin{matrix}\lambda+\nu+\frac{1}{2},\,-\lambda-\nu+\frac{1}{2}\\ 0,\,\frac{1}{2}-\lambda\end{matrix}\right)$$

07.14.26.0019.01

$$\theta(1-|z|)\,(1-z)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda\left(\frac{2}{z}-1\right) \;\text{==}\; \frac{(2\lambda)_\nu\,\Gamma\left(\lambda+\frac{1}{2}\right)}{\Gamma(\nu+1)}\,G_{2,2}^{2,0}\!\left(z\,\middle|\,\begin{matrix}\lambda+\frac{1}{2},\,2\lambda\\ 2\lambda+\nu,\,-\nu\end{matrix}\right)$$

07.14.26.0020.01

$$\theta(|z|-1)\,(z-1)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda\left(\frac{2}{z}-1\right)=\frac{(2\,\lambda)_\nu\,\Gamma\left(\lambda+\frac{1}{2}\right)}{\Gamma(\nu+1)}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\lambda+\frac{1}{2},\,2\,\lambda\\2\,\lambda+\nu,\,-\nu\end{array}\right)/;\,z\notin(-\infty,\,-1)$$

07.14.26.0021.01

$$\theta(1-|z|)\,(1-z)^{\lambda-\frac{1}{2}}\,C_\nu^{(\lambda)}\left(\sqrt{z}\right)=\frac{(2\,\lambda)_\nu\,\Gamma\left(\lambda+\frac{1}{2}\right)}{\Gamma(\nu+1)}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}\frac{\nu+1}{2}+\lambda,\,\frac{1-\nu}{2}\\0,\,\frac{1}{2}\end{array}\right)/;\,z\notin(-1,\,0)$$

07.14.26.0022.01

$$\theta(|z|-1)\,(z-1)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda\left(\sqrt{z}\right)=\frac{\Gamma\left(\lambda+\frac{1}{2}\right)(2\,\lambda)_\nu}{\Gamma(\nu+1)}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\frac{1-\nu}{2},\,\lambda+\frac{\nu+1}{2}\\0,\,\frac{1}{2}\end{array}\right)$$

07.14.26.0023.01

$$\theta(1-|z|)\,(1-z)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda\left(\frac{1}{\sqrt{z}}\right)=\frac{\Gamma\left(\lambda+\frac{1}{2}\right)(2\,\lambda)_\nu}{\Gamma(\nu+1)}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}\lambda,\,\lambda+\frac{1}{2}\\\lambda+\frac{\nu}{2},\,-\frac{\nu}{2}\end{array}\right)/;\,z\notin(-1,\,0)$$

07.14.26.0024.01

$$\theta(|z|-1)\,(z-1)^{\lambda-\frac{1}{2}}\,C_\nu^\lambda\left(\frac{1}{\sqrt{z}}\right)=\frac{(2\,\lambda)_\nu\,\Gamma\left(\lambda+\frac{1}{2}\right)}{\Gamma(\nu+1)}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\lambda+\frac{1}{2},\,\lambda\\-\frac{\nu}{2},\,\lambda+\frac{\nu}{2}\end{array}\right)$$

07.14.26.0025.01

$$\theta(1-|z|)\,(1-z)^{-\frac{n}{2}-\lambda}\,C_n^\lambda\left(\frac{1}{\sqrt{1-z}}\right)=\frac{(-2)^n\,\Gamma(1-\lambda)}{n!}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}\frac{1-n}{2}-\lambda,\,1-\frac{n}{2}-\lambda\\0,\,\frac{1}{2}-\lambda\end{array}\right)/;\,n\in\mathbb{N}$$

07.14.26.0026.01

$$\theta(|z|-1)\,(z-1)^{-\frac{n}{2}-\lambda}\,C_n^\lambda\left(\sqrt{\frac{z}{z-1}}\right)=\frac{(-2)^n\,\Gamma(1-\lambda)}{n!}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}-\frac{n}{2}-\lambda+1,\,\frac{1-n}{2}\\0,\,\frac{1}{2}\end{array}\right)/;\,n\in\mathbb{N}$$

07.14.26.0027.01

$$\theta(1-|z|)\,(1-z)^{-\frac{n}{2}-\lambda}\,C_n^\lambda\left(\frac{2-z}{2\sqrt{1-z}}\right)=\frac{(-1)^n\,\Gamma(1-\lambda)}{n!}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}-n-2\,\lambda+1,\,1-\lambda\\0,\,1-2\,\lambda\end{array}\right)/;\,n\in\mathbb{N}$$

07.14.26.0028.01

$$\theta(|z|-1)\,(z-1)^{-\frac{n}{2}-\lambda}\,C_n^\lambda\left(\frac{2\,z-1}{2\sqrt{z}\,\sqrt{z-1}}\right)=\frac{(-1)^n\,\Gamma(1-\lambda)}{n!}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}1-\frac{n}{2}-\lambda,\,\lambda-\frac{n}{2}\\\frac{n}{2}+\lambda,\,-\frac{n}{2}\end{array}\right)/;\,n\in\mathbb{N}$$

07.14.26.0029.01

$$\theta(1-|z|)\,(1-z)^{2\,\lambda-1}\,C_\nu^\lambda\left(\frac{z+1}{2\sqrt{z}}\right)=\frac{\Gamma(2\,\lambda+\nu)}{\Gamma(\nu+1)}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}\lambda-\frac{\nu}{2},\,2\,\lambda+\frac{\nu}{2}\\\lambda+\frac{\nu}{2},\,-\frac{\nu}{2}\end{array}\right)/;\,z\notin(-1,\,0)$$

07.14.26.0030.01

$$\theta(|z|-1)\,(z-1)^{2\,\lambda-1}\,C_\nu^\lambda\left(\frac{z+1}{2\sqrt{z}}\right)=\frac{\Gamma(2\,\lambda+\nu)}{\Gamma(\nu+1)}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\lambda-\frac{\nu}{2},\,2\,\lambda+\frac{\nu}{2}\\\lambda+\frac{\nu}{2},\,-\frac{\nu}{2}\end{array}\right)$$

Generalized cases involving algebraic functions

07.14.26.0031.01

$$\left(z^2+1\right)^{-\lambda-\frac{\nu}{2}} C_\nu^\lambda\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{2^\nu}{\Gamma(\lambda)\,\Gamma(\nu+1)} G_{2,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 1-\lambda-\frac{\nu}{2}, \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.14.26.0032.01

$$\left(z^2+1\right)^{-\lambda-\frac{\nu}{2}} C_\nu^\lambda\left(\frac{2z^2+1}{2z\sqrt{z^2+1}}\right) = \frac{1}{\Gamma(\lambda)\,\Gamma(\nu+1)} G_{2,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 1-\lambda-\frac{\nu}{2}, \lambda-\frac{\nu}{2} \\ -\frac{\nu}{2}, \lambda+\frac{\nu}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

Generalized cases involving unit step θ

07.14.26.0033.01

$$\theta(1-|z|)\left(1-z^2\right)^{\lambda-\frac{1}{2}} C_\nu^\lambda(z) = \frac{(2\lambda)_\nu}{\Gamma(\nu+1)} \Gamma\left(\lambda+\frac{1}{2}\right) G_{2,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{\nu+1}{2}+\lambda, \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right)$$

07.14.26.0034.01

$$\theta(|z|-1)\left(z^2-1\right)^{\lambda-\frac{1}{2}} C_\nu^\lambda(z) = \frac{\Gamma\left(\lambda+\frac{1}{2}\right)(2\lambda)_\nu}{\Gamma(\nu+1)} G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1-\nu}{2}, \lambda+\frac{\nu+1}{2} \\ 0, \frac{1}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.14.26.0035.01

$$\theta(1-|z|)\left(1-z^2\right)^{\lambda-\frac{1}{2}} C_\nu^\lambda\left(\frac{1}{z}\right) = \frac{\Gamma\left(\lambda+\frac{1}{2}\right)(2\lambda)_\nu}{\Gamma(\nu+1)} G_{2,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \lambda, \lambda+\frac{1}{2} \\ \lambda+\frac{\nu}{2}, -\frac{\nu}{2} \end{array}\right)$$

07.14.26.0036.01

$$\theta(|z|-1)\left(z^2-1\right)^{\lambda-\frac{1}{2}} C_\nu^\lambda\left(\frac{1}{z}\right) = \frac{(2\lambda)_\nu\,\Gamma\left(\lambda+\frac{1}{2}\right)}{\Gamma(\nu+1)} G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \lambda+\frac{1}{2}, \lambda \\ -\frac{\nu}{2}, \lambda+\frac{\nu}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.14.26.0037.01

$$\theta(|z|-1)\left(z^2-1\right)^{-\frac{n}{2}-\lambda} C_n^\lambda\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{(-2)^n\,\Gamma(1-\lambda)}{n!} G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 1-\frac{n}{2}-\lambda, \frac{1-n}{2} \\ 0, \frac{1}{2} \end{array}\right) /; n \in \mathbb{N} \land \operatorname{Re}(z) > 0$$

07.14.26.0038.01

$$\theta(|z|-1)\left(z^2-1\right)^{-\frac{n}{2}-\lambda} C_n^\lambda\left(\frac{2z^2-1}{2z\sqrt{z^2-1}}\right) = \frac{(-1)^n\,\Gamma(1-\lambda)}{n!} G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 1-\frac{n}{2}-\lambda, \lambda-\frac{n}{2} \\ \frac{n}{2}+\lambda, -\frac{n}{2} \end{array}\right) /; n \in \mathbb{N} \land \operatorname{Re}(z) > 0$$

07.14.26.0039.01

$$\theta(1-|z|)\left(1-z^2\right)^{2\lambda-1} C_\nu^\lambda\left(\frac{z^2+1}{2z}\right) = \frac{\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)} G_{2,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \lambda-\frac{\nu}{2}, 2\lambda+\frac{\nu}{2} \\ \lambda+\frac{\nu}{2}, -\frac{\nu}{2} \end{array}\right)$$

07.14.26.0040.01

$$\theta(|z|-1)\left(z^2-1\right)^{2\lambda-1} C_\nu^\lambda\left(\frac{z^2+1}{2z}\right) = \frac{\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)} G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \lambda-\frac{\nu}{2}, 2\lambda+\frac{\nu}{2} \\ \lambda+\frac{\nu}{2}, -\frac{\nu}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

Through other functions

Involving some hypergeometric-type functions

07.14.26.0041.01

$$C_\nu^\lambda(z) = \frac{(2\lambda)_\nu}{\left(\lambda + \frac{1}{2}\right)_\nu} P_\nu^{\left(\lambda - \frac{1}{2}, \lambda - \frac{1}{2}\right)}(z)$$

07.14.26.0042.01

$$C_\nu^\lambda(z) = \frac{\sqrt{\pi}\,\Gamma\left(\lambda + \frac{\nu}{2}\right)}{\Gamma(\lambda)\,\Gamma\left(\frac{\nu+1}{2}\right)} P_{\frac{\nu}{2}}^{\left(\lambda - \frac{1}{2}, -\frac{1}{2}\right)}\left(2z^2 - 1\right)$$

07.14.26.0043.01

$$C_\nu^\lambda(z) = \frac{\sqrt{\pi}\,z\,\Gamma\left(\lambda + \frac{\nu+1}{2}\right)}{\Gamma(\lambda)\,\Gamma\left(\frac{\nu}{2} + 1\right)} P_{\frac{\nu-1}{2}}^{\left(\lambda - \frac{1}{2}, \frac{1}{2}\right)}\left(2z^2 - 1\right)$$

Representations through equivalent functions

With related functions

07.14.27.0001.01

$$C_\nu^\lambda(z) = \frac{2^{\frac{1}{2}-\lambda}\sqrt{\pi}\,\Gamma(\nu + 2\lambda)}{\Gamma(\nu+1)\,\Gamma(\lambda)} \left(1 - z^2\right)^{\frac{1-2\lambda}{4}} P_{\lambda+\nu-\frac{1}{2}}^{\frac{1}{2}-\lambda}(z)$$

07.14.27.0002.01

$$C_\nu^\lambda(z) = \frac{2^{\frac{1}{2}-\lambda}\sqrt{\pi}\,\Gamma(2\lambda + \nu)}{\Gamma(\nu+1)\,\Gamma(\lambda)} (z+1)^{\frac{1-2\lambda}{4}} (z-1)^{\frac{1-2\lambda}{4}} \mathcal{P}_{\lambda+\nu-\frac{1}{2}}^{\frac{1}{2}-\lambda}(z)$$

07.14.27.0003.01

$$C_\nu^\lambda(z) = \frac{\pi\,2^{1-\lambda}}{(\lambda+\nu)\,\Gamma(\lambda)} \sqrt{\frac{(\lambda+\nu)\,\Gamma(2\lambda+\nu)}{\Gamma(\nu+1)}} \left(1 - z^2\right)^{\frac{1-2\lambda}{4}} Y_{\lambda+\nu-1/2}^{1/2-\lambda}\left(\cos^{-1}(z), 0\right)$$

Theorems

Expansions in generalized Fourier series

$$f(x) = \sum_{k=0}^{\infty} c_k\,\psi_k(x) /; \quad c_k = \int_{-1}^{1} f(t)\,\psi_k(t)\,dt, \;\; \psi_k(x) = \frac{2^{1/2-\lambda}\sqrt{\pi\,\Gamma(2\lambda+n)}}{\Gamma(\lambda)\sqrt{n!\,(\lambda+n)}} \left(1 - x^2\right)^{\frac{2\lambda-1}{4}} C_k^\lambda(x), \;\; k \in \mathbb{N}.$$

Eigenfunctions of the angular part of a *d*-dimensional Laplace operator

The eigenfunctions of the angular part $L^2 = -\sum_{i>j}^{d}\left(x_i \frac{\partial}{\partial x_j} - x_j \frac{\partial}{\partial x_i}\right)^2$ of a d-dimensional Laplace operator $\Delta = \frac{1}{r^{d-1}} \frac{\partial}{\partial r} r^{d-1} \frac{\partial}{\partial r} + \frac{L^2}{r^2}$, where $\mathbf{u}, \mathbf{u}'$ are two unit vectors in $\mathbb{R}^d$, has the representation $L^2\,C_n^{d/2-1}(\mathbf{u}.\mathbf{u}') = n(n+d-2)\,C_n^{d/2-1}(\mathbf{u}.\mathbf{u}')$.

Removing Gibbs oscillations from Fourier series

Let $f(x)$ be a doubly periodic function with $f(-1) \neq f(1)$. Let $\hat{f}_k$ be its Fourier components $\hat{f}_k = \frac{1}{2} \int_{-1}^{1} f(x)\, e^{-i k \pi x}\, dx$. Then the Fourier sum $\sum_{k=-n}^{n} \hat{f}_k\, e^{i k \pi x}$ exhibits Gibbs oscillations.
It is possible to recover the original function $f(x)$ as a sum without such Gibbs oscillations in the following manner: $\sum_{k=0}^{n} g_k^n\, C_k^{\lambda}(x)$, where $g_k^n = \delta_{0k}\, \hat{f}_0 + \Gamma(\lambda)\, i^k\, (k+\lambda) \sum_{l=-n}^{n} (1-\delta_{0l})\, J_{k+\lambda}(\pi l) \left(\frac{2}{l \pi}\right)^{\lambda} \hat{f}_l \,/;\, \lambda = \left\lfloor \frac{2 \pi e n}{27} \right\rfloor$.
This sum converges pointwise to $f(x)$.

Quantum mechanical eigenfunctions of the hydrogen atom

The quantum mechanical eigenfunctions $\psi_{n m l}(p, \theta, \phi)$ of the hydrogen atom in the momentum representation are:

$$\psi_{n m l}(p, \theta, \phi) = 16 \pi \sqrt{m}\, \kappa_n^2\, 2^l\, l! \sqrt{\frac{(n-l-1)!}{(n+l)!}}\, \frac{1}{\left(\kappa_n^2 + p^2\right)^2} \left(\frac{\kappa_n\, p}{\kappa_n^2 + p^2}\right)^l C_{n-l-1}^{l+1}\!\left(\frac{\kappa_n^2 - p^2}{\kappa_n^2 + p^2}\right) Y_l^m(\theta, \phi) \,/;\, \kappa_n = \frac{\tau}{n} \,/;\, \tau > 0,\; n, l \in \mathbb{N},\; l \leq n,\; m \in \mathbb{Z},\; |m| \leq l.$$

History

– L. Gegenbauer (1893)

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