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HermiteHGeneral

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Notations

Traditional name

Hermite function

Traditional notation

$H_\nu(z)$

Mathematica StandardForm notation

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HermiteH[ν, z]
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Primary definition

07.01.02.0001.01

$$H_\nu(z) = 2^\nu \sqrt{\pi} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} \, {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - \frac{2 z}{\Gamma\left(-\frac{\nu}{2}\right)} \, {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) \right)$$

Specific values

Specialized values

For fixed ν

07.01.03.0001.01

$$H_\nu(0) = \frac{2^\nu \sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)}$$

For fixed z

Explicit integer ν

07.01.03.0002.01

$$H_0(z) = 1$$

07.01.03.0003.01

$$H_1(z) = 2 z$$

07.01.03.0004.01

$$H_2(z) = -2 + 4 z^2$$

07.01.03.0005.01

$$H_3(z) = -12\,z + 8\,z^3$$

07.01.03.0006.01

$$H_4(z) = 12 - 48\,z^2 + 16\,z^4$$

07.01.03.0007.01

$$H_5(z) = 120\,z - 160\,z^3 + 32\,z^5$$

07.01.03.0008.01

$$H_6(z) = -120 + 720\,z^2 - 480\,z^4 + 64\,z^6$$

07.01.03.0009.01

$$H_7(z) = -1680\,z + 3360\,z^3 - 1344\,z^5 + 128\,z^7$$

07.01.03.0010.01

$$H_8(z) = 1680 - 13\,440\,z^2 + 13\,440\,z^4 - 3584\,z^6 + 256\,z^8$$

07.01.03.0011.01

$$H_9(z) = 30\,240\,z - 80\,640\,z^3 + 48\,384\,z^5 - 9216\,z^7 + 512\,z^9$$

07.01.03.0012.01

$$H_{10}(z) = -30\,240 + 302\,400\,z^2 - 403\,200\,z^4 + 161\,280\,z^6 - 23\,040\,z^8 + 1024\,z^{10}$$

Symbolic integer ν

07.01.03.0013.01

$$H_n(z) = n!\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k\,(2\,z)^{n-2\,k}}{k!\,(n-2\,k)!} \;/;\; n \in \mathbb{N}$$

07.01.03.0014.01

$$H_{-n}(z) = -\frac{i^{n+1}\,2^{-n}\,\sqrt{\pi}}{(n-1)!}\,e^{z^2}\,H_{n-1}(i\,z) - \frac{2^{-2\left\lfloor \frac{n}{2} \right\rfloor-1}\,z^{2\left\lfloor \frac{n}{2} \right\rfloor-n+1}}{\left(\frac{1}{2}\right)_{\left\lfloor \frac{n}{2} \right\rfloor}}\,e^{z^2}$$

$$\sum_{k=0}^{n-\left\lfloor \frac{n}{2} \right\rfloor-1} \frac{(-1)^k}{k!}\,L_{n-\left\lfloor \frac{n}{2} \right\rfloor-k-1}^{k}\!\left(-z^2\right)\left(\Gamma\!\left(k+n-2\left\lfloor \frac{n}{2} \right\rfloor-\frac{1}{2}\right)\operatorname{erf}(z) - (-1)^{k+n}\,z\,e^{-z^2}\sum_{j=0}^{k+n-2\left\lfloor \frac{n}{2} \right\rfloor-2} \frac{(-1)^j\,\Gamma\!\left(-j-\frac{1}{2}\right)z^{2\,j}}{\Gamma\!\left(2\left\lfloor \frac{n}{2} \right\rfloor-n-k+\frac{3}{2}\right)}\right) +$$

$$\frac{4^{-\left\lfloor \frac{n}{2} \right\rfloor-1}}{\left(\frac{1}{2}\right)_{\left\lfloor \frac{n}{2} \right\rfloor}}\,L_{n-\left\lfloor \frac{n}{2} \right\rfloor-1}\!\left(-z^2\right)\left((-1)^n\,z^{2\left\lfloor \frac{n}{2} \right\rfloor-n+2}\sum_{j=0}^{n-2\left\lfloor \frac{n}{2} \right\rfloor-2} \frac{(-1)^j\,\Gamma\!\left(-j-\frac{1}{2}\right)z^{2\,j}}{\Gamma\!\left(2\left\lfloor \frac{n}{2} \right\rfloor-n+\frac{3}{2}\right)} + 4\left(2\left\lfloor \frac{n}{2} \right\rfloor-n+1\right)\right) /;\; n \in \mathbb{N}^+$$

General characteristics

Domain and analyticity

$H_\nu(z)$ is an analytical function of ν and z which is defined over $\mathbb{C}^2$. For fixed ν, it is an entire function of z. For fixed z, it is an entire function of ν. For integer ν, $H_\nu(z)$ degenerates to a polynomial in z.

07.01.04.0001.01

$$(\nu * z) \longrightarrow H_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

07.01.04.0002.01

$H_n(-z) = (-1)^n H_n(z) /; n \in \mathbb{N}$

Mirror symmetry

07.01.04.0003.01

$H_{\bar{\nu}}(\bar{z}) = \overline{H_\nu(z)}$

Periodicity

No periodicity

Poles and essential singularities

With respect to *z*

For fixed $\nu /; \nu \notin \mathbb{N}$, the function $H_\nu(z)$ has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

07.01.04.0004.01

$\mathcal{S}ing_z(H_\nu(z)) = \{\{\tilde{\infty}, \infty\}\} /; \nu \notin \mathbb{N}$

For positive integer ν, the function $H_\nu(z)$ is polynomial and has pole of order ν at $z = \tilde{\infty}$.

07.01.04.0005.01

$\mathcal{S}ing_z(H_\nu(z)) = \{\{\tilde{\infty}, \nu\}\} /; \nu \in \mathbb{N}^+$

With respect to *ν*

For fixed z, the function $H_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

07.01.04.0006.01

$\mathcal{S}ing_\nu(H_\nu(z)) = \{\{\tilde{\infty}, \infty\}\}$

Branch points

With respect to *z*

For fixed ν, the function $H_\nu(z)$ does not have branch points.

07.01.04.0007.01

$\mathcal{BP}_z(H_\nu(z)) = \{\}$

With respect to *ν*

For fixed z, the function $H_\nu(z)$ does not have branch points.

07.01.04.0008.01

$\mathcal{BP}_\nu(H_\nu(z)) = \{\}$

Branch cuts

With respect to *z*

For fixed integer ν, the function $H_\nu(z)$ does not have branch cuts.

07.01.04.0009.01

$$\mathcal{BC}_z(H_\nu(z)) = \{\}$$

With respect to ν

For fixed z, the function $H_\nu(z)$ does not have branch cuts.

07.01.04.0010.01

$$\mathcal{BC}_\nu(H_\nu(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

07.01.06.0010.01

$$H_\nu(z) = H_\nu(z_0) + 2\,\nu\, H_{\nu-1}(z_0)\,(z-z_0) + (\nu-1)\,\nu\, H_{\nu-2}(z_0)\,(z-z_0)^2 + \ldots \;/;\; (z \to z_0)$$

07.01.06.0011.01

$$H_\nu(z) = H_\nu(z_0) + 2\,\nu\, H_{\nu-1}(z_0)\,(z-z_0) + (\nu-1)\,\nu\, H_{\nu-2}(z_0)\,(z-z_0)^2 + O\big((z-z_0)^3\big)$$

07.01.06.0012.01

$$H_\nu(z) = \sum_{k=0}^{\infty} \frac{2^k}{k!} \binom{\nu}{k} H_{\nu-k}(z_0)\,(z-z_0)^k$$

07.01.06.0013.01

$$H_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{2^{k+\nu}\,\pi\, z_0^{-k}}{\Gamma\left(\frac{1-\nu}{2}\right)}\, {}_2\tilde{F}_2\left(1, -\frac{\nu}{2}; \frac{1-k}{2}, 1-\frac{k}{2}; z_0^2\right) - \frac{2^{k+\nu}\,\pi\, z_0^{1-k}}{\Gamma\left(-\frac{\nu}{2}\right)}\, {}_2\tilde{F}_2\left(1, \frac{1-\nu}{2}; 1-\frac{k}{2}, \frac{3-k}{2}; z_0^2\right)\right)(z-z_0)^k$$

07.01.06.0014.01

$$H_\nu(z) \propto H_\nu(z_0)\,\left(1 + O(z-z_0)\right)$$

Expansions at $z = 0$

For the function itself

General case

07.01.06.0001.02

$$H_\nu(z) \propto \frac{2^\nu \sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)} \left(1 - \nu\, z^2 - \frac{\nu\,(2-\nu)}{6}\, z^4 + \ldots\right) - \frac{2^{\nu+1}\sqrt{\pi}}{\Gamma\left(-\frac{\nu}{2}\right)}\, z \left(1 + \frac{1-\nu}{3}\, z^2 + \frac{(1-\nu)(3-\nu)}{30}\, z^4 + \ldots\right) /;\; (z \to 0)$$

07.01.06.0015.01

$$H_\nu(z) \propto \frac{2^\nu \sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)} \left(1 - \nu z^2 - \frac{\nu(2-\nu)}{6} z^4 + O(z^6)\right) - \frac{2^{\nu+1}\sqrt{\pi}}{\Gamma\left(-\frac{\nu}{2}\right)} z \left(1 + \frac{1-\nu}{3} z^2 + \frac{(1-\nu)(3-\nu)}{30} z^4 + O(z^6)\right)$$

07.01.06.0002.01

$$H_\nu(z) = 2^\nu \sqrt{\pi} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} \sum_{k=0}^{\infty} \frac{\left(-\frac{\nu}{2}\right)_k z^{2k}}{\left(\frac{1}{2}\right)_k k!} - \frac{2z}{\Gamma\left(-\frac{\nu}{2}\right)} \sum_{k=0}^{\infty} \frac{\left(\frac{1-\nu}{2}\right)_k z^{2k}}{\left(\frac{3}{2}\right)_k k!} \right)$$

07.01.06.0003.01

$$H_\nu(z) = 2^\nu \sqrt{\pi} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - \frac{2z}{\Gamma\left(-\frac{\nu}{2}\right)} {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) \right)$$

07.01.06.0004.02

$$H_\nu(z) \propto \frac{2^\nu \sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)} \left(1 + O(z^2)\right) - \frac{2^{\nu+1}\sqrt{\pi}\, z}{\Gamma\left(-\frac{\nu}{2}\right)} \left(1 + O(z^2)\right)$$

07.01.06.0016.01

$$H_\nu(z) = F_\infty(z, \nu) /;$$

$$\left(\left(F_m(z, \nu) = 2^\nu \sqrt{\pi} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} \sum_{k=0}^{m} \frac{\left(-\frac{\nu}{2}\right)_k z^{2k}}{\left(\frac{1}{2}\right)_k k!} - \frac{2z}{\Gamma\left(-\frac{\nu}{2}\right)} \sum_{k=0}^{m} \frac{\left(\frac{1-\nu}{2}\right)_k z^{2k}}{\left(\frac{3}{2}\right)_k k!} \right) = H_\nu(z) - \frac{2^\nu \sqrt{\pi}\, z^{2m+2}}{(m+1)!} \left(\frac{\left(-\frac{\nu}{2}\right)_{m+1}}{\Gamma\left(\frac{1-\nu}{2}\right) \left(\frac{1}{2}\right)_{m+1}} {}_2F_2\left(1, m - \frac{\nu}{2} + 1; m + \frac{3}{2}, m+2; z^2\right) - \frac{2\left(\frac{1-\nu}{2}\right)_{m+1} z}{\Gamma\left(-\frac{\nu}{2}\right)\left(\frac{3}{2}\right)_{m+1}} {}_2F_2\left(1, m - \frac{\nu}{2} + \frac{3}{2}; m+2, m + \frac{5}{2}; z^2\right) \right)\right) \bigwedge m \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

07.01.06.0005.01

$$H_n(z) = n! \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k (2z)^{n-2k}}{k!\,(n-2k)!} /; n \in \mathbb{N}$$

07.01.06.0017.01

$$H_n(z) \propto \frac{(-1)^{\left\lfloor \frac{n}{2} \right\rfloor} n!\, 2^{n-2\left\lfloor \frac{n}{2} \right\rfloor} z^{n-2\left\lfloor \frac{n}{2} \right\rfloor}}{\left(n - 2\left\lfloor \frac{n}{2} \right\rfloor\right)! \left\lfloor \frac{n}{2} \right\rfloor!} \left(1 + O(z^2)\right) /; n \in \mathbb{N}$$

Expansions at $z = \infty$

For the function itself

Special cases

07.01.06.0018.01

$$H_n(z) \propto 2^n z^n \left(1 - \frac{(n-1)\,n}{4\,z^2} + \frac{(n-3)\,(n-2)\,(n-1)\,n}{32\,z^4} - \ldots\right) /;\ (|z| \to \infty) \land n \in \mathbb{N}$$

07.01.06.0019.01

$$H_n(z) \propto 2^n z^n \left(1 - \frac{(n-1)\,n}{4\,z^2} + \frac{(n-3)\,(n-2)\,(n-1)\,n}{32\,z^4} - O\left(\frac{1}{z^6}\right)\right) /;\ (|z| \to \infty) \land n \in \mathbb{N}$$

07.01.06.0020.01

$$H_n(z) = 2^n z^n \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k \left(-\frac{n}{2}\right)_k \left(\frac{1-n}{2}\right)_k z^{-2k}}{k!} /;\ n \in \mathbb{N}$$

07.01.06.0021.01

$$H_n(z) = n! \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k (2\,z)^{n-2k}}{k!\,(n-2\,k)!} /;\ n \in \mathbb{N}$$

07.01.06.0022.01

$$H_n(z) = 2^n z^n\, {}_2F_0\left(-\frac{n}{2}, \frac{1-n}{2};;-\frac{1}{z^2}\right) /;\ n \in \mathbb{N}$$

07.01.06.0023.01

$$H_n(z) \propto 2^n z^n \left(1 + O\left(\frac{1}{z^2}\right)\right) /;\ n \in \mathbb{N}$$

Asymptotic series expansions

07.01.06.0024.01

$$H_\nu(z) \propto -\frac{\sin(\nu\,\pi)\left(z^2\right)^{-\frac{\nu}{2}-1}}{2\sqrt{\pi}} \left(2^\nu \sqrt{\pi}\sqrt{-z^2}\left(-z^4\right)^{\nu/2}\left(\sqrt{-z^2}\csc\left(\frac{\nu\,\pi}{2}\right) + z\sec\left(\frac{\nu\,\pi}{2}\right)\right)\left(1 + \frac{(1-\nu)\,\nu}{4\,z^2} + \frac{(-3+\nu)\,(-2+\nu)\,(-1+\nu)\,\nu}{32\,z^4} + \ldots\right) + e^{z^2}\left(\sqrt{z^2} - z\right)\Gamma(\nu+1)\left(1 + \frac{(1+\nu)\,(2+\nu)}{4\,z^2} + \frac{(1+\nu)\,(2+\nu)\,(3+\nu)\,(4+\nu)}{32\,z^4} + \ldots\right)\right) /;\ (|z| \to \infty)$$

07.01.06.0025.01

$$H_\nu(z) \propto \begin{cases} 2^\nu z^\nu \left(1 - \frac{(\nu-1)\,\nu}{4\,z^2} + \frac{(\nu-3)\,(\nu-2)\,(\nu-1)\,\nu}{32\,z^4} + O\left(\frac{1}{z^6}\right)\right) & -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} \\ 2^\nu z^\nu \left(1 - \frac{(\nu-1)\,\nu}{4\,z^2} + \frac{(\nu-3)\,(\nu-2)\,(\nu-1)\,\nu}{32\,z^4} + O\left(\frac{1}{z^6}\right)\right) - \frac{e^{z^2+i\pi\nu}\sqrt{\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\left(1 + \frac{(\nu+1)\,(\nu+2)}{4\,z^2} + \frac{(\nu+1)\,(\nu+3)\,(\nu+4)\,(\nu+2)}{32\,z^4} + O\left(\frac{1}{z^6}\right)\right) & \arg(z) > \frac{\pi}{2} \\ 2^\nu z^\nu \left(1 - \frac{(\nu-1)\,\nu}{4\,z^2} + \frac{(\nu-3)\,(\nu-2)\,(\nu-1)\,\nu}{32\,z^4} + O\left(\frac{1}{z^6}\right)\right) - \frac{e^{z^2-i\pi\nu}\sqrt{\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\left(1 + \frac{(\nu+1)\,(\nu+2)}{4\,z^2} + \frac{(\nu+1)\,(\nu+3)\,(\nu+4)\,(\nu+2)}{32\,z^4} + O\left(\frac{1}{z^6}\right)\right) & \text{True} \end{cases} /;\ (|z| \to \infty)$$

07.01.06.0026.01

$$H_\nu(z) \propto -\frac{\sin(\nu\pi)\,(z^2)^{-\frac{\nu}{2}-1}}{2\sqrt{\pi}}\left(2^\nu\sqrt{\pi}\sqrt{-z^2}\,(-z^4)^{\nu/2}\left(\sqrt{-z^2}\csc\left(\frac{\nu\pi}{2}\right)+z\sec\left(\frac{\nu\pi}{2}\right)\right)\left(\sum_{k=0}^{n}\frac{(-1)^k\left(-\frac{\nu}{2}\right)_k\left(\frac{1-\nu}{2}\right)_k z^{-2k}}{k!}+O\left(z^{-2n-2}\right)\right)+ e^{z^2}\left(\sqrt{z^2}-z\right)\Gamma(\nu+1)\left(\sum_{k=0}^{n}\frac{\left(\frac{\nu+1}{2}\right)_k\left(\frac{\nu+2}{2}\right)_k z^{-2k}}{k!}+O\left(z^{-2n-2}\right)\right)\right) /;\ (|z|\to\infty)$$

07.01.06.0027.01

$$H_\nu(z) \propto \begin{cases} 2^\nu z^\nu\left(\sum_{k=0}^{n}\frac{(-1)^k\left(-\frac{\nu}{2}\right)_k\left(\frac{1-\nu}{2}\right)_k z^{-2k}}{k!}+O\left(z^{-2n-2}\right)\right) & -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} \\ 2^\nu z^\nu\left(\sum_{k=0}^{n}\frac{(-1)^k\left(-\frac{\nu}{2}\right)_k\left(\frac{1-\nu}{2}\right)_k z^{-2k}}{k!}+O\left(z^{-2n-2}\right)\right)-\frac{e^{z^2+i\pi\nu}\sqrt{\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\left(\sum_{k=0}^{n}\frac{\left(\frac{\nu+1}{2}\right)_k\left(\frac{\nu+2}{2}\right)_k z^{-2k}}{k!}+O\left(z^{-2n-2}\right)\right) & \arg(z)>\frac{\pi}{2} \\ 2^\nu z^\nu\left(\sum_{k=0}^{n}\frac{(-1)^k\left(-\frac{\nu}{2}\right)_k\left(\frac{1-\nu}{2}\right)_k z^{-2k}}{k!}+O\left(z^{-2n-2}\right)\right)-\frac{e^{z^2-i\pi\nu}\sqrt{\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\left(\sum_{k=0}^{n}\frac{\left(\frac{\nu+1}{2}\right)_k\left(\frac{\nu+2}{2}\right)_k z^{-2k}}{k!}+O\left(z^{-2n-2}\right)\right) & \text{True} \end{cases} /;\ (|z|\to\infty)$$

07.01.06.0006.01

$$H_\nu(z) \propto -\frac{\sin(\nu\pi)\,(z^2)^{-\frac{\nu}{2}-1}}{2\sqrt{\pi}}\left(e^{z^2}\left(\sqrt{z^2}-z\right)\Gamma(\nu+1)\,{}_2F_0\left(\frac{\nu+1}{2},\frac{\nu+2}{2};;\frac{1}{z^2}\right)+ 2^\nu\sqrt{\pi}\sqrt{-z^2}\left(\sqrt{-z^2}\csc\left(\frac{\nu\pi}{2}\right)+z\sec\left(\frac{\nu\pi}{2}\right)\right)(-z^4)^{\nu/2}\,{}_2F_0\left(-\frac{\nu}{2},\frac{1-\nu}{2};;-\frac{1}{z^2}\right)\right) /;\ (|z|\to\infty)$$

07.01.06.0028.01

$$H_\nu(z) \propto \begin{cases} 2^\nu z^\nu\,{}_2F_0\left(\frac{1-\nu}{2},-\frac{\nu}{2};;-\frac{1}{z^2}\right) & -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} \\ 2^\nu z^\nu\,{}_2F_0\left(\frac{1-\nu}{2},-\frac{\nu}{2};;-\frac{1}{z^2}\right)-\frac{e^{z^2+i\pi\nu}\sqrt{\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\,{}_2F_0\left(\frac{\nu+1}{2},\frac{\nu}{2}+1;;\frac{1}{z^2}\right) & \arg(z)>\frac{\pi}{2} \\ 2^\nu z^\nu\,{}_2F_0\left(\frac{1-\nu}{2},-\frac{\nu}{2};;-\frac{1}{z^2}\right)-\frac{e^{z^2-i\pi\nu}\sqrt{\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\,{}_2F_0\left(\frac{\nu+1}{2},\frac{\nu}{2}+1;;\frac{1}{z^2}\right) & \text{True} \end{cases} /;\ (|z|\to\infty)$$

07.01.06.0007.02

$$H_\nu(z) \propto (z^2)^{-\frac{\nu}{2}-1}\left(\frac{\sqrt{\pi}\,e^{z^2}\left(\sqrt{z^2}-z\right)}{2\,\Gamma(-\nu)}\left(1+O\left(\frac{1}{z^2}\right)\right)-2^\nu\sqrt{-z^2}\,(-z^4)^{\nu/2}\left(\sqrt{-z^2}\cos\left(\frac{\nu\pi}{2}\right)+z\sin\left(\frac{\nu\pi}{2}\right)\right)\left(1+O\left(\frac{1}{z^2}\right)\right)\right) /;\ (|z|\to\infty)$$

07.01.06.0029.01

$$H_\nu(z) \propto \begin{cases} 2^\nu z^\nu\left(1+O\left(\frac{1}{z^2}\right)\right) & -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}\bigvee\nu\in\mathbb{N} \\ 2^\nu z^\nu\left(1+O\left(\frac{1}{z^2}\right)\right)-\frac{e^{z^2+i\pi\nu}\sqrt{\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\left(1+O\left(\frac{1}{z^2}\right)\right) & \arg(z)>\frac{\pi}{2} \\ 2^\nu z^\nu\left(1+O\left(\frac{1}{z^2}\right)\right)-\frac{e^{z^2-i\pi\nu}\sqrt{\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\left(1+O\left(\frac{1}{z^2}\right)\right) & \text{True} \end{cases} /;\ (|z|\to\infty)$$

07.01.06.0008.01

$$H_\nu(z) \propto 2^\nu z^\nu\,{}_2F_0\left(-\frac{\nu}{2},\frac{1-\nu}{2};;-\frac{1}{z^2}\right) /;\ -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}\bigwedge(|z|\to\infty)$$

07.01.06.0009.01

$$H_\nu(z) \propto 2^\nu z^\nu \left(1 + O\left(\frac{1}{z^2}\right)\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2} \bigwedge (|z| \to \infty)$$

Integral representations

On the real axis

07.01.07.0001.01

$$H_\nu(z) = \frac{2^{\nu+1}}{\sqrt{\pi}} e^{z^2} \int_0^\infty e^{-t^2} t^\nu \cos\left(2 z t - \frac{\pi \nu}{2}\right) dt /; \operatorname{Re}(\nu) > -1$$

07.01.07.0002.01

$$H_n(z) = \frac{e^{3 \pi i n/2} 2^n}{\sqrt{\pi}} \int_{-\infty}^\infty e^{-(t - i z)^2} t^n \, dt /; n \in \mathbb{N}$$

Integral representations of negative integer order

Rodrigues-type formula.

07.01.07.0003.01

$$H_n(z) = (-1)^n e^{z^2} \frac{\partial^n e^{-z^2}}{\partial z^n} /; n \in \mathbb{N}$$

Limit representations

07.01.09.0001.01

$$H_\nu(z) = \lim_{\lambda \to \infty} 2^{\nu/2} \Gamma(\nu + 1) \lambda^{-\frac{\nu}{2}} L_\nu^\lambda\left(\lambda - \sqrt{2 \lambda} \, z\right)$$

07.01.09.0002.01

$$H_\nu(z) = \Gamma(\nu + 1) \lim_{\lambda \to \infty} \lambda^{-\frac{\nu}{2}} C_\nu^\lambda\left(\frac{z}{\sqrt{\lambda}}\right) /; |z| < 1$$

07.01.09.0003.01

$$H_\nu(z) = 2^\nu \Gamma(\nu + 1) \lim_{a \to \infty} a^{-\frac{\nu}{2}} P_\nu^{(a,a)}\left(\frac{z}{\sqrt{a}}\right)$$

Generating functions

07.01.11.0001.01

$$H_n(z) = n! \left([t^n] \, e^{2 z t - t^2}\right) /; n \in \mathbb{N}$$

Differential equations

Ordinary linear differential equations and wronskians

07.01.13.0005.01

$$w''(z) - 2 z w'(z) + 2 \nu w(z) = 0 /; w(z) = c_1 H_\nu(z) + c_2 e^{z^2} H_{-\nu-1}(i z)$$

07.01.13.0006.01

$$W_z\left(H_\nu(z),\ e^{z^2} H_{-\nu-1}(i z)\right) == -i\, e^{z^2-\frac{i \pi \nu}{2}}$$

07.01.13.0007.01

$$w''(z) - 2 z\, w'(z) + 2 \nu\, w(z) == 0 /; w(z) == c_1 H_\nu(z) + c_2 \left(z\, {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) + {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right)\right)$$

07.01.13.0008.01

$$W_z\left(H_\nu(z),\ z\, {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) + {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right)\right) == \frac{2^{\nu+1} e^{z^2} \sqrt{\pi}}{\Gamma\left(-\frac{\nu}{2}\right)} + \frac{2^\nu e^{z^2} \sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)}$$

07.01.13.0001.01

$$w''(z) - 2 z\, w'(z) + 2 \nu\, w(z) == 0 /; w(z) == c_1 H_\nu(z) + c_2\, {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) \bigwedge \neg (\nu \in \mathbb{Z})$$

07.01.13.0002.02

$$W_z\left(H_\nu(z),\ {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right)\right) == \frac{2^{\nu+1} e^{z^2} \sqrt{\pi}}{\Gamma\left(-\frac{\nu}{2}\right)}$$

07.01.13.0003.01

$$w''(z) - 2 z\, w'(z) + 2 \nu\, w(z) == 0 /; w(z) == c_1 H_\nu(z) + c_2\, z\, {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right)$$

07.01.13.0004.02

$$W_z\left(H_\nu(z),\ z\, {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right)\right) == \frac{2^\nu e^{z^2} \sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)}$$

07.01.13.0009.01

$$w''(z) - \left(2 g(z) g'(z) + \frac{g''(z)}{g'(z)}\right) w'(z) + 2 \nu\, g'(z)^2 w(z) = 0 /; w(z) = c_1 H_\nu(g(z)) + c_2\, e^{g(z)^2} H_{-\nu-1}(i\, g(z))$$

07.01.13.0010.01

$$W_z\left(H_\nu(g(z)),\ e^{g(z)^2} H_{-\nu-1}(i\, g(z))\right) == -i\, e^{-\frac{1}{2} i \pi \nu} e^{g(z)^2} g'(z)$$

07.01.13.0011.01

$$w''(z) - \left(2 g(z) g'(z) + \frac{2 h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) + \left(2 \nu\, g'(z)^2 + \frac{2 g(z) h'(z) g'(z)}{h(z)} + \frac{h'(z) g''(z)}{h(z) g'(z)} + \frac{2 h'(z)^2 - h(z) h''(z)}{h(z)^2}\right) w(z) = 0 /;$$
$$w(z) = c_1 h(z) H_\nu(g(z)) + c_2 h(z)\, e^{g(z)^2} H_{-\nu-1}(i\, g(z))$$

07.01.13.0012.01

$$W_z\left(h(z) H_\nu(g(z)),\ h(z)\, e^{g(z)^2} H_{-\nu-1}(i\, g(z))\right) == -i\, e^{-\frac{1}{2} i \pi \nu} e^{g(z)^2} h(z)^2 g'(z)$$

07.01.13.0013.01

$$z^2 w''(z) - \left(2 a^2 r z^{2r} + r + 2 s - 1\right) z\, w'(z) + \left(2 a^2 r (s + r \nu) z^{2r} + s (r + s)\right) w(z) == 0 /;$$
$$w(z) = c_1 z^s H_\nu(a z^r) + c_2 z^s e^{a^2 z^{2r}} H_{-\nu-1}(i\, a z^r)$$

07.01.13.0014.01

$$W_z\left(z^s H_\nu(a z^r),\ z^s e^{a^2 z^{2r}} H_{-\nu-1}(i\, a z^r)\right) == -i\, a\, e^{a^2 z^{2r} - \frac{i \pi \nu}{2}} r z^{r+2s-1}$$

07.01.13.0015.01

$$w''(z)+\left(-\left(2a^2 r^{2z}+1\right)\log(r)-2\log(s)\right)w'(z)+\left(2a^2\log(r)\left(\nu\log(r)+\log(s)\right)r^{2z}+\log(s)\left(\log(r)+\log(s)\right)\right)w(z)=0\;/;$$
$$w(z)=c_1 s^z H_\nu(a r^z)+c_2 s^z e^{a^2 r^{2z}} H_{-\nu-1}(i a r^z)$$

07.01.13.0016.01

$$W_z\left(s^z H_\nu(a r^z),\, s^z e^{a^2 r^{2z}} H_{-\nu-1}(i a r^z)\right)=-i a e^{a^2 r^{2z}-\frac{i\pi\nu}{2}} r^z s^{2z}\log(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.01.16.0007.01

$$H_\nu\left(\sqrt{z^2}\right)=H_\nu(z)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.01.16.0008.01

$$H_\nu\left(\sqrt{z^2}\right)=H_\nu(z)+2^\nu\left(\sqrt{z^2}-z\right)\Gamma\left(\frac{\nu+1}{2}\right)\sin\left(\frac{\pi\nu}{2}\right)L_{\frac{\nu-1}{2}}^{\frac{1}{2}}\left(z^2\right)$$

07.01.16.0009.01

$$H_\nu\left(\sqrt{z^2}\right)=H_\nu(z)+2^{\nu+1}\sqrt{\pi}\,\frac{z-\sqrt{z^2}}{\Gamma\left(-\frac{\nu}{2}\right)}\,{}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};z^2\right)$$

07.01.16.0010.01

$$H_{2n}\left(\sqrt{z^2}\right)=H_{2n}(z)\;/;\;n\in\mathbb{N}$$

07.01.16.0011.01

$$H_\nu(-z)=H_\nu(z)-2^{\nu+1}z\,\Gamma\left(\frac{\nu+1}{2}\right)\sin\left(\frac{\pi\nu}{2}\right)L_{\frac{\nu-1}{2}}^{\frac{1}{2}}\left(z^2\right)$$

07.01.16.0012.01

$$H_\nu(-z)=H_\nu(z)+\frac{2^{\nu+2}\sqrt{\pi}\,z}{\Gamma\left(-\frac{\nu}{2}\right)}\,{}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};z^2\right)$$

07.01.16.0001.01

$$H_n(-z)=(-1)^n H_n(z)\;/;\;n\in\mathbb{N}$$

Addition formulas

07.01.16.0013.01

$$H_\nu(z_1-z_2)=e^{z_2(z_2-2z_1)}\sum_{k=0}^{\infty}\frac{z_2^k}{k!}H_{k+\nu}(z_1)$$

07.01.16.0014.01

$$H_\nu(z_1-z_2)=\sum_{k=0}^{\infty}\frac{2^k z_2^k(-\nu)_k}{k!}H_{\nu-k}(z_1)$$

07.01.16.0002.01

$$H_n(z_1+z_2) == 2^{-\frac{n}{2}} \sum_{k=0}^{n} \binom{n}{k} H_k\left(z_1 \sqrt{2}\right) H_{n-k}\left(z_2 \sqrt{2}\right) /; n \in \mathbb{N}$$

07.01.16.0015.01

$$H_\nu(\cos(\alpha)\, z_1 + \sin(\alpha)\, z_2) = \cos^\nu(\alpha) \sum_{k=0}^{\infty} \frac{(-\nu)_k\, (-\tan(\alpha))^k}{k!} H_k(z_2)\, H_{\nu-k}(z_1)$$

07.01.16.0003.01

$$H_n(\cos(\alpha)\, z_1 + \sin(\alpha)\, z_2) == n! \sum_{k=0}^{n} \frac{\cos^k(\alpha) \sin^{n-k}(\alpha)\, H_k(z_1)\, H_{n-k}(z_2)}{k!\, (n-k)!} /; n \in \mathbb{N}$$

Multiple arguments

07.01.16.0004.01

$$H_n(z_1\, z_2) == z_1^n \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{n!}{k!\, (n-2k)!} \left(1 - \frac{1}{z_1^2}\right)^k H_{n-2k}(z_2) /; n \in \mathbb{N}$$

Products, sums, and powers of the direct function

Products of the direct function

07.01.16.0005.01

$$H_n(z)\, H_m(z) == n!\, m! \sum_{k=0}^{\min(n,m)} \frac{2^k\, H_{-2k+m+n}(z)}{k!\, (n-k)!\, (m-k)!} /; n \in \mathbb{N} \land m \in \mathbb{N}$$

07.01.16.0006.01

$$H_n(z)\, H_m(z) == \sum_{k=0}^{\min(n,m)} \binom{m}{k} \binom{n}{k} H_{-2k+m+n}(z)\, 2^k\, k! /; n \in \mathbb{N} \land m \in \mathbb{N}$$

Identities

Recurrence identities

Consecutive neighbors

07.01.17.0001.01

$$H_\nu(z) == \frac{z}{\nu+1} H_{\nu+1}(z) - \frac{1}{2(\nu+1)} H_{\nu+2}(z)$$

07.01.17.0002.01

$$H_\nu(z) == 2\, z\, H_{\nu-1}(z) - 2\, (\nu-1)\, H_{\nu-2}(z)$$

Distant neighbors

07.01.17.0006.01

$$H_\nu(z) == C_n(\nu, z)\, H_{n+\nu}(z) - \frac{1}{2(n+\nu)} C_{n-1}(\nu, z)\, H_{n+\nu+1}(z) /;$$

$$C_0(\nu, z) == 1 \bigwedge C_1(\nu, z) == \frac{z}{\nu+1} \bigwedge C_n(\nu, z) == \frac{z}{n+\nu} C_{n-1}(\nu, z) - \frac{1}{2(n+\nu-1)} C_{n-2}(\nu, z) \bigwedge n \in \mathbb{N}^+$$

07.01.17.0007.01

$$H_\nu(z) = \mathcal{C}_n(\nu, z)\, H_{\nu-n}(z) - 2(\nu - n)\, \mathcal{C}_{n-1}(\nu, z)\, H_{\nu-n-1}(z) \;/;\; \mathcal{C}_0(\nu, z) = 1 \land \mathcal{C}_1(\nu, z) = 2z \land \mathcal{C}_n(\nu, z) = 2z\, \mathcal{C}_{n-1}(\nu, z) - 2(\nu - n + 1)\, \mathcal{C}_{n-2}(\nu, z) \land n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

Recurrence relations

07.01.17.0003.01

$$2\nu H_{\nu-1}(z) + H_{\nu+1}(z) = 2z H_\nu(z)$$

07.01.17.0004.01

$$H_\nu(z) = \frac{2\nu H_{\nu-1}(z) + H_{\nu+1}(z)}{2z}$$

Normalized recurrence relation

07.01.17.0005.01

$$z\, p(\nu, z) = p(\nu + 1, z) + \frac{1}{2} \nu\, p(\nu - 1, z) \;/;\; p(\nu, z) = 2^{-\nu} H_\nu(z)$$

Complex characteristics

Real part

07.01.19.0001.01

$$\operatorname{Re}(H_n(x + i y)) = \sum_{j=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^j\, 2^{2j}\, (-n)_{2j}\, y^{2j}}{(2j)!} H_{n-2j}(x) \;/;\; x \in \mathbb{R} \land y \in \mathbb{R} \land n \in \mathbb{N}$$

Imaginary part

07.01.19.0002.01

$$\operatorname{Im}(H_n(x + i y)) = \sum_{j=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(-1)^{j-1}\, 2^{2j+1}\, (-n)_{2j+1}\, y^{2j+1}}{(2j+1)!} H_{n-2j-1}(x) \;/;\; x \in \mathbb{R} \land y \in \mathbb{R} \land n \in \mathbb{N}$$

Differentiation

Low-order differentiation

With respect to ν

07.01.20.0001.01

$$\frac{\partial H_\nu(z)}{\partial \nu} = \log(2)\, H_\nu(z) + \frac{1}{4\,\Gamma(-\nu)} \left(\Gamma\left(-\frac{\nu}{2}\right) \psi\left(\frac{1-\nu}{2}\right) {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - 2\, z\, \Gamma\left(\frac{1-\nu}{2}\right) \psi\left(-\frac{\nu}{2}\right) {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right)\right) - \frac{z^2}{6\,\Gamma(-\nu)} \left(3\, \Gamma\left(-\frac{\nu}{2}\right) F_{2\times 0\times 1}^{1\times 1\times 2}\left(\begin{matrix} 1-\frac{\nu}{2}; 1; 1, -\frac{\nu}{2}; \\ 2, \frac{3}{2};; 1-\frac{\nu}{2}; \end{matrix}\; z^2, z^2\right) - 2\, z\, \Gamma\left(\frac{1-\nu}{2}\right) F_{2\times 0\times 1}^{1\times 1\times 2}\left(\begin{matrix} \frac{3-\nu}{2}; 1; 1, \frac{1-\nu}{2}; \\ 2, \frac{5}{2};; \frac{3-\nu}{2}; \end{matrix}\; z^2, z^2\right)\right)$$

07.01.20.0002.01

$$\frac{\partial H_\nu(z)}{\partial \nu} = \log(2)\, H_\nu(z) + \frac{1}{4\,\Gamma(-\nu)} \left(\Gamma\left(-\frac{\nu}{2}\right) \psi\left(\frac{1-\nu}{2}\right) {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - 2\, z\, \Gamma\left(\frac{1-\nu}{2}\right) \psi\left(-\frac{\nu}{2}\right) {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right)\right) + \frac{2^{\nu-1} \sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)} \left(\psi\left(-\frac{\nu}{2}\right) {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - \sum_{k=0}^{\infty} \frac{\left(-\frac{\nu}{2}\right)_k \psi\left(k-\frac{\nu}{2}\right) z^{2k}}{k!\, \left(\frac{1}{2}\right)_k}\right) - \frac{2^{\nu} \sqrt{\pi}\, z}{\Gamma\left(-\frac{\nu}{2}\right)} \left(\psi\left(\frac{1-\nu}{2}\right) {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) - \sum_{k=0}^{\infty} \frac{\left(\frac{1-\nu}{2}\right)_k \psi\left(k+\frac{1-\nu}{2}\right) z^{2k}}{k!\, \left(\frac{3}{2}\right)_k}\right)$$

07.01.20.0003.01

$$\frac{\partial^2 H_\nu(z)}{\partial \nu^2} = \log^2(2)\; H_\nu(z) + 2^{\nu-2} \sqrt{\pi} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} \left(\psi\left(\frac{1-\nu}{2}\right)^2 + \log(16)\, \psi\left(\frac{1-\nu}{2}\right) - \psi^{(1)}\left(\frac{1-\nu}{2}\right)\right) {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - \frac{2\, z}{\Gamma\left(-\frac{\nu}{2}\right)} \left(\psi\left(-\frac{\nu}{2}\right)^2 + \log(16)\, \psi\left(-\frac{\nu}{2}\right) - \psi^{(1)}\left(-\frac{\nu}{2}\right)\right) {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) + \frac{2\left(\log(4) + \psi\left(\frac{1-\nu}{2}\right)\right)}{\Gamma\left(\frac{1-\nu}{2}\right)} \left(\psi\left(-\frac{\nu}{2}\right) {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - \sum_{k=0}^{\infty} \frac{\left(-\frac{\nu}{2}\right)_k \psi\left(k-\frac{\nu}{2}\right) z^{2k}}{k!\, \left(\frac{1}{2}\right)_k}\right) + \frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} \left(\left(\psi\left(-\frac{\nu}{2}\right)^2 - \psi^{(1)}\left(-\frac{\nu}{2}\right)\right) {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) + \sum_{k=0}^{\infty} \frac{\left(-\frac{\nu}{2}\right)_k z^{2k}}{k!\, \left(\frac{1}{2}\right)_k} \left(\psi\left(k-\frac{\nu}{2}\right)^2 - 2\, \psi\left(-\frac{\nu}{2}\right) \psi\left(k-\frac{\nu}{2}\right) + \psi^{(1)}\left(k-\frac{\nu}{2}\right)\right)\right) - \frac{4\, z\left(\log(4) + \psi\left(-\frac{\nu}{2}\right)\right)}{\Gamma\left(-\frac{\nu}{2}\right)} \left(\psi\left(\frac{1-\nu}{2}\right) {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) - \sum_{k=0}^{\infty} \frac{\left(\frac{1-\nu}{2}\right)_k \psi\left(k+\frac{1-\nu}{2}\right) z^{2k}}{k!\, \left(\frac{3}{2}\right)_k}\right) - \frac{2\, z}{\Gamma\left(-\frac{\nu}{2}\right)} \left(\left(\psi\left(\frac{1-\nu}{2}\right)^2 - \psi^{(1)}\left(\frac{1-\nu}{2}\right)\right) {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) + \sum_{k=0}^{\infty} \frac{\left(\frac{1-\nu}{2}\right)_k z^{2k}}{k!\, \left(\frac{3}{2}\right)_k} \left(\psi\left(k+\frac{1-\nu}{2}\right)^2 - 2\, \psi\left(\frac{1-\nu}{2}\right) \psi\left(k+\frac{1-\nu}{2}\right) + \psi^{(1)}\left(k+\frac{1-\nu}{2}\right)\right)\right)\right)$$

07.01.20.0011.01

$$H_{2n}^{(1,0)}(z) = 2^{2n-1}\, n!\left(\frac{1}{2}\right)_n \left(\frac{1}{(2n)!}\left(-2\,{}_2F_2\left(1,1;\frac{3}{2},2;z^2\right)z^2+\pi\,\text{erfi}(z)+\psi\left(\frac{1}{2}-n\right)-\gamma\right)H_{2n}(z)-(-1)^n\sum_{k=0}^{n}\frac{\psi\left(\frac{1}{2}-k\right)(-4)^k z^{2k}}{(n-k)!\,(2k)!}+\frac{e^{z^2}(-1)^n\sqrt{\pi}}{\left(\frac{1}{2}\right)_n}\sum_{k=0}^{n-1}\frac{1}{k+1}\left(z\,L_k^{-k-\frac{1}{2}}(-z^2)-\frac{\sqrt{\pi}\,z^{2k+2}}{k!}L_{-\frac{1}{2}}^{k+\frac{1}{2}}(-z^2)\right)L_{-k+n-1}^{k+\frac{1}{2}}(z^2)\right)\,/;\, n\in\mathbb{N}$$

Brychkov Yu.A. (2006)

07.01.20.0012.01

$$H_0^{(1,0)}(z) = \frac{1}{2}\left(-2\,{}_2F_2\left(1,1;\frac{3}{2},2;z^2\right)z^2+\pi\,\text{erfi}(z)-\gamma\right)$$

Brychkov Yu.A. (2006)

07.01.20.0013.01

$$H_2^{(1,0)}(z) = 2\left(1-2z^2\right){}_2F_2\left(1,1;\frac{3}{2},2;z^2\right)z^2-2\,e^{z^2}\sqrt{\pi}\,\text{erfc}(z)\,z+\pi\left(2z^2-1\right)\text{erfi}(z)-\left(2z^2-1\right)\gamma-2$$

Brychkov Yu.A. (2006)

07.01.20.0014.01

$$\frac{\partial H_\nu(0)}{\partial \nu} = \begin{cases} i^{\nu+1}\,2^{\nu-1}\sqrt{\pi}\,\frac{\nu-1}{2}! & \frac{\nu-1}{2}\in\mathbb{Z}\bigwedge \nu>0 \\ \frac{2^{\nu-1}\sqrt{\pi}\left(\log(4)+\psi\left(\frac{1-\nu}{2}\right)\right)}{\Gamma\left(\frac{1-\nu}{2}\right)} & \text{True}\end{cases}$$

With respect to z

07.01.20.0004.01

$$\frac{\partial H_\nu(z)}{\partial z} = 2\,\nu\,H_{\nu-1}(z)$$

07.01.20.0005.01

$$\frac{\partial^2 H_\nu(z)}{\partial z^2} = 4(\nu-1)\,\nu\,H_{\nu-2}(z)$$

Backward shift operator:

07.01.20.0006.01

$$\frac{\partial H_\nu(z)}{\partial z} = 2\,z\,H_\nu(z)-H_{\nu+1}(z)$$

07.01.20.0007.01

$$\frac{\partial\left(e^{-z^2}H_\nu(z)\right)}{\partial z} = -e^{-z^2}H_{\nu+1}(z)$$

Symbolic differentiation

With respect to ν

07.01.20.0015.01

$$\frac{\partial^m H_\nu(z)}{\partial \nu^m} = \sqrt{\pi}\, 2^\nu \sum_{k=0}^{m} \binom{m}{k} \log^{m-k}(2) \left(\sum_{j=0}^{\infty} \frac{\partial^k \frac{\left(-\frac{\nu}{2}\right)_j}{\Gamma\left(\frac{1-\nu}{2}\right)}}{\partial \nu^k} \frac{(2z)^{2j}}{(2j)!} - 2z \sum_{j=0}^{\infty} \frac{\partial^k \frac{\left(\frac{1-\nu}{2}\right)_j}{\Gamma\left(-\frac{\nu}{2}\right)}}{\partial \nu^k} \frac{(2z)^{2j}}{(2j+1)!} \right) /; m \in \mathbb{N}$$

With respect to *z*

07.01.20.0008.02

$$\frac{\partial^m H_\nu(z)}{\partial z^m} == \frac{2^m \nu!}{(\nu - m)!} H_{\nu-m}(z) /; m \in \mathbb{N}$$

07.01.20.0009.02

$$\frac{\partial^m H_\nu(z)}{\partial z^m} == \frac{2^{m+\nu} \pi z^{-m}}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_2\tilde{F}_2\left(1, -\frac{\nu}{2}; \frac{1-m}{2}, 1-\frac{m}{2}; z^2\right) - \frac{2^{m+\nu} \pi z^{1-m}}{\Gamma\left(-\frac{\nu}{2}\right)} {}_2\tilde{F}_2\left(1, \frac{1-\nu}{2}; 1-\frac{m}{2}, \frac{3-m}{2}; z^2\right) /; m \in \mathbb{N}$$

Fractional integro-differentiation

With respect to *z*

07.01.20.0010.01

$$\frac{\partial^\alpha H_\nu(z)}{\partial z^\alpha} == \frac{2^{\nu+\alpha} \pi z^{-\alpha}}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_2\tilde{F}_2\left(1, -\frac{\nu}{2}; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; z^2\right) - \frac{2^{\nu+\alpha} \pi z^{1-\alpha}}{\Gamma\left(-\frac{\nu}{2}\right)} {}_2\tilde{F}_2\left(1, \frac{1-\nu}{2}; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; z^2\right)$$

Integration

Indefinite integration

Involving only one direct function

07.01.21.0001.01

$$\int H_\nu(z)\, dz == \frac{1}{2(\nu+1)} H_{\nu+1}(z)$$

Involving one direct function and elementary functions

Involving power function

07.01.21.0002.01

$$\int z^{\alpha-1} H_\nu(z)\, dz == 2^{\nu-1} \pi z^\alpha \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} \Gamma\left(\frac{\alpha}{2}\right) {}_2\tilde{F}_2\left(-\frac{\nu}{2}, \frac{\alpha}{2}; \frac{1}{2}, \frac{\alpha}{2}+1; z^2\right) - \frac{z}{\Gamma\left(-\frac{\nu}{2}\right)} \Gamma\left(\frac{\alpha+1}{2}\right) {}_2\tilde{F}_2\left(\frac{1-\nu}{2}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; z^2\right) \right)$$

07.01.21.0003.01

$$\int z\, H_\nu(z)\, dz == \frac{1}{3} 2^{\nu-1} \sqrt{\pi} \left(\frac{3\left({}_1F_1\left(-\frac{\nu}{2}-1; -\frac{1}{2}; z^2\right) - 1\right)}{(\nu+2)\,\Gamma\left(\frac{1-\nu}{2}\right)} - \frac{4 z^3\, {}_1F_1\left(\frac{1}{2}-\frac{\nu}{2}; \frac{5}{2}; z^2\right)}{\Gamma\left(-\frac{\nu}{2}\right)} \right)$$

07.01.21.0004.01

$$\int z^m H_\nu(a z)\, dz = 2^{\nu-1} \pi z^{m+1} \left(\frac{\Gamma\left(\frac{m+1}{2}\right) {}_2\tilde{F}_2\left(-\frac{\nu}{2}, \frac{m+1}{2}; \frac{1}{2}, \frac{m+3}{2}; a^2 z^2\right)}{\Gamma\left(\frac{1-\nu}{2}\right)} - \frac{a z\, \Gamma\left(\frac{m}{2}+1\right) {}_2\tilde{F}_2\left(\frac{1}{2}-\frac{\nu}{2}, \frac{m}{2}+1; \frac{3}{2}, \frac{m}{2}+2; a^2 z^2\right)}{\Gamma\left(-\frac{\nu}{2}\right)} \right)$$

07.01.21.0005.01

$$\int z^{-\nu-3} H_\nu(z)\, dz = \frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)\Gamma\left(-\frac{\nu}{2}\right)} \left(2^{\nu-1} \sqrt{\pi}\, z^{-\nu-2} \left(\Gamma\left(-\frac{\nu}{2}-1\right) {}_1F_1\left(-\frac{\nu}{2}-1; \frac{1}{2}; z^2\right) - 2 z\, \Gamma\left(-\frac{\nu}{2}-\frac{1}{2}\right) {}_1F_1\left(-\frac{\nu}{2}-\frac{1}{2}; \frac{3}{2}; z^2\right)\right)\right)$$

Involving exponential function

07.01.21.0006.01

$$\int e^{-z^2} H_\nu(z)\, dz = 2^\nu \sqrt{\pi} \left(\frac{z\, {}_1F_1\left(\frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}; -z^2\right)}{\Gamma\left(\frac{1-\nu}{2}\right)} + \frac{{}_1F_1\left(\frac{\nu}{2}; \frac{1}{2}; -z^2\right) - 1}{\nu\, \Gamma\left(-\frac{\nu}{2}\right)} \right)$$

Involving exponential function and a power function

07.01.21.0007.01

$$\int z^{\alpha-1} e^{-p z} H_\nu(z)\, dz = 2^\nu \sqrt{\pi} \left(\frac{2 z^{\alpha+1}}{\Gamma\left(-\frac{\nu}{2}\right)(p z)^{\alpha+1}} \sum_{k=0}^{\infty} \frac{\left(\frac{1-\nu}{2}\right)_k \Gamma(2k+\alpha+1, p z)}{\left(\frac{3}{2}\right)_k k!\, p^{2k}} - \frac{z^\alpha}{\Gamma\left(\frac{1-\nu}{2}\right)(p z)^\alpha} \sum_{k=0}^{\infty} \frac{\left(-\frac{\nu}{2}\right)_k \Gamma(2k+\alpha, p z)}{\left(\frac{1}{2}\right)_k k!\, p^{2k}} \right)$$

07.01.21.0008.01

$$\int z^{\alpha-1} e^{-p z^2} H_\nu(z)\, dz = 2^\nu \sqrt{\pi} \left(\frac{z^{\alpha+1}}{\Gamma\left(-\frac{\nu}{2}\right)\left(p z^2\right)^{\frac{\alpha+1}{2}}} \sum_{k=0}^{\infty} \frac{\left(\frac{1-\nu}{2}\right)_k \Gamma\left(\frac{\alpha+1}{2}+k, p z^2\right)}{\left(\frac{3}{2}\right)_k k!\, p^k} - \frac{z^\alpha}{2\, \Gamma\left(\frac{1-\nu}{2}\right)\left(p z^2\right)^{\alpha/2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{\nu}{2}\right)_k \Gamma\left(\frac{\alpha}{2}+k, p z^2\right)}{\left(\frac{1}{2}\right)_k k!\, p^k} \right)$$

07.01.21.0009.01

$$\int z^{\alpha-1} e^{-a^2 z^2} H_\nu(a z)\, dz =$$

$$2^{\nu-1} \pi z^\alpha \left(\frac{\Gamma\left(\frac{\alpha}{2}\right)}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_2\tilde{F}_2\left(\frac{\nu+1}{2}, \frac{\alpha}{2}; \frac{1}{2}, \frac{\alpha+2}{2}; -a^2 z^2\right) - \frac{a z\, \Gamma\left(\frac{\alpha+1}{2}\right)}{\Gamma\left(-\frac{\nu}{2}\right)} {}_2\tilde{F}_2\left(\frac{\nu+2}{2}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; -a^2 z^2\right) \right)$$

07.01.21.0010.01

$$\int z^{\nu-2} e^{-z^2} H_\nu(z)\, dz = \frac{z^{\nu-1} \sin(\pi \nu)}{\pi} \left(z\, \Gamma\left(\frac{1}{2}-\frac{\nu}{2}\right) \Gamma(\nu)\, {}_1F_1\left(\frac{\nu}{2}; \frac{3}{2}; -z^2\right) + \Gamma\left(1-\frac{\nu}{2}\right) \Gamma(\nu-1)\, {}_1F_1\left(\frac{\nu-1}{2}; \frac{1}{2}; -z^2\right) \right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

07.01.21.0011.01

$$\int z H_\nu(z)^2\, dz = \frac{1}{8} \left(2 H_\nu(z)^2 + \frac{H_{\nu+1}(z)^2}{\nu+1} \right)$$

Involving powers of the direct function, power and exponential functions

07.01.21.0012.01

$$\int e^{-2 z^2} z\, H_\nu(z)^2 \, dz = -\frac{1}{4} e^{-2 z^2} \left(2 \nu H_{\nu-1}(z)^2 + H_\nu(z)^2\right)$$

Involving direct function and Gamma-, Beta-, Erf-type functions

Involving probability integral-type functions

Involving erf

07.01.21.0013.01

$$\int \operatorname{erf}(a z) H_\nu(a z) \, dz = \frac{1}{2 a(\nu+1)} \left(\frac{2}{\sqrt{\pi}} e^{-a^2 z^2} H_\nu(a z) + \operatorname{erf}(a z) H_{\nu+1}(a z)\right)$$

Involving erfi

07.01.21.0014.01

$$\int e^{-z^2} \operatorname{erfi}(z) H_\nu(z) \, dz = \frac{H_\nu(z)}{\nu \sqrt{\pi}} - e^{-z^2} \operatorname{erfi}(z) H_{\nu-1}(z)$$

Definite integration

Involving the direct function

07.01.21.0015.01

$$\int_0^\infty t^{\alpha-1} e^{-a t^2} H_\nu(t) \, dt = 2^{\nu-1} a^{-\frac{\alpha}{2}} \sqrt{\pi} \left(\frac{\Gamma\left(\frac{\alpha}{2}\right)}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_2F_1\left(\frac{\alpha}{2}, -\frac{\nu}{2}; \frac{1}{2}; \frac{1}{a}\right) + \frac{\Gamma\left(\frac{\alpha+1}{2}\right)}{\sqrt{a}\, \alpha\, \Gamma\left(1-\frac{\nu}{2}\right)} \left((a-1)\, {}_2F_1\left(\frac{\alpha+1}{2}, \frac{1-\nu}{2}; -\frac{1}{2}; \frac{1}{a}\right) - (a-\alpha+\nu-2)\, {}_2F_1\left(\frac{\alpha+1}{2}, \frac{1-\nu}{2}; \frac{1}{2}; \frac{1}{a}\right)\right)\right) /; \operatorname{Re}(a) > 0 \land \operatorname{Re}(\alpha) > 0$$

07.01.21.0016.01

$$\int_{-\infty}^\infty e^{-(z-t)^2} H_n(t) \, dt = 2^n \sqrt{\pi}\, z^n /; n \in \mathbb{N}$$

Orthogonality:

07.01.21.0017.01

$$\int_{-\infty}^\infty e^{-t^2} H_m(t) H_n(t) \, dt = 2^n n! \sqrt{\pi}\, \delta_{n,m} /; n \in \mathbb{N} \land m \in \mathbb{N}$$

07.01.21.0018.01

$$\int_{-\infty}^\infty e^{-t^2} H_l(t) H_m(t) H_n(t) \, dt = \frac{2^{\frac{1}{2}(l+m+n)} l!\, m!\, n! \sqrt{\pi}}{\left(\frac{1}{2}(l+m-n)\right)! \left(\frac{1}{2}(-l+m+n)\right)! \left(\frac{1}{2}(l-m+n)\right)!} /;$$

$$l \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge n \in \mathbb{N} \bigwedge \frac{1}{2}(l+m+n) \in \mathbb{Z} \bigwedge l+m \geq n \bigwedge m+n \geq l \bigwedge l+n \geq m$$

07.01.21.0019.01

$$\int_{-\infty}^{\infty} e^{-t^2} H_l(t)\, H_m(t)\, H_n(t)\, dt = 0 \;/;\; l \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge n \in \mathbb{N} \bigwedge \neg\left(\frac{1}{2}(l+m+n) \in \mathbb{Z} \bigwedge l+m \geq n \bigwedge m+n \geq l \bigwedge l+n \geq m\right)$$

Summation

Finite summation

07.01.23.0001.01

$$\sum_{k=0}^{n} \frac{H_k(x)\, H_k(y)}{2^k\, k!} = \frac{H_{n+1}(x)\, H_n(y) - H_n(x)\, H_{n+1}(y)}{2^{n+1}\, n!\, (x-y)} \;/;\; n \in \mathbb{N}$$

07.01.23.0002.01

$$\sum_{k=0}^{n} \binom{n}{k} i^k\, H_{n-k}(x)\, H_k(y) = 2^n\, (x + i\, y)^n \;/;\; n \in \mathbb{N}$$

07.01.23.0003.01

$$\sum_{k=0}^{n} \binom{n}{k} \cos\left(\frac{k\,\pi}{2}\right) H_{n-k}(x)\, H_k(y) = 2^n\, \mathrm{Re}\left((x + i\, y)^n\right) \;/;\; n \in \mathbb{N}$$

07.01.23.0004.01

$$\sum_{k=0}^{n} \binom{n}{k} \sin\left(\frac{k\,\pi}{2}\right) H_{n-k}(x)\, H_k(y) = 2^n\, \mathrm{Im}\left((x + i\, y)^n\right) \;/;\; n \in \mathbb{N}$$

Infinite summation

07.01.23.0005.01

$$\sum_{n=0}^{\infty} \frac{H_n(z)\, w^n}{n!} = e^{2\,z\,w - w^2}$$

07.01.23.0006.01

$$\sum_{n=0}^{\infty} \frac{H_{2n}(z)\, w^n}{(2\,n)!} = e^{-w} \cos\left(2\,z\,\sqrt{-w}\right)$$

07.01.23.0007.01

$$\sum_{n=0}^{\infty} \frac{H_{2n+1}(z)\, w^n}{(2\,n+1)!} = \frac{e^{-w} \sin\left(2\,z\,\sqrt{-w}\right)}{\sqrt{-w}}$$

07.01.23.0008.01

$$\sum_{n=0}^{\infty} \frac{(c)_n\, H_{2n}(z)\, w^{2n}}{(2\,n)!} = \left(w^2+1\right)^{-c} {}_1F_1\left(c;\, \frac{1}{2};\, \frac{z^2\, w^2}{w^2+1}\right)$$

07.01.23.0009.01

$$\sum_{n=0}^{\infty} \frac{\left(c+\frac{1}{2}\right)_n H_{2n+1}(z)\, w^{2n}}{(2\,n+1)!} = 2\left(w^2+1\right)^{-c-\frac{1}{2}} z\, {}_1F_1\left(c+\frac{1}{2};\, \frac{3}{2};\, \frac{w^2\, z^2}{w^2+1}\right)$$

07.01.23.0010.01

$$\sum_{n=0}^{\infty} \frac{H_n(z)\, w^n}{\left\lfloor \frac{n}{2} \right\rfloor!} = \left(4\,w^2 + 2\,z\,w + 1\right)\left(4\,w^2+1\right)^{-\frac{3}{2}} \exp\left(\frac{4\,w^2\, z^2}{4\,w^2+1}\right)$$

07.01.23.0011.01

$$\sum_{n=0}^{\infty} \frac{H_n(x+n\,y)\,t^n}{n!} = \frac{e^{2\,x\,\alpha-\alpha^2}}{1-2\,y\,\alpha} \;/;\; \alpha = -\frac{W(-2\,t\,y)}{2\,y} \bigwedge -1 < t < 1 \bigwedge -1 < y < 1$$

07.01.23.0012.01

$$\sum_{n=0}^{\infty} \frac{H_{k+j\,n}(z)\,w^{k+j\,n}}{(k+j\,n)!} = \frac{1}{j} \sum_{l=1}^{j} \exp\left(2\,e^{\frac{2\,i\,\pi\,l}{j}}\,z\,w + e^{\frac{4\,i\,\pi\,l}{j}}\left(-w^2\right) - \frac{2\,i\,\pi\,k\,l}{j}\right) /;\; k \in \mathbb{N} \bigwedge j \in \mathbb{N}$$

07.01.23.0013.01

$$\sum_{n=0}^{\infty} \frac{H_n(z)\,H_n(z_1)\,w^n}{n!} = \frac{1}{\sqrt{1-4\,w^2}} \exp\left(\frac{2\,w\left(2\,w\left(z^2+z_1^2\right)-2\,z\,z_1\right)}{4\,w^2-1}\right) /;\; |w| < \frac{1}{2}$$

07.01.23.0014.01

$$\sum_{n=0}^{\infty} \frac{H_n(x)\,H_n(y)}{2^n\,n!} = \sqrt{\pi}\; e^{\frac{1}{2}\left(x^2+y^2\right)}\,\delta(x-y) \;/;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R}$$

07.01.23.0015.01

$$\sum_{n=0}^{\infty} \binom{m}{n} H_n(z)\,H_{m-n}(z_1) = 2^{m/2}\,H_m\left(\frac{z+z_1}{\sqrt{2}}\right) /;\; m \in \mathbb{N}$$

Operations

Limit operation

07.01.25.0001.01

$$\lim_{n\to\infty} \frac{(-1)^n}{4^n\,n!} H_{2\,n+1}\left(\frac{z}{2\,\sqrt{n}}\right) = \frac{2\sin(z)}{\sqrt{\pi}} \;/;\; n \in \mathbb{N}$$

07.01.25.0002.01

$$\lim_{n\to\infty} \frac{(-1)^n\,\sqrt{n}}{4^n\,n!} H_{2\,n}\left(\frac{z}{2\,\sqrt{n}}\right) = \frac{\cos(z)}{\sqrt{\pi}} \;/;\; n \in \mathbb{N}$$

Orthogonality, completeness, and Fourier expansions

The set of functions $H_n(x)$, $n = 0, 1, \ldots$, forms a complete, orthogonal (with weight $\frac{1}{2^n\,n!\,\sqrt{\pi}}\,e^{-x^2}$) system on the interval $(-\infty, \infty)$.

07.01.25.0003.01

$$\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{\sqrt{\pi}\;2^n\,n!}}\,e^{-\frac{x^2}{2}}\,H_n(x)\right)\left(\frac{1}{\sqrt{\sqrt{\pi}\;2^n\,n!}}\,e^{-\frac{y^2}{2}}\,H_n(y)\right) = \delta(x-y)$$

07.01.25.0004.01

$$\int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{\sqrt{\pi}\;2^m\,m!}}\,e^{-\frac{t^2}{2}}\,H_m(t)\right)\left(\frac{1}{\sqrt{\sqrt{\pi}\;2^n\,n!}}\,e^{-\frac{t^2}{2}}\,H_n(t)\right) dt = \delta_{n,m}$$

Any sufficiently smooth function $f(x)$ can be expanded in the system $\{H_n(x)\}_{n=0,1,\ldots}$ as a generalized Fourier series, with its sum converging to $f(x)$ almost everywhere.

07.01.25.0005.01

$$f(x) = \sum_{n=0}^{\infty} c_n \psi_n(x) \;/;\; c_n = \int_{-\infty}^{\infty} \psi_n(t) f(t)\, dt \bigwedge \psi_n(x) = \frac{1}{\sqrt{\sqrt{\pi}\, 2^n\, n!}} e^{-\frac{x^2}{2}} H_n(x) \;/;\; x \in \mathbb{R}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_1\tilde{F}_1$

07.01.26.0001.01

$$H_\nu(z) = 2^\nu \pi \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_1\tilde{F}_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - \frac{z}{\Gamma\left(-\frac{\nu}{2}\right)} {}_1\tilde{F}_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) \right)$$

Involving ${}_1F_1$

07.01.26.0002.01

$$H_\nu(z) = 2^\nu \sqrt{\pi} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - \frac{2z}{\Gamma\left(-\frac{\nu}{2}\right)} {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) \right)$$

Involving ${}_pF_q$

07.01.26.0003.01

$$H_n(z) = (2z)^n \, {}_2F_0\left(-\frac{n}{2}, \frac{1-n}{2};; -\frac{1}{z^2}\right) /;\; n \in \mathbb{N}$$

Involving hypergeometric U

07.01.26.0004.01

$$H_n(z) = 2^n U\left(-\frac{n}{2}, \frac{1}{2}, z^2\right) /;\; n \in \mathbb{N} \land \operatorname{Re}(z) > 0$$

07.01.26.0005.01

$$H_n(z) = 2^n z\, U\left(\frac{1-n}{2}, \frac{3}{2}, z^2\right) /;\; n \in \mathbb{N} \land \operatorname{Re}(z) > 0$$

07.01.26.0006.01

$$H_\nu(z) = \frac{2^\nu}{\sqrt{z^2}\, \Gamma\left(\frac{1-\nu}{2}\right)} \left(\sqrt{\pi} \left(\sqrt{z^2} - z\right) {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) + z\, \Gamma\left(\frac{1-\nu}{2}\right) U\left(-\frac{\nu}{2}, \frac{1}{2}, z^2\right) \right)$$

07.01.26.0007.01

$$H_\nu(z) = \frac{2^\nu}{z\, \Gamma\left(\frac{1-\nu}{2}\right)} \left(\Gamma\left(\frac{1-\nu}{2}\right) U\left(\frac{1-\nu}{2}, \frac{3}{2}, z^2\right) z^2 + \sqrt{\pi} \left(z - \sqrt{z^2}\right) {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) \right)$$

07.01.26.0008.01

$$H_\nu(z) = \frac{2^\nu}{\Gamma\left(-\frac{\nu}{2}\right)} \left(2\sqrt{\pi} \left(\sqrt{z^2} - z\right) {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right) + \Gamma\left(-\frac{\nu}{2}\right) U\left(-\frac{\nu}{2}, \frac{1}{2}, z^2\right) \right)$$

07.01.26.0009.01

$$\mathbf{H}_\nu(z) = \frac{2^\nu}{\Gamma\left(-\frac{\nu}{2}\right)}\left(2\sqrt{\pi}\left(\sqrt{z^2}-z\right){}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};z^2\right)+\sqrt{z^2}\,\Gamma\left(-\frac{\nu}{2}\right)U\left(\frac{1-\nu}{2},\frac{3}{2},z^2\right)\right)$$

Through Meijer G

Classical cases for the direct function itself

07.01.26.0019.01

$$\mathbf{H}_\nu\left(\sqrt{z}\right) = \frac{1}{2\sqrt{\pi}\,\Gamma(-\nu)}\,G_{1,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{\nu}{2}+1\\ 0,\frac{1}{2}\end{matrix}\right)\,/;\,\nu\notin\mathbb{Z}$$

07.01.26.0021.01

$$\mathbf{H}_\nu\left(-\sqrt{z}\right) = \frac{\sqrt{2}\,\pi^{3/2}}{\Gamma(-\nu)}\,G_{3,4}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{8},\frac{5}{8}\\ 0,\frac{1}{2},\frac{1}{8},\frac{5}{8}\end{matrix}\right)$$

07.01.26.0022.01

$$\mathbf{H}_\nu(z) = \frac{1}{2\sqrt{\pi}\,\Gamma(-\nu)}\,G_{1,2}^{2,1}\left(z^2\,\middle|\,\begin{matrix}\frac{\nu}{2}+1\\ 0,\frac{1}{2}\end{matrix}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.01.26.0023.01

$$\mathbf{H}_\nu\left(-\sqrt{z}\right)+\mathbf{H}_\nu\left(\sqrt{z}\right) = \frac{\sqrt{2}\,\pi^{3/2}}{\Gamma(-\nu)}\,G_{2,3}^{1,1}\left(z\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{4}\\ 0,\frac{1}{2},\frac{1}{4}\end{matrix}\right)$$

07.01.26.0024.01

$$\mathbf{H}_\nu\left(\sqrt{z}\right)-\mathbf{H}_\nu\left(-\sqrt{z}\right) = \frac{\sqrt{2}\,\pi^{3/2}}{\Gamma(-\nu)}\,G_{2,3}^{1,1}\left(z\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{4}\\ \frac{1}{2},0,\frac{1}{4}\end{matrix}\right)$$

Classical cases involving exp

07.01.26.0010.01

$$e^{-z^2}\,\mathbf{H}_\nu(z) = 2^\nu\,G_{1,2}^{2,0}\left(z^2\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{matrix}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.01.26.0011.01

$$e^{-z}\,\mathbf{H}_\nu\left(\sqrt{z}\right) = 2^\nu\,G_{1,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{matrix}\right)$$

07.01.26.0025.01

$$e^{-z}\,\mathbf{H}_\nu\left(-\sqrt{z}\right) = 2^\nu\,G_{2,3}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{\nu+1}{2}\\ 0,\frac{1}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

07.01.26.0026.01

$$e^{-z}\left(\mathbf{H}_\nu\left(-\sqrt{z}\right)+\mathbf{H}_\nu\left(\sqrt{z}\right)\right) = 2^{\nu+1}\cos\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{matrix}\right)$$

07.01.26.0027.01

$$e^{-z}\left(H_\nu\left(\sqrt{z}\right)-H_\nu\left(-\sqrt{z}\right)\right)=2^{\nu+1}\sin\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z\left|\begin{array}{c}\frac{1-\nu}{2}\\ \frac{1}{2},0\end{array}\right.\right)$$

Classical cases involving exp and cosh

07.01.26.0028.01

$$e^{-\frac{z^2}{2}}\cosh\left(\frac{z^2}{2}\right)H_\nu(z)=2^{\nu-1}G_{1,2}^{2,0}\left(z^2\left|\begin{array}{c}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{array}\right.\right)+\frac{1}{4\sqrt{\pi}\,\Gamma(-\nu)}G_{1,2}^{2,1}\left(z^2\left|\begin{array}{c}\frac{\nu}{2}+1\\ 0,\frac{1}{2}\end{array}\right.\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.01.26.0029.01

$$e^{-\frac{z^2}{2}}\cosh\left(\frac{z^2}{2}\right)H_\nu(-z)=2^{\nu-1}G_{2,3}^{2,1}\left(z^2\left|\begin{array}{c}\frac{1-\nu}{2},\frac{\nu+1}{2}\\ 0,\frac{1}{2},\frac{\nu+1}{2}\end{array}\right.\right)+\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{3,4}^{2,1}\left(z^2\left|\begin{array}{c}\frac{\nu}{2}+1,\frac{1}{8},\frac{5}{8}\\ 0,\frac{1}{2},\frac{1}{8},\frac{5}{8}\end{array}\right.\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.01.26.0030.01

$$e^{-\frac{z^2}{2}}\cosh\left(\frac{z^2}{2}\right)(H_\nu(-z)+H_\nu(z))=2^{\nu}\cos\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z^2\left|\begin{array}{c}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{array}\right.\right)+\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{2,3}^{1,1}\left(z^2\left|\begin{array}{c}\frac{\nu}{2}+1,\frac{1}{4}\\ 0,\frac{1}{2},\frac{1}{4}\end{array}\right.\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.01.26.0031.01

$$e^{-\frac{z^2}{2}}\cosh\left(\frac{z^2}{2}\right)(H_\nu(z)-H_\nu(-z))=\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{2,3}^{1,1}\left(z^2\left|\begin{array}{c}\frac{\nu}{2}+1,\frac{1}{4}\\ \frac{1}{2},0,\frac{1}{4}\end{array}\right.\right)+2^{\nu}G_{1,2}^{1,1}\left(z^2\left|\begin{array}{c}\frac{1-\nu}{2}\\ \frac{1}{2},0\end{array}\right.\right)\sin\left(\frac{\pi\nu}{2}\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.01.26.0032.01

$$e^{-\frac{z}{2}}\cosh\left(\frac{z}{2}\right)H_\nu\left(\sqrt{z}\right)=2^{\nu-1}G_{1,2}^{2,0}\left(z\left|\begin{array}{c}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{array}\right.\right)+\frac{1}{4\sqrt{\pi}\,\Gamma(-\nu)}G_{1,2}^{2,1}\left(z\left|\begin{array}{c}\frac{\nu}{2}+1\\ 0,\frac{1}{2}\end{array}\right.\right)$$

07.01.26.0033.01

$$e^{-\frac{z}{2}}\cosh\left(\frac{z}{2}\right)H_\nu\left(-\sqrt{z}\right)=2^{\nu-1}G_{2,3}^{2,1}\left(z\left|\begin{array}{c}\frac{1-\nu}{2},\frac{\nu+1}{2}\\ 0,\frac{1}{2},\frac{\nu+1}{2}\end{array}\right.\right)+\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{3,4}^{2,1}\left(z\left|\begin{array}{c}\frac{\nu}{2}+1,\frac{1}{8},\frac{5}{8}\\ 0,\frac{1}{2},\frac{1}{8},\frac{5}{8}\end{array}\right.\right)$$

07.01.26.0034.01

$$e^{-\frac{z}{2}}\cosh\left(\frac{z}{2}\right)\left(H_\nu\left(-\sqrt{z}\right)+H_\nu\left(\sqrt{z}\right)\right)=2^{\nu}\cos\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z\left|\begin{array}{c}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{array}\right.\right)+\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{2,3}^{1,1}\left(z\left|\begin{array}{c}\frac{\nu}{2}+1,\frac{1}{4}\\ 0,\frac{1}{2},\frac{1}{4}\end{array}\right.\right)$$

07.01.26.0035.01

$$e^{-\frac{z}{2}}\cosh\left(\frac{z}{2}\right)\left(H_\nu\left(\sqrt{z}\right)-H_\nu\left(-\sqrt{z}\right)\right)=\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{2,3}^{1,1}\left(z\left|\begin{array}{c}\frac{\nu}{2}+1,\frac{1}{4}\\ \frac{1}{2},0,\frac{1}{4}\end{array}\right.\right)+2^{\nu}G_{1,2}^{1,1}\left(z\left|\begin{array}{c}\frac{1-\nu}{2}\\ \frac{1}{2},0\end{array}\right.\right)\sin\left(\frac{\pi\nu}{2}\right)$$

Classical cases involving exp and sinh

07.01.26.0036.01

$$e^{-\frac{z^2}{2}}\sinh\left(\frac{z^2}{2}\right)H_\nu(z)=\frac{1}{4\sqrt{\pi}\,\Gamma(-\nu)}G_{1,2}^{2,1}\left(z^2\left|\begin{array}{c}\frac{\nu}{2}+1\\ 0,\frac{1}{2}\end{array}\right.\right)-2^{\nu-1}G_{1,2}^{2,0}\left(z^2\left|\begin{array}{c}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{array}\right.\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.01.26.0037.01

$$e^{-\frac{z^2}{2}} \sinh\left(\frac{z^2}{2}\right) H_\nu(-z) = \frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)} G_{3,4}^{2,1}\left(z^2 \left| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{array} \right.\right) - 2^{\nu-1} G_{2,3}^{2,1}\left(z^2 \left| \begin{array}{c} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{array} \right.\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.01.26.0038.01

$$e^{-\frac{z^2}{2}} \sinh\left(\frac{z^2}{2}\right) (H_\nu(-z) + H_\nu(z)) = \frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)} G_{2,3}^{1,1}\left(z^2 \left| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{array} \right.\right) - 2^{\nu} \cos\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(z^2 \left| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array} \right.\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.01.26.0039.01

$$e^{-\frac{z^2}{2}} \sinh\left(\frac{z^2}{2}\right) (H_\nu(z) - H_\nu(-z)) = \frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)} G_{2,3}^{1,1}\left(z^2 \left| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{array} \right.\right) - 2^{\nu} \sin\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(z^2 \left| \begin{array}{c} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{array} \right.\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.01.26.0040.01

$$e^{-\frac{z}{2}} \sinh\left(\frac{z}{2}\right) H_\nu\left(\sqrt{z}\right) = \frac{1}{4\sqrt{\pi}\,\Gamma(-\nu)} G_{1,2}^{2,1}\left(z \left| \begin{array}{c} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{array} \right.\right) - 2^{\nu-1} G_{1,2}^{2,0}\left(z \left| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array} \right.\right)$$

07.01.26.0041.01

$$e^{-\frac{z}{2}} \sinh\left(\frac{z}{2}\right) H_\nu\left(-\sqrt{z}\right) = \frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)} G_{3,4}^{2,1}\left(z \left| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{array} \right.\right) - 2^{\nu-1} G_{2,3}^{2,1}\left(z \left| \begin{array}{c} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{array} \right.\right)$$

07.01.26.0042.01

$$e^{-\frac{z}{2}} \sinh\left(\frac{z}{2}\right) \left(H_\nu\left(-\sqrt{z}\right) + H_\nu\left(\sqrt{z}\right)\right) = \frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)} G_{2,3}^{1,1}\left(z \left| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{array} \right.\right) - 2^{\nu} \cos\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(z \left| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array} \right.\right)$$

07.01.26.0043.01

$$e^{-\frac{z}{2}} \sinh\left(\frac{z}{2}\right) \left(H_\nu\left(\sqrt{z}\right) - H_\nu\left(-\sqrt{z}\right)\right) = \frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)} G_{2,3}^{1,1}\left(z \left| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{array} \right.\right) - 2^{\nu} \sin\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(z \left| \begin{array}{c} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{array} \right.\right)$$

Classical cases for products of *H*

07.01.26.0044.01

$$H_\nu\left(e^{-\frac{i\pi}{4}} \sqrt[4]{z}\right) H_\nu\left(e^{\frac{i\pi}{4}} \sqrt[4]{z}\right) = \frac{2^{\nu-\frac{3}{2}}}{\pi\,\Gamma(-\nu)} G_{2,4}^{4,1}\left(\frac{z}{4} \left| \begin{array}{c} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array} \right.\right)$$

Classical cases involving Exp and products of *H*

07.01.26.0045.01

$$e^{-\sqrt{z}}\, H_{-\nu-1}\left(\sqrt[4]{z}\right) H_\nu\left(\sqrt[4]{z}\right) = \frac{1}{2^{3/2}\sqrt{\pi}} G_{2,4}^{4,0}\left(\frac{z}{4} \left| \begin{array}{c} \frac{1-\nu}{2}, \frac{\nu}{2}+1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array} \right.\right)$$

Classical cases involving Exp and parabolic cylinder *D*

07.01.26.0046.01

$$e^{-\frac{z^2}{2}} D_\nu\left(i\sqrt{2}\, z\right) H_\nu(z) = \frac{2^{\frac{\nu-3}{2}}}{\pi\,\Gamma(-\nu)} G_{2,4}^{4,1}\left(-\frac{z^4}{4} \left| \begin{array}{c} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array} \right.\right) /; -\frac{\pi}{2} \leq \arg(z) \leq 0$$

07.01.26.0047.01

$$e^{-\frac{z^2}{2}} D_{-\nu-1}\left(\sqrt{2}\, z\right) H_\nu(z) = \frac{2^{\frac{\nu}{2}-1}}{\sqrt{\pi}} G_{2,4}^{4,0}\left(\frac{z^4}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu}{2}+1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

Generalized cases for the direct function itself

07.01.26.0020.01

$$H_\nu(z) = \frac{1}{2\sqrt{\pi}\, \Gamma(-\nu)} G_{1,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right) /; \nu \notin \mathbb{Z}$$

07.01.26.0012.01

$$H_\nu(z) = \frac{1}{2\sqrt{\pi}\, z\, \Gamma(-\nu)} G_{1,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu+3}{2} \\ 1, \frac{1}{2} \end{matrix}\right) /; \nu \notin \mathbb{Z}$$

07.01.26.0048.01

$$H_\nu(z) = \frac{\sqrt{2}\, \pi^{3/2}}{\Gamma(-\nu)} G_{3,4}^{2,1}\left(-z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{matrix}\right)$$

07.01.26.0013.01

$$H_n(z) = \sqrt{\pi}\, n!\, (-1)^{\left\lfloor \frac{n}{2} \right\rfloor} G_{1,2}^{1,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{n}{2}+1 \\ \frac{n}{2} - \left\lfloor \frac{n}{2} \right\rfloor, -\frac{n}{2} + \left\lfloor \frac{n}{2} \right\rfloor + \frac{1}{2} \end{matrix}\right) /; n \in \mathbb{N}$$

07.01.26.0014.01

$$H_n(z) = \frac{1}{2\sqrt{\pi}\, z} \lim_{m\to n} \frac{1}{\Gamma(-m)} G_{1,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{m+3}{2} \\ 1, \frac{1}{2} \end{matrix}\right) /; n \in \mathbb{Z}$$

07.01.26.0049.01

$$H_\nu(z) + H_\nu(-z) = \frac{\sqrt{2}\, \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{matrix}\right)$$

07.01.26.0050.01

$$H_\nu(z) - H_\nu(-z) = \frac{\sqrt{2}\, \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{matrix}\right)$$

Generalized cases involving exp

07.01.26.0015.01

$$e^{-z^2} H_\nu(z) = 2^\nu G_{1,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.01.26.0051.01

$$e^{-z^2} H_\nu(-z) = 2^\nu G_{2,3}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

07.01.26.0052.01

$$e^{-z^2} (H_\nu(z) + H_\nu(-z)) = 2^{\nu+1} \cos\left(\frac{\pi \nu}{2}\right) G_{1,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.01.26.0053.01

$$e^{-z^2}\left(H_\nu(z)-H_\nu(-z)\right)=2^{\nu+1}\sin\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ \frac{1}{2},0\end{matrix}\right)$$

Generalized cases involving exp and cosh

07.01.26.0054.01

$$e^{-\frac{z^2}{2}}\cosh\left(\frac{z^2}{2}\right)H_\nu(z)=2^{\nu-1}G_{1,2}^{2,0}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{matrix}\right)+\frac{1}{4\sqrt{\pi}\,\Gamma(-\nu)}G_{1,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1\\ 0,\frac{1}{2}\end{matrix}\right)$$

07.01.26.0055.01

$$e^{-\frac{z^2}{2}}\cosh\left(\frac{z^2}{2}\right)H_\nu(-z)=2^{\nu-1}G_{2,3}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{\nu+1}{2}\\ 0,\frac{1}{2},\frac{\nu+1}{2}\end{matrix}\right)+\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{3,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{8},\frac{5}{8}\\ 0,\frac{1}{2},\frac{1}{8},\frac{5}{8}\end{matrix}\right)$$

07.01.26.0056.01

$$e^{-\frac{z^2}{2}}\cosh\left(\frac{z^2}{2}\right)\left(H_\nu(z)+H_\nu(-z)\right)=2^{\nu}\cos\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{matrix}\right)+\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{2,3}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{4}\\ 0,\frac{1}{2},\frac{1}{4}\end{matrix}\right)$$

07.01.26.0057.01

$$e^{-\frac{z^2}{2}}\cosh\left(\frac{z^2}{2}\right)\left(H_\nu(z)-H_\nu(-z)\right)=\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{2,3}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{4}\\ \frac{1}{2},0,\frac{1}{4}\end{matrix}\right)+2^{\nu}\sin\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ \frac{1}{2},0\end{matrix}\right)$$

Generalized cases involving exp and sinh

07.01.26.0058.01

$$e^{-\frac{z^2}{2}}\sinh\left(\frac{z^2}{2}\right)H_\nu(z)=\frac{1}{4\sqrt{\pi}\,\Gamma(-\nu)}G_{1,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1\\ 0,\frac{1}{2}\end{matrix}\right)-2^{\nu-1}G_{1,2}^{2,0}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{matrix}\right)$$

07.01.26.0059.01

$$e^{-\frac{z^2}{2}}\sinh\left(\frac{z^2}{2}\right)H_\nu(-z)=\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{3,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{8},\frac{5}{8}\\ 0,\frac{1}{2},\frac{1}{8},\frac{5}{8}\end{matrix}\right)-2^{\nu-1}G_{2,3}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{\nu+1}{2}\\ 0,\frac{1}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

07.01.26.0060.01

$$e^{-\frac{z^2}{2}}\sinh\left(\frac{z^2}{2}\right)\left(H_\nu(-z)+H_\nu(z)\right)=\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{2,3}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{4}\\ 0,\frac{1}{2},\frac{1}{4}\end{matrix}\right)-2^{\nu}\cos\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2}\end{matrix}\right)$$

07.01.26.0061.01

$$e^{-\frac{z^2}{2}}\sinh\left(\frac{z^2}{2}\right)\left(H_\nu(z)-H_\nu(-z)\right)=\frac{\pi^{3/2}}{\sqrt{2}\,\Gamma(-\nu)}G_{2,3}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1}{4}\\ \frac{1}{2},0,\frac{1}{4}\end{matrix}\right)-2^{\nu}\sin\left(\frac{\pi\nu}{2}\right)G_{1,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ \frac{1}{2},0\end{matrix}\right)$$

Generalized cases for products of *H*

07.01.26.0062.01

$$H_\nu\left(e^{-\frac{i\pi}{4}}z\right)H_\nu\left(e^{\frac{i\pi}{4}}z\right)=\frac{2^{\nu-\frac{3}{2}}}{\pi\,\Gamma(-\nu)}G_{2,4}^{4,1}\left(\frac{z}{\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\frac{1-\nu}{2}\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4}\end{matrix}\right)$$

Generalized cases involving Exp and products of *H*

07.01.26.0063.01

$$e^{-z} H_{-\nu-1}\left(\sqrt{z}\right) H_\nu\left(\sqrt{z}\right) = \frac{1}{2\sqrt{2\pi}} G_{2,4}^{4,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu}{2}+1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right)$$

07.01.26.0064.01

$$e^{-z^2} H_{-\nu-1}(z) H_\nu(z) = \frac{1}{2\sqrt{2\pi}} G_{2,4}^{4,0}\left(\frac{z}{\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu}{2}+1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right)$$

Generalized cases involving Exp and parabolic cylinder *D*

07.01.26.0065.01

$$e^{-\frac{z^2}{2}} D_\nu\left(i\sqrt{2}\, z\right) H_\nu(z) = \frac{2^{\frac{\nu-3}{2}}}{\pi\,\Gamma(-\nu)} G_{2,4}^{4,1}\left(\frac{e^{\frac{i\pi}{4}} z}{\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right)$$

07.01.26.0066.01

$$e^{-\frac{z^2}{2}} D_{-\nu-1}\left(\sqrt{2}\, z\right) H_\nu(z) = \frac{2^{\frac{\nu}{2}-1}}{\sqrt{\pi}} G_{2,4}^{4,0}\left(\frac{z}{\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu}{2}+1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right)$$

Through other functions

Involving some hypergeometric-type functions

07.01.26.0067.01

$$H_\nu(z) = 2^\nu \left(\cos\left(\frac{\pi\nu}{2}\right) \Gamma\left(\frac{\nu}{2}+1\right) L_{\frac{\nu}{2}}^{-\frac{1}{2}}\left(z^2\right) + z\,\Gamma\left(\frac{\nu+1}{2}\right) L_{\frac{\nu-1}{2}}^{\frac{1}{2}}\left(z^2\right) \sin\left(\frac{\pi\nu}{2}\right)\right)$$

07.01.26.0068.01

$$H_{2n}(z) = (-1)^n\, n!\, 2^{2n} L_n^{-\frac{1}{2}}\left(z^2\right) /;\, n \in \mathbb{N}$$

07.01.26.0069.01

$$H_{2n+1}(z) = (-1)^n\, 2^{2n+1} z\, n!\, L_n^{\frac{1}{2}}\left(z^2\right) /;\, n \in \mathbb{N}$$

07.01.26.0016.01

$$H_\nu(z) = \lim_{\lambda\to\infty} 2^{\nu/2}\, \Gamma(\nu+1)\, \lambda^{-\frac{\nu}{2}} L_\nu^\lambda\left(\lambda - \sqrt{2\lambda}\, z\right)$$

07.01.26.0017.01

$$H_\nu(z) = \Gamma(\nu+1) \lim_{\lambda\to\infty} \lambda^{-\frac{\nu}{2}} C_\nu^\lambda\left(\frac{z}{\sqrt{\lambda}}\right) /;\, |z| < 1$$

07.01.26.0018.01

$$H_\nu(z) = 2^\nu\, \Gamma(\nu+1) \lim_{a\to\infty} a^{-\frac{\nu}{2}} P_\nu^{(a,a)}\left(\frac{z}{\sqrt{a}}\right)$$

Representations through equivalent functions

With related functions

07.01.27.0001.01

$$H_\nu(z) == 2^\nu \left(\cos\left(\frac{\nu\pi}{2}\right)\Gamma\left(\frac{\nu}{2}+1\right) L_{\frac{\nu}{2}}^{-\frac{1}{2}}(z^2) + z\sin\left(\frac{\nu\pi}{2}\right)\Gamma\left(\frac{\nu+1}{2}\right) L_{\frac{\nu-1}{2}}^{\frac{1}{2}}(z^2)\right)$$

07.01.27.0002.01

$$H_{2n}(z) == (-1)^n\, 2^{2n}\, n!\, L_n^{-\frac{1}{2}}(z^2) \text{ /; } n \in \mathbb{N}$$

07.01.27.0003.01

$$H_{2n+1}(z) == (-1)^n\, 2^{2n+1}\, n!\, z\, L_n^{\frac{1}{2}}(z^2) \text{ /; } n \in \mathbb{N}$$

07.01.27.0004.01

$$H_\nu(z) = 2^{\nu/2}\, e^{\frac{z^2}{2}}\, D_\nu\left(\sqrt{2}\, z\right)$$

Zeros

07.01.30.0001.01

$$\sum_{\substack{k=1\\k\neq j}}^{n} \frac{1}{x_j - x_k} == x_j \text{ /; } H_n(x_k) == 0$$

Theorems

Expansions in generalized Fourier series

$$f(x) == \sum_{k=0}^{\infty} c_k\, \psi_k(x) \text{ /; } \quad c_k == \int_{-\infty}^{\infty} f(t)\, \psi_k(t)\, dt, \ \ \psi_k(x) == \frac{1}{2^{k/2}\sqrt{k!}\,\sqrt[4]{\pi}}\, e^{-x^2/2}\, H_k(x), \ \ k \in \mathbb{N}.$$

Fourier transform eigenfunctions

Hermite polynomials together with their weighting function are eigenfunctions of the Fourier and inverse Fourier transforms:

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i t x}\, e^{-x^2/2}\, H_n(x)\, dx == -i^n\, e^{-t^2/2}\, H_n(t) \text{ /; } n \in \mathbb{N}$$

Zeros of Hermite polynomials

For any given interval (a, b), there exists some $n \in \mathbb{N}$ such that $H_n(x)$ has a zero in this interval.

The number of simple graphs

The number of simple graphs with no cycles and n vertices is $H_n(n+1) - n\, H_{n-1}(n+1)$.

History

– P. S. Laplace (1810)
– Ch. Hermite (1864)
– N. J. Sonine (1880)

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