Hypergeometric0F1Regularized

Notations

Traditional name

Regularized confluent hypergeometric function ${}_0\tilde{F}_1$

Traditional notation

${}_0\tilde{F}_1(;b;z)$

Mathematica StandardForm notation

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Hypergeometric0F1Regularized[b, z]
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Primary definition

07.18.02.0001.01

$${}_0\tilde{F}_1(;b;z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(b+k)\,k!}$$

Specific values

Specialized values

For fixed b

07.18.03.0001.01

$${}_0\tilde{F}_1(;b;0) = \frac{1}{\Gamma(b)}$$

For fixed z and symbolic parameter

07.18.03.0002.01

$${}_0\tilde{F}_1(;b;z) = -\frac{1}{\sqrt{\pi}} \exp\left(\frac{\pi i}{2}\left(\frac{3}{2}-b\right)\right) z^{\frac{1-2b}{4}} \left(\sinh\left(\frac{\pi i}{2}\left(\frac{3}{2}-b\right) - 2\sqrt{z}\right) \sum_{k=0}^{\left\lfloor \frac{1}{4}(2|b-1|-1)\right\rfloor} \frac{\left(2k+|b-1|-\frac{1}{2}\right)!}{2^{4k}\,(2k)!\,\left(|b-1|-2k-\frac{1}{2}\right)!\,z^k} + \frac{1}{\sqrt{z}} \cosh\left(\frac{\pi i}{2}\left(\frac{3}{2}-b\right) - 2\sqrt{z}\right) \sum_{k=0}^{\left\lfloor \frac{1}{4}(2|b-1|-3)\right\rfloor} \frac{\left(2k+|b-1|+\frac{1}{2}\right)!}{2^{4k+2}\,(2k+1)!\,\left(|b-1|-2k-\frac{3}{2}\right)!\,z^k} \right) /;\, b-\frac{1}{2} \in \mathbb{Z}$$

07.18.03.0003.01

$${}_0\tilde{F}_1(;b;z) = \frac{z^{\frac{1}{2}(-b-|b-1|+1)}\,\Gamma\left(-\frac{1}{3}\right)}{2\,3^{5/6}\,\Gamma(1-|b-1|)} \left(\sqrt[6]{3}\,\sqrt[3]{z}\left(\sqrt{3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right) - 3\,\mathrm{sgn}(b-1)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\right) \sum_{k=0}^{|b-1|-\frac{4}{3}} \frac{\left(|b-1|-k-\frac{4}{3}\right)!\,(-z)^k}{k!\left(\frac{4}{3}\right)_k\left(|b-1|-2k-\frac{4}{3}\right)!\,(1-|b-1|)_k} + \left(\sqrt{3}\,\mathrm{sgn}(b-1)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right) - \mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) \sum_{k=0}^{|b-1|-\frac{1}{3}} \frac{\left(|b-1|-k-\frac{1}{3}\right)!\,(-z)^k}{k!\left(|b-1|-2k-\frac{1}{3}\right)!\left(\frac{1}{3}\right)_k(1-|b-1|)_k}\right) /;\ |b-1|+\frac{2}{3}\in\mathbb{Z}$$

07.18.03.0004.01

$${}_0\tilde{F}_1(;b;z) = \frac{z^{\frac{1}{2}(1-b-|b-1|)}\,\Gamma\left(-\frac{2}{3}\right)\mathrm{sgn}(b-1)}{6\,3^{5/6}\,\Gamma(1-|b-1|)} \left(9\,z^{2/3}\left(\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right) + \mathrm{sgn}(b-1)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) \sum_{k=0}^{|b-1|-\frac{5}{3}} \frac{\left(|b-1|-k-\frac{5}{3}\right)!\,(-1)^k z^k}{k!\left(|b-1|-2k-\frac{5}{3}\right)!\left(\frac{5}{3}\right)_k(1-|b-1|)_k} - 2\left(3\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right) + 3^{2/3}\,\mathrm{sgn}(b-1)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) \sum_{k=0}^{|b-1|-\frac{2}{3}} \frac{\left(|b-1|-k-\frac{2}{3}\right)!\,(-1)^k z^k}{k!\left(|b-1|-2k-\frac{2}{3}\right)!\left(\frac{2}{3}\right)_k(1-|b-1|)_k}\right) /;\ |b-1|-\frac{2}{3}\in\mathbb{Z}$$

07.18.03.0005.01

$${}_0\tilde{F}_1\left(;n+\frac{2}{3};z\right) = \frac{\sqrt[6]{3}}{2}\,\frac{\partial^n\left(\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right)}{\partial z^n} /;\ n\in\mathbb{N}$$

07.18.03.0006.01

$${}_0\tilde{F}_1\left(;n+\frac{4}{3};z\right) = \frac{\sqrt[6]{3}}{2}\,\frac{\partial^n\left(\frac{1}{\sqrt[3]{z}}\left(\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)\right)\right)}{\partial z^n} /;\ n\in\mathbb{N}$$

07.18.03.0007.01

$${}_0\tilde{F}_1\left(;\frac{2}{3}-n;z\right) = \frac{1}{2}\,3^{1/6}\,z^{\frac{1}{3}+n}\,\frac{\partial^n\left(\frac{1}{\sqrt[3]{z}}\left(\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right)\right)}{\partial z^n} /;\ n\in\mathbb{N}$$

07.18.03.0008.01

$${}_0\tilde{F}_1\left(;\frac{4}{3}-n;z\right) = \frac{1}{2}\,3^{1/6}\,z^{-\frac{1}{3}+n}\,\frac{\partial^n\left(\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)\right)}{\partial z^n} /;\ n\in\mathbb{N}$$

For fixed *z*

For fixed *z* and $b = \frac{m}{2}$

07.18.03.0028.01

$${}_0\tilde{F}_1\left(;-\frac{11}{2};z\right)=\frac{(4\,z\,(8\,z\,(2\,z+105)+4725)+10\,395)\cosh\left(2\sqrt{z}\right)-42\sqrt{z}\,(16\,z\,(z+15)+495)\sinh\left(2\sqrt{z}\right)}{64\sqrt{\pi}}$$

07.18.03.0029.01

$${}_0\tilde{F}_1\left(;-\frac{11}{2};-z\right)=\frac{(10\,395-4\,z\,(8\,z\,(2\,z-105)+4725))\cos\left(2\sqrt{z}\right)+42\sqrt{z}\,(16\,(z-15)\,z+495)\sin\left(2\sqrt{z}\right)}{64\sqrt{\pi}}$$

07.18.03.0030.01

$${}_0\tilde{F}_1\left(;-\frac{9}{2};z\right)=\frac{2\sqrt{z}\,(4\,z\,(4\,z+105)+945)\sinh\left(2\sqrt{z}\right)-15\,(16\,z\,(z+7)+63)\cosh\left(2\sqrt{z}\right)}{32\sqrt{\pi}}$$

07.18.03.0031.01

$${}_0\tilde{F}_1\left(;-\frac{9}{2};-z\right)=\frac{-15\,(16\,(z-7)\,z+63)\cos\left(2\sqrt{z}\right)-2\sqrt{z}\,(4\,z\,(4\,z-105)+945)\sin\left(2\sqrt{z}\right)}{32\sqrt{\pi}}$$

07.18.03.0032.01

$${}_0\tilde{F}_1\left(;-\frac{7}{2};z\right)=\frac{(4\,z\,(4\,z+45)+105)\cosh\left(2\sqrt{z}\right)-10\sqrt{z}\,(8\,z+21)\sinh\left(2\sqrt{z}\right)}{16\sqrt{\pi}}$$

07.18.03.0033.01

$${}_0\tilde{F}_1\left(;-\frac{7}{2};-z\right)=\frac{(4\,z\,(4\,z-45)+105)\cos\left(2\sqrt{z}\right)+10\,(21-8\,z)\sqrt{z}\,\sin\left(2\sqrt{z}\right)}{16\sqrt{\pi}}$$

07.18.03.0009.01

$${}_0\tilde{F}_1\left(;-\frac{5}{2};z\right)=\frac{2\sqrt{z}\,(4\,z+15)\sinh\left(2\sqrt{z}\right)-3\,(8\,z+5)\cosh\left(2\sqrt{z}\right)}{8\sqrt{\pi}}$$

07.18.03.0034.01

$${}_0\tilde{F}_1\left(;-\frac{5}{2};-z\right)=\frac{3\,(8\,z-5)\cos\left(2\sqrt{z}\right)+2\sqrt{z}\,(4\,z-15)\sin\left(2\sqrt{z}\right)}{8\sqrt{\pi}}$$

07.18.03.0010.01

$${}_0\tilde{F}_1\left(;-\frac{3}{2};z\right)=\frac{(4\,z+3)\cosh\left(2\sqrt{z}\right)-6\sqrt{z}\,\sinh\left(2\sqrt{z}\right)}{4\sqrt{\pi}}$$

07.18.03.0035.01

$${}_0\tilde{F}_1\left(;-\frac{3}{2};-z\right)=\frac{(3-4\,z)\cos\left(2\sqrt{z}\right)+6\sqrt{z}\,\sin\left(2\sqrt{z}\right)}{4\sqrt{\pi}}$$

07.18.03.0011.01

$${}_0\tilde{F}_1\left(;-\frac{1}{2};z\right)=-\frac{\cosh\left(2\sqrt{z}\right)-2\sqrt{z}\,\sinh\left(2\sqrt{z}\right)}{2\sqrt{\pi}}$$

07.18.03.0036.01

$${}_0\tilde{F}_1\left(;-\frac{1}{2};-z\right)=-\frac{\cos\left(2\sqrt{z}\right)+2\sqrt{z}\,\sin\left(2\sqrt{z}\right)}{2\sqrt{\pi}}$$

07.18.03.0012.01

$${}_0\tilde{F}_1\left(;\frac{1}{2};z\right)=\frac{\cosh\left(2\sqrt{z}\right)}{\sqrt{\pi}}$$

07.18.03.0037.01

$${}_0\tilde{F}_1\left(;\frac{1}{2};-z\right)=\frac{\cos\left(2\sqrt{z}\right)}{\sqrt{\pi}}$$

07.18.03.0013.01

$${}_0\tilde{F}_1\left(;\frac{3}{2};z\right)=\frac{\sinh\left(2\sqrt{z}\right)}{\sqrt{\pi}\sqrt{z}}$$

07.18.03.0038.01

$${}_0\tilde{F}_1\left(;\frac{3}{2};-z\right)=\frac{\sin\left(2\sqrt{z}\right)}{\sqrt{\pi}\sqrt{z}}$$

07.18.03.0014.01

$${}_0\tilde{F}_1\left(;\frac{5}{2};z\right)=\frac{1}{2\sqrt{\pi}\,z}\left(2\cosh\left(2\sqrt{z}\right)-\frac{\sinh\left(2\sqrt{z}\right)}{\sqrt{z}}\right)$$

07.18.03.0039.01

$${}_0\tilde{F}_1\left(;\frac{5}{2};-z\right)=\frac{\sin\left(2\sqrt{z}\right)-2\sqrt{z}\cos\left(2\sqrt{z}\right)}{2\sqrt{\pi}\,z^{3/2}}$$

07.18.03.0015.01

$${}_0\tilde{F}_1\left(;\frac{7}{2};z\right)=\frac{1}{4\sqrt{\pi}\,z^{5/2}}\left((4z+3)\sinh\left(2\sqrt{z}\right)-6\sqrt{z}\cosh\left(2\sqrt{z}\right)\right)$$

07.18.03.0040.01

$${}_0\tilde{F}_1\left(;\frac{7}{2};-z\right)=\frac{(3-4z)\sin\left(2\sqrt{z}\right)-6\sqrt{z}\cos\left(2\sqrt{z}\right)}{4\sqrt{\pi}\,z^{5/2}}$$

07.18.03.0016.01

$${}_0\tilde{F}_1\left(;\frac{9}{2};z\right)=\frac{1}{8\sqrt{\pi}\,z^{7/2}}\left(2\sqrt{z}\,(4z+15)\cosh\left(2\sqrt{z}\right)-3(8z+5)\sinh\left(2\sqrt{z}\right)\right)$$

07.18.03.0041.01

$${}_0\tilde{F}_1\left(;\frac{9}{2};-z\right)=\frac{2\sqrt{z}\,(4z-15)\cos\left(2\sqrt{z}\right)+3(5-8z)\sin\left(2\sqrt{z}\right)}{8\sqrt{\pi}\,z^{7/2}}$$

07.18.03.0042.01

$${}_0\tilde{F}_1\left(;\frac{11}{2};z\right)=\frac{(4z(4z+45)+105)\sinh\left(2\sqrt{z}\right)-10\sqrt{z}\,(8z+21)\cosh\left(2\sqrt{z}\right)}{16\sqrt{\pi}\,z^{9/2}}$$

07.18.03.0043.01

$${}_0\tilde{F}_1\left(;\frac{11}{2};-z\right)=\frac{10\sqrt{z}\,(8z-21)\cos\left(2\sqrt{z}\right)+(4z(4z-45)+105)\sin\left(2\sqrt{z}\right)}{16\sqrt{\pi}\,z^{9/2}}$$

For fixed z and $b = \frac{m}{3}$

07.18.03.0044.01

$$ {}_0\tilde{F}_1\left(;-\frac{17}{3};z\right) = \frac{1}{1458\, 3^{5/6}} \left(99\sqrt{3}\, z^{2/3}\left(243 z^2+4032 z+9520\right)\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right) - 3\sqrt[6]{3}\left(729 z^3+42768 z^2+277200 z+209440\right)\operatorname{Ai}'\left(3^{2/3}\sqrt[3]{z}\right) - 99 z^{2/3}\left(243 z^2+4032 z+9520\right)\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right) + 3^{2/3}\left(729 z^3+42768 z^2+277200 z+209440\right)\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0045.01

$$ {}_0\tilde{F}_1\left(;-\frac{17}{3};-z\right) = -\frac{1}{2916\, 3^{5/6}} \left(99(-1)^{2/3}\left(3i+\sqrt{3}\right) z^{2/3}\left(243 z^2-4032 z+9520\right)\operatorname{Ai}\left(-3^{2/3}\sqrt[3]{z}\right) - 6\sqrt[6]{3}\left(729 z^3-42768 z^2+277200 z-209440\right)\operatorname{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right) - 99\sqrt[6]{-1}\left(i-\sqrt{3}\right) z^{2/3}\left(243 z^2-4032 z+9520\right)\operatorname{Bi}\left(-3^{2/3}\sqrt[3]{z}\right) + 2\, 3^{2/3}\left(729 z^3-42768 z^2+277200 z-209440\right)\operatorname{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0046.01

$$ {}_0\tilde{F}_1\left(;-\frac{16}{3};z\right) = -\frac{1}{486\, 3^{5/6}} \left(-\sqrt{3}\left(729 z^3+34020 z^2+163800 z+58240\right)\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right) + 30\sqrt[6]{3}\sqrt[3]{z}\left(243 z^2+3276 z+5824\right)\operatorname{Ai}'\left(3^{2/3}\sqrt[3]{z}\right) + \left(-729 z^3-34020 z^2-163800 z-58240\right)\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right) + 10\, 3^{2/3}\sqrt[3]{z}\left(243 z^2+3276 z+5824\right)\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0047.01

$$ {}_0\tilde{F}_1\left(;-\frac{16}{3};-z\right) = -\frac{1}{486\, 3^{5/6}} \left(\sqrt{3}\left(729 z^3-34020 z^2+163800 z-58240\right)\operatorname{Ai}\left(-3^{2/3}\sqrt[3]{z}\right) + 15\sqrt[3]{-1}\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\left(243 z^2-3276 z+5824\right)\operatorname{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right) + \left(729 z^3-34020 z^2+163800 z-58240\right)\operatorname{Bi}\left(-3^{2/3}\sqrt[3]{z}\right) + 5\sqrt[3]{-1}\sqrt[6]{3}\left(3i-\sqrt{3}\right)\sqrt[3]{z}\left(243 z^2-3276 z+5824\right)\operatorname{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0048.01

$$ {}_0\tilde{F}_1\left(;-\frac{14}{3};z\right) = -\frac{1}{486\, 3^{5/6}} \left(9\sqrt{3}\, z^{2/3}\left(81 z^2+2376 z+6160\right)\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right) - 24\sqrt[6]{3}\left(243 z^2+1980 z+1540\right)\operatorname{Ai}'\left(3^{2/3}\sqrt[3]{z}\right) - 9 z^{2/3}\left(81 z^2+2376 z+6160\right)\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right) + 8\, 3^{2/3}\left(243 z^2+1980 z+1540\right)\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0049.01

$$ {}_0\tilde{F}_1\left(;-\frac{14}{3};-z\right) = \frac{1}{972\, 3^{5/6}} \left(9(-1)^{2/3}\left(3i+\sqrt{3}\right) z^{2/3}\left(81 z^2-2376 z+6160\right)\operatorname{Ai}\left(-3^{2/3}\sqrt[3]{z}\right) + 48\sqrt[6]{3}\left(243 z^2-1980 z+1540\right)\operatorname{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right) - 9\sqrt[6]{-1}\left(i-\sqrt{3}\right) z^{2/3}\left(81 z^2-2376 z+6160\right)\operatorname{Bi}\left(-3^{2/3}\sqrt[3]{z}\right) - 16\, 3^{2/3}\left(243 z^2-1980 z+1540\right)\operatorname{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0050.01

$$ {}_0\tilde{F}_1\left(;-\frac{13}{3};z\right)=\frac{1}{162\,3^{5/6}}\left(-7\sqrt{3}\left(243z^2+1440z+520\right)\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+3\sqrt[6]{3}\sqrt[3]{z}\left(81z^2+1890z+3640\right)\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-7\left(243z^2+1440z+520\right)\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\sqrt[3]{z}\left(81z^2+1890z+3640\right)\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0051.01

$$ {}_0\tilde{F}_1\left(;-\frac{13}{3};-z\right)=\frac{1}{324\,3^{5/6}}\left(-14\sqrt{3}\left(243z^2-1440z+520\right)\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\left(81z^2-1890z+3640\right)\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-14\left(243z^2-1440z+520\right)\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\sqrt[6]{3}\left(3i-\sqrt{3}\right)\sqrt[3]{z}\left(81z^2-1890z+3640\right)\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0052.01

$$ {}_0\tilde{F}_1\left(;-\frac{11}{3};z\right)=\frac{1}{162\,3^{5/6}}\left(72\sqrt{3}\,z^{2/3}(18z+55)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\left(81z^2+1080z+880\right)\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-72z^{2/3}(18z+55)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\left(81z^2+1080z+880\right)\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0053.01

$$ {}_0\tilde{F}_1\left(;-\frac{11}{3};-z\right)=\frac{1}{162\,3^{5/6}}\left(36(-1)^{2/3}\left(3i+\sqrt{3}\right)z^{2/3}(18z-55)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\left(81z^2-1080z+880\right)\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-36\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}(18z-55)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\left(81z^2-1080z+880\right)\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0054.01

$$ {}_0\tilde{F}_1\left(;-\frac{10}{3};z\right)=-\frac{1}{54\,3^{5/6}}\left(-\sqrt{3}\left(81z^2+756z+280\right)\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+42\sqrt[6]{3}\sqrt[3]{z}\,(9z+20)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+\left(-81z^2-756z-280\right)\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+14\,3^{2/3}\sqrt[3]{z}\,(9z+20)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0055.01

$$ {}_0\tilde{F}_1\left(;-\frac{10}{3};-z\right)=\frac{1}{54\,3^{5/6}}\left(\sqrt{3}\left(81z^2-756z+280\right)\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+21\sqrt[3]{-1}\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,(9z-20)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+\left(81z^2-756z+280\right)\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+7\sqrt[3]{-1}\sqrt[6]{3}\left(3i-\sqrt{3}\right)\sqrt[3]{z}\,(9z-20)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0056.01

$$ {}_0\tilde{F}_1\left(;-\frac{8}{3};z\right)=-\frac{1}{54\,3^{5/6}}\left(9\sqrt{3}\,z^{2/3}(9z+40)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-30\sqrt[6]{3}\,(9z+8)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-9z^{2/3}(9z+40)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+10\,3^{2/3}(9z+8)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0057.01

$$ {}_0\tilde{F}_1\left(;-\frac{8}{3};-z\right)=-\frac{1}{108\,3^{5/6}}\left(9(-1)^{2/3}\left(3i+\sqrt{3}\right)z^{2/3}(9z-40)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+60\sqrt[6]{3}\,(9z-8)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-9\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}(9z-40)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)-20\,3^{2/3}(9z-8)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0017.01

$${}_0\tilde{F}_1\left(;-\frac{7}{3};z\right)=\frac{1}{18\,3^{5/6}}\left(3\sqrt[6]{3}\,(9z+28)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}+3^{2/3}\,(9z+28)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}-4\sqrt{3}\,(18z+7)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-4\,(18z+7)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0058.01

$${}_0\tilde{F}_1\left(;-\frac{7}{3};-z\right)=-\frac{1}{36\,3^{5/6}}\left(-8\sqrt{3}\,(18z-7)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,(9z-28)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-8\,(18z-7)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\,\sqrt[6]{3}\left(3i-\sqrt{3}\right)\sqrt[3]{z}\,(9z-28)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0018.01

$${}_0\tilde{F}_1\left(;-\frac{5}{3};z\right)=\frac{1}{18\,3^{5/6}}\left(45\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)z^{2/3}-45\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)z^{2/3}-3\sqrt[6]{3}\,(9z+10)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\,(9z+10)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0059.01

$${}_0\tilde{F}_1\left(;-\frac{5}{3};-z\right)=-\frac{1}{36\,3^{5/6}}\left(45\,(-1)^{2/3}\left(3i+\sqrt{3}\right)z^{2/3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-6\sqrt[6]{3}\,(9z-10)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-45\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+2\,3^{2/3}\,(9z-10)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0019.01

$${}_0\tilde{F}_1\left(;-\frac{4}{3};z\right)=\frac{1}{6\,3^{5/6}}\left(-12\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}-4\,3^{2/3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}+\sqrt{3}\,(9z+4)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+(9z+4)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0060.01

$${}_0\tilde{F}_1\left(;-\frac{4}{3};-z\right)=-\frac{1}{6\,3^{5/6}}\left(\sqrt{3}\,(9z-4)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+6\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+(9z-4)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+2\sqrt[3]{-1}\,\sqrt[6]{3}\left(3i-\sqrt{3}\right)\sqrt[3]{z}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0020.01

$${}_0\tilde{F}_1\left(;-\frac{2}{3};z\right)=\frac{1}{6\,3^{5/6}}\left(9\left(\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)\right)z^{2/3}+6\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-2\,3^{2/3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0061.01

$${}_0\tilde{F}_1\left(;-\frac{2}{3};-z\right)=\frac{1}{12\,3^{5/6}}\left(9\,(-1)^{2/3}\left(3i+\sqrt{3}\right)z^{2/3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+12\sqrt[6]{3}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-9\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)-4\,3^{2/3}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0021.01

$${}_0\tilde{F}_1\left(;-\frac{1}{3};z\right)=\frac{1}{2\,3^{5/6}}\left(\sqrt[6]{3}\left(3\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+\sqrt{3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\sqrt[3]{z}-\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right)$$

07.18.03.0062.01

$$ {}_0\tilde{F}_1\left(;-\frac{1}{3};-z\right)=\frac{1}{4\,3^{5/6}}\left(-2\sqrt{3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-2\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\,\sqrt[6]{3}\left(3\,i-\sqrt{3}\right)\sqrt[3]{z}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0022.01

$$ {}_0\tilde{F}_1\left(;\frac{1}{3};z\right)=\frac{1}{6}\left(3^{5/6}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)-3\sqrt[3]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0063.01

$$ {}_0\tilde{F}_1\left(;\frac{1}{3};-z\right)=\frac{\left(1-i\sqrt[6]{3}\right)\left(-1-i\sqrt[6]{3}+\sqrt[3]{3}\right)\left(3\left(1-i\sqrt{3}\right)\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+\left(3\,i-\sqrt{3}\right)\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)}{8\,3^{2/3}} $$

07.18.03.0023.01

$$ {}_0\tilde{F}_1\left(;\frac{2}{3};z\right)=\frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0064.01

$$ {}_0\tilde{F}_1\left(;\frac{2}{3};-z\right)=\frac{1}{8}\sqrt[6]{3}\left(-i+\sqrt[6]{3}\right)\left(-1+i\sqrt[6]{3}+\sqrt[3]{3}\right)\left(\left(3-i\sqrt{3}\right)\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+\left(-i+\sqrt{3}\right)\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0024.01

$$ {}_0\tilde{F}_1\left(;\frac{4}{3};z\right)=\frac{1}{2}\sqrt[6]{3}\left(\frac{1}{\sqrt[3]{z}}\left(\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)\right)\right) $$

07.18.03.0065.01

$$ {}_0\tilde{F}_1\left(;\frac{4}{3};-z\right)=-\frac{\sqrt[6]{-3}\left(-i+\sqrt[6]{3}\right)\left(-1+i\sqrt[6]{3}+\sqrt[3]{3}\right)\left(\left(3\,i-\sqrt{3}\right)\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+\left(1-i\sqrt{3}\right)\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)\right)}{8\sqrt[3]{z}} $$

07.18.03.0025.01

$$ {}_0\tilde{F}_1\left(;\frac{5}{3};z\right)=\frac{3\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)}{2\,3^{5/6}\,z^{2/3}} $$

07.18.03.0066.01

$$ {}_0\tilde{F}_1\left(;\frac{5}{3};-z\right)=\frac{\sqrt[3]{-1}\left(1-i\sqrt[6]{3}\right)\left(-1-i\sqrt[6]{3}+\sqrt[3]{3}\right)\left(3\left(1+i\sqrt{3}\right)\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+\left(3\,i+\sqrt{3}\right)\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)}{8\,3^{2/3}\,z^{2/3}} $$

07.18.03.0026.01

$$ {}_0\tilde{F}_1\left(;\frac{7}{3};z\right)=\frac{1}{2\,3^{5/6}\,z^{4/3}}\left(-3\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}+3^{2/3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}+\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0067.01

$$ {}_0\tilde{F}_1\left(;\frac{7}{3};-z\right)=-\frac{1}{4\,3^{5/6}\,z^{4/3}}\left((-1)^{2/3}\left(3\,i+\sqrt{3}\right)\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-6\sqrt[6]{3}\,\sqrt[3]{z}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-\sqrt[6]{-1}\left(i-\sqrt{3}\right)\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+2\,3^{2/3}\sqrt[3]{z}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0068.01

$$ {}_0\tilde{F}_1\left(;\frac{8}{3};z\right)=\frac{9\sqrt{3}\,z^{2/3}\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-6\sqrt[6]{3}\operatorname{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+9z^{2/3}\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-2\,3^{2/3}\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)}{6\,3^{5/6}z^{5/3}} $$

07.18.03.0069.01

$$ {}_0\tilde{F}_1\left(;\frac{8}{3};-z\right)=-\frac{1}{6\,3^{5/6}z^{5/3}}\left(9\sqrt{3}\,z^{2/3}\operatorname{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\operatorname{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+9z^{2/3}\operatorname{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\sqrt[6]{3}\left(3i-\sqrt{3}\right)\operatorname{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0070.01

$$ {}_0\tilde{F}_1\left(;\frac{10}{3};z\right)=-\frac{1}{6\,3^{5/6}z^{7/3}}\left(\sqrt{3}\,(9z+4)\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-12\sqrt[6]{3}\sqrt[3]{z}\operatorname{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+(-9z-4)\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+4\,3^{2/3}\sqrt[3]{z}\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0071.01

$$ {}_0\tilde{F}_1\left(;\frac{10}{3};-z\right)=\frac{1}{12\,3^{5/6}z^{7/3}}\left((-1)^{2/3}\left(3i+\sqrt{3}\right)(9z-4)\operatorname{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+24\sqrt[6]{3}\sqrt[3]{z}\operatorname{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-\sqrt[6]{-1}\left(i-\sqrt{3}\right)(9z-4)\operatorname{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)-8\,3^{2/3}\sqrt[3]{z}\operatorname{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0072.01

$$ {}_0\tilde{F}_1\left(;\frac{11}{3};z\right)=\frac{1}{18\,3^{5/6}z^{8/3}}\left(-45\sqrt{3}\,z^{2/3}\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+3\sqrt[6]{3}\,(9z+10)\operatorname{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-45z^{2/3}\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}(9z+10)\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0073.01

$$ {}_0\tilde{F}_1\left(;\frac{11}{3};-z\right)=\frac{1}{36\,3^{5/6}z^{8/3}}\left(-90\sqrt{3}\,z^{2/3}\operatorname{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)(9z-10)\operatorname{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-90z^{2/3}\operatorname{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\sqrt[6]{3}\left(3i-\sqrt{3}\right)(9z-10)\operatorname{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0074.01

$$ {}_0\tilde{F}_1\left(;\frac{13}{3};z\right)=\frac{1}{18\,3^{5/6}z^{10/3}}\left(4\sqrt{3}\,(18z+7)\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\sqrt[3]{z}\,(9z+28)\operatorname{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-4(18z+7)\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\sqrt[3]{z}\,(9z+28)\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0075.01

$$ {}_0\tilde{F}_1\left(;\frac{13}{3};-z\right)=\frac{1}{18\,3^{5/6}z^{10/3}}\left(2(-1)^{2/3}\left(3i+\sqrt{3}\right)(18z-7)\operatorname{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\sqrt[3]{z}\,(9z-28)\operatorname{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-2\sqrt[6]{-1}\left(i-\sqrt{3}\right)(18z-7)\operatorname{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\sqrt[3]{z}\,(9z-28)\operatorname{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0076.01

$$ {}_0\tilde{F}_1\left(;\frac{14}{3};z\right)=-\frac{1}{54\,3^{5/6}\,z^{11/3}}\left(-9\sqrt{3}\,z^{2/3}\,(9z+40)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+30\sqrt[6]{3}\,(9z+8)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-9z^{2/3}\,(9z+40)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+10\,3^{2/3}\,(9z+8)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0077.01

$$ {}_0\tilde{F}_1\left(;\frac{14}{3};-z\right)=\frac{1}{54\,3^{5/6}\,z^{11/3}}\left(9\sqrt{3}\,z^{2/3}\,(9z-40)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+15\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)(9z-8)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+9z^{2/3}\,(9z-40)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+5\sqrt[3]{-1}\,\sqrt[6]{3}\left(3i-\sqrt{3}\right)(9z-8)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0078.01

$$ {}_0\tilde{F}_1\left(;\frac{16}{3};z\right)=-\frac{1}{54\,3^{5/6}\,z^{13/3}}\left(\sqrt{3}\left(81z^2+756z+280\right)\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-42\sqrt[6]{3}\,\sqrt[3]{z}\,(9z+20)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+\left(-81z^2-756z-280\right)\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+14\,3^{2/3}\sqrt[3]{z}\,(9z+20)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0079.01

$$ {}_0\tilde{F}_1\left(;\frac{16}{3};-z\right)=-\frac{1}{108\,3^{5/6}\,z^{13/3}}\left((-1)^{2/3}\left(3i+\sqrt{3}\right)\left(81z^2-756z+280\right)\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+84\sqrt[6]{3}\,\sqrt[3]{z}\,(9z-20)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-\sqrt[6]{-1}\left(i-\sqrt{3}\right)\left(81z^2-756z+280\right)\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)-28\,3^{2/3}\sqrt[3]{z}\,(9z-20)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0080.01

$$ {}_0\tilde{F}_1\left(;\frac{17}{3};z\right)=\frac{1}{162\,3^{5/6}\,z^{14/3}}\left(-72\sqrt{3}\,z^{2/3}\,(18z+55)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+3\sqrt[6]{3}\left(81z^2+1080z+880\right)\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-72z^{2/3}\,(18z+55)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\left(81z^2+1080z+880\right)\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.18.03.0081.01

$$ {}_0\tilde{F}_1\left(;\frac{17}{3};-z\right)=-\frac{1}{324\,3^{5/6}\,z^{14/3}}\left(-144\sqrt{3}\,z^{2/3}\,(18z-55)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\left(81z^2-1080z+880\right)\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-144z^{2/3}\,(18z-55)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\,\sqrt[6]{3}\left(3i-\sqrt{3}\right)\left(81z^2-1080z+880\right)\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right) $$

Values at infinities

07.18.03.0027.01

$$ {}_0\tilde{F}_1(;b;\infty)=\tilde{\infty} $$

General characteristics

Domain and analyticity

${}_0\tilde{F}_1(;b;z)$ is an analytical entire function of b and z which is defined in $\mathbb{C}^2$. For fixed b, it is an entire function of z.

07.18.04.0001.01

$$(b*z)\longrightarrow {}_0\tilde{F}_1(;b;z)::(\mathbb{C}\otimes\mathbb{C})\longrightarrow\mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.18.04.0002.01

$${}_0\tilde{F}_1\left(;\bar{b};\bar{z}\right)=\overline{{}_0\tilde{F}_1(;b;z)}$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed b, the function ${}_0\tilde{F}_1(;b;z)$ has only one singular point at $z=\tilde{\infty}$. It is an essential singular point.

07.18.04.0003.01

$$\mathcal{S}ing_z\left({}_0\tilde{F}_1(;b;z)\right)=\{\{\tilde{\infty},\infty\}\}$$

With respect to b

For fixed z, the function ${}_0\tilde{F}_1(;b;z)$ has only one singular point at $b=\tilde{\infty}$. It is an essential singular point.

07.18.04.0004.01

$$\mathcal{S}ing_b\left({}_0\tilde{F}_1(;b;z)\right)=\{\{\tilde{\infty},\infty\}\}$$

Branch points

With respect to z

The function ${}_0\tilde{F}_1(;b;z)$ does not have branch points with respect to z.

07.18.04.0005.01

$$\mathcal{BP}_z\left({}_0\tilde{F}_1(;b;z)\right)=\{\}$$

With respect to b

The function ${}_0\tilde{F}_1(;b;z)$ does not have branch points with respect to b.

07.18.04.0006.01

$$\mathcal{BP}_b\left({}_0\tilde{F}_1(;b;z)\right)=\{\}$$

Branch cuts

With respect to z

The function ${}_0\tilde{F}_1(;b;z)$ does not have branch cuts with respect to z.

07.18.04.0007.01

$$\mathcal{BC}_z\left({}_0\tilde{F}_1(;b;z)\right) == \{\}$$

With respect to *b*

The function ${}_0\tilde{F}_1(;b;z)$ does not have branch cuts with respect to b.

07.18.04.0008.01

$$\mathcal{BC}_b\left({}_0\tilde{F}_1(;b;z)\right) == \{\}$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

07.18.06.0017.01

$${}_0\tilde{F}_1(;b;z) \propto {}_0\tilde{F}_1(;b;z_0) + {}_0\tilde{F}_1(;b+1;z_0)(z-z_0) + \frac{1}{2}\,{}_0\tilde{F}_1(;b+2;z_0)(z-z_0)^2 + \ldots /; (z \to z_0)$$

07.18.06.0018.01

$${}_0\tilde{F}_1(;b;z) \propto {}_0\tilde{F}_1(;b;z_0) + {}_0\tilde{F}_1(;b+1;z_0)(z-z_0) + \frac{1}{2}\,{}_0\tilde{F}_1(;b+2;z_0)(z-z_0)^2 + O\left((z-z_0)^3\right)$$

07.18.06.0019.01

$${}_0\tilde{F}_1(;b;z) == \sum_{k=0}^{\infty} \frac{1}{k!}\,{}_0\tilde{F}_1(;b+k;z_0)(z-z_0)^k$$

07.18.06.0020.01

$${}_0\tilde{F}_1(;b;z) = \tilde{F}_{1\times 0\times 0}^{0\times 0\times 0}\left(\begin{matrix} ;;; \\ b;;; \end{matrix}\, z_0, z-z_0\right)$$

07.18.06.0021.01

$${}_0\tilde{F}_1(;b;z) \propto {}_0\tilde{F}_1(;b;z_0)\,(1+O(z-z_0))$$

Expansions at $z == 0$

For the function itself

General case

07.18.06.0001.02

$${}_0\tilde{F}_1(;b;z) \propto \frac{1}{\Gamma(b)}\left(1 + \frac{z}{b} + \frac{z^2}{2\,b\,(1+b)} + \ldots\right) /; (z \to 0)$$

07.18.06.0022.01

$${}_0\tilde{F}_1(;b;z) \propto \frac{1}{\Gamma(b)}\left(1 + \frac{z}{b} + \frac{z^2}{2\,b\,(1+b)} + O\left(z^3\right)\right)$$

07.18.06.0002.01

$${}_0\tilde{F}_1(;b;z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(b+k)\,k!}$$

07.18.06.0003.02

$${}_0\tilde{F}_1(;b;z) \propto \frac{1}{\Gamma(b)}\,(1+O(z))$$

07.18.06.0023.01

$${}_0\tilde{F}_1(;b;z) = F_\infty(z,b) \,/;\, \left(\left(F_m(z,b) = \sum_{k=0}^{n} \frac{z^k}{\Gamma(b+k)\,k!} = {}_0\tilde{F}_1(;b;z) - z^{n+1}\,{}_1\tilde{F}_2(1;n+2,b+n+1;z)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

07.18.06.0024.01

$${}_0\tilde{F}_1(;0;z) \propto z + \frac{z^2}{2} + \frac{z^3}{12} + O\left(z^4\right)$$

07.18.06.0025.01

$${}_0\tilde{F}_1(;-1;z) \propto \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{48} + O\left(z^5\right)$$

07.18.06.0026.01

$${}_0\tilde{F}_1(;-2;z) \propto \frac{z^3}{6} + \frac{z^4}{24} + \frac{z^5}{240} + O\left(z^6\right)$$

07.18.06.0027.01

$${}_0\tilde{F}_1(;-n;z) \propto \frac{z^{n+1}}{(n+1)!}\left(1 + \frac{z}{(n+2)} + \frac{z^2}{2\,(n+2)\,(n+3)} + O\left(z^3\right)\right) /;\, n \in \mathbb{N}$$

07.18.06.0028.01

$${}_0\tilde{F}_1(;-n;z) = z^{n+1} \sum_{k=0}^{\infty} \frac{z^k}{(n+k+1)!\,k!} \,/;\, n \in \mathbb{N}$$

07.18.06.0029.01

$${}_0\tilde{F}_1(;-n;z) \propto \frac{z^{n+1}}{(n+1)!}\,(1+O(z)) \,/;\, n \in \mathbb{N}$$

Asymptotic series expansions

Expansions for |Arg(z)| < π

07.18.06.0004.01

$${}_0\tilde{F}_1(;b;z) \propto \frac{1}{2\sqrt{\pi}}\, z^{\frac{1-2b}{4}}\, e^{2\sqrt{z}}\, {}_2F_0\left(b-\frac{1}{2}, \frac{3}{2}-b;;\frac{1}{4\sqrt{z}}\right) /;\, |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

07.18.06.0005.01

$${}_0\tilde{F}_1(;b;z) \propto \frac{1}{2\sqrt{\pi}}\, z^{\frac{1-2b}{4}}\, e^{2\sqrt{z}} \left(1 + O\left(\frac{1}{\sqrt{z}}\right)\right) /;\, |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

The general formulas

07.18.06.0006.01

$$ {}_0\tilde{F}_1(;b;z) \propto \mathcal{A}_{\tilde{F}}\left(\begin{matrix} ; \\ b; \end{matrix} \{z, \tilde{\infty}, \infty\}\right) /; (|z| \to \infty) $$

07.18.06.0007.01

$$ {}_0\tilde{F}_1(;b;z) \propto \mathcal{A}_{\tilde{F}}^{(\text{trig})}\left(\begin{matrix} ; \\ b; \end{matrix} \{z, \tilde{\infty}, \infty\}\right) /; (|z| \to \infty) $$

Expansions for any z in exponential form

07.18.06.0008.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{1}{2\sqrt{\pi}} (-z)^{\frac{1-2b}{4}} \left(\exp\left(i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right)\left(1 + \frac{i(2b-3)(2b-1)}{16\sqrt{-z}} + \frac{(2b-5)(2b-3)(2b-1)(2b+1)}{512z} + \ldots\right) + \exp\left(-i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right)\left(1 - \frac{i(2b-3)(2b-1)}{16\sqrt{-z}} + \frac{(2b-5)(2b-3)(2b-1)(2b+1)}{512z} + \ldots\right)\right) /; (|z| \to \infty) $$

07.18.06.0009.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{1}{2\sqrt{\pi}} (-z)^{\frac{1-2b}{4}} \left(\exp\left(i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right) {}_2F_0\left(b-\frac{1}{2}, \frac{3}{2}-b;; \frac{1}{4i\sqrt{-z}}\right) + \exp\left(-i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right) {}_2F_0\left(b-\frac{1}{2}, \frac{3}{2}-b;; -\frac{1}{4i\sqrt{-z}}\right)\right) /; (|z| \to \infty) $$

07.18.06.0010.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{1}{2\sqrt{\pi}} (-z)^{\frac{1-2b}{4}} \left(\exp\left(i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right)\left(1 + O\left(\frac{1}{\sqrt{z}}\right)\right) + \exp\left(-i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right)\left(1 + O\left(\frac{1}{\sqrt{z}}\right)\right)\right) /; (|z| \to \infty) $$

07.18.06.0011.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{1}{2\sqrt{\pi}} \left(e^{-2i\sqrt{-z}} \left(-i\sqrt{-z}\right)^{\frac{1}{2}-b} {}_2F_0\left(b-\frac{1}{2}, \frac{3}{2}-b;; -\frac{1}{4i\sqrt{-z}}\right) + e^{2i\sqrt{-z}} \left(i\sqrt{-z}\right)^{\frac{1}{2}-b} {}_2F_0\left(b-\frac{1}{2}, \frac{3}{2}-b;; \frac{1}{4i\sqrt{-z}}\right)\right) /; (|z| \to \infty) $$

07.18.06.0012.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{1}{2\sqrt{\pi}} \left(e^{-2i\sqrt{-z}} \left(-i\sqrt{-z}\right)^{\frac{1}{2}-b} \left(1 + O\left(\frac{1}{\sqrt{z}}\right)\right) + e^{2i\sqrt{-z}} \left(i\sqrt{-z}\right)^{\frac{1}{2}-b} \left(1 + O\left(\frac{1}{\sqrt{z}}\right)\right)\right) /; (|z| \to \infty) $$

Expansions for any z in trigonometric form

07.18.06.0013.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{(-z)^{\frac{1}{4}(1-2b)}}{2\sqrt{\pi}} \left(\left(1+\frac{(3-2b)(5-2b)(-1+2b)(1+2b)}{512z}+\frac{1}{1\,572\,864z^2}((-9+2b)(-7+2b)(-5+2b)(-3+2b)(-1+2b)(1+2b)(3+2b)(5+2b))+\ldots\right)\cos\left(\frac{(2b-1)\pi}{4}-2\sqrt{-z}\right)+\frac{(2b-3)(2b-1)}{16\sqrt{-z}}\left(1+\frac{(5-2b)(7-2b)(1+2b)(3+2b)}{1536z}+\frac{1}{7\,864\,320z^2}((-11+2b)(-9+2b)(-7+2b)(-5+2b)(1+2b)(3+2b)(5+2b)(7+2b))+\ldots\right)\sin\left(\frac{(2b-1)\pi}{4}-2\sqrt{-z}\right)\right)\; /; (|z|\to\infty) $$

07.18.06.0030.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{1}{\sqrt{\pi}}(-z)^{\frac{1}{4}(1-2b)} \left(\cos\left(\frac{1}{4}(2b-1)\pi-2\sqrt{-z}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{4}(3-2b)\right)_k\left(\frac{1}{4}(5-2b)\right)_k\left(\frac{1}{4}(2b-1)\right)_k\left(\frac{1}{4}(2b+1)\right)_k}{\left(\frac{1}{2}\right)_k}\left(\frac{1}{4z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)+\frac{(2b-1)(2b-3)}{16\sqrt{-z}}\sin\left(\frac{1}{4}(2b-1)\pi-2\sqrt{-z}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{4}(5-2b)\right)_k\left(\frac{1}{4}(7-2b)\right)_k\left(\frac{1}{4}(2b+1)\right)_k\left(\frac{1}{4}(2b+3)\right)_k}{\left(\frac{3}{2}\right)_k}\left(\frac{1}{4z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)\right)\; /; (|z|\to\infty) $$

07.18.06.0014.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{1}{\sqrt{\pi}}(-z)^{\frac{1-2b}{4}}\left(\cos\left(\frac{2b-1}{4}\pi-2\sqrt{-z}\right){}_4F_1\left(\frac{3-2b}{4},\frac{5-2b}{4},\frac{2b-1}{4},\frac{2b+1}{4};\frac{1}{2};\frac{1}{4z}\right)+\frac{(2b-1)(2b-3)}{16\sqrt{-z}}\sin\left(\frac{2b-1}{4}\pi-2\sqrt{-z}\right){}_4F_1\left(\frac{5-2b}{4},\frac{7-2b}{4},\frac{2b+1}{4},\frac{2b+3}{4};\frac{3}{2};\frac{1}{4z}\right)\right)\; /; (|z|\to\infty) $$

07.18.06.0015.01

$$ {}_0\tilde{F}_1(;b;z) \propto \frac{1}{\sqrt{\pi}}(-z)^{\frac{1-2b}{4}}\left(\cos\left(\frac{2b-1}{4}\pi-2\sqrt{-z}\right)\left(1+O\left(\frac{1}{z}\right)\right)+\frac{(2b-1)(2b-3)}{16\sqrt{-z}}\sin\left(\frac{2b-1}{4}\pi-2\sqrt{-z}\right)\left(1+O\left(\frac{1}{z}\right)\right)\right)\; /; (|z|\to\infty) $$

Residue representations

07.18.06.0016.01

$$ {}_0\tilde{F}_1(;b;z) = \sum_{j=0}^{\infty}\operatorname{res}_s\left(\frac{(-z)^{-s}}{\Gamma(b-s)}\Gamma(s)\right)(-j) $$

Limit representations

07.18.09.0001.01

$$ {}_0\tilde{F}_1(;\, b;\, z) = z^{\frac{1-b}{2}} \lim_{\lambda\to\infty} \lambda^{b-1}\, \mathrm{P}_\lambda^{1-b}\left(\cosh\left(\frac{2\sqrt{z}}{\lambda}\right)\right) $$

07.18.09.0002.01

$$ {}_0\tilde{F}_1(;\, b;\, z) = \lim_{n\to\infty} \frac{1}{n^{b-1}} L_n^{b-1}\left(-\frac{z}{n}\right) $$

07.18.09.0003.01

$$ {}_0\tilde{F}_1(;\, b;\, z) = \frac{1}{\Gamma(b)} \lim_{a\to\infty} {}_1F_1\left(a;\, b;\, \frac{z}{a}\right) $$

07.18.09.0004.01

$$ {}_0\tilde{F}_1(;\, b;\, z) = \frac{1}{\Gamma(b)} \lim_{n\to\infty} \lim_{m\to\infty} {}_2F_1\left(m,\, n;\, b;\, \frac{z}{m\,n}\right) $$

Continued fraction representations

07.18.10.0001.01

$$ {}_1\tilde{F}_1(a;\, b;\, z) = \frac{1}{\Gamma(b)} \left(1 + (a\,z/b) \middle/ \left(1 + -\frac{(1+a)\,z}{2\,(1+b)} \middle/ \left(1 + \frac{(1+a)\,z}{2\,(1+b)} + \cfrac{-\frac{(2+a)\,z}{3\,(2+b)}}{1 + \frac{(2+a)\,z}{3\,(2+b)} + \cfrac{-\frac{(3+a)\,z}{4\,(3+b)}}{1 + \frac{(3+a)\,z}{4\,(3+b)} + \cfrac{-\frac{(4+a)\,z}{5\,(4+b)}}{1 + \frac{(4+a)\,z}{5\,(4+b)} + \ldots}}}\right)\right)\right) $$

07.18.10.0002.01

$$ {}_1\tilde{F}_1(a;\, b;\, z) = \frac{1}{\Gamma(b)} \left(1 + \frac{a\,z}{b\left(1 + \mathrm{K}_k\left(-\frac{(a+k)\,z}{(k+1)\,(b+k)},\, \frac{(a+k)\,z}{(k+1)\,(b+k)} + 1\right)_1^\infty\right)}\right) $$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

07.18.13.0003.01

$$ z\,w''(z) + b\,w'(z) - w(z) = 0 \,/;\, w(z) = c_1\, {}_0\tilde{F}_1(;\, b;\, z) + c_2\, z^{\frac{1-b}{2}}\, K_{1-b}\left(2\sqrt{z}\right) $$

07.18.13.0004.02

$$ W_z\left({}_0\tilde{F}_1(;\, b;\, z),\, z^{\frac{1-b}{2}}\, K_{1-b}\left(2\sqrt{z}\right)\right) = -\frac{z^{-b}}{2} $$

07.18.13.0001.01

$$ z\,w''(z) + b\,w'(z) - w(z) = 0 \,/;\, w(z) = c_1\, {}_0\tilde{F}_1(;\, b;\, z) + c_2\, z^{1-b}\, {}_0\tilde{F}_1(;\, 2-b;\, z) \bigwedge b \notin \mathbb{Z} $$

07.18.13.0002.02

$$W_z\left({}_0\tilde{F}_1(;b;z),\, z^{1-b}\,{}_0\tilde{F}_1(;2-b;z)\right) = \frac{z^{-b}\sin(b\,\pi)}{\pi}$$

07.18.13.0005.01

$$w''(z) + \left(\frac{b\,g'(z)}{g(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) - \frac{g'(z)^2\,w(z)}{g(z)} = 0 \,/;\, w(z) = c_1\,{}_0\tilde{F}_1(;b;g(z)) + c_2\,g(z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{g(z)}\right)$$

07.18.13.0006.01

$$W_z\left({}_0\tilde{F}_1(;b;g(z)),\, g(z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{g(z)}\right)\right) = -\frac{1}{2}\,g'(z)\,g(z)^{-b}$$

07.18.13.0007.01

$$h(z)^2\,w''(z) + h(z)^2\left(\frac{b\,g'(z)}{g(z)} - \frac{2\,h'(z)}{h(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) + \left(2\,h'(z)^2 + \frac{h(z)\,g''(z)\,h'(z)}{g'(z)} - h(z)\,h''(z) - \frac{h(z)\,g'(z)\,(h(z)\,g'(z) + b\,h'(z))}{g(z)}\right) w(z) = 0 \,/;$$
$$w(z) = c_1\,h(z)\,{}_0\tilde{F}_1(;b;g(z)) + c_2\,h(z)\,g(z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{g(z)}\right)$$

07.18.13.0008.01

$$W_z\left(h(z)\,{}_0\tilde{F}_1(;b;g(z)),\, h(z)\,g(z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{g(z)}\right)\right) = -\frac{1}{2}\,h(z)^2\,g'(z)\,g(z)^{-b}$$

07.18.13.0009.01

$$z^2\,w''(z) + ((b-1)\,r - 2\,s + 1)\,z\,w'(z) + \left(s\,(-b\,r + r + s) - a\,r^2\,z^r\right) w(z) = 0 \,/;$$
$$w(z) = c_1\,z^s\,{}_0\tilde{F}_1(;b;a\,z^r) + c_2\,z^s\,(a\,z^r)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{a\,z^r}\right)$$

07.18.13.0010.01

$$W_z\left(z^s\,{}_0\tilde{F}_1(;b;a\,z^r),\, z^s\,(a\,z^r)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{a\,z^r}\right)\right) = -\frac{1}{2}\,a\,r\,z^{r+2\,s-1}\,(a\,z^r)^{-b}$$

07.18.13.0011.01

$$w''(z) + ((b-1)\log(r) - 2\log(s))\,w'(z) + \left(-a\log^2(r)\,r^z + \log^2(s) - (b-1)\log(r)\log(s)\right) w(z) = 0 \,/;$$
$$w(z) = c_1\,s^z\,{}_0\tilde{F}_1(;b;a\,r^z) + c_2\,s^z\,(a\,r^z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{a\,r^z}\right)$$

07.18.13.0012.01

$$W_z\left(s^z\,{}_0\tilde{F}_1(;b;a\,r^z),\, s^z\,(a\,r^z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{a\,r^z}\right)\right) = -\frac{1}{2}\,a\,r^z\,(a\,r^z)^{-b}\,s^{2\,z}\log(r)$$

Transformations

Products, sums, and powers of the direct function

Products of the direct function

07.18.16.0001.01

$${}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;c;z) = \Gamma(b+c-1)\,{}_2\tilde{F}_3\left(\frac{b+c}{2}, \frac{b+c-1}{2}; b, c, b+c-1; 4\,z\right)$$

07.18.16.0002.01

$$_0\tilde{F}_1(;b;z) = {}_0\tilde{F}_1(;2-b;z)\,z^{1-b} + \frac{2\sin(b\pi)}{\pi}\,z^{\frac{1-b}{2}}\,K_{b-1}\left(2\sqrt{z}\right)$$

Identities

Recurrence identities

Consecutive neighbors

07.18.17.0001.01

$$_0\tilde{F}_1(;b;z) = b\,{}_0\tilde{F}_1(;b+1;z) + z\,{}_0\tilde{F}_1(;b+2;z)$$

07.18.17.0002.01

$$_0\tilde{F}_1(;b;z) = -\frac{1}{z}\left((b-2)\,{}_0\tilde{F}_1(;b-1;z) - {}_0\tilde{F}_1(;b-2;z)\right)$$

Distant neighbors

Increasing

07.18.17.0003.01

$$_0\tilde{F}_1(;b;z) = (b)_n \sum_{k=0}^{n-1} \frac{(n-k)!\,(-z)^k}{k!\,(b)_k\,(1-b-n)_k\,(n-2k)!}\,{}_0\tilde{F}_1(;b+n;z) + z\,(b)_{n+1} \sum_{k=0}^{n-1} \frac{(n-k-1)!\,(-z)^k}{k!\,(b)_k\,(-b-n)_{k+2}\,(n-2k-1)!}\,{}_0\tilde{F}_1(;b+n+1;z)\;/;\;n\in\mathbb{N}^+$$

07.18.17.0011.01

$$_0\tilde{F}_1(;b;z) = C_n(b,z)\,{}_0\tilde{F}_1(;b+n;z) + z\,C_{n-1}(b,z)\,{}_0\tilde{F}_1(;b+n+1;z)\;/;\; C_0(b,z) = 1 \bigwedge C_1(b,z) = b \bigwedge C_2(b,z) = b^2+b+z \bigwedge C_n(b,z) = (b+n-1)\,C_{n-1}(b,z) + z\,C_{n-2}(b,z) \bigwedge n\in\mathbb{N}^+$$

07.18.17.0012.01

$$_0\tilde{F}_1(;b;z) = C_n(b,z)\,{}_0\tilde{F}_1(;b+n;z) + z\,C_{n-1}(b,z)\,{}_0\tilde{F}_1(;b+n+1;z)\;/;\; C_n(\nu,z) = (b)_n\,{}_2F_3\left(\frac{1-n}{2},-\frac{n}{2};b,-n,1-b-n;4z\right) \bigwedge n\in\mathbb{N}^+$$

Decreasing

07.18.17.0004.01

$$_0\tilde{F}_1(;b;z) = z^{-n}\,(2-b)_{n-1} \left(\sum_{k=0}^{n-1} \frac{(n-k-1)!\,(-z)^k}{k!\,(n-2k-1)!\,(2-b)_k\,(b-n)_k}\,{}_0\tilde{F}_1(;b-n-1;z) - (b-n-1)\sum_{k=0}^{n-1} \frac{(n-k)!\,(-z)^k}{k!\,(n-2k)!\,(2-b)_k\,(b-n-1)_k}\,{}_0\tilde{F}_1(;b-n;z)\right)$$

07.18.17.0013.01

$$_0\tilde{F}_1(;b;z) = \frac{1}{z}\,C_{n-1}(b,z)\,{}_0\tilde{F}_1(;b-n-1;z) + C_n(b,z)\,{}_0\tilde{F}_1(;b-n;z)\;/;\; C_0(b,z) = 1 \bigwedge C_1(b,z) = \frac{2-b}{z} \bigwedge C_n(b,z) = -\frac{b-n-1}{z}\,C_{n-1}(b,z) + \frac{1}{z}\,C_{n-2}(b,z) \bigwedge n\in\mathbb{N}^+$$

07.18.17.0014.01

$$ {}_0\tilde{F}_1(;b;z) = C_n(b,z)\,{}_0\tilde{F}_1(;b-n;z) + \frac{1}{z}\,C_{n-1}(b,z)\,{}_0\tilde{F}_1(;b-n-1;z)\ /;\ C_n(\nu,z) = z^{-n}\,(2-b)_n\,{}_2F_3\left(\frac{1-n}{2}, -\frac{n}{2}; 2-b, -n, b-n-1; 4z\right) \bigwedge n \in \mathbb{N}^+ $$

Functional identities

Relations between contiguous functions

07.18.17.0005.01

$$ {}_0\tilde{F}_1(;b-1;z) - (b-1)\,{}_0\tilde{F}_1(;b;z)\,z\,{}_0\tilde{F}_1(;b+1;z) = 0 $$

Relations of special kind

07.18.17.0006.01

$$ {}_0\tilde{F}_1(;-n;z) = z^{n+1}\,{}_0\tilde{F}_1(;n+2;z)\ /;\ n \in \mathbb{N} $$

07.18.17.0007.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;1-b;z) - z\,{}_0\tilde{F}_1(;b+1;z)\,{}_0\tilde{F}_1(;2-b;z) = \frac{\sin(b\pi)}{\pi} $$

Division on even and odd parts and generalization

07.18.17.0008.01

$$ {}_0\tilde{F}_1(;b;z) = A^+(z) + A^-(z)\ /;\ A^+(z) = \frac{1}{2}\left({}_0\tilde{F}_1(;b;z) + {}_0\tilde{F}_1(;b;-z)\right) \bigwedge A^-(z) = \frac{1}{2}\left({}_0\tilde{F}_1(;b;z) - {}_0\tilde{F}_1(;b;-z)\right) $$

07.18.17.0009.01

$$ {}_0\tilde{F}_1(;b;z) = A^+(z) + A^-(z)\ /;\ A^+(z) = 2^{1-b}\,\pi\,{}_0\tilde{F}_3\left(;\frac{1}{2},\frac{b}{2},\frac{b+1}{2};\frac{z^2}{16}\right) \bigwedge A^-(z) = 2^{-b-1}\,\pi\,z\,{}_0\tilde{F}_3\left(;\frac{3}{2},\frac{b+1}{2},\frac{b+2}{2};\frac{z^2}{16}\right) $$

07.18.17.0010.01

$$ {}_0F_1(;b;z) = \sum_{k=0}^{n-1} \frac{z^k}{(b)_k\,k!}\,{}_1F_{2n}\left(1;\frac{k+1}{n},\ldots,\frac{k+n}{n},\frac{b+k}{n},\ldots,\frac{b+k+n-1}{n};n^{-2n}\,z^n\right) $$

Differentiation

Low-order differentiation

With respect to *b*

07.18.20.0001.01

$$ {}_0\tilde{F}_1^{(1,0)}(;b;z) = -\sum_{k=0}^{\infty} \frac{\psi(b+k)\,z^k}{k!\,\Gamma(b+k)} $$

07.18.20.0002.01

$$ {}_0\tilde{F}_1^{(1,0)}(;b;z) = -z\,\Gamma(b)\,\tilde{F}_{2\times0\times1}^{0\times1\times2}\left(\begin{matrix} ;1;1,b; \\ 2,b+1;;b+1; \end{matrix}\ z,z\right) - \psi(b)\,{}_0\tilde{F}_1(;b;z) $$

07.18.20.0003.01

$${}_0\tilde{F}_1^{(1,0)}(;n;z) == (-1)^n z^{-\frac{n-1}{2}} \left(K_{n-1}\left(2\sqrt{z}\right) - \frac{(n-1)!}{2} z^{-\frac{n-1}{2}} \sum_{k=0}^{n-2} \frac{(-1)^k I_k\left(2\sqrt{z}\right) z^{k/2}}{(n-k-1)\,k!} \right) - \frac{\log(z)}{2}\, {}_0\tilde{F}_1(;n;z) \,/;\, n \in \mathbb{N}^+ \bigwedge z \neq 0$$

07.18.20.0012.01

$${}_0\tilde{F}_1^{(1,0)}(;n;z) = z^{-\frac{n-1}{2}} \left((-1)^n K_{n-1}\left(2\sqrt{z}\right) - \frac{1}{2}\log(z)\, I_{n-1}\left(2\sqrt{z}\right) \right) - \frac{(-1)^n (n-1)!}{2} z^{1-n} \sum_{k=0}^{n-2} \frac{(-1)^k}{(n-k-1)\,k!} I_k\left(2\sqrt{z}\right) z^{k/2} \,/;$$
$$n \in \mathbb{N}^+ \bigwedge z \neq 0$$

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07.18.20.0013.01

$${}_0\tilde{F}_1^{(1,0)}(;n;-z) = \frac{z^{-\frac{n-1}{2}}}{2} \left(\pi\, Y_{n-1}\left(2\sqrt{z}\right) - \log(z)\, J_{n-1}\left(2\sqrt{z}\right) \right) + \frac{(n-1)!}{2} z^{1-n} \sum_{k=0}^{n-2} \frac{1}{(n-k-1)\,k!} J_k\left(2\sqrt{z}\right) z^{k/2} \,/;\, n \in \mathbb{N}^+ \bigwedge z \neq 0$$

Brychkov Yu.A. (2007)

07.18.20.0014.01

$${}_0\tilde{F}_1^{(1,0)}(;-n;z) = z^{\frac{n+1}{2}} \left((-1)^n K_{n+1}\left(2\sqrt{z}\right) - \frac{1}{2}\log(z)\, I_{-n-1}\left(2\sqrt{z}\right) \right) + \frac{(-1)^n (n+1)!}{2} \sum_{k=0}^{n} \frac{(-1)^k}{(n-k+1)\,k!} I_k\left(2\sqrt{z}\right) z^{k/2} \,/;$$
$$n \in \mathbb{N} \land z \neq 0$$

Brychkov Yu.A. (2007)

07.18.20.0015.01

$${}_0\tilde{F}_1^{(1,0)}(;-n;-z) = \frac{1}{2} z^{\frac{n+1}{2}} \left((-1)^{n-1} \pi\, Y_{n+1}\left(2\sqrt{z}\right) - \log(z)\, J_{-n-1}\left(2\sqrt{z}\right) \right) + \frac{(-1)^n (n+1)!}{2} \sum_{k=0}^{n} \frac{1}{(n-k+1)\,k!} J_k\left(2\sqrt{z}\right) z^{k/2} \,/;$$
$$n \in \mathbb{N} \land z \neq 0$$

Brychkov Yu.A. (2007)

07.18.20.0016.01

$${}_0\tilde{F}_1^{(1,0)}\left(;n+\frac{1}{2};z\right) = \delta_n \frac{\cosh\left(2\sqrt{z}\right)\mathrm{Chi}\left(4\sqrt{z}\right) - \sinh\left(2\sqrt{z}\right)\mathrm{Shi}\left(4\sqrt{z}\right)}{\sqrt{\pi}} -$$
$$\frac{1}{2} z^{\frac{1-2n}{4}} \log(z)\, I_{n-\frac{1}{2}}\left(2\sqrt{z}\right) - \frac{(-1)^n 2^{2-2n}}{(n-1)!\sqrt{\pi}} z^{\frac{1}{2}-n} \left(4\sqrt{z} \sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1} \binom{n-1}{2k+1} (-2k+2n-3)! \right.$$
$$\left(-\cosh\left(2\sqrt{z}\right)\mathrm{Chi}\left(4\sqrt{z}\right) + \cosh\left(2\sqrt{z}\right)\left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{3}{2}\right)\right) + \sinh\left(2\sqrt{z}\right)\mathrm{Shi}\left(4\sqrt{z}\right)\right)(16z)^k +$$
$$\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} \binom{n-1}{2k} (-2k+2n-2)! \left(\mathrm{Chi}\left(4\sqrt{z}\right)\sinh\left(2\sqrt{z}\right) + \left(\psi\left(k-n+\frac{3}{2}\right) - \psi\left(k+\frac{1}{2}\right)\right)\sinh\left(2\sqrt{z}\right) - \right.$$
$$\left.\left.\cosh\left(2\sqrt{z}\right)\mathrm{Shi}\left(4\sqrt{z}\right)\right)(16z)^k \right) /;\, n \in \mathbb{N} \land z \neq 0$$

Brychkov Yu.A. (2007)

07.18.20.0017.01

$$ {}_0\tilde{F}_1^{(1,0)}\left(;n+\frac{1}{2};-z\right)= $$

$$ \delta_n\frac{\cos\left(2\sqrt{z}\right)\operatorname{Ci}\left(4\sqrt{z}\right)+\sin\left(2\sqrt{z}\right)\operatorname{Si}\left(4\sqrt{z}\right)}{\sqrt{\pi}}-\frac{1}{2}\log(z)\,z^{\frac{1-2n}{4}}\,J_{n-\frac{1}{2}}\left(2\sqrt{z}\right)-\frac{2^{2-2n}}{(n-1)!\sqrt{\pi}}\,z^{\frac{1}{2}-n}\Bigg(4\sqrt{z}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\binom{n-1}{2k+1} $$

$$ (2n-2k-3)!\left(\cos\left(2\sqrt{z}\right)\left(\operatorname{Ci}\left(4\sqrt{z}\right)-\psi\left(k+\frac{3}{2}\right)+\psi\left(k-n+\frac{3}{2}\right)\right)+\sin\left(2\sqrt{z}\right)\operatorname{Si}\left(4\sqrt{z}\right)\right)(-16z)^k+ $$

$$ \sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n-1}{2k}(2n-2k-2)!\left(\left(-\operatorname{Ci}\left(4\sqrt{z}\right)+\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{3}{2}\right)\right)\sin\left(2\sqrt{z}\right)+\cos\left(2\sqrt{z}\right)\operatorname{Si}\left(4\sqrt{z}\right)\right) $$

$$ (-16z)^k\Bigg)\;/;\;n\in\mathbb{N}\land z\neq 0 $$

Brychkov Yu.A. (2007)

07.18.20.0018.01

$$ {}_0\tilde{F}_1^{(1,0)}\left(;\frac{1}{2}-n;z\right)=\frac{(-1)^n\,2^{-2n}}{n!\sqrt{\pi}} $$

$$ \Bigg(4\sqrt{z}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n}{2k+1}(2n-2k-1)!\left(\left(-\operatorname{Chi}\left(4\sqrt{z}\right)+\psi\left(k-n+\frac{1}{2}\right)-\psi\left(k+\frac{3}{2}\right)\right)\sinh\left(2\sqrt{z}\right)+\cosh\left(2\sqrt{z}\right)\operatorname{Shi}\left(4\sqrt{z}\right)\right) $$

$$ (16z)^k+\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\binom{n}{2k}(2n-2k)!\left(\cosh\left(2\sqrt{z}\right)\operatorname{Chi}\left(4\sqrt{z}\right)+\cosh\left(2\sqrt{z}\right)\left(\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)-\right. $$

$$ \left.\sinh\left(2\sqrt{z}\right)\operatorname{Shi}\left(4\sqrt{z}\right)\right)(16z)^k\Bigg)-\frac{1}{2}\,z^{\frac{2n+1}{4}}\log(z)\,I_{-n-\frac{1}{2}}\left(2\sqrt{z}\right)\;/;\;n\in\mathbb{N}\land z\neq 0 $$

Brychkov Yu.A. (2007)

07.18.20.0019.01

$$ {}_0\tilde{F}_1^{(1,0)}\left(;\frac{1}{2}-n;-z\right)= \frac{(-1)^n\,2^{-2n}}{n!\,\sqrt{\pi}}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\binom{n}{2k}(2n-2k)!\left(\cos\left(2\sqrt{z}\right)\left(\operatorname{Ci}\left(4\sqrt{z}\right)+\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)+\sin\left(2\sqrt{z}\right)\operatorname{Si}\left(4\sqrt{z}\right)\right)(-16z)^k+ 4\sqrt{z}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n}{2k+1}(-2k+2n-1)!\left(\left(\operatorname{Ci}\left(4\sqrt{z}\right)-\psi\left(k-n+\frac{1}{2}\right)+\psi\left(k+\frac{3}{2}\right)\right)\sin\left(2\sqrt{z}\right)-\cos\left(2\sqrt{z}\right)\operatorname{Si}\left(4\sqrt{z}\right)\right) (-16z)^k\right)-\frac{1}{2}z^{\frac{2n+1}{4}}\log(z)\,J_{-n-\frac{1}{2}}\left(2\sqrt{z}\right)\;/;\;n\in\mathbb{N}\land z\neq 0 $$

Brychkov Yu.A. (2007)

07.18.20.0020.01

$$ {}_0\tilde{F}_1^{(1,0)}(;n;0)=\frac{(-1)^n}{|n|!}\;/;\;n\in\mathbb{Z} $$

07.18.20.0021.01

$$ {}_0\tilde{F}_1^{(1,0)}\left(;n+\frac{1}{2};0\right)=\frac{(-1)^{n-1}\,\Gamma\left(\frac{1}{2}-n\right)\psi\left(\frac{1}{2}+n\right)}{\pi}\;/;\;n\in\mathbb{Z} $$

With respect to *z*

07.18.20.0004.01

$$ \frac{\partial\,{}_0\tilde{F}_1(;b;z)}{\partial z}={}_0\tilde{F}_1(;b+1;z) $$

07.18.20.0005.01

$$ \frac{\partial^2\,{}_0\tilde{F}_1(;b;z)}{\partial z^2}={}_0\tilde{F}_1(;b+2;z) $$

Symbolic differentiation

With respect to *b*

07.18.20.0006.02

$$ {}_0\tilde{F}_1^{(n,0)}(;b;z)=\sum_{k=0}^{\infty}\frac{1}{k!}\,\frac{\partial^n\frac{1}{\Gamma(b+k)}}{\partial b^n}\,z^k\;/;\;n\in\mathbb{N} $$

With respect to *z*

07.18.20.0022.01

$$ \frac{\partial^n\,{}_0\tilde{F}_1(;b;z)}{\partial z^n}=z^{-n}\,(-1)^{n-1}\,(b-1)_n \left(\sum_{k=0}^{n-1}\frac{(n-k-1)!\,(-z)^k}{k!\,(n-2k-1)!\,(2-b-n)_k\,(b-1)_{k+1}}\,{}_0\tilde{F}_1(;b-1;z)-\sum_{k=0}^{n}\frac{(n-k)!\,(-z)^k}{k!\,(n-2k)!\,(2-b-n)_k\,(b-1)_k}\,{}_0\tilde{F}_1(;b;z)\right)/;\;n\in\mathbb{N} $$

07.18.20.0007.02

$$\frac{\partial^n {}_0\tilde{F}_1(;b;z)}{\partial z^n} = {}_0\tilde{F}_1(;b+n;z) \;/;\; n \in \mathbb{N}$$

07.18.20.0008.02

$$\frac{\partial^n {}_0\tilde{F}_1(;b;z)}{\partial z^n} = z^{-n}\, {}_1\tilde{F}_2(1;b,1-n;z) \;/;\; n \in \mathbb{N}$$

07.18.20.0009.02

$$\frac{\partial^n \left(z^\alpha\, {}_0\tilde{F}_1(;b;z)\right)}{\partial z^n} = \Gamma(\alpha+1)\, z^{\alpha-n}\, {}_1\tilde{F}_2(\alpha+1;-n+\alpha+1,b;z) \;/;\; n \in \mathbb{N}$$

07.18.20.0010.02

$$\frac{\partial^n \left(z^{b-1}\, {}_0\tilde{F}_1(;b;z)\right)}{\partial z^n} = z^{b-n-1}\, {}_0\tilde{F}_1(;b-n;z) \;/;\; n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to *z*

07.18.20.0011.01

$$\frac{\partial^\alpha {}_0\tilde{F}_1(;b;z)}{\partial z^\alpha} = z^{-\alpha}\, {}_1\tilde{F}_2(1;b,1-\alpha;z)$$

Integration

Indefinite integration

Involving only one direct function

07.18.21.0001.01

$$\int {}_0\tilde{F}_1(;b;z)\, dz = {}_0\tilde{F}_1(;b-1;z)$$

Involving one direct function and elementary functions

Involving power function

07.18.21.0002.01

$$\int z^{\alpha-1}\, {}_0\tilde{F}_1(;b;a\,z)\, dz = z^\alpha\, \Gamma(\alpha)\, {}_1\tilde{F}_2(\alpha;b,\alpha+1;a\,z)$$

07.18.21.0003.01

$$\int z^{\alpha-1}\, {}_0\tilde{F}_1(;b;z)\, dz = z^\alpha\, \Gamma(\alpha)\, {}_1\tilde{F}_2(\alpha;b,\alpha+1;z)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

07.18.21.0004.01

$$\int z^{b-\frac{3}{2}}\,{}_0\tilde{F}_1(;b;z)^2\,dz = \Gamma\left(b-\frac{1}{2}\right)\Gamma(2b-1)\,z^{b-\frac{1}{2}}\,{}_2\tilde{F}_3\left(b-\frac{1}{2},\,b-\frac{1}{2};\,b,\,2b-1,\,b+\frac{1}{2};\,4z\right)$$

Involving products of the direct function and a power function

07.18.21.0005.01

$$\int z^{\frac{b+c-3}{2}}\,{}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;c;z)\,dz = \Gamma\left(\frac{b+c-1}{2}\right)\Gamma(b+c-1)\,z^{\frac{b+c-1}{2}}\,{}_3\tilde{F}_4\left(\frac{b+c-1}{2},\,\frac{b+c-1}{2},\,\frac{b+c}{2};\,b,\,c,\,b+c-1,\,\frac{b+c+1}{2};\,4z\right)$$

Definite integration

For the direct function itself

07.18.21.0006.01

$$\int_0^1 t^{\alpha-1}\,{}_0\tilde{F}_1(;b;t)\,dt = \Gamma(\alpha)\,{}_1\tilde{F}_2(\alpha;\,b,\,\alpha+1;\,1)\,/;\,\mathrm{Re}(\alpha)>0$$

07.18.21.0007.01

$$\int_0^\infty t^{\alpha-1}\,{}_0\tilde{F}_1(;b;-t)\,dt = \frac{\Gamma(\alpha)}{\Gamma(b-\alpha)}\,/;\,0<\mathrm{Re}(\alpha)<\frac{2\,\mathrm{Re}(b)+1}{4}$$

Involving the direct function

07.18.21.0008.01

$$\int_0^\infty t^{\alpha-1}\,e^{-a t}\,{}_0\tilde{F}_1(;b;t)\,dt = a^{-\alpha}\,\Gamma(\alpha)\,{}_1\tilde{F}_1\left(\alpha;\,b;\,\frac{1}{a}\right)\,/;\,\mathrm{Re}(a)>0\land\mathrm{Re}(\alpha)>0$$

Integral transforms

Laplace transforms

07.18.22.0001.01

$$\mathcal{L}_t\left[{}_0\tilde{F}_1(;b;t)\right](z) = e^{1/z}\,z^{b-2}\left(1-Q\left(b-1,\,\frac{1}{z}\right)\right)\,/;\,\mathrm{Re}(z)>0$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

07.18.26.0001.01

$${}_0\tilde{F}_1(;b;z) = {}_p\tilde{F}_q(a_1,\,a_2,\,\ldots,\,a_p;\,b_1,\,b_2,\,\ldots,\,b_q;\,z)\,/;\,p=0\bigwedge q=1\bigwedge b_1=b$$

Involving ${}_pF_q$

07.18.26.0002.01

$${}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;c;z) = \Gamma(b+c-1)\,{}_2\tilde{F}_3\left(\frac{b+c}{2},\,\frac{b+c-1}{2};\,b,\,c,\,b+c-1;\,4z\right)$$

Involving ${}_0F_1$

07.18.26.0003.01

$${}_0\tilde{F}_1(;b;z) = \frac{{}_0F_1(;b;z)}{\Gamma(b)} \;/;\; -b \notin \mathbb{N}$$

07.18.26.0004.01

$${}_0\tilde{F}_1(;-n;z) = \lim_{b\to -n} \frac{1}{\Gamma(b)}\, {}_0F_1(;b;z) \;/;\; n \in \mathbb{N}$$

Involving ${}_1\tilde{F}_1$

07.18.26.0005.01

$${}_0\tilde{F}_1(;b;z) = \frac{\Gamma(2b-1)}{\Gamma(b)}\, e^{-2\sqrt{z}}\, {}_1\tilde{F}_1\left(b-\frac{1}{2};\, 2b-1;\, 4\sqrt{z}\right)$$

Involving ${}_1F_1$

07.18.26.0006.01

$${}_0\tilde{F}_1(;b;z) = \frac{1}{\Gamma(b)}\, e^{-2\sqrt{z}}\, {}_1F_1\left(b-\frac{1}{2};\, 2b-1;\, 4\sqrt{z}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

07.18.26.0007.01

$${}_0\tilde{F}_1(;b;z) = G_{0,2}^{1,0}(-z \mid 0,\, 1-b)$$

07.18.26.0008.01

$${}_0\tilde{F}_1(;b;z) = \pi\, G_{1,3}^{1,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0,\, 1-b,\, \frac{1}{2} \end{matrix}\right)$$

Classical cases involving exp

07.18.26.0009.01

$$e^{-2\sqrt{z}}\, {}_0\tilde{F}_1(;b;z) = \frac{4^{b-1}}{\sqrt{\pi}}\, G_{1,2}^{1,1}\left(4\sqrt{z} \,\middle|\, \begin{matrix} \frac{3}{2}-b \\ 0,\, 2-2b \end{matrix}\right)$$

07.18.26.0010.01

$$e^{2\sqrt{z}}\, {}_0\tilde{F}_1(;b;z) = 4^{b-1}\sqrt{\pi}\, \csc(b\pi)\, G_{2,3}^{1,1}\left(4\sqrt{z} \,\middle|\, \begin{matrix} \frac{3}{2}-b,\, 1-b \\ 0,\, 2-2b,\, 1-b \end{matrix}\right)$$

07.18.26.0011.01

$$e^{-z}\, {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = \frac{4^{b-1}}{\sqrt{\pi}}\, G_{1,2}^{1,1}\left(2z \,\middle|\, \begin{matrix} \frac{3}{2}-b \\ 0,\, 2-2b \end{matrix}\right)$$

07.18.26.0012.01

$$e^{z}\, {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = 4^{b-1}\sqrt{\pi}\, \csc(b\pi)\, G_{2,3}^{1,1}\left(2z \,\middle|\, \begin{matrix} \frac{3}{2}-b,\, 1-b \\ 0,\, 2-2b,\, 1-b \end{matrix}\right)$$

Classical cases involving cos

07.18.26.0013.01

$$\cos\left(2\sqrt{z}\right)\,_0\tilde{F}_1(;b;-z) = 2^{b-\frac{3}{2}}\,G_{2,4}^{1,2}\left(4z\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4}\\0,\frac{1}{2},1-b,\frac{3}{2}-b\end{matrix}\right)$$

07.18.26.0015.01

$$\cos\left(a+2\sqrt{z}\right)\,_0\tilde{F}_1(;b;-z) = 2^{b-\frac{3}{2}}\,G_{3,5}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4},\frac{a}{\pi}+\frac{1}{2}\\0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{a}{\pi}+\frac{1}{2}\end{matrix}\right)$$

07.18.26.0014.01

$$\cos(z)\,_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4}\\0,\frac{1}{2},1-b,\frac{3}{2}-b\end{matrix}\right)$$

07.18.26.0090.01

$$\cos(a+z)\,_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4},\frac{a}{\pi}+\frac{1}{2}\\0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{a}{\pi}+\frac{1}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.18.26.0091.01

$$\cos(a+z)\,_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\left(\cos(a)\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b)\\0,\frac{1}{2},1-b,\frac{3}{2}-b\end{matrix}\right)-\frac{\sin(a)}{z}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{5}{4}-\frac{b}{2},\frac{7}{4}-\frac{b}{2}\\1,\frac{1}{2},\frac{3}{2}-b,2-b\end{matrix}\right)\right)$$

Classical cases involving sin

07.18.26.0016.01

$$\sin\left(2\sqrt{z}\right)\,_0\tilde{F}_1(;b;-z) = 2^{b-\frac{3}{2}}\,G_{2,4}^{1,2}\left(4z\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4}\\\frac{1}{2},0,1-b,\frac{3}{2}-b\end{matrix}\right)$$

07.18.26.0017.01

$$\sin\left(a+2\sqrt{z}\right)\,_0\tilde{F}_1(;b;-z) = -2^{b-\frac{3}{2}}\,G_{3,5}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4},\frac{a}{\pi}+1\\0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{a}{\pi}+1\end{matrix}\right)$$

07.18.26.0092.01

$$\sin(z)\,_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b)\\\frac{1}{2},0,1-b,\frac{3}{2}-b\end{matrix}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.18.26.0093.01

$$\sin(z)\,_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) = \frac{2^{b-\frac{3}{2}}}{z}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{5}{4}-\frac{b}{2},\frac{7}{4}-\frac{b}{2}\\1,\frac{1}{2},\frac{3}{2}-b,2-b\end{matrix}\right)$$

07.18.26.0094.01

$$\sin(a+z)\,_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) = -2^{b-\frac{3}{2}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{a}{\pi}+1\\0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{a}{\pi}+1\end{matrix}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.18.26.0095.01

$$\sin(a+z)\,_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\left(\frac{\cos(a)}{z}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{5}{4}-\frac{b}{2},\frac{7}{4}-\frac{b}{2}\\1,\frac{1}{2},\frac{3}{2}-b,2-b\end{matrix}\right)+\sin(a)\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b)\\0,\frac{1}{2},1-b,\frac{3}{2}-b\end{matrix}\right)\right)$$

Classical cases involving cosh

07.18.26.0018.01

$$\cosh\left(2\sqrt{z}\right)\,{}_0\tilde{F}_1(;b;z) = 2^{b-\frac{3}{2}}\,\pi\sec\left(\frac{b\pi}{2}\right)G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{array}{c}\frac{3-2b}{4},\frac{5-2b}{4},\frac{1-b}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1-b}{2}\end{array}\right)$$

07.18.26.0096.01

$$\cosh\left(a+2\sqrt{z}\right)\,{}_0\tilde{F}_1(;b;z) = \frac{2^{2b-3}\,e^{-a}}{\sqrt{\pi}}\left(G_{1,2}^{1,1}\left(4\sqrt{z}\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\ 0,2-2b\end{array}\right)+e^{2a}\,\pi\csc(b\pi)\,G_{2,3}^{1,1}\left(4\sqrt{z}\,\middle|\,\begin{array}{c}\frac{3}{2}-b,1-b\\ 0,2-2b,1-b\end{array}\right)\right)$$

07.18.26.0019.01

$$\cosh(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,\pi\sec\left(\frac{b\pi}{2}\right)G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{3-2b}{4},\frac{5-2b}{4},\frac{1-b}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1-b}{2}\end{array}\right)$$

07.18.26.0097.01

$$\cosh(a+z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,G_{3,5}^{2,2}\left(-z^2\,\middle|\,\begin{array}{c}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{1}{2}+\frac{ia}{\pi}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1}{2}+\frac{ia}{\pi}\end{array}\right)\,/;\,-\pi<\arg(z)\leq 0$$

07.18.26.0083.01

$$\cosh(a+z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = \frac{2^{2b-3}}{\sqrt{\pi}}\left(e^{-a}\,G_{1,2}^{1,1}\left(2z\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\ 0,2-2b\end{array}\right)+e^{a}\,\pi\csc(b\pi)\,G_{2,3}^{1,1}\left(2z\,\middle|\,\begin{array}{c}\frac{3}{2}-b,1-b\\ 0,2-2b,1-b\end{array}\right)\right)$$

Classical cases involving sinh

07.18.26.0020.01

$$\sinh\left(2\sqrt{z}\right)\,{}_0\tilde{F}_1(;b;z) = -2^{b-\frac{3}{2}}\,\pi\csc\left(\frac{b\pi}{2}\right)G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{array}{c}\frac{3-2b}{4},\frac{5-2b}{4},\frac{1-b}{2}\\ \frac{1}{2},0,\frac{1-b}{2},1-b,\frac{3}{2}-b\end{array}\right)$$

07.18.26.0098.01

$$\sinh\left(a+2\sqrt{z}\right)\,{}_0\tilde{F}_1(;b;z) = \frac{2^{2b-3}\,e^{-a}}{\sqrt{\pi}}\left(e^{2a}\,\pi\csc(b\pi)\,G_{2,3}^{1,1}\left(4\sqrt{z}\,\middle|\,\begin{array}{c}\frac{3}{2}-b,1-b\\ 0,2-2b,1-b\end{array}\right)-G_{1,2}^{1,1}\left(4\sqrt{z}\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\ 0,2-2b\end{array}\right)\right)$$

07.18.26.0099.01

$$\sinh(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = -2^{b-\frac{3}{2}}\,\pi\csc\left(\frac{b\pi}{2}\right)G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{1-b}{2}\\ \frac{1}{2},0,\frac{1-b}{2},1-b,\frac{3}{2}-b\end{array}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.18.26.0100.01

$$\sinh(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = -\frac{2^{b-\frac{3}{2}}}{z}\,G_{2,4}^{1,2}\left(-z^2\,\middle|\,\begin{array}{c}\frac{5}{4}-\frac{b}{2},\frac{7}{4}-\frac{b}{2}\\ 1,\frac{1}{2},\frac{3}{2}-b,2-b\end{array}\right)$$

07.18.26.0084.01

$$\sinh(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = \frac{2^{2b-3}}{\sqrt{\pi}}\left(\pi\csc(b\pi)\,G_{2,3}^{1,1}\left(2z\,\middle|\,\begin{array}{c}\frac{3}{2}-b,1-b\\ 0,2-2b,1-b\end{array}\right)-G_{1,2}^{1,1}\left(2z\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\ 0,2-2b\end{array}\right)\right)$$

07.18.26.0101.01

$$\sinh(a+z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,i\,G_{3,5}^{2,2}\left(-z^2\,\middle|\,\begin{array}{c}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{ia}{\pi}+1\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{ia}{\pi}+1\end{array}\right)\,/;\,-\pi<\arg(z)\leq 0$$

07.18.26.0085.01

$$\sinh(a+z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)=\frac{2^{2b-3}}{\sqrt{\pi}}\left(e^a\,\pi\csc(b\,\pi)\,G_{2,3}^{1,1}\left(2\,z\,\middle|\,\begin{matrix}\frac{3}{2}-b,\,1-b\\0,\,2-2\,b,\,1-b\end{matrix}\right)-e^{-a}\,G_{1,2}^{1,1}\left(2\,z\,\middle|\,\begin{matrix}\frac{3}{2}-b\\0,\,2-2\,b\end{matrix}\right)\right)$$

Classical cases involving Ai

07.18.26.0086.01

$$\mathrm{Ai}\left(3^{2/3}\,\sqrt[3]{z}\right)\,{}_0\tilde{F}_1(;b;z)=\frac{2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1}{6}(4-3\,b),\,\frac{1}{6}(7-3\,b)\\0,\,\frac{1}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right)$$

07.18.26.0102.01

$$\mathrm{Ai}(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=\frac{2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\left(\frac{4\,z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3\,b),\,\frac{1}{6}(7-3\,b)\\0,\,\frac{1}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right)\,/;\,-\frac{\pi}{3}<\arg(z)\leq\frac{\pi}{3}$$

Classical cases involving Ai′

07.18.26.0087.01

$$\mathrm{Ai}'\left(3^{2/3}\,\sqrt[3]{z}\right)\,{}_0\tilde{F}_1(;b;z)=-\frac{2^{b-\frac{8}{3}}\,\sqrt[6]{3}}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1}{6}(5-3\,b),\,\frac{1}{6}(8-3\,b)\\0,\,\frac{2}{3},\,1-b,\,\frac{5}{3}-b\end{matrix}\right)$$

07.18.26.0103.01

$$\mathrm{Ai}'(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=-\frac{2^{b-\frac{8}{3}}\,\sqrt[6]{3}}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(\frac{4\,z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{6}(5-3\,b),\,\frac{1}{6}(8-3\,b)\\0,\,\frac{2}{3},\,1-b,\,\frac{5}{3}-b\end{matrix}\right)\,/;\,-\frac{\pi}{3}<\arg(z)\leq\frac{\pi}{3}$$

Classical cases involving Bi

07.18.26.0088.01

$$\mathrm{Bi}\left(3^{2/3}\,\sqrt[3]{z}\right)\,{}_0\tilde{F}_1(;b;z)=\frac{2^{b-\frac{1}{3}}\,\sqrt{\pi}}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1}{6}(4-3\,b),\,\frac{1}{6}(7-3\,b),\,\frac{1}{6},\,\frac{2}{3}\\0,\,\frac{1}{3},\,\frac{1}{6},\,\frac{2}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right)$$

07.18.26.0104.01

$$\mathrm{Bi}(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=\frac{2^{b-\frac{1}{3}}\,\sqrt{\pi}}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(\frac{4\,z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3\,b),\,\frac{1}{6}(7-3\,b),\,\frac{1}{6},\,\frac{2}{3}\\0,\,\frac{1}{3},\,\frac{1}{6},\,\frac{2}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right)\,/;\,-\frac{\pi}{3}<\arg(z)\leq\frac{\pi}{3}$$

Classical cases involving Bi′

07.18.26.0089.01

$$\mathrm{Bi}'\left(3^{2/3}\,\sqrt[3]{z}\right)\,{}_0\tilde{F}_1(;b;z)=2^{b-\frac{2}{3}}\,\sqrt[6]{3}\,\sqrt{\pi}\,G_{4,6}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1}{6}(5-3\,b),\,\frac{1}{6}(8-3\,b),\,\frac{1}{3},\,\frac{5}{6}\\0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{5}{6},\,1-b,\,\frac{5}{3}-b\end{matrix}\right)$$

07.18.26.0105.01

$$\mathrm{Bi}'(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=2^{b-\frac{2}{3}}\,\sqrt[6]{3}\,\sqrt{\pi}\,G_{4,6}^{2,2}\left(\frac{4\,z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{6}(5-3\,b),\,\frac{1}{6}(8-3\,b),\,\frac{1}{3},\,\frac{5}{6}\\0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{5}{6},\,1-b,\,\frac{5}{3}-b\end{matrix}\right)\,/;\,-\frac{\pi}{3}<\arg(z)\leq\frac{\pi}{3}$$

Classical cases for powers of ${}_0\tilde{F}_1$

07.18.26.0021.01

$$ {}_0\tilde{F}_1(;b;z)^2 = \sqrt{\pi}\; 2^{2b-2}\, G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{array}{c} \frac{3}{2}-b,\ \frac{1}{2} \\ 0,\ 1-b,\ 2-2b,\ \frac{1}{2} \end{array}\right) $$

07.18.26.0022.01

$$ {}_0\tilde{F}_1(;b;z)^2 = \frac{2^{2b-2}}{\sqrt{\pi}}\, G_{1,3}^{1,1}\left(-4z \,\middle|\, \begin{array}{c} \frac{3}{2}-b \\ 0,\ 1-b,\ 2-2b \end{array}\right) $$

Classical cases for products of ${}_0\tilde{F}_1$

07.18.26.0023.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;c;z) = \sqrt{\pi}\; 2^{b+c-2}\, G_{3,5}^{1,2}\left(4z \,\middle|\, \begin{array}{c} 1-\frac{b+c}{2},\ \frac{3-b-c}{2},\ \frac{1}{2} \\ 0,\ 1-b,\ 1-c,\ 2-b-c,\ \frac{1}{2} \end{array}\right) /;\ 1-b-c \notin \mathbb{N} $$

07.18.26.0024.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;c;z) = \frac{2^{b+c-2}}{\sqrt{\pi}}\, G_{2,4}^{1,2}\left(-4z \,\middle|\, \begin{array}{c} 1-\frac{b+c}{2},\ \frac{1}{2}(-b-c+3) \\ 0,\ 1-b,\ 1-c,\ -b-c+2 \end{array}\right) /;\ 1-b-c \notin \mathbb{N} $$

07.18.26.0106.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;-b-n+1;z) = \frac{1}{2^{n+1}\sqrt{\pi}} \left(2\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{\left((-1)^{\lfloor \frac{n+1}{2} \rfloor}\,\Gamma\!\left(k-n+\lfloor \frac{n}{2} \rfloor+\frac{1}{2}\right)\right)\left(1-k+\lfloor \frac{n}{2} \rfloor\right)_{n-\lfloor \frac{n}{2} \rfloor}(4z)^k}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)} - (-1)^n\,\pi\, G_{3,5}^{1,2}\left(4z \,\middle|\, \begin{array}{c} \frac{n+1}{2},\ \frac{n+2}{2},\ \frac{1}{2} \\ n+1,\ b+n,\ 0,\ 1-b,\ \frac{1}{2} \end{array}\right) \right) /;\ n \in \mathbb{N} $$

07.18.26.0107.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;1-b;z) = \frac{\sin(b\pi)}{\pi} - \frac{1}{2}\sqrt{\pi}\; G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{array}{c} 1,\ \frac{1}{2} \\ 1,\ b,\ 0,\ 1-b \end{array}\right) $$

07.18.26.0108.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;-b;z) = \frac{1}{4}\sqrt{\pi}\; G_{3,5}^{1,2}\left(4z \,\middle|\, \begin{array}{c} 1,\ \frac{3}{2},\ \frac{1}{2} \\ 2,\ b+1,\ 0,\ 1-b,\ \frac{1}{2} \end{array}\right) - \frac{b\sin(b\pi)}{\pi} $$

07.18.26.0025.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;2-b;z) = \sqrt{\pi}\; G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{array}{c} \frac{1}{2},\ \frac{1}{2} \\ 0,\ 1-b,\ b-1,\ \frac{1}{2} \end{array}\right) $$

07.18.26.0026.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;2-b;z) = \frac{1}{\sqrt{\pi}}\, G_{1,3}^{1,1}\left(-4z \,\middle|\, \begin{array}{c} \frac{1}{2} \\ 0,\ 1-b,\ b-1 \end{array}\right) $$

07.18.26.0027.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;b-1;z) = 2^{2b-3}\sqrt{\pi}\; G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{array}{c} \frac{3}{2}-b,\ \frac{1}{2} \\ 0,\ 1-b,\ 3-2b,\ \frac{1}{2} \end{array}\right) $$

07.18.26.0028.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;b-1;z) = \frac{2^{2b-3}}{\sqrt{\pi}} G_{1,3}^{1,1}\left(-4z \,\middle|\, \begin{matrix} \frac{3}{2}-b \\ 0,1-b,3-2b \end{matrix}\right) $$

07.18.26.0029.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;b+1;z) = 2^{2b-1}\sqrt{\pi}\, G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{matrix} \frac{1}{2}-b,\frac{1}{2} \\ 0,-b,1-2b,\frac{1}{2} \end{matrix}\right) $$

07.18.26.0030.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;b+1;z) = \frac{2^{2b-1}}{\sqrt{\pi}} G_{1,3}^{1,1}\left(-4z \,\middle|\, \begin{matrix} \frac{1}{2}-b \\ 0,-b,1-2b \end{matrix}\right) $$

07.18.26.0031.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;b;-z) = \sqrt{\pi}\, 2^{\frac{1-b}{2}} \left(z^2\right)^{\frac{1-b}{4}} G_{0,4}^{1,0}\left(\frac{z^2}{4} \,\middle|\, \frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{3(1-b)}{4}\right) $$

07.18.26.0032.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;2-b;-z) = \sqrt{\pi}\, G_{1,5}^{2,0}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} 1-\frac{b}{2} \\ 0,\frac{1}{2},1-\frac{b}{2},\frac{1-b}{2},\frac{b-1}{2} \end{matrix}\right) /;\ \mathrm{Re}(z) < 0 $$

Classical cases involving Bessel J

07.18.26.0033.01

$$ {}_0\tilde{F}_1(;b;-z)\,J_\nu\left(2\sqrt{z}\right) = \frac{2^{b-1}}{\sqrt{\pi}} G_{2,4}^{1,2}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2},1-\frac{b}{2} \\ \frac{\nu}{2},-\frac{\nu}{2},1-b+\frac{\nu}{2},1-b-\frac{\nu}{2} \end{matrix}\right) /;\ 1-b-\nu \notin \mathbb{N} $$

07.18.26.0109.01

$$ {}_0\tilde{F}_1(;b;-z)\,J_{-b-n}\left(2\sqrt{z}\right) = \frac{1}{\sqrt{\pi}} \left(2^{-n} z^{-\frac{b+n}{2}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left((-1)^{k+\left\lfloor \frac{n+1}{2} \right\rfloor} \Gamma\left(k-n+\left\lfloor \frac{n}{2} \right\rfloor+\frac{1}{2}\right)\right) \left(1-k+\left\lfloor \frac{n}{2} \right\rfloor\right)_{n-\left\lfloor \frac{n}{2} \right\rfloor} (4z)^k}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)} - \right. $$

$$ \left. (-1)^n 2^{b-1} G_{2,4}^{1,2}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2},1-\frac{b}{2} \\ \frac{n-b}{2}+1,\frac{b+n}{2},-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n) \end{matrix}\right) \right) /;\ n \in \mathbb{N} $$

07.18.26.0110.01

$$ {}_0\tilde{F}_1(;b;-z)\,J_{-b}\left(2\sqrt{z}\right) = \frac{1}{2\pi}\left(2z^{-\frac{b}{2}}\sin(b\pi) - 2^b\sqrt{\pi}\, G_{3,5}^{2,2}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2},1-\frac{b}{2},\frac{b}{2} \\ 1-\frac{b}{2},\frac{b}{2},\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2} \end{matrix}\right)\right) $$

07.18.26.0111.01

$$ {}_0\tilde{F}_1(;b;-z)\,J_{-b-1}\left(2\sqrt{z}\right) = \frac{2^{b-1}}{\sqrt{\pi}} G_{2,4}^{1,2}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2},1-\frac{b}{2} \\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(-b-1),\frac{1}{2}(1-3b) \end{matrix}\right) - \frac{b\,z^{-\frac{b}{2}-\frac{1}{2}}\sin(b\pi)}{\pi} $$

07.18.26.0034.01

$$ {}_0\tilde{F}_1(;b;-z)\,J_b\left(2\sqrt{z}\right) = \frac{2^{b-1}}{\sqrt{\pi}} G_{1,3}^{1,1}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2} \\ \frac{b}{2},-\frac{b}{2},1-\frac{3b}{2} \end{matrix}\right) $$

07.18.26.0035.01

$${}_0\tilde{F}_1(;b;z)\,J_{b-1}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,z^{\frac{b-1}{2}}\left(z^2\right)^{\frac{1-b}{4}} G_{0,4}^{1,0}\left(\frac{z^2}{4}\,\middle|\,\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{3-3b}{4}\right)$$

07.18.26.0036.01

$${}_0\tilde{F}_1(;b;z)\,J_{1-b}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{1,5}^{2,0}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{b+1}{4}\\ \frac{1-b}{4},\frac{3-b}{4},\frac{3-3b}{4},\frac{b-1}{4},\frac{b+1}{4}\end{matrix}\right)/;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.18.26.0112.01

$${}_0F_1\left(;b;-\frac{z^2}{4}\right)J_\nu(z) = \frac{\Gamma(b)\,2^{b-1}}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{\nu}{2},-\frac{\nu}{2},-b+\frac{\nu}{2}+1,-b-\frac{\nu}{2}+1\end{matrix}\right)/;-b-\nu\notin\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.18.26.0113.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_{-b-n}(z) = \frac{2^{b-1}}{\sqrt{\pi}}\left(2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(-k+\left\lfloor\frac{n}{2}\right\rfloor+1\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}-\right.$$
$$\left.(-1)^n\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{n-b}{2}+1,\frac{b+n}{2},-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n)\end{matrix}\right)\right)/;n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.18.26.0114.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_{-b}(z) = \frac{1}{2\pi}\left(2^{b+1}z^{-b}\sin(b\pi)-2^b\sqrt{\pi}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ 1-\frac{b}{2},\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2}\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.18.26.0115.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_{-b-1}(z) = 2^{b-1}\left(\frac{1}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(-b-1),\frac{1}{2}(1-3b)\end{matrix}\right)-\frac{4\,b\,z^{-b-1}\sin(b\pi)}{\pi}\right)/;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.18.26.0116.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_b(z) = \frac{2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2}\\ \frac{b}{2},-\frac{b}{2},1-\frac{3b}{2}\end{matrix}\right)/;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.18.26.0117.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_{b-1}(z) = \frac{2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z^2\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ \frac{b-1}{2},\frac{1-b}{2},\frac{3(1-b)}{2}\end{matrix}\right)/;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.18.26.0118.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_{1-b}(z) = \frac{2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z^2\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ \frac{1-b}{2},\frac{b-1}{2},\frac{3(1-b)}{2}\end{matrix}\right)/;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

Classical cases involving Bessel *I*

07.18.26.0037.01

$${}_0\tilde{F}_1(;b;z)\,I_\nu\left(2\sqrt{z}\right) = \sqrt{\pi}\,\csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right)2^{b-1}\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{3-2b}{4}\\ \frac{\nu}{2},-\frac{\nu}{2},1-b+\frac{\nu}{2},1-b-\frac{\nu}{2},\frac{3-2b}{4}\end{matrix}\right)/;-b-\nu\notin\mathbb{N}$$

07.18.26.0119.01

$$ {}_0\tilde{F}_1(;b;z)\, I_{-b-n}\left(2\sqrt{z}\right) = \frac{1}{\sqrt{\pi}} \left(2^{-n} z^{-\frac{1}{2}(b+n)} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} \Gamma\left(k-n+\left\lfloor \frac{n}{2} \right\rfloor+\frac{1}{2}\right) \left(1-k+\left\lfloor \frac{n}{2} \right\rfloor\right)_{n-\left\lfloor \frac{n}{2} \right\rfloor} (4z)^k}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)} - (-1)^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{b-\frac{1}{2}} \pi\, G_{3,5}^{1,2}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1}{4}(3-2b) \\ \frac{n-b}{2}+1, \frac{b+n}{2}, -\frac{1}{2}(b+n), 1-\frac{1}{2}(3b+n), \frac{1}{4}(3-2b) \end{matrix}\right) \right) /;\, n \in \mathbb{N} $$

07.18.26.0120.01

$$ {}_0\tilde{F}_1(;b;z)\, I_{-b}\left(2\sqrt{z}\right) = \frac{1}{\sqrt{\pi}} \left(\frac{z^{-\frac{b}{2}} \sin(b\pi)}{\sqrt{\pi}} - 2^{b-\frac{1}{2}} \pi\, G_{3,5}^{1,2}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1}{4}(3-2b) \\ 1-\frac{b}{2}, \frac{b}{2}, -\frac{b}{2}, 1-\frac{3b}{2}, \frac{1}{4}(3-2b) \end{matrix}\right) \right) $$

07.18.26.0121.01

$$ {}_0\tilde{F}_1(;b;z)\, I_{-b-1}\left(2\sqrt{z}\right) = \frac{1}{\sqrt{\pi}} \left(-\frac{b \sin(b\pi)\, z^{-\frac{1}{2}(b+1)}}{\sqrt{\pi}} - 2^{b-\frac{1}{2}} \pi\, G_{3,5}^{1,2}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1}{4}(3-2b) \\ \frac{3-b}{2}, \frac{b+1}{2}, \frac{1}{2}(-b-1), \frac{1}{2}(1-3b), \frac{1}{4}(3-2b) \end{matrix}\right) \right) $$

07.18.26.0122.01

$$ {}_0\tilde{F}_1(;b;z)\, I_b\left(2\sqrt{z}\right) = 2^{b-1}\sqrt{\pi}\, \csc\left(\left(b+\frac{1}{4}\right)\pi\right) G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{matrix} \frac{1-b}{2}, \frac{1}{4}(3-2b) \\ \frac{b}{2}, -\frac{b}{2}, 1-\frac{3b}{2}, \frac{1}{4}(3-2b) \end{matrix}\right) $$

07.18.26.0038.01

$$ {}_0\tilde{F}_1(;b;-z)\, I_{b-1}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}} \sqrt{\pi}\, z^{\frac{b-1}{2}} \left(z^2\right)^{-\frac{b-1}{4}} G_{0,4}^{1,0}\left(\frac{z^2}{4} \,\middle|\, \frac{b-1}{4}, \frac{1-b}{4}, \frac{3-b}{4}, \frac{3-3b}{4}\right) $$

07.18.26.0039.01

$$ {}_0\tilde{F}_1(;b;-z)\, I_{1-b}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}} \sqrt{\pi}\, G_{1,5}^{2,0}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{5-3b}{4} \\ \frac{3-b}{4}, \frac{1-b}{4}, \frac{5-3b}{4}, \frac{3-3b}{4}, \frac{b-1}{4} \end{matrix}\right) /;\, -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2} $$

07.18.26.0123.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) I_\nu(z) = \sqrt{\pi}\, \csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right) 2^{b-1} G_{3,5}^{1,2}\left(z^2 \,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1}{4}(3-2b) \\ \frac{\nu}{2}, -\frac{\nu}{2}, -b+\frac{\nu}{2}+1, -b-\frac{\nu}{2}+1, \frac{1}{4}(3-2b) \end{matrix}\right) /; $$
$$ \neg(-b-\nu \in \mathbb{Z} \land -b-\nu \geq 0) \bigwedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2} $$

07.18.26.0124.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) I_{-b-n}(z) = \frac{2^{b-1}}{\sqrt{\pi}} \left(2 \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} z^{-b+2k-n} \Gamma\left(k-n+\left\lfloor \frac{n}{2} \right\rfloor+\frac{1}{2}\right) \left(1-k+\left\lfloor \frac{n}{2} \right\rfloor\right)_{n-\left\lfloor \frac{n}{2} \right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)} - (-1)^{\left\lfloor \frac{n}{2} \right\rfloor} \sqrt{2}\, \pi\, G_{3,5}^{1,2}\left(z^2 \,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1}{4}(3-2b) \\ \frac{n-b}{2}+1, \frac{b+n}{2}, -\frac{1}{2}(b+n), 1-\frac{1}{2}(3b+n), \frac{1}{4}(3-2b) \end{matrix}\right) \right) /;\, n \in \mathbb{N} \bigwedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2} $$

07.18.26.0125.01

$${}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)I_{-b}(z)=\frac{2^{b-1}}{\sqrt{\pi}}\left(\frac{2\,z^{-b}\sin(b\,\pi)}{\sqrt{\pi}}-\sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\1-\frac{b}{2},\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2},\frac{1}{4}(3-2b)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.18.26.0126.01

$${}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)I_{-b-1}(z)=-\frac{2^{b-1}}{\sqrt{\pi}}\left(\frac{4\,b\sin(b\,\pi)\,z^{-b-1}}{\sqrt{\pi}}+\sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\\frac{1-b}{2}+1,\frac{b+1}{2},\frac{1}{2}(-b-1),\frac{1}{2}(1-3b),\frac{1}{4}(3-2b)\end{matrix}\right)\right)/;$$

$$-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.18.26.0127.01

$${}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)I_b(z)=2^{b-1}\sqrt{\pi}\,\csc\left(\left(b+\frac{1}{4}\right)\pi\right)G_{2,4}^{1,1}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\frac{1}{4}(3-2b)\\\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2},\frac{1}{4}(3-2b)\end{matrix}\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.18.26.0128.01

$${}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)I_{b-1}(z)=\frac{2^{b-1}\,e^{\frac{1}{2}\pi i(1-b)}}{\sqrt{\pi}}G_{1,3}^{1,1}\left(-z^2\,\middle|\,\begin{matrix}1-\frac{b}{2}\\\frac{b-1}{2},\frac{1-b}{2},\frac{3(1-b)}{2}\end{matrix}\right)/;-\pi<\arg(z)\leq 0$$

07.18.26.0129.01

$${}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)I_{1-b}(z)=\frac{2^{b-1}\,e^{\frac{1}{2}\pi i(b-1)}}{\sqrt{\pi}}G_{1,3}^{1,1}\left(-z^2\,\middle|\,\begin{matrix}1-\frac{b}{2}\\\frac{1-b}{2},\frac{b-1}{2},\frac{3(1-b)}{2}\end{matrix}\right)/;-\pi<\arg(z)\leq 0$$

Classical cases involving Bessel Y

07.18.26.0040.01

$${}_0\tilde{F}_1(;b;-z)\,Y_\nu\left(2\sqrt{z}\right)=-\frac{2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1-\nu}{2}\\-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},1-b-\frac{\nu}{2},1-b+\frac{\nu}{2}\end{matrix}\right)/;-b-\nu\notin\mathbb{N}\land -b+\nu\notin\mathbb{N}$$

07.18.26.0130.01

$${}_0\tilde{F}_1(;b;-z)\,Y_{b+n}\left(2\sqrt{z}\right)=\frac{2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(-b+n+1)\\\frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(-b+n+1)\end{matrix}\right)+$$

$$\frac{(-1)^{n+1}\,2^{-n}\,z^{\frac{1}{2}(-b-n)}\csc(b\,\pi)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}/;n\in\mathbb{N}$$

07.18.26.0131.01

$${}_0\tilde{F}_1(;b;-z)\,Y_{-b-n}\left(2\sqrt{z}\right)=\frac{(-1)^n\,2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(b+n+1)\\\frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(b+n+1)\end{matrix}\right)-$$

$$\frac{2^{-n}\,z^{\frac{1}{2}(-b-n)}\cot(b\,\pi)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}/;n\in\mathbb{N}$$

07.18.26.0132.01

$${}_0\tilde{F}_1(;b;-z)\,Y_b\left(2\sqrt{z}\right)=\frac{2^{b-1}}{\sqrt{\pi}}G_{2,4}^{2,1}\left(4\,z\,\middle|\,\begin{matrix}1-\frac{b}{2},\frac{1-b}{2}\\1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2}\end{matrix}\right)-\frac{z^{-\frac{b}{2}}}{\pi}$$

07.18.26.0133.01

$$ {}_0\tilde{F}_1(;b;-z)\,Y_{b+1}\left(2\sqrt{z}\right)=\frac{2^{b-1}}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2}\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}(1-3\,b),\,\frac{1}{2}(-b-1)\end{matrix}\right)-\frac{b\,z^{-\frac{b}{2}-\frac{1}{2}}}{\pi} $$

07.18.26.0134.01

$$ {}_0\tilde{F}_1(;b;-z)\,Y_{-b}\left(2\sqrt{z}\right)=-\frac{\cos(b\,\pi)\,z^{-\frac{b}{2}}}{2\,\pi}-\frac{2^{b-1}}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,\frac{b+1}{2}\\ -\frac{b}{2},\,\frac{b}{2},\,1-\frac{3\,b}{2},\,\frac{b+1}{2}\end{matrix}\right) $$

07.18.26.0135.01

$$ {}_0\tilde{F}_1(;b;-z)\,Y_{-b}\left(2\sqrt{z}\right)=\frac{2^{b-1}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{b+1}{2}\\ 1-\frac{b}{2},\,\frac{b}{2},\,1-\frac{3\,b}{2},\,-\frac{b}{2},\,\frac{b+1}{2}\end{matrix}\right)-\frac{z^{-\frac{b}{2}}\cos(b\,\pi)}{\pi} $$

07.18.26.0136.01

$$ {}_0\tilde{F}_1(;b;-z)\,Y_{-b-1}\left(2\sqrt{z}\right)=\frac{b\,z^{-\frac{b}{2}-\frac{1}{2}}\cos(b\,\pi)}{\pi}-\frac{2^{b-1}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{b+2}{2}\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}(1-3\,b),\,\frac{1}{2}(-b-1),\,\frac{b+2}{2}\end{matrix}\right) $$

07.18.26.0041.01

$$ {}_0\tilde{F}_1(;b;-z)\,Y_{b-1}\left(2\sqrt{z}\right)=-\frac{2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{2,0}\left(4\,z\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ \frac{1-b}{2},\,\frac{b-1}{2},\,\frac{3-3\,b}{2}\end{matrix}\right) $$

07.18.26.0042.01

$$ {}_0\tilde{F}_1(;b;-z)\,Y_{b-2}\left(2\sqrt{z}\right)=\frac{2^{b-1}}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(4\,z\,\middle|\,\begin{matrix}1-\frac{b}{2},\,\frac{1-b}{2}\\ 1-\frac{b}{2},\,\frac{b}{2}-1,\,2-\frac{3\,b}{2},\,-\frac{b}{2}\end{matrix}\right) $$

07.18.26.0043.01

$$ {}_0\tilde{F}_1(;b;-z)\,Y_{b-3}\left(2\sqrt{z}\right)=\frac{2^{b-1}}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2}\\ \frac{3-b}{2},\,\frac{b-3}{2},\,\frac{5-3\,b}{2},\,-\frac{b+1}{2}\end{matrix}\right) $$

07.18.26.0045.01

$$ {}_0\tilde{F}_1(;b;z)\,Y_{b-1}\left(2\sqrt{z}\right)=-2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}1-\frac{b}{4},\,\frac{2-b}{4}\\ \frac{3-b}{4},\,\frac{1-b}{4},\,\frac{b-1}{4},\,1-\frac{b}{4},\,\frac{2-b}{4},\,\frac{3-3\,b}{4}\end{matrix}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0044.01

$$ {}_0\tilde{F}_1(;b;z)\,Y_{1-b}\left(2\sqrt{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{b-2}{4},\,\frac{b}{4}\\ \frac{3-b}{4},\,\frac{1-b}{4},\,\frac{b-1}{4},\,\frac{b-2}{4},\,\frac{3-3\,b}{4},\,\frac{b}{4}\end{matrix}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0137.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_\nu(z)=-\frac{2^{b-1}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1-\nu}{2}\\ -\frac{\nu}{2},\,\frac{\nu}{2},\,\frac{1-\nu}{2},\,-b-\frac{\nu}{2}+1,\,-b+\frac{\nu}{2}+1\end{matrix}\right)/; $$

$$ \neg(-b-\nu\in\mathbb{Z}\land -b-\nu\ge 0)\bigwedge\neg(\nu-b\in\mathbb{Z}\land \nu-b\ge 0)\bigwedge -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0138.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{b+n}(z)=\frac{2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(-b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(-b+n+1)\end{array}\right)+ $$

$$ \frac{(-1)^{n+1}2^b z^{-b-n}\csc(b\pi)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\;/;\;n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0139.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{-b-n}(z)=\frac{(-1)^n 2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(b+n+1)\end{array}\right)- $$

$$ \frac{2^b z^{-b-n}\cot(b\pi)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}\;/;\;n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0140.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_b(z)=\frac{2^{b-1}}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{array}{c}1-\frac{b}{2},\frac{1-b}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2}\end{array}\right)-\frac{2^b z^{-b}}{\pi}\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0141.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{b+1}(z)=\frac{2^{b-1}}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1)\end{array}\right)-\frac{2^{b+1}b\,z^{-b-1}}{\pi}\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0142.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{-b}(z)=-\frac{2^{b-1}\cos(b\pi)\,z^{-b}}{\pi}-\frac{2^{b-1}}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},\frac{b+1}{2}\\ -\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},\frac{b+1}{2}\end{array}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0143.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{-b}(z)=\frac{2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{b+1}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2},\frac{b+1}{2}\end{array}\right)-\frac{2^b z^{-b}\cos(b\pi)}{\pi}\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0144.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{-b-1}(z)=\frac{2^{b+1}b\,z^{-b-1}\cos(b\pi)}{\pi}-\frac{2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{b+2}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1),\frac{b+2}{2}\end{array}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.18.26.0046.01

$$ {}_0\tilde{F}_1\left(;b;\sqrt{z}\right)Y_{1-b}\left(2\sqrt[4]{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z}{4}\,\middle|\,\begin{array}{c}\frac{b-2}{4},\frac{b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{b-2}{4},\frac{3-3b}{4},\frac{b}{4}\end{array}\right) $$

07.18.26.0047.01

$$ {}_0\tilde{F}_1\left(;b;\sqrt{z}\right)Y_{b-1}\left(2\sqrt[4]{z}\right)=-2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z}{4}\,\middle|\,\begin{array}{c}1-\frac{b}{4},\frac{2-b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},1-\frac{b}{4},\frac{2-b}{4},\frac{3-3b}{4}\end{array}\right) $$

Classical cases involving Bessel *K*

07.18.26.0048.01

$$ {}_0\tilde{F}_1(;b;z)\,K_\nu\left(2\sqrt{z}\right)=\frac{2^{b-2}}{\sqrt{\pi}}\,G_{2,4}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},1-b-\frac{\nu}{2},1-b+\frac{\nu}{2}\end{matrix}\right)/;\,-b-\nu\notin\mathbb{N}\bigwedge -b+\nu\notin\mathbb{N} $$

07.18.26.0145.01

$$ {}_0\tilde{F}_1(;b;z)\,K_{b+n}\left(2\sqrt{z}\right)= $$
$$ \pi(-1)^{n-1}2^{b-\frac{3}{2}}\left(\sqrt{\pi}\,\csc\left(\frac{1}{4}(4b+(-1)^n)\pi\right)G_{4,6}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b)\\ 1-\frac{b-n}{2},\frac{b+n}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b),-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n)\end{matrix}\right)-\right. $$
$$ \left.\frac{\csc(b\pi)\,2^{-b-n+\frac{1}{2}}z^{-\frac{b+n}{2}}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}4^k z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)/;\,n\in\mathbb{N} $$

07.18.26.0146.01

$$ {}_0\tilde{F}_1(;b;z)\,K_{-b-n}\left(2\sqrt{z}\right)= $$
$$ \pi(-1)^{n-1}2^{b-\frac{3}{2}}\left(\sqrt{\pi}\,\csc\left(\frac{1}{4}(4b+(-1)^n)\pi\right)G_{4,6}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b)\\ 1-\frac{b-n}{2},\frac{b+n}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b),-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n)\end{matrix}\right)-\right. $$
$$ \left.\frac{\csc(b\pi)\,2^{-b-n+\frac{1}{2}}z^{-\frac{b+n}{2}}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}4^k z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)/;\,n\in\mathbb{N} $$

07.18.26.0147.01

$$ {}_0\tilde{F}_1(;b;z)\,K_b\left(2\sqrt{z}\right)=\frac{1}{2}\left(z^{-\frac{b}{2}}-2^{b-\frac{1}{2}}\pi^{3/2}\csc\left(\frac{1}{4}(4b+1)\pi\right)G_{4,6}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b),\frac{1}{4}(2b+1)\\ 1-\frac{b}{2},\frac{b}{2},\frac{1}{4}(3-2b),1-\frac{3b}{2},-\frac{b}{2},\frac{1}{4}(2b+1)\end{matrix}\right)\right) $$

07.18.26.0148.01

$$ {}_0\tilde{F}_1(;b;z)\,K_{b+1}\left(2\sqrt{z}\right)= $$
$$ \frac{1}{2}\left(b\,z^{-\frac{1}{2}(b+1)}+2^{b-\frac{1}{2}}\pi^{3/2}\csc\left(\frac{1}{4}(4b-1)\pi\right)G_{4,6}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b),\frac{1}{4}(2b+1)\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{4}(3-2b),-\frac{1}{2}(b+1),\frac{1}{4}(2b+1)\end{matrix}\right)\right) $$

07.18.26.0149.01

$$ {}_0\tilde{F}_1(;b;z)\,K_{-b}\left(2\sqrt{z}\right)=\frac{1}{2}\left(z^{-\frac{b}{2}}-2^{b-\frac{1}{2}}\pi^{3/2}\csc\left(\frac{1}{4}(4b+1)\pi\right)G_{4,6}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b),\frac{1}{4}(2b+1)\\ 1-\frac{b}{2},\frac{b}{2},\frac{1}{4}(3-2b),1-\frac{3b}{2},-\frac{b}{2},\frac{1}{4}(2b+1)\end{matrix}\right)\right) $$

07.18.26.0150.01

$$ {}_0\tilde{F}_1(;b;z)\,K_{-b-1}\left(2\sqrt{z}\right)= $$
$$ \frac{1}{2}\left(b\,z^{-\frac{1}{2}(b+1)}+2^{b-\frac{1}{2}}\pi^{3/2}\csc\left(\frac{1}{4}(4b-1)\pi\right)G_{4,6}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b),\frac{1}{4}(2b+1)\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{4}(3-2b),-\frac{1}{2}(b+1),\frac{1}{4}(2b+1)\end{matrix}\right)\right) $$

07.18.26.0049.01

$$ {}_0\tilde{F}_1(;b;-z)\,K_{b-1}\left(2\sqrt{z}\right) = \frac{2^{-\frac{b+3}{2}}}{\sqrt{\pi}}\, G_{0,4}^{3,0}\left(\frac{z^2}{4}\,\middle|\, \frac{3-b}{4}, \frac{1-b}{4}, \frac{b-1}{4}, \frac{3-3b}{4}\right) /;\ \mathrm{Re}(z) > 0 $$

07.18.26.0050.01

$$ {}_0\tilde{F}_1(;b;-z)\,K_{1-b}\left(2\sqrt{z}\right) = \frac{2^{-\frac{b+3}{2}}}{\sqrt{\pi}}\, G_{0,4}^{3,0}\left(\frac{z^2}{4}\,\middle|\, \frac{3-b}{4}, \frac{1-b}{4}, \frac{b-1}{4}, \frac{3-3b}{4}\right) /;\ \mathrm{Re}(z) > 0 $$

07.18.26.0151.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) K_\nu(z) = \frac{2^{b-2}}{\sqrt{\pi}}\, G_{2,4}^{2,2}\left(z^2\,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, -b-\frac{\nu}{2}+1, -b+\frac{\nu}{2}+1 \end{matrix}\right) /;\ -b-\nu \notin \mathbb{N} \bigwedge \nu-b \notin \mathbb{N} \bigwedge -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} $$

07.18.26.0152.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) K_{b+n}(z) = $$

$$ \pi(-1)^{n-1} 2^{b-\frac{3}{2}} \left(\sqrt{\pi}\,\csc\left(\frac{1}{4}(4b+(-1)^n)\pi\right) G_{4,6}^{2,2}\left(z^2\,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1}{4}(2b+1), \frac{1}{4}(3-2b) \\ 1-\frac{b-n}{2}, \frac{b+n}{2}, \frac{1}{4}(2b+1), \frac{1}{4}(3-2b), -\frac{1}{2}(b+n), 1-\frac{1}{2}(3b+n) \end{matrix}\right) - \right. $$

$$ \left. \frac{\csc(b\pi)\sqrt{2}}{\sqrt{\pi}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} z^{-b+2k-n}\, \Gamma\left(k-n+\left\lfloor \frac{n}{2} \right\rfloor+\frac{1}{2}\right) \left(1-k+\left\lfloor \frac{n}{2} \right\rfloor\right)_{n-\left\lfloor \frac{n}{2} \right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)} \right) /;\ n \in \mathbb{N} \bigwedge -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} $$

07.18.26.0153.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) K_{-b-n}(z) = $$

$$ \pi(-1)^{n-1} 2^{b-\frac{3}{2}} \left(\sqrt{\pi}\,\csc\left(\frac{1}{4}(4b+(-1)^n)\pi\right) G_{4,6}^{2,2}\left(z^2\,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1}{4}(2b+1), \frac{1}{4}(3-2b) \\ 1-\frac{b-n}{2}, \frac{b+n}{2}, \frac{1}{4}(2b+1), \frac{1}{4}(3-2b), -\frac{1}{2}(b+n), 1-\frac{1}{2}(3b+n) \end{matrix}\right) - \right. $$

$$ \left. \frac{\csc(b\pi)\sqrt{2}}{\sqrt{\pi}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} z^{-b+2k-n}\, \Gamma\left(k-n+\left\lfloor \frac{n}{2} \right\rfloor+\frac{1}{2}\right) \left(1-k+\left\lfloor \frac{n}{2} \right\rfloor\right)_{n-\left\lfloor \frac{n}{2} \right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)} \right) /;\ n \in \mathbb{N} \bigwedge -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} $$

07.18.26.0154.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) K_b(z) = 2^{b-1}\left(z^{-b} - \frac{\pi^{3/2}\csc\left(\frac{1}{4}(4b+1)\pi\right)}{\sqrt{2}}\, G_{4,6}^{2,2}\left(z^2\,\middle|\, \begin{matrix} \frac{1-b}{2}, 1-\frac{b}{2}, \frac{1}{4}(3-2b), \frac{1}{4}(2b+1) \\ 1-\frac{b}{2}, \frac{b}{2}, \frac{1}{4}(3-2b), 1-\frac{3b}{2}, -\frac{b}{2}, \frac{1}{4}(2b+1) \end{matrix}\right)\right) /; $$

$$ -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} $$

07.18.26.0155.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_{b+1}(z) = 2^{b-1}\left(2\,b\,z^{-b-1}+\sqrt{\frac{\pi}{2}}\,\pi\csc\left(\frac{1}{4}(4\,b-1)\,\pi\right)G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2\,b),\frac{1}{4}(2\,b+1)\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3\,b),\frac{1}{4}(3-2\,b),-\frac{1}{2}(b+1),\frac{1}{4}(2\,b+1)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.18.26.0156.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_{-b}(z) = 2^{b-1}\left(z^{-b}-\frac{\pi^{3/2}\csc\left(\frac{1}{4}(4\,b+1)\,\pi\right)}{\sqrt{2}}G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2\,b),\frac{1}{4}(2\,b+1)\\ 1-\frac{b}{2},\frac{b}{2},\frac{1}{4}(3-2\,b),1-\frac{3\,b}{2},-\frac{b}{2},\frac{1}{4}(2\,b+1)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.18.26.0157.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_{-b-1}(z) = 2^{b-1}\left(2\,b\,z^{-b-1}+\sqrt{\frac{\pi}{2}}\,\pi\csc\left(\frac{1}{4}(4\,b-1)\,\pi\right)G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2\,b),\frac{1}{4}(2\,b+1)\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3\,b),\frac{1}{4}(3-2\,b),-\frac{1}{2}(b+1),\frac{1}{4}(2\,b+1)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.18.26.0051.01

$$ {}_0\tilde{F}_1\left(;b;-\sqrt{z}\right)K_{b-1}\left(2\sqrt[4]{z}\right) = \frac{2^{-\frac{b+3}{2}}}{\sqrt{\pi}}G_{0,4}^{3,0}\left(\frac{z}{4}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{3-3\,b}{4}\right) $$

07.18.26.0052.01

$$ {}_0\tilde{F}_1\left(;b;-\sqrt{z}\right)K_{1-b}\left(2\sqrt[4]{z}\right) = \frac{2^{-\frac{b+3}{2}}}{\sqrt{\pi}}G_{0,4}^{3,0}\left(\frac{z}{4}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{3-3\,b}{4}\right) $$

Classical cases involving ${}_0F_1$

07.18.26.0053.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;c;z) = \sqrt{\pi}\,2^{b+c-2}\,\Gamma(c)\,G_{3,5}^{1,2}\left(4\,z\,\middle|\,\begin{matrix}1-\frac{b+c}{2},\frac{3-b-c}{2},\frac{1}{2}\\ 0,1-b,1-c,2-b-c,\frac{1}{2}\end{matrix}\right)/;1-b-c\notin\mathbb{N} $$

07.18.26.0054.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;c;z) = \frac{2^{b+c-2}\,\Gamma(c)}{\sqrt{\pi}}G_{2,4}^{1,2}\left(-4\,z\,\middle|\,\begin{matrix}1-\frac{b+c}{2},\frac{3-b-c}{2}\\ 0,1-b,1-c,2-b-c\end{matrix}\right)/;1-b-c\notin\mathbb{N} $$

07.18.26.0158.01

$$ {}_0\tilde{F}_1(;-b-n+1;z)\,{}_0F_1(;b;z) = \frac{\Gamma(b)}{2^{n+1}\sqrt{\pi}} \left(2\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{\left((-1)^{\lfloor \frac{n+1}{2} \rfloor}\Gamma\left(k-n+\lfloor \frac{n}{2} \rfloor+\frac{1}{2}\right)\right)\left(1-k+\lfloor \frac{n}{2} \rfloor\right)_{n-\lfloor \frac{n}{2} \rfloor}(4z)^k}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)} - (-1)^n\,\pi\, G_{3,5}^{1,2}\left(4z \,\middle|\, \begin{matrix} \frac{n+1}{2}, \frac{n+2}{2}, \frac{1}{2} \\ n+1, b+n, 0, 1-b, \frac{1}{2} \end{matrix}\right)\right) /; n \in \mathbb{N} $$

07.18.26.0159.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;1-b;z) = \frac{1}{\Gamma(b)} - \frac{\pi^{3/2}\csc(b\pi)}{2\,\Gamma(b)} G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{matrix} 1, \frac{1}{2} \\ 1, b, 0, 1-b \end{matrix}\right) $$

07.18.26.0160.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;-b;z) = \frac{1}{4}\sqrt{\pi}\;\Gamma(-b)\,G_{3,5}^{1,2}\left(4z \,\middle|\, \begin{matrix} 1, \frac{3}{2}, \frac{1}{2} \\ 2, b+1, 0, 1-b, \frac{1}{2} \end{matrix}\right) + \frac{1}{\Gamma(b)} $$

07.18.26.0055.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;2-b;z) = \sqrt{\pi}\;\Gamma(2-b)\,G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, b-1, 1-b, \frac{1}{2} \end{matrix}\right) $$

07.18.26.0056.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;2-b;z) = \frac{\Gamma(2-b)}{\sqrt{\pi}} G_{1,3}^{1,1}\left(-4z \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, b-1, 1-b \end{matrix}\right) $$

07.18.26.0057.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;b+1;z) = 2^{2b-1}\sqrt{\pi}\;\Gamma(b+1)\,G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{matrix} \frac{1}{2}-b, \frac{1}{2} \\ 0, -b, 1-2b, \frac{1}{2} \end{matrix}\right) $$

07.18.26.0058.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;b+1;z) = \frac{2^{2b-1}\,\Gamma(b+1)}{\sqrt{\pi}} G_{1,3}^{1,1}\left(-4z \,\middle|\, \begin{matrix} \frac{1}{2}-b \\ 0, -b, 1-2b \end{matrix}\right) $$

07.18.26.0059.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;b;z) = 2^{2b-2}\sqrt{\pi}\;\Gamma(b)\,G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{matrix} \frac{3}{2}-b, \frac{1}{2} \\ 0, 1-b, 2-2b, \frac{1}{2} \end{matrix}\right) $$

07.18.26.0060.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;b;z) = \frac{2^{2b-2}\,\Gamma(b)}{\sqrt{\pi}} G_{1,3}^{1,1}\left(-4z \,\middle|\, \begin{matrix} \frac{3}{2}-b \\ 0, 1-b, 2-2b \end{matrix}\right) $$

07.18.26.0061.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;b-1;z) = 2^{2b-3}\sqrt{\pi}\;\Gamma(b-1)\,G_{2,4}^{1,1}\left(4z \,\middle|\, \begin{matrix} \frac{3}{2}-b, \frac{1}{2} \\ 0, 1-b, 3-2b, \frac{1}{2} \end{matrix}\right) $$

07.18.26.0062.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;b-1;z) = \frac{2^{2b-3}\,\Gamma(b-1)}{\sqrt{\pi}} G_{1,3}^{1,1}\left(-4z \,\middle|\, \begin{matrix} \frac{3}{2}-b \\ 0, 1-b, 3-2b \end{matrix}\right) $$

07.18.26.0063.01

$${}_0\tilde{F}_1(;b;z)\,{}_0F_1(;b;-z) = \sqrt{\pi}\;2^{\frac{1-b}{2}}\left(z^2\right)^{\frac{1-b}{4}}\Gamma(b)\,G_{0,4}^{1,0}\!\left(\frac{z^2}{4}\,\middle|\,\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{3(1-b)}{4}\right)$$

07.18.26.0064.01

$${}_0\tilde{F}_1(;b;z)\,{}_0F_1(;2-b;-z) = \sqrt{\pi}\;\Gamma(2-b)\,G_{1,5}^{2,0}\!\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{b}{2}\\ 0,\frac{1}{2},\frac{1-b}{2},\frac{b-1}{2},\frac{b}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{2}\;\arg(z)\le\frac{\pi}{2}$$

Generalized cases involving cos

07.18.26.0161.01

$$\cos(z)\,{}_0\tilde{F}_1\!\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{1}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1}{2}\end{matrix}\right)$$

07.18.26.0065.01

$$\cos(a+z)\,{}_0\tilde{F}_1\!\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4},\frac{a}{\pi}+\frac{1}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{a}{\pi}+\frac{1}{2}\end{matrix}\right)$$

Generalized cases involving sin

07.18.26.0066.01

$$\sin(z)\,{}_0\tilde{F}_1\!\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,G_{2,4}^{1,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4}\\ \frac{1}{2},0,1-b,\frac{3}{2}-b\end{matrix}\right)$$

07.18.26.0067.01

$$\sin(a+z)\,{}_0\tilde{F}_1\!\left(;\nu+1;-\frac{z^2}{4}\right) = 2^{\nu-\frac{1}{2}}\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-2\nu}{4},\frac{3-2\nu}{4},\frac{a}{\pi}\\ 0,\frac{1}{2},-\nu,\frac{1}{2}-\nu,\frac{a}{\pi}\end{matrix}\right)$$

Generalized cases involving cosh

07.18.26.0162.01

$$\cosh(z)\,{}_0\tilde{F}_1\!\left(;b;\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,\pi\sec\!\left(\frac{b\pi}{2}\right)G_{3,5}^{1,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{1-b}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1-b}{2}\end{matrix}\right)$$

07.18.26.0163.01

$$\cosh(a+z)\,{}_0\tilde{F}_1\!\left(;b;\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,G_{3,5}^{2,2}\!\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{1}{2}+\frac{i\,a}{\pi}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1}{2}+\frac{i\,a}{\pi}\end{matrix}\right)$$

Generalized cases involving sinh

07.18.26.0068.01

$$\sinh(z)\,{}_0\tilde{F}_1\!\left(;b;\frac{z^2}{4}\right) = -2^{b-\frac{3}{2}}\,\pi\csc\!\left(\frac{b\pi}{2}\right)G_{3,5}^{1,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4},\frac{1-b}{2}\\ \frac{1}{2},0,\frac{1-b}{2},1-b,\frac{3}{2}-b\end{matrix}\right)$$

07.18.26.0164.01

$$\sinh(z)\,{}_0\tilde{F}_1\!\left(;b;\frac{z^2}{4}\right) = -\frac{2^{b-\frac{3}{2}}}{z}\,G_{2,4}^{1,2}\!\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{5}{4}-\frac{b}{2},\frac{7}{4}-\frac{b}{2}\\ 1,\frac{1}{2},\frac{3}{2}-b,2-b\end{matrix}\right)$$

07.18.26.0165.01

$$\sinh(a+z)\,{}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)=-2^{b-\frac{3}{2}}\,i\,G_{3,5}^{2,2}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{i\,a}{\pi}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{i\,a}{\pi}\end{array}\right)$$

Generalized cases involving Ai

07.18.26.0069.01

$$\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z)=\frac{2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b)\\ 0,\frac{1}{3},1-b,\frac{4}{3}-b\end{array}\right)$$

07.18.26.0166.01

$$\mathrm{Ai}(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=\frac{2^{b-\frac{7}{3}}}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b)\\ 0,\frac{1}{3},1-b,\frac{4}{3}-b\end{array}\right)$$

Generalized cases involving Ai′

07.18.26.0070.01

$$\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z)=-\frac{2^{b-\frac{8}{3}}\sqrt[6]{3}}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\frac{1}{6}(8-3b)\\ 0,\frac{2}{3},1-b,\frac{5}{3}-b\end{array}\right)$$

07.18.26.0167.01

$$\mathrm{Ai}'(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=-\frac{2^{b-\frac{8}{3}}\sqrt[6]{3}}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\frac{1}{6}(8-3b)\\ 0,\frac{2}{3},1-b,\frac{5}{3}-b\end{array}\right)$$

Generalized cases involving Bi

07.18.26.0071.01

$$\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z)=\frac{2^{b-\frac{1}{3}}\sqrt{\pi}}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b),\frac{1}{6},\frac{2}{3}\\ 0,\frac{1}{3},\frac{1}{6},\frac{2}{3},1-b,\frac{4}{3}-b\end{array}\right)$$

07.18.26.0168.01

$$\mathrm{Bi}(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=\frac{2^{b-\frac{1}{3}}\sqrt{\pi}}{\sqrt[6]{3}}\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b),\frac{1}{6},\frac{2}{3}\\ 0,\frac{1}{3},\frac{1}{6},\frac{2}{3},1-b,\frac{4}{3}-b\end{array}\right)$$

Generalized cases involving Bi′

07.18.26.0072.01

$$\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z)=2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,G_{4,6}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\frac{1}{6}(8-3b),\frac{1}{3},\frac{5}{6}\\ 0,\frac{2}{3},\frac{1}{3},\frac{5}{6},1-b,\frac{5}{3}-b\end{array}\right)$$

07.18.26.0169.01

$$\mathrm{Bi}'(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\frac{1}{6}(8-3b),\frac{1}{3},\frac{5}{6}\\ 0,\frac{2}{3},\frac{1}{3},\frac{5}{6},1-b,\frac{5}{3}-b\end{array}\right)$$

Classical cases for products of ${}_0\tilde{F}_1$

07.18.26.0170.01

$${}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;b;-z)=\sqrt{\pi}\;2^{1-b}\,G_{0,4}^{1,0}\!\left(\frac{z}{2},\frac{1}{2}\;\middle|\;0,\tfrac{1}{2}-\tfrac{b}{2},1-\tfrac{b}{2},1-b\right)$$

07.18.26.0171.01

$${}_0\tilde{F}_1(;b;z)\,{}_0\tilde{F}_1(;2-b;-z)=\sqrt{\pi}\;G_{1,5}^{2,0}\!\left(-\frac{z}{2},\frac{1}{2}\;\middle|\;\begin{matrix}1-\frac{b}{2}\\0,\frac{1}{2},1-\frac{b}{2},\frac{1-b}{2},\frac{b-1}{2}\end{matrix}\right)$$

Generalized cases involving Bessel J

07.18.26.0074.01

$${}_0\tilde{F}_1(;b;z)\,J_{b-1}\left(2\sqrt{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\;G_{0,4}^{1,0}\!\left(\frac{z}{2},\frac{1}{2}\;\middle|\;\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{3-3b}{4}\right)$$

07.18.26.0075.01

$${}_0\tilde{F}_1(;b;z)\,J_{1-b}\left(2\sqrt{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\;G_{1,5}^{2,0}\!\left(\frac{z}{2},\frac{1}{2}\;\middle|\;\begin{matrix}\frac{b+1}{4}\\\frac{3-b}{4},\frac{1-b}{4},\frac{3-3b}{4},\frac{b-1}{4},\frac{b+1}{4}\end{matrix}\right)$$

07.18.26.0172.01

$${}_0\tilde{F}_1\!\left(;b;\frac{z^2}{4}\right)J_{b-1}(z)=2^{\frac{1-b}{2}}\sqrt{\pi}\;G_{0,4}^{1,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\;\middle|\;\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{1}{4}(3-3b)\right)$$

07.18.26.0173.01

$${}_0\tilde{F}_1\!\left(;b;\frac{z^2}{4}\right)J_{1-b}(z)=2^{\frac{1-b}{2}}\sqrt{\pi}\;G_{1,5}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\;\middle|\;\begin{matrix}\frac{b+1}{4}\\\frac{1-b}{4},\frac{3-b}{4},\frac{1}{4}(3-3b),\frac{b-1}{4},\frac{b+1}{4}\end{matrix}\right)$$

07.18.26.0073.01

$${}_0\tilde{F}_1\!\left(;b;-\frac{z^2}{4}\right)J_\nu(z)=\frac{2^{b-1}}{\sqrt{\pi}}\,G_{2,4}^{1,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\\frac{\nu}{2},-\frac{\nu}{2},1-b+\frac{\nu}{2},1-b-\frac{\nu}{2}\end{matrix}\right)/;\,-b-\nu\notin\mathbb{N}$$

07.18.26.0174.01

$${}_0\tilde{F}_1\!\left(;b;-\frac{z^2}{4}\right)J_{-b-n}(z)=\frac{2^{b-1}}{\sqrt{\pi}}\left(2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(-k+\left\lfloor\frac{n}{2}\right\rfloor+1\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}-\right.$$
$$\left.(-1)^n\,G_{2,4}^{1,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\\frac{n-b}{2}+1,\frac{b+n}{2},-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n)\end{matrix}\right)\right)/;\,n\in\mathbb{N}$$

07.18.26.0175.01

$${}_0\tilde{F}_1\!\left(;b;-\frac{z^2}{4}\right)J_{-b}(z)=\frac{1}{2\pi}\left(2^{b+1}z^{-b}\sin(b\pi)-2^b\sqrt{\pi}\;G_{2,4}^{1,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\1-\frac{b}{2},\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2}\end{matrix}\right)\right)$$

07.18.26.0176.01

$${}_0\tilde{F}_1\!\left(;b;-\frac{z^2}{4}\right)J_{-b-1}(z)=2^{b-1}\left(\frac{1}{\sqrt{\pi}}\,G_{2,4}^{1,2}\!\left(z,\frac{1}{2}\;\middle|\;\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\\frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(-b-1),\frac{1}{2}(1-3b)\end{matrix}\right)-\frac{4\,b\,z^{-b-1}\sin(b\pi)}{\pi}\right)$$

07.18.26.0177.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_b(z)=\frac{2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2}\\ \frac{b}{2},-\frac{b}{2},1-\frac{3b}{2}\end{array}\right)$$

07.18.26.0178.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_{b-1}(z)=\frac{2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\frac{b}{2}\\ \frac{b-1}{2},\frac{1-b}{2},\frac{3(1-b)}{2}\end{array}\right)$$

07.18.26.0179.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)J_{1-b}(z)=\frac{2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\frac{b}{2}\\ \frac{1-b}{2},\frac{b-1}{2},\frac{3(1-b)}{2}\end{array}\right)$$

Generalized cases involving Bessel *I*

07.18.26.0076.01

$${}_0\tilde{F}_1(;b;-z)\,I_{b-1}\left(2\sqrt{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{0,4}^{1,0}\!\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{3-3b}{4}\right)$$

07.18.26.0077.01

$${}_0\tilde{F}_1(;b;-z)\,I_{1-b}\left(2\sqrt{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{1,5}^{2,0}\!\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{5-3b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{5-3b}{4},\frac{3-3b}{4},\frac{b-1}{4}\end{array}\right)$$

07.18.26.0180.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)I_{b-1}(z)=2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{0,4}^{1,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{1}{4}(3-3b)\right)$$

07.18.26.0181.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)I_{1-b}(z)=2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{1,5}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4}(5-3b)\\ \frac{3-b}{4},\frac{1-b}{4},\frac{1}{4}(5-3b),\frac{1}{4}(3-3b),\frac{b-1}{4}\end{array}\right)$$

07.18.26.0182.01

$${}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)I_\nu(z)=\sqrt{\pi}\,\csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right)2^{b-1}\,G_{3,5}^{1,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{\nu}{2},-\frac{\nu}{2},-b+\frac{\nu}{2}+1,-b-\frac{\nu}{2}+1,\frac{1}{4}(3-2b)\end{array}\right)/;\,-b-\nu\notin\mathbb{N}$$

07.18.26.0183.01

$${}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)I_{-b-n}(z)=\frac{2^{b-1}}{\sqrt{\pi}}\left(2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}-\right.$$
$$\left.(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\sqrt{2}\,\pi\,G_{3,5}^{1,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{n-b}{2}+1,\frac{b+n}{2},-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n),\frac{1}{4}(3-2b)\end{array}\right)\right)/;\,n\in\mathbb{N}$$

07.18.26.0184.01

$${}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)I_{-b}(z)=\frac{2^b z^{-b}\sin(b\pi)}{\pi}-2^{b-\frac{1}{2}}\sqrt{\pi}\,G_{4,6}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{b}{2},\frac{1}{4}(3-2b)\\ 1-\frac{b}{2},\frac{b}{2},\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2},\frac{1}{4}(3-2b)\end{array}\right)$$

07.18.26.0185.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) I_{-b-1}(z) = -\frac{2^{b-1}}{\sqrt{\pi}}\left(\frac{4\,b\sin(b\,\pi)\,z^{-b-1}}{\sqrt{\pi}} + \sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2\,b)\\ \frac{1-b}{2}+1,\frac{b+1}{2},\frac{1}{2}(-b-1),\frac{1}{2}(1-3\,b),\frac{1}{4}(3-2\,b)\end{matrix}\right)\right) $$

07.18.26.0186.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) I_b(z) = 2^{b-1}\sqrt{\pi}\,\csc\left(\left(b+\frac{1}{4}\right)\pi\right) G_{2,4}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},\frac{1}{4}(3-2\,b)\\ \frac{b}{2},-\frac{b}{2},1-\frac{3\,b}{2},\frac{1}{4}(3-2\,b)\end{matrix}\right) $$

07.18.26.0187.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) I_{b-1}(z) = 2^{b-1}\sqrt{\pi}\,\csc\left(\left(b-\frac{1}{4}\right)\pi\right) G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2\,b)\\ \frac{b-1}{2},\frac{1-b}{2},\frac{1-b}{2},\frac{1}{2}(-3)(b-1),\frac{1}{4}(3-2\,b)\end{matrix}\right) $$

07.18.26.0188.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) I_{1-b}(z) = 2^{b-\frac{1}{2}}\sqrt{\pi}\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2\,b)\\ \frac{1-b}{2},\frac{b-1}{2},\frac{1}{2}(-3)(b-1),\frac{1-b}{2},\frac{1}{4}(3-2\,b)\end{matrix}\right) $$

07.18.26.0189.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) I_{b-1}(z) = 2^{\frac{1}{2}-\frac{b}{2}}\sqrt{\pi}\,G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{3(1-b)}{4}\right) $$

07.18.26.0190.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) I_{1-b}(z) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{1,5}^{2,0}\left(\frac{z}{2^{3/2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(5-3\,b)\\ \frac{3-b}{4},\frac{1-b}{4},\frac{1}{4}(5-3\,b),\frac{1}{4}(3-3\,b),\frac{b-1}{4}\end{matrix}\right) $$

Generalized cases involving Bessel *Y*

07.18.26.0078.01

$$ {}_0\tilde{F}_1(;b;z)\,Y_{b-1}\left(2\sqrt{z}\right) = -2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{b}{4},\frac{2-b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},1-\frac{b}{4},\frac{2-b}{4},\frac{3-3\,b}{4}\end{matrix}\right) $$

07.18.26.0079.01

$$ {}_0\tilde{F}_1(;b;z)\,Y_{1-b}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{b-2}{4},\frac{b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{b-2}{4},\frac{3-3\,b}{4},\frac{b}{4}\end{matrix}\right) $$

07.18.26.0191.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) Y_{b-1}(z) = -2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}1-\frac{b}{4},\frac{2-b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},1-\frac{b}{4},\frac{2-b}{4},\frac{1}{4}(3-3\,b)\end{matrix}\right) $$

07.18.26.0192.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right) Y_{1-b}(z) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{b-2}{4},\frac{b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{b-2}{4},\frac{1}{4}(3-3\,b),\frac{b}{4}\end{matrix}\right) $$

07.18.26.0193.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) Y_\nu(z) = -\frac{2^{b-1}}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1-\nu}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},-b-\frac{\nu}{2}+1,-b+\frac{\nu}{2}+1\end{matrix}\right) /;\ -b-\nu\notin\mathbb{N}\land\nu-b\notin\mathbb{N} $$

07.18.26.0194.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{b+n}(z)=\frac{2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(-b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(-b+n+1)\end{array}\right)+ $$

$$ \frac{(-1)^{n+1}2^b z^{-b-n}\csc(b\pi)}{\sqrt{\pi}}\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{(-1)^{k+\lfloor\frac{n+1}{2}\rfloor}z^{2k}\,\Gamma\left(k-n+\lfloor\frac{n}{2}\rfloor+\frac{1}{2}\right)\left(1-k+\lfloor\frac{n}{2}\rfloor\right)_{n-\lfloor\frac{n}{2}\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\;/;\;n\in\mathbb{N} $$

07.18.26.0195.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{-b-n}(z)=\frac{(-1)^n2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(b+n+1)\end{array}\right)- $$

$$ \frac{2^b z^{-b-n}\cot(b\pi)}{\sqrt{\pi}}\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{(-1)^{k+\lfloor\frac{n+1}{2}\rfloor}z^{2k}\,\Gamma\left(k-n+\lfloor\frac{n}{2}\rfloor+\frac{1}{2}\right)\left(1-k+\lfloor\frac{n}{2}\rfloor\right)_{n-\lfloor\frac{n}{2}\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}\;/;\;n\in\mathbb{N} $$

07.18.26.0196.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_b(z)=\frac{2^{b-1}}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\frac{b}{2},\frac{1-b}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2}\end{array}\right)-\frac{2^b z^{-b}}{\pi} $$

07.18.26.0197.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{b+1}(z)=\frac{2^{b-1}}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1)\end{array}\right)-\frac{2^{b+1}b\,z^{-b-1}}{\pi} $$

07.18.26.0198.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{-b}(z)=-\frac{2^{b-1}\cos(b\pi)\,z^{-b}}{\pi}-\frac{2^{b-1}}{\sqrt{\pi}}G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},\frac{b+1}{2}\\ -\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},\frac{b+1}{2}\end{array}\right) $$

07.18.26.0199.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{-b}(z)=\frac{2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{b+1}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2},\frac{b+1}{2}\end{array}\right)-\frac{2^b z^{-b}\cos(b\pi)}{\pi} $$

07.18.26.0200.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)Y_{-b-1}(z)=\frac{2^{b+1}b\,z^{-b-1}\cos(b\pi)}{\pi}-\frac{2^{b-1}}{\sqrt{\pi}}G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{b+2}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1),\frac{b+2}{2}\end{array}\right) $$

Generalized cases involving Bessel *K*

07.18.26.0080.01

$$ {}_0\tilde{F}_1(;b;-z)\,K_{b-1}\left(2\sqrt{z}\right)=\frac{2^{-\frac{b+3}{2}}}{\sqrt{\pi}}G_{0,4}^{3,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{3-3b}{4}\right) $$

07.18.26.0081.01

$$ {}_0\tilde{F}_1(;b;-z)\,K_{1-b}\left(2\sqrt{z}\right)=\frac{2^{-\frac{b+3}{2}}}{\sqrt{\pi}}G_{0,4}^{3,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{3-3b}{4}\right) $$

07.18.26.0201.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)K_{b-1}(z)=\frac{2^{-\frac{1}{2}(b+3)}}{\sqrt{\pi}}G_{0,4}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{1}{4}(3-3b)\right) $$

07.18.26.0202.01

$$ {}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right)K_{1-b}(z)=\frac{2^{-\frac{1}{2}(b+3)}}{\sqrt{\pi}}G_{0,4}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{1}{4}(3-3b)\right) $$

07.18.26.0203.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_\nu(z)=\frac{2^{b-2}}{\sqrt{\pi}}G_{2,4}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},-b-\frac{\nu}{2}+1,-b+\frac{\nu}{2}+1\end{matrix}\right)\,/;\,-b-\nu\notin\mathbb{N}\land\nu-b\notin\mathbb{N} $$

07.18.26.0204.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_{b+n}(z)= $$

$$ \pi(-1)^{n-1}2^{b-\frac{3}{2}}\left(\sqrt{\pi}\csc\left(\frac{1}{4}(4b+(-1)^n)\pi\right)G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b)\\ 1-\frac{b-n}{2},\frac{b+n}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b),-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n)\end{matrix}\right)-\right. $$

$$ \left.\frac{\csc(b\pi)\sqrt{2}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)\,/;\,n\in\mathbb{N} $$

07.18.26.0205.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_{-b-n}(z)= $$

$$ \pi(-1)^{n-1}2^{b-\frac{3}{2}}\left(\sqrt{\pi}\csc\left(\frac{1}{4}(4b+(-1)^n)\pi\right)G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b)\\ 1-\frac{b-n}{2},\frac{b+n}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b),-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n)\end{matrix}\right)-\right. $$

$$ \left.\frac{\csc(b\pi)\sqrt{2}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)\,/;\,n\in\mathbb{N} $$

07.18.26.0206.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_b(z)=2^{b-1}\left(z^{-b}-\frac{\pi^{3/2}\csc\left(\frac{1}{4}(4b+1)\pi\right)}{\sqrt{2}}G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b),\frac{1}{4}(2b+1)\\ 1-\frac{b}{2},\frac{b}{2},\frac{1}{4}(3-2b),1-\frac{3b}{2},-\frac{b}{2},\frac{1}{4}(2b+1)\end{matrix}\right)\right) $$

07.18.26.0207.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_{b+1}(z)= $$

$$ 2^{b-1}\left(2b\,z^{-b-1}+\sqrt{\frac{\pi}{2}}\,\pi\csc\left(\frac{1}{4}(4b-1)\pi\right)G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b),\frac{1}{4}(2b+1)\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{4}(3-2b),-\frac{1}{2}(b+1),\frac{1}{4}(2b+1)\end{matrix}\right)\right) $$

07.18.26.0208.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_{-b}(z)=2^{b-1}\left(z^{-b}-\frac{\pi^{3/2}\csc\left(\frac{1}{4}(4b+1)\pi\right)}{\sqrt{2}}G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b),\frac{1}{4}(2b+1)\\ 1-\frac{b}{2},\frac{b}{2},\frac{1}{4}(3-2b),1-\frac{3b}{2},-\frac{b}{2},\frac{1}{4}(2b+1)\end{matrix}\right)\right) $$

07.18.26.0209.01

$$ {}_0\tilde{F}_1\left(;b;\frac{z^2}{4}\right)K_{-b-1}(z)= $$

$$ 2^{b-1}\left(2\,b\,z^{-b-1}+\sqrt{\frac{\pi}{2}}\,\pi\csc\left(\frac{1}{4}(4b-1)\pi\right)G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b),\frac{1}{4}(2b+1)\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{4}(3-2b),-\frac{1}{2}(b+1),\frac{1}{4}(2b+1)\end{matrix}\right)\right) $$

Generalized cases involving ${}_0F_1$

07.18.26.0210.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;b;-z)=\sqrt{\pi}\;2^{1-b}\,\Gamma(b)\,G_{0,4}^{1,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,0,\frac{1}{2}-\frac{b}{2},1-\frac{b}{2},1-b\right) $$

07.18.26.0211.01

$$ {}_0\tilde{F}_1(;b;z)\,{}_0F_1(;2-b;-z)=-\frac{\pi^{3/2}\csc(b\,\pi)}{\Gamma(b-1)}\,G_{1,5}^{2,0}\left(-\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ 0,\frac{1}{2},1-\frac{b}{2},\frac{1-b}{2},\frac{b-1}{2}\end{matrix}\right) $$

Through other functions

Involving some hypergeometric-type functions

07.18.26.0082.01

$$ {}_0\tilde{F}_1(;b;z)=z^{\frac{1-b}{2}}\,\boldsymbol{L}_{1-b}\left(2\sqrt{z}\right)\;/;\;b-\frac{3}{2}\in\mathbb{N} $$

Representations through equivalent functions

With related functions

07.18.27.0001.01

$$ {}_0\tilde{F}_1(;b;z)=\frac{{}_0F_1(;b;z)}{\Gamma(b)}\;/;\;-b\notin\mathbb{N} $$

07.18.27.0002.01

$$ {}_0\tilde{F}_1(;b;z)=(-z)^{\frac{1-b}{2}}\,J_{b-1}\left(2\sqrt{-z}\right) $$

07.18.27.0003.01

$$ {}_0\tilde{F}_1(;b;z)=z^{\frac{1-b}{2}}\,I_{b-1}\left(2\sqrt{z}\right) $$

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