Hypergeometric2F1

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Notations

Traditional name

Gauss hypergeometric function ${}_2F_1$

Traditional notation

${}_2F_1(a, b; c; z)$

Mathematica StandardForm notation

`Hypergeometric2F1[a, b, c, z]`

Primary definition

Basic definition

07.23.02.0001.01

$${}_2F_1(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{(c)_k k!} /; |z| < 1 \bigvee |z| = 1 \bigwedge \operatorname{Re}(c - a - b) > 0$$

For $|z| < 1$ and generic parameters a, b, c, the hypergeometric function ${}_2F_1(a, b; c; z)$ is defined by the above infinite sum (that is convergent). Outside of the unit circle $|z| < 1$the function ${}_2F_1(a, b; c; z)$ is defined as the analytic continuation with respect to z of this sum, with the parameters a, b, c held fixed.

Complete definition

07.23.02.0004.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(b-a)\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)} \sum_{k=0}^{\infty} \frac{(a)_k (a-c+1)_k z^{-k}}{k!\,(a-b+1)_k} + \frac{\Gamma(a-b)\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(b)_k (b-c+1)_k z^{-k}}{k!\,(-a+b+1)_k} /;$$
$$|z| > 1 \bigwedge a - b \notin \mathbb{Z}$$

Outside of the unit circle $|z| < 1$the function ${}_2F_1(a, b; c; z)$ can be defined by the above formula. Under the stated restrictions, all occurring sums are convergent.

07.23.02.0005.01

$${}_2F_1(a, b; c; z) = \lim_{\epsilon \to 0} {}_2F_1(a, b + \epsilon; c; z) /; |z| > 1 \bigwedge a - b \in \mathbb{Z}$$

07.23.02.0006.01

$${}_2F_1(a, b; c; z) = \lim_{r \to 1} {}_2F_1(a, b; c; r\, z) /; |z| = 1 \land r \in \mathbb{R}$$

For $a == -n,\ c == -m\ /;\ m \geq n$ being nonpositive integers, the function ${}_2F_1(a, b; c; z)$ cannot be uniquely defined by a limiting procedure based on the above definition because the two variables a, c can approach nonpositive integers $-n$, $-m\ /;\ m \geq n$ at different speeds. For nonpositive integers $a == -n,\ c == -m\ /;\ m \geq n$ one defines:

07.23.02.0002.01

$${}_2F_1(-n, b; -m; z) == \sum_{k=0}^{n} \frac{(-n)_k\,(b)_k\, z^k}{(-m)_k\, k!}\ /;\ m \in \mathbb{N} \land n \in \mathbb{N} \land m \geq n$$

Using the symmetry ${}_2F_1(a, b; c; z) == {}_2F_1(b, a; c; z)$, we have analogeously for $b == -n,\ c == -m\ /;\ m \geq n$ nonpositive integers:

07.23.02.0003.01

$${}_2F_1(a, -n; -m; z) == \sum_{k=0}^{n} \frac{(a)_k\,(-n)_k\, z^k}{(-m)_k\, k!}\ /;\ m \in \mathbb{N} \land n \in \mathbb{N} \land m \geq n$$

For $a == -n,\ c == -m\ /;\ m < n$ or $b == -n,\ c == -m\ /;\ m < n$ being nonpositive integers, the function ${}_2F_1(a, b; c; z)$ is not finite:

07.23.02.0007.01

$${}_2F_1(-n, b; -m; z) = \tilde{\infty}\ /;\ m \in \mathbb{N} \land n \in \mathbb{N} \land m < n$$

07.23.02.0007.01

$${}_2F_1(a, -n; -m; z) = \tilde{\infty}\ /;\ m \in \mathbb{N} \land n \in \mathbb{N} \land m < n$$

Specific values

Values at $z == 0$

07.23.03.0001.01

$${}_2F_1(a, b; c; 0) == 1$$

Values at $z = 1$

07.23.03.0002.01

$${}_2F_1(a, b; c; 1) == \frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\ /;\ \mathrm{Re}(c-a-b) > 0$$

07.23.03.0003.01

$${}_2F_1(-n, b; c; 1) == \frac{(c-b)_n}{(c)_n}\ /;\ n \in \mathbb{N}$$

Values at $z == -1$

For fixed a, b

07.23.03.0004.01

$${}_2F_1(a, b; a-b-m; -1) == \frac{2^{-2b-m-1}\,\Gamma(a-b-m)}{\Gamma(a-2b-m)} \sum_{k=0}^{m+1} \binom{m+1}{k} \frac{\Gamma\left(\frac{a+k-m}{2}-b\right)}{\Gamma\left(\frac{a+k-m}{2}\right)}\ /;\ m \in \mathbb{N}$$

07.23.03.0005.01

$$ {}_2F_1(a,\, b;\, a-b-1;\, -1) = \frac{2^{-2(b+1)}\,\Gamma(a-b-1)}{\Gamma(a-2b-1)} \left(\frac{\Gamma\left(\frac{a-1}{2}-b\right)}{\Gamma\left(\frac{a-1}{2}\right)} + \frac{\Gamma\left(\frac{a+1}{2}-b\right)}{\Gamma\left(\frac{a+1}{2}\right)} + \frac{2\,\Gamma\left(\frac{a}{2}-b\right)}{\Gamma\left(\frac{a}{2}\right)} \right) $$

07.23.03.0006.01

$$ {}_2F_1(a,\, b;\, a-b;\, -1) = 2^{-a}\sqrt{\pi}\;\Gamma(a-b) \left(\frac{1}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{1}{2}(a-2b+1)\right)} + \frac{1}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a}{2}-b\right)} \right) $$

07.23.03.0007.01

$$ {}_2F_1(a,\, b;\, a-b+1;\, -1) = \frac{2^{-a}\sqrt{\pi}\;\Gamma(a-b+1)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a}{2}-b+1\right)} $$

07.23.03.0008.01

$$ {}_2F_1(a,\, b;\, a-b+2;\, -1) = \frac{2^{-a}\sqrt{\pi}\;\Gamma(a-b+2)}{b-1} \left(\frac{1}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{a+3}{2}-b\right)} - \frac{1}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a}{2}-b+1\right)} \right) $$

07.23.03.0009.01

$$ {}_2F_1(a,\, b;\, a-b+3;\, -1) = \frac{2^{-a}\sqrt{\pi}\;\Gamma(a-b+3)}{(b-1)(b-2)} \left(\frac{a(a+1)}{4\left(\Gamma\left(\frac{a+3}{2}\right)\Gamma\left(\frac{a}{2}-b+2\right)\right)} + \frac{1}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a}{2}-b+1\right)} - \frac{a}{\Gamma\left(\frac{a}{2}+1\right)\Gamma\left(\frac{a+3}{2}-b\right)} \right) $$

For fixed *b*, *c*

07.23.03.0010.01

$$ {}_2F_1(-n,\, b;\, c;\, -1) = \frac{n!}{(c)_n}\, P_n^{(c-1,b-c-n)}(3) \,/;\, n \in \mathbb{N} $$

07.23.03.0011.01

$$ {}_2F_1(-n,\, b;\, c;\, -1) = \frac{n!\,(-2)^n}{(c)_n}\, P_n^{(-b-n,c-1)}(0) \,/;\, n \in \mathbb{N} $$

For fixed *b*

07.23.03.0012.01

$$ {}_2F_1(1,\, b;\, -b-n;\, -1) = \frac{2^{-2b-n-2}\,\Gamma(1-b)\,\Gamma(-b-n)}{\Gamma(-2b-n)} + \frac{1}{2}\sum_{k=0}^{n+1} \frac{(-1)^k\,(b)_k}{(-b-n)_k} \,/;\, n \in \mathbb{N} $$

07.23.03.0013.01

$$ {}_2F_1(1,\, b;\, n-b;\, -1) = \frac{2^{n-2b-2}\,\Gamma(1-b)\,\Gamma(n-b)}{\Gamma(n-2b)} - \frac{1}{2}\sum_{k=1}^{n-2} \frac{(-1)^k\,(b-n+1)_k}{(1-b)_k} \,/;\, n \in \mathbb{N} $$

07.23.03.0014.01

$$ {}_2F_1(1,\, b;\, b+1;\, -1) = \frac{b}{2}\left(\psi\left(\frac{b+1}{2}\right) - \psi\left(\frac{b}{2}\right)\right) $$

07.23.03.0015.01

$$ {}_2F_1(1,\, b;\, b+2;\, -1) = (b+1)\left(\psi\left(\frac{b+1}{2}\right) - \psi\left(\frac{b}{2}\right)\right) b - b - 1 $$

07.23.03.0016.01

$$ {}_2F_1(1, b; b+3; -1) == b(b+1)(b+2)\left(\psi\left(\frac{b+1}{2}\right) - \psi\left(\frac{b}{2}\right)\right) - b^2 - \frac{7b}{2} - 3 $$

07.23.03.0017.01

$$ {}_2F_1(1, b; b+4; -1) == b(b+1)(b+2)(b+3)\left(\frac{2}{3}\left(\psi\left(\frac{b+1}{2}\right) - \psi\left(\frac{b}{2}\right)\right) + \frac{2}{3(b+1)} - \frac{1}{6(b+2)} - \frac{7}{6b}\right) $$

07.23.03.0018.01

$$ {}_2F_1(2, b; b+1; -1) == \frac{b(b-1)}{2}\left(\psi\left(\frac{b}{2}\right) - \psi\left(\frac{b-1}{2}\right)\right) - \frac{b}{2} $$

For other parameters

07.23.03.0019.01

$$ b\,{}_2F_1(a, a+b; a+1; -1) + a\,{}_2F_1(b, a+b; b+1; -1) == \frac{\Gamma(a+1)\,\Gamma(b+1)}{\Gamma(a+b)} $$

07.23.03.0020.01

$$ (1-a)\,{}_2F_1(1, a; 2-b; -1) + (1-b)\,{}_2F_1(1, b; 2-a; -1) == \frac{2^{-a-b+1}\,\Gamma(2-a)\,\Gamma(2-b)}{\Gamma(2-a-b)} $$

07.23.03.0021.01

$$ {}_2F_1(2, a; b; -1) - \frac{(b-1)(a+b-3)}{2(a-1)}\,{}_2F_1(1, a-1; b-1; -1) == \frac{(b-2)(b-1)}{2(1-a)} $$

Values at $z = \frac{1}{2}$

For fixed a, b

07.23.03.0022.01

$$ {}_2F_1\left(a, b; \frac{a+b-m}{2}; \frac{1}{2}\right) == \frac{2^{b-1}\,\Gamma\left(\frac{a+b-m}{2}\right)}{\Gamma(b)} \sum_{k=0}^{m+1} \binom{m+1}{k} \frac{\Gamma\left(\frac{b+k}{2}\right)}{\Gamma\left(\frac{a+k-m}{2}\right)} \;/;\; m \in \mathbb{N} $$

07.23.03.0023.01

$$ {}_2F_1\left(a, b; \frac{a+b-1}{2}; \frac{1}{2}\right) == \frac{2^{b-1}}{\Gamma(b)}\,\Gamma\left(\frac{a+b-1}{2}\right)\left(\frac{\Gamma\left(\frac{b}{2}\right)}{\Gamma\left(\frac{a-1}{2}\right)} + \frac{2\,\Gamma\left(\frac{b+1}{2}\right)}{\Gamma\left(\frac{a}{2}\right)} + \frac{\Gamma\left(\frac{b}{2}+1\right)}{\Gamma\left(\frac{a+1}{2}\right)}\right) $$

07.23.03.0024.01

$$ {}_2F_1\left(a, b; \frac{a+b}{2}; \frac{1}{2}\right) == \sqrt{\pi}\;\Gamma\left(\frac{a+b}{2}\right)\left(\frac{1}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b}{2}\right)} + \frac{1}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{b+1}{2}\right)}\right) $$

07.23.03.0025.01

$$ {}_2F_1\left(a, b; \frac{a+b+1}{2}; \frac{1}{2}\right) == \frac{\sqrt{\pi}\;\Gamma\left(\frac{a+b+1}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b+1}{2}\right)} $$

07.23.03.0026.01

$$ {}_2F_1\left(a, b; \frac{a+b}{2}+1; \frac{1}{2}\right) == \frac{2\sqrt{\pi}\;\Gamma\left(\frac{a+b}{2}+1\right)}{a-b}\left(\frac{1}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{b+1}{2}\right)} - \frac{1}{\Gamma\left(\frac{b}{2}\right)\Gamma\left(\frac{a+1}{2}\right)}\right) $$

For fixed a, c

07.23.03.0027.01

$$ {}_2F_1\left(a, -a; c; \frac{1}{2}\right) = \frac{\sqrt{\pi}\,\Gamma(c)}{2^c}\left(\frac{1}{\Gamma\left(\frac{a+c+1}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} + \frac{1}{\Gamma\left(\frac{a+c}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}\right) $$

07.23.03.0028.01

$$ {}_2F_1\left(a, 1-a; c; \frac{1}{2}\right) = \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma\left(\frac{a+c}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

07.23.03.0029.01

$$ {}_2F_1\left(a, 2-a; c; \frac{1}{2}\right) = \frac{2^{2-c}\sqrt{\pi}\,\Gamma(c)}{a-1}\left(\frac{1}{\Gamma\left(\frac{a+c}{2}-1\right)\Gamma\left(\frac{c-a+1}{2}\right)} - \frac{1}{\Gamma\left(\frac{a+c-1}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}\right) $$

07.23.03.0030.01

$$ {}_2F_1\left(a, 3-a; c; \frac{1}{2}\right) = \frac{2^{3-c}\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)}\left(\frac{c-2}{\Gamma\left(\frac{a+c}{2}-1\right)\Gamma\left(\frac{c-a+1}{2}\right)} - \frac{2}{\Gamma\left(\frac{a+c-3}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}\right) $$

07.23.03.0031.01

$$ {}_2F_1\left(a, 4-a; c; \frac{1}{2}\right) = \frac{2^{4-c}\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)(a-3)}\left(\frac{2c-a-3}{\Gamma\left(\frac{a+c}{2}-2\right)\Gamma\left(\frac{c-a+1}{2}\right)} - \frac{a+2c-7}{\Gamma\left(\frac{a+c-3}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}\right) $$

07.23.03.0032.01

$$ {}_2F_1\left(a, 5-a; c; \frac{1}{2}\right) = \frac{2^{5-c}\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)(a-3)(a-4)}\left(\frac{2(c-3)^2-(a-2)(a-3)}{\Gamma\left(\frac{a+c}{2}-2\right)\Gamma\left(\frac{c-a+1}{2}\right)} - \frac{4(c-3)}{\Gamma\left(\frac{a+c-5}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}\right) $$

07.23.03.0033.01

$$ {}_2F_1\left(a, 6-a; c; \frac{1}{2}\right) = $$
$$ \frac{2^{6-c}\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)(a-3)(a-4)(a-5)}\left(\frac{-a^2-2ca+13a+4c^2-22c+20}{\Gamma\left(\frac{a+c}{2}-3\right)\Gamma\left(\frac{c-a+1}{2}\right)} - \frac{-a^2+2ca-a+4c^2-34c+62}{\Gamma\left(\frac{a+c-5}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}\right) $$

For fixed a

07.23.03.0034.01

$$ {}_2F_1\left(a, a; a+1; \frac{1}{2}\right) = 2^{a-1}\,a\left(\psi\left(\frac{a+1}{2}\right) - \psi\left(\frac{a}{2}\right)\right) $$

For fixed b

07.23.03.0035.01

$$ {}_2F_1\left(1, b; \frac{b-m}{2}; \frac{1}{2}\right) = \frac{2^{b-1}}{\Gamma(b)}\,\Gamma\left(\frac{b+m}{2}+1\right)\Gamma\left(\frac{b-m}{2}\right) + \sum_{k=0}^{m+1}\frac{(-1)^k\left(-\frac{b+m}{2}\right)_k}{\left(\frac{b-m}{2}\right)_k} \;/;\; m \in \mathbb{N} $$

07.23.03.0036.01

$${}_2F_1\left(1,\, b;\, \frac{b+m}{2};\, \frac{1}{2}\right) = \frac{2^{b-1}}{\Gamma(b)}\,\Gamma\left(\frac{b-m}{2}+1\right)\Gamma\left(\frac{b+m}{2}\right) - \sum_{k=1}^{m-2} \frac{(-1)^k \left(1-\frac{b+m}{2}\right)_k}{\left(\frac{b-m}{2}+1\right)_k} \;/;\; m \in \mathbb{N}$$

For fixed c

07.23.03.0037.01

$${}_2F_1\left(1,\, 1;\, c;\, \frac{1}{2}\right) = (c-1)\left(\psi\left(\frac{c}{2}\right) - \psi\left(\frac{c-1}{2}\right)\right)$$

07.23.03.0038.01

$${}_2F_1\left(1,\, 2;\, c;\, \frac{1}{2}\right) = 2\,(c-1)\left(1-(c-2)\left(\psi\left(\frac{c}{2}\right) - \psi\left(\frac{c-1}{2}\right)\right)\right)$$

07.23.03.0039.01

$${}_2F_1\left(1,\, 3;\, c;\, \frac{1}{2}\right) = (c-1)\left(7-2\,c+2\,(c-3)\,(c-2)\left(\psi\left(\frac{c}{2}\right) - \psi\left(\frac{c-1}{2}\right)\right)\right)$$

07.23.03.0040.01

$${}_2F_1\left(1,\, 4;\, c;\, \frac{1}{2}\right) = \frac{2}{3}\,(c-1)\left(2\,c^2-15\,c+29-2\,(c-2)\,(c-3)\,(c-4)\left(\psi\left(\frac{c}{2}\right) - \psi\left(\frac{c-1}{2}\right)\right)\right)$$

07.23.03.0041.01

$${}_2F_1\left(1,\, 5;\, c;\, \frac{1}{2}\right) = \frac{c-1}{6}\left(293-4\,c^3+50\,c^2-208\,c+4\,(c-5)\,(c-4)\,(c-3)\,(c-2)\left(\psi\left(\frac{c}{2}\right) - \psi\left(\frac{c-1}{2}\right)\right)\right)$$

07.23.03.0042.01

$${}_2F_1\left(2,\, 2;\, c;\, \frac{1}{2}\right) = 2\,(c-1)\left(6-2\,c+(2\,c-5)\,(c-2)\left(\psi\left(\frac{c}{2}\right) - \psi\left(\frac{c-1}{2}\right)\right)\right)$$

07.23.03.0043.01

$${}_2F_1\left(2,\, 3;\, c;\, \frac{1}{2}\right) = 2\,(c-1)\,(c-2)\left(2\,(c-3)^2\left(\psi\left(\frac{c-1}{2}\right) - \psi\left(\frac{c-2}{2}\right)\right) - 2\,c+7\right)$$

07.23.03.0044.01

$${}_2F_1\left(2,\, 4;\, c;\, \frac{1}{2}\right) = \frac{2}{3}\,(c-1)\,(c-2)\left(4\,c^2-32\,c+65-2\,(c-3)\,(c-4)\,(2\,c-7)\left(\psi\left(\frac{c-1}{2}\right) - \psi\left(\frac{c-2}{2}\right)\right)\right)$$

07.23.03.0045.01

$${}_2F_1\left(3,\, 3;\, c;\, \frac{1}{2}\right) = 2\,(c-1)\,(c-2)\,(c-3)\left(7-2\,c+\left(2\,c^2-14\,c+25\right)\left(\psi\left(\frac{c-2}{2}\right) - \psi\left(\frac{c-3}{2}\right)\right)\right)$$

Values at other z

Values at $z = -\frac{1}{2}$

07.23.03.0046.01

$${}_2F_1\left(2,\, b;\, \frac{5-b}{2};\, -\frac{1}{2}\right) = 1-\frac{b}{3}$$

Values at $z = -\frac{1}{3}$

07.23.03.0047.01

$$ {}_2F_1\left(a, 1-\frac{a}{2}; a+\frac{1}{2}; -\frac{1}{3}\right) = \frac{\sqrt{\pi}\, 3^{-\frac{a}{2}}\, \Gamma\left(a+\frac{1}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)^2} $$

07.23.03.0048.01

$$ {}_2F_1\left(a, a+\frac{1}{2}; \frac{3}{2}-2a; -\frac{1}{3}\right) = \frac{2}{\sqrt{\pi}\, \Gamma\left(\frac{4}{3}-2a\right)} \left(\frac{9}{8}\right)^{2a} \Gamma\left(\frac{4}{3}\right) \Gamma\left(\frac{3}{2}-2a\right) $$

Values at $z = -\frac{1}{8}$

07.23.03.0049.01

$$ {}_2F_1\left(a, a+\frac{1}{3}; \frac{4}{3}-a; -\frac{1}{8}\right) = \left(\frac{2}{3}\right)^{3a} \frac{\Gamma\left(\frac{2}{3}-a\right)\Gamma\left(\frac{4}{3}-a\right)}{\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}-2a\right)} $$

07.23.03.0050.01

$$ {}_2F_1\left(a, \frac{1-a}{3}; \frac{4a+5}{6}; -\frac{1}{8}\right) = \frac{2^{-a}\sqrt{\pi}\, \Gamma\left(\frac{4a+2}{3}\right)}{\Gamma\left(a+\frac{1}{2}\right)\Gamma\left(\frac{a+2}{3}\right)} $$

07.23.03.0051.01

$$ {}_2F_1\left(a, \frac{2-a}{3}; \frac{4a+7}{6}; -\frac{1}{8}\right) = \frac{3\, 2^{-a}\sqrt{\pi}}{2a-1} \left(\frac{\Gamma\left(\frac{4a+4}{3}\right)}{\Gamma(a)\,\Gamma\left(\frac{2a+5}{6}\right)} - \frac{\Gamma\left(\frac{4a+4}{3}\right)}{\Gamma\left(a+\frac{1}{2}\right)\Gamma\left(\frac{a+1}{3}\right)}\right) $$

Values at $z = \frac{1}{9}$

07.23.03.0052.01

$$ {}_2F_1\left(a, 1-2a; \frac{4}{3}-a; \frac{1}{9}\right) = \frac{3^{-a}\,\Gamma\left(\frac{2}{3}-a\right)\Gamma\left(\frac{4}{3}-a\right)}{\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}-2a\right)} $$

Values at $z = \frac{1}{4}$

07.23.03.0053.01

$$ {}_2F_1\left(\frac{1}{2}, b; \frac{5}{2}-2b; \frac{1}{4}\right) = \frac{2^{2b}\sqrt{\pi}\, \Gamma\left(\frac{5}{2}-2b\right)}{3\,\Gamma\left(\frac{3}{2}-b\right)^2} $$

Values at $z = \frac{1}{3}$

07.23.03.0054.01

$$ {}_2F_1\left(a, 1-2a; a+2; \frac{1}{3}\right) = \left(\frac{2}{3}\right)^{2a} (a+1) $$

Values at $z = \frac{8}{9}$

07.23.03.0055.01

$$ {}_2F_1\left(a, a+\frac{1}{2}; \frac{4a+2}{3}; \frac{8}{9}\right) = \left(\frac{3}{2}\right)^{2a} \frac{\sqrt{\pi}}{\Gamma\left(\frac{a+2}{3}\right)\Gamma\left(a+\frac{1}{2}\right)} \Gamma\left(\frac{4a+2}{3}\right) $$

07.23.03.0056.01

$$ {}_2F_1\left(a, a+\frac{1}{2}; \frac{4a}{3}+1; \frac{8}{9}\right) = 2^{1-2a}\, 3^{2a} \sqrt{\pi} \left(\frac{\Gamma\left(\frac{4a}{3}\right)}{\Gamma(a)\,\Gamma\left(\frac{2a+3}{6}\right)} - \frac{\Gamma\left(\frac{4a}{3}\right)}{\Gamma\left(a+\frac{1}{2}\right)\Gamma\left(\frac{a}{3}\right)}\right) $$

07.23.03.0057.01

$$ {}_2F_1\left(a, a+\frac{1}{2}; 4a-1; \frac{8}{9}\right) = 2^{2-6a}\, 3^{2a} \sqrt{\pi} \left(\frac{3\,\Gamma(4a-1)}{\Gamma(a)\,\Gamma\left(3a-\frac{1}{2}\right)} - \frac{\Gamma(4a-1)}{\Gamma\left(a+\frac{1}{2}\right)\Gamma(3a-1)}\right) $$

07.23.03.0058.01

$$ {}_2F_1\left(a, a+\frac{1}{2}; 4a; \frac{8}{9}\right) = \frac{2^{1-6a}\, 3^{2a} \sqrt{\pi}\; \Gamma(4a)}{\Gamma\left(a+\frac{1}{2}\right)\Gamma(3a)} $$

Values at $z = 2$

07.23.03.0059.01

$$ {}_2F_1(-n, b; 2b-1; 2) = \frac{\Gamma\left(b-\frac{1}{2}\right)}{\sqrt{\pi}} \left(\frac{(1+(-1)^n)\,\Gamma\left(\frac{n+1}{2}\right)}{2\,\Gamma\left(\frac{n-1}{2}+b\right)} - \frac{(1-(-1)^n)\,\Gamma\left(\frac{n+2}{2}\right)}{2\,\Gamma\left(b+\frac{n}{2}\right)}\right) /; n \in \mathbb{N} $$

07.23.03.0060.01

$$ {}_2F_1(-n, b; 2b; 2) = \frac{\left(n!\, 2^{-n-1}\right)(1+(-1)^n)\,\Gamma\left(b+\frac{1}{2}\right)}{\frac{n}{2}!\,\Gamma\left(\frac{n+1}{2}+b\right)} /; n \in \mathbb{N} $$

07.23.03.0061.01

$$ {}_2F_1(-n, b; 2b+1; 2) = \frac{\Gamma\left(b+\frac{1}{2}\right)}{\sqrt{\pi}} \left(\frac{(1+(-1)^n)\,\Gamma\left(\frac{n+1}{2}\right)}{2\,\Gamma\left(\frac{n+1}{2}+b\right)} + \frac{(1-(-1)^n)\,\Gamma\left(\frac{n+2}{2}\right)}{2\,\Gamma\left(\frac{n+2}{2}+b\right)}\right) /; n \in \mathbb{N} $$

07.23.03.0062.01

$$ {}_2F_1(-n, b; -2n-2; 2) = \frac{2^{2n+2}\, n!}{(2n+2)!} \left((b+n+1)\left(\frac{b+1}{2}\right)_{n+1} - 2\left(\frac{b}{2}\right)_{n+2}\right) /; n \in \mathbb{N} $$

07.23.03.0063.01

$$ {}_2F_1(-n, b; -2n-1; 2) = \frac{2^{2n+1}\, n!}{(2n+1)!} \left(\left(\frac{b+1}{2}\right)_{n+1} - \left(\frac{b}{2}\right)_{n+1}\right) /; n \in \mathbb{N} $$

07.23.03.0064.01

$$ {}_2F_1(-n, b; -2n; 2) = \frac{2^{2n}\, n!}{(2n)!} \left(\frac{b+1}{2}\right)_n /; n \in \mathbb{N}^+ $$

Values at $z = \frac{4-3\sqrt{2}}{8}$

07.23.03.0065.01

$$ {}_2F_1\left(a, \frac{2-a}{3}; \frac{2a+5}{6}; \frac{4-3\sqrt{2}}{8}\right) = \left(\frac{2}{3}\right)^{a/2} \frac{\sqrt{\pi}\; \Gamma\left(\frac{2a+5}{6}\right)}{\Gamma\left(\frac{a+3}{6}\right)\Gamma\left(\frac{a+5}{6}\right)} $$

07.23.03.0066.01

$$ {}_2F_1\left(a,\frac{4-a}{3};\frac{2a+7}{6};\frac{4-3\sqrt{2}}{8}\right)=\frac{3\,2^{-\frac{a}{2}}\sqrt{\pi}}{a-1}\left(\frac{\Gamma\left(\frac{2a+4}{3}\right)}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{a+5}{6}\right)}-\frac{\Gamma\left(\frac{2a+4}{3}\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a+2}{6}\right)}\right) $$

Values at $z=\frac{1-\sqrt{2}}{2}$

07.23.03.0067.01

$$ {}_2F_1\left(\frac{1}{2},b;\frac{2b+3}{4};\frac{1-\sqrt{2}}{2}\right)=\frac{2^{-\frac{b}{2}}\sqrt{\pi}}{\Gamma\left(\frac{b+2}{4}\right)\Gamma\left(\frac{b+3}{4}\right)}\Gamma\left(\frac{2b+3}{4}\right) $$

07.23.03.0068.01

$$ {}_2F_1\left(\frac{3}{2},b;\frac{2b+5}{4};\frac{1-\sqrt{2}}{2}\right)=2^{2-\frac{b}{2}}\sqrt{\pi}\left(\frac{\Gamma\left(\frac{2b+5}{4}\right)}{\Gamma\left(\frac{b+1}{4}\right)\Gamma\left(\frac{b+2}{4}\right)}-\frac{\Gamma\left(\frac{2b+5}{4}\right)}{\Gamma\left(\frac{b}{4}\right)\Gamma\left(\frac{b+3}{4}\right)}\right) $$

Values at $z=\frac{2-\sqrt{2}}{4}$

07.23.03.0069.01

$$ {}_2F_1\left(a,4-a;\frac{5}{2};\frac{2-\sqrt{2}}{4}\right)=\frac{3\pi}{2^{3/2}(a-2)}\left(\frac{1}{\Gamma\left(\frac{a+1}{4}\right)\Gamma\left(\frac{7-a}{4}\right)}-\frac{1}{\Gamma\left(\frac{a+3}{4}\right)\Gamma\left(\frac{5-a}{4}\right)}\right) $$

Values at $z=2\sqrt{2}-2$

07.23.03.0070.01

$$ {}_2F_1\left(a,\frac{2a+1}{4};a+\frac{1}{2};2\sqrt{2}-2\right)=\frac{\sqrt{\pi}}{\Gamma\left(\frac{a+2}{4}\right)\Gamma\left(\frac{a+3}{4}\right)}\Gamma\left(\frac{2a+3}{4}\right)\left(\frac{2+\sqrt{2}}{2}\right)^a $$

07.23.03.0071.01

$$ {}_2F_1\left(a,\frac{2a+3}{4};a+\frac{3}{2};2\sqrt{2}-2\right)=\frac{\sqrt{\pi}\left(2+\sqrt{2}\right)^a}{2^{a-2}}\left(\frac{\Gamma\left(\frac{2a+5}{4}\right)}{\Gamma\left(\frac{a+1}{4}\right)\Gamma\left(\frac{a+2}{4}\right)}-\frac{\Gamma\left(\frac{2a+5}{4}\right)}{\Gamma\left(\frac{a}{4}\right)\Gamma\left(\frac{a+3}{4}\right)}\right) $$

Values at $z=12\sqrt{2}-16$

07.23.03.0072.01

$$ {}_2F_1\left(a,\frac{4a+1}{6};\frac{4a+1}{3};12\sqrt{2}-16\right)=\frac{\sqrt{\pi}\,\Gamma\left(\frac{2a+2}{3}\right)}{\Gamma\left(\frac{a+4}{6}\right)\Gamma\left(\frac{a+1}{2}\right)}\left(\frac{2+\sqrt{2}}{2}\right)^{2a} $$

07.23.03.0073.01

$$ {}_2F_1\left(a,\frac{4a+3}{6};\frac{4a}{3}+1;12\sqrt{2}-16\right)=\frac{\sqrt{\pi}}{2}\left(\frac{2+\sqrt{2}}{2}\right)^{2a}\left(\frac{3\,\Gamma\left(\frac{2a}{3}+1\right)}{\Gamma\left(\frac{a}{2}+1\right)\Gamma\left(\frac{a+3}{6}\right)}-\frac{\Gamma\left(\frac{2a}{3}+1\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{a}{6}+1\right)}\right) $$

07.23.03.0074.01

$$ {}_2F_1\left(a,2a-\frac{3}{2};4a-3;12\sqrt{2}-16\right)=\frac{\left(2+\sqrt{2}\right)^{2a}\sqrt{\pi}}{2^{4a-2}}\left(\frac{3\,\Gamma(2a-1)}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{3a-1}{2}\right)}-\frac{\Gamma(2a-1)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{3a}{2}-1\right)}\right) $$

07.23.03.0075.01

$$ {}_2F_1\left(a,\, 2\,a-\frac{1}{2};\, 4\,a-1;\, 12\,\sqrt{2}\,-16\right)=\frac{\left(2+\sqrt{2}\right)^{2\,a}\,\sqrt{\pi}\,\Gamma(2\,a)}{2^{4\,a-1}\,\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{3\,a}{2}\right)} $$

Values at $z=\frac{2-\sqrt{3}}{4}$

07.23.03.0076.01

$$ {}_2F_1\left(a,\, 2-3\,a;\, \frac{3}{2}-a;\, \frac{2-\sqrt{3}}{4}\right)=\frac{3^{\frac{3\,a}{2}}\,\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{3}{2}-a\right)}{2^{2\,a-1}\,\sqrt{\pi}\,\Gamma\left(\frac{4}{3}-a\right)} $$

Values at $z=e^{\pm\frac{\pi\,i}{3}}$

07.23.03.0077.01

$$ {}_2F_1\left(a,\, \frac{a+1}{3};\, \frac{2\,(a+1)}{3};\, e^{\frac{\pi\,i}{3}}\right)=\frac{3^{\frac{a}{2}-1}\,e^{\frac{\pi\,a\,i}{6}}}{\Gamma\left(\frac{2}{3}\right)\Gamma(a)}\,\Gamma\left(\frac{2\,(a+1)}{3}\right)\Gamma\left(\frac{a}{3}\right) $$

07.23.03.0078.01

$$ {}_2F_1\left(a,\, \frac{a+1}{3};\, \frac{2\,(a+1)}{3};\, e^{-\frac{\pi\,i}{3}}\right)=\frac{3^{\frac{a}{2}-1}\,e^{-\frac{\pi\,a\,i}{6}}}{\Gamma\left(\frac{2}{3}\right)\Gamma(a)}\,\Gamma\left(\frac{2\,(a+1)}{3}\right)\Gamma\left(\frac{a}{3}\right) $$

07.23.03.0663.01

$$ {}_2F_1\left(a,\, 3\,a;\, 2\,a;\, e^{\frac{\pi\,i}{3}}\right)=(4\,i)^a\,3^{-\frac{1}{2}(3\,a+1)}\,\sqrt{\pi}\,\Gamma\left(a+\frac{1}{2}\right)\left(\frac{1}{\Gamma\left(\frac{1}{3}\right)\Gamma\left(a+\frac{2}{3}\right)}+\frac{1}{\Gamma\left(\frac{2}{3}\right)\Gamma\left(a+\frac{1}{3}\right)}\right) $$

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07.23.03.0664.01

$$ {}_2F_1\left(a,\, 3\,a;\, 2\,a;\, e^{-\frac{\pi\,i}{3}}\right)=(-4\,i)^a\,3^{-\frac{1}{2}(3\,a+1)}\,\sqrt{\pi}\,\Gamma\left(a+\frac{1}{2}\right)\left(\frac{1}{\Gamma\left(\frac{1}{3}\right)\Gamma\left(a+\frac{2}{3}\right)}+\frac{1}{\Gamma\left(\frac{2}{3}\right)\Gamma\left(a+\frac{1}{3}\right)}\right) $$

Bill Gosper

Values at z including ϕ

07.23.03.0665.01

$$ {}_2F_1\left(a,\, 1-a;\, \frac{3\,a}{2}-\frac{1}{2};\, 1-\phi\right)=3^{\frac{3\,a}{2}-1}\,5^{\frac{1}{2}-\frac{5\,a}{4}}\,\phi^{\frac{3\,a}{2}-2}\,\Gamma\left(\frac{a}{2}-\frac{1}{6}\right)\Gamma\left(\frac{a}{2}+\frac{1}{6}\right)\left(\frac{1}{\Gamma\left(\frac{a}{2}-\frac{3}{10}\right)\Gamma\left(\frac{a}{2}+\frac{3}{10}\right)}+\frac{\phi}{\Gamma\left(\frac{a}{2}-\frac{1}{10}\right)\Gamma\left(\frac{a}{2}+\frac{1}{10}\right)}\right) $$

Bill Gosper

07.23.03.0666.01

$$ {}_2F_1\left(a,\, 1-a;\, \frac{3\,a}{2};\, 1-\phi\right)=\frac{5^{\frac{1}{4}-\frac{5\,a}{4}}\left(3+\sqrt{5}\right)\phi^{\frac{3\,a}{2}-\frac{5}{2}}\,\pi\,\Gamma\left(\frac{3\,a}{2}\right)}{\Gamma\left(\frac{a}{2}+\frac{2}{5}\right)\Gamma\left(\frac{a}{2}+\frac{3}{5}\right)\Gamma\left(\frac{a}{2}\right)} $$

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07.23.03.0667.01

$${}_2F_1\left(a,\,1-a;\,\frac{3\,a}{2}+\frac{1}{2};\,1-\phi\right)=\frac{3^{\frac{3a}{2}}\,5^{-\frac{5a}{4}}\,\phi^{\frac{3a}{2}-1}\,\Gamma\left(\frac{a}{2}+\frac{1}{6}\right)\Gamma\left(\frac{a}{2}+\frac{5}{6}\right)}{\Gamma\left(\frac{a}{2}+\frac{3}{10}\right)\Gamma\left(\frac{a}{2}+\frac{7}{10}\right)}$$

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07.23.03.0668.01

$${}_2F_1\left(a,\,-a;\,-\frac{3\,a}{2};\,-\frac{1}{\phi}\right)=\frac{5^{-\frac{5a}{4}-\frac{3}{4}}\,\phi^{\frac{3a}{2}-\frac{1}{2}}\,(1+\phi)^{\frac{1}{2}(-3a-1)}\,\Gamma\left(-\frac{3a}{2}\right)}{2\,\pi\,\Gamma\left(-\frac{5a}{2}\right)}\left(\Gamma\left(\frac{2}{5}-\frac{a}{2}\right)\Gamma\left(\frac{3}{5}-\frac{a}{2}\right)+\left(2+\sqrt{5}\right)\Gamma\left(\frac{1}{5}-\frac{a}{2}\right)\Gamma\left(\frac{4}{5}-\frac{a}{2}\right)\right)$$

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07.23.03.0669.01

$${}_2F_1\left(a,\,\frac{5\,a}{3}-\frac{5}{6};\,2\,a;\,\frac{4}{\phi^3}\right)=3^a\,5^{\frac{1}{6}-\frac{5a}{6}}\,\phi^{5a-3}\,\Gamma\left(\frac{a}{3}+\frac{1}{6}\right)\Gamma\left(\frac{a}{3}+\frac{1}{2}\right)\left(\frac{\phi}{\Gamma\left(\frac{a}{3}+\frac{7}{30}\right)\Gamma\left(\frac{a}{3}+\frac{13}{30}\right)}-\frac{1}{\Gamma\left(\frac{a}{3}+\frac{1}{30}\right)\Gamma\left(\frac{a}{3}+\frac{19}{30}\right)}\right)$$

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Specialized values

For fixed *a*, *b*, *z*

07.23.03.0079.01

$${}_2F_1(a,\,b;\,a;\,z)=(1-z)^{-b}$$

07.23.03.0080.01

$${}_2F_1(a,\,b;\,b;\,z)=(1-z)^{-a}$$

07.23.03.0081.01

$${}_2F_1(a,\,b;\,b-1;\,z)=\frac{(1-z)^{-a-1}\,(b+(a-b+1)\,z-1)}{b-1}$$

07.23.03.0082.01

$${}_2F_1(a,\,b;\,b-n;\,z)=(1-z)^{-a-n}\sum_{k=0}^{n}\frac{(-n)_k\,(b-a-n)_k\,z^k}{(b-n)_k\,k!}\;/;\;n\in\mathbb{N}$$

07.23.03.0083.01

$${}_2F_1(a,\,b;\,b-n;\,z)=\frac{(1-z)^{-a}}{(1-b)_n}\sum_{k=0}^{n}(-1)^k\,(1-b)_{n-k}\,(a)_k\binom{n}{k}\left(\frac{z}{1-z}\right)^k\;/;\;n\in\mathbb{N}$$

07.23.03.0084.01

$${}_2F_1(a,\,b;\,b+1;\,z)=b\,z^{-b}\,\mathrm{B}_z(b,\,1-a)$$

07.23.03.0085.01

$${}_2F_1\left(a,\,b;\,a+b-\frac{1}{2};\,z\right)=\frac{2^{a+b-\frac{3}{2}}}{\sqrt{1-z}}\,\Gamma\left(a+b-\frac{1}{2}\right)z^{\frac{3-2a-2b}{4}}\,P_{-a+b-\frac{1}{2}}^{-a-b+\frac{3}{2}}\left(\sqrt{1-z}\right)$$

07.23.03.0086.01

$${}_2F_1\left(a,b;a+b-\frac{1}{2};z\right)=\frac{2^{a+b-\frac{3}{2}}}{\sqrt{1-z}}\,\Gamma\left(a+b-\frac{1}{2}\right)(-z)^{\frac{3-2a-2b}{4}}\,\mathfrak{P}_{b-a-\frac{1}{2}}^{\frac{3}{2}-a-b}\left(\sqrt{1-z}\right)$$

07.23.03.0087.01

$${}_2F_1\left(a,b;a+b+\frac{1}{2};z\right)=2^{a+b-\frac{1}{2}}\,\Gamma\left(a+b+\frac{1}{2}\right)z^{\frac{1-2a-2b}{4}}\,P_{b-a-\frac{1}{2}}^{\frac{1}{2}-a-b}\left(\sqrt{1-z}\right)$$

07.23.03.0088.01

$${}_2F_1\left(a,b;a+b+\frac{1}{2};z\right)=2^{a+b-\frac{1}{2}}\,\Gamma\left(a+b+\frac{1}{2}\right)(-z)^{\frac{1-2a-2b}{4}}\,\mathfrak{P}_{b-a-\frac{1}{2}}^{\frac{1}{2}-a-b}\left(\sqrt{1-z}\right)\,/;\,z\notin(0,1)$$

07.23.03.0089.01

$${}_2F_1\left(a,b;a+b+\frac{3}{2};z\right)=\frac{1}{2a+1}\,\Gamma\left(a+b+\frac{3}{2}\right)\left(\frac{z}{4}\right)^{-\frac{1+2a+2b}{4}}\left(\sqrt{z}\,P_{b-a-\frac{1}{2}}^{\frac{1}{2}-a-b}\left(\sqrt{1-z}\right)-2\,b\,P_{b-a+\frac{1}{2}}^{-a-b-\frac{1}{2}}\left(\sqrt{1-z}\right)\right)$$

07.23.03.0090.01

$${}_2F_1\left(a,b;a+b+\frac{3}{2};z\right)=\frac{1}{2a+1}\,\Gamma\left(a+b+\frac{3}{2}\right)\left(-\frac{z}{4}\right)^{-\frac{1+2a+2b}{4}}\left(\sqrt{-z}\,\mathfrak{P}_{b-a-\frac{1}{2}}^{\frac{1}{2}-a-b}\left(\sqrt{1-z}\right)-2\,b\,\mathfrak{P}_{b-a+\frac{1}{2}}^{-\frac{1}{2}-a-b}\left(\sqrt{1-z}\right)\right)/;\,z\notin(0,1)$$

07.23.03.0091.01

$${}_2F_1(a,b;a-b+1;z)=\Gamma(a-b+1)\,z^{\frac{b-a}{2}}\,(1-z)^{-b}\,\mathfrak{P}_{-b}^{b-a}\left(\frac{1+z}{1-z}\right)/;\,z\notin(-\infty,0)\bigwedge z\notin(1,\infty)$$

07.23.03.0092.01

$${}_2F_1(a,b;a-b+1;z)=\Gamma(a-b+1)\,(-z)^{\frac{b-a}{2}}\,(1-z)^{-b}\,P_{-b}^{b-a}\left(\frac{1+z}{1-z}\right)/;\,z\notin(1,\infty)$$

07.23.03.0093.01

$${}_2F_1(a,b;a-b+1;z)=\Gamma(a-b+1)\,z^{\frac{b-a}{2}}\,(1-z)^{-b}\,\mathfrak{P}_{-b}^{b-a}\left(\frac{1+z}{1-z}\right)/;\,z\notin(-\infty,0)\bigwedge z\notin(1,\infty)$$

07.23.03.0094.01

$${}_2F_1(a,b;a-b+1;z)=\frac{e^{\frac{\pi i}{2}(1-2b)}\,\Gamma(a-b+1)}{\sqrt{\pi}\,\Gamma(a)}\,z^{\frac{2b-2a-1}{4}}\,(1-z)^{\frac{1}{2}-b}\,\mathfrak{Q}_{a-b-\frac{1}{2}}^{b-\frac{1}{2}}\left(\frac{z+1}{2\sqrt{z}}\right)/;\,|z|<1$$

07.23.03.0095.01

$${}_2F_1(a,b;a-b+2;z)=\frac{\Gamma(a-b+2)}{b-1}\,z^{\frac{b-a-1}{2}}\,(1-z)^{-b}\left(a\,\mathfrak{P}_{-b}^{b-a-1}\left(\frac{1+z}{1-z}\right)-\sqrt{z}\,\mathfrak{P}_{-b}^{b-a}\left(\frac{1+z}{1-z}\right)\right)/;\,z\notin(-\infty,0)\bigwedge z\notin(1,\infty)$$

07.23.03.0096.01

$${}_2F_1(a,b;a-b+2;z)=\frac{\Gamma(a-b+2)}{b-1}\,(-z)^{\frac{b-a-1}{2}}\,(1-z)^{-b}\left(a\,P_{-b}^{b-a-1}\left(\frac{1+z}{1-z}\right)-\sqrt{-z}\,P_{-b}^{b-a}\left(\frac{1+z}{1-z}\right)\right)/;\,z\notin(1,\infty)$$

07.23.03.0097.01

$${}_2F_1\left(a,b;\frac{a+b}{2};z\right)=\frac{z}{2}\,\Gamma\left(\frac{a+b}{2}\right)\left(z-z^2\right)^{-\frac{a+b+2}{4}}\left(P_{\frac{a-b}{2}}^{1-\frac{a+b}{2}}(1-2z)+P_{\frac{a-b}{2}-1}^{1-\frac{a+b}{2}}(1-2z)\right)$$

07.23.03.0098.01

$${}_2F_1\left(a,b;\frac{a+b}{2};z\right)=-\frac{z}{2}\,\Gamma\left(\frac{a+b}{2}\right)\left(z^2-z\right)^{-\frac{a+b+2}{4}}\left(\mathfrak{P}_{\frac{a-b}{2}}^{1-\frac{a+b}{2}}(1-2z)+\mathfrak{P}_{\frac{a-b}{2}-1}^{1-\frac{a+b}{2}}(1-2z)\right)/;\,\mathrm{Re}(z)\le 0$$

07.23.03.0099.01

$${}_2F_1\left(a, b; \frac{a+b+1}{2}; z\right) = \frac{\Gamma(1-a)\,\Gamma(a+b)}{\Gamma(b)}\, C_{-a}^{\frac{a+b}{2}}(1-2z)$$

07.23.03.0100.01

$${}_2F_1\left(a, b; \frac{a+b+1}{2}; z\right) = \Gamma\left(\frac{a+b+1}{2}\right)\left(z-z^2\right)^{\frac{1-a-b}{4}} P_{\frac{a-b-1}{2}}^{\frac{1-a-b}{2}}(1-2z)$$

07.23.03.0101.01

$${}_2F_1\left(a, b; \frac{a+b+1}{2}; z\right) = \Gamma\left(\frac{a+b+1}{2}\right)\left(z^2-z\right)^{\frac{1-a-b}{4}} \mathfrak{P}_{\frac{a-b-1}{2}}^{\frac{1-a-b}{2}}(1-2z) \;/;\; \operatorname{Re}(z) \le 0$$

07.23.03.0102.01

$${}_2F_1\left(a, b; \frac{a+b}{2}+1; z\right) = \frac{\Gamma\left(\frac{a+b}{2}+1\right)}{z\,(b-a)}\left(z-z^2\right)^{\frac{2-a-b}{4}}\left(P_{\frac{a-b}{2}}^{1-\frac{a+b}{2}}(1-2z) - P_{\frac{a-b}{2}-1}^{1-\frac{a+b}{2}}(1-2z)\right)$$

07.23.03.0103.01

$${}_2F_1\left(a, b; \frac{a+b}{2}+1; z\right) = \frac{\Gamma\left(\frac{a+b}{2}+1\right)}{z\,(b-a)}\left(z^2-z\right)^{\frac{2-a-b}{4}}\left(\mathfrak{P}_{\frac{a-b}{2}}^{1-\frac{a+b}{2}}(1-2z) - \mathfrak{P}_{\frac{a-b}{2}-1}^{1-\frac{a+b}{2}}(1-2z)\right) \;/;\; \operatorname{Re}(z) \le 0$$

07.23.03.0104.01

$${}_2F_1(a, b; 2b; z) = 2^{2b-1}\,\Gamma\left(b+\frac{1}{2}\right) z^{\frac{1}{2}-b}\,(1-z)^{\frac{2b-2a-1}{4}}\, \mathfrak{P}_{a-b-\frac{1}{2}}^{\frac{1}{2}-b}\left(\frac{2-z}{2\sqrt{1-z}}\right) \;/;\; \operatorname{Re}\left(\frac{2-z}{2\sqrt{1-z}}\right) > 0$$

07.23.03.0105.01

$${}_2F_1(a, b; 2b; z) = \frac{2^{2b}\, e^{\pi i (a-b)}}{\sqrt{\pi}\;\Gamma(2b-a)}\,\Gamma\left(b+\frac{1}{2}\right) z^{-b}(1-z)^{\frac{b-a}{2}}\, \mathfrak{Q}_{b-1}^{b-a}\left(\frac{2}{z}-1\right) \;/;\; z \notin (-\infty, 0) \land z \notin (1, \infty)$$

07.23.03.0106.01

$${}_2F_1(a, b; 2b; z) = \frac{2^{2b}\, e^{\pi i (b-a)}}{\sqrt{\pi}\;\Gamma(a)}\,\Gamma\left(b+\frac{1}{2}\right)(-z)^{-b}\,(1-z)^{\frac{b-a}{2}}\, \mathfrak{Q}_{b-1}^{a-b}\left(1-\frac{2}{z}\right) \;/;\; z \notin (0, 1)$$

07.23.03.0107.01

$${}_2F_1\left(a, b; \frac{1}{2}; z\right) = \frac{2^{a+b-\frac{3}{2}}}{\sqrt{\pi}}\,\Gamma\left(a+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)(1-z)^{\frac{1-2a-2b}{4}}\left(P_{a-b-\frac{1}{2}}^{\frac{1}{2}-a-b}\left(-\sqrt{z}\right) + P_{a-b-\frac{1}{2}}^{\frac{1}{2}-a-b}\left(\sqrt{z}\right)\right)$$

07.23.03.0108.01

$${}_2F_1\left(a, b; \frac{1}{2}; z\right) = \frac{2^{a-b-1}}{\sqrt{\pi}}\,\Gamma\left(a+\frac{1}{2}\right)\Gamma(1-b)\,(1-z)^{-\frac{a+b}{2}}\left(P_{a+b-1}^{b-a}\left(-\sqrt{\frac{z}{z-1}}\right) + P_{a+b-1}^{b-a}\left(\sqrt{\frac{z}{z-1}}\right)\right) \;/;\; z \notin (1, \infty)$$

For fixed *a*, *c*, *z*

07.23.03.0109.01

$${}_2F_1\left(a, a+\frac{1}{2}; c; z\right) = 2^{c-1}\,\Gamma(c)\,(-z)^{\frac{1-c}{2}}\,(1-z)^{\frac{c-1}{2}-a}\, P_{2a-c}^{1-c}\left(\frac{1}{\sqrt{1-z}}\right)$$

07.23.03.0110.01

$${}_2F_1\left(a, a+\frac{1}{2}; c; z\right) = 2^{c-1}\,\Gamma(c)\,z^{\frac{1-c}{2}}\,(1-z)^{\frac{c-1}{2}-a}\, \mathfrak{P}_{2a-c}^{1-c}\left(\frac{1}{\sqrt{1-z}}\right) \;/;\; z \notin (-\infty, 0)$$

07.23.03.0111.01

$$ {}_2F_1\left(a, a+\frac{1}{2}; c; z\right) = \frac{2^{c-\frac{1}{2}}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(2a)} e^{\pi i\left(c-2a-\frac{1}{2}\right)} z^{\frac{1-2c}{4}} (1-z)^{\frac{2c-1}{4}-a} \mathfrak{Q}_{c-\frac{3}{2}}^{2a-c+\frac{1}{2}}\left(\frac{1}{\sqrt{z}}\right) /; z \notin (1, \infty) $$

07.23.03.0112.01

$$ {}_2F_1(a, 1-a; c; z) = \frac{(1-z)^{\frac{c-1}{2}}}{z^{\frac{c-1}{2}}} \Gamma(c)\, P_{-a}^{1-c}(1-2z) $$

07.23.03.0113.01

$$ {}_2F_1(a, 1-a; c; z) = \frac{(z-1)^{\frac{c-1}{2}}}{z^{\frac{c-1}{2}}} \Gamma(c)\, \mathcal{P}_{-a}^{1-c}(1-2z) $$

07.23.03.0114.01

$$ {}_2F_1(a, 2-a; c; z) = \frac{\Gamma(c)}{a-1} \left(\frac{1}{z}-1\right)^{\frac{c-2}{2}} \left(\frac{1}{z}\right)^{-\frac{c}{2}} z^{-\frac{c}{2}} \left((a+c-2)\sqrt{\frac{1}{z}-1}\,\sqrt{\frac{1}{z}}\,\sqrt{z}\, P_{-a}^{1-c}(1-2z) - P_{-a}^{2-c}(1-2z)\right) $$

07.23.03.0115.01

$$ {}_2F_1(a, 3-a; c; z) = -\frac{\Gamma(c)}{2(a-2)(a-1)} (1-z)^{\frac{c}{2}-2} z^{-\frac{c}{2}} \left((2za-a-c-2z+3)\, P_{1-a}^{2-c}(1-2z) + (a+c-3)\, P_{-a}^{2-c}(1-2z)\right) $$

07.23.03.0116.01

$$ {}_2F_1(a, 3-a; c; z) = \frac{\Gamma(c)}{2(a-1)(a-2)z^2} \left(\frac{z-1}{z}\right)^{\frac{c}{2}-2} \left((a+c-3)\, \mathcal{P}_{-a}^{2-c}(1-2z) - (a+c-3+2(1-a)z)\, \mathcal{P}_{1-a}^{2-c}(1-2z)\right) $$

For fixed *b*, *c*, *z*

07.23.03.0117.01

$$ {}_2F_1(-n, b; c; z) = \sum_{k=0}^{n} \frac{(-n)_k\,(b)_k\,z^k}{(c)_k\,k!} /; n \in \mathbb{N} $$

07.23.03.0118.01

$$ {}_2F_1(-n, b; c; z) = \frac{z^{1-c}\,(1-z)^{c+n-b}}{(c)_n} \frac{\partial^n\left(z^{c+n-1}\,(1-z)^{b-c}\right)}{\partial z^n} /; n \in \mathbb{N} $$

07.23.03.0119.01

$$ {}_2F_1(-n, b; c; z) = \frac{n!}{(c)_n} P_n^{(c-1, b-c-n)}(1-2z) /; n \in \mathbb{N} $$

07.23.03.0120.01

$$ {}_2F_1(-n, b; c; z) = \frac{n!\,z^n}{(c)_n} P_n^{(-b-n, b-c-n)}\left(1-\frac{2}{z}\right) /; n \in \mathbb{N} $$

07.23.03.0121.01

$$ {}_2F_1(-n, b; c; z) = \frac{n!\,(1-z)^n}{(c)_n} P_n^{(c-1, -b-n)}\left(\frac{1+z}{1-z}\right) /; n \in \mathbb{N} $$

07.23.03.0122.01

$$ {}_2F_1(1, b; c; z) = (c-1)\, z^{1-c}\, (1-z)^{-b+c-1}\, \mathrm{B}_z(c-1, b-c+1) $$

For fixed *a*, *z*

07.23.03.0123.01

$$_2F_1\left(a, \frac{a}{2}+1; \frac{a}{2}; z\right) = (1-z)^{-a-1}(z+1)$$

07.23.03.0124.01

$$_2F_1\left(a, a+\frac{1}{2}; 2a-1; z\right) = \frac{2^{2a-2}\left((2a-3)\left(\sqrt{1-z}-1\right)+2(a-1)z\right)}{(2a-1)z(1-z)^{3/2}}\left(\sqrt{1-z}+1\right)^{3-2a}$$

07.23.03.0125.01

$$_2F_1\left(a, a+\frac{1}{2}; 2a; z\right) = \frac{2^{2a-1}\left(\sqrt{1-z}+1\right)^{1-2a}}{\sqrt{1-z}}$$

07.23.03.0126.01

$$_2F_1\left(a, a+\frac{1}{2}; 2a+1; z\right) = 2^{2a}\left(\sqrt{1-z}+1\right)^{-2a}$$

07.23.03.0127.01

$$_2F_1\left(a, a+\frac{1}{2}; \frac{1}{2}; z\right) = \frac{1}{2}\left(\left(1-\sqrt{z}\right)^{-2a}+\left(1+\sqrt{z}\right)^{-2a}\right)$$

07.23.03.0128.01

$$_2F_1\left(a, a+\frac{1}{2}; \frac{1}{2}; z\right) = (1-z)^{-a}\cosh\left(2a\tanh^{-1}\left(\sqrt{z}\right)\right)$$

07.23.03.0129.01

$$_2F_1\left(a, a+\frac{1}{2}; \frac{3}{2}; z\right) = \frac{\left(1-\sqrt{z}\right)^{1-2a}-\left(\sqrt{z}+1\right)^{1-2a}}{2(2a-1)\sqrt{z}}$$

07.23.03.0130.01

$$_2F_1\left(a, a+\frac{1}{2}; \frac{3}{2}; z\right) = \frac{(1-z)^{\frac{1}{2}-a}}{(2a-1)\sqrt{z}}\sinh\left((2a-1)\tanh^{-1}\left(\sqrt{z}\right)\right)$$

07.23.03.0131.01

$$_2F_1\left(a, -a; \frac{1}{2}; z\right) = T_a(1-2z)$$

07.23.03.0132.01

$$_2F_1\left(a, -a; \frac{1}{2}; z\right) = \frac{1}{2}\left(\left(\sqrt{1-z}-\sqrt{-z}\right)^{2a}+\left(\sqrt{1-z}+\sqrt{-z}\right)^{2a}\right)$$

07.23.03.0133.01

$$_2F_1\left(a, -a; \frac{1}{2}; z\right) = \cos\left(2a\sin^{-1}\left(\sqrt{z}\right)\right)$$

07.23.03.0134.01

$$_2F_1\left(a, 1-a; \frac{1}{2}; z\right) = \frac{1}{\sqrt{1-z}}\cos\left((2a-1)\sin^{-1}\left(\sqrt{z}\right)\right)$$

07.23.03.0135.01

$$_2F_1\left(a, 1-a; \frac{1}{2}; z\right) = \frac{1}{2\sqrt{1-z}}\left(\left(\sqrt{1-z}-\sqrt{-z}\right)^{2a-1}+\left(\sqrt{1-z}+\sqrt{-z}\right)^{2a-1}\right)$$

07.23.03.0136.01

$$_2F_1(a, 1-a; 1; z) = P_{-a}(1-2z)$$

07.23.03.0137.01

$$ {}_2F_1\left(a, 1-a; \frac{3}{2}; z\right) = \frac{1}{(2a-1)\sqrt{z}} \sin\left((2a-1)\sin^{-1}\left(\sqrt{z}\right)\right) $$

07.23.03.0138.01

$$ {}_2F_1\left(a, 1-a; \frac{3}{2}; z\right) = \frac{1}{2(2a-1)\sqrt{-z}} \left(\left(\sqrt{1-z} + \sqrt{-z}\right)^{2a-1} - \left(\sqrt{1-z} - \sqrt{-z}\right)^{2a-1}\right) $$

07.23.03.0139.01

$$ {}_2F_1\left(a, 2-a; \frac{3}{2}; z\right) = \frac{1}{1-a} U_{-a}(1-2z) $$

07.23.03.0140.01

$$ {}_2F_1\left(a, 2-a; \frac{3}{2}; z\right) = \frac{1}{a-1} U_{a-2}(1-2z) $$

07.23.03.0141.01

$$ {}_2F_1\left(a, 2-a; \frac{3}{2}; z\right) = \frac{1}{2(a-1)\sqrt{z}\sqrt{1-z}} \sin\left(2(a-1)\sin^{-1}\left(\sqrt{z}\right)\right) $$

For fixed b, z

07.23.03.0142.01

$$ {}_2F_1(n, b; b-m; z) = (1-z)^{-m-n} \sum_{k=0}^{m} \frac{(-m)_k (b-m-n)_k z^k}{(b-m)_k k!} /; m \in \mathbb{N} \land n \in \mathbb{N} $$

07.23.03.0143.01

$$ {}_2F_1(n, b; b-m; z) = (1-z)^{-n} \sum_{k=0}^{m} \frac{(-m)_k (n)_k}{(b-m)_k k!} \left(\frac{z}{z-1}\right)^k /; m \in \mathbb{N} \land n \in \mathbb{N} $$

07.23.03.0144.01

$$ {}_2F_1(n, b; m; z) = (1-z)^{-b} \sum_{k=0}^{n-m} \frac{(m-n)_k (b)_k}{(m)_k k!} \left(\frac{z}{z-1}\right)^k /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N} \bigwedge m \leq n $$

07.23.03.0145.01

$$ {}_2F_1(n, b; m; z) = (1-z)^{m-n-b} \sum_{k=0}^{n-m} \frac{(m-n)_k (m-b)_k z^k}{(m)_k k!} /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N} \bigwedge m \leq n $$

07.23.03.0146.01

$$ {}_2F_1(n, b; m; z) = \frac{(m-1)!\,\Gamma(b-m+n)}{(n-1)!\,\Gamma(b)} (1-z)^{m-n-b} z^{n-m} \sum_{k=0}^{n-1} \frac{(1-n)_k (m-n)_k}{(m-b-n+1)_k k!} \left(\frac{z-1}{z}\right)^k + \\ \frac{z^{-n}(m-1)!\,\Gamma(m-n-b)}{(m-n-1)!\,\Gamma(m-b)} \sum_{k=0}^{m-n-1} \frac{(n-m+1)_k (n)_k}{(b-m+n+1)_k k!} \left(\frac{z-1}{z}\right)^k /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N} \bigwedge m > n $$

07.23.03.0147.01

$$ {}_2F_1(-n, b; -2n-2; z) = -\frac{(-1)^n n!\,\Gamma(1-b)}{2(2n+2)!} (z-1)^{-\frac{b+n+3}{2}} \left(\frac{1}{z}\right)^{\frac{n+b+1}{2} - \left\lfloor \frac{n}{2} \right\rfloor} \\ z^{\frac{b+3n+5}{2} - \left\lfloor \frac{n}{2} \right\rfloor} \left(\left(b+2n+2-\frac{2(n+1)}{z}\right) P_{n+1}^{b+n+1}\left(1-\frac{2}{z}\right) - (b+2n+2) P_n^{b+n+1}\left(1-\frac{2}{z}\right)\right) /; n \in \mathbb{N} $$

07.23.03.0148.01

$$\,_2F_1(-n,b;-2n-2;z)=\frac{n!\,\Gamma(1-b)}{2(2n+2)!}(-z)^{n+2}(1-z)^{-\frac{b+n+3}{2}}\left(\left(b+2n+2-\frac{2(n+1)}{z}\right)\mathfrak{P}_{n+1}^{b+n+1}\left(1-\frac{2}{z}\right)-(b+2n+2)\,\mathfrak{P}_n^{b+n+1}\left(1-\frac{2}{z}\right)\right)/;\,n\in\mathbb{N}$$

07.23.03.0149.01

$$\,_2F_1(-n,b;-2n-1;z)=\frac{(-1)^n\,n!\,\Gamma(1-b)}{(2n+1)!}(z-1)^{-\frac{b+n+1}{2}}\left(\frac{1}{z}\right)^{\frac{b+n}{2}}z^{\frac{b+3n}{2}}\left((b+2n+1)\sqrt{z-1}\,P_n^{b+n}\left(\frac{z-2}{z}\right)+\sqrt{\frac{1}{z}}\,\sqrt{z}\,P_n^{b+n+1}\left(\frac{z-2}{z}\right)\right)/;\,n\in\mathbb{N}$$

07.23.03.0150.01

$$\,_2F_1(-n,b;-2n-1;z)=\frac{n!\,\Gamma(1-b)}{(2n+1)!}(-z)^n(1-z)^{-\frac{b+n}{2}}\left(\mathfrak{P}_n^{b+n+1}\left(1-\frac{2}{z}\right)\frac{1}{\sqrt{1-z}}+(b+2n+1)\,\mathfrak{P}_n^{b+n}\left(1-\frac{2}{z}\right)\right)/;\,n\in\mathbb{N}$$

07.23.03.0151.01

$$\,_2F_1(-n,b;-2n;z)=\frac{(-1)^n\,n!\,\Gamma(1-b)}{(2n)!}(z-1)^{\frac{1}{2}(-b-n)}\left(\frac{1}{z}\right)^{\frac{b+n}{2}}z^{\frac{1}{2}(b+3n)}\,P_n^{b+n}\left(\frac{z-2}{z}\right)/;\,n\in\mathbb{N}^+$$

07.23.03.0152.01

$$\,_2F_1(-n,b;-2n;z)=\frac{n!\,\Gamma(1-b)\,(-z)^n}{(2n)!}(1-z)^{-\frac{b+n}{2}}\,\mathfrak{P}_n^{b+n}\left(1-\frac{2}{z}\right)/;\,n\in\mathbb{N}^+$$

07.23.03.0153.01

$$\,_2F_1(-n,b;2b-1;z)=\frac{\Gamma(1-b)}{2(2b-1)_n}(z-1)^{\frac{b+n}{2}-1}\left(\frac{1}{z}\right)^{\frac{b+n}{2}}z^{1+\frac{n-b}{2}}\left(P_{b-2}^{b+n}\left(\frac{z-2}{z}\right)+P_{b-1}^{b+n}\left(\frac{z-2}{z}\right)\right)/;\,n\in\mathbb{N}$$

07.23.03.0154.01

$$\,_2F_1(-n,b;2b-1;z)=\frac{(-1)^n\,(1-z)^{\frac{n-b}{2}}\,\Gamma(1-b)}{2(2b-1)_n}\left(\frac{z-1}{z}\right)^{b-1}\left(\mathfrak{P}_{b-1}^{b+n}\left(1-\frac{2}{z}\right)+\mathfrak{P}_{b-2}^{b+n}\left(1-\frac{2}{z}\right)\right)/;\,n\in\mathbb{N}$$

07.23.03.0155.01

$$\,_2F_1(-n,b;2b;z)=\frac{n!\,2^{-2n}\,z^n}{\left(b+\frac{1}{2}\right)_n}\,C_n^{\frac{1}{2}-b-n}\left(1-\frac{2}{z}\right)/;\,n\in\mathbb{N}$$

07.23.03.0156.01

$$\,_2F_1(-n,b;-b-n;z)=\frac{n!\,z^{n/2}}{(b+1)_n}\left(C_n^{b+1}\left(\frac{z+1}{2\sqrt{z}}\right)-\sqrt{z}\,C_{n-1}^{b+1}\left(\frac{z+1}{2\sqrt{z}}\right)\right)/;\,n\in\mathbb{N}$$

07.23.03.0157.01

$$\,_2F_1(-n,b;1-b-n;z)=\frac{n!\,z^{n/2}}{(b)_n}\,C_n^b\left(\frac{z+1}{2\sqrt{z}}\right)/;\,n\in\mathbb{N}$$

07.23.03.0158.01

$$\,_2F_1(-n,b;1-b-n;z)=\frac{n!\,(1-z)^n}{(1-2b-2n)_n}\,C_n^{\frac{1}{2}-b-n}\left(\frac{1+z}{1-z}\right)/;\,n\in\mathbb{N}$$

07.23.03.0159.01

$$\,_2F_1(-n,b;2-b-n;z)=\frac{n!\,z^{\frac{n-1}{2}}}{(b-1)_n}\left(\sqrt{z}\,C_n^b\left(\frac{z+1}{2\sqrt{z}}\right)-C_{n-1}^b\left(\frac{z+1}{2\sqrt{z}}\right)\right)/;\,n\in\mathbb{N}$$

07.23.03.0160.01

$${}_2F_1\left(-n,\, b;\, b-n-\frac{1}{2};\, z\right) = \frac{(-1)^n\,(2\,n+1)!\,z^{n+\frac{1}{2}}}{2^{2\,n+1}\left(\frac{1}{2}-b-n\right)_{2\,n+1}\sqrt{z-1}}\,C_{2\,n+1}^{\frac{1}{2}-b-n}\left(\sqrt{\frac{z-1}{z}}\right) /;\, n \in \mathbb{N}$$

07.23.03.0161.01

$${}_2F_1\left(-n,\, b;\, b-n-\frac{1}{2};\, z\right) = -\frac{(2\,n+1)!}{(2-2\,b)_{2\,n+1}\sqrt{1-z}}\,C_{2\,n+1}^{b-n-1}\left(\sqrt{1-z}\right) /;\, n \in \mathbb{N}$$

07.23.03.0162.01

$${}_2F_1\left(-n,\, b;\, b-n+\frac{1}{2};\, z\right) = \frac{(2\,n)!}{(1-2\,b)_{2\,n}}\,C_{2\,n}^{b-n}\left(\sqrt{1-z}\right) /;\, n \in \mathbb{N}$$

07.23.03.0163.01

$${}_2F_1\left(-n,\, b;\, b-n+\frac{1}{2};\, z\right) = \frac{(2\,n)!}{\left(b-n+\frac{1}{2}\right)_{2\,n}}\left(-\frac{z}{4}\right)^n C_{2\,n}^{\frac{1}{2}-b-n}\left(\sqrt{\frac{z-1}{z}}\right) /;\, n \in \mathbb{N}$$

07.23.03.0164.01

$${}_2F_1\left(-n,\, b;\, \frac{b-n+1}{2};\, z\right) = \frac{(-1)^n\,n!}{(1-b)_n}\,C_n^{\frac{b-n}{2}}(1-2\,z) /;\, n \in \mathbb{N}$$

07.23.03.0165.01

$${}_2F_1\left(-\frac{n}{2},\, b;\, b+\frac{1-n}{2};\, z\right) = \frac{(-1)^n\,n!}{(1-2\,b)_n}\,C_n^{b-\frac{n}{2}}\left(\sqrt{1-z}\right) /;\, n \in \mathbb{N}$$

07.23.03.0166.01

$${}_2F_1(1,\, b;\, b+1;\, z) = b\,\Phi(z,\, 1,\, b)$$

07.23.03.0167.01

$${}_2F_1\left(-n,\, b;\, \frac{1}{2};\, z\right) = \frac{n!}{(1-b)_n}\,C_{2\,n}^{b-n}\left(\sqrt{z}\right) /;\, n \in \mathbb{N}$$

07.23.03.0168.01

$${}_2F_1\left(-n,\, b;\, \frac{3}{2};\, z\right) = -\frac{n!}{2\,(1-b)_{n+1}\sqrt{z}}\,C_{2\,n+1}^{b-n-1}\left(\sqrt{z}\right) /;\, n \in \mathbb{N}$$

07.23.03.0169.01

$${}_2F_1\left(1,\, b;\, \frac{3}{2};\, z\right) = \frac{2^{\frac{1}{2}-b}\,\Gamma(1-b)}{\sqrt{\pi}\,\sqrt{z}}\,(1-z)^{\frac{1-2\,b}{4}}\,Q_{\frac{1}{2}-b}^{b-\frac{1}{2}}\left(\sqrt{z}\right)$$

07.23.03.0170.01

$${}_2F_1(1,\, b;\, 2;\, z) = \frac{(1-z)^{1-b}-1}{(b-1)\,z}$$

For fixed *c*, *z*

07.23.03.0171.01

$${}_2F_1\left(-\frac{n}{2},\, \frac{1-n}{2};\, c;\, z\right) = \frac{n!\,(1-z)^{n/2}}{(2\,c-1)_n}\,C_n^{c-\frac{1}{2}}\left(\frac{1}{\sqrt{1-z}}\right) /;\, n \in \mathbb{N}$$

07.23.03.0172.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; c; z\right) = \frac{(-1)^n\, n!}{(c)_n} \left(\frac{z}{4}\right)^{n/2} C_n^{1-c-n}\left(\frac{1}{\sqrt{z}}\right) /; n \in \mathbb{N} $$

07.23.03.0173.01

$$ {}_2F_1(-n, 1; c; z) = (1-c)\, z^{1-c}\, (z-1)^{c+n-1}\, \mathrm{B}_{1-\frac{1}{z}}(1-c-n, n+1) /; n \in \mathbb{N}^+ $$

For fixed z and with symbolical integers in parameters

For fixed z and $a = 1$, $b = \frac{m}{n}$, $c = \frac{m}{n} + 1$

07.23.03.0178.01

$$ {}_2F_1\left(1, \frac{m}{n}; \frac{m}{n}+1; z\right) = -\frac{m}{n} z^{-\frac{m}{n}} \sum_{k=0}^{n-1} e^{-\frac{2\pi i k m}{n}} \log\left(1 - z^{1/n} e^{\frac{2\pi i k}{n}}\right) /; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge n > m $$

07.23.03.0179.01

$$ {}_2F_1\left(1, \frac{m}{n}; \frac{m}{n}+1; z\right) = -\frac{m\,(-z)^{-\frac{m}{n}}}{n} \sum_{k=0}^{n-1} \exp\left(-\frac{\pi (2k+1)\, i\, m}{n}\right) \log\left(1 - (-z)^{1/n} \exp\left(\frac{\pi\, i\, (2k+1)}{n}\right)\right) /; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge n > m $$

07.23.03.0180.01

$$ {}_2F_1\left(1, \frac{m}{n}; \frac{m}{n}+1; z\right) = -\frac{m}{n} z^{-\frac{m}{n}} \left(\log\left(1 - z^{1/n}\right) + \frac{(-1)^m (1+(-1)^n)}{2} \log\left(1 + z^{1/n}\right) + \sum_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \left(\cos\left(\frac{2\pi k m}{n}\right) \log\left(1 - 2\cos\left(\frac{2\pi k}{n}\right) z^{1/n} + z^{2/n}\right) - 2\sin\left(\frac{2\pi k m}{n}\right) \tan^{-1}\left(\frac{z^{1/n} \sin\left(\frac{2\pi k}{n}\right)}{1 - z^{1/n} \cos\left(\frac{2\pi k}{n}\right)}\right)\right)\right) /; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge n > m $$

For fixed z and $a = 1$, $b = n$, $c = p$

07.23.03.0174.01

$$ {}_2F_1(1, n; m; z) = \frac{(1-m)_n}{(n-1)!} \left((-1)^m z^{1-m} \log(1-z)\, (1-z)^{m-n-1} + (-z)^{1-n} \sum_{k=0}^{m-n-1} \frac{(n-m+1)_k}{k!} \sum_{j=1}^{k+n-1} \frac{z^{j-k-1}}{j}\right) /; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge n < m $$

07.23.03.0175.01

$$ {}_2F_1(1, n; m; z) = \frac{(m-1)!}{(m-n-1)!\, z} \left(\sum_{k=1}^{m-n-1} \frac{(m-n-k-1)!}{(m-k-1)!} \left(\frac{z-1}{z}\right)^{k-1} - \frac{z}{(n-1)!} \left(\frac{z-1}{z}\right)^{m-n-1} \left(\log(1-z)\, z^{-n} + \sum_{k=1}^{n-1} \frac{z^{-k}}{n-k}\right)\right) /; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge n < m $$

07.23.03.0176.01

$$ {}_2F_1(1, n; m; z) = (m-1)\, (1-z)^{m-n-1} \sum_{k=0}^{n-m} \frac{(m-n)_k\, z^k}{(k+m-1)\, k!} /; m-1 \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge n \geq m $$

07.23.03.0224.01

$${}_2F_1(1,\, n;\, n+1;\, z) = -\frac{n}{z^n}\left(\log(1-z) + \sum_{k=1}^{n-1}\frac{z^k}{k}\right) /;\, n \in \mathbb{N}^+$$

07.23.03.0231.01

$${}_2F_1(1,\, 1;\, m;\, z) = \frac{(m-1)\, z}{(z-1)^2}\left(\sum_{k=2}^{m-1}\frac{1}{m-k}\left(\frac{z-1}{z}\right)^k - \left(\frac{z-1}{z}\right)^m \log(1-z)\right) /;\, m-1 \in \mathbb{N}^+$$

07.23.03.0670.01

$${}_2F_1(1,\, 2;\, m;\, z) = \frac{1}{(1-z)^{3-m}} /;\, m = 1 \lor m = 2$$

07.23.03.0671.01

$${}_2F_1(1,\, 2;\, m;\, z) = -((m-3)\, m+2)\left(z^{1-m}\,(z-1)^{m-3}\log(1-z) + \frac{1}{z}\sum_{k=0}^{m-3}\frac{(3-m)_k}{k!}\sum_{j=1}^{k+1}\frac{z^{j-k-1}}{j}\right) /;\, m \in \mathbb{Z} \land m \geq 3$$

For fixed z and $a = -m$, $b = n$, $c = p$

07.23.03.0182.01

$${}_2F_1(-n,\, 1;\, m;\, z) = \frac{n!\,(z-1)^{m-2}}{(m)_n\, z^{m-1}}\left(\sum_{k=0}^{m-2}\frac{(n+1)_k}{k!}\left(\frac{z}{z-1}\right)^k - (1-z)^{n+1}\right) /;\, m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}$$

07.23.03.0206.01

$${}_2F_1(-n,\, n+1;\, 1;\, z) = P_n(1-2z) /;\, n \in \mathbb{N}$$

07.23.03.0227.01

$${}_2F_1(-n,\, 1;\, 2;\, z) = \frac{z\,(1-z)^n - (1-z)^n + 1}{n\, z + z} /;\, n \in \mathbb{N}$$

07.23.03.0228.01

$${}_2F_1(-n,\, 2;\, 3;\, z) = \frac{2\left(z^2\,(1-z)^n - (1-z)^n - n\, z\,(1-z)^{n+1} + 1\right)}{(n+1)\,(n+2)\, z^2} /;\, n \in \mathbb{N}$$

07.23.03.0229.01

$${}_2F_1(-n,\, 3;\, 4;\, z) = \left(6 - 6\,(1-z)^n - 3\, n^2\,(1-z)^{1+n}\, z^2 + 6\,(1-z)^n\, z^3 + 3\, n\,(1-z)^n\, z\left(-2 - z + 3\, z^2\right)\right) \Big/ \left((n+1)\,(n+2)\,(n+3)\, z^3\right) /;\, n \in \mathbb{N}$$

For fixed z and $a = -m$, $b = n$, $c = -p$

07.23.03.0181.01

$${}_2F_1(-n,\, 1;\, -m;\, z) = \frac{z^{m+1}\,(-1)^n\, n!\,(z-1)^{n-m-1}}{(-m)_n} + \frac{m+1}{n+1}\sum_{k=0}^{m-n}\frac{(n-m)_k\,(1-z)^{-k-1}}{(n+2)_k} /;\, m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+ \bigwedge n < m$$

For fixed z and $a = -m$, $b = -n$, $c = p$

07.23.03.0195.01

$${}_2F_1(-n,\, -n;\, 1;\, z) = (1-z)^n\, P_n\!\left(\frac{1+z}{1-z}\right) /;\, n \in \mathbb{N}$$

For fixed z and $a = -m$, $b = -n$, $c = -p$

07.23.03.0185.01

$$ {}_2F_1(-n, -n; -2n; z) = \frac{(-1)^n\, n!\, z^n}{2^{2n} \left(\frac{1}{2}\right)_n} P_n\left(1 - \frac{2}{z}\right) /;\ n \in \mathbb{N}^+ $$

For fixed z and $a = 1$, $b = n$, $c = \frac{1}{2} + p$

07.23.03.0177.02

$$ {}_2F_1\left(1, n; m + \frac{1}{2}; z\right) = \frac{\Gamma\left(\frac{1}{2} - m + n\right)\Gamma\left(m + \frac{1}{2}\right)}{\pi\,(n-1)!} z^{-m}(1-z)^{m-n} \left(\frac{2\sqrt{z}}{\sqrt{1-z}} \sin^{-1}\left(\sqrt{z}\right) - \sum_{k=1}^{m-1} \frac{(k-1)!\, z^k}{\left(\frac{1}{2}\right)_k}\right) + \frac{2m-1}{2(n-1)} \sum_{k=0}^{n-m-1} \frac{\left(m - n + \frac{1}{2}\right)_k}{(2-n)_k (1-z)^{k+1}} /;\ m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+ \bigwedge ((m = 1 \land n > 2) \lor 1 < m \leq n) $$

07.23.03.0672.01

$$ {}_2F_1\left(1, n; \frac{1}{2}; z\right) = \frac{2\left(\frac{1}{2}\right)_n}{(n-1)!\,(1-z)^n} \left(1 + \frac{\sqrt{z}\, \sin^{-1}\left(\sqrt{z}\right)}{\sqrt{1-z}}\right) + \frac{1}{2} \sum_{k=0}^{n-2} \frac{\left(\frac{1}{2} - n\right)_k}{(1-n)_{k+1} (1-z)^{k+1}} /;\ n \in \mathbb{N}^+ $$

07.23.03.0673.01

$$ {}_2F_1\left(1, n; \frac{3}{2}; z\right) = -\frac{(1-z)^{-n}\left(-\frac{1}{2}\right)_n}{(n-1)!} \left(\frac{2\sqrt{1-z}\, \sin^{-1}\left(\sqrt{z}\right)}{\sqrt{z}} + \sum_{k=1}^{n-1} \frac{(1-z)^k (k-1)!}{\left(\frac{1}{2}\right)_k}\right) /;\ n \in \mathbb{N}^+ $$

07.23.03.0674.01

$$ {}_2F_1\left(1, n; \frac{5}{2}; z\right) = \frac{1}{z} \left(\frac{\pi\left(-\frac{3}{2}\right)_n (1-z)^{\frac{3}{2}-n}}{(n-1)!\,\sqrt{z}} - \frac{(1-z)^{1-n}\left(-\frac{3}{2}\right)_n}{(n-1)!} \left(\frac{2\sqrt{1-z}\, \sin^{-1}\left(\sqrt{1-z}\right)}{\sqrt{z}} - \sum_{j=1}^{n-2} \frac{(j-1)!\,(1-z)^j}{\left(\frac{1}{2}\right)_j}\right) - \frac{3}{2(n-1)}\right) /;\ n - 1 \in \mathbb{N}^+ $$

07.23.03.0675.01

$$ {}_2F_1\left(1, n; \frac{5}{2}; z\right) = \frac{3}{(2n-3)\,z} \left(-\frac{2(1-z)^{1-n}\left(\frac{1}{2}\right)_n}{(n-1)!} \left(1 - \frac{\sqrt{1-z}\, \sin^{-1}\left(\sqrt{z}\right)}{(2n-1)\sqrt{z}}\right) - \frac{(2(n-1)z+1)\left(-\frac{1}{2}\right)_n}{(n-1)!\,(1-z)} \sum_{k=0}^{n-2} \frac{(1-z)^{-k}(n-k-2)!}{\left(\frac{1}{2}\right)_{n-k-1}} - \frac{1}{2} \sum_{k=0}^{n-2} \frac{(1-z)^{-k}\left(\frac{1}{2} - n\right)_k}{(1-n)_{k+1}}\right) /;\ n \in \mathbb{N}^+ $$

07.23.03.0676.01

$$ {}_2F_1\left(1, n; n - \frac{1}{2}; z\right) = \frac{3-2n}{2(n-1)(z-1)} + \frac{\left(-\frac{1}{2}\right)_n z^{1-n}}{(z-1)(n-1)!} \left(\frac{2\sqrt{z}}{\sqrt{1-z}} \sin^{-1}\left(\sqrt{z}\right) - \sum_{k=1}^{n-2} \frac{(k-1)!\, z^k}{\left(\frac{1}{2}\right)_k}\right) /;\ n - 1 \in \mathbb{N}^+ $$

07.23.03.0223.01

$$ {}_2F_1\left(1, n; n+\frac{1}{2}; z\right) = \frac{\left(\frac{1}{2}\right)_n}{(n-1)!\, z^n} \left(2 \sqrt{\frac{z}{1-z}} \sin^{-1}\left(\sqrt{z}\right) - \sum_{k=1}^{n-1} \frac{(k-1)!\, z^k}{\left(\frac{1}{2}\right)_k}\right) /; n \in \mathbb{N}^+ $$

07.23.03.0232.01

$$ {}_2F_1\left(1, 1; n+\frac{1}{2}; z\right) = \frac{(2n-1)(z-1)^{n-1}}{z^n} \left(\sqrt{\frac{z}{1-z}} \sin^{-1}\left(\sqrt{z}\right) + \sum_{k=1}^{n-1} \frac{1}{2k-1} \left(\frac{z}{z-1}\right)^k\right) /; n \in \mathbb{N}^+ $$

07.23.03.0677.01

$$ {}_2F_1\left(1, 1; n+\frac{3}{2}; z\right) = -(2n+1)(z-1)^{n-1} z^{-n} \left(\frac{\sqrt{1-z}}{\sqrt{z}} \sin^{-1}\left(\sqrt{z}\right) - \sum_{k=1}^{|n|} \frac{1}{2k-1} \left(\frac{z}{z-1}\right)^{\operatorname{sgn}(n)\, k - \theta(n)}\right) /; n \in \mathbb{Z} $$

07.23.03.0678.01

$$ {}_2F_1\left(1, 2; n+\frac{3}{2}; z\right) = \frac{2n+1}{2(1-z)} - \frac{1}{2}\left((2n+1)(2n-1)(z-1)^{n-2} z^{-n}\right) \left(\frac{\sqrt{1-z}}{\sqrt{z}} \sin^{-1}\left(\sqrt{z}\right) - \sum_{k=1}^{|n|} \frac{1}{2k-1} \left(\frac{z}{z-1}\right)^{\operatorname{sgn}(n)\, k - \theta(n)}\right) /; n \in \mathbb{Z} $$

For fixed z and $a = -m$, $b = n$, $c = \frac{1}{2} + p$

07.23.03.0198.01

$$ {}_2F_1\left(-n, n; \frac{1}{2}; z\right) = T_n(1-2z) /; n \in \mathbb{N} $$

07.23.03.0199.01

$$ {}_2F_1\left(-n, n; \frac{1}{2}; z\right) = \frac{(-1)^n}{2} \left(\left(\sqrt{z} - \sqrt{z-1}\right)^{2n} + \left(\sqrt{z-1} + \sqrt{z}\right)^{2n}\right) /; n \in \mathbb{N} $$

07.23.03.0200.01

$$ {}_2F_1\left(-n, n; \frac{1}{2}; z\right) = (-1)^n T_{2n}\left(\sqrt{z}\right) /; n \in \mathbb{N} $$

07.23.03.0201.01

$$ {}_2F_1\left(-n, n; \frac{1}{2}; z\right) = T_{2n}\left(\sqrt{1-z}\right) /; n \in \mathbb{N} $$

07.23.03.0204.01

$$ {}_2F_1\left(-n, n+1; \frac{1}{2}; z\right) = (-1)^n U_{2n}\left(\sqrt{z}\right) /; n \in \mathbb{N} $$

07.23.03.0205.01

$$ {}_2F_1\left(-n, n+1; \frac{1}{2}; z\right) = \frac{1}{\sqrt{1-z}} T_{2n+1}\left(\sqrt{1-z}\right) /; n \in \mathbb{N} $$

07.23.03.0207.01

$$ {}_2F_1\left(-n, n+1; \frac{3}{2}; z\right) = \frac{1}{2n+1} U_{2n}\left(\sqrt{1-z}\right) /; n \in \mathbb{N} $$

07.23.03.0208.01

$$ {}_2F_1\left(-n, n+1; \frac{3}{2}; z\right) = \frac{(-1)^n}{(2n+1)\sqrt{z}} T_{2n+1}\left(\sqrt{z}\right) /; n \in \mathbb{N} $$

07.23.03.0212.01

$${}_2F_1\left(-n,\, n+2;\, \frac{3}{2};\, z\right) = \frac{1}{n+1}\, U_n(1-2\,z) \,/;\, n \in \mathbb{N}$$

07.23.03.0213.01

$${}_2F_1\left(-n,\, n+2;\, \frac{3}{2};\, z\right) = \frac{(-1)^n}{2\,(n+1)\,\sqrt{z}}\, U_{2\,n+1}\left(\sqrt{z}\right) \,/;\, n \in \mathbb{N}$$

For fixed z and $a = -m$, $b = -n$, $c = \frac{1}{2} \pm p$

07.23.03.0184.01

$${}_2F_1\left(-n,\, -n;\, -2\,n - \frac{1}{2};\, z\right) = \frac{(-1)^n\,(2\,n+1)!\,\sqrt{\pi}}{\Gamma\left(2\,n + \frac{3}{2}\right)\sqrt{z-1}} \left(\frac{z}{4}\right)^{n+\frac{1}{2}} P_{2\,n+1}\left(\sqrt{\frac{z-1}{z}}\right) \,/;\, n \in \mathbb{N}$$

07.23.03.0186.01

$${}_2F_1\left(-n,\, -n;\, \frac{1}{2} - 2\,n;\, z\right) = \frac{(-1)^n\,(2\,n)!^2\,(4\,z)^n}{(4\,n)!}\, P_{2\,n}\left(\sqrt{\frac{z-1}{z}}\right) \,/;\, n \in \mathbb{N}$$

07.23.03.0194.01

$${}_2F_1\left(-n,\, -n;\, \frac{1}{2};\, z\right) = \frac{n!\,(z-1)^n}{\left(\frac{1}{2}\right)_n}\, P_{2\,n}\left(\sqrt{\frac{z}{z-1}}\right) \,/;\, n \in \mathbb{N}$$

07.23.03.0196.01

$${}_2F_1\left(-n,\, -n;\, \frac{3}{2};\, z\right) = \frac{(-1)^n\,n!\,(1-z)^{n+\frac{1}{2}}}{\left(\frac{3}{2}\right)_n\sqrt{-z}}\, P_{2\,n+1}\left(\sqrt{\frac{z}{z-1}}\right) \,/;\, n \in \mathbb{N}$$

For fixed z and $a = 1$, $b = \frac{1}{2} + n$, $c = p$

07.23.03.0679.01

$${}_2F_1\left(1,\, \frac{1}{2};\, n;\, z\right) = \frac{(-1)^n\,z^{1-n}\,(n-1)!\,(1-z)^{n-\frac{3}{2}}}{2\left(-\frac{1}{2}\right)_n} + \frac{2\,(n-1)}{z} \sum_{k=0}^{n-2} \frac{(2-n)_k\,z^{-k}}{\left(\frac{3}{2}\right)_k} \,/;\, n \in \mathbb{N}^+$$

07.23.03.0680.01

$${}_2F_1\left(1,\, n + \frac{1}{2};\, n+1;\, z\right) = \frac{z^{-n}\,n!}{\sqrt{1-z}\,\left(\frac{1}{2}\right)_n} - \frac{2\,n}{z} \sum_{k=0}^{n-1} \frac{(1-n)_k\left(1-\frac{1}{z}\right)^k}{\left(\frac{3}{2}\right)_k} \,/;\, n \in \mathbb{N}$$

For fixed z and $a = -m$, $b = \frac{1}{2} - n$, $c = -p$

07.23.03.0183.01

$${}_2F_1\left(-n,\, -n - \frac{1}{2};\, -2\,n-1;\, z\right) = (-1)^n\,2^{-2\,n}\,z^n\,U_n\left(1 - \frac{2}{z}\right) \,/;\, n \in \mathbb{N}$$

07.23.03.0187.01

$${}_2F_1\left(-n,\, \frac{1}{2} - n;\, 1 - 2\,n;\, z\right) = (-1)^n\,2^{1-2\,n}\,z^n\,T_n\left(1 - \frac{2}{z}\right) \,/;\, n - 1 \in \mathbb{N}^+$$

For fixed z and $a = -m$, $b = n + \frac{1}{2}$, $c = 1$

07.23.03.0226.01

$$ {}_2F_1\left(-n, \frac{1}{2}; 1; z\right) = (1-z)^{n/2} P_n\left(\frac{2-z}{2\sqrt{1-z}}\right) /; n \in \mathbb{N} $$

07.23.03.0203.01

$$ {}_2F_1\left(-n, n+\frac{1}{2}; 1; z\right) = P_{2n}\left(\sqrt{1-z}\right) /; n \in \mathbb{N} $$

07.23.03.0210.01

$$ {}_2F_1\left(-n, n+\frac{3}{2}; 1; z\right) = \frac{1}{\sqrt{1-z}} P_{2n+1}\left(\sqrt{1-z}\right) /; n \in \mathbb{N} $$

For fixed z and $a = \frac{1}{2}$, $b = n$, $c = p$

07.23.03.0681.01

$$ {}_2F_1\left(\frac{1}{2}, n; n+1; z\right) = \frac{z^{-n}\, n!}{\left(\frac{1}{2}\right)_n} - \frac{2n\sqrt{1-z}}{z} \sum_{k=0}^{n-1} \frac{(1-n)_k \left(1-\frac{1}{z}\right)^k}{\left(\frac{3}{2}\right)_k} /; n \in \mathbb{N} $$

For fixed z and $a = \frac{1}{2}$, $b = \frac{1}{2} + n$, $c = p$

07.23.03.0682.01

$$ {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; n+1; z\right) = \frac{2\, n!\, (1-z)^n}{\pi \left(\frac{1}{2}\right)_n^2} \frac{\partial^n K(z)}{\partial z^n} /; n \in \mathbb{N} $$

07.23.03.0683.01

$$ {}_2F_1\left(\frac{1}{2}, \frac{3}{2}; n+1; z\right) = \frac{2\, n!\, (1-z)^{n-1}}{\pi \left(-\frac{1}{2}\right)_n \left(\frac{1}{2}\right)_n} \frac{\partial^n E(z)}{\partial z^n} /; n \in \mathbb{N} $$

For fixed z and $a = \frac{1}{2} + m$, $b = n$, $c = \frac{1}{2} + p$

07.23.03.0684.01

$$ {}_2F_1\left(\frac{1}{2}, n+1; \frac{3}{2}; z\right) = \frac{\left(\frac{1}{2}\right)_n}{2\, n!} \left(\frac{2\tanh^{-1}\left(\sqrt{z}\right)}{\sqrt{z}} + \sum_{k=1}^{n} \frac{(1-z)^{-k}\, (k-1)!}{\left(\frac{1}{2}\right)_k}\right) /; n \in \mathbb{N} $$

07.23.03.0685.01

$$ {}_2F_1\left(\frac{3}{2}, 2; n+\frac{3}{2}; z\right) = $$

$$ \frac{(2n+1)\left(\frac{1}{2}\right)_n z^{-n-1} (z-1)^{n-2}}{(n-1)!} \left(\left((2n-z-1)\sqrt{z}\, \tanh^{-1}\left(\sqrt{z}\right) - z\right) - \sum_{k=1}^{n-1} \frac{(k-1)!\, (k-n+z)}{\left(\frac{1}{2}\right)_k} \left(\frac{z}{z-1}\right)^k\right) /; n \in \mathbb{N}^+ $$

07.23.03.0686.01

$${}_2F_1\left(n+\frac{1}{2}, n+1; n+\frac{3}{2}; z\right) - \frac{(-1)^n \left(\frac{1}{2}\right)_{n+1}}{n!\, z^{n+1}} \left(2\sqrt{z}\, \tanh^{-1}\left(\sqrt{z}\right) + \sum_{j=1}^{n} \frac{(j-1)!}{\left(\frac{1}{2}\right)_j} \left(\frac{z}{z-1}\right)^j\right) /; n \in \mathbb{N}$$

For fixed z and $a = -m$, $b = \frac{1}{2} - n$, $c = \frac{1}{2} + p$

07.23.03.0197.01

$${}_2F_1\left(-n, \frac{1}{2} - n; \frac{1}{2}; z\right) = (1-z)^n\, T_n\left(\frac{1+z}{1-z}\right) /; n \in \mathbb{N}$$

07.23.03.0193.01

$${}_2F_1\left(-n, -n - \frac{1}{2}; \frac{3}{2}; z\right) = \frac{(1-z)^n}{n+1}\, U_n\left(\frac{1+z}{1-z}\right) /; n \in \mathbb{N}$$

For fixed z and $a = -m$, $b = n + \frac{1}{2}$, $c = \frac{1}{2} \pm p$

07.23.03.0202.01

$${}_2F_1\left(-n, n + \frac{1}{2}; \frac{1}{2}; z\right) = \frac{(-1)^n\, n!}{\left(\frac{1}{2}\right)_n}\, P_{2n}\left(\sqrt{z}\right) /; n \in \mathbb{N}$$

07.23.03.0209.01

$${}_2F_1\left(-n, n + \frac{3}{2}; \frac{1}{2}; z\right) = \frac{2\,(-1)^n\, n!\, \sqrt{z}}{\left(\frac{3}{2}\right)_n}\, \frac{\partial P_{2n+1}\left(\sqrt{z}\right)}{\partial z} /; n \in \mathbb{N}$$

07.23.03.0211.01

$${}_2F_1\left(-n, n + \frac{3}{2}; \frac{3}{2}; z\right) = \frac{(-1)^n\, n!}{\left(\frac{3}{2}\right)_n \sqrt{z}}\, P_{2n+1}\left(\sqrt{z}\right) /; n \in \mathbb{N}$$

07.23.03.0214.01

$${}_2F_1\left(-n, n + \frac{5}{2}; \frac{3}{2}; z\right) = \frac{2\,(-1)^n\, n!}{3\left(\frac{5}{2}\right)_n}\, \frac{\partial P_{2n+2}\left(\sqrt{z}\right)}{\partial z} /; n \in \mathbb{N}$$

07.23.03.0221.01

$${}_2F_1\left(-n, \frac{1}{2}; \frac{1}{2} - n; z\right) = \frac{2^{2n}\, z^{n/2}\, n!^2}{(2n)!}\, P_n\left(\frac{z+1}{2\sqrt{z}}\right) /; n \in \mathbb{N}$$

For fixed z and $a = 1$, $b = \frac{1}{2} + n$, $c = \frac{1}{2} + p$

07.23.03.0222.01

$${}_2F_1\left(1, n - \frac{1}{2}; n + \frac{1}{2}; z\right) = \frac{2n-1}{z^n} \left(\sqrt{z}\, \tanh^{-1}\left(\sqrt{z}\right) - \sum_{k=1}^{n-1} \frac{z^k}{2k-1}\right) /; n \in \mathbb{N}^+$$

07.23.03.0230.01

$$ {}_2F_1\left(1, \frac{1}{2}; n+\frac{1}{2}; z\right) = \frac{\left(\frac{1}{2}\right)_n (z-1)^{n-1}}{(n-1)!\, z^n} \left(2\sqrt{z}\, \tanh^{-1}\left(\sqrt{z}\right) + \sum_{k=1}^{n-1} \frac{(k-1)!}{\left(\frac{1}{2}\right)_k} \left(\frac{z}{z-1}\right)^k\right) /;\, n \in \mathbb{N}^+ $$

07.23.03.0687.01

$$ {}_2F_1\left(1, \frac{3}{2}; n+\frac{3}{2}; z\right) = \frac{2n+1}{2n-z-1} \left(1 + \frac{2\left(\frac{1}{2}\right)_n (z-1)^{n-1} z^{-n-1}}{(n-1)!} \left(-z + (2n-z-1)\tanh^{-1}\left(\sqrt{z}\right)\sqrt{z} - \sum_{k=1}^{n-1} \frac{(k-1)!\, (z-1)^{-k} z^k (k-n+z)}{\left(\frac{1}{2}\right)_k}\right)\right) /;\, n \in \mathbb{N}^+ $$

For fixed z and $a = 1$, $b = \frac{1}{2} - n$, $c = \frac{1}{2} + p$

07.23.03.0225.02

$$ {}_2F_1\left(1, \frac{1}{2}-n; \frac{3}{2}; z\right) = \frac{\left(\frac{1}{2}\right)_n (1-z)^n}{2\, n!} \left(\frac{2\tanh^{-1}\left(\sqrt{z}\right)}{\sqrt{z}} + \sum_{k=1}^{n} \frac{(1-z)^{-k} (k-1)!}{\left(\frac{1}{2}\right)_k}\right) /;\, n \in \mathbb{N} $$

For fixed z and $a = 1$, $b = \frac{1}{2} - n$, $c = \frac{3}{2} - p$

07.23.03.0688.01

$$ {}_2F_1\left(1, \frac{1}{2}-n; \frac{3}{2}-n; z\right) = (1-2n)\, z^n \left(\frac{\tanh^{-1}\left(\sqrt{z}\right)}{\sqrt{z}} - \sum_{k=1}^{n} \frac{z^{-k}}{2k-1}\right) /;\, n \in \mathbb{N} $$

For fixed z and $a = \frac{1}{2}$, $b = \frac{1}{2} + n$, $c = \frac{1}{2} + p$

07.23.03.0689.01

$$ {}_2F_1\left(\frac{1}{2}, n+\frac{1}{2}; n+\frac{3}{2}; z\right) = \frac{\left(\frac{1}{2}\right)_{n+1}}{n!\, z^{n+1}} \left(2\sqrt{z}\, \sin^{-1}\left(\sqrt{z}\right) - \sqrt{1-z} \sum_{j=1}^{n} \frac{(j-1)!\, z^j}{\left(\frac{1}{2}\right)_j}\right) /;\, n \in \mathbb{N} $$

07.23.03.0690.01

$$ {}_2F_1\left(\frac{1}{2}, n+\frac{1}{2}; \frac{3}{2}; z\right) = \frac{(1-z)^{\frac{1}{2}-n}}{2n-1} \sum_{k=0}^{n-1} \frac{(1-n)_k (1-z)^k}{\left(\frac{3}{2}-n\right)_k} /;\, n \in \mathbb{N}^+ $$

For fixed z and $a = \frac{1}{2}$, $b = \frac{1}{2} - n$, $c = \frac{1}{2} + p$

07.23.03.0691.01

$$ {}_2F_1\left(\frac{1}{2}, \frac{1}{2}-n; \frac{3}{2}; z\right) = \frac{\left(\frac{1}{2}\right)_n}{(2\, n!)\sqrt{1-z}} \left(\frac{\left(2\sqrt{1-z}\right)\sin^{-1}\left(\sqrt{z}\right)}{\sqrt{z}} + \sum_{k=1}^{n} \frac{(1-z)^k (k-1)!}{\left(\frac{1}{2}\right)_k}\right) /;\, n \in \mathbb{N} $$

For fixed z and $a = -\frac{n}{2}$, $b = \frac{1-n}{2}$, $c = \pm p$

07.23.03.0216.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; 1; z\right) = (1-z)^{n/2} P_n\left(\frac{1}{\sqrt{1-z}}\right) /; n \in \mathbb{N} $$

07.23.03.0189.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; -n; z\right) = 2^{-n} z^{n/2} U_n\left(\frac{1}{\sqrt{z}}\right) /; n \in \mathbb{N}^+ $$

07.23.03.0191.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; 1-n; z\right) = 2^{1-n} z^{n/2} T_n\left(\frac{1}{\sqrt{z}}\right) /; n-2 \in \mathbb{N}^+ $$

07.23.03.0192.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; 1-n; z\right) = 2^{-n} \left(\left(1-\sqrt{1-z}\right)^n + \left(1+\sqrt{1-z}\right)^n\right) /; n-2 \in \mathbb{N}^+ $$

For fixed z and $a = -\frac{n}{2}$, $b = \frac{1-n}{2}$, $c = \frac{1}{2} \pm p$

07.23.03.0188.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; -n-\frac{1}{2}; z\right) = -\frac{2^{n+1} n!^2}{(2n+1)!} z^{\frac{n+3}{2}} \frac{\partial P_{n+1}\left(\frac{1}{\sqrt{z}}\right)}{\partial z} /; n \in \mathbb{N} $$

07.23.03.0190.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; \frac{1}{2}-n; z\right) = \frac{n!\, 2^{-n} z^{n/2}}{\left(\frac{1}{2}\right)_n} P_n\left(\frac{1}{\sqrt{z}}\right) /; n \in \mathbb{N} $$

07.23.03.0215.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; \frac{1}{2}; z\right) = (1-z)^{n/2} T_n\left(\frac{1}{\sqrt{1-z}}\right) /; n \in \mathbb{N} $$

07.23.03.0217.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; \frac{3}{2}; z\right) = \frac{(1-z)^{n/2}}{n+1} U_n\left(\frac{1}{\sqrt{1-z}}\right) /; n \in \mathbb{N} $$

For fixed z and $a = -\frac{m}{2}$, $b = \frac{n}{2}$, $c = p + \frac{1}{2}$

07.23.03.0218.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{n}{2}; \frac{1}{2}; z\right) = T_n\left(\sqrt{1-z}\right) /; n \in \mathbb{N} $$

07.23.03.0220.01

$$ {}_2F_1\left(-\frac{n}{2}, \frac{n}{2}+1; \frac{3}{2}; z\right) = \frac{1}{n+1} U_n\left(\sqrt{1-z}\right) /; n \in \mathbb{N} $$

For fixed z and $a = -\frac{m}{2}$, $b = \frac{n}{2}$, $c = 1$

07.23.03.0219.01

$${}_2F_1\left(-\frac{n}{2}, \frac{n+1}{2}; 1; z\right) = P_n\left(\sqrt{1-z}\right) /; n \in \mathbb{N}$$

For integer and half-integer parameters and fixed z

For rational parameters with denominators 3 and fixed z

For rational parameters with denominators 4 and fixed z

For rational parameters with denominators 5 and fixed z

For rational parameters with denominators 6 and fixed z

For rational parameters with denominators 8 and fixed z

For rational parameters with denominators 8 and fixed z and $a > 0$

Values at fixed points

For $z = -1$ and integer parameters

07.23.03.0599.01

$${}_2F_1(-n, -m; 1-m; -1) = \frac{(-1)^{m-n}\, n!\, (m-n-1)!}{(m-1)!} \left(1 - 2^m \sum_{k=0}^{m-n-1} \frac{(-m)_k\, 2^{-k}}{k!}\right) /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N} \bigwedge m \geq n+1$$

07.23.03.0600.01

$${}_2F_1(-n, m; m+1; -1) = \frac{(-1)^m\, n!\, m!}{(m+n)!} - 2^n \sum_{k=1}^{m} \frac{(-m)_k\, 2^k}{(n+1)_k} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

07.23.03.0601.01

$${}_2F_1(-n, 2; 1; -1) = 2^{n-1}\,(n+2) /; n \in \mathbb{N}$$

07.23.03.0602.01

$${}_2F_1(1, m; m+1; -1) = (-1)^{m-1}\, m \left(\log(2) + \sum_{k=1}^{m-1} \frac{(-1)^k}{k}\right) /; m \in \mathbb{N}^+$$

Values at $z = -1$

07.23.03.0603.01

$${}_2F_1\left(\frac{1}{6}, 1; \frac{13}{6}; -1\right) = \frac{7}{18}\left(\sqrt{3}\, \log\left(2+\sqrt{3}\right) + \pi - 3\right)$$

07.23.03.0604.01

$${}_2F_1\left(\frac{1}{4}, 1; \frac{5}{4}; -1\right) = \frac{\sqrt{2}}{8}\left(2\log\left(1+\sqrt{2}\right) + \pi\right)$$

07.23.03.0605.01

$${}_2F_1\left(\frac{1}{4}, 1; \frac{9}{4}; -1\right) = \frac{5\sqrt{2}}{16}\left(2\log\left(1+\sqrt{2}\right) - 2\sqrt{2} + \pi\right)$$

07.23.03.0606.01

$${}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; -1\right) = \frac{1}{9}\left(3\log(2) + \sqrt{3}\,\pi\right)$$

07.23.03.0607.01

$${}_2F_1\left(\frac{1}{3}, 1; \frac{7}{3}; -1\right) = \frac{4}{27}\left(6\log(2) + 2\sqrt{3}\,\pi - 9\right)$$

07.23.03.0608.01

$${}_2F_1\left(\frac{1}{3}, 1; \frac{10}{3}; -1\right) = \frac{7}{162}\left(48\log(2) + 16\sqrt{3}\,\pi - 99\right)$$

07.23.03.0609.01

$${}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; -1\right) = \frac{\pi}{4}$$

07.23.03.0610.01

$${}_2F_1\left(\frac{1}{2}, 1; \frac{5}{2}; -1\right) = \frac{3}{4}(\pi - 2)$$

07.23.03.0611.01

$${}_2F_1\left(\frac{1}{2}, 1; \frac{7}{2}; -1\right) = \frac{15\pi}{8} - 5$$

07.23.03.0612.01

$${}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; -1\right) = \frac{2}{9}\left(\sqrt{3}\,\pi - 3\log(2)\right)$$

07.23.03.0613.01

$${}_2F_1\left(\frac{2}{3}, 1; \frac{8}{3}; -1\right) = \frac{5}{27}\left(4\sqrt{3}\,\pi - 3(4\log(2) + 3)\right)$$

07.23.03.0614.01

$${}_2F_1\left(\frac{2}{3}, 1; \frac{11}{3}; -1\right) = \frac{4}{81}\left(40\sqrt{3}\,\pi - 3(40\log(2) + 39)\right)$$

07.23.03.0615.01

$${}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -1\right) = \frac{3\sqrt{2}}{8}\left(\pi - 2\log\left(1 + \sqrt{2}\right)\right)$$

07.23.03.0616.01

$${}_2F_1\left(\frac{3}{4}, 1; \frac{11}{4}; -1\right) = \frac{7\sqrt{2}}{16}\left(3\pi - 6\log\left(1 + \sqrt{2}\right) - 2\sqrt{2}\right)$$

07.23.03.0617.01

$${}_2F_1\left(\frac{5}{6}, 1; \frac{17}{6}; -1\right) = \frac{11}{18}\left(5\pi - 5\sqrt{3}\log\left(2 + \sqrt{3}\right) - 3\right)$$

07.23.03.0618.01

$${}_2F_1(1, 1; 2; -1) = \log(2)$$

07.23.03.0619.01

$${}_2F_1(1, 1; 3; -1) = \log(16) - 2$$

07.23.03.0620.01

$${}_2F_1(1, 1; 4; -1) = 6\log(4) - \frac{15}{2}$$

07.23.03.0621.01

$${}_2F_1(1,\ 1;\ 5;\ -1) = 32\log(2) - \frac{64}{3}$$

07.23.03.0622.01

$${}_2F_1(1,\ 2;\ 3;\ -1) = 2 - \log(4)$$

Values at $z = \frac{1}{2}$

07.23.03.0623.01

$${}_2F_1\left(\frac{1}{3},\ \frac{2}{3};\ 1;\ \frac{1}{2}\right) = \frac{3\,2^{2/3}}{8\,\pi^2}\,\Gamma\left(\frac{1}{3}\right)^3$$

07.23.03.0624.01

$${}_2F_1\left(1,\ \frac{5}{4};\ 2;\ \frac{1}{2}\right) = 8\left(\sqrt[4]{2} - 1\right)$$

07.23.03.0625.01

$${}_2F_1\left(1,\ 3;\ 2;\ \frac{1}{2}\right) = 3$$

07.23.03.0626.01

$${}_2F_1\left(1,\ 4;\ 2;\ \frac{1}{2}\right) = \frac{14}{3}$$

07.23.03.0627.01

$${}_2F_1\left(1,\ 4;\ 3;\ \frac{1}{2}\right) = \frac{8}{3}$$

07.23.03.0628.01

$${}_2F_1\left(1,\ 5;\ 2;\ \frac{1}{2}\right) = \frac{15}{2}$$

For $z = \frac{1}{9}$ and integer parameters

07.23.03.0629.01

$${}_2F_1\left(-n,\ \frac{1}{4} - n;\ 2\,n + \frac{5}{4};\ \frac{1}{9}\right) = \frac{2^{6\,n}\,3^{-5\,n}}{\left(\frac{2}{3}\right)_n \left(\frac{13}{12}\right)_n} \left(\frac{5}{4}\right)_{2\,n} \;/;\ n \in \mathbb{N}$$

07.23.03.0630.01

$${}_2F_1\left(-n,\ \frac{1}{4} - n;\ 2\,n + \frac{9}{4};\ \frac{1}{9}\right) = \frac{2^{6\,n}\,3^{-5\,n}}{\left(\frac{4}{3}\right)_n \left(\frac{17}{12}\right)_n} \left(\frac{9}{4}\right)_{2\,n} \;/;\ n \in \mathbb{N}$$

For $z = \frac{1}{4}$ and integer parameters

07.23.03.0631.01

$${}_2F_1\left(-2\,n,\ -3\,n - 1;\ -2\,n - \frac{1}{2};\ \frac{1}{4}\right) = \frac{2^{-4\,n}\,(3\,n + 1)!}{n!\left(\frac{3}{2}\right)_{2\,n}} \;/;\ n \in \mathbb{N}$$

For $z = \frac{8}{9}$ and integer parameters

07.23.03.0632.01

$$_2F_1\left(-n, \frac{n}{2}+1; \frac{4}{3}; \frac{8}{9}\right) = \frac{(1+(-1)^n)(-3)^{-\frac{n}{2}}}{2\left(\frac{7}{6}\right)_{\frac{n}{2}}} \left(\frac{1}{2}\right)_{\frac{n}{2}} /; n \in \mathbb{N}$$

For $z = \frac{4}{3}$ and integer parameters

07.23.03.0633.01

$$_2F_1\left(a, 2a-\frac{1}{2}; 4a-1; 12\sqrt{2}-16\right) = \frac{\left(2+\sqrt{2}\right)^{2a}\sqrt{\pi}\,\Gamma(2a)}{2^{4a-1}\,\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{3a}{2}\right)} /; n \in \mathbb{N}$$

For $z = 2$ and integer parameters

07.23.03.0634.01

$$_2F_1(-n, 1; m; 2) = 2^{1-m}\left(\frac{(-1)^n\, n!}{(m)_n} - \sum_{k=0}^{m-2} \frac{(-1)^k (1-m)_{k+1}}{k!\,(k+n+1)}\right) /; n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+$$

07.23.03.0635.01

$$_2F_1(-n, 1; -2n-1; 2) = \frac{2^{2n+1}\, n!\,(n+1)!}{(2n+1)!} - 1 /; n \in \mathbb{N}$$

07.23.03.0636.01

$$_2F_1(-n, 1; -2n; 2) = \frac{2^{2n}\, n!^2}{(2n)!} /; n \in \mathbb{N}^+$$

07.23.03.0637.01

$$_2F_1(-n, 2; -2n-2; 2) = \frac{(2n+3)(n+3)}{n+1} - \frac{2^{2n+3}\, n!\,(n+2)!}{(2n+2)!} /; n \in \mathbb{N}$$

07.23.03.0638.01

$$_2F_1(-n, 2; -2n-1; 2) = 2n+3 - \frac{2^{2n+2}\,(n+1)!^2}{(2n+2)!} /; n \in \mathbb{N}$$

For $z = 4$ and integer parameters

07.23.03.0639.01

$$_2F_1\left(-2n-1, \frac{1}{2}; n; 4\right) = -\frac{5n+2}{n} /; n \in \mathbb{N}^+$$

07.23.03.0640.01

$$_2F_1\left(-2n-1, \frac{1}{2}; n+1; 4\right) = -1 /; n \in \mathbb{N}$$

07.23.03.0641.01

$$_2F_1\left(-2n-1, \frac{1}{2}; n+2; 4\right) = 0 /; n \in \mathbb{N}$$

07.23.03.0642.01

$$_2F_1\left(-2n-1, \frac{1}{2}; n+3; 4\right) = \frac{n+2}{2(2n+3)} /; n \in \mathbb{N}$$

07.23.03.0643.01

$${}_2F_1\left(-2n-1, \frac{1}{2}; n+4; 4\right) = \frac{5(n+2)(n+3)}{4(2n+3)(2n+5)} /; n \in \mathbb{N}$$

07.23.03.0644.01

$${}_2F_1\left(-2n-1, \frac{1}{2}; n+5; 4\right) = \frac{21(n+2)(n+3)(n+4)}{8(2n+3)(2n+5)(2n+7)} /; n \in \mathbb{N}$$

07.23.03.0645.01

$${}_2F_1\left(-2n, \frac{1}{2}; n; 4\right) = 3 /; n \in \mathbb{N}^+$$

07.23.03.0646.01

$${}_2F_1\left(-2n, \frac{1}{2}; n+1; 4\right) = 1 /; n \in \mathbb{N}$$

07.23.03.0647.01

$${}_2F_1\left(-2n, \frac{1}{2}; n+\frac{3}{2}; 4\right) = \frac{\left(\frac{1}{2}\right)_n \left(\frac{3}{2}\right)_n}{\left(\frac{5}{6}\right)_n \left(\frac{7}{6}\right)_n} /; n \in \mathbb{N}$$

07.23.03.0648.01

$${}_2F_1\left(-2n, \frac{1}{2}; n+2; 4\right) = \frac{n+1}{2n+1} /; n \in \mathbb{N}$$

07.23.03.0649.01

$${}_2F_1\left(-2n, \frac{1}{2}; n+3; 4\right) = \frac{3(n+1)(n+2)}{2(2n+1)(2n+3)} /; n \in \mathbb{N}$$

07.23.03.0650.01

$${}_2F_1\left(-2n, \frac{1}{2}; n+4; 4\right) = \frac{(n+2)(n+3)(11n+10)}{4(2n+1)(2n+3)(2n+5)} /; n \in \mathbb{N}$$

07.23.03.0651.01

$${}_2F_1\left(-2n, \frac{1}{2}; n+5; 4\right) = \frac{(n+2)(n+3)(n+4)(43n+35)}{8(2n+1)(2n+3)(2n+5)(2n+7)} /; n \in \mathbb{N}$$

07.23.03.0652.01

$${}_2F_1\left(-2n, \frac{3}{2}; n+2; 4\right) = n+1 /; n \in \mathbb{N}$$

For $z = -8$ and integer parameters

07.23.03.0653.01

$${}_2F_1\left(-n, -2n-\frac{2}{3}; \frac{4}{3}; -8\right) = \frac{(-27)^n}{\left(\frac{3}{2}\right)_n} \left(\frac{5}{6}\right)_n /; n \in \mathbb{N}$$

07.23.03.0654.01

$${}_2F_1\left(-n, -2n-\frac{1}{3}; \frac{2}{3}; -8\right) = (-27)^n /; n \in \mathbb{N}$$

07.23.03.0655.01

$${}_2F_1\left(-n, \frac{2}{3}-2n; \frac{2}{3}; -8\right) = \frac{2^{2n-1}(1-3n)_n}{\left(\frac{1}{3}\right)_n \left(\frac{2}{3}\right)_n} \left(\left(\frac{1}{2}\right)_n + 2\left(\frac{1}{6}\right)_n\right) /; n \in \mathbb{N}^+$$

07.23.03.0656.01

$${}_2F_1\left(-n,\frac{4}{3}-2n;\frac{1}{3};-8\right)=\frac{2^{2n-1}(2-3n)_n\left(\left(-\frac{1}{2}\right)_n+2\left(-\frac{1}{6}\right)_n\right)}{\left(-\frac{1}{3}\right)_n\left(\frac{1}{3}\right)_n}\;/;\;n\in\mathbb{N}^+$$

07.23.03.0657.01

$${}_2F_1\left(-n,\frac{8}{3}-2n;-\frac{1}{3};-8\right)=\frac{2^{2n}(4-3n)_n\left(\left(\frac{n}{3}-\frac{3}{4}\right)\left(-\frac{1}{2}\right)_{n-1}+\left(-\frac{5}{6}\right)_n\right)}{\left(-\frac{1}{3}\right)_n\left(-\frac{2}{3}\right)_n}\;/;\;n-1\in\mathbb{N}^+$$

07.23.03.0658.01

$${}_2F_1\left(-n,\frac{1}{6}-\frac{n}{2};\frac{2}{3};-8\right)=2\,3^{\frac{3n-1}{2}}\cos\left(\frac{\pi}{6}(3n+1)\right)\;/;\;n\in\mathbb{N}$$

07.23.03.0659.01

$${}_2F_1\left(-n,\frac{1}{3}-\frac{n}{2};\frac{1}{3};-8\right)=(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}3^{2n-\left\lfloor\frac{n}{2}\right\rfloor-1}\left(\frac{2}{\left(\frac{1}{6}\right)_{\frac{n}{2}}}\left(\frac{1}{2}\right)_{\frac{n}{2}}+1\right)^{2\left\lfloor\frac{n}{2}\right\rfloor-n+1}\;/;\;n\in\mathbb{N}$$

07.23.03.0660.01

$${}_2F_1\left(-2n-1,-n-\frac{7}{6};\frac{7}{3};-8\right)=\frac{(-1)^{n+1}\,5\,3^{3n+2}}{(2n+3)(2n+5)}\;/;\;n\in\mathbb{N}$$

07.23.03.0661.01

$${}_2F_1\left(-2n-1,-n-\frac{5}{6};\frac{5}{3};-8\right)=\frac{(-1)^{n+1}\,3^{3n+2}}{2n+3}\;/;\;n\in\mathbb{N}$$

07.23.03.0662.01

$${}_2F_1\left(-2n,-n-\frac{1}{6};\frac{4}{3};-8\right)=\frac{(-1)^n\,3^{3n}}{2n+1}\;/;\;n\in\mathbb{N}$$

General characteristics

Some abbreviations

07.23.04.0001.01

$$\mathcal{NT}(\{a_1,a_2\})=\neg(-a_1\in\mathbb{N}\lor -a_2\in\mathbb{N})$$

Domain and analyticity

${}_2F_1(a,b;c;z)$ is an analytical function of a, b, c and z which is defined in $\mathbb{C}^4$. For fixed b, c, z, it is an entire function of a. For fixed a, c, z, it is an entire function of b. For negative integer $a$or b, ${}_2F_1(a,b;c;z)$ degenerates to a polynomial in z of order $-a$or $-b$.

07.23.04.0002.01

$$(a*b*c*z)\longrightarrow{}_2F_1(a,b;c;z)::(\mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C})\longrightarrow\mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.23.04.0003.02

$${}_2F_1(\overline{a}, \overline{b}; \overline{c}; \overline{z}) \equiv \overline{{}_2F_1(a, b; c; z)} \;/;\; z \notin (1, \infty)$$

Permutation symmetry

07.23.04.0004.01

$${}_2F_1(a, b; c; z) \equiv {}_2F_1(b, a; c; z)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed a, b, c in nonpolynomial cases (when $\neg\,(-a \in \mathbb{N} \lor -b \in \mathbb{N})$), the function ${}_2F_1(a, b; c; z)$ does not have poles and essential singularities.

07.23.04.0005.01

$$\mathcal{Sing}_z({}_2F_1(a, b; c; z)) \equiv \{\} \;/;\; \mathcal{NT}(\{a, b\})$$

For negative integer a or $b$and fixed c, the function ${}_2F_1(a, b; c; z)$ is a polynomial and has pole of order $-a$or $-b$at $z = \tilde{\infty}$.

07.23.04.0006.01

$$\mathcal{Sing}_z({}_2F_1(a, b; c; z)) \equiv \{\{\tilde{\infty}, -\alpha\}\} \;/;\; (-a \in \mathbb{N}^+ \land \alpha \equiv a) \lor (-b \in \mathbb{N}^+ \land \alpha \equiv b) \lor (-a \in \mathbb{N}^+ \land -b \in \mathbb{N}^+ \land \alpha \equiv \min(-a, -b))$$

With respect to c

For fixed a, b, z, the function ${}_2F_1(a, b; c; z)$ has an infinite set of singular points:

a) $c \equiv -k \;/;\; k \in \mathbb{N}$, are the simple poles with residues $\frac{(-1)^k}{k!}\, {}_2\tilde{F}_1(a, b; -k; z)$;

b) $c \equiv \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

07.23.04.0007.01

$$\mathcal{Sing}_c({}_2F_1(a, b; c; z)) \equiv \{\{\{-k, 1\} \;/;\; k \in \mathbb{N}\}, \{\tilde{\infty}, \infty\}\}$$

07.23.04.0008.01

$$\operatorname{res}_c({}_2F_1(a, b; c; z))(-k) \equiv \frac{(-1)^k}{k!}\, {}_2\tilde{F}_1(a, b; -k; z) \;/;\; k \in \mathbb{N}$$

With respect to b

For fixed a, c, z, the function ${}_2F_1(a, b; c; z)$ has only one singular point at $b = \tilde{\infty}$. It is an essential singular point.

07.23.04.0009.01

$$\mathcal{Sing}_b({}_2F_1(a, b; c; z)) \equiv \{\{\tilde{\infty}, \infty\}\}$$

With respect to a

For fixed b, c, z, the function ${}_2F_1(a, b; c; z)$ has only one singular point at $a = \tilde{\infty}$. It is an essential singular point.

07.23.04.0010.01

$$\mathcal{Sing}_a({}_2F_1(a, b; c; z)) \equiv \{\{\tilde{\infty}, \infty\}\}$$

Branch points

With respect to z

For fixed a, b, c, in nonpolynomial cases (when $\neg(-a \in \mathbb{N} \bigvee -b \in \mathbb{N})$), the function ${}_2F_1(a, b; c; z)$ has two branch points: $z == 1$, $z == \tilde{\infty}$.

07.23.04.0011.01

$$\mathcal{BP}_z({}_2F_1(a, b; c; z)) == \{1, \tilde{\infty}\} /; \mathcal{NT}(\{a, b\})$$

07.23.04.0012.01

$$\mathcal{R}_z({}_2F_1(a, b; c; z), 1) == \log /; c - a - b \in \mathbb{Z} \bigvee c - a - b \notin \mathbb{Q} \bigwedge \mathcal{NT}(\{a, b\})$$

07.23.04.0013.01

$$\mathcal{R}_z({}_2F_1(a, b; c; z), 1) == s /; c - a - b == \frac{r}{s} \bigwedge r \in \mathbb{Z} \bigwedge s - 1 \in \mathbb{N}^+ \bigwedge \gcd(r, s) == 1 \bigwedge \mathcal{NT}(\{a, b\})$$

07.23.04.0014.01

$$\mathcal{R}_z({}_2F_1(a, b; c; z), \tilde{\infty}) == \log /; a - b \in \mathbb{Z} \bigvee \neg(a \in \mathbb{Q} \bigwedge b \in \mathbb{Q})$$

07.23.04.0015.01

$$\mathcal{R}_z({}_2F_1(a, b; c; z), \tilde{\infty}) == \operatorname{lcm}(s, u) /;$$
$$a == \frac{r}{s} \bigwedge b == \frac{t}{u} \bigwedge \{r, s, t, u\} \in \mathbb{Z} \bigwedge s > 1 \bigwedge u > 1 \bigwedge \gcd(r, s) == 1 \bigwedge \gcd(t, u) == 1 \bigwedge \mathcal{NT}(\{a, b\})$$

With respect to c

For fixed a, b, z, the function ${}_2F_1(a, b; c; z)$ does not have branch points.

07.23.04.0016.01

$$\mathcal{BP}_c({}_2F_1(a, b; c; z)) == \{\}$$

With respect to b

For fixed a, c, z, the function ${}_2F_1(a, b; c; z)$ does not have branch points.

07.23.04.0017.01

$$\mathcal{BP}_b({}_2F_1(a, b; c; z)) == \{\}$$

With respect to a

For fixed b, c, z, the function ${}_2F_1(a, b; c; z)$ does not have branch points.

07.23.04.0018.01

$$\mathcal{BP}_a({}_2F_1(a, b; c; z)) == \{\}$$

Branch cuts

With respect to z

For fixed a, b, c, in nonpolynomial cases (when $\neg(-a \in \mathbb{N} \bigvee -b \in \mathbb{N})$), the function ${}_2F_1(a, b; c; z)$ is a single-valued function on the z-plane cut along the interval $(1, \infty)$, where it is continuous from below.

07.23.04.0019.01

$$\mathcal{BC}_z({}_2F_1(a, b; c; z)) == \{\{(1, \infty), i\}\} /; \mathcal{NT}(\{a, b\})$$

07.23.04.0020.01

$$\lim_{\epsilon \to +0} {}_2F_1(a, b; c; x - i\,\epsilon) == {}_2F_1(a, b; c; x) \;/;\; x > 1$$

07.23.04.0021.01

$$\lim_{\epsilon \to +0} {}_2F_1(a, b; c; x + i\,\epsilon) == \frac{2\,\pi\,i\,e^{i\,(a+b-c)\,\pi}\,\Gamma(c)}{\Gamma(c-a)\,\Gamma(c-b)\,\Gamma(a+b-c+1)}\,{}_2F_1(a, b; a+b-c+1; 1-x) + e^{2\,\pi\,i\,(a+b-c)}\,{}_2F_1(a, b; c; x) \;/;\; x > 1$$

With respect to *c*

For fixed a, b, z, the function ${}_2F_1(a, b; c; z)$ does not have branch cuts.

07.23.04.0022.01

$$\mathcal{BC}_c({}_2F_1(a, b; c; z)) == \{\}$$

With respect to *b*

For fixed a, c, z, the function ${}_2F_1(a, b; c; z)$ does not have branch cuts.

07.23.04.0023.01

$$\mathcal{BC}_b({}_2F_1(a, b; c; z)) == \{\}$$

With respect to *a*

For fixed b, c, z, the function ${}_2F_1(a, b; c; z)$ does not have branch cuts.

07.23.04.0024.01

$$\mathcal{BC}_a({}_2F_1(a, b; c; z)) == \{\}$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

07.23.06.0036.01

$$
{}_2F_1(a,b;c;z)\propto\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\Bigg(G_{2,2}^{2,2}\left(1-z_0\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,-a-b+c\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}-
2\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}i\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a)\,\Gamma(b)\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-z_0)+
\Bigg(G_{2,2}^{2,2}\left(1-z_0\,\middle|\,\begin{matrix}-a,\,-b\\0,\,-a-b+c-1\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}+
2\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}i\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a+1)\,\Gamma(b+1)\,{}_2\tilde{F}_1(a+1,b+1;a+b-c+2;1-z_0)\Bigg)
(z-z_0)+\frac{1}{2}\Bigg(G_{2,2}^{2,2}\left(1-z_0\,\middle|\,\begin{matrix}-a-1,\,-b-1\\0,\,-a-b+c-2\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}-
2\,i\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a+2)\,\Gamma(b+2)
{}_2\tilde{F}_1(a+2,b+2;a+b-c+3;1-z_0)\Bigg)(z-z_0)^2+\ldots\Bigg)\,/;\,(z\to z_0)
$$

07.23.06.0037.01

$$
{}_2F_1(a,b;c;z)\propto\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\Bigg(G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,-a-b+c\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}-2\,e^{i\,(c-a-b)\,\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\,i\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a)\,\Gamma(b)\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-z_0)+\left(G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}-a,\,-b\\0,\,-a-b+c-1\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}+2\,e^{i\,(c-a-b)\,\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\,i\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a+1)\,\Gamma(b+1)\,{}_2\tilde{F}_1(a+1,b+1;a+b-c+2;1-z_0)\right)(z-z_0)+\frac{1}{2}\left(G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}-a-1,\,-b-1\\0,\,-a-b+c-2\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}-2\,i\,e^{i\,(c-a-b)\,\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a+2)\,\Gamma(b+2)\,{}_2\tilde{F}_1(a+2,b+2;a+b-c+3;1-z_0)\right)(z-z_0)^2+O\!\left((z-z_0)^3\right)\Bigg)
$$

07.23.06.0038.01

$$
{}_2F_1(a,b;c;z)=\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}-a-k+1,\,-b-k+1\\0,\,-a-b+c-k\end{matrix}\right)-2\,\pi\,i\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor e^{i\,(c-a-b)\,\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor(-1)^k\,\Gamma(a+k)\,\Gamma(b+k)\,{}_2\tilde{F}_1(a+k,b+k;a+b-c+k+1;1-z_0)\right)(z-z_0)^k
$$

07.23.06.0039.01

$$
{}_2F_1(a,b;c;z) == \frac{\pi\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \Bigg(\Gamma(a+k)\,\Gamma(b+k)\Bigg(\csc((c-a-b)\pi)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor} - 2\,i\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Bigg)\,{}_2\tilde{F}_1(a+k,b+k;a+b-c+k+1;1-z_0) - \Gamma(c-a)\,\Gamma(c-b)\,(1-z_0)^{c-a-b-k}\csc((c-a-b)\pi)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\,{}_2\tilde{F}_1(c-a,c-b;-a-b+c-k+1;1-z_0)\Bigg)(z-z_0)^k \;/;\; c-a-b\notin\mathbb{Z}
$$

07.23.06.0040.01

$$
{}_2F_1(a,b;c;z) \propto \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\Bigg(G_{2,2}^{2,2}\!\left(1-z_0\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,-a-b+c\end{matrix}\right)\left(\frac{1}{1-z_0}\right)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}(1-z_0)^{(c-a-b)\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor} - 2\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor}\,i\,\pi\left\lfloor\frac{\arg(1-z_0)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0-z)}{2\pi}\right\rfloor\Gamma(a)\,\Gamma(b)\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-z_0) + O(z-z_0)\Bigg)
$$

Expansions on branch cuts

For the function itself

07.23.06.0041.01

$$
{}_2F_1(a,b;c;z) \propto \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\Bigg(e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\left(e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\!\left(1-x\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,-a-b+c\end{matrix}\right) - 2\,i\,\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\Gamma(a)\,\Gamma(b)\,{}_2\tilde{F}_1(a,b;a+b-c+1;1-x)\right) + e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\left(2\,i\,\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\Gamma(a+1)\,\Gamma(b+1)\,{}_2\tilde{F}_1(a+1,b+1;a+b-c+2;1-x) + e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\!\left(1-x\,\middle|\,\begin{matrix}-a,\,-b\\0,\,-a-b+c-1\end{matrix}\right)\right)(z-x) + \frac{1}{2}\,e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\left(e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\!\left(1-x\,\middle|\,\begin{matrix}-a-1,\,-b-1\\0,\,-a-b+c-2\end{matrix}\right) - 2\,i\,\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\Gamma(a+2)\,\Gamma(b+2)\,{}_2\tilde{F}_1(a+2,b+2;a+b-c+3;1-x)\right)(z-x)^2 + \ldots\Bigg) \;/;\; (z\to x)\land x\in\mathbb{R}\land x>1
$$

07.23.06.0042.01

$$\,_2F_1(a,b;c;z)\propto\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\Bigg(e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\Bigg(e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(1-x\,\middle|\,\begin{array}{c}1-a,\,1-b\\0,\,-a-b+c\end{array}\right)-2\,i\,\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\Gamma(a)\,\Gamma(b)\,_2\tilde{F}_1(a,b;a+b-c+1;1-x)\Bigg)+$$
$$e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\Bigg(2\,i\,\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\Gamma(a+1)\,\Gamma(b+1)\,_2\tilde{F}_1(a+1,b+1;a+b-c+2;1-x)+$$
$$e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(1-x\,\middle|\,\begin{array}{c}-a,\,-b\\0,\,-a-b+c-1\end{array}\right)\Bigg)(z-x)+$$
$$\frac{1}{2}\,e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\Bigg(e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(1-x\,\middle|\,\begin{array}{c}-a-1,\,-b-1\\0,\,-a-b+c-2\end{array}\right)-2\,i\,\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\Gamma(a+2)$$
$$\Gamma(b+2)\,_2\tilde{F}_1(a+2,b+2;a+b-c+3;1-x)\Bigg)(z-x)^2+O\left((z-x)^3\right)\Bigg)\;/;\;x\in\mathbb{R}\land x>1$$

07.23.06.0043.01

$$\,_2F_1(a,b;c;z)=$$
$$\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,e^{(c-a-b)\pi i\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\sum_{k=0}^{\infty}\frac{1}{k!}\Bigg(e^{i(c-a-b)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}G_{2,2}^{2,2}\left(1-x\,\middle|\,\begin{array}{c}-a-k+1,\,-b-k+1\\0,\,-a-b+c-k\end{array}\right)-2\,\pi\,i$$
$$\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor(-1)^k\,\Gamma(a+k)\,\Gamma(b+k)\,_2\tilde{F}_1(a+k,b+k;a+b-c+k+1;1-x)\Bigg)(z-x)^k\;/;\;x\in\mathbb{R}\land x>1$$

07.23.06.0044.01

$$\,_2F_1(a,b;c;z)=\frac{\pi\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,e^{2\,i(c-a-b)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}$$
$$\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\Bigg((1-x)^{c-a-b-k}\csc((a+b-c)\pi)\,\Gamma(c-a)\,\Gamma(c-b)\,_2\tilde{F}_1(c-a,c-b;c-a-b-k+1;1-x)-$$
$$\left(\csc((a+b-c)\pi)+2\,e^{i(a+b-c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}i\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\right)\Gamma(a+k)\,\Gamma(b+k)$$
$$\,_2\tilde{F}_1(a+k,b+k;a+b-c+k+1;1-x)\Bigg)(z-x)^k\;/;\;c-a-b\notin\mathbb{Z}\bigwedge x\in\mathbb{R}\bigwedge x>1$$

07.23.06.0045.01

$$\,_2F_1(a,b;c;z)=\frac{\pi^2\;\Gamma(c)\csc(c\,\pi)\,(1-x)^{c-a-b+c}}{\Gamma(a)\,\Gamma(b)}\,e^{i(c-a-b)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}$$
$$\sum_{k=0}^{\infty}\frac{(1-x)^{-k}}{k!}\Bigg(\frac{2\,i\,x^{-c-k+1}(1-x)^{a+b-c+k}}{\Gamma(c-a)\,\Gamma(c-b)}\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\,_2\tilde{F}_1(a-c+1,b-c+1;-c-k+2;x)\;+$$
$$\Bigg(\frac{\csc((a+b-c)\pi)}{\Gamma(1-a-k)\,\Gamma(1-b-k)}\,e^{i(-a-b+c)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}-$$
$$\frac{\sin((c-a)\pi)\sin((c-b)\pi)\,\Gamma(a+k)\,\Gamma(b+k)}{\pi^2}\left(e^{i(c-a-b)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\csc((a+b-c)\pi)+2\,i\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\right)\Bigg)$$
$$\,_2\tilde{F}_1(c-a,c-b;c+k;x)\Bigg)(z-x)^k\;/;\;c-a-b\notin\mathbb{Z}\bigwedge c\notin\mathbb{Z}\bigwedge x\in\mathbb{R}\bigwedge x>1$$

07.23.06.0046.01

$$_2F_1(a, b; c; z) \propto \frac{1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, e^{i(c-a-b)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor}\,\Gamma(c)\left(e^{i(c-a-b)\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor} G_{2,2}^{2,2}\!\left(1-x\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,-a-b+c\end{matrix}\right) - 2\,i\,\pi\left\lfloor\frac{\arg(x-z)}{2\pi}\right\rfloor\Gamma(a)\,\Gamma(b)\,{}_2\tilde{F}_1(a, b; a+b-c+1; 1-x)\right) + O(z-x)\;/;\; x\in\mathbb{R}\land x>1$$

Expansions at $z == 0$

For the function itself

General case

07.23.06.0001.02

$$_2F_1(a, b; c; z) \propto 1 + \frac{a\,b}{c}\,z + \frac{a\,(1+a)\,b\,(1+b)}{2\,c\,(1+c)}\,z^2 + \ldots\;/;\;(z\to 0)$$

07.23.06.0047.01

$$_2F_1(a, b; c; z) \propto 1 + \frac{a\,b}{c}\,z + \frac{a\,(1+a)\,b\,(1+b)}{2\,c\,(1+c)}\,z^2 + O(z^3)$$

07.23.06.0002.01

$$_2F_1(a, b; c; z) == \sum_{k=0}^{\infty}\frac{(a)_k\,(b)_k\,z^k}{(c)_k\,k!}\;/;\;|z|<1$$

07.23.06.0003.02

$$_2F_1(a, b; c; z) \propto 1 + O(z)$$

07.23.06.0048.01

$$_2F_1(a, b; c; z) == F_\infty(z, a, b, c)\;/;\;\left(\left(F_n(z, a, b, c) == \sum_{k=0}^{n}\frac{(a)_k\,(b)_k\,z^k}{(c)_k\,k!} == {}_2F_1(a, b; c; z) - \frac{z^{n+1}\,(a)_{n+1}\,(b)_{n+1}}{(n+1)!\,(c)_{n+1}}\,{}_3F_2(1, a+n+1, b+n+1; n+2, c+n+1; z)\right)\bigwedge n\in\mathbb{N}\right)$$

Summed form of the truncated series expansion.

Generic formulas for main term

07.23.06.0049.01

$$_2F_1(a, b; c; z) \propto \begin{cases}\tilde{\infty} & -c\in\mathbb{N}\bigwedge((-a\in\mathbb{N}\land c-a>0)\bigvee(-b\in\mathbb{N}\bigwedge c-b>0))\\ 1 & \text{True}\end{cases}\;/;\;(z\to 0)$$

Expansions at $z == 1$

For the function itself

General case

07.23.06.0004.01

$${}_2F_1(a,b;c;z) = \Gamma(c)\,\mathcal{A}_{\tilde{F}}\left(\begin{matrix} a, b; \\ c; \end{matrix}\ \{z, 1, \infty\}\right)$$

07.23.06.0005.02

$${}_2F_1(a,b;c;z) \propto \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\left(1+\frac{(a-c)(-b+c)(z-1)}{1-a-b+c}+\frac{(-a+c)(1-a+c)(-b+c)(1-b+c)(z-1)^2}{2(1-a-b+c)(2-a-b+c)}+\ldots\right)+\frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\left(1-\frac{a\,b\,(z-1)}{1+a+b-c}+\frac{a(1+a)\,b\,(1+b)(z-1)^2}{2(1+a+b-c)(2+a+b-c)}+\ldots\right)\text{/; }(z\to 1)\bigwedge c-a-b\notin\mathbb{Z}$$

07.23.06.0050.01

$${}_2F_1(a,b;c;z) \propto \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\left(1+\frac{(a-c)(-b+c)(z-1)}{1-a-b+c}+\frac{(-a+c)(1-a+c)(-b+c)(1-b+c)(z-1)^2}{2(1-a-b+c)(2-a-b+c)}+O\left((z-1)^3\right)\right)+\frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\left(1-\frac{a\,b\,(z-1)}{1+a+b-c}+\frac{a(1+a)\,b\,(1+b)(z-1)^2}{2(1+a+b-c)(2+a+b-c)}+O\left((z-1)^3\right)\right)\text{/; }(z\to 1)\bigwedge c-a-b\notin\mathbb{Z}$$

07.23.06.0006.01

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\sum_{k=0}^{\infty}\frac{(c-a)_k\,(c-b)_k\,(1-z)^k}{(c-a-b+1)_k\,k!}+\frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(a)_k\,(b)_k\,(1-z)^k}{(a+b-c+1)_k\,k!}\text{/; }|z-1|<1\bigwedge c-a-b\notin\mathbb{Z}$$

07.23.06.0007.01

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\,{}_2F_1(c-a,c-b;c-a-b+1;1-z)+\frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\,{}_2F_1(a,b;a+b-c+1;1-z)\text{/; }c-a-b\notin\mathbb{Z}$$

07.23.06.0008.01

$${}_2F_1(a,b;c;z) \propto \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}(1+O(z-1))+\frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}(1+O(z-1))\text{/; }(z\to 1)\bigwedge c-a-b\notin\mathbb{Z}$$

07.23.06.0051.01

$${}_2F_1(a,b;c;z) = F_\infty(z,a,b,c)\text{/; }\left(\left(F_n(z,a,b,c) = \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\sum_{k=0}^{n}\frac{(c-a)_k\,(c-b)_k}{(c-a-b+1)_k\,k!}(1-z)^k+\frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\sum_{k=0}^{n}\frac{(a)_k\,(b)_k}{(a+b-c+1)_k\,k!}(1-z)^k = {}_2F_1(a,b;c;z)-\frac{\Gamma(c)\,\Gamma(c-a-b)\,(a)_{n+1}\,(b)_{n+1}}{(n+1)!\,\Gamma(c-a)\,\Gamma(c-b)\,(a+b-c+1)_{n+1}}(1-z)^{n+1}\,{}_3F_2(1,a+n+1,b+n+1;n+2,a+b-c+n+2;1-z)-\frac{\Gamma(a+b-c)\,\Gamma(c)\,(c-a)_{n+1}\,(c-b)_{n+1}}{(n+1)!\,\Gamma(a)\,\Gamma(b)\,(c-a-b+1)_{n+1}}(1-z)^{c-a-b+n+1}\,{}_3F_2(1,-a+c+n+1,-b+c+n+1;n+2,c-a-b+n+2;1-z)\right)\bigwedge n\in\mathbb{N}\right)\bigwedge c-a-b\notin\mathbb{Z}$$

Summed form of the truncated series expansion.

Logarithmic cases

07.23.06.0009.01

$$ {}_2F_1(a, b; a+b-n; z) = \frac{(n-1)!\,\Gamma(a+b-n)}{\Gamma(a)\,\Gamma(b)} (1-z)^{-n} \sum_{k=0}^{n-1} \frac{(a-n)_k\,(b-n)_k\,(1-z)^k}{k!\,(1-n)_k} + \frac{(-1)^n\,\Gamma(a+b-n)}{\Gamma(a-n)\,\Gamma(b-n)} \sum_{k=0}^{\infty} \frac{(a)_k\,(b)_k}{k!\,(k+n)!} (-\log(1-z) + \psi(k+1) + \psi(k+n+1) - \psi(a+k) - \psi(b+k))\,(1-z)^k \;/;\; n \in \mathbb{N}^+ \bigwedge |1-z| < 1 $$

07.23.06.0010.01

$$ {}_2F_1(a, b; a+b-n; z) = \frac{(n-1)!\,\Gamma(a+b-n)}{\Gamma(a)\,\Gamma(b)} (1-z)^{-n} \sum_{k=0}^{n-1} \frac{(a-n)_k\,(b-n)_k\,(1-z)^k}{k!\,(1-n)_k} + \frac{(-1)^{n-1}\,\Gamma(a+b-n)}{\Gamma(a-n)\,\Gamma(b-n)\,n!} \log(1-z)\,{}_2F_1(a, b; n+1; 1-z) + \frac{(-1)^n\,\Gamma(a+b-n)}{\Gamma(a-n)\,\Gamma(b-n)} \sum_{k=0}^{\infty} \frac{(a)_k\,(b)_k}{k!\,(k+n)!} (\psi(k+1) + \psi(k+n+1) - \psi(a+k) - \psi(b+k))\,(1-z)^k \;/;\; n \in \mathbb{N}^+ \bigwedge |1-z| < 1 $$

07.23.06.0011.01

$$ {}_2F_1(a, b; a+b-n; z) \propto \frac{(n-1)!\,\Gamma(a+b-n)\,(1-z)^{-n}}{\Gamma(a)\,\Gamma(b)} (1 + O(z-1)) + \frac{(-1)^{n-1}\,\Gamma(a+b-n)}{n!\,\Gamma(a-n)\,\Gamma(b-n)} (\log(1-z) + \psi(a) + \psi(b) - \psi(n+1) + \gamma)\,(1 + O(z-1)) \;/;\; (z \to 1) \bigwedge n \in \mathbb{N}^+ $$

07.23.06.0012.01

$$ {}_2F_1(a, b; a+b; z) = \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} \left(\sum_{k=0}^{\infty} \frac{(a)_k\,(b)_k}{k!^2} (-\log(1-z) + 2\,\psi(k+1) - \psi(a+k) - \psi(b+k))\,(1-z)^k \right) \;/;\; |1-z| < 1 $$

07.23.06.0013.01

$$ {}_2F_1(a, b; a+b; z) = \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} \left(\sum_{k=0}^{\infty} \frac{(a)_k\,(b)_k\,(2\,\psi(k+1) - \psi(a+k) - \psi(b+k))\,(1-z)^k}{k!^2} - \log(1-z)\,{}_2F_1(a, b; 1; 1-z) \right) \;/;\; |1-z| < 1 $$

07.23.06.0014.01

$$ {}_2F_1(a, b; a+b; z) \propto -\frac{\Gamma(a+b)\,(\log(1-z) + \psi(a) + \psi(b) + 2\,\gamma)}{\Gamma(a)\,\Gamma(b)} (1 + O(z-1)) \;/;\; (z \to 1) $$

07.23.06.0015.01

$$ {}_2F_1(a, b; a+b+n; z) = \frac{(n-1)!\,\Gamma(a+b+n)}{\Gamma(a+n)\,\Gamma(b+n)} \sum_{k=0}^{n-1} \frac{(a)_k\,(b)_k\,(1-z)^k}{k!\,(1-n)_k} + \frac{\Gamma(a+b+n)}{\Gamma(a)\,\Gamma(b)} (z-1)^n \sum_{k=0}^{\infty} \frac{(a+n)_k\,(b+n)_k}{k!\,(k+n)!} (-\log(1-z) + \psi(k+1) + \psi(k+n+1) - \psi(a+k+n) - \psi(b+k+n))\,(1-z)^k \;/;\; n \in \mathbb{N}^+ \bigwedge |1-z| < 1 $$

07.23.06.0016.01

$$
{}_2F_1(a,\,b;\,a+b+n;\,z) = \frac{(n-1)!\,\Gamma(a+b+n)}{\Gamma(a+n)\,\Gamma(b+n)} \sum_{k=0}^{n-1} \frac{(a)_k\,(b)_k\,(1-z)^k}{k!\,(1-n)_k} - \frac{\Gamma(a+b+n)}{\Gamma(a)\,\Gamma(b)\,n!} \log(1-z)\,(z-1)^n\,{}_2F_1(a+n,\,b+n;\,n+1;\,1-z) + \frac{\Gamma(a+b+n)}{\Gamma(a)\,\Gamma(b)} (z-1)^n \sum_{k=0}^{\infty} \frac{(a+n)_k\,(b+n)_k}{k!\,(k+n)!} (\psi(k+1)+\psi(k+n+1)-\psi(a+k+n)-\psi(b+k+n))\,(1-z)^k \;/;\; n\in\mathbb{N}^+ \bigwedge |1-z|<1
$$

07.23.06.0017.01

$$
{}_2F_1(a,\,b;\,a+b+n;\,z) \propto \frac{(n-1)!\,\Gamma(a+b+n)}{\Gamma(a+n)\,\Gamma(b+n)} (1+O(z-1)) - \frac{\Gamma(a+b+n)}{n!\,\Gamma(a)\,\Gamma(b)} (z-1)^n\,(\log(1-z)-\psi(n+1)+\psi(a+n)+\psi(b+n)+\gamma)\,(1+O(z-1)) \;/;\; (z\to 1) \bigwedge n\in\mathbb{N}^+
$$

07.23.06.0052.01

$$
{}_2F_1(a,\,b;\,a+b-n;\,z) = F_\infty(z,\,a,\,b,\,n) \;/;\; \left(\left(F_m(z,\,a,\,b,\,n) = \frac{(n-1)!\,\Gamma(a+b-n)\,(1-z)^{-n}}{\Gamma(a)\,\Gamma(b)} \sum_{k=0}^{n-1} \frac{(a-n)_k\,(b-n)_k\,(1-z)^k}{k!\,(1-n)_k} + \frac{(-1)^n\,\Gamma(a+b-n)}{\Gamma(a-n)\,\Gamma(b-n)} \sum_{k=0}^{m} \frac{(a)_k\,(b)_k}{k!\,(k+n)!} (-\log(1-z)+\psi(k+1)+\psi(k+n+1)-\psi(a+k)-\psi(b+k))\,(1-z)^k = {}_2F_1(a,\,b;\,a+b-n;\,z) - \frac{(-1)^n\,\Gamma(a+b-n)}{\Gamma(a)\,\Gamma(b)\,\Gamma(a-n)\,\Gamma(b-n)} G_{4,4}^{2,4}\!\left(1-z \,\middle|\, \begin{matrix} m+1,\,m+1,\,1-a,\,1-b \\ m+1,\,m+1,\,0,\,-n \end{matrix}\right)\right) \bigwedge m\in\mathbb{N}\right) \bigwedge n\in\mathbb{N}
$$

Summed form of the truncated series expansion.

07.23.06.0053.01

$$
{}_2F_1(a,\,b;\,a+b+n;\,z) = F_\infty(z,\,a,\,b,\,n) \;/;\; \left(\left(F_m(z,\,a,\,b,\,n) = \frac{\Gamma(a+b+n)}{\Gamma(a+n)\,\Gamma(b+n)} \sum_{k=0}^{n-1} \frac{\Gamma(n-k)\,(a)_k\,(b)_k}{k!} (z-1)^k + \frac{(z-1)^n\,\Gamma(a+b+n)}{\Gamma(a)\,\Gamma(b)} \sum_{k=0}^{m} \frac{(a+n)_k\,(b+n)_k}{k!\,(k+n)!} (-\log(1-z)+\psi(k+1)+\psi(k+n+1)-\psi(a+k+n)-\psi(b+k+n))\,(1-z)^k = {}_2F_1(a,\,b;\,a+b+n;\,z) - \frac{(-1)^n\,\Gamma(a+b+n)}{\Gamma(a)\,\Gamma(b)\,\Gamma(a+n)\,\Gamma(b+n)} G_{4,4}^{2,4}\!\left(1-z \,\middle|\, \begin{matrix} m+n+1,\,m+n+1,\,1-a,\,1-b \\ m+n+1,\,m+n+1,\,0,\,n \end{matrix}\right)\right) \bigwedge m\in\mathbb{N}\right) \bigwedge n\in\mathbb{N}
$$

Summed form of the truncated series expansion.

Generic formulas for main term

07.23.06.0054.01

$$ {}_2F_1(a,\,b;\,c;\,z) \propto \begin{cases} \tilde{\infty} & -c \in \mathbb{N} \bigwedge ((-a \in \mathbb{N} \land c-a>0) \bigvee (-b \in \mathbb{N} \bigwedge c-b>0)) \\ \frac{(a+b-c-1)!\,\Gamma(c)\,(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)} + \frac{(-1)^{a+b-c-1}\,\Gamma(c)\,(\log(1-z)+\psi(a)+\psi(b)-\psi(a+b-c+1)+\gamma)}{(a+b-c)!\,\Gamma(c-a)\,\Gamma(c-b)} & a+b-c \in \mathbb{N}^+ \\ -\frac{\Gamma(a+b)\,(\log(1-z)+\psi(a)+\psi(b)+2\,\gamma)}{\Gamma(a)\,\Gamma(b)} & c = a+b \\ \frac{(c-a-b-1)!\,\Gamma(c)}{\Gamma(c-a)\,\Gamma(c-b)} - \frac{\Gamma(c)\;(\log(1-z)+\psi(c-a)-\psi(c-a-b+1)+\psi(c-b)+\gamma)\,(z-1)^{c-a-b}}{(c-a-b)!\,\Gamma(a)\,\Gamma(b)} & c-a-b \in \mathbb{N}^+ \\ \frac{\Gamma(c)\,\Gamma(a+b-c)\,(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)} + \frac{\Gamma(c-a-b)\,\Gamma(c)}{\Gamma(c-a)\,\Gamma(c-b)} & \text{True} \end{cases} $$

$$ /;\ (z \to 1) $$

07.23.06.0055.01

$$ {}_2F_1(a,\,b;\,c;\,z) \propto \begin{cases} \tilde{\infty} & -c \in \mathbb{N} \bigwedge ((-a \in \mathbb{N} \land c-a>0) \bigvee (-b \in \mathbb{N} \bigwedge c-b>0)) \\ \frac{\Gamma(c-a-b)\,\Gamma(c)}{\Gamma(c-a)\,\Gamma(c-b)} & \operatorname{Re}(c-a-b) > 0 \\ \frac{\Gamma(a+b-c)\,\Gamma(c)\,(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)} & \operatorname{Re}(c-a-b) < 0 \\ -\frac{\Gamma(a+b)\log(1-z)}{\Gamma(a)\,\Gamma(b)} & c = a+b \\ \frac{\Gamma(a+b-c)\,\Gamma(c)\,(1-z)^{c-a-b}}{\Gamma(a)\,\Gamma(b)} + \frac{\Gamma(c-a-b)\,\Gamma(c)}{\Gamma(c-a)\,\Gamma(c-b)} & \text{True} \end{cases} \quad /;\ (z \to 1) $$

Expansions at $z == \infty$

For the function itself

The general formulas

07.23.06.0018.01

$$ {}_2F_1(a,\,b;\,c;\,z) == \Gamma(c)\,\mathcal{A}_{\tilde{F}}\left(\begin{matrix} a,\, b; \\ c; \end{matrix}\ \{z,\,\tilde{\infty},\,\infty\}\right) /;\ z \notin (0,\,1) $$

07.23.06.0019.01

$$ {}_2F_1(a,\,b;\,c;\,z) == \Gamma(c)\,\mathcal{A}_{\tilde{F}}^{(\text{power})}\left(\begin{matrix} a,\, b; \\ c; \end{matrix}\ \{z,\,\tilde{\infty},\,\infty\}\right) /;\ z \notin (0,\,1) $$

Case of simple poles

07.23.06.0020.02

$$ {}_2F_1(a,\,b;\,c;\,z) \propto \frac{\Gamma(b-a)\,\Gamma(c)}{\Gamma(b)\,\Gamma(c-a)}\,(-z)^{-a}\left(1 + \frac{a\,(1+a-c)}{(1+a-b)\,z} + \frac{a\,(1+a)\,(1+a-c)\,(2+a-c)}{2\,(1+a-b)\,(2+a-b)\,z^2} + \ldots\right) + \frac{\Gamma(a-b)\,\Gamma(c)}{\Gamma(a)\,\Gamma(c-b)}\,(-z)^{-b}\left(1 + \frac{b\,(1+b-c)}{(1-a+b)\,z} + \frac{b\,(1+b)\,(1+b-c)\,(2+b-c)}{2\,(1-a+b)\,(2-a+b)\,z^2} + \ldots\right) /;\ (|z| \to \infty) \bigwedge a-b \notin \mathbb{Z} $$

07.23.06.0021.01

$$ {}_2F_1(a,\,b;\,c;\,z) == \frac{\Gamma(b-a)\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)} \sum_{k=0}^{\infty} \frac{(a)_k\,(a-c+1)_k\,z^{-k}}{k!\,(a-b+1)_k} + \frac{\Gamma(a-b)\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(b)_k\,(b-c+1)_k\,z^{-k}}{k!\,(-a+b+1)_k} /; $$

$$ |z| > 1 \bigwedge a-b \notin \mathbb{Z} $$

07.23.06.0022.01

$$ {}_2F_1(a,b;c;z) = \frac{\Gamma(b-a)\,\Gamma(c)}{\Gamma(b)\,\Gamma(c-a)}\,(-z)^{-a}\,{}_2F_1\!\left(a,a-c+1;a-b+1;\frac{1}{z}\right) + \frac{\Gamma(a-b)\,\Gamma(c)}{\Gamma(a)\,\Gamma(c-b)}\,(-z)^{-b}\,{}_2F_1\!\left(b,b-c+1;-a+b+1;\frac{1}{z}\right) /;\ a-b\notin\mathbb{Z}\bigwedge z\notin(0,1) $$

07.23.06.0023.01

$$ {}_2F_1(a,b;c;z) \propto \frac{\Gamma(b-a)\,\Gamma(c)}{\Gamma(b)\,\Gamma(c-a)}\,(-z)^{-a}\left(1+O\!\left(\frac{1}{z}\right)\right) + \frac{\Gamma(a-b)\,\Gamma(c)}{\Gamma(a)\,\Gamma(c-b)}\,(-z)^{-b}\left(1+O\!\left(\frac{1}{z}\right)\right) /;\ (|z|\to\infty)\bigwedge a\neq b $$

07.23.06.0056.01

$$ {}_2F_1(a,b;c;z) = F_\infty(z,a,b,c) /; \left(\left(F_n(z,a,b,c) = \frac{\Gamma(b-a)\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)}\sum_{k=0}^{n}\frac{(a)_k\,(a-c+1)_k\,z^{-k}}{k!\,(a-b+1)_k} + \frac{\Gamma(a-b)\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)}\sum_{k=0}^{n}\frac{(b)_k\,(b-c+1)_k\,z^{-k}}{k!\,(-a+b+1)_k} = {}_2F_1(a,b;c;z) - \frac{\Gamma(b-a)\,\Gamma(c)\,(a)_{n+1}\,(a-c+1)_{n+1}\,z^{-n-1}}{(n+1)!\,\Gamma(b)\,\Gamma(c-a)\,(a-b+1)_{n+1}}\,(-z)^{-a}\,{}_3F_2\!\left(1,a+n+1,a-c+n+2;n+2,a-b+n+2;\frac{1}{z}\right) - \frac{\Gamma(a-b)\,\Gamma(c)\,(b)_{n+1}\,(b-c+1)_{n+1}\,z^{-n-1}}{(n+1)!\,\Gamma(a)\,\Gamma(c-b)\,(-a+b+1)_{n+1}}\,(-z)^{-b}\,{}_3F_2\!\left(1,b+n+1,b-c+n+2;n+2,-a+b+n+2;\frac{1}{z}\right)\right)\bigwedge n\in\mathbb{N}\right)\bigwedge \neg\, a-b\in\mathbb{Z} $$

Summed form of the truncated series expansion.

Case of double poles

07.23.06.0057.01

$$ {}_2F_1(a,a+n;c;z) = \frac{\Gamma(c)\ (-z)^{-a}}{\Gamma(a+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,\Gamma(n-k)\,z^{-k}}{k!\,\Gamma(c-a-k)} + \frac{\sin((c-a)\,\pi)\,\Gamma(c)\,\Gamma(a-c+n+1)}{\pi\,n!\,\Gamma(a)}\log(-z)\,(-z)^{-a-n}\sum_{k=0}^{\infty}\frac{(a+n)_k\,(a-c+n+1)_k}{k!\,(n+1)_k}\,z^{-k} + \frac{\Gamma(c)\,(-z)^{-a-n}}{\Gamma(a+n)\,\Gamma(c-a)}\sum_{k=0}^{\infty}\frac{(a)_{k+n}\,(a-c+1)_{k+n}}{k!\,(k+n)!}\,(\psi(k+1)+\psi(k+n+1)-\psi(c-a-k-n)-\psi(a+k+n))\,z^{-k} /;\ |z|>1\bigwedge n\in\mathbb{N}\bigwedge c-a\notin\mathbb{Z} $$

07.23.06.0024.01

$$ {}_2F_1(a,a+n;c;z) = \frac{\Gamma(c)\ (-z)^{-a}}{\Gamma(a+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,\Gamma(n-k)\,z^{-k}}{k!\,\Gamma(c-a-k)} + \frac{\sin((c-a)\,\pi)\,\Gamma(c)\,\Gamma(a-c+n+1)}{\pi\,n!\,\Gamma(a)}\log(-z)\,(-z)^{-a-n}\,{}_2F_1\!\left(a+n,a-c+n+1;n+1;\frac{1}{z}\right) + \frac{\Gamma(c)\,(-z)^{-a-n}}{\Gamma(a+n)\,\Gamma(c-a)}\sum_{k=0}^{\infty}\frac{(a)_{k+n}\,(a-c+1)_{k+n}}{k!\,(k+n)!}\,(\psi(k+1)+\psi(k+n+1)-\psi(c-a-k-n)-\psi(a+k+n))\,z^{-k} /;\ |z|>1\bigwedge n\in\mathbb{N}\bigwedge c-a\notin\mathbb{Z} $$

07.23.06.0025.01

$$
{}_2F_1(a,\,a+n;\,c;\,z)\propto \frac{\Gamma(c)\,(n-1)!\,(-z)^{-a}}{\Gamma(a+n)\,\Gamma(c-a)}\left(1+O\left(\frac{1}{z}\right)\right)+\frac{\sin((c-a)\,\pi)\,\Gamma(c)\,\Gamma(a-c+n+1)}{\pi\,n!\,\Gamma(a)}
(\log(-z)-\psi(c-a-n)+\psi(n+1)-\psi(a+n)-\gamma)\,(-z)^{-a-n}\left(1+O\left(\frac{1}{z}\right)\right)\;/;\;(|z|\to\infty)\bigwedge n\in\mathbb{N}^+\bigwedge c-a\notin\mathbb{Z}
$$

07.23.06.0058.01

$$
{}_2F_1(a,\,a;\,c;\,z)=\frac{\Gamma(c)\,(-z)^{-a}}{\Gamma(a)\,\Gamma(c-a)}\,(\log(-z)-\psi(c-a)-\psi(a)-2\,\gamma)\left(1+O\left(\frac{1}{z}\right)\right)\;/;\;(|z|\to\infty)\land\neg\, c-a\in\mathbb{Z}
$$

07.23.06.0059.01

$$
{}_2F_1(a,\,a+n;\,a+m;\,z)=\frac{\Gamma(n)\,\Gamma(a+m)\,(-z)^{-a}}{\Gamma(m)\,\Gamma(a+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,(1-m)_k\,z^{-k}}{k!\,(1-n)_k}+
\frac{(-1)^n\,\Gamma(a+m)^2\,(-z)^{-a-m}}{\Gamma(a)\,\Gamma(a+n)\,m!\,(m-n)!}\sum_{k=0}^{\infty}\frac{k!\,(a+m)_k\,z^{-k}}{(m+1)_k\,(m-n+1)_k}+\frac{(-1)^n\,\Gamma(a+m)}{\Gamma(a)\,(m-n-1)!}\,(-z)^{-a-n}\sum_{k=0}^{m-n-1}\frac{(a+n)_k\,(1-m+n)_k}{k!\,(k+n)!}
(\log(-z)-\psi(m-n-k)+\psi(k+1)+\psi(k+n+1)-\psi(a+k+n))\,z^{-k}\;/;\;n\in\mathbb{N}\bigwedge m\in\mathbb{N}\bigwedge m>n\bigwedge z\notin(0,\,1)
$$

07.23.06.0026.01

$$
{}_2F_1(a,\,a+n;\,a+m;\,z)=\frac{\Gamma(n)\,\Gamma(a+m)\,(-z)^{-a}}{\Gamma(m)\,\Gamma(a+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,(1-m)_k\,z^{-k}}{k!\,(1-n)_k}+
\frac{(-1)^n\,\Gamma(a+m)^2\,(-z)^{-a-m}}{\Gamma(a)\,\Gamma(a+n)\,m!\,(m-n)!}\,{}_3F_2\left(1,\,1,\,a+m;\,m+1,\,m-n+1;\,\frac{1}{z}\right)+\frac{(-1)^n\,\Gamma(a+m)}{\Gamma(a)\,(m-n-1)!}\,(-z)^{-a-n}\sum_{k=0}^{m-n-1}\frac{(a+n)_k\,(1-m+n)_k}{k!\,(k+n)!}
(\log(-z)-\psi(m-n-k)+\psi(k+1)+\psi(k+n+1)-\psi(a+k+n))\,z^{-k}\;/;\;n\in\mathbb{N}\bigwedge m\in\mathbb{N}\bigwedge m>n\bigwedge z\notin(0,\,1)
$$

07.23.06.0027.01

$$
{}_2F_1(a,\,a+n;\,a+m;\,z)\propto\frac{\Gamma(n)\,\Gamma(a+m)\,(-z)^{-a}}{\Gamma(m)\,\Gamma(a+n)}\left(1+O\left(\frac{1}{z}\right)\right)+\frac{(-1)^n\,\Gamma(a+m)^2\,(-z)^{-a-m}}{\Gamma(a)\,\Gamma(a+n)\,m!\,(m-n)!}\left(1+O\left(\frac{1}{z}\right)\right)+\frac{(-1)^n\,\Gamma\,(a+m)}{\Gamma(a)\,n!\,(m-n-1)!}
(\log(-z)-\psi(m-n)+\psi(n+1)-\psi(a+n)-\gamma)\,(-z)^{-a-n}\left(1+O\left(\frac{1}{z}\right)\right)\;/;\;(|z|\to\infty)\bigwedge n\in\mathbb{N}^+\bigwedge m\in\mathbb{N}^+\bigwedge m>n
$$

07.23.06.0028.01

$$
{}_2F_1(a,\,a;\,a+m;\,z)\propto\frac{\Gamma(a+m)\,(\log(-z)-\psi(a)-\psi(m)-2\,\gamma)}{\Gamma(a)\,(m-1)!}\,(-z)^{-a}\left(1+O\left(\frac{1}{z}\right)\right)+\frac{\Gamma(a+m)^2}{\Gamma(a)^2\,m!^2}\,(-z)^{-a-m}\left(1+O\left(\frac{1}{z}\right)\right)\;/;
(|z|\to\infty)\bigwedge m\in\mathbb{N}^+
$$

07.23.06.0060.01

$$
{}_2F_1(a,\,a+n;\,c;\,z)=F_\infty(z,\,a,\,a+n,\,c)\;/;
\left(\left(F_n(z,\,a,\,a+n,\,c)=\frac{\Gamma(c)\,(-z)^{-a}}{\Gamma(a+n)}\sum_{k=0}^{n-1}\frac{(a)_k\,\Gamma(n-k)\,z^{-k}}{k!\,\Gamma(-a+c-k)}+\frac{\Gamma(c)\,(-z)^{-a-n}}{\Gamma(a+n)\,\Gamma(c-a)}\sum_{k=0}^{m}\frac{(a)_{k+n}\,(a-c+1)_{k+n}}{k!\,(k+n)!}
(\log(-z)+\psi(k+1)+\psi(k+n+1)-\psi(-a+c-k-n)-\psi(a+k+n))\,z^{-k}={}_2F_1(a,\,a+n;\,c;\,z)-
\frac{(-1)^n\,\Gamma(c)}{\Gamma(a)\,\Gamma(a+n)}\,G_{4,4}^{3,2}\left(-z\,\middle|\,\begin{array}{c}-a-m-n,\,-a-m-n,\,1-a,\,-a-n+1\\0,\,-a-m-n,\,-a-m-n,\,1-c\end{array}\right)\right)\bigwedge m\in\mathbb{N}\right)\bigwedge n\in\mathbb{N}\bigwedge\neg\,c-a\in\mathbb{Z}
$$

Summed form of the truncated series expansion.

Case of canceled double poles

07.23.06.0061.01

$${}_2F_1(a,\, a+n;\, a-m;\, z) = \frac{(-1)^m\, \Gamma(a-m)\, (m+n)!\, (-z)^{-a-n}}{\Gamma(a)\, n!} \sum_{k=0}^{\infty} \frac{((a+n)_k\, (m+n+1)_k}{k!\, (n+1)_k}\, z^{-k} \;/;\; n \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge z \notin (0,\, 1)$$

07.23.06.0029.01

$${}_2F_1(a,\, a+n;\, a-m;\, z) = \frac{(-1)^m\, \Gamma(a-m)\, (m+n)!\, (-z)^{-a-n}}{\Gamma(a)\, n!}\, {}_2F_1\!\left(a+n,\, m+n+1;\, n+1;\, \frac{1}{z}\right) /;\; n \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge z \notin (0,\, 1)$$

07.23.06.0030.01

$${}_2F_1(a,\, a+n;\, a-m;\, z) \propto \frac{(-1)^m\, \Gamma(a-m)\, (m+n)!}{\Gamma(a)\, n!}\, (-z)^{-a-n} \left(1 + O\!\left(\frac{1}{z}\right)\right) /;\; (|z| \to \infty) \land n \in \mathbb{N} \land m \in \mathbb{N}$$

07.23.06.0062.01

$${}_2F_1(a,\, a+n;\, a+m;\, z) = \frac{\Gamma(a+m)\, \Gamma(n)\, (-z)^{-a}}{\Gamma(m)\, \Gamma(a+n)} \sum_{k=0}^{m-1} \frac{(a)_k\, (1-m)_k\, z^{-k}}{k!\, (1-n)_k} + \frac{(-1)^m\, (a)_m\, (n-m)!\, (-z)^{-a-n}}{n!} \sum_{k=0}^{\infty} \frac{(a+n)_k\, (-m+n+1)_k}{k!\, (n+1)_k}\, z^{-k} \;/;\; n \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge m \le n \bigwedge z \notin (0,\, 1)$$

07.23.06.0031.01

$${}_2F_1(a,\, a+n;\, a+m;\, z) = \frac{\Gamma(a+m)\, \Gamma(n)\, (-z)^{-a}}{\Gamma(m)\, \Gamma(a+n)} \sum_{k=0}^{m-1} \frac{(a)_k\, (1-m)_k\, z^{-k}}{k!\, (1-n)_k} + \frac{(-1)^m\, (a)_m\, (n-m)!\, (-z)^{-a-n}}{n!}\, {}_2F_1\!\left(a+n,\, 1-m+n;\, n+1;\, \frac{1}{z}\right) /;\; n \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge m \le n \bigwedge z \notin (0,\, 1)$$

07.23.06.0032.01

$${}_2F_1(a,\, a+n;\, a+m;\, z) \propto \frac{\Gamma(a+m)\, \Gamma(n)}{\Gamma(m)\, \Gamma(a+n)}\, (-z)^{-a} \left(1 + O\!\left(\frac{1}{z}\right)\right) /;\; (|z| \to \infty) \bigwedge n \in \mathbb{N}^+ \bigwedge m \in \mathbb{N}^+ \bigwedge m \le n$$

Generic formulas for main term

07.23.06.0063.01

$$
{}_2F_1(a,\,b;\,c;\,z)\propto
\begin{cases}
\frac{(b-a-1)!\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)}+\frac{\sin((c-a)\,\pi)\,\Gamma(b-c+1)\,\Gamma(c)\,(\log(-z)+\psi(b-a+1)-\psi(c-b)-\psi(b)-\gamma)\,(-z)^{-b}}{\pi\,(b-a)!\,\Gamma(a)} & b-a\in\mathbb{N}^+\bigwedge c-a\notin\mathbb{Z}\\
\frac{(-z)^{-a}\,\Gamma(c)\,(\log(-z)-\psi(c-a)-\psi(a)-2\,\gamma)}{\Gamma(a)\,\Gamma(c-a)} & b=a\bigwedge c-a\notin\mathbb{Z}\\
\frac{(a-b-1)!\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)}+\frac{\sin((c-b)\,\pi)\,\Gamma(a-c+1)\,\Gamma(c)\,(\log(-z)-\psi(c-a)-\psi(a)+\psi(a-b+1)-\gamma)\,(-z)^{-a}}{\pi\,(a-b)!\,\Gamma(b)} & a-b\in\mathbb{N}^+\bigwedge c-b\notin\mathbb{Z}\\
\frac{(-1)^{a-c}\,(b-c)!\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,(b-a)!} & b-a\in\mathbb{N}\bigwedge a-c\in\mathbb{N}\\
\frac{(-1)^{b-c}\,(a-c)!\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,(a-b)!} & a-b\in\mathbb{N}\bigwedge b-c\in\mathbb{N}\\
\frac{\Gamma(b-a)\,\Gamma(c)\,(-z)^{-a}}{\Gamma(c-a)\,\Gamma(b)}+\frac{(-1)^{b-a}\,\Gamma(c)\,(\log(-z)+\psi(b-a+1)-\psi(c-b)-\psi(b)-\gamma)\,(-z)^{-b}}{\Gamma(a)\,(b-a)!\,(c-b-1)!} & b-a\in\mathbb{N}^+\bigwedge c-a\in\mathbb{N}^+\bigwedge c-b>0\\
\frac{\Gamma(a-b)\,\Gamma(c)\,(-z)^{-b}}{\Gamma(c-b)\,\Gamma(a)}+\frac{(-1)^{a-b}\,\Gamma(c)\,(\log(-z)-\psi(c-a)-\psi(a)+\psi(a-b+1)-\gamma)\,(-z)^{-a}}{\Gamma(b)\,(a-b)!\,(c-a-1)!} & a-b\in\mathbb{N}^+\bigwedge c-b\in\mathbb{N}^+\bigwedge c-a>0\\
\frac{\Gamma(c)\,(\log(-z)-\psi(c-a)-\psi(a)-2\,\gamma)\,(-z)^{-a}}{\Gamma(a)\,(c-a-1)!}+\frac{\Gamma(c)^2\,(-z)^{-c}}{\Gamma(a)^2\,((c-a)!)^2} & b=a\bigwedge c-a\in\mathbb{N}^+\\
\frac{(-1)^a\,(b)_{-a}\,z^{-a}}{(c)_{-a}} & -c\in\mathbb{N}\land -a\in\mathbb{N}\land c-a\le 0\\
\frac{(-1)^b\,(a)_{-b}\,z^{-b}}{(c)_{-b}} & -c\in\mathbb{N}\bigwedge -b\in\mathbb{N}\bigwedge c-b\le 0\\
\tilde{\infty} & -c\in\mathbb{N}\bigwedge((-a\in\mathbb{N}\land c-a>0)\bigvee(-b\in\mathbb{N}\bigwedge c-b>0))\\
\frac{\Gamma(b-a)\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)}+\frac{\Gamma(a-b)\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} & \text{True}
\end{cases}
\;/;\;(|z|\to\infty)
$$

07.23.06.0064.01

$$
{}_2F_1(a,\,b;\,c;\,z)\propto
\begin{cases}
\frac{(-1)^{a-c}\,(b-c)!\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,(b-a)!} & b-a\in\mathbb{N}\bigwedge a-c\in\mathbb{N}\\
\frac{\Gamma(c)\,\log(-z)\,(-z)^{-a}}{\Gamma(a)\,(c-a-1)!} & b=a\bigwedge a-c\notin\mathbb{N}\\
\frac{(-1)^{b-c}\,(a-c)!\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,(a-b)!} & a-b\in\mathbb{N}\bigwedge b-c\in\mathbb{N}\\
\frac{\Gamma(b-a)\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)} & \operatorname{Re}(b-a)>0\bigvee(-c\in\mathbb{N}\land -a\in\mathbb{N}\land a-c\ge 0)\\
\frac{\Gamma(a-b)\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} & \operatorname{Re}(b-a)>0\bigvee(-c\in\mathbb{N}\bigwedge -b\in\mathbb{N}\bigwedge b-c\ge 0)\\
\tilde{\infty} & -c\in\mathbb{N}\bigwedge((-a\in\mathbb{N}\land c-a>0)\bigvee(-b\in\mathbb{N}\bigwedge c-b>0))\\
\frac{\Gamma(b-a)\,\Gamma(c)\,(-z)^{-a}}{\Gamma(b)\,\Gamma(c-a)}+\frac{\Gamma(a-b)\,\Gamma(c)\,(-z)^{-b}}{\Gamma(a)\,\Gamma(c-b)} & \text{True}
\end{cases}
\;/;\;(|z|\to\infty)
$$

Expansions at $z == \infty$ for polynomial cases

For the function itself

07.23.06.0033.01

$$
{}_2F_1(-n,\,b;\,c;\,z)=\frac{(b)_n\,(-z)^n}{(c)_n}\,{}_2F_1\!\left(-n,\,-c-n+1;\,1-b-n;\,\frac{1}{z}\right)/;\,n\in\mathbb{N}\land\neg\,(-c\in\mathbb{N}\land c+n>0)
$$

Residue representations

General case

07.23.06.0034.01

$$ {}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(a-s)\,\Gamma(b-s)\,(-z)^{-s}}{\Gamma(c-s)}\,\Gamma(s)\right)(-j) \,/;\, |z|<1 $$

07.23.06.0035.01

$$ {}_2F_1(a,b;c;z) = -\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)} \left(\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s)\,\Gamma(b-s)\,(-z)^{-s}}{\Gamma(c-s)}\,\Gamma(a-s)\right)(a+j) + \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s)\,\Gamma(a-s)\,(-z)^{-s}}{\Gamma(c-s)}\,\Gamma(b-s)\right)(b+j)\right) /; $$
$$ |z|>1 \bigwedge a-b \notin \mathbb{Z} $$

07.23.06.0065.01

$$ {}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} \left(\sum_{j=0}^{\infty} \operatorname{res}_s((\Gamma(-a-b+c+s)\,\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s})\,\Gamma(s))(-j) + \right. $$
$$ \left. \sum_{j=0}^{\infty} \operatorname{res}_s((\Gamma(s)\,\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s})\,\Gamma(-a-b+c+s))\,(a+b-c-j)\right) /;\, |1-z|<1 \bigwedge c-a-b \notin \mathbb{Z} $$

07.23.06.0066.01

$$ {}_2F_1(a,b;c;z) = -\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)} \left(\sum_{j=0}^{\infty} \operatorname{res}_s((\Gamma(s)\,\Gamma(-a-b+c+s)\,\Gamma(b-s)\,(1-z)^{-s})\,\Gamma(a-s))\,(a+j) + \right. $$
$$ \left. \sum_{j=0}^{\infty} \operatorname{res}_s((\Gamma(s)\,\Gamma(-a-b+c+s)\,\Gamma(a-s)\,(1-z)^{-s})\,\Gamma(b-s))\,(b+j)\right) /;\, |1-z|>1 \bigwedge a-b \notin \mathbb{Z} $$

07.23.06.0067.01

$$ {}_2F_1(a,b;c;z) = -\frac{\csc((a-b)\,\pi)\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)} $$
$$ \left(\sin((a-c)\,\pi)\,(-z)^{-a}\left(\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s)\,\Gamma(a-c-s+1)\left(-\frac{1}{z}\right)^{-s}}{\Gamma(a-b-s+1)}\,\Gamma(a-s)\right)(a+j) + \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s)\,\Gamma(a-s)\left(-\frac{1}{z}\right)^{-s}}{\Gamma(a-b-s+1)}\,\Gamma(a-c-s+1)\right)\right.\right. $$
$$ \left.(a-c+j+1)\right) + \sin((b-c)\,\pi)\,(-z)^{-b}\left(\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s)\,\Gamma(b-c-s+1)\left(-\frac{1}{z}\right)^{-s}}{\Gamma(-a+b-s+1)}\,\Gamma(b-s)\right)(b+j) + \right. $$
$$ \left.\left.\sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(b-s)\,\Gamma(s)\left(-\frac{1}{z}\right)^{-s}}{\Gamma(-a+b-s+1)}\,\Gamma(b-c-s+1)\right)(b-c+j+1)\right)\right) /;\, |z|<1 \bigwedge a-b \notin \mathbb{Z} \bigwedge c \notin \mathbb{Z} $$

07.23.06.0068.01

$$ {}_2F_1(a,b;c;z) = \frac{\csc((a-b)\,\pi)\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)} \left(\sin((a-c)\,\pi)\,(-z)^{-a} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(a-s)\,\Gamma(a-c-s+1)\left(-\frac{1}{z}\right)^{-s}}{\Gamma(a-b-s+1)}\,\Gamma(s)\right)(-j) + \right. $$
$$ \left. \sin((b-c)\,\pi)\,(-z)^{-b} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(b-s)\,\Gamma(b-c-s+1)\left(-\frac{1}{z}\right)^{-s}}{\Gamma(-a+b-s+1)}\,\Gamma(s)\right)(-j)\right) /;\, |z|>1 \bigwedge a-b \notin \mathbb{Z} $$

Logarithmic cases

07.23.06.0069.01

$$ {}_2F_1(a,\, a+n;\, c;\, z) = -\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(a+n)} \left(\sum_{j=0}^{n-1} \operatorname{res}_s\!\left(\frac{\Gamma(s)\,\Gamma(a+n-s)\,(-z)^{-s}}{\Gamma(c-s)}\, \Gamma(a-s) \right)(a+j) + \sum_{j=0}^{\infty} \operatorname{res}_s\!\left(\frac{\Gamma(s)\,(-z)^{-s}}{\Gamma(c-s)}\, \Gamma(a-s)\,\Gamma(a+n-s) \right)(a+n+j) \right) /;\, |z| > 1 \land n \in \mathbb{N} $$

07.23.06.0070.01

$$ {}_2F_1(a,\, b;\, a+b+n;\, z) = \frac{\Gamma(a+b+n)}{\Gamma(a)\,\Gamma(b)\,\Gamma(a+n)\,\Gamma(b+n)} \left(\sum_{j=0}^{n-1} \operatorname{res}_s\!\left(\left(\Gamma(n+s)\,\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s}\right) \Gamma(s) \right)(-j) + \sum_{j=0}^{\infty} \operatorname{res}_s\!\left(\left(\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s}\right) \Gamma(s)\,\Gamma(n+s) \right)(-n-j) \right) /;\, |1-z| < 1 \land n \in \mathbb{N} $$

07.23.06.0071.01

$$ {}_2F_1(a,\, b;\, a+b-n;\, z) = \frac{\Gamma(a+b-n)}{\Gamma(a)\,\Gamma(b)\,\Gamma(a-n)\,\Gamma(b-n)} \left(\sum_{j=0}^{n-1} \operatorname{res}_s\!\left(\left(\Gamma(s)\,\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s}\right) \Gamma(s-n) \right)(n-j) + \sum_{j=0}^{\infty} \operatorname{res}_s\!\left(\left(\Gamma(a-s)\,\Gamma(b-s)\,(1-z)^{-s}\right) \Gamma(s)\,\Gamma(s-n) \right)(-j) \right) /;\, |1-z| < 1 \bigwedge n \in \mathbb{N}^+ $$

Limit representations

07.23.09.0001.01

$$ {}_2F_1(a,\, b;\, c;\, z) = \lim_{p\to\infty} {}_3F_2(a,\, b,\, p\,z;\, c,\, p;\, 1) /;\, \operatorname{Re}(c-a-b+p\,(1-z)) > 0 $$

07.23.09.0002.01

$$ {}_2F_1(a,\, b;\, c;\, z) = \lim_{p\to\infty} {}_2F_2(a,\, b;\, c,\, p;\, p\,z) $$

07.23.09.0003.01

$$ {}_2F_1(a,\, b;\, c;\, z) = \lim_{q\to\infty} \lim_{p\to\infty} {}_2F_3(a,\, b;\, c,\, p,\, q;\, p\,q\,z) $$

Continued fraction representations

07.23.10.0001.01

$$ {}_2F_1(a,\, b;\, c;\, z) = 1 + \frac{a\,b}{c}\, z \Bigg/ \left(1 + \frac{-\frac{(a+1)\,(b+1)}{2\,(c+1)}\, z}{\left(1 + \frac{(a+1)\,(b+1)}{2\,(c+1)}\, z + \frac{-\frac{(a+2)\,(b+2)}{3\,(c+2)}\, z}{1+\frac{(a+2)\,(b+2)}{3\,(c+2)}\, z+\ldots} \right)} \right) $$

07.23.10.0002.01

$$ {}_2F_1(a,\, b;\, c;\, z) = 1 + \frac{a\,b\,z}{c\left(1 + \mathrm{K}_k\!\left(-\frac{(a+k)\,(b+k)\,z}{(k+1)\,(c+k)},\, 1 + \frac{(a+k)\,(b+k)\,z}{(k+1)\,(c+k)}\right)_1^{\infty}\right)} $$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

Representation of fundamental system solutions near zero

07.23.13.0028.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;\;w(z)=c_1\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)\,+c_2\,G_{2,2}^{2,2}\!\left(z\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)$$

07.23.13.0029.01

$$W_z\!\left({}_2\tilde{F}_1(a,\,b;\,c;\,z),\,G_{2,2}^{2,2}\!\left(z\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)\right)=-(1-z)^{-a-b+c-1}\,z^{-c}\,\Gamma(a-c+1)\,\Gamma(b-c+1)$$

07.23.13.0030.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;$$
$$w(z)=c_1\,{}_2\tilde{F}_1(a,\,b;\,c;\,z)+\,c_2\,z^{1-c}\,{}_2\tilde{F}_1(a-c+1,\,b-c+1;\,2-c;\,z)\bigwedge c\notin\mathbb{Z}$$

07.23.13.0031.01

$$W_z\!\left({}_2\tilde{F}_1(a,\,b;\,c;\,z),\,z^{1-c}\,{}_2\tilde{F}_1(a-c+1,\,b-c+1;\,2-c;\,z)\right)=\frac{\sin(c\,\pi)}{\pi}\,(1-z)^{c-a-b-1}\,z^{-c}$$

07.23.13.0001.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;$$
$$w(z)=c_1\,{}_2F_1(a,\,b;\,c;\,z)+\,c_2\,z^{1-c}\,{}_2F_1(a-c+1,\,b-c+1;\,2-c;\,z)\bigwedge c\notin\mathbb{Z}$$

07.23.13.0002.02

$$W_z\!\left({}_2F_1(a,\,b;\,c;\,z),\,z^{1-c}\,{}_2F_1(a-c+1,\,b-c+1;\,2-c;\,z)\right)=(1-c)\,(1-z)^{c-a-b-1}\,z^{-c}$$

07.23.13.0032.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;\;w(z)=c_1\,{}_2F_1(a,\,b;\,c;\,z)+\,c_2\,U(a,\,b,\,c,\,z)$$

07.23.13.0033.01

$$W_z({}_2F_1(a,\,b;\,c;\,z),\,U(a,\,b,\,c,\,z))=-\frac{\Gamma(a+b-c+1)\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\,(1-z)^{c-a-b-1}\,z^{-c}$$

07.23.13.0034.01

$$w''(z)-\left(\frac{(c-(a+b+1)\,g(z))\,g'(z)}{(g(z)-1)\,g(z)}+\frac{g''(z)}{g'(z)}\right)w'(z)+\frac{a\,b\,g'(z)^2}{(g(z)-1)\,g(z)}\,w(z)=0\,/;$$
$$w(z)=\;c_1\,{}_2\tilde{F}_1(a,\,b;\,c;\,g(z))+c_2\,G_{2,2}^{2,2}\!\left(g(z)\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)$$

07.23.13.0035.01

$$W_z\!\left({}_2\tilde{F}_1(a,\,b;\,c;\,g(z)),\,G_{2,2}^{2,2}\!\left(g(z)\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)\right)=-(1-g(z))^{c-a-b-1}\,\Gamma(a-c+1)\,\Gamma(b-c+1)\,g'(z)\,g(z)^{-c}$$

07.23.13.0036.01

$$h(z)^2\,w''(z)\,-h(z)\left(h(z)\left(\frac{(c-(a+b+1)\,g(z))\,g'(z)}{(g(z)-1)\,g(z)}+\frac{g''(z)}{g'(z)}\right)+2\,h'(z)\right)w'(z)\,+$$
$$\left(\frac{a\,b\,h(z)^2\,g'(z)^2}{(g(z)-1)\,g(z)}+2\,h'(z)^2+h(z)\left(\frac{(c-(a+b+1)\,g(z))\,g'(z)\,h'(z)}{(g(z)-1)\,g(z)}+\frac{g''(z)\,h'(z)}{g'(z)}-h''(z)\right)\right)w(z)\,=0\,/;$$
$$w(z)=c_1\,h(z)\,{}_2\tilde{F}_1(a,\,b;\,c;\,g(z))\,+c_2\,h(z)\,G_{2,2}^{2,2}\!\left(g(z)\,\middle|\,\begin{matrix}1-a,\,1-b\\0,\,1-c\end{matrix}\right)$$

07.23.13.0037.01

$$W_z\left(h(z)\,{}_2\tilde{F}_1(a,\,b;\,c;\,g(z)),\,h(z)\,G_{2,2}^{2,2}\!\left(g(z)\,\middle|\,\begin{array}{c}1-a,\,1-b\\0,\,1-c\end{array}\right)\right)=-(1-g(z))^{c-a-b-1}\,\Gamma(a-c+1)\,\Gamma(b-c+1)\,h(z)^2\,g'(z)\,g(z)^{-c}$$

07.23.13.0038.01

$$z^2(1-d\,z^r)\,w''(z)+z\,((1-2\,s)\,(1-d\,z^r)-r\,((a+b)\,d\,z^r-c+1))\,w'(z)+\left(-a\,b\,d\,r^2\,z^r+r\,s\,((a+b)\,d\,z^r-c+1)+s^2\,(1-d\,z^r)\right)w(z)=0\,/;$$
$$w(z)=c_1\,z^s\,{}_2\tilde{F}_1(a,\,b;\,c;\,d\,z^r)+c_2\,z^s\,G_{2,2}^{2,2}\!\left(d\,z^r\,\middle|\,\begin{array}{c}1-a,\,1-b\\0,\,1-c\end{array}\right)$$

07.23.13.0039.01

$$W_z\left(z^s\,{}_2\tilde{F}_1(a,\,b;\,c;\,d\,z^r),\,z^s\,G_{2,2}^{2,2}\!\left(d\,z^r\,\middle|\,\begin{array}{c}1-a,\,1-b\\0,\,1-c\end{array}\right)\right)=-d\,r\,z^{r+2\,s-1}\,(d\,z^r)^{-c}\,(1-d\,z^r)^{-a-b+c-1}\,\Gamma(a-c+1)\,\Gamma(b-c+1)$$

07.23.13.0040.01

$$(1-d\,r^z)\,w''(z)-(((a+b)\,d\,r^z-c+1)\log(r)+2\,(1-d\,r^z)\log(s))\,w'(z)+\left(-a\,b\,d\log^2(r)\,r^z+(1-d\,r^z)\log^2(s)+((a+b)\,d\,r^z-c+1)\log(r)\log(s)\right)w(z)=0\,/;$$
$$w(z)=c_1\,s^z\,{}_2\tilde{F}_1(a,\,b;\,c;\,d\,r^z)+c_2\,s^z\,G_{2,2}^{2,2}\!\left(d\,r^z\,\middle|\,\begin{array}{c}1-a,\,1-b\\0,\,1-c\end{array}\right)$$

07.23.13.0041.01

$$W_z\left(s^z\,{}_2\tilde{F}_1(a,\,b;\,c;\,d\,r^z),\,s^z\,G_{2,2}^{2,2}\!\left(d\,r^z\,\middle|\,\begin{array}{c}1-a,\,1-b\\0,\,1-c\end{array}\right)\right)=-d\,r^z\,(d\,r^z)^{-c}\,(1-d\,r^z)^{c-a-b-1}\,s^{2\,z}\,\Gamma(a-c+1)\,\Gamma(b-c+1)\log(r)$$

Representation of fundamental system solutions near unit

07.23.13.0042.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;$$
$$w(z)=c_1\,{}_2\tilde{F}_1(a,\,b;\,a+b-c+1;\,1-z)+c_2\,G_{2,2}^{2,2}\!\left(1-z\,\middle|\,\begin{array}{c}1-a,\,1-b\\0,\,-a-b+c\end{array}\right)$$

07.23.13.0043.01

$$W_z\left({}_2\tilde{F}_1(a,\,b;\,a+b-c+1;\,1-z),\,G_{2,2}^{2,2}\!\left(1-z\,\middle|\,\begin{array}{c}1-a,\,1-b\\0,\,c-a-b\end{array}\right)\right)=(1-z)^{c-a-b-1}\,z^{-c}\,\Gamma(c-a)\,\Gamma(c-b)$$

07.23.13.0044.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;$$
$$w(z)=c_1(1-z)^{c-a-b}\,{}_2\tilde{F}_1(c-a,\,c-b;\,-a-b+c+1;\,1-z)+c_2\,{}_2\tilde{F}_1(a,\,b;\,a+b-c+1;\,1-z)\bigwedge c-a-b\notin\mathbb{Z}$$

07.23.13.0045.01

$$W_z\left((1-z)^{-a-b+c}\,{}_2\tilde{F}_1(c-a,\,c-b;\,-a-b+c+1;\,1-z),\,{}_2\tilde{F}_1(a,\,b;\,a+b-c+1;\,1-z)\right)=\frac{\sin((c-a-b)\,\pi)}{\pi}\,(1-z)^{c-a-b-1}\,z^{-c}$$

07.23.13.0046.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\,/;$$
$$w(z)=c_1(1-z)^{c-a-b}\,{}_2F_1(c-a,\,c-b;\,1-a-b+c;\,1-z)+c_2\,{}_2F_1(a,\,b;\,a+b-c+1;\,1-z)\bigwedge c-a-b\notin\mathbb{Z}$$

07.23.13.0047.01

$$W_z\left((1-z)^{c-a-b}\,{}_2F_1(c-a,\,c-b;\,-a-b+c+1;\,1-z),\,{}_2F_1(a,\,b;\,a+b-c+1;\,1-z)\right)=(c-a-b)\,(1-z)^{-a-b+c-1}\,z^{-c}$$

Representation of fundamental system solutions near infinity

07.23.13.0048.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\;/;\;w(z)=c_1\,z^{-a}\,{}_2\tilde{F}_1\!\left(a,\,a-c+1;\,a-b+1;\,\frac{1}{z}\right)+c_2\,G_{2,2}^{2,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}c,\,1\\a,\,b\end{matrix}\right)$$

07.23.13.0049.01

$$W_z\!\left(z^{-a}\,{}_2\tilde{F}_1\!\left(a,\,a-c+1;\,a-b+1;\,\frac{1}{z}\right),\,G_{2,2}^{2,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}c,\,1\\a,\,b\end{matrix}\right)\right)=(z-1)^{-a-b+c-1}\,z^{-c}\,\Gamma(b-c+1)\,\Gamma(b)$$

07.23.13.0050.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\;/;$$
$$w(z)=c_1\,z^{-a}\,{}_2\tilde{F}_1\!\left(a,\,a-c+1;\,a-b+1;\,\frac{1}{z}\right)+c_2\,z^{-b}\,{}_2\tilde{F}_1\!\left(b,\,b-c+1;\,-a+b+1;\,\frac{1}{z}\right)\bigwedge a-b\notin\mathbb{Z}$$

07.23.13.0051.01

$$W_z\!\left(z^{-a}\,{}_2\tilde{F}_1\!\left(a,\,a-c+1;\,a-b+1;\,\frac{1}{z}\right),\,z^{-b}\,{}_2\tilde{F}_1\!\left(b,\,b-c+1;\,-a+b+1;\,\frac{1}{z}\right)\right)=\frac{\sin((a-b)\,\pi)}{\pi}\,(z-1)^{-a-b+c-1}\,z^{-c}$$

07.23.13.0052.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\;/;$$
$$w(z)=c_1\,z^{-a}\,{}_2F_1\!\left(a,\,a-c+1;\,a-b+1;\,\frac{1}{z}\right)+c_2\,z^{-b}\,{}_2F_1\!\left(b,\,b-c+1;\,1-a+b;\,\frac{1}{z}\right)\bigwedge a-b\notin\mathbb{Z}$$

07.23.13.0053.01

$$W_z\!\left(z^{-a}\,{}_2F_1\!\left(a,\,a-c+1;\,a-b+1;\,\frac{1}{z}\right),\,z^{-b}\,{}_2F_1\!\left(b,\,b-c+1;\,-a+b+1;\,\frac{1}{z}\right)\right)=(a-b)\,(z-1)^{c-a-b-1}\,z^{-c}$$

List of solutions

07.23.13.0003.01

$$(1-z)\,z\,w''(z)+(c-(a+b+1)\,z)\,w'(z)-a\,b\,w(z)=0\;/;$$
$$w\,(z)=c_1\,u_{j,k}(z)+c_2\,u_{l,m}(z)\bigwedge 1\le j\le 6\bigwedge 1\le k\le 4\bigwedge 1\le l\le 6\bigwedge 1\le m\le 4\bigwedge j\ne l$$

where $u_{j,k}(z)$ are arbitrary functions from the following list of 24 functions

07.23.13.0004.01

$$u_{1,1}(z)={}_2F_1(a,\,b;\,c;\,z)$$

07.23.13.0005.01

$$u_{1,2}(z)=(1-z)^{c-a-b}\,{}_2F_1(c-a,\,c-b;\,c;\,z)$$

07.23.13.0006.01

$$u_{1,3}(z)=(1-z)^{-a}\,{}_2F_1\!\left(a,\,c-b;\,c;\,\frac{z}{z-1}\right)$$

07.23.13.0007.01

$$u_{1,4}(z)=(1-z)^{-b}\,{}_2F_1\!\left(c-a,\,b;\,c;\,\frac{z}{z-1}\right)$$

07.23.13.0008.01

$$u_{2,1}(z)={}_2F_1(a,\,b;\,a+b-c+1;\,1-z)$$

07.23.13.0009.01

$$u_{2,2}(z)=z^{1-c}\,{}_2F_1(b-c+1,\,a-c+1;\,a+b-c+1;\,1-z)$$

07.23.13.0010.01

$$u_{2,3}(z) = z^{-a}\, {}_2F_1\left(a, a-c+1; a+b-c+1; \frac{z-1}{z}\right)$$

07.23.13.0011.01

$$u_{2,4}(z) = z^{-b}\, {}_2F_1\left(b-c+1, b; a+b-c+1; \frac{z-1}{z}\right)$$

07.23.13.0012.01

$$u_{3,1}(z) = (-z)^{-a}\, {}_2F_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right)$$

07.23.13.0013.01

$$u_{3,2}(z) = \left(\frac{z-1}{z}\right)^{c-a-b} (-z)^{-a}\, {}_2F_1\left(1-b, c-b; a-b+1; \frac{1}{z}\right)$$

07.23.13.0014.01

$$u_{3,3}(z) = \left(\frac{z-1}{z}\right)^{-a} (-z)^{-a}\, {}_2F_1\left(a, c-b; a-b+1; \frac{1}{1-z}\right)$$

07.23.13.0015.01

$$u_{3,4}(z) = \left(\frac{z-1}{z}\right)^{c-a-1} (-z)^{-a}\, {}_2F_1\left(1-b, a-c+1; a-b+1; \frac{1}{1-z}\right)$$

07.23.13.0016.01

$$u_{4,1}(z) = (-z)^{-b}\, {}_2F_1\left(b-c+1, b; -a+b+1; \frac{1}{z}\right)$$

07.23.13.0017.01

$$u_{4,2}(z) = \left(\frac{z-1}{z}\right)^{c-a-b} (-z)^{-b}\, {}_2F_1\left(c-a, 1-a; b-a+1; \frac{1}{z}\right)$$

07.23.13.0018.01

$$u_{4,3}(z) = \left(\frac{z-1}{z}\right)^{c-b-1} (-z)^{-b}\, {}_2F_1\left(b-c+1, 1-a; b-a+1; \frac{1}{1-z}\right)$$

07.23.13.0019.01

$$u_{4,4}(z) = \left(\frac{z-1}{z}\right)^{-b} (-z)^{-b}\, {}_2F_1\left(c-a, b; b-a+1; \frac{1}{1-z}\right)$$

07.23.13.0020.01

$$u_{5,1}(z) = z^{1-c}\, {}_2F_1(a-c+1, b-c+1; 2-c; z)$$

07.23.13.0021.01

$$u_{5,2}(z) = (1-z)^{c-a-b}\, z^{1-c}\, {}_2F_1(1-a, 1-b; 2-c; z)$$

07.23.13.0022.01

$$u_{5,3}(z) = (1-z)^{c-a-1}\, z^{1-c}\, {}_2F_1\left(a-c+1, 1-b; 2-c; \frac{z}{z-1}\right)$$

07.23.13.0023.01

$$u_{5,4}(z) = (1-z)^{c-b-1}\, z^{1-c}\, {}_2F_1\left(1-a, b-c+1; 2-c; \frac{z}{z-1}\right)$$

07.23.13.0024.01

$$u_{6,1}(z) = (1-z)^{c-a-b}\, {}_2F_1(c-a, c-b; c-a-b+1; 1-z)$$

07.23.13.0025.01

$$u_{6,2}(z) == (1-z)^{c-a-b}\, z^{1-c}\, {}_2F_1(1-b,\, 1-a;\, c-a-b+1;\, 1-z)$$

07.23.13.0026.01

$$u_{6,3}(z) == (1-z)^{c-a-b}\, z^{a-c}\, {}_2F_1\left(c-a,\, 1-a;\, c-a-b+1;\, \frac{z-1}{z}\right)$$

07.23.13.0027.01

$$u_{6,4}(z) == (1-z)^{c-a-b}\, z^{b-c}\, {}_2F_1\left(1-b,\, c-b;\, c-a-b+1;\, \frac{z-1}{z}\right)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.23.16.0001.01

$${}_2F_1(c-a,\, c-b;\, c;\, z) == (1-z)^{a+b-c}\, {}_2F_1(a,\, b;\, c;\, z)$$

07.23.16.0002.01

$${}_2F_1\left(a,\, c-b;\, c;\, \frac{z}{z-1}\right) == (1-z)^a\, {}_2F_1(a,\, b;\, c;\, z) \;/;\; z \notin (1,\, \infty)$$

07.23.16.0003.01

$${}_2F_1\left(c-a,\, b;\, c;\, \frac{z}{z-1}\right) == (1-z)^b\, {}_2F_1(a,\, b;\, c;\, z) \;/;\; z \notin (1,\, \infty)$$

07.23.16.0004.01

$${}_2F_1\left(a,\, a+\frac{1}{2};\, c;\, 2\,z-z^2\right) == \left(1-\frac{z}{2}\right)^{-2\,a}\, {}_2F_1\left(2\,a,\, 2\,a-c+1;\, c;\, \frac{z}{2-z}\right) \;/;\; \mathrm{Re}(z) < 1$$

07.23.16.0005.01

$${}_2F_1\left(a,\, b;\, 2\,b;\, \frac{4\,z}{(z+1)^2}\right) == (z+1)^{2\,a}\, {}_2F_1\left(a,\, a-b+\frac{1}{2};\, b+\frac{1}{2};\, z^2\right) \;/;\; |z| < 1$$

Products, sums, and powers of the direct function

Products of the direct function

07.23.16.0006.01

$${}_2F_1(a,\, b;\, c;\, g\,z)\, {}_2F_1(\alpha,\, \beta;\, \gamma;\, h\,z) == \sum_{k=0}^{\infty} c_k\, z^k \;/;\; c_k == \frac{h^k\, (\alpha)_k\, (\beta)_k}{k!\, (\gamma)_k}\, {}_4F_3\left(-k,\, 1-k-\gamma,\, a,\, b;\, 1-k-\alpha,\, 1-k-\beta,\, c;\, \frac{g}{h}\right) \bigvee$$

$$c_k == \frac{g^k\, (a)_k\, (b)_k}{k!\, (c)_k}\, {}_4F_3\left(-k,\, 1-c-k,\, \alpha,\, \beta;\, 1-a-k,\, 1-b-k,\, \gamma;\, \frac{h}{g}\right)$$

07.23.16.0007.01

$${}_2F_1(a,\, b;\, c;\, g\,z)\, {}_2F_1(\alpha,\, \beta;\, \gamma;\, h\,z) == \sum_{k=0}^{\infty} \sum_{m=0}^{k} \frac{(a)_m\, (b)_m\, (\alpha)_{k-m}\, (\beta)_{k-m}\, g^m\, h^{k-m}\, z^k}{(c)_m\, (\gamma)_{k-m}\;\; m!\, (k-m)!}$$

07.23.16.0008.01

$${}_2F_1(a,\, b;\, c;\, g\,z)\, {}_2F_1(\alpha,\, \beta;\, \gamma;\, h\,z) == F_{0;1;1}^{0:2;2}\left(\begin{matrix} :a,\, b;\, \alpha,\, \beta; \\ :c;\, \gamma; \end{matrix}\; g\,z,\, h\,z\right)$$

Identities

Recurrence identities

Consecutive neighbors

07.23.17.0001.01

$${}_2F_1(a, b; c; z) = \frac{2b - c + 2 + (a - b - 1)z}{b - c + 1}\, {}_2F_1(a, b + 1; c; z) + \frac{(b + 1)(z - 1)}{b - c + 1}\, {}_2F_1(a, b + 2; c; z)$$

07.23.17.0002.01

$${}_2F_1(a, b; c; z) = \frac{c - 2b + 2 + (b - a - 1)z}{(b - 1)(z - 1)}\, {}_2F_1(a, b - 1; c; z) + \frac{b - c - 1}{(b - 1)(z - 1)}\, {}_2F_1(a, b - 2; c; z)$$

07.23.17.0003.01

$${}_2F_1(a, b; c; z) = \frac{(2c - a - b + 1)z - c}{c(z - 1)}\, {}_2F_1(a, b; c + 1; z) + \frac{(a - c - 1)(c - b + 1)z}{c(c + 1)(z - 1)}\, {}_2F_1(a, b; c + 2; z)$$

07.23.17.0004.01

$${}_2F_1(a, b; c; z) = \frac{(c - 1)(2 - c - (a + b - 2c + 3)z)}{(a - c + 1)(b - c + 1)z}\, {}_2F_1(a, b; c - 1; z) + \frac{(c - 1)(c - 2)(1 - z)}{(a - c + 1)(b - c + 1)z}\, {}_2F_1(a, b; c - 2; z)$$

Consecutive neighbors (nine basic relations)

07.23.17.0134.01

$$(a - c)\, {}_2F_1(a - 1, b; c; z) + (c - 2a + (a - b)z)\, {}_2F_1(a, b; c; z) = a(z - 1)\, {}_2F_1(a + 1, b; c; z)$$

07.23.17.0135.01

$$(b - c)\, {}_2F_1(a, b - 1; c; z) + (c - 2b + (b - a)z)\, {}_2F_1(a, b; c; z) = b(z - 1)\, {}_2F_1(a, b + 1; c; z)$$

07.23.17.0136.01

$$(c - 1)c(1 - z)\, {}_2F_1(a, b; c - 1; z) + c(1 - c + (2c - a - b - 1)z)\, {}_2F_1(a, b; c; z) = (a - c)(b - c)z\, {}_2F_1(a, b; c + 1; z)$$

07.23.17.0137.01

$$(b - c)\, {}_2F_1(a, b - 1; c; z) + (c - a - b)\, {}_2F_1(a, b; c; z) = a(z - 1)\, {}_2F_1(a + 1, b; c; z)$$

07.23.17.0138.01

$$(a - c)\, {}_2F_1(a - 1, b; c; z) + (-a - b + c)\, {}_2F_1(a, b; c; z) = b(z - 1)\, {}_2F_1(a, b + 1; c; z)$$

07.23.17.0139.01

$$(c - b)(a - c)z\, {}_2F_1(a, b; c + 1; z) + ((c - b)z - a)c\, {}_2F_1(a, b; c; z) = ac(z - 1)\, {}_2F_1(a + 1, b; c; z)$$

07.23.17.0140.01

$$((c - b)z - a)\, {}_2F_1(a + 1, b; c + 1; z) + (a - c)\, {}_2F_1(a, b; c + 1; z) = c(z - 1)\, {}_2F_1(a + 1, b; c; z)$$

07.23.17.0141.01

$$(c - a)(b - c)z\, {}_2F_1(a, b; c + 1; z) + ((c - a)z - b)c\, {}_2F_1(a, b; c; z) = bc(z - 1)\, {}_2F_1(a, b + 1; c; z)$$

07.23.17.0142.01

$$((c - a)z - b)\, {}_2F_1(a, b + 1; c + 1; z) + (b - c)\, {}_2F_1(a, b; c + 1; z) = c(z - 1)\, {}_2F_1(a, b + 1; c; z)$$

Distant neighbors

07.23.17.0143.01

$$ {}_2F_1(a,\,b;\,c;\,z) = C_n(a,\,b,\,c,\,z)\,{}_2F_1(a,\,b+n;\,c;\,z) + \frac{(b+n)\,(z-1)}{b-c+n}\,C_{n-1}(a,\,b,\,c,\,z)\,{}_2F_1(a,\,b+n+1;\,c;\,z)\;/;\; C_0(a,\,b,\,c,\,z) = 1 \bigwedge C_1(a,\,b,\,c,\,z) = \frac{2\,a-b+z+2}{a-b+1} \bigwedge C_n(a,\,b,\,c,\,z) = \frac{2\,b-c+2\,n+(a-b-n)\,z}{b-c+n}\,C_{n-1}(a,\,b,\,c,\,z) + \frac{(b+n-1)\,(z-1)}{b-c+n-1}\,C_{n-2}(a,\,b,\,c,\,z) \bigwedge n \in \mathbb{N}^+ $$

07.23.17.0144.01

$$ {}_2F_1(a,\,b;\,c;\,z) = \frac{n-b+c}{(n-b)\,(z-1)}\,C_{n-1}(a,\,b,\,c,\,z)\,{}_2F_1(a,\,b-n-1;\,c;\,z) + C_n(a,\,b,\,c,\,z)\,{}_2F_1(a,\,b-n;\,c;\,z)\;/;\; C_0(a,\,b,\,c,\,z) = 1 \bigwedge C_1(a,\,b,\,c,\,z) = \frac{2-2\,b+c+(b-a-1)\,z}{(b-1)\,(z-1)} \bigwedge C_n(a,\,b,\,c,\,z) = -\frac{-2\,b+c+2\,n+(b-a-n)\,z}{(n-b)\,(z-1)}\,C_{n-1}(a,\,b,\,c,\,z) + \frac{n-b+c-1}{(n-b-1)\,(z-1)}\,C_{n-2}(a,\,b,\,c,\,z) \bigwedge n \in \mathbb{N}^+ $$

07.23.17.0145.01

$$ {}_2F_1(a,\,b;\,c;\,z) = C_n(a,\,b,\,c,\,z)\,{}_2F_1(a,\,b;\,c+n;\,z) - \frac{(a-n-c)\,(b-n-c)\,z}{(n+c-1)\,(n+c)\,(z-1)}\,C_{n-1}(a,\,b,\,c,\,z)\,{}_2F_1(a,\,b;\,c+n+1;\,z)\;/;\; C_0(a,\,b,\,c,\,z) = 1 \bigwedge C_1(a,\,b,\,c,\,z) = \frac{(1-a-b+2\,c)\,z-c}{c\,(z-1)} \bigwedge C_n(a,\,b,\,c,\,z) = \frac{1-c-n+(1-a-b+2\,(c+n-1))\,z}{(c+n-1)\,(z-1)}\,C_{n-1}(a,\,b,\,c,\,z) - \frac{(a-c-n+1)\,(b-c-n+1)\,z}{(c+n-2)\,(c+n-1)\,(z-1)}\,C_{n-2}(a,\,b,\,c,\,z) \bigwedge n \in \mathbb{N}^+ $$

07.23.17.0146.01

$$ {}_2F_1(a,\,b;\,c;\,z) = \frac{(n-c+1)\,(n-c)\,(1-z)}{(n+a-c)\,(n+b-c)\,z}\,C_{n-1}(a,\,b,\,c,\,z)\,{}_2F_1(a,\,b;\,c-n-1;\,z) + C_n(a,\,b,\,c,\,z)\,{}_2F_1(a,\,b;\,c-n;\,z)\;/;\; C_0(a,\,b,\,c,\,z) = 1 \bigwedge C_1(a,\,b,\,c,\,z) = \frac{(c-1)\,(2-c-(a+b-2\,c+3)\,z)}{(a-c+1)\,(b-c+1)\,z} \bigwedge C_n(a,\,b,\,c,\,z) = \frac{(n-c)\,(n-c-1)\,(1-z)}{(a-c+n-1)\,(b-c+n-1)\,z}\,C_{n-2}(a,\,b,\,c,\,z) + \frac{(c-n)\,(1-c+n-(a+b+2\,(n-c-1)+3)\,z)}{(a-c+n)\,(b-c+n)\,z}\,C_{n-1}(a,\,b,\,c,\,z) \bigwedge n \in \mathbb{N}^+ $$

07.23.17.0005.01

$$ {}_2F_1(a,\,b;\,c;\,z) = \frac{(1-c)_m\,z^{-m}}{(a-c+1)_m} \sum_{k=0}^{m} \binom{m}{k} (z-1)^{m-k}\,{}_2F_1(a,\,b-k;\,c-m;\,z)\;/;\; m \in \mathbb{N} $$

07.23.17.0006.01

$$ {}_2F_1(a,\,b;\,c;\,z) = \frac{(1-c)_m\,z^{-m}}{(1-a)_m} \sum_{k=0}^{m} (-1)^k \binom{m}{k}\,{}_2F_1(a-m,\,b-k;\,c-m;\,z)\;/;\; m \in \mathbb{N} $$

07.23.17.0007.01

$$ {}_2F_1(1,\,b;\,c;\,z) = \frac{(c-b)_m\,(1-z)^{-m}}{(1-b)_m}\,{}_2F_1(1,\,b-m;\,c;\,z) + \frac{c-1}{b-1} \sum_{k=0}^{m-1} \frac{(c-b)_k\,(1-z)^{-k-1}}{(2-b)_k}\;/;\; m \in \mathbb{N}^+ $$

07.23.17.0008.01

$$ {}_2F_1(1,\,b;\,c;\,z) = \frac{(1-c)_m}{(b-c+1)_m} \left(\frac{z-1}{z}\right)^m {}_2F_1(1,\,b;\,c-m;\,z) + \frac{1}{z} \sum_{k=1}^{m} \frac{(1-c)_k}{(b-c+1)_k} \left(\frac{z-1}{z}\right)^{k-1}\;/;\; m \in \mathbb{N}^+ $$

Functional identities

Relations between contiguous functions

07.23.17.0009.01

$$(a-c)\,{}_2F_1(a-1,b;c;z)-a(z-1)\,{}_2F_1(a+1,b;c;z)+(c-2a+(a-b)z)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0010.01

$$(a-c)\,{}_2F_1(a-1,b;c;z)+(c-b)\,{}_2F_1(a,b-1;c;z)+(z-1)(a-b)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0011.01

$$(b-c)\,{}_2F_1(a,b-1;c;z)-a(z-1)\,{}_2F_1(a+1,b;c;z)+(c-a-b)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0012.01

$$(a-c)\,{}_2F_1(a-1,b;c;z)-b(z-1)\,{}_2F_1(a,b+1;c;z)+(c-a-b)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0013.01

$$b\,{}_2F_1(a,b+1;c;z)-a\,{}_2F_1(a+1,b;c;z)+(a-b)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0014.01

$$(b-c)\,{}_2F_1(a,b-1;c;z)-b(z-1)\,{}_2F_1(a,b+1;c;z)+(c-2b+(b-a)z)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0015.01

$$(c-1)(z-1)c\,{}_2F_1(a,b;c-1;z)+(a-c)(b-c)z\,{}_2F_1(a,b;c+1;z)+c(c+(a+b-2c+1)z-1)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0016.01

$$(c-a)\,{}_2F_1(a-1,b;c;z)+(c-1)(z-1)\,{}_2F_1(a,b;c-1;z)+(a+(b-c+1)z-1)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0017.01

$$c\,{}_2F_1(a-1,b;c;z)+(b-c)z\,{}_2F_1(a,b;c+1;z)+(z-1)c\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0018.01

$$(c-1)\,{}_2F_1(a,b;c-1;z)-a\,{}_2F_1(a+1,b;c;z)+(a-c+1)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0019.01

$$(z-1)ac\,{}_2F_1(a+1,b;c;z)+(a-c)(b-c)z\,{}_2F_1(a,b;c+1;z)+c(a+(b-c)z)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0020.01

$$(c-b)\,{}_2F_1(a,b-1;c;z)+(c-1)(z-1)\,{}_2F_1(a,b;c-1;z)+(b+(a-c+1)z-1)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0021.01

$$c\,{}_2F_1(a,b-1;c;z)+(a-c)z\,{}_2F_1(a,b;c+1;z)+(z-1)c\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0022.01

$$(c-1)\,{}_2F_1(a,b;c-1;z)-b\,{}_2F_1(a,b+1;c;z)+(b-c+1)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0023.01

$$(z-1)bc\,{}_2F_1(a,b+1;c;z)+(a-c)(b-c)z\,{}_2F_1(a,b;c+1;z)+c(b+(a-c)z)\,{}_2F_1(a,b;c;z)=0$$

07.23.17.0024.01

$$(a-b)(a-c)\,{}_2F_1(a-1,b;c;z)+a(c-a-b)\,{}_2F_1(a+1,b;c;z)+b(2a-c+(b-a)z)\,{}_2F_1(a,b+1;c;z)=0$$

07.23.17.0025.01

$$a(c-2b+(b-a)z)\,{}_2F_1(a+1,b;c;z)+(a-b)(b-c)\,{}_2F_1(a,b-1;c;z)+b(a+b-c)\,{}_2F_1(a,b+1;c;z)=0$$

07.23.17.0026.01

$$(a-c+1)(a-c)\,{}_2F_1(a-1,b;c;z)-\\ a(a+(b-c+1)z-1)\,{}_2F_1(a+1,b;c;z)-(c-1)(c-2a+(a-b)z)\,{}_2F_1(a,b;c-1;z)=0$$

07.23.17.0027.01

$$a\,c\,(1-c+(2\,c-a-b-1)\,z)\,{}_2F_1(a+1,\,b;\,c;\,z)+(c-1)\,c\,(a+(b-c)\,z)\,{}_2F_1(a,\,b;\,c-1;\,z)-(a-c)\,(a-c+1)\,(b-c)\,z\,{}_2F_1(a,\,b;\,c+1;\,z)=0$$

07.23.17.0028.01

$$(b-c+1)\,(b-c)\,{}_2F_1(a,\,b-1;\,c;\,z)+b\,(1-b+(c-a-1)\,z)\,{}_2F_1(a,\,b+1;\,c;\,z)-(c-1)\,(z\,(b-a)-2\,b+c)\,{}_2F_1(a,\,b;\,c-1;\,z)=0$$

07.23.17.0029.01

$$(c-1)\,c\,(b+(a-c)\,z)\,{}_2F_1(a,\,b;\,c-1;\,z)-b\,c\,(c+(a+b-2\,c+1)\,z-1)\,{}_2F_1(a,\,b+1;\,c;\,z)-(a-c)\,(b-c)\,(b-c+1)\,z\,{}_2F_1(a,\,b;\,c+1;\,z)=0$$

07.23.17.0030.01

$$c\,{}_2F_1(a-1,\,b;\,c;\,z)-c\,{}_2F_1(a,\,b-1;\,c;\,z)-(a-b)\,z\,{}_2F_1(a,\,b;\,c+1;\,z)=0$$

07.23.17.0031.01

$$(b-c+1)\,(a-c)\,{}_2F_1(a-1,\,b;\,c;\,z)+b\,(1-a+(c-b-1)\,z)\,{}_2F_1(a,\,b+1;\,c;\,z)+(c-1)\,(a+b-c)\,{}_2F_1(a,\,b;\,c-1;\,z)=0$$

07.23.17.0032.01

$$a\,(1-b+(c-a-1)\,z)\,{}_2F_1(a+1,\,b;\,c;\,z)+(a-c+1)\,(b-c)\,{}_2F_1(a,\,b-1;\,c;\,z)+(c-1)\,(a+b-c)\,{}_2F_1(a,\,b;\,c-1;\,z)=0$$

07.23.17.0033.01

$$(b-c+1)\,a\,{}_2F_1(a+1,\,b;\,c;\,z)-b\,(a-c+1)\,{}_2F_1(a,\,b+1;\,c;\,z)+(c-1)\,(a-b)\,{}_2F_1(a,\,b;\,c-1;\,z)=0$$

07.23.17.0034.01

$$a\,c\,(b+(a-c)\,z)\,{}_2F_1(a+1,\,b;\,c;\,z)-b\,c\,(a+(b-c)\,z)\,{}_2F_1(a,\,b+1;\,c;\,z)+(a-b)\,(a-c)\,(b-c)\,z\,{}_2F_1(a,\,b;\,c+1;\,z)=0$$

Additional relations between contiguous functions

07.23.17.0035.01

$$c\,{}_2F_1(a,\,b;\,c;\,z)-a\,{}_2F_1(a+1,\,b;\,c+1;\,z)+(a-c)\,{}_2F_1(a,\,b;\,c+1;\,z)=0$$

07.23.17.0036.01

$$(a-b)\,c\,{}_2F_1(a,\,b;\,c;\,z)-a\,(c-b)\,{}_2F_1(a+1,\,b;\,c+1;\,z)+(c-a)\,b\,{}_2F_1(a,\,b+1;\,c+1;\,z)=0$$

Relations for fixed z

07.23.17.0037.01

$${}_2F_1(a,\,b;\,c;\,-1)=2^{-a}\,{}_2F_1\left(a,\,c-b;\,c;\,\frac{1}{2}\right)$$

07.23.17.0038.01

$${}_2F_1\left(a,\,b;\,c;\,\frac{1}{2}\right)=2^a\,{}_2F_1(a,\,c-b;\,c;\,-1)$$

07.23.17.0039.01

$${}_2F_1(-n,\,b;\,1;\,2)=\frac{(-1)^n\,(b)_n}{n!}\,{}_2F_1(-n,\,1-b;\,1-b-n;\,-1)\;/;\;n\in\mathbb{N}$$

Relations of special kind

07.23.17.0040.01

$${}_2F_1(a,\,b;\,a+1;\,z)+{}_2F_1(-a,\,b;\,1-a;\,z)=2\,{}_3F_2(a,\,-a,\,b;\,a+1,\,1-a;\,z)$$

07.23.17.0041.01

$${}_2F_1(-a,\,a+1;\,c;\,z)+{}_2F_1(a,\,1-a;\,c;\,z)=2\,{}_2F_1(a,\,-a;\,c;\,z)$$

07.23.17.0042.01

$$ {}_2F_1(a, b; c; z)\, {}_2F_1(-a, -b; -c; z) + \frac{a\, b\, (a-c)\, (b-c)}{c^2 \left(1-c^2\right)} z^2\, {}_2F_1(1-a, 1-b; 2-c; z)\, {}_2F_1(a+1, b+1; c+2; z) = 1 $$

07.23.17.0043.01

$$ {}_2F_1\left(-\lambda - \frac{1}{2}, \nu + \frac{1}{2}; \mu + \nu + 1; 1-z\right) {}_2F_1\left(\lambda + \frac{1}{2}, \frac{1}{2} - \nu; \lambda + \mu + 1; z\right) + {}_2F_1\left(\frac{1}{2} - \lambda, \nu + \frac{1}{2}; \mu + \nu + 1; 1-z\right) {}_2F_1\left(\lambda + \frac{1}{2}, -\nu - \frac{1}{2}; \lambda + \mu + 1; z\right) - {}_2F_1\left(\frac{1}{2} - \lambda, \nu + \frac{1}{2}; \mu + \nu + 1; 1-z\right) {}_2F_1\left(\lambda + \frac{1}{2}, \frac{1}{2} - \nu; \lambda + \mu + 1; z\right) = \frac{\Gamma(\lambda + \mu + 1)\, \Gamma(\mu + \nu + 1)}{\Gamma\left(\mu + \frac{1}{2}\right) \Gamma\left(\lambda + \mu + \nu + \frac{3}{2}\right)} $$

Generalized Legendre identity, Elliott 1904

07.23.17.0044.01

$$ {}_2F_1(a-1, 1-a; c; z)\, {}_2F_1(a, 1-a; c; 1-z) + {}_2F_1(a-1, 1-a; c; 1-z)\, {}_2F_1(a, 1-a; c; z) - {}_2F_1(a, 1-a; c; 1-z)\, {}_2F_1(a, 1-a; c; z) = \frac{\Gamma(c)^2}{\Gamma(-a+c+1)\, \Gamma(a+c-1)} $$

Generalized Legendre identity, Anderson, Qiu, Vamanamurthy, Vuorinen 2000

Reduction to polynomial

07.23.17.0045.01

$$ {}_2F_1(a, b; b-n; z) = (1-z)^{-a}\, {}_2F_1\left(-n, a; b-n; \frac{z}{z-1}\right) /; n \in \mathbb{N} $$

07.23.17.0046.01

$$ {}_2F_1(a, b; b-n; z) = \frac{(-1)^n\, (a)_n}{(1-b)_n} (1-z)^{-a-n}\, {}_2F_1(-n, b-a-n; 1-a-n; 1-z) /; n \in \mathbb{N} $$

07.23.17.0047.01

$$ {}_2F_1(a, b; b-n; z) = \frac{(-1)^n\, (a-b+1)_n}{(1-b)_n} z^n\, (1-z)^{-a-n}\, {}_2F_1\left(-n, 1-b; a-b+1; \frac{1}{z}\right) /; n \in \mathbb{N} $$

07.23.17.0048.01

$$ {}_2F_1(a, b; b-n; z) = \frac{(a-b+1)_n}{(1-b)_n} (1-z)^{-a}\, {}_2F_1\left(-n, a; a-b+1; \frac{1}{1-z}\right) /; n \in \mathbb{N} $$

07.23.17.0049.01

$$ {}_2F_1(a, b; b-n; z) = \frac{(a)_n}{(1-b)_n} (-z)^n\, (1-z)^{-a-n}\, {}_2F_1\left(-n, 1-b; 1-a-n; 1-\frac{1}{z}\right) /; n \in \mathbb{N} $$

07.23.17.0050.01

$$ {}_2F_1(a, b; b-n; z) = (1-z)^{-a-n}\, {}_2F_1(-n, -a+b-n; b-n; z) /; n \in \mathbb{N} $$

Division on even and odd parts and generalization

07.23.17.0051.01

$$ {}_2F_1(a, b; c; z) = A^+(z) + A^-(z) /; A^+(z) = \frac{1}{2} \left({}_2F_1(a, b; c; z) + {}_2F_1(a, b; c; -z)\right) \bigwedge A^-(z) = \frac{1}{2} \left({}_2F_1(a, b; c; z) - {}_2F_1(a, b; c; -z)\right) $$

07.23.17.0052.01

$${}_2F_1(a, b; c; z) = A^+(z) + A^-(z) /; A^+(z) = {}_4F_3\left(\frac{a}{2}, \frac{b}{2}, \frac{a+1}{2}, \frac{b+1}{2}; \frac{1}{2}, \frac{c}{2}, \frac{c+1}{2}; z^2\right) \bigwedge A^-(z) = \frac{a\,b\,z}{c}\, {}_4F_3\left(\frac{a+1}{2}, \frac{b+1}{2}, \frac{a+2}{2}, \frac{b+2}{2}; \frac{3}{2}, \frac{c+1}{2}, \frac{c+2}{2}; z^2\right)$$

07.23.17.0053.01

$${}_2F_1(a, b; c; z) = \sum_{k=0}^{n-1} \frac{(a)_k\,(b)_k\,z^k}{k!\,(c)_k}\, {}_{2n+1}F_{2n}\left(1, \frac{a+k}{n}, \ldots, \frac{a+k+n-1}{n}, \frac{b+k}{n}, \ldots, \frac{b+k+n-1}{n}; \frac{k+1}{n}, \ldots, \frac{k+n}{n}, \frac{c+k}{n}, \ldots, \frac{c+k+n-1}{n}; z^n\right)$$

Generic general cases

07.23.17.0054.01

$${}_2F_1(a, b; c; z) = (1-z)^{c-a-b}\, {}_2F_1(c-a, c-b; c; z)$$

07.23.17.0055.01

$${}_2F_1(a, b; c; z) = (1-z)^{-a}\, {}_2F_1\left(a, c-b; c; \frac{z}{z-1}\right) /; z \notin (1, \infty)$$

07.23.17.0147.01

$${}_2F_1(a, b; c; z) = e^{2 i (c-a-b) \pi} \left(\frac{1}{1-z}\right)^a {}_2F_1\left(a, c-b; c; \frac{z}{z-1}\right) - \frac{2\,i\,\pi\, e^{i (c-a-b) \pi}\,\Gamma(c)}{\Gamma(a+b-c+1)\,\Gamma(c-a)\,\Gamma(c-b)}\, {}_2F_1(a, b; a+b-c+1; 1-z) /; z \in (1, \infty)$$

07.23.17.0056.01

$${}_2F_1(a, b; c; z) = (1-z)^{-b}\, {}_2F_1\left(c-a, b; c; \frac{z}{z-1}\right) /; z \notin (1, \infty)$$

07.23.17.0057.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(b-a)\,\Gamma(c)}{\Gamma(b)\,\Gamma(c-a)} (-z)^{-a}\, {}_2F_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right) + \frac{\Gamma(a-b)\,\Gamma(c)}{\Gamma(a)\,\Gamma(c-b)} (-z)^{-b}\, {}_2F_1\left(b, b-c+1; -a+b+1; \frac{1}{z}\right) /; a-b \notin \mathbb{Z} \bigwedge z \notin (0, 1)$$

07.23.17.0058.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}\, {}_2F_1(c-a, c-b; c-a-b+1; 1-z)\,(1-z)^{c-a-b} + \frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\, {}_2F_1(a, b; a+b-c+1; 1-z) /; c-a-b \notin \mathbb{Z}$$

07.23.17.0059.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(c)\,\Gamma(b-a)}{\Gamma(b)\,\Gamma(c-a)} (1-z)^{-a}\, {}_2F_1\left(a, c-b; a-b+1; \frac{1}{1-z}\right) + \frac{\Gamma(c)\,\Gamma(a-b)}{\Gamma(a)\,\Gamma(c-b)} (1-z)^{-b}\, {}_2F_1\left(b, c-a; b-a+1; \frac{1}{1-z}\right) /; a-b \notin \mathbb{Z}$$

07.23.17.0060.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(c)\,\Gamma(c-a-b)\,z^{-a}}{\Gamma(c-a)\,\Gamma(c-b)}\, {}_2F_1\left(a, a-c+1; a+b-c+1; 1-\frac{1}{z}\right) + \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)} (1-z)^{c-a-b}\, z^{a-c}\, {}_2F_1\left(c-a, 1-a; c-a-b+1; 1-\frac{1}{z}\right) /; c-a-b \notin \mathbb{Z} \bigwedge z \notin (-\infty, 0)$$

07.23.17.0148.01

$${}_2F_1(a, b; c; z) = U(a, b, a+b-c+1, 1-z)$$

Confluent general cases

07.23.17.0149.01

$$ {}_2F_1(a, b; c; z) = \frac{\Gamma(\nu - c + 1)\,\Gamma(c)\sin((c-\mu)\pi)}{\pi\, n!\,\Gamma(\mu)} \log(-z)\,(-z)^{-\nu}\,{}_2F_1\!\left(\nu, \nu - c + 1; n + 1; \frac{1}{z}\right) + \frac{\Gamma(c)}{\Gamma(\nu)}(-z)^{-\mu}\sum_{k=0}^{n-1}\frac{(\mu)_k\,\Gamma(n-k)\,z^{-k}}{k!\,\Gamma(c-k-\mu)} + \frac{\Gamma(c)\,(-z)^{-\nu}}{\Gamma(\nu)\,\Gamma(c-\mu)}\sum_{k=0}^{\infty}\frac{1}{k!\,(k+n)!}\left((\mu)_{k+n}\,(\mu - c + 1)_{k+n}\,(\psi(k+1) + \psi(k+n+1) - \psi(c-k-\nu) - \psi(k+\nu))\,z^{-k}\right) /; $$

$|z| > 1 \land b - a \in \mathbb{Z} \land n = |b - a| \land \mu = a + \min(0, b - a) \land \nu = a + \max(0, b - a) \land \neg\, c - a \in \mathbb{Z}$

07.23.17.0150.01

$$ {}_2F_1(a, b; c; z) = \frac{(-z)^{-\mu}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\left((-1)^n\,\Gamma(c)\,(-z)^{\mu-c}\,{}_3\tilde{F}_2\!\left(1, 1, c; c - \mu + 1, c - n - \mu + 1; \frac{1}{z}\right) + \sum_{k=0}^{n-1}\frac{z^{-k}\,\Gamma(k+\mu)\,(n-k-1)!}{k!\,(c-k-\mu-1)!} + (-1)^n\sum_{k=n}^{c-\mu-1}\frac{(-z)^{-k}\,\Gamma(k+\mu)}{k!\,(k-n)!\,(c-k-\mu-1)!}\,(\log(-z) + \psi(k+1) + \psi(k-n+1) - \psi(c-k-\mu) - \psi(k+\mu))\right) /; $$

$|z| > 1 \land b - a \in \mathbb{Z} \land n == |b - a| \land c - a \in \mathbb{Z} \land \mu == a + \min(0, b - a)$

07.23.17.0151.01

$$ {}_2F_1(a, b; c; z) = \frac{(-1)^{m-n-1}\,\Gamma(c)}{\Gamma(a+m-n)\,\Gamma(b+m-n)\,n!}\log(1-z)\,(z-1)^m\,{}_2F_1(a+m, b+m; n+1; 1-z) + \frac{(-1)^{m-n}\,\Gamma(c)}{\Gamma(a+m-n)\,\Gamma(b+m-n)}(z-1)^m\sum_{k=0}^{\infty}\frac{(a+m)_k\,(b+m)_k\,(\psi(k+1) + \psi(k+n+1) - \psi(a+k+m) - \psi(b+k+m))\,(1-z)^k}{k!\,(k+n)!} + \frac{\Gamma(c)\,(1-z)^{m-n}}{\Gamma(a+m)\,\Gamma(b+m)}\sum_{k=0}^{n-1}\frac{(n-1)!\,(a+m-n)_k\,(b+m-n)_k\,(1-z)^k}{k!\,(1-n)_k} /; $$

$c - a - b \in \mathbb{Z} \land |1 - z| < 1 \land n = |c - a - b| \land m = \max(c - a - b, 0)$

Relations including three Kummer's solutions

Below relations hold for $z \in \mathbb{C} \setminus \mathbb{R}$ and all values of parameters a, b, c, for which the gamma factors are finite.

07.23.17.0061.01

$$ {}_2F_1(a, b; c; z) = \frac{\Gamma(a+b-c)\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)}(1-z)^{c-a-b}\,{}_2F_1(c-a, c-b; c-a-b+1; 1-z) + \frac{\Gamma(c-a-b)\,\Gamma(c)}{\Gamma(c-a)\,\Gamma(c-b)}\,{}_2F_1(a, b; a+b-c+1; 1-z) $$

07.23.17.0062.01

$$ {}_2F_1(a, b; c; z) = \frac{\Gamma(b-a)\,\Gamma(c)}{\Gamma(c-a)\,\Gamma(b)}(-z)^{-a}\,{}_2F_1\!\left(a, a-c+1; a-b+1; \frac{1}{z}\right) + \frac{\Gamma(a-b)\,\Gamma(c)}{\Gamma(c-b)\,\Gamma(a)}(-z)^{-b}\,{}_2F_1\!\left(b-c+1, b; -a+b+1; \frac{1}{z}\right) /; $$

$z \notin (0, 1)$

07.23.17.0063.01

$$ {}_2F_1(a, b; c; z) = \frac{\Gamma(1-b)\,\Gamma(a-c+1)}{\Gamma(a-b+1)\,\Gamma(1-c)}(-z)^{-a}\,{}_2F_1\!\left(a, a-c+1; a-b+1; \frac{1}{z}\right) + \frac{\Gamma(1-b)\,\Gamma(a-c+1)\,\Gamma(c)}{\Gamma(a)\,\Gamma(2-c)\,\Gamma(c-b)}(-z)^{1-c}\,{}_2F_1(a-c+1, b-c+1; 2-c; z) /; z \notin (0, 1) $$

07.23.17.0064.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(1-b)\,\Gamma(c)}{\Gamma(a)\,\Gamma(c-a-b+1)} (-z)^{a-c} z^{c-a} (1-z)^{c-a-b} {}_2F_1(c-a, c-b; c-a-b+1; 1-z) + \frac{\Gamma(1-b)\,\Gamma(c)}{\Gamma(a-b+1)\,\Gamma(c-a)} z^{-a} {}_2F_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right) /; z \notin (0, 1)$$

07.23.17.0065.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(a-c+1)\,\Gamma(c)}{\Gamma(a-b+1)\,\Gamma(b)} z^{b-c} (-z)^{c-a-b} {}_2F_1\left(a, a-c+1; a-b+1; \frac{1}{z}\right) + \frac{\Gamma(a-c+1)\,\Gamma(c)}{\Gamma(a+b-c+1)\,\Gamma(c-b)} z^{b} (-z)^{-b} {}_2F_1(a, b; a+b-c+1; 1-z) /; z \notin (0, 1)$$

07.23.17.0066.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(b-c+1)\,\Gamma(c)}{\Gamma(a+b-c+1)\,\Gamma(c-a)} z^{a} (-z)^{-a} {}_2F_1(a, b; a+b-c+1; 1-z) + \frac{\Gamma(b-c+1)\,\Gamma(c)}{\Gamma(a)\,\Gamma(b-a+1)} z^{a-c} (-z)^{c-a-b} {}_2F_1\left(b-c+1, b; b-a+1; \frac{1}{z}\right) /; z \notin (0, 1)$$

07.23.17.0067.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(a-c+1)\,\Gamma(b-c+1)}{\Gamma(1-c)\,\Gamma(a+b-c+1)} {}_2F_1(a, b; a+b-c+1; 1-z) - \frac{\Gamma(a-c+1)\,\Gamma(b-c+1)\,\Gamma(c-1)}{\Gamma(a)\,\Gamma(b)\,\Gamma(1-c)} z^{1-c} {}_2F_1(a-c+1, b-c+1; 2-c; z)$$

07.23.17.0068.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(b-a+1)\,\Gamma(1-c)} (-z)^{-b} {}_2F_1\left(b-c+1, b; b-a+1; \frac{1}{z}\right) + \frac{\Gamma(1-a)\,\Gamma(b-c+1)\,\Gamma(c)}{\Gamma(b)\,\Gamma(2-c)\,\Gamma(c-a)} (-z)^{1-c} {}_2F_1(a-c+1, b-c+1; 2-c; z) /; z \notin (0, 1)$$

07.23.17.0069.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(1-a)\,\Gamma(c)}{\Gamma(b)\,\Gamma(c-a-b+1)} (-z)^{b-c} z^{c-b} (1-z)^{c-a-b} {}_2F_1(c-a, c-b; c-a-b+1; 1-z) + \frac{\Gamma(1-a)\,\Gamma(c)}{\Gamma(b-a+1)\,\Gamma(c-b)} z^{-b} {}_2F_1\left(b-c+1, b; b-a+1; \frac{1}{z}\right) /; z \notin (0, 1)$$

07.23.17.0070.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(1-c)\,\Gamma(c-a-b+1)} (1-z)^{c-a-b} {}_2F_1(c-a, c-b; c-a-b+1; 1-z) - \frac{\Gamma(1-a)\,\Gamma(1-b)\,\Gamma(c-1)}{\Gamma(1-c)\,\Gamma(c-a)\,\Gamma(c-b)} z^{1-c} {}_2F_1(a-c+1, b-c+1; 2-c; z)$$

07.23.17.0071.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(b-a)\,\Gamma(c)}{\Gamma(b)\,\Gamma(c-a)} (1-z)^{-a} {}_2F_1\left(a, c-b; a-b+1; \frac{1}{1-z}\right) + \frac{\Gamma(a-b)\,\Gamma(c)}{\Gamma(a)\,\Gamma(c-b)} (1-z)^{-b} {}_2F_1\left(c-a, b; b-a+1; \frac{1}{1-z}\right)$$

07.23.17.0072.01

$${}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-b)\,\Gamma(c)}{\Gamma(a-b+1)\,\Gamma(c-a)}\,(z-1)^{-a}\,{}_2F_1\!\left(a,\,c-b;\,a-b+1;\,\frac{1}{1-z}\right)+ \frac{\Gamma(1-b)\,\Gamma(c)}{\Gamma(a)\,\Gamma(c-a-b+1)}\,(1-z)^{-b}\,(z-1)^{c-a}\,{}_2F_1(c-b,\,c-a;\,c-a-b+1;\,1-z)\;/;\;z\notin(0,\,1)$$

07.23.17.0073.01

$${}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(c)}{\Gamma(b-a+1)\,\Gamma(c-b)}\,(z-1)^{-b}\,{}_2F_1\!\left(c-a,\,b;\,b-a+1;\,\frac{1}{1-z}\right)+ \frac{\Gamma(1-a)\,\Gamma(c)}{\Gamma(b)\,\Gamma(c-a-b+1)}\,(1-z)^{-a}\,(z-1)^{c-b}\,{}_2F_1(c-b,\,c-a;\,c-a-b+1;\,1-z)\;/;\;z\notin(0,\,1)$$

07.23.17.0074.01

$${}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-b)\,\Gamma(a-c+1)\,\Gamma(c)}{\Gamma(a)\,\Gamma(2-c)\,\Gamma(c-b)}\,(1-z)^{1-c}\,(z-1)^{c-1}\,z^{1-c}\,{}_2F_1(b-c+1,\,a-c+1;\,2-c;\,z)+ \frac{\Gamma(1-b)\,\Gamma(a-c+1)}{\Gamma(a-b+1)\,\Gamma(1-c)}\,(1-z)^{-a}\,{}_2F_1\!\left(a,\,c-b;\,a-b+1;\,\frac{1}{1-z}\right)\;/;\;z\notin(0,\,1)$$

07.23.17.0075.01

$${}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(b-a+1)\,\Gamma(1-c)}\,(1-z)^{-b}\,{}_2F_1\!\left(c-a,\,b;\,b-a+1;\,\frac{1}{1-z}\right)- \frac{\Gamma(1-a)\,\Gamma(b-c+1)\,\Gamma(c)}{\Gamma(b)\,\Gamma(2-c)\,\Gamma(c-a)}\,(z-1)^{c}\,(1-z)^{-c}\,z^{1-c}\,{}_2F_1(b-c+1,\,a-c+1;\,2-c;\,z)\;/;\;z\notin(0,\,1)$$

07.23.17.0076.01

$${}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(1-c)\,\Gamma(c-a-b+1)}\,(1-z)^{c-a-b}\,{}_2F_1(c-b,\,c-a;\,c-a-b+1;\,1-z)+ \frac{\Gamma(1-a)\,\Gamma(1-b)\,\Gamma(c)}{\Gamma(2-c)\,\Gamma(c-a)\,\Gamma(c-b)}\,z^{1-c}\,{}_2F_1(b-c+1,\,a-c+1;\,2-c;\,z)$$

07.23.17.0077.01

$${}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(b-c+1)}{\Gamma(b-a+1)\,\Gamma(1-c)}\left(-\frac{1}{z}\right)^{b}{}_2F_1\!\left(b,\,b-c+1;\,b-a+1;\,\frac{1}{z}\right)+ \frac{\Gamma(1-a)\,\Gamma(b-c+1)\,\Gamma(c)}{\Gamma(b)\,\Gamma(2-c)\,\Gamma(c-a)}\left(-\frac{1}{z}\right)^{c-1}{}_2F_1(b-c+1,\,a-c+1;\,2-c;\,z)\;/;\;z\notin(1,\,\infty)$$

07.23.17.0078.01

$${}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(1-b)}{\Gamma(1-c)\,\Gamma(c-a-b+1)}\left(1-\frac{1}{z}\right)^{c-a-b}\left(-\frac{1}{z}\right)^{a+b-c}\left(\frac{1}{z}\right)^{c-b}{}_2F_1\!\left(1-b,\,c-b;\,c-a-b+1;\,1-\frac{1}{z}\right)+ \frac{\Gamma(1-a)\,\Gamma(1-b)\,\Gamma(c)}{\Gamma(2-c)\,\Gamma(c-a)\,\Gamma(c-b)}\left(\frac{1}{z}\right)^{c-1}{}_2F_1(b-c+1,\,a-c+1;\,2-c;\,z)\;/;\;z\notin(1,\,\infty)$$

07.23.17.0079.01

$${}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(1-a)\,\Gamma(c)}{\Gamma(b)\,\Gamma(c-a-b+1)}\left(-\frac{1}{z}\right)^{a}\left(1-\frac{1}{z}\right)^{c-a-b}{}_2F_1\!\left(1-b,\,c-b;\,-a-b+c+1;\,1-\frac{1}{z}\right)+ \frac{\Gamma(1-a)\,\Gamma(c)}{\Gamma(b-a+1)\,\Gamma(c-b)}\left(\frac{1}{z}\right)^{b}{}_2F_1\!\left(b,\,b-c+1;\,b-a+1;\,\frac{1}{z}\right)\;/;\;z\notin(1,\,\infty)$$

07.23.17.0080.01

$$ {}_2F_1(a, b; c; z) = \frac{\Gamma(1-b)\,\Gamma(c)}{\Gamma(a-b+1)\,\Gamma(c-a)} \left(\frac{1}{z}\right)^a {}_2F_1\left(a-c+1, a; a-b+1; \frac{1}{z}\right) + \frac{\Gamma(1-b)\,\Gamma(c)}{\Gamma(a)\,\Gamma(c-a-b+1)} \left(1-\frac{1}{z}\right)^{c-a-b} \left(-\frac{1}{z}\right)^b {}_2F_1\left(c-a, 1-a; c-a-b+1; 1-\frac{1}{z}\right) /; z \notin (1, \infty) $$

Quadratic transformations with fixed *a*, *b*, *z*

07.23.17.0081.01

$$ {}_2F_1\left(a, b; a+b-\frac{1}{2}; z\right) = \frac{1}{\sqrt{1-z}}\, {}_2F_1\left(2a-1, 2b-1; a+b-\frac{1}{2}; \frac{1}{2}\left(1-\sqrt{1-z}\right)\right) $$

07.23.17.0082.01

$$ {}_2F_1\left(a, b; a+b-\frac{1}{2}; z\right) = \frac{1}{\sqrt{1-z}} \left(\frac{\sqrt{1-z}+1}{2}\right)^{1-2a} {}_2F_1\left(2a-1, a-b+\frac{1}{2}; a+b-\frac{1}{2}; \frac{\sqrt{1-z}-1}{\sqrt{1-z}+1}\right) $$

07.23.17.0083.01

$$ {}_2F_1\left(a, b; a+b-\frac{1}{2}; z\right) = \frac{1}{\sqrt{1-z}} \left(\sqrt{1-z}+\sqrt{-z}\right)^{1-2a} {}_2F_1\left(2a-1, a+b-1; 2a+2b-2; 2z+2\sqrt{z^2-z}\right) /; \operatorname{Re}(z) < \frac{1}{2} $$

07.23.17.0084.01

$$ {}_2F_1\left(a, b; a+b-\frac{1}{2}; z\right) = \frac{1}{\sqrt{1-z}} \left(\sqrt{1-z}-\sqrt{-z}\right)^{1-2a} {}_2F_1\left(2a-1, a+b-1; 2a+2b-2; 2z+2\sqrt{z^2-z}\right) /; \operatorname{Re}(z) > \frac{1}{2} $$

07.23.17.0085.01

$$ {}_2F_1\left(a, b; a+b+\frac{1}{2}; z\right) = {}_2F_1\left(2a, 2b; a+b+\frac{1}{2}; \frac{1-\sqrt{1-z}}{2}\right) $$

07.23.17.0086.01

$$ {}_2F_1\left(a, b; a+b+\frac{1}{2}; z\right) = \left(\frac{\sqrt{1-z}+1}{2}\right)^{-2a} {}_2F_1\left(2a, a-b+\frac{1}{2}; a+b+\frac{1}{2}; \frac{\sqrt{1-z}-1}{\sqrt{1-z}+1}\right) $$

07.23.17.0087.01

$$ {}_2F_1\left(a, b; a+b+\frac{1}{2}; z\right) = \left(\sqrt{1-z}+\sqrt{-z}\right)^{-2a} {}_2F_1\left(2a, a+b; 2a+2b; 2z+2\sqrt{z^2-z}\right) /; \operatorname{Re}(z) < \frac{1}{2} $$

07.23.17.0088.01

$$ {}_2F_1\left(a, b; a+b+\frac{1}{2}; z\right) = \left(\sqrt{1-z}-\sqrt{-z}\right)^{-2a} {}_2F_1\left(2a, a+b; 2a+2b; 2z+2\sqrt{z^2-z}\right) /; \operatorname{Re}(z) > \frac{1}{2} $$

07.23.17.0089.01

$$ {}_2F_1\left(a, b; a+b+\frac{1}{2}; 4z(1-z)\right) = {}_2F_1\left(2a, 2b; a+b+\frac{1}{2}; z\right) /; \operatorname{Re}(z) < \frac{1}{2} $$

07.23.17.0090.01

$$ {}_2F_1(a, b; a-b+1; z) = (1-z)^{-a}\, {}_2F_1\left(\frac{a}{2}, \frac{a+1}{2}-b; a-b+1; -\frac{4z}{(1-z)^2}\right) /; |z| < 1 $$

07.23.17.0091.01

$$ {}_2F_1(a, b; a-b+1; z) = \frac{z+1}{(1-z)^{a+1}}\, {}_2F_1\left(\frac{a+1}{2}, \frac{a}{2}-b+1; a-b+1; -\frac{4z}{(1-z)^2}\right) /; |z| < 1 $$

07.23.17.0092.01

$$_2F_1(a, b; a-b+1; z) = (1+z)^{-a} \, _2F_1\left(\frac{a}{2}, \frac{a+1}{2}; a-b+1; \frac{4z}{(z+1)^2}\right) /; |z| < 1$$

07.23.17.0093.01

$$_2F_1(a, b; a-b+1; z) = (1-z)^{1-2b} (1+z)^{2b-a-1} \, _2F_1\left(\frac{a+1}{2} - b, \frac{a}{2} - b + 1; a-b+1; \frac{4z}{(z+1)^2}\right) /; |z| < 1$$

07.23.17.0094.01

$$_2F_1(a, b; a-b+1; z) = \left(1+\sqrt{z}\right)^{-2a} \, _2F_1\left(a, a-b+\frac{1}{2}; 2a-2b+1; \frac{4\sqrt{z}}{\left(1+\sqrt{z}\right)^2}\right) /; |z| < 1$$

07.23.17.0095.01

$$_2F_1(a, b; a-b+1; z) = \left(1-\sqrt{z}\right)^{-2a} \, _2F_1\left(a, a-b+\frac{1}{2}; 2a-2b+1; -\frac{4\sqrt{z}}{\left(1-\sqrt{z}\right)^2}\right) /; |z| < 1$$

07.23.17.0096.01

$$_2F_1\left(a, b; \frac{a+b+1}{2}; z\right) = \, _2F_1\left(\frac{a}{2}, \frac{b}{2}; \frac{a+b+1}{2}; 4z(1-z)\right) /; \mathrm{Re}(z) < \frac{1}{2}$$

07.23.17.0097.01

$$_2F_1\left(a, b; \frac{a+b+1}{2}; z\right) = (1-2z) \, _2F_1\left(\frac{a+1}{2}, \frac{b+1}{2}; \frac{a+b+1}{2}; 4z(1-z)\right) /; \mathrm{Re}(z) < \frac{1}{2}$$

07.23.17.0098.01

$$_2F_1\left(a, b; \frac{a+b+1}{2}; z\right) = (1-2z)^{-a} \, _2F_1\left(\frac{a}{2}, \frac{a+1}{2}; \frac{a+b+1}{2}; \frac{4z(z-1)}{(2z-1)^2}\right) /; \mathrm{Re}(z) < \frac{1}{2}$$

07.23.17.0099.01

$$_2F_1\left(a, b; \frac{a+b+1}{2}; z\right) = \left(\sqrt{1-z} + \sqrt{-z}\right)^{-2a} \, _2F_1\left(a, \frac{a+b}{2}; a+b; \frac{4\sqrt{z^2-z}}{\left(\sqrt{1-z} + \sqrt{-z}\right)^2}\right) /; \mathrm{Re}(z) < \frac{1}{2}$$

07.23.17.0100.01

$$_2F_1\left(a, b; \frac{a+b+1}{2}; z\right) = \frac{\sqrt{\pi}\, \Gamma\left(\frac{a+b+1}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b+1}{2}\right)} \, _2F_1\left(\frac{a}{2}, \frac{b}{2}; \frac{1}{2}; (2z-1)^2\right) + \frac{2\sqrt{\pi}\,(2z-1)\,\Gamma\left(\frac{a+b+1}{2}\right)}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{b}{2}\right)} \, _2F_1\left(\frac{a+1}{2}, \frac{b+1}{2}; \frac{3}{2}; (2z-1)^2\right)$$

07.23.17.0101.01

$$_2F_1(a, b; 2b; z) = (1-z)^{-\frac{a}{2}} \, _2F_1\left(\frac{a}{2}, b-\frac{a}{2}; b+\frac{1}{2}; \frac{z^2}{4(z-1)}\right)$$

07.23.17.0102.01

$$_2F_1(a, b; 2b; z) = \left(1-\frac{z}{2}\right)(1-z)^{-\frac{a+1}{2}} \, _2F_1\left(\frac{1-a}{2} + b, \frac{a+1}{2}; b+\frac{1}{2}; \frac{z^2}{4(z-1)}\right)$$

07.23.17.0103.01

$$_2F_1(a, b; 2b; z) = \left(1-\frac{z}{2}\right)^{-a} \, _2F_1\left(\frac{a}{2}, \frac{a+1}{2}; b+\frac{1}{2}; \frac{z^2}{(2-z)^2}\right) /; z \notin (2, \infty)$$

07.23.17.0104.01

$$ {}_2F_1(a, b; 2b; z) = (1-z)^{b-a}\left(1-\frac{z}{2}\right)^{a-2b} {}_2F_1\left(b-\frac{a}{2}, \frac{1-a}{2}+b; b+\frac{1}{2}; \frac{z^2}{(2-z)^2}\right) /; z \notin (2, \infty) $$

07.23.17.0105.01

$$ {}_2F_1(a, b; 2b; z) = (1-z)^{-\frac{a}{2}} {}_2F_1\left(a, 2b-a; b+\frac{1}{2}; -\frac{\left(1-\sqrt{1-z}\right)^2}{4\sqrt{1-z}}\right) $$

07.23.17.0106.01

$$ {}_2F_1(a, b; 2b; z) = \left(\frac{\sqrt{1-z}+1}{2}\right)^{-2a} {}_2F_1\left(a, a-b+\frac{1}{2}; b+\frac{1}{2}; \left(\frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right)^2\right) $$

07.23.17.0107.01

$$ {}_2F_1\left(a, b; \frac{1}{2}; z\right) = \frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)}{2\sqrt{\pi}\,\Gamma\left(a+b+\frac{1}{2}\right)}\left({}_2F_1\left(2a, 2b; a+b+\frac{1}{2}; \frac{1-\sqrt{z}}{2}\right) + {}_2F_1\left(2a, 2b; a+b+\frac{1}{2}; \frac{1+\sqrt{z}}{2}\right)\right) $$

07.23.17.0108.01

$$ {}_2F_1\left(a, b; \frac{1}{2}; z\right) = \frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma(1-b)}{2\sqrt{\pi}\,\Gamma(a-b+1)}(1-z)^{-a} $$
$$ \left({}_2F_1\left(2a, 1-2b; a-b+1; \frac{\sqrt{1-z}-\sqrt{-z}}{2\sqrt{1-z}}\right) + {}_2F_1\left(2a, 1-2b; a-b+1; \frac{\sqrt{1-z}+\sqrt{-z}}{2\sqrt{1-z}}\right)\right) /; z \notin (1, \infty) $$

07.23.17.0109.01

$$ {}_2F_1\left(a, b; \frac{3}{2}; z\right) = $$
$$ \frac{\Gamma\left(a-\frac{1}{2}\right)\Gamma\left(b-\frac{1}{2}\right)}{4\sqrt{\pi}\,\Gamma\left(a+b-\frac{1}{2}\right)} z^{-\frac{1}{2}}\left({}_2F_1\left(2a-1, 2b-1; a+b-\frac{1}{2}; \frac{1+\sqrt{z}}{2}\right) - {}_2F_1\left(2a-1, 2b-1; a+b-\frac{1}{2}; \frac{1-\sqrt{z}}{2}\right)\right) $$

Quadratic transformations with fixed *a*, *c*, *z*

07.23.17.0110.01

$$ {}_2F_1(a, 1-a; c; z) = (1-z)^{c-1} {}_2F_1\left(\frac{c-a}{2}, \frac{a+c-1}{2}; c; 4z(1-z)\right) /; \mathrm{Re}(z) < \frac{1}{2} $$

07.23.17.0111.01

$$ {}_2F_1(a, 1-a; c; z) = (1-z)^{c-1}(1-2z) {}_2F_1\left(\frac{a+c}{2}, \frac{1+c-a}{2}; c; 4z(1-z)\right) /; \mathrm{Re}(z) < \frac{1}{2} $$

07.23.17.0112.01

$$ {}_2F_1(a, 1-a; c; z) = (1-z)^{c-1}(1-2z)^{a-c} {}_2F_1\left(\frac{c-a}{2}, \frac{1+c-a}{2}; c; \frac{4z(z-1)}{(1-2z)^2}\right) /; \mathrm{Re}(z) < \frac{1}{2} $$

07.23.17.0113.01

$$ {}_2F_1(a, 1-a; c; z) = (1-z)^{c-1}\left(\sqrt{1-z}+\sqrt{-z}\right)^{2-2a-2c} {}_2F_1\left(a+c-1, c-\frac{1}{2}; 2c-1; \frac{4\sqrt{z(z-1)}}{\left(\sqrt{1-z}+\sqrt{-z}\right)^2}\right) /; \mathrm{Re}(z) < \frac{1}{2} $$

07.23.17.0114.01

$${}_2F_1\left(a, a+\frac{1}{2}; c; z\right) = (1-z)^{-a}\, {}_2F_1\left(2a, 2c-2a-1; c; \frac{\sqrt{1-z}-1}{2\sqrt{1-z}}\right)$$

07.23.17.0115.01

$${}_2F_1\left(a, a+\frac{1}{2}; c; z\right) = \left(\frac{\sqrt{1-z}+1}{2}\right)^{-2a} {}_2F_1\left(2a, 2a-c+1; c; \frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right)$$

07.23.17.0116.01

$${}_2F_1\left(a, a+\frac{1}{2}; c; z\right) = \left(1+\sqrt{z}\right)^{-2a} {}_2F_1\left(2a, c-\frac{1}{2}; 2c-1; \frac{2\sqrt{z}}{1+\sqrt{z}}\right)$$

07.23.17.0117.01

$${}_2F_1\left(a, a+\frac{1}{2}; c; z\right) = \left(1-\sqrt{z}\right)^{-2a} {}_2F_1\left(2a, c-\frac{1}{2}; 2c-1; -\frac{2\sqrt{z}}{1-\sqrt{z}}\right) /; z \notin (1, \infty)$$

Cubic transformations

07.23.17.0118.01

$${}_2F_1\left(a, a+\frac{1}{2}; \frac{4a+2}{3}; z\right) = \left(1-\frac{9z}{8}\right)^{-\frac{2a}{3}} {}_2F_1\left(\frac{a}{3}, \frac{a}{3}+\frac{1}{2}; \frac{4a+5}{6}; -\frac{27z^2(1-z)}{(8-9z)^2}\right) /; |z-1| > \frac{1}{3}$$

07.23.17.0119.01

$${}_2F_1\left(a, a-\frac{1}{2}; \frac{4a}{3}; z\right) = \left(1-\frac{9z}{8}\right)^{\frac{1-2a}{3}} {}_2F_1\left(\frac{2a-1}{6}, \frac{a+1}{3}; \frac{4a+3}{6}; \frac{27(z-1)z^2}{(8-9z)^2}\right) /; |z-1| > \frac{1}{3}$$

07.23.17.0120.01

$${}_2F_1\left(a, a+\frac{1}{2}; \frac{4a+5}{6}; z\right) = (1-9z)^{-\frac{2a}{3}} {}_2F_1\left(\frac{a}{3}, \frac{a}{3}+\frac{1}{2}; \frac{4a+5}{6}; -\frac{27z(1-z)^2}{(1-9z)^2}\right) /; |z| < \frac{1}{9}$$

07.23.17.0121.01

$${}_2F_1\left(a, a-\frac{1}{2}; \frac{4a+3}{6}; z\right) = (1-9z)^{\frac{1-2a}{3}} {}_2F_1\left(\frac{2a-1}{6}, \frac{a+1}{3}; \frac{4a+3}{6}; -\frac{27(z-1)^2 z}{(1-9z)^2}\right) /; |z| < \frac{1}{9}$$

07.23.17.0122.01

$${}_2F_1\left(a, \frac{2a+1}{6}; \frac{4a+2}{3}; z\right) = \left(1-\frac{z}{4}\right)^{-a} {}_2F_1\left(\frac{a}{3}, \frac{a+1}{3}; \frac{4a+5}{6}; -\frac{27z^2}{(z-4)^3}\right) /; z \notin (4, \infty)$$

07.23.17.0123.01

$${}_2F_1\left(a, 3a-\frac{1}{2}; 4a; z\right) = \left(1-\frac{z}{4}\right)^{\frac{1}{2}-3a} {}_2F_1\left(a-\frac{1}{6}, a+\frac{1}{6}; 2a+\frac{1}{2}; -\frac{27z^2}{(z-4)^3}\right) /; z \notin (4, \infty)$$

07.23.17.0124.01

$${}_2F_1\left(a, \frac{2a+1}{6}; \frac{4a+2}{3}; z\right) = \left(1-\frac{z}{4}\right)^{-a} {}_2F_1\left(\frac{a}{3}, \frac{a+1}{3}; \frac{4a+5}{6}; -\frac{27z^2}{(z-4)^3}\right) /; |z| < \frac{1}{8}$$

07.23.17.0125.01

$${}_2F_1\left(a, 1-3a; \frac{3}{2}-2a; z\right) = (1-4z)^{3a-1} {}_2F_1\left(\frac{1}{3}-a, \frac{2}{3}-a; \frac{3}{2}-2a; \frac{27z}{(4z-1)^3}\right) /; |z| < \frac{1}{8}$$

07.23.17.0126.01

$$ {}_2F_1\left(a, \frac{2a+1}{6}; \frac{1}{2}; z\right) = (1-z)^{-\frac{2a}{3}} \, {}_2F_1\left(\frac{a}{3}, \frac{1-2a}{6}; \frac{1}{2}; -\frac{z(z-9)^2}{27(1-z)^2}\right) /; |z| < 1 $$

07.23.17.0127.01

$$ {}_2F_1\left(a, 3a-\frac{1}{2}; \frac{1}{2}; z\right) = (1-z)^{\frac{1}{3}-2a} \, {}_2F_1\left(a-\frac{1}{6}, \frac{1}{3}-a; \frac{1}{2}; -\frac{(z-9)^2 z}{27(z-1)^2}\right) /; |z| < 1 $$

07.23.17.0128.01

$$ {}_2F_1\left(a, \frac{1-a}{3}; \frac{1}{2}; z\right) = (1-z)^{-\frac{a}{3}} \, {}_2F_1\left(\frac{a}{3}, \frac{1-2a}{6}; \frac{1}{2}; \frac{z(9-8z)^2}{27(1-z)}\right) /; \operatorname{Re}(z) < \frac{5}{8} \bigvee |z| < \frac{3}{4} $$

07.23.17.0129.01

$$ {}_2F_1\left(a, 1-3a; \frac{1}{2}; z\right) = (1-z)^{a-\frac{1}{3}} \, {}_2F_1\left(\frac{1}{3}-a, a-\frac{1}{6}; \frac{1}{2}; \frac{(9-8z)^2 z}{27(1-z)}\right) /; \operatorname{Re}(z) < \frac{5}{8} \bigvee |z| < \frac{3}{4} $$

07.23.17.0130.01

$$ {}_2F_1\left(a, \frac{2a+3}{6}; \frac{3}{2}; z\right) = \left(1-\frac{z}{9}\right)\left(1+\frac{z}{3}\right)^{-a-1} \, {}_2F_1\left(\frac{a+1}{3}, \frac{a+2}{3}; \frac{3}{2}; \frac{z(z-9)^2}{(z+3)^3}\right) /; |z| < 1 $$

07.23.17.0131.01

$$ {}_2F_1\left(a, 3a-\frac{3}{2}; \frac{3}{2}; z\right) = \left(1-\frac{z}{9}\right)\left(\frac{z}{3}+1\right)^{\frac{1}{2}-3a} \, {}_2F_1\left(a-\frac{1}{6}, a+\frac{1}{6}; \frac{3}{2}; \frac{z(z-9)^2}{(z+3)^3}\right) /; \operatorname{Abs}[z] < 1 $$

07.23.17.0132.01

$$ {}_2F_1\left(a, 1-\frac{a}{3}; \frac{3}{2}; z\right) = \left(1-\frac{4z}{3}\right)^{-a-1} \left(1-\frac{8z}{9}\right) {}_2F_1\left(\frac{a+1}{3}, \frac{a+2}{3}; \frac{3}{2}; \frac{z(9-8z)^2}{(4z-3)^3}\right) /; |z| < \frac{3}{4} \bigvee \operatorname{Re}(z) < \frac{5}{8} $$

07.23.17.0133.01

$$ {}_2F_1\left(a, 3-3a; \frac{3}{2}; z\right) = \left(1-\frac{8z}{9}\right)\left(1-\frac{4z}{3}\right)^{3a-4} \, {}_2F_1\left(\frac{4}{3}-a, \frac{5}{3}-a; \frac{3}{2}; \frac{z(9-8z)^2}{(4z-3)^3}\right) /; \operatorname{Re}(z) < \frac{3}{4} \bigvee |z| < \frac{5}{8} $$

Differentiation

Low-order differentiation

With respect to *a*

07.23.20.0001.01

$$ {}_2F_1^{(1,0,0,0)}(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k \psi(a+k) z^k}{(c)_k \, k!} - \psi(a) \, {}_2F_1(a, b; c; z) /; |z| < 1 $$

07.23.20.0002.01

$$ {}_2F_1^{(1,0,0,0)}(a, b; c; z) = \frac{z b}{c} F_{2\times0\times1}^{2\times1\times2}\left(\begin{matrix} a+1, b+1; 1; 1, a; \\ 2, c+1;; a+1; \end{matrix} z, z\right) $$

With respect to *b*

07.23.20.0003.01

$$ {}_2F_1^{(0,1,0,0)}(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k \psi(b+k) z^k}{(c)_k \, k!} - \psi(b) \, {}_2F_1(a, b; c; z) /; |z| < 1 $$

07.23.20.0004.01

$$ {}_2F_1^{(0,1,0,0)}(a,\,b;\,c;\,z) \equiv \frac{z\,a}{c}\,F_{2\times0\times1}^{2\times1\times2}\left(\begin{matrix} a+1,\,b+1;\,1;\,1,\,b; \\ 2,\,c+1;;\,b+1; \end{matrix}\,z,\,z\right) $$

07.23.20.0051.01

$$ \text{Hypergeometric2F1}^{(0,1,0,0)}(1,\,c,\,c,\,z) = \frac{z}{1-z}\,\hat{\Phi}(z,\,1,\,c) $$

Brychkov Yu.A. (2006)

With respect to *c*

07.23.20.0005.01

$$ {}_2F_1^{(0,0,1,0)}(a,\,b;\,c;\,z) \equiv \psi(c)\,{}_2F_1(a,\,b;\,c;\,z) - \sum_{k=0}^{\infty}\frac{(a)_k\,(b)_k\,\psi(c+k)\,z^k}{(c)_k\,k!}\,/;\,|z|<1 $$

07.23.20.0006.01

$$ {}_2F_1^{(0,0,1,0)}(a,\,b;\,c;\,z) \equiv -\frac{z\,a\,b}{c^2}\,F_{2\times0\times1}^{2\times1\times2}\left(\begin{matrix} a+1,\,b+1;\,1;\,1,\,c; \\ 2,\,c+1;;\,c+1; \end{matrix}\,z,\,z\right) $$

With respect to element of parameters ||| With respect to element of parameters

07.23.20.0007.01

$$ \frac{\partial\,{}_2F_1(a,\,b;\,a+1;\,z)}{\partial a} \equiv \frac{b\,z}{(a+1)^2}\,{}_3F_2(a+1,\,a+1,\,b+1;\,a+2,\,a+2;\,z) $$

07.23.20.0008.01

$$ \frac{\partial\,{}_2F_1(a+1,\,b;\,a;\,z)}{\partial a} \equiv -\frac{b\,z}{a^2}\,(1-z)^{-b-1} $$

07.23.20.0009.01

$$ \frac{\partial\,{}_2F_1(1,\,b;\,b+c;\,z)}{\partial b} \equiv \frac{c\,z\,(1-z)^{c-1}}{(b+c)^2}\,{}_3F_2(b+c,\,b+c,\,c+1;\,b+c+1,\,b+c+1;\,z) $$

With respect to *z*

07.23.20.0010.01

$$ \frac{\partial\,{}_2F_1(a,\,b;\,c;\,z)}{\partial z} \equiv \frac{a\,b}{c}\,{}_2F_1(a+1,\,b+1;\,c+1;\,z) $$

07.23.20.0011.01

$$ \frac{\partial^2\,{}_2F_1(a,\,b;\,c;\,z)}{\partial z^2} \equiv \frac{a\,(a+1)\,b\,(b+1)}{c\,(c+1)}\,{}_2F_1(a+2,\,b+2;\,c+2;\,z) $$

Symbolic differentiation

With respect to *a*

07.23.20.0012.02

$$ {}_2F_1^{(n,0,0,0)}(a,\,b;\,c;\,z) \equiv \sum_{k=0}^{\infty}\frac{(b)_k}{(c)_k\,k!}\,\frac{\partial^n\,(a)_k}{\partial a^n}\,z^k\,/;\,|z|<1\land n\in\mathbb{N} $$

With respect to *b*

07.23.20.0013.02

$${}_2F_1^{(0,n,0,0)}(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k}{(c)_k\, k!} \frac{\partial^n (b)_k}{\partial b^n} z^k \;/;\; |z| < 1 \wedge n \in \mathbb{N}$$

With respect to c

07.23.20.0014.02

$${}_2F_1^{(0,0,n,0)}(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k\, (b)_k}{k!} \frac{\partial^n \frac{1}{(c)_k}}{\partial c^n} z^k \;/;\; |z| < 1 \wedge n \in \mathbb{N}$$

With respect to element of parameters ||| With respect to element of parameters

07.23.20.0049.02

$$\frac{\partial^n {}_2F_1(a, b; a+1; z)}{\partial a^n} = \frac{(-1)^{n-1}\, n!\, b\, z}{(a+1)^{n+1}} \, {}_{n+2}F_{n+1}(a+1, \ldots, a+1, b+1; a+2, \ldots, a+2; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0050.02

$$\frac{\partial^n {}_2F_1(a+1, b; a; z)}{\partial a^n} = (-1)^n\, a^{-n-1}\, b\, (1-z)^{-b-1}\, z\, n! \;/;\; n \in \mathbb{N}$$

With respect to z

07.23.20.0015.02

$$\frac{\partial^n {}_2F_1(a, b; c; z)}{\partial z^n} = \frac{(a)_n\, (b)_n}{(c)_n} \, {}_2F_1(a+n, b+n; c+n; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0016.02

$$\frac{\partial^n {}_2F_1(a, b; c; z)}{\partial z^n} = z^{-n}\, \Gamma(c)\, {}_3\tilde{F}_2(1, a, b; 1-n, c; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0017.02

$$\frac{\partial^n (z^{\alpha}\, {}_2F_1(a, b; c; z))}{\partial z^n} = (-1)^n\, (-\alpha)_n\, z^{\alpha-n}\, {}_3F_2(\alpha+1, a, b; 1-n+\alpha, c; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0018.02

$$\frac{\partial^n \left(z^{a+n-1}\, {}_2F_1(a, b; c; z)\right)}{\partial z^n} = (a)_n\, z^{a-1}\, {}_2F_1(a+n, b; c; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0019.02

$$\frac{\partial^n \left(z^{c-1}\, {}_2F_1(a, b; c; z)\right)}{\partial z^n} = (c-n)_n\, z^{c-n-1}\, {}_2F_1(a, b; c-n; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0020.02

$$\frac{\partial^n \left((1-z)^{a+n-1}\, {}_2F_1(a, b; c; z)\right)}{\partial z^n} = \frac{(-1)^n\, (a)_n\, (c-b)_n}{(c)_n} (1-z)^{a-1}\, {}_2F_1(a+n, b; c+n; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0021.02

$$\frac{\partial^n \left((1-z)^{a+b-c}\, {}_2F_1(a, b; c; z)\right)}{\partial z^n} = \frac{(c-a)_n\, (c-b)_n}{(c)_n} (1-z)^{a+b-c-n}\, {}_2F_1(a, b; c+n; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0022.02

$$\frac{\partial^n \left(z^{c-1}\, (1-z)^{b-c+n}\, {}_2F_1(a, b; c; z)\right)}{\partial z^n} = (-1)^n\, (1-c)_n\, z^{c-n-1}\, (1-z)^{b-c}\, {}_2F_1(a-n, b; c-n; z) \;/;\; n \in \mathbb{N}$$

07.23.20.0023.02

$$\frac{\partial^n\left(z^{c-1}\,(1-z)^{a+b-c}\,{}_2F_1(a,\,b;\,c;\,z)\right)}{\partial z^n} = (-1)^n\,(1-c)_n\,z^{c-n-1}\,(1-z)^{a+b-c-n}\,{}_2F_1(a-n,\,b-n;\,c-n;\,z)\,/;\,n\in\mathbb{N}$$

07.23.20.0024.02

$$\frac{\partial^n\left(z^{c-a+n-1}\,(1-z)^{a+b-c}\,{}_2F_1(a,\,b;\,c;\,z)\right)}{\partial z^n} = (c-a)_n\,z^{c-a-1}\,(1-z)^{a+b-c-n}\,{}_2F_1(a-n,\,b;\,c;\,z)\,/;\,n\in\mathbb{N}$$

07.23.20.0025.02

$$\frac{\partial^n\left(z^n\,{}_2F_1\left(-n,\,b;\,\frac{1}{2};\,z\right)\right)}{\partial z^n} = n!\,{}_3F_2\left(-n,\,n+1,\,b;\,\frac{1}{2},\,1;\,z\right)/;\,n\in\mathbb{N}$$

07.23.20.0026.02

$$\frac{\partial^n\left(z^\alpha\,{}_2F_1(-n,\,b;\,c;\,z)\right)}{\partial z^n} = (-1)^n\,(-\alpha)_n\,z^{\alpha-n}\,{}_3F_2(-n,\,\alpha+1,\,b;\,1-n+\alpha,\,c;\,z)\,/;\,n\in\mathbb{N}$$

07.23.20.0027.02

$$\frac{\partial^n\left(z^{-a}\,{}_2F_1\left(a,\,b;\,c;\,\frac{1}{z}\right)\right)}{\partial z^n} = (-1)^n\,(a)_n\,z^{-a-n}\,{}_2F_1\left(a+n,\,b;\,c;\,\frac{1}{z}\right)/;\,n\in\mathbb{N}$$

07.23.20.0028.02

$$\frac{\partial^n\left(z^{-a}\,(z-1)^{a+b-c}\,{}_2F_1\left(a,\,b;\,c;\,\frac{1}{z}\right)\right)}{\partial z^n} = (-1)^n\,(c-b)_n\,z^{-a}\,(z-1)^{a+b-c-n}\,{}_2F_1\left(a,\,b-n;\,c;\,\frac{1}{z}\right)/;\,n\in\mathbb{N}$$

07.23.20.0029.02

$$\frac{\partial^n\left(z^{-a}\,(z-1)^{a+n-1}\,{}_2F_1\left(a,\,b;\,c;\,\frac{1}{z}\right)\right)}{\partial z^n} = \frac{(a)_n\,(c-b)_n}{(c)_n}\,z^{-a-n}\,(z-1)^{a-1}\,{}_2F_1\left(a+n,\,b;\,c+n;\,\frac{1}{z}\right)/;\,n\in\mathbb{N}$$

07.23.20.0030.02

$$\frac{\partial^n\left(z^{-a}\,(z-1)^{a-c+n}\,{}_2F_1\left(a,\,b;\,c;\,\frac{1}{z}\right)\right)}{\partial z^n} = (1-c)_n\,z^{-a}\,(z-1)^{a-c}\,{}_2F_1\left(a,\,b-n;\,c-n;\,\frac{1}{z}\right)/;\,n\in\mathbb{N}$$

07.23.20.0031.02

$$\frac{\partial^n\,{}_2F_1\left(\frac{1}{2}-\left\lfloor\frac{n}{2}\right\rfloor,\,b;\,c;\,z^2\right)}{\partial z^n} =$$

$$\frac{(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\,2^{2\left\lfloor\frac{n}{2}\right\rfloor}}{(c)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}\left(\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}\left(n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}(b)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\,z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2F_1\left(n-\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2},\,b+n-\left\lfloor\frac{n}{2}\right\rfloor;\,c+n-\left\lfloor\frac{n}{2}\right\rfloor;\,z^2\right)/;\,n\in\mathbb{N}$$

07.23.20.0032.02

$$\frac{\partial^n\left(z\,{}_2F_1\left(\left\lfloor\frac{n}{2}\right\rfloor+\frac{3}{2}-n,\,b;\,c;\,z^2\right)\right)}{\partial z^n} =$$

$$\frac{(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\,2^{2\left\lfloor\frac{n}{2}\right\rfloor}}{(c)_{\left\lfloor\frac{n}{2}\right\rfloor}}\left(\frac{3}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}\left(n-2\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}(b)_{\left\lfloor\frac{n}{2}\right\rfloor}\,z^{1-n+2\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2F_1\left(\left\lfloor\frac{n}{2}\right\rfloor+\frac{3}{2},\,b+\left\lfloor\frac{n}{2}\right\rfloor;\,c+\left\lfloor\frac{n}{2}\right\rfloor;\,z^2\right)/;\,n\in\mathbb{N}$$

07.23.20.0033.02

$$\frac{\partial^n\left(z^{2c-1}\,{}_2F_1\left(c-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2},\,b;\,c;\,z^2\right)\right)}{\partial z^n} = (-1)^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\,(1-2c)_n\,z^{2c-n-1}\,{}_2F_1\left(c+\frac{1}{2},\,b;\,c-\left\lfloor\frac{n}{2}\right\rfloor;\,z^2\right)/;\,n\in\mathbb{N}$$

07.23.20.0034.02

$$\frac{\partial^n\left(z^{2c-2}\,{}_2F_1\left(c-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2},b;c;z^2\right)\right)}{\partial z^n}=(-1)^{n-2\left\lfloor\frac{n}{2}\right\rfloor}(2-2c)_n\,z^{2c-n-2}\,{}_2F_1\left(c-\frac{1}{2},b;c-n+\left\lfloor\frac{n}{2}\right\rfloor;z^2\right)/;n\in\mathbb{N}$$

07.23.20.0035.02

$$\frac{\partial^n\left(\left(1-z^2\right)^{b+\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}}\,{}_2F_1\left(c+\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2},b;c;z^2\right)\right)}{\partial z^n}==$$

$$\frac{(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}2^{2\left\lfloor\frac{n}{2}\right\rfloor}}{(c)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}\left(\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}\left(n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}(c-b)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\,z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\left(1-z^2\right)^{b-n+\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}}\,{}_2F_1\left(c-\frac{1}{2},b;c+n-\left\lfloor\frac{n}{2}\right\rfloor;z^2\right)/;n\in\mathbb{N}$$

07.23.20.0036.02

$$\frac{\partial^n\left(z\left(1-z^2\right)^{b+n-\left\lfloor\frac{n}{2}\right\rfloor-\frac{3}{2}}\,{}_2F_1\left(c+n-\left\lfloor\frac{n}{2}\right\rfloor-\frac{3}{2},b;c;z^2\right)\right)}{\partial z^n}==$$

$$\frac{(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}2^{2\left\lfloor\frac{n}{2}\right\rfloor}}{(c)_{\left\lfloor\frac{n}{2}\right\rfloor}}\left(\frac{3}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}\left(n-2\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}\right)_{\left\lfloor\frac{n}{2}\right\rfloor}(c-b)_{\left\lfloor\frac{n}{2}\right\rfloor}\,z^{1-n+2\left\lfloor\frac{n}{2}\right\rfloor}\left(1-z^2\right)^{b-\left\lfloor\frac{n}{2}\right\rfloor-\frac{3}{2}}\,{}_2F_1\left(c-\frac{3}{2},b;c+\left\lfloor\frac{n}{2}\right\rfloor;z^2\right)/;n\in\mathbb{N}$$

07.23.20.0037.02

$$\frac{\partial^n\left(z^{2c-1}\left(1-z^2\right)^{b-c+n-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}}\,{}_2F_1\left(n-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2},b;c;z^2\right)\right)}{\partial z^n}==$$

$$(-1)^{n-2\left\lfloor\frac{n}{2}\right\rfloor}(1-2c)_n\,z^{2c-n-1}\left(1-z^2\right)^{b-c-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2}}\,{}_2F_1\left(-\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2},b-\left\lfloor\frac{n}{2}\right\rfloor;c-\left\lfloor\frac{n}{2}\right\rfloor;z^2\right)/;n\in\mathbb{N}$$

07.23.20.0038.02

$$\frac{\partial^n\left(z^{2c-2}\left(1-z^2\right)^{b-c+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}}\,{}_2F_1\left(\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2},b;c;z^2\right)\right)}{\partial z^n}==$$

$$(-1)^{n-2\left\lfloor\frac{n}{2}\right\rfloor}(2-2c)_n\,z^{2c-n-2}\left(1-z^2\right)^{b-c-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}}\,{}_2F_1\left(\frac{1}{2}-n+\left\lfloor\frac{n}{2}\right\rfloor,b-n+\left\lfloor\frac{n}{2}\right\rfloor;c-n+\left\lfloor\frac{n}{2}\right\rfloor;z^2\right)/;n\in\mathbb{N}$$

07.23.20.0039.02

$$\frac{\partial^n\,{}_2F_1\left(a,b;\frac{1}{2};z^2\right)}{\partial z^n}==2^{2n-2\left\lfloor\frac{n}{2}\right\rfloor}(a)_{n-\left\lfloor\frac{n}{2}\right\rfloor}(b)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\,z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2F_1\left(a+n-\left\lfloor\frac{n}{2}\right\rfloor,b+n-\left\lfloor\frac{n}{2}\right\rfloor;n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};z^2\right)/;n\in\mathbb{N}$$

07.23.20.0040.02

$$\frac{\partial^n\left(z\,{}_2F_1\left(a,b;\frac{3}{2};z^2\right)\right)}{\partial z^n}==2^{2\left\lfloor\frac{n}{2}\right\rfloor}(a)_{\left\lfloor\frac{n}{2}\right\rfloor}(b)_{\left\lfloor\frac{n}{2}\right\rfloor}\,z^{1-n+2\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2F_1\left(a+\left\lfloor\frac{n}{2}\right\rfloor,b+\left\lfloor\frac{n}{2}\right\rfloor;\frac{3}{2}-n+2\left\lfloor\frac{n}{2}\right\rfloor;z^2\right)/;n\in\mathbb{N}$$

07.23.20.0041.02

$$\frac{\partial^n\left(\left(1-z^2\right)^{a+b-\frac{1}{2}}\,{}_2F_1\left(a,b;\frac{1}{2};z^2\right)\right)}{\partial z^n}==$$

$$2^{2n-2\left\lfloor\frac{n}{2}\right\rfloor}\left(\frac{1}{2}-a\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\left(\frac{1}{2}-b\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\left(1-z^2\right)^{a+b-n-\frac{1}{2}}\,{}_2F_1\left(a-\left\lfloor\frac{n}{2}\right\rfloor,b-\left\lfloor\frac{n}{2}\right\rfloor;n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};z^2\right)/;n\in\mathbb{N}$$

07.23.20.0042.02

$$\frac{\partial^n\left(z\left(1-z^2\right)^{a+b-\frac{3}{2}}\,{}_2F_1\left(a,b;\frac{3}{2};z^2\right)\right)}{\partial z^n}= 2^{2\left\lfloor\frac{n}{2}\right\rfloor}\left(\frac{3}{2}-a\right)_{\left\lfloor\frac{n}{2}\right\rfloor}\left(\frac{3}{2}-b\right)_{\left\lfloor\frac{n}{2}\right\rfloor} z^{1-n+2\left\lfloor\frac{n}{2}\right\rfloor}\left(1-z^2\right)^{a+b-n-\frac{3}{2}}\,{}_2F_1\left(a-n+\left\lfloor\frac{n}{2}\right\rfloor,b-n+\left\lfloor\frac{n}{2}\right\rfloor;\frac{3}{2}-n+2\left\lfloor\frac{n}{2}\right\rfloor;z^2\right)\;/;\;n\in\mathbb{N}$$

07.23.20.0043.02

$$\frac{\partial^n\left(\left(1-z^2\right)^{a+n-\left\lfloor\frac{n}{2}\right\rfloor-1}\,{}_2F_1\left(a,a+n-2\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2};\frac{1}{2};z^2\right)\right)}{\partial z^n}=(-1)^{n-\left\lfloor\frac{n}{2}\right\rfloor}2^{2n-2\left\lfloor\frac{n}{2}\right\rfloor}(a)_{n-\left\lfloor\frac{n}{2}\right\rfloor}\left(1-a-n+2\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor} z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\left(1-z^2\right)^{a-\left\lfloor\frac{n}{2}\right\rfloor-1}\,{}_2F_1\left(a+n-2\left\lfloor\frac{n}{2}\right\rfloor,a+n-2\left\lfloor\frac{n}{2}\right\rfloor-\frac{1}{2};n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};z^2\right)\;/;\;n\in\mathbb{N}$$

07.23.20.0044.02

$$\frac{\partial^n\left(z\left(1-z^2\right)^{a+\left\lfloor\frac{n}{2}\right\rfloor-1}\,{}_2F_1\left(a,a-n+2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};\frac{3}{2};z^2\right)\right)}{\partial z^n}=(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}2^{2\left\lfloor\frac{n}{2}\right\rfloor}(a)_{\left\lfloor\frac{n}{2}\right\rfloor}\left(1-a+n-2\left\lfloor\frac{n}{2}\right\rfloor\right)_{\left\lfloor\frac{n}{2}\right\rfloor} z^{1-n+2\left\lfloor\frac{n}{2}\right\rfloor}\left(1-z^2\right)^{a-n+\left\lfloor\frac{n}{2}\right\rfloor-1}\,{}_2F_1\left(a-n+2\left\lfloor\frac{n}{2}\right\rfloor,a-n+2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};\frac{3}{2}-n+2\left\lfloor\frac{n}{2}\right\rfloor;z^2\right)\;/;\;n\in\mathbb{N}$$

07.23.20.0045.02

$$\frac{\partial^n\left(\left(1-z^2\right)^{n-\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2F_1\left(1,a;\frac{1}{2};z^2\right)\right)}{\partial z^n}= (-1)^{n-\left\lfloor\frac{n}{2}\right\rfloor}2^{2n-2\left\lfloor\frac{n}{2}\right\rfloor}\left(n-\left\lfloor\frac{n}{2}\right\rfloor\right)!\left(\frac{1}{2}-a\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor} z^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\,{}_2F_1\left(a,n-\left\lfloor\frac{n}{2}\right\rfloor+1;n-2\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2};z^2\right)\;/;\;n\in\mathbb{N}$$

07.23.20.0046.02

$$\frac{\partial^n\left(z^\alpha\,{}_2F_1\left(-\frac{n}{2},\frac{1-n}{2};c;z^m\right)\right)}{\partial z^n}=(-1)^n(-\alpha)_n z^{\alpha-n}\,{}_{m+2}F_{m+1}\left(-\frac{n}{2},\frac{1-n}{2},\frac{\alpha+1}{m},\frac{\alpha+2}{m},\ldots,\frac{\alpha+m}{m};\frac{\alpha-n+1}{m},\frac{\alpha-n+2}{m},\ldots,\frac{\alpha-n+m}{m},c;z^m\right)\;/;\;m\in\mathbb{N}^+\land n\in\mathbb{N}$$

07.23.20.0047.02

$$\frac{\partial^n\left(e^{-z}\,{}_2F_1(-n,b;c;z)\right)}{\partial z^n}=(-1)^n e^{-z}\sum_{k=0}^{n}\frac{(-n)_k z^k}{k!\,(c)_k}\,{}_3F_1(-n,k-n,b+k;c+k;z)\;/;\;n\in\mathbb{N}$$

Fractional integro-differentiation

With respect to z

07.23.20.0048.01

$$\frac{\partial^\alpha\,{}_2F_1(a,b;c;z)}{\partial z^\alpha}=z^{-\alpha}\,\Gamma(c)\,{}_3\tilde{F}_2(1,a,b;1-\alpha,c;z)$$

Integration

Indefinite integration

Involving only one direct function

07.23.21.0001.01

$$\int {}_2F_1(a,b;c;z)\,dz = \frac{c-1}{(a-1)(b-1)}\,{}_2F_1(a-1,b-1;c-1;z)$$

Involving one direct function and elementary functions

Involving power function

07.23.21.0002.01

$$\int z^{\alpha-1}\,{}_2F_1(a,b;c;a\,z)\,dz = \frac{z^\alpha}{\alpha}\,{}_3F_2(a,b,\alpha;c,\alpha+1;a\,z)$$

07.23.21.0003.01

$$\int z^{\alpha-1}\,{}_2F_1(a,b;c;z)\,dz = \frac{z^\alpha}{\alpha}\,{}_3F_2(a,b,\alpha;c,\alpha+1;z)$$

07.23.21.0004.01

$$\int z^{\alpha-1}\,{}_2F_1(a,b;c;-z)\,dz = \frac{z^\alpha}{\alpha}\,{}_3F_2(a,b,\alpha;c,\alpha+1;-z)$$

07.23.21.0005.01

$$\int z^{a-2}\,{}_2F_1(a,b;c;z)\,dz = \frac{z^{a-1}\,\Gamma(a-1)\,\Gamma(c)\,{}_2\tilde{F}_1(b,a-1;c;z)}{\Gamma(a)}$$

07.23.21.0006.01

$$\int z^{c-1}\,{}_2F_1(a,b;c;z)\,dz = z^c\,\Gamma(c)\,{}_2\tilde{F}_1(a,b;c+1;z)$$

Involving algebraic functions

07.23.21.0007.01

$$\int (1-z)^{\alpha-1}\,{}_2F_1(a,b;c;z)\,dz = -\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{3,3}^{2,3}\!\left(1-z\,\middle|\,\begin{matrix}1,-a+\alpha+1,-b+\alpha+1\\ \alpha,-a-b+c+\alpha,0\end{matrix}\right)$$

07.23.21.0008.01

$$\int (1-z)^{a+b-c}\,{}_2F_1(a,b;c;z)\,dz = -\frac{\Gamma(c)\,{}_2\tilde{F}_1(-a+c-1,-b+c-1;c-1;z)}{(a-c+1)(-b+c-1)}$$

07.23.21.0009.01

$$\int (1-z)^{a-2}\,{}_2F_1(a,b;c;z)\,dz = \frac{(1-z)^{a-1}\,\Gamma(c)\,{}_2\tilde{F}_1(a-1,b;c-1;z)}{(a-1)(b-c+1)}$$

07.23.21.0010.01

$$\int z^{\alpha-1}\,(1-z)^{a+b-c}\,{}_2F_1(a,b;c;z)\,dz = \frac{z^\alpha\,{}_3F_2(c-a,c-b,\alpha;c,\alpha+1;z)}{\alpha}$$

07.23.21.0011.01

$$\int z^{c-1}(1-z)^{b-c-1}\,{}_2F_1(a,b;c;z)\,dz = (1-z)^{b-c}\,z^c\,\Gamma(c)\,{}_2\tilde{F}_1(a+1,b;c+1;z)$$

07.23.21.0012.01

$$\int z^{c-1}(1-z)^{a+b-c}\,{}_2F_1(a,\,b;\,c;\,z)\,dz = z^c\,\Gamma(c)\,{}_2\tilde{F}_1(c-a,\,c-b;\,c+1;\,z)$$

07.23.21.0013.01

$$\int z^{c-a-2}(1-z)^{a+b-c}\,{}_2F_1(a,\,b;\,c;\,z)\,dz = \frac{z^{-a+c-1}\,\Gamma(c)\,\Gamma(-a+c-1)\,{}_2\tilde{F}_1(c-b,\,-a+c-1;\,c;\,z)}{\Gamma(c-a)}$$

Definite integration

For the direct function itself

07.23.21.0014.01

$$\int_0^\infty t^{\alpha-1}\,{}_2F_1(a,\,b;\,c;\,-t)\,dt = \frac{\Gamma(c)\,\Gamma(\alpha)\,\Gamma(a-\alpha)\,\Gamma(b-\alpha)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-\alpha)} \;/;\; 0 < \mathrm{Re}(\alpha) < \min(\mathrm{Re}(a),\,\mathrm{Re}(b))$$

Involving the direct function

07.23.21.0015.01

$$\int_0^\infty t^{\alpha-1}\,e^{-d\,t}\,{}_2F_1(a,\,b;\,c;\,-t)\,dt = \frac{\Gamma(c)\,\Gamma(a-\alpha)\,\Gamma(b-\alpha)\,\Gamma(\alpha)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-\alpha)}\,{}_2F_2(\alpha,\,1-c+\alpha;\,1-a+\alpha,\,1-b+\alpha;\,d) + \frac{\Gamma(c)\,\Gamma(a-b)\,\Gamma(\alpha-b)}{\Gamma(a)\,\Gamma(c-b)}\,d^{b-\alpha}\,{}_2F_2(b,\,b-c+1;\,1-a+b,\,b-\alpha+1;\,d) + \frac{\Gamma(c)\,\Gamma(b-a)\,\Gamma(\alpha-a)}{\Gamma(b)\,\Gamma(c-a)}\,d^{a-\alpha}\,{}_2F_2(a,\,a-c+1;\,a-b+1,\,a-\alpha+1;\,d) \;/;\; \mathrm{Re}(\alpha) > 0 \bigwedge \mathrm{Re}(d) > 0$$

Integral transforms

Laplace transforms

07.23.22.0001.01

$$\mathcal{L}_t[{}_2F_1(a,\,b;\,c;\,-t)]\,(z) = \frac{\Gamma(1-a)\,\Gamma(b-a)\,\Gamma(c)\,z^{a-1}}{\Gamma(b)\,\Gamma(c-a)}\,{}_1F_1(a-c+1;\,a-b+1;\,z) + \frac{\Gamma(1-b)\,\Gamma(a-b)\,\Gamma(c)\;z^{b-1}}{\Gamma(a)\,\Gamma(c-b)}\,{}_1F_1(b-c+1;\,1-a+b;\,z) + \frac{c-1}{(a-1)\,(b-1)}\,{}_2F_2(1,\,2-c;\,2-a,\,2-b;\,z) \;/;\; \mathrm{Re}(z) > 0$$

Summation

Finite summation

07.23.23.0001.01

$$\sum_{k=0}^{m}\binom{m}{k}(z-1)^{-k}\,{}_2F_1(a,\,b-k;\,c;\,z) = \frac{(c-a)_m}{(c)_m}\left(\frac{z}{z-1}\right)^m {}_2F_1(a,\,b;\,c+m;\,z) \;/;\; m \in \mathbb{N}$$

07.23.23.0002.01

$$\sum_{k=0}^{m}(-1)^k\binom{m}{k}{}_2F_1(a,\,b-k;\,c;\,z) = \frac{(a)_m}{(c)_m}\,z^m\,{}_2F_1(a+m,\,b;\,c+m;\,z) \;/;\; m \in \mathbb{N}$$

Infinite summation

07.23.23.0003.01

$$\sum_{k=0}^{\infty} \frac{(a)_k}{k!} \, {}_2F_1(-k, b; c; w) \, z^k = \left(\frac{1}{1-z}\right)^a {}_2F_1\left(a, b; c; \frac{z\,w}{z-1}\right)$$

Operations

Limit operation

07.23.25.0001.01

$$\lim_{z \to 1} (1-z)^{a+b-c} \, {}_2F_1(a, b; c; z) = \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)} \,/;\, \mathrm{Re}(c-a-b) < 0$$

07.23.25.0002.01

$$\lim_{c \to -n} \frac{1}{\Gamma(c)} \, {}_2F_1(a, b; c; z) = z^{n+1} \, (a)_{n+1} \, (b)_{n+1} \, {}_2\tilde{F}_1(a+n+1, b+n+1; n+2; z) \,/;\, n \in \mathbb{N}$$

07.23.25.0003.01

$$\lim_{a \to \infty} {}_2F_1\left(a, b; c; \frac{z}{a}\right) = {}_1F_1(b; c; z)$$

07.23.25.0004.01

$$\lim_{c \to \infty} {}_2F_1\left(a, b; c; 1 - \frac{c}{z}\right) = z^a \, U(a, a-b+1, z)$$

07.23.25.0005.01

$$\lim_{b \to \infty} \lim_{a \to \infty} {}_2F_1\left(a, b; c; \frac{z}{a\,b}\right) = {}_0F_1(; c; z)$$

07.23.25.0006.01

$$\lim_{a \to \infty} {}_2F_1\left(a, b; \frac{a}{z}; 1\right) = (1-z)^{-b} \,/;\, \mathrm{Re}\left(\frac{a\,(1-z)}{z} - b\right) > 0$$

07.23.25.0007.01

$$\lim_{a \to n} \frac{1}{a\,\Gamma(1-a)} \, {}_2F_1(a, a; a+1; 1) = (-1)^{n-1} \, S_n^{(1)} \,/;\, n \in \mathbb{N}$$

07.23.25.0008.01

$$\lim_{z \to 1} {}_2F_1(1-m, 2; 1; z) = (-1)^{m-1} \, (m-1)! \, \mathcal{S}_2^{(m)} \,/;\, m \in \mathbb{N}^+$$

07.23.25.0009.01

$$\lim_{a \to 0} \frac{{}_2F_1(a, a; 1; z) - 1}{a^2} = \mathrm{Li}_2(z)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_pF_q$

07.23.26.0001.01

$${}_2F_1(a, b; c; z) = {}_pF_q\left(a_1, \ldots, a_p; b_1, \ldots, b_q; z\right) \,/;\, p = 2 \land q = 1 \land a_1 = a \land a_2 = b \land b_1 = c$$

07.23.26.0002.01

$${}_2F_1(a, b; c; z) = {}_3F_2(a, b, a_3; c, a_3; z)$$

Involving ${}_2\tilde{F}_1$

07.23.26.0003.01

$${}_2F_1(a, b; c; z) = \Gamma(c)\, {}_2\tilde{F}_1(a, b; c; z)$$

Through Meijer *G*

Classical cases for the direct function itself

07.23.26.0004.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\, G_{2,2}^{1,2}\!\left(-z \,\middle|\, \begin{matrix} 1-a, 1-b \\ 0, 1-c \end{matrix}\right)$$

07.23.26.0005.01

$${}_2F_1(a, b; c; z) = \frac{\pi\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\, G_{3,3}^{1,2}\!\left(z \,\middle|\, \begin{matrix} 1-a, 1-b, \frac{1}{2} \\ 0, 1-c, \frac{1}{2} \end{matrix}\right) /;\ |z| < 1$$

07.23.26.0006.01

$${}_2F_1(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{2,2}^{2,2}\!\left(1-z \,\middle|\, \begin{matrix} 1-a, 1-b \\ 0, c-a-b \end{matrix}\right)$$

07.23.26.0007.01

$${}_2F_1(a, b; c; 1-z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{2,2}^{2,2}\!\left(z \,\middle|\, \begin{matrix} 1-a, 1-b \\ 0, c-a-b \end{matrix}\right)$$

07.23.26.0273.01

$${}_2F_1(a, b; c; z) = \frac{\csc((a-b)\pi)\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)} \left(\sin((a-c)\pi)\,(-z)^{-a}\, G_{2,2}^{1,2}\!\left(-\frac{1}{z} \,\middle|\, \begin{matrix} 1-a, c-a \\ 0, b-a \end{matrix}\right) - \sin((b-c)\pi)\,(-z)^{-b}\, G_{2,2}^{1,2}\!\left(-\frac{1}{z} \,\middle|\, \begin{matrix} 1-b, c-b \\ 0, a-b \end{matrix}\right)\right)$$

07.23.26.0008.01

$${}_2F_1(a, b; c; z) - {}_2F_1(a, b; c; -z) = \frac{2^{a+b-c}\,\Gamma(c)\,z}{\Gamma(a)\,\Gamma(b)}\, G_{4,4}^{1,4}\!\left(-z^2 \,\middle|\, \begin{matrix} -\frac{a}{2}, -\frac{b}{2}, \frac{1-a}{2}, \frac{1-b}{2} \\ 0, -\frac{1}{2}, -\frac{c}{2}, \frac{1-c}{2} \end{matrix}\right)$$

07.23.26.0009.01

$${}_2F_1(a, b; c; z) + {}_2F_1(a, b; c; -z) = \frac{2^{a+b-c}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\, G_{4,4}^{1,4}\!\left(-z^2 \,\middle|\, \begin{matrix} \frac{1-a}{2}, \frac{1-b}{2}, 1-\frac{a}{2}, 1-\frac{b}{2} \\ 0, \frac{1}{2}, \frac{1-c}{2}, 1-\frac{c}{2} \end{matrix}\right)$$

07.23.26.0010.01

$${}_2F_1(a, b; c; z) = 1 - \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\, G_{3,3}^{1,3}\!\left(-z \,\middle|\, \begin{matrix} 1, 1-a, 1-b \\ 1, 0, 1-c \end{matrix}\right)$$

07.23.26.0011.01

$${}_2F_1(a, b; c; z) = 1 - \frac{\pi\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\, G_{4,4}^{1,3}\!\left(z \,\middle|\, \begin{matrix} 1, 1-a, 1-b, \frac{1}{2} \\ 1, 0, \frac{1}{2}, 1-c \end{matrix}\right) /;\ |z| < 1$$

Classical cases involving algebraic functions with linear arguments

07.23.26.0012.01

$$(1-z)^{a+b-c}\,{}_2F_1(a,\,b;\,c;\,z) = \frac{\Gamma(c)}{\Gamma(c-a)\,\Gamma(c-b)}\,G_{2,2}^{1,2}\!\left(-z\,\middle|\,\begin{array}{c} a-c+1,\,b-c+1 \\ 0,\,1-c \end{array}\right)$$

07.23.26.0013.01

$$(1-z)^{a+b-c}\,{}_2F_1(a,\,b;\,c;\,z) = \frac{\pi\,\Gamma(c)}{\Gamma(c-a)\,\Gamma(c-b)}\,G_{3,3}^{1,2}\!\left(z\,\middle|\,\begin{array}{c} a-c+1,\,b-c+1,\,\frac{1}{2} \\ 0,\,1-c,\,\frac{1}{2} \end{array}\right) /;\ |z|<1$$

07.23.26.0014.01

$$(1-z)\,{}_2F_1(a,\,b;\,a-b+1;\,z) = \frac{2\,\Gamma(a-b+1)}{\Gamma(a)\,\Gamma(b-1)}\,G_{3,3}^{1,3}\!\left(-z\,\middle|\,\begin{array}{c} 2-a,\,2-b,\,\frac{1-a}{2} \\ 0,\,\frac{3-a}{2},\,b-a \end{array}\right)$$

07.23.26.0015.01

$$(1-z)\,{}_2F_1\!\left(a,\,b;\,\frac{b-1}{2}+a;\,z\right) = \frac{\Gamma\!\left(\frac{b-1}{2}+a\right)}{2\,\Gamma(a)\,\Gamma(b-1)}\,G_{3,3}^{1,3}\!\left(-z\,\middle|\,\begin{array}{c} 2-2\,a,\,2-a,\,2-b \\ 0,\,3-2\,a,\,\frac{3-b}{2}-a \end{array}\right)$$

07.23.26.0016.01

$$(1-z)^{2\,b}\,{}_2F_1(a,\,b;\,a-b+1;\,z) = \frac{2\,\Gamma(a-b+1)}{\Gamma(a-2\,b+1)\,\Gamma(-b)}\,G_{3,3}^{1,3}\!\left(-z\,\middle|\,\begin{array}{c} b+1,\,b-\frac{a}{2},\,1-a+2\,b \\ 0,\,b-a,\,1-\frac{a}{2}+b \end{array}\right)$$

07.23.26.0017.01

$$(1-z)\,{}_2F_1(a,\,b;\,a-b+1;\,z) = \frac{2\,\pi\,\Gamma(a-b+1)}{\Gamma(a)\,\Gamma(b-1)}\,G_{4,4}^{1,3}\!\left(z\,\middle|\,\begin{array}{c} 2-a,\,2-b,\,\frac{1-a}{2},\,\frac{1}{2} \\ 0,\,\frac{3-a}{2},\,b-a,\,\frac{1}{2} \end{array}\right) /;\ |z|<1$$

07.23.26.0018.01

$$(1-z)^{-b}\,{}_2F_1(b,\,a;\,a+2\,b+1;\,z) = \frac{\Gamma(a+2\,b+1)}{2\,\Gamma(2\,b)\,\Gamma(a+b+1)}\,G_{3,3}^{1,3}\!\left(-z\,\middle|\,\begin{array}{c} -2\,(a+b),\,1-2\,b,\,1-a-b \\ 0,\,1-2\,a-2\,b,\,-a-2\,b \end{array}\right)$$

07.23.26.0019.01

$$(1-z)^{c-1}\,{}_2F_1(a,\,b;\,c;\,1-z) = \Gamma(c)\left((z-1)^{1-c}\,(1-z)^{c-1}\,G_{2,2}^{0,2}\!\left(z\,\middle|\,\begin{array}{c} c-a,\,c-b \\ 0,\,c-a-b \end{array}\right) + G_{2,2}^{2,0}\!\left(z\,\middle|\,\begin{array}{c} c-a,\,c-b \\ 0,\,c-a-b \end{array}\right)\right) /;\ z\notin(-1,\,0)$$

Classical cases involving algebraic functions with rational arguments

07.23.26.0020.01

$$(1-z)^{c-1}\,{}_2F_1\!\left(a,\,b;\,c;\,\frac{z-1}{z}\right) = \Gamma(c)\left(G_{2,2}^{2,0}\!\left(z\,\middle|\,\begin{array}{c} a+b,\,c \\ a,\,b \end{array}\right) - (1-z)^{c}\,(z-1)^{-c}\,G_{2,2}^{0,2}\!\left(z\,\middle|\,\begin{array}{c} a+b,\,c \\ a,\,b \end{array}\right)\right) /;\ z\notin(-\infty,\,-1)$$

07.23.26.0021.01

$$(1+z)^{-a}\,{}_2F_1\!\left(a,\,b;\,c;\,\frac{z}{z+1}\right) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(c-b)}\,G_{2,2}^{1,2}\!\left(z\,\middle|\,\begin{array}{c} 1-a,\,b-c+1 \\ 0,\,1-c \end{array}\right) /;\ z\notin(-\infty,\,-1)$$

07.23.26.0022.01

$$(1+z)^{-b}\,{}_2F_1\!\left(a,\,b;\,c;\,\frac{z}{z+1}\right) = \frac{\Gamma(c)}{\Gamma(b)\,\Gamma(c-a)}\,G_{2,2}^{1,2}\!\left(z\,\middle|\,\begin{array}{c} 1-b,\,a-c+1 \\ 0,\,1-c \end{array}\right) /;\ z\notin(-\infty,\,-1)$$

07.23.26.0023.01

$$(z+1)^{1-a}\,{}_2F_1\!\left(a,\,1-a;\,c;\,\frac{z}{z+1}\right) = \frac{2\,\Gamma(c)}{\Gamma(a-1)\,\Gamma(a+c-1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c} 2-a,\,3-a-c,\,1-\frac{a+c}{2} \\ 0,\,1-c,\,2-\frac{a+c}{2} \end{array}\right) /;\ z\notin(-\infty,\,-1)$$

07.23.26.0024.01

$$(z+1)^{1-a}\,{}_2F_1\left(a,\,b;\,\frac{1}{2}(a+b+1);\,\frac{z}{z+1}\right)=\frac{2\,\Gamma\left(\frac{1+a+b}{2}\right)}{\Gamma(a)\,\Gamma\left(\frac{a-b-1}{2}\right)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-a,\,\frac{1-a}{2},\,\frac{3-a+b}{2}\\0,\,\frac{3-a}{2},\,\frac{1-a-b}{2}\end{array}\right)\,/;\,z\notin(-\infty,\,-1)$$

07.23.26.0025.01

$$(z+1)^{1-b}\,{}_2F_1\left(a,\,b;\,2\,b-a-1;\,\frac{z}{z+1}\right)=\frac{\Gamma(2\,b-a-1)}{2\,\Gamma(b)\,\Gamma(2\,b-2\,a-2)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-2\,b,\,2\,a-2\,b+3,\,2-b\\0,\,3-2\,b,\,a-2\,b+2\end{array}\right)\,/;\,z\notin(-\infty,\,-1)$$

07.23.26.0026.01

$$(z+1)^{-2\,a}\,{}_2F_1\left(a,\,2\,a+1;\,c;\,\frac{z}{z+1}\right)=\frac{\Gamma(c)}{2\,\Gamma(2\,a)\,\Gamma(c-a)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}1-2\,a,\,2\,(a-c+1),\,a-c+2\\0,\,2\,a-2\,c+3,\,1-c\end{array}\right)\,/;\,z\notin(-\infty,\,-1)$$

Classical cases involving algebraic functions with quadratic arguments

07.23.26.0027.01

$$(z+1)^{-2\,a}\,{}_2F_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{4\,z}{(z+1)^2}\right)=\frac{\Gamma(c)\,\Gamma(c-2\,a)}{\Gamma(2\,a)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}1-2\,a,\,c-2\,a\\0,\,1-c\end{array}\right)\,/;\,z\notin(-1,\,0)$$

07.23.26.0028.01

$$(z+1)^{2\,a-2\,c+1}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{2\,a-c+\frac{1}{2}}\,{}_2F_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{4\,z}{(z+1)^2}\right)=\frac{\Gamma(2\,a-c+1)\,\Gamma(c)}{\Gamma(2\,c-2\,a-1)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}2\,(a-c+1),\,2\,a-c+1\\0,\,1-c\end{array}\right)$$

07.23.26.0029.01

$$(z+1)^{1-2\,a}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{-a}\,{}_2F_1\left(a,\,b;\,a+b-\frac{1}{2};\,-\frac{4\,z}{(z-1)^2}\right)=\frac{\Gamma\left(b-a+\frac{1}{2}\right)\Gamma\left(a+b-\frac{1}{2}\right)}{\Gamma(2\,a-1)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}2-2\,a,\,b-a+\frac{1}{2}\\0,\,\frac{3}{2}-a-b\end{array}\right)$$

07.23.26.0030.01

$$(z+1)^{-2\,a}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{-a}\,{}_2F_1\left(a,\,b;\,a+b+\frac{1}{2};\,-\frac{4\,z}{(z-1)^2}\right)=\frac{\Gamma\left(-a+b+\frac{1}{2}\right)\Gamma\left(a+b+\frac{1}{2}\right)}{\Gamma(2\,a)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}1-2\,a,\,-a+b+\frac{1}{2}\\0,\,-a-b+\frac{1}{2}\end{array}\right)$$

07.23.26.0031.01

$$\left(\sqrt{z}+1\right)^{-2\,a}\,{}_2F_1\left(a,\,b;\,2\,b;\,\frac{4\sqrt{z}}{\left(\sqrt{z}+1\right)^2}\right)=\frac{\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}1-a,\,b-a+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{array}\right)\,/;\,z\notin(-1,\,0)$$

07.23.26.0032.01

$$\left(\sqrt{z}+1\right)^{2\,(a-2\,b)}\left(\frac{\left(\sqrt{z}-1\right)^2}{\left(\sqrt{z}+1\right)^2}\right)^{a-b}\,{}_2F_1\left(a,\,b;\,2\,b;\,\frac{4\sqrt{z}}{\left(\sqrt{z}+1\right)^2}\right)=\frac{\Gamma\left(a-b+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)}{\Gamma(2\,b-a)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}a-2\,b+1,\,a-b+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{array}\right)$$

07.23.26.0033.01

$$\left(\sqrt{z}+1\right)^{-2\,a}\left(\frac{\left(\sqrt{z}-1\right)^2}{\left(\sqrt{z}+1\right)^2}\right)^{-a}\,{}_2F_1\left(a,\,b;\,2\,b;\,-\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right)=\frac{\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}1-a,\,b-a+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{array}\right)$$

07.23.26.0034.01

$$\left(\sqrt{z}+1\right)^{2\,(a-2\,b)}\left(\frac{\left(\sqrt{z}-1\right)^2}{\left(\sqrt{z}+1\right)^2}\right)^{-b}\,{}_2F_1\left(a,\,b;\,2\,b;\,-\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right)=\frac{\Gamma\left(a-b+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)}{\Gamma(2\,b-a)}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}a-2\,b+1,\,a-b+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{array}\right)$$

Classical cases involving algebraic functions with squares in arguments

07.23.26.0035.01

$$\left(\sqrt{1-z}+1\right)^{-b} {}_2F_1\left(a, b; b+1; \frac{1-\sqrt{1-z}}{2}\right) = \frac{b\, 2^{a-1}}{\sqrt{\pi}} G_{3,3}^{1,3}\left(-z \,\middle|\, \begin{matrix} \frac{1-a-b}{2}, 1-\frac{a+b}{2}, 1-b \\ 0, 1-a-b, -b \end{matrix}\right)$$

07.23.26.0036.01

$$\left(\sqrt{1-z}+1\right)^{b-2c+2} {}_2F_1\left(1, b; c; \frac{1-\sqrt{1-z}}{2}\right) = \frac{c-1}{\sqrt{\pi}} G_{3,3}^{1,3}\left(-z \,\middle|\, \begin{matrix} \frac{b}{2}+1-c, \frac{b+3}{2}-c, 2-c \\ 0, b-2c+2, 1-c \end{matrix}\right)$$

07.23.26.0037.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{-b} {}_2F_1\left(a, b; b+1; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{a-1} b}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 1-\frac{b}{2}, \frac{b+2}{2}, \frac{b}{2}+a \\ \frac{a+1}{2}, \frac{a}{2}, \frac{b}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

07.23.26.0038.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{b-2c+2} {}_2F_1\left(1, b; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{c-1}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} \frac{b}{2}-c+2, \frac{b}{2}+1, c-\frac{b}{2} \\ 1, \frac{1}{2}, \frac{b}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

07.23.26.0039.01

$$\left(\sqrt{z+1}-1\right)^{2b} {}_2F_1\left(a, b; b+1; \frac{z-2\sqrt{z+1}+2}{z}\right) = \frac{2^{-a} b}{\sqrt{\pi}} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} \frac{a}{2}+b, \frac{a+1}{2}+b, b+1 \\ 2b, a, b \end{matrix}\right)$$

07.23.26.0040.01

$$\left(\sqrt{z+1}-1\right)^{b+c-1} {}_2F_1\left(1, b; c; \frac{z-2\sqrt{z+1}+2}{z}\right) = \frac{c-1}{2\sqrt{\pi}} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} \frac{b+c}{2}, \frac{b+c+1}{2}, b+1 \\ b+c-1, 1, b \end{matrix}\right)$$

07.23.26.0041.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^{2a} {}_2F_1\left(a, b; a+1; 2z-2\sqrt{z+1}\sqrt{z}+1\right) = \frac{2^{-b} a}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 1-a, 1, a-b+1 \\ \frac{1-b}{2}, 1-\frac{b}{2}, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

07.23.26.0042.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^{b+c-1} {}_2F_1\left(1, b; c; 2z-2\sqrt{z+1}\sqrt{z}+1\right) = \frac{c-1}{2\sqrt{\pi}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} \frac{3-b-c}{2}, \frac{c-b+1}{2}, \frac{b+c-1}{2} \\ 0, \frac{1}{2}, \frac{c-b-1}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

07.23.26.0043.01

$$\left(\sqrt{1-z}+1\right)^{1-b-c} {}_2F_1\left(1, b; c; \frac{\sqrt{1-z}-1}{\sqrt{1-z}+1}\right) = \frac{c-1}{2\sqrt{\pi}} G_{3,3}^{1,3}\left(-z \,\middle|\, \begin{matrix} 1-\frac{b+c}{2}, \frac{3-b-c}{2}, 2-c \\ 0, 2-b-c, 1-c \end{matrix}\right)$$

07.23.26.0044.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-b-c} {}_2F_1\left(1, b; c; \frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right) = \frac{c-1}{2\sqrt{\pi}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} \frac{3-b-c}{2}, \frac{1+c-b}{2}, \frac{b+c-1}{2} \\ 0, \frac{1}{2}, \frac{c-b-1}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

07.23.26.0045.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{-2b} {}_2F_1\left(a, b; b+1; \frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right) = \frac{2^{-a} b}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 1-b, 1, 1-a+b \\ 0, \frac{1-a}{2}, 1-\frac{a}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

07.23.26.0046.01

$$\left(\sqrt{z+1}-1\right)^{b} {}_2F_1\left(a, b; b+1; \frac{z-2\sqrt{z+1}+2}{2-2\sqrt{z+1}}\right) = \frac{2^{a-1} b}{\sqrt{\pi}} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} \frac{1+b-a}{2}, \frac{b-a}{2}+1, 1 \\ b, 0, 1-a \end{matrix}\right)$$

07.23.26.0047.01

$$\left(\sqrt{z+1}-1\right)^{2c-b-2}{}_2F_1\left(1,b;c;\frac{z-2\sqrt{z+1}+2}{2-2\sqrt{z+1}}\right)=\frac{c-1}{\sqrt{\pi}}G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}c-\frac{b}{2}-1,c-\frac{b+1}{2},c-b\\2c-b-2,0,c-b-1\end{matrix}\right)$$

07.23.26.0048.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^{a}{}_2F_1\left(a,b;a+1;\frac{2z-2\sqrt{z+1}\sqrt{z}+1}{2z-2\sqrt{z}\sqrt{z+1}}\right)=\frac{2^{b-1}a}{\sqrt{\pi}}G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}1-\frac{a}{2},\frac{a+2}{2},\frac{a}{2}+b\\\frac{b}{2},\frac{b+1}{2},\frac{a}{2}\end{matrix}\right)/;z\notin(-1,0)$$

07.23.26.0049.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)^{2c-b-2}{}_2F_1\left(1,b;c;\frac{2z-2\sqrt{z+1}\sqrt{z}+1}{2z-2\sqrt{z}\sqrt{z+1}}\right)=\frac{c-1}{\sqrt{\pi}}G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{b}{2}-c+2,\frac{b}{2}+1,c-\frac{b}{2}\\\frac{1}{2},1,\frac{b}{2}\end{matrix}\right)/;z\notin(-1,0)$$

Classical cases involving algebraic functions with cubics in arguments

07.23.26.0050.01

$$(4z+1)^{-3a}{}_2F_1\left(a,a+\frac{1}{3};2a+\frac{5}{6};\frac{27z}{(4z+1)^3}\right)=\frac{2^{-6a}\Gamma\left(4a+\frac{2}{3}\right)}{\Gamma(3a)\Gamma\left(a+\frac{1}{6}\right)}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}1-3a,a+\frac{2}{3}\\\frac{1}{6}-2a,0\end{matrix}\right)$$

07.23.26.0051.01

$$(z+4)^{-3a}{}_2F_1\left(a,a+\frac{1}{3};2a+\frac{5}{6};\frac{27z^2}{(z+4)^3}\right)=\frac{2^{-6a}\Gamma\left(4a+\frac{2}{3}\right)}{\Gamma(3a)\Gamma\left(a+\frac{1}{6}\right)}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{5}{6}-a,1-3a\\0,\frac{1}{3}-4a\end{matrix}\right)$$

07.23.26.0052.01

$$(3z+4)^{-3a}(9z+8)\,{}_2F_1\left(a,a+\frac{1}{3};\frac{3}{2};\frac{(9z+8)^2}{(3z+4)^3}\right)=\frac{3^{2-3a}\sqrt{\pi}}{2\Gamma\left(\frac{4}{3}-a\right)\Gamma(3a-1)}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}2-3a,\frac{5}{2}-3a\\\frac{7}{3}-4a,0\end{matrix}\right)/;|z|>1\lor\operatorname{Re}(z)\geq 0$$

07.23.26.0053.01

$$(3z-1)^{-3a}(9z+1)\,{}_2F_1\left(a,a+\frac{1}{3};\frac{3}{2};\frac{(9z+1)^2}{(1-3z)^3}\right)=\frac{\sqrt{\pi}}{2\,3^{3a-2}\Gamma\left(a+\frac{1}{6}\right)\Gamma(3a-1)}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}2-3a,\frac{5}{2}-3a\\\frac{7}{6}-2a,0\end{matrix}\right)/;|z|>1\lor\operatorname{Re}(z)>0$$

07.23.26.0054.01

$$(3-z)^{-3a}(z+9)\,{}_2F_1\left(a,a+\frac{1}{3};\frac{3}{2};\frac{z(z+9)^2}{(z-3)^3}\right)=\frac{\sqrt{\pi}\,3^{2-3a}}{2\Gamma\left(a+\frac{1}{6}\right)\Gamma(3a-1)}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{5}{6}-a,2-3a\\0,-\frac{1}{2}\end{matrix}\right)/;|z|<1\lor\operatorname{Re}(z)>0$$

07.23.26.0055.01

$$(4z+3)^{-3a}(8z+9)\,{}_2F_1\left(a,a+\frac{1}{3};\frac{3}{2};\frac{z(8z+9)^2}{(4z+3)^3}\right)=\frac{\sqrt{\pi}\,3^{2-3a}}{2\Gamma\left(\frac{4}{3}-a\right)\Gamma(3a-1)}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}a-\frac{1}{3},2-3a\\0,-\frac{1}{2}\end{matrix}\right)/;|z|<1\lor\operatorname{Re}(z)\geq 0$$

07.23.26.0056.01

$$(z+1)^{-a}{}_2F_1\left(a,\frac{1}{6}-a;\frac{1}{2};-\frac{(9z+8)^2}{27z^2(z+1)}\right)=\frac{\sqrt{\pi}}{\Gamma\left(\frac{1}{3}-a\right)\Gamma(3a)}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}1-a,\frac{1}{2}-a\\\frac{1}{3}-2a,2a\end{matrix}\right)/;|z|>1\lor\operatorname{Re}(z)>0$$

07.23.26.0057.01

$$(z+1)^{-a}{}_2F_1\left(a,\frac{1}{6}-a;\frac{1}{2};-\frac{z(8z+9)^2}{27(z+1)}\right)=\frac{\sqrt{\pi}}{\Gamma\left(\frac{1}{3}-a\right)\Gamma(3a)}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}a+\frac{2}{3},1-3a\\0,\frac{1}{2}\end{matrix}\right)/;|z|<1\lor\operatorname{Re}(z)>0$$

Classical cases with rational arguments and unit step θ

07.23.26.0058.01

$$\theta(1-|z|)\,{}_2F_1(a,b;c;-z) = \Gamma(1-a)\,\Gamma(1-b)\,\Gamma(c)\,G_{2,2}^{1,0}\!\left(z \,\middle|\, \begin{matrix} 1-a,\,1-b \\ 0,\,1-c \end{matrix}\right)$$

07.23.26.0059.01

$$\theta(1-|z|)\,{}_2F_1(a,b;c;z) = \pi\,\Gamma(1-a)\,\Gamma(1-b)\,\Gamma(c)\,G_{3,3}^{1,0}\!\left(z \,\middle|\, \begin{matrix} 1-a,\,1-b,\,\frac{1}{2} \\ 0,\,1-c,\,\frac{1}{2} \end{matrix}\right)$$

07.23.26.0060.01

$$\theta(|z|-1)\,{}_2F_1\!\left(a,b;c;-\frac{1}{z}\right) = \Gamma(1-a)\,\Gamma(1-b)\,\Gamma(c)\,G_{2,2}^{0,1}\!\left(z \,\middle|\, \begin{matrix} 1,\,c \\ a,\,b \end{matrix}\right)$$

07.23.26.0061.01

$$\theta(|z|-1)\,{}_2F_1\!\left(a,b;c;\frac{1}{z}\right) = \pi\,\Gamma(1-a)\,\Gamma(1-b)\,\Gamma(c)\,G_{3,3}^{0,1}\!\left(z \,\middle|\, \begin{matrix} 1,\,c,\,\frac{1}{2} \\ a,\,b,\,\frac{1}{2} \end{matrix}\right)$$

Classical cases involving algebraic functions with linear arguments and unit step θ

07.23.26.0062.01

$$\theta(1-|z|)\,(z+1)^{a+b-c}\,{}_2F_1(a,b;c;-z) = \Gamma(a-c+1)\,\Gamma(b-c+1)\,\Gamma(c)\,G_{2,2}^{1,0}\!\left(z \,\middle|\, \begin{matrix} a-c+1,\,b-c+1 \\ 0,\,1-c \end{matrix}\right)$$

07.23.26.0063.01

$$\theta(1-|z|)\,(1-z)^{a+b-c}\,{}_2F_1(a,b;c;z) = \pi\,\Gamma(a-c+1)\,\Gamma(b-c+1)\,\Gamma(c)\,G_{3,3}^{1,0}\!\left(z \,\middle|\, \begin{matrix} \frac{1}{2},\,a-c+1,\,b-c+1 \\ 0,\,\frac{1}{2},\,1-c \end{matrix}\right)$$

07.23.26.0064.01

$$\theta(1-|z|)\,(1-z)^{c-1}\,{}_2F_1(a,b;c;1-z) = \Gamma(c)\,G_{2,2}^{2,0}\!\left(z \,\middle|\, \begin{matrix} c-a,\,c-b \\ 0,\,c-a-b \end{matrix}\right) /;\ \notin(-1,0)\land \mathrm{Re}(c)>0$$

07.23.26.0065.01

$$\theta(|z|-1)\,(z-1)^{c-1}\,{}_2F_1(a,b;c;1-z) = \Gamma(c)\,G_{2,2}^{0,2}\!\left(z \,\middle|\, \begin{matrix} c-a,\,c-b \\ 0,\,c-a-b \end{matrix}\right) /;\ \mathrm{Re}(c)>0$$

Classical cases involving algebraic functions with rational arguments and unit step θ

07.23.26.0066.01

$$\theta(|z|-1)\,(z+1)^{a+b-c}\,{}_2F_1\!\left(a,b;c;-\frac{1}{z}\right) = \Gamma(a-c+1)\,\Gamma(b-c+1)\,\Gamma(c)\,G_{2,2}^{0,1}\!\left(z \,\middle|\, \begin{matrix} a+b-c+1,\,a+b \\ a,\,b \end{matrix}\right)$$

07.23.26.0067.01

$$\theta(|z|-1)\,(z-1)^{a+b-c}\,{}_2F_1\!\left(a,b;c;\frac{1}{z}\right) = \pi\,\Gamma(a-c+1)\,\Gamma(b-c+1)\,\Gamma(c)\,G_{3,3}^{0,1}\!\left(z \,\middle|\, \begin{matrix} a+b-c+1,\,a+b-c+\frac{1}{2},\,a+b \\ a+b-c+\frac{1}{2},\,a,\,b \end{matrix}\right)$$

07.23.26.0068.01

$$\theta(1-|z|)\,(1-z)^{c-1}\,{}_2F_1\!\left(a,b;c;\frac{z-1}{z}\right) = \Gamma(c)\,G_{2,2}^{2,0}\!\left(z \,\middle|\, \begin{matrix} a+b,\,c \\ a,\,b \end{matrix}\right) /;\ \mathrm{Re}(c)>0$$

07.23.26.0069.01

$$\theta(1-|z|)\,(z+1)^{-a}\,{}_2F_1\!\left(a,b;c;\frac{z}{z+1}\right) = \Gamma(1-a)\,\Gamma(b-c+1)\,\Gamma(c)\,G_{2,2}^{1,0}\!\left(z \,\middle|\, \begin{matrix} 1-a,\,b-c+1 \\ 0,\,1-c \end{matrix}\right)$$

07.23.26.0070.01

$$\theta(1-|z|)\,(1-z)^{-a}\,{}_2F_1\!\left(a,b;c;\frac{z}{z-1}\right)=\pi\,\Gamma(1-a)\,\Gamma(b-c+1)\,\Gamma(c)\,G_{3,3}^{1,0}\!\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1-a,b-c+1\\0,\frac{1}{2},1-c\end{matrix}\right)$$

07.23.26.0071.01

$$\theta(1-|z|)\,(z+1)^{-b}\,{}_2F_1\!\left(a,b;c;\frac{z}{z+1}\right)=\Gamma(1-b)\,\Gamma(a-c+1)\,\Gamma(c)\,G_{2,2}^{1,0}\!\left(z\,\middle|\,\begin{matrix}1-b,a-c+1\\0,1-c\end{matrix}\right)$$

07.23.26.0072.01

$$\theta(1-|z|)\,(1-z)^{-b}\,{}_2F_1\!\left(a,b;c;\frac{z}{z-1}\right)=\pi\,\Gamma(1-b)\,\Gamma(a-c+1)\,\Gamma(c)\,G_{3,3}^{1,0}\!\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1-b,a-c+1\\0,\frac{1}{2},1-c\end{matrix}\right)$$

07.23.26.0073.01

$$\theta(|z|-1)\,(z-1)^{c-1}\,{}_2F_1\!\left(a,b;c;\frac{z-1}{z}\right)=\Gamma(c)\,G_{2,2}^{0,2}\!\left(z\,\middle|\,\begin{matrix}a+b,c\\a,b\end{matrix}\right)/;\ \notin(-\infty,-1)\land\operatorname{Re}(c)>0$$

07.23.26.0074.01

$$\theta(|z|-1)\,(z+1)^{-a}\,{}_2F_1\!\left(a,b;c;\frac{1}{z+1}\right)=\Gamma(1-a)\,\Gamma(b-c+1)\,\Gamma(c)\,G_{2,2}^{0,1}\!\left(z\,\middle|\,\begin{matrix}1-a,c-a\\0,c-a-b\end{matrix}\right)$$

07.23.26.0075.01

$$\theta(|z|-1)\,(z+1)^{-b}\,{}_2F_1\!\left(a,b;c;\frac{1}{z+1}\right)=\Gamma(1-b)\,\Gamma(a-c+1)\,\Gamma(c)\,G_{2,2}^{0,1}\!\left(z\,\middle|\,\begin{matrix}1-b,c-b\\0,c-a-b\end{matrix}\right)$$

07.23.26.0076.01

$$\theta(|z|-1)\,(z-1)^{-a}\,{}_2F_1\!\left(a,b;c;\frac{1}{1-z}\right)=\pi\,\Gamma(1-a)\,\Gamma(b-c+1)\,\Gamma(c)\,G_{3,3}^{0,1}\!\left(z\,\middle|\,\begin{matrix}1-a,\frac{1}{2}-a,c-a\\\frac{1}{2}-a,0,c-a-b\end{matrix}\right)$$

07.23.26.0077.01

$$\theta(|z|-1)\,(z-1)^{-b}\,{}_2F_1\!\left(a,b;c;\frac{1}{1-z}\right)=\pi\,\Gamma(1-b)\,\Gamma(a-c+1)\,\Gamma(c)\,G_{3,3}^{0,1}\!\left(z\,\middle|\,\begin{matrix}1-b,\frac{1}{2}-b,c-b\\\frac{1}{2}-b,0,c-a-b\end{matrix}\right)$$

Classical cases involving sgn

07.23.26.0078.01

$$(z+1)^{-2a}\,((1-z)\operatorname{sgn}(1-|z|))^{4a-2c+1}\,{}_2F_1\!\left(a,a+\frac{1}{2};c;\frac{4z}{(z+1)^2}\right)=\frac{\Gamma(2a-c+1)\,\Gamma(c)}{\Gamma(2c-2a-1)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}2(a-c+1),2a-c+1\\0,1-c\end{matrix}\right)$$

07.23.26.0079.01

$$((1-z)\operatorname{sgn}(1-|z|))^{-2a}\,{}_2F_1\!\left(a,b;a+b+\frac{1}{2};-\frac{4z}{(z-1)^2}\right)=\frac{\Gamma\!\left(b-a+\frac{1}{2}\right)\Gamma\!\left(a+b+\frac{1}{2}\right)}{\Gamma(2a)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}1-2a,b-a+\frac{1}{2}\\0,\frac{1}{2}-a-b\end{matrix}\right)$$

07.23.26.0080.01

$$((1-z)\operatorname{sgn}(1-|z|))^{-2b}\,{}_2F_1\!\left(a,b;a+b+\frac{1}{2};-\frac{4z}{(z-1)^2}\right)=\frac{\Gamma\!\left(a-b+\frac{1}{2}\right)\Gamma\!\left(a+b+\frac{1}{2}\right)}{\Gamma(2b)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}1-2b,a-b+\frac{1}{2}\\0,\frac{1}{2}-a-b\end{matrix}\right)$$

07.23.26.0081.01

$$(z+1)\,((1-z)\operatorname{sgn}(1-|z|))^{-2a}\,{}_2F_1\!\left(a,b;a+b-\frac{1}{2};-\frac{4z}{(z-1)^2}\right)=\frac{\Gamma\!\left(b-a+\frac{1}{2}\right)\Gamma\!\left(a+b-\frac{1}{2}\right)}{\Gamma(2a-1)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}2-2a,b-a+\frac{1}{2}\\0,\frac{3}{2}-a-b\end{matrix}\right)$$

07.23.26.0082.01

$$(z+1)\left((1-z)\operatorname{sgn}(1-|z|)\right)^{-2b}\,{}_2F_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-\frac{4z}{(z-1)^2}\right)=\frac{\Gamma\!\left(a-b+\frac{1}{2}\right)\Gamma\!\left(a+b-\frac{1}{2}\right)}{\Gamma(2b-1)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}2-2b,\,a-b+\frac{1}{2}\\0,\,\frac{3}{2}-a-b\end{matrix}\right)$$

07.23.26.0083.01

$$\left(1+\sqrt{z}\right)^{-2b}\left(\left(1-\sqrt{z}\right)\operatorname{sgn}(1-|z|)\right)^{2a-2b}\,{}_2F_1\!\left(a,\,b;\,2b;\,\frac{4\sqrt{z}}{\left(\sqrt{z}+1\right)^2}\right)=\frac{\Gamma\!\left(a-b+\frac{1}{2}\right)\Gamma\!\left(b+\frac{1}{2}\right)}{\Gamma(2b-a)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}a-2b+1,\,a-b+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{matrix}\right)$$

07.23.26.0084.01

$$\left(\left(1-\sqrt{z}\right)\operatorname{sgn}(1-|z|)\right)^{-2a}\,{}_2F_1\!\left(a,\,b;\,2b;\,-\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right)=\frac{\Gamma\!\left(b+\frac{1}{2}\right)\Gamma\!\left(b-a+\frac{1}{2}\right)}{\Gamma(a)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}1-a,\,b-a+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{matrix}\right)$$

07.23.26.0085.01

$$\left(\sqrt{z}+1\right)^{2a-2b}\left(\left(1-\sqrt{z}\right)\operatorname{sgn}(1-|z|)\right)^{-2b}\,{}_2F_1\!\left(a,\,b;\,2b;\,-\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right)=\frac{\Gamma\!\left(a-b+\frac{1}{2}\right)\Gamma\!\left(b+\frac{1}{2}\right)}{\Gamma(2b-a)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{matrix}a-2b+1,\,a-b+\frac{1}{2}\\0,\,\frac{1}{2}-b\end{matrix}\right)$$

Classical cases involving powers of ${}_2F_1$

07.23.26.0086.01

$${}_2F_1\!\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right)^2=\frac{2^{2a+2b-1}\,\Gamma\!\left(a+b+\frac{1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}1-2a,\,1-2b,\,1-a-b\\0,\,\frac{1}{2}-a-b,\,1-2a-2b\end{matrix}\right)$$

07.23.26.0087.01

$$(1+z)\,{}_2F_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right)^2=\frac{2^{2a+2b-3}\,\Gamma\!\left(a+b-\frac{1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}2-2a,\,2-2b,\,2-a-b\\0,\,\frac{3}{2}-a-b,\,3-2a-2b\end{matrix}\right)$$

07.23.26.0088.01

$$(z+1)^{-2a}\,{}_2F_1\!\left(a,\,a+\frac{1}{2};\,c;\,\frac{1}{z+1}\right)^2=\frac{4^{c-1}\,\Gamma(c)^2}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2c-2a-1)}\,G_{3,3}^{3,1}\!\left(z\,\middle|\,\begin{matrix}1-2a,\,c-2a,\,2c-2a-1\\0,\,2c-4a-1,\,c-2a-\frac{1}{2}\end{matrix}\right)/;\,z\notin(-1,\,0)$$

07.23.26.0089.01

$$(1+z)^{-2a}\,{}_2F_1\!\left(a,\,a+\frac{1}{2};\,c;\,\frac{z}{z+1}\right)^2=\frac{4^{c-1}\,\Gamma(c)^2}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2c-2a-1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}1-2a,\,2a-2c+2,\,\frac{3}{2}-c\\0,\,1-c,\,2-2c\end{matrix}\right)/;\,z\notin(-\infty,\,-1)$$

07.23.26.0090.01

$${}_2F_1\!\left(a,\,b;\,\frac{a+b+1}{2};\,\frac{1-\sqrt{1+z}}{2}\right)^2=\frac{2^{a+b-1}\,\Gamma\!\left(\frac{a+b+1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}1-a,\,1-b,\,1-\frac{a+b}{2}\\0,\,\frac{1-a-b}{2},\,1-a-b\end{matrix}\right)$$

07.23.26.0091.01

$$\left(\sqrt{1+z}+1\right)^{2-2c}\,{}_2F_1\!\left(a,\,1-a;\,c;\,\frac{1-\sqrt{1+z}}{2}\right)^2=\frac{\Gamma(c)^2}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{matrix}a-c+1,\,2-a-c,\,\frac{3}{2}-c\\0,\,1-c,\,2-2c\end{matrix}\right)$$

07.23.26.0092.01

$$ {}_2F_1\left(a,\, b;\, \frac{a+b+1}{2};\, \left(\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)^2\right) = \frac{2^{a+b-1}\,\Gamma\left(\frac{a+b+1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 1,\, \frac{1+a+b}{2},\, a+b \\ b,\, a,\, \frac{a+b}{2} \end{array}\right) /;\, z \notin (-1,\, 0) $$

07.23.26.0093.01

$$ \left(\sqrt{z}+\sqrt{z+1}\right)^{2-2c}\, {}_2F_1\left(a,\, 1-a;\, c;\, \left(\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)^2\right) = \frac{\Gamma(c)^2}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 2-c,\, 1,\, c \\ a,\, 1-a,\, \frac{1}{2} \end{array}\right) /;\, z \notin (-1,\, 0) $$

07.23.26.0094.01

$$ \left(\sqrt{1+z}+1\right)^{-2a}\, {}_2F_1\left(a,\, b;\, a-b+1;\, \left(\frac{\sqrt{1+z}-1}{\sqrt{1+z}+1}\right)^2\right) = \frac{4^{-b}\,\Gamma(a-b+1)^2}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} 1-a,\, 2b-a,\, \frac{1}{2}-a+b \\ 0,\, b-a,\, 2b-2a \end{array}\right) $$

07.23.26.0095.01

$$ \left(\sqrt{z}+\sqrt{z+1}\right)^{-2a}\, {}_2F_1\left(a,\, b;\, a-b+1;\, \left(\frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right)^2\right) = $$
$$ \frac{4^{-b}\,\Gamma(a-b+1)^2}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 1-a,\, 1-b,\, a-2b+1 \\ 0,\, 1-2b,\, \frac{1}{2}-b \end{array}\right) /;\, z \notin (-1,\, 0) $$

Classical cases for products of ${}_2F_1$ with linear arguments

07.23.26.0096.01

$$ {}_2F_1\left(a,\, b;\, a+b-\frac{1}{2};\, -z\right) {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) = \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)^2}{(2a+2b-1)\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} 1-2a,\, 1-2b,\, 1-a-b \\ 0,\, \frac{1}{2}-a-b,\, 2(1-a-b) \end{array}\right) $$

07.23.26.0097.01

$$ {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2F_1\left(a+1,\, b;\, a+b+\frac{1}{2};\, -z\right) = \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(2b)\,\Gamma(2a+1)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} -2a,\, 1-2b,\, 1-a-b \\ 0,\, \frac{1}{2}-a-b,\, 1-2a-2b \end{array}\right) $$

07.23.26.0098.01

$$ {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2F_1\left(a,\, b+1;\, a+b+\frac{1}{2};\, -z\right) = \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b+1)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} 1-2a,\, -2b,\, 1-a-b \\ 0,\, \frac{1}{2}-a-b,\, 1-2a-2b \end{array}\right) $$

07.23.26.0099.01

$$ {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2F_1\left(a+1,\, b+1;\, a+b+\frac{3}{2};\, -z\right) = $$
$$ \frac{2^{2a+2b+1}\,\Gamma\left(a+b+\frac{3}{2}\right)^2}{(2a+2b+1)\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b+1)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} -2a,\, -2b,\, -a-b \\ 0,\, -\frac{1}{2}-a-b,\, -2(a+b) \end{array}\right) $$

07.23.26.0100.01

$$ {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2F_1\left(\frac{1}{2}-a,\, \frac{1}{2}-b;\, \frac{3}{2}-a-b;\, -z\right) = \frac{(1-2a-2b)\cos(\pi(a-b))}{2\sqrt{\pi}\,\cos(\pi(a+b))}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} \frac{1}{2},\, a-b+\frac{1}{2},\, \frac{1}{2}-a+b \\ 0,\, \frac{1}{2}-a-b,\, a+b-\frac{1}{2} \end{array}\right) $$

Classical cases involving products of ${}_2F_1$ with linear arguments

07.23.26.0101.01

$$\frac{1}{\sqrt{1+z}}\,{}_2F_1\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right){}_2F_1\left(a+\frac{1}{2},\,b-\frac{1}{2};\,a+b+\frac{1}{2};\,-z\right)=\frac{\Gamma(2\,(a+b))\,\Gamma\left(a+b+\frac{1}{2}\right)}{\Gamma(2\,a+1)\,\Gamma(2\,b)\,\Gamma(a+b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}-2\,a,\,1-2\,b,\,1-a-b\\0,\,\frac{1}{2}-a-b,\,1-2\,a-2\,b\end{array}\right)$$

07.23.26.0102.01

$$\frac{1}{\sqrt{1+z}}\,{}_2F_1\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right){}_2F_1\left(a+\frac{1}{2},\,b+\frac{1}{2};\,a+b+\frac{3}{2};\,-z\right)=\frac{2^{2\,a+2\,b+1}\,\Gamma\left(a+b+\frac{3}{2}\right)^2}{(2\,a+2\,b+1)\,\sqrt{\pi}\,\Gamma(2\,a+1)\,\Gamma(2\,b+1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}-2\,a,\,-2\,b,\,-a-b\\0,\,-a-b-\frac{1}{2},\,-2\,(a+b)\end{array}\right)$$

07.23.26.0103.01

$$\sqrt{1+z}\,{}_2F_1\left(a-\frac{1}{2},\,b-\frac{1}{2};\,a+b-\frac{1}{2};\,-z\right){}_2F_1\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right)=\frac{2^{2\,a+2\,b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(2\,a-1)\,\Gamma(2\,b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-2\,a,\,2-2\,b,\,2-a-b\\0,\,\frac{3}{2}-a-b,\,3-2\,a-2\,b\end{array}\right)$$

07.23.26.0104.01

$$\sqrt{1+z}\,{}_2F_1\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right){}_2F_1\left(a-\frac{1}{2},\,b+\frac{1}{2};\,a+b-\frac{1}{2};\,-z\right)=\frac{\Gamma(2\,(a+b-1))\,\Gamma\left(a+b-\frac{1}{2}\right)}{\Gamma(2\,a-1)\,\Gamma(2\,b)\,\Gamma(a+b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}2-2\,a,\,1-2\,b,\,2-a-b\\0,\,\frac{3}{2}-a-b,\,3-2\,a-2\,b\end{array}\right)$$

07.23.26.0105.01

$$\sqrt{1+z}\,{}_2F_1\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right){}_2F_1\left(1-a,\,1-b;\,\frac{5}{2}-a-b;\,-z\right)=\frac{(2\,a+2\,b-3)\cos((a-b)\,\pi)}{2\,\sqrt{\pi}\,\cos((a+b)\,\pi)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\,a-b+\frac{1}{2},\,\frac{1}{2}-a+b\\0,\,\frac{3}{2}-a-b,\,a+b-\frac{3}{2}\end{array}\right)$$

07.23.26.0106.01

$$\sqrt{1+z}\,{}_2F_1\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right){}_2F_1\left(a+\frac{1}{2},\,b+\frac{1}{2};\,a+b+\frac{1}{2};\,-z\right)=\frac{2^{2\,a+2\,b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(2\,a)\,\Gamma(2\,b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}1-2\,a,\,1-2\,b,\,1-a-b\\0,\,\frac{1}{2}-a-b,\,1-2\,a-2\,b\end{array}\right)$$

07.23.26.0107.01

$$\sqrt{1+z}\,{}_2F_1\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right){}_2F_1\left(1-a,\,1-b;\,\frac{3}{2}-a-b;\,-z\right)=\frac{(1-2\,a-2\,b)\cos((a-b)\,\pi)}{2\,\sqrt{\pi}\,\cos((a+b)\,\pi)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\,a-b+\frac{1}{2},\,b-a+\frac{1}{2}\\0,\,\frac{1}{2}-a-b,\,a+b-\frac{1}{2}\end{array}\right)$$

07.23.26.0108.01

$$(1+z)\,{}_2F_1\left(a,\, b;\, a+b-\frac{1}{2};\, -z\right) {}_2F_1\left(\frac{3}{2}-a,\, \frac{3}{2}-b;\, \frac{5}{2}-a-b;\, -z\right) = \frac{(2\,a+2\,b-3)\cos((a-b)\,\pi)}{2\,\sqrt{\pi}\,\cos((a+b)\,\pi)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} \frac{1}{2},\, a-b+\frac{1}{2},\, b-a+\frac{1}{2} \\ 0,\, \frac{3}{2}-a-b,\, a+b-\frac{3}{2} \end{array}\right)$$

Classical cases for products of ${}_2F_1$ with algebraic arguments

07.23.26.0109.01

$${}_2F_1\left(a,\, b;\, c;\, -2\left(z+\sqrt{z+1}\,\sqrt{z}\right)\right) {}_2F_1\left(a,\, b;\, c;\, -2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right) = \frac{2^{1-c}\,\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{array}{c} 1-a,\, 1-b,\, a-c+1,\, b-c+1 \\ 0,\, \frac{1-c}{2},\, 1-\frac{c}{2},\, 1-c \end{array}\right)$$

07.23.26.0110.01

$${}_2F_1\left(a,\, b;\, c;\, -\frac{2\left(1+\sqrt{z+1}\right)}{z}\right) {}_2F_1\left(a,\, b;\, c;\, -\frac{2\left(1-\sqrt{z+1}\right)}{z}\right) = \frac{2^{1-c}\,\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{4,1}\left(z \,\middle|\, \begin{array}{c} 1,\, \frac{c}{2},\, \frac{c+1}{2},\, c \\ a,\, b,\, c-a,\, c-b \end{array}\right) /;\, z \notin (-1,\, 0)$$

07.23.26.0111.01

$${}_2F_1\left(a,\, b;\, c;\, \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(a,\, b;\, a+b-c+1;\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{2^{a+b-1}\,\Gamma(a+b-c+1)\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{array}{c} 1-a,\, 1-b,\, 1-\frac{a+b}{2},\, \frac{1-a-b}{2} \\ 0,\, 1-a-b,\, 1-c,\, c-a-b \end{array}\right)$$

07.23.26.0112.01

$${}_2F_1\left(a,\, a+\frac{1}{2};\, c;\, \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(a,\, a+\frac{1}{2};\, 2\,a-c+\frac{3}{2};\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{2^{4\,a-\frac{3}{2}}\,\Gamma\left(2\,a-c+\frac{3}{2}\right)\Gamma(c)}{\pi\,\Gamma(2\,a)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{array}{c} \frac{1}{4}-a,\, \frac{1}{2}-a,\, \frac{3}{4}-a,\, 1-a \\ 0,\, 1-c,\, \frac{1}{2}-2\,a,\, \frac{1}{2}-2\,a+c \end{array}\right)$$

07.23.26.0113.01

$${}_2F_1\left(a,\, b;\, \frac{a+b+1}{2};\, \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(1-a,\, 1-b;\, \frac{3-a-b}{2};\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{1-a-b}{2\,\sqrt{\pi}\,\cos\left(\frac{1}{2}\,(a+b)\,\pi\right)} \cos\left(\frac{1}{2}\,(a-b)\,\pi\right) G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} \frac{1}{2},\, \frac{a-b+1}{2},\, \frac{b-a+1}{2} \\ 0,\, \frac{1-a-b}{2},\, \frac{a+b-1}{2} \end{array}\right)$$

07.23.26.0114.01

$${}_2F_1\left(a,\, 1-a;\, c;\, \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(a,\, 1-a;\, 2-c;\, \frac{1-\sqrt{1+z}}{2}\right) = \frac{\sin(a\,\pi)\,(1-c)}{\sqrt{\pi}\,\sin(c\,\pi)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} \frac{1}{2},\, 1-a,\, a \\ 0,\, 1-c,\, c-1 \end{array}\right)$$

07.23.26.0115.01

$${}_2F_1\left(a,\,b;\,c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(a,\,b;\,a+b-c+1;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{a+b-1}\,\Gamma(a+b-c+1)\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\!\left(z\left|\begin{array}{c}1,\,c,\,a+b,\,a+b-c+1\\ a,\,b,\,\frac{a+b}{2},\,\frac{a+b+1}{2}\end{array}\right.\right)/;\,z\notin(-1,\,0)$$

07.23.26.0116.01

$${}_2F_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(a,\,a+\frac{1}{2};\,2a-c+\frac{3}{2};\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{4a-\frac{3}{2}}\,\Gamma\left(2a-c+\frac{3}{2}\right)\Gamma(c)}{\pi\,\Gamma(2a)}\,G_{4,4}^{4,1}\!\left(z\left|\begin{array}{c}1,\,c,\,2a+\frac{1}{2},\,2a-c+\frac{3}{2}\\ a,\,a+\frac{1}{4},\,a+\frac{1}{2},\,a+\frac{3}{4}\end{array}\right.\right)/;\,z\notin(-1,\,0)$$

07.23.26.0117.01

$${}_2F_1\left(a,\,b;\,\frac{1+a+b}{2};\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(1-a,\,1-b;\,\frac{3-a-b}{2};\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{(1-a-b)\cos\left(\frac{1}{2}(a-b)\pi\right)}{2\sqrt{\pi}\,\cos\left(\frac{1}{2}(a+b)\pi\right)}\,G_{3,3}^{3,1}\!\left(z\left|\begin{array}{c}1,\,\frac{1+a+b}{2},\,\frac{3-a-b}{2}\\ \frac{1}{2},\,\frac{1+a-b}{2},\,\frac{1-a+b}{2}\end{array}\right.\right)/;\,z\notin(-1,\,0)$$

07.23.26.0118.01

$${}_2F_1\left(a,\,1-a;\,c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\left(a,\,1-a;\,2-c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{(1-c)\sin(a\pi)}{\sqrt{\pi}\,\sin(c\pi)}\,G_{3,3}^{3,1}\!\left(z\left|\begin{array}{c}1,\,c,\,2-c\\ a,\,1-a,\,\frac{1}{2}\end{array}\right.\right)/;\,z\notin(-1,\,0)$$

Classical cases involving products of ${}_2F_1$ with algebraic arguments

07.23.26.0119.01

$$\left(2z+2\sqrt{z+1}\,\sqrt{z}+1\right)^{c-a-b}{}_2F_1\left(a,\,b;\,c;\,-2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right){}_2F_1\left(c-a,\,c-b;\,c;\,-2\left(z+\sqrt{z}\,\sqrt{z+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\left|\begin{array}{c}1-a,\,1-b,\,a-c+1,\,b-c+1\\ 0,\,\frac{1-c}{2},\,1-\frac{c}{2},\,1-c\end{array}\right.\right)$$

07.23.26.0120.01

$$\left(2z-2\sqrt{z+1}\,\sqrt{z}+1\right)^{c-a-b}{}_2F_1\left(a,\,b;\,c;\,-2\left(z+\sqrt{z}\,\sqrt{z+1}\right)\right){}_2F_1\left(c-a,\,c-b;\,c;\,-2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\left|\begin{array}{c}1-a,\,1-b,\,a-c+1,\,b-c+1\\ 0,\,\frac{1-c}{2},\,1-\frac{c}{2},\,1-c\end{array}\right.\right)$$

07.23.26.0121.01

$$\left(z+2\sqrt{z+1}+2\right)^{c-a-b}{}_2F_1\left(a,\,b;\,c;\,-\frac{2\left(1-\sqrt{1+z}\right)}{z}\right){}_2F_1\left(c-a,\,c-b;\,c;\,-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\!\left(z\left|\begin{array}{c}c-a-b+1,\,\frac{3c}{2}-a-b,\,\frac{3c+1}{2}-a-b,\,2c-a-b\\ c-a,\,c-b,\,2c-2a-b,\,2c-2b-a\end{array}\right.\right)/;\,z\notin(-1,\,0)$$

07.23.26.0122.01

$$\left(z-2\sqrt{z+1}+2\right)^{c-a-b}\,{}_2F_1\!\left(a,b;c;-\frac{2\left(1+\sqrt{1+z}\right)}{z}\right){}_2F_1\!\left(c-a,c-b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{matrix}c-a-b+1,\frac{3c}{2}-a-b,\frac{3c+1}{2}-a-b,2c-a-b\\ c-a,c-b,2c-2a-b,2c-2b-a\end{matrix}\right)/;\,z\notin(-1,0)$$

07.23.26.0123.01

$$\left(1-2\left(z+\sqrt{z-1}\sqrt{z}\right)\right)^{a}\,{}_2F_1\!\left(a,b;c;2\left(z+\sqrt{z-1}\sqrt{z}\right)\right){}_2F_1\!\left(a,c-b;c;2\left(z+\sqrt{z-1}\sqrt{z}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(-z\,\middle|\,\begin{matrix}1-a,a-c+1,1-b,b-c+1\\ 0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)$$

07.23.26.0124.01

$$\left(1-2z-2\sqrt{z-1}\sqrt{z}\right)^{b}\,{}_2F_1\!\left(a,b;c;2\left(z+\sqrt{z}\sqrt{z-1}\right)\right){}_2F_1\!\left(c-a,b;c;2\left(z+\sqrt{z}\sqrt{z-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(-z\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\ 0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)$$

07.23.26.0125.01

$$\left(z-2\left(\sqrt{1-z}+1\right)\right)^{a}\,{}_2F_1\!\left(a,b;c;\frac{2\left(\sqrt{1-z}+1\right)}{z}\right){}_2F_1\!\left(a,c-b;c;\frac{2\left(\sqrt{1-z}+1\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,z^a\,G_{4,4}^{4,1}\!\left(-z\,\middle|\,\begin{matrix}1,\frac{c}{2},\frac{c+1}{2},c\\ a,b,c-a,c-b\end{matrix}\right)$$

07.23.26.0126.01

$$\left(z-2\left(\sqrt{1-z}+1\right)\right)^{b}\,{}_2F_1\!\left(a,b;c;\frac{2\left(1+\sqrt{1-z}\right)}{z}\right){}_2F_1\!\left(c-a,b;c;\frac{2\left(1+\sqrt{1-z}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,z^a\,G_{4,4}^{4,1}\!\left(-z\,\middle|\,\begin{matrix}1,\frac{c}{2},\frac{c+1}{2},c\\ a,b,c-a,c-b\end{matrix}\right)$$

07.23.26.0127.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2F_1\!\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(b-c+1,a-c+1;a+b-c+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{a+b-c}\,\Gamma(a+b-c+1)\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}1-a,1-b,\frac{1-a-b}{2},1-\frac{a+b}{2}\\ 0,1-a-b,1-c,c-a-b\end{matrix}\right)$$

07.23.26.0128.01

$$\left(\sqrt{1+z}+1\right)^{a+b-2c+1}\,{}_2F_1\!\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(1-b,1-a;c-a-b+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{\Gamma(c)\,\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}a-c+1,b-c+1,\frac{a+b+1}{2}-c,\frac{a+b}{2}-c+1\\ 0,a+b-2c+1,1-c,a+b-c\end{matrix}\right)$$

07.23.26.0129.01

$$\left(\sqrt{1+z}+1\right)^{a+b-c} {}_2F_1\left(a, b; c; \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(c-a, c-b; c-a-b+1; \frac{1-\sqrt{1+z}}{2}\right) = \frac{2^{c-1}\,\Gamma(c)\,\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{array}{c} a-c+1,\, b-c+1,\, \frac{a+b}{2}-c+1,\, \frac{a+b+1}{2}-c \\ 0,\, a+b-2c+1,\, 1-c,\, a+b-c \end{array}\right)$$

07.23.26.0130.01

$$\left(\sqrt{1+z}+1\right)^{1-c} {}_2F_1\left(a, a+\frac{1}{2}; c; \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(a-c+a, a-c+\frac{3}{2}; 2a-c+\frac{3}{2}; \frac{1-\sqrt{1+z}}{2}\right) = \frac{2^{4a-c-\frac{1}{2}}\,\Gamma\left(2a-c+\frac{3}{2}\right)\Gamma(c)}{\pi\,\Gamma(2a)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{array}{c} \frac{1}{4}-a,\, \frac{1}{2}-a,\, \frac{3}{4}-a,\, 1-a \\ 0,\, c-2a-\frac{1}{2},\, \frac{1}{2}-2a,\, 1-c \end{array}\right)$$

07.23.26.0131.01

$$\left(\sqrt{1+z}+1\right)^{2a-2c+\frac{3}{2}} {}_2F_1\left(a, a+\frac{1}{2}; c; \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(1-a, \frac{1}{2}-a; \frac{1}{2}-2a+c; \frac{1-\sqrt{1+z}}{2}\right) = \frac{4^{c-a-1}\,\Gamma(c)\,\Gamma\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{array}{c} a-c+\frac{3}{4},\, a-c+1,\, a-c+\frac{5}{4},\, a-c+\frac{3}{2} \\ 0,\, 1-c,\, 2a-2c+\frac{3}{2},\, 2a-c+\frac{1}{2} \end{array}\right)$$

07.23.26.0132.01

$$\left(\sqrt{1+z}+1\right)^{1-c} {}_2F_1\left(a, 1-a; c; \frac{1-\sqrt{1+z}}{2}\right) {}_2F_1\left(2-a-c, a-c+1; 2-c; \frac{1-\sqrt{1+z}}{2}\right) = \frac{2^{1-c}\,(1-c)\csc(c\pi)\sin(a\pi)}{\sqrt{\pi}}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} \frac{1}{2},\, 1-a,\, a \\ 0,\, c-1,\, 1-c \end{array}\right)$$

07.23.26.0133.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c} {}_2F_1\left(a, b; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(b-c+1, a-c+1; a+b-c+1; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{a+b-c}\,\Gamma(a+b-c+1)\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\, G_{4,4}^{4,1}\left(z \,\middle|\, \begin{array}{c} \frac{3-c}{2},\, a+b+\frac{1-c}{2},\, \frac{c+1}{2},\, a+b+\frac{3-3c}{2} \\ a+\frac{1-c}{2},\, b+\frac{1-c}{2},\, \frac{a+b-c+1}{2},\, \frac{a+b-c}{2}+1 \end{array}\right) /;\, z \notin (-1, 0)$$

07.23.26.0134.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{a+b-2c+1} {}_2F_1\left(a, b; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(1-b, 1-a; -a-b+c+1; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\Gamma(c)\,\Gamma(-a-b+c+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{4,1}\left(z \,\middle|\, \begin{array}{c} \frac{a+b+3}{2}-c,\, \frac{1-a-b}{2}+c,\, \frac{1+a+b}{2},\, \frac{3-a-b}{2} \\ \frac{1}{2},\, 1,\, \frac{1-a+b}{2},\, \frac{1+a-b}{2} \end{array}\right) /;\, z \notin (-1, 0)$$

07.23.26.0135.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{a+b-c} {}_2F_1\left(a, b; c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2F_1\left(c-a, c-b; 1-a-b+c; \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{c-1}\,\Gamma(c)\,\Gamma(1-a-b+c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{4,1}\left(z \,\middle|\, \begin{array}{c} \frac{a+b-c}{2}+1,\, \frac{3c-a-b}{2},\, \frac{a+b+c}{2},\, \frac{c-a-b}{2}+1 \\ \frac{b-a+c}{2},\, \frac{a-b+c}{2},\, \frac{c}{2},\, \frac{c+1}{2} \end{array}\right) /;\, z \notin (-1, 0)$$

07.23.26.0136.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c}\,{}_2F_1\!\left(a,\,a+\frac{1}{2};\,c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\!\left(a-c+1,\,a-c+\frac{3}{2};\,2a-c+\frac{3}{2};\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{4a-c-\frac{1}{2}}\,\Gamma\!\left(2a-c+\frac{3}{2}\right)\Gamma(c)}{\pi\,\Gamma(2a)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}\frac{3-c}{2},\,2a-\frac{3c}{2}+2,\,2a-\frac{c}{2}+1,\,\frac{c+1}{2}\\ a+\frac{1-c}{2},\,a+\frac{3-2c}{4},\,a-\frac{c}{2}+1,\,a+\frac{5-2c}{4}\end{array}\right)/;\,z\notin(-1,0)$$

07.23.26.0137.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2a-2c+\frac{3}{2}}\,{}_2F_1\!\left(a,\,a+\frac{1}{2};\,c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2F_1\!\left(1-a,\,\frac{1}{2}-a;\,\frac{1}{2}-2a+c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{4^{c-a-1}\,\Gamma(c)\,\Gamma\!\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}a-c+\frac{7}{4},\,a+\frac{3}{4},\,c-a+\frac{1}{4},\,\frac{5}{4}-a\\ \frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,1\end{array}\right)/;\,z\notin(-1,0)$$

07.23.26.0138.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2F_1\!\left(a,\,1-a;\,c;\,\frac{1-\sqrt{1+z}}{2}\right){}_2F_1\!\left(2-a-c,\,a-c+1;\,2-c;\,\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{1-c}\,(1-c)\csc(c\pi)\sin(a\pi)}{\sqrt{\pi}}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\,1-a,\,a\\ 0,\,c-1,\,1-c\end{array}\right)/;\,z\notin(-1,0)$$

Classical cases involving ${}_2\tilde{F}_1$ with linear arguments

07.23.26.0139.01

$${}_2F_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right){}_2\tilde{F}_1\!\left(a-1,\,b-1;\,a+b-\frac{3}{2};\,-z\right)=\frac{4^{a+b-2}\,\Gamma\!\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}2-2a,\,2-2b,\,2-a-b\\ 0,\,\frac{3}{2}-a-b,\,2(2-a-b)\end{array}\right)$$

07.23.26.0140.01

$${}_2F_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right){}_2\tilde{F}_1\!\left(a-1,\,b;\,a+b-\frac{1}{2};\,-z\right)=\frac{2^{2a+2b-3}\,\Gamma\!\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}2-2a,\,1-2b,\,2-a-b\\ 0,\,\frac{3}{2}-a-b,\,3-2a-2b\end{array}\right)$$

07.23.26.0141.01

$${}_2F_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right){}_2\tilde{F}_1\!\left(a,\,b-1;\,a+b-\frac{1}{2};\,-z\right)=\frac{2^{2a+2b-3}\,\Gamma\!\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}1-2a,\,2-2b,\,2-a-b\\ 0,\,\frac{3}{2}-a-b,\,3-2a-2b\end{array}\right)$$

07.23.26.0142.01

$${}_2F_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right){}_2\tilde{F}_1\!\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right)=\frac{2^{2a+2b-2}\,\Gamma\!\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}1-2a,\,1-2b,\,1-a-b\\ 0,\,\frac{1}{2}-a-b,\,2(1-a-b)\end{array}\right)$$

07.23.26.0143.01

$${}_2F_1\!\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right){}_2\tilde{F}_1\!\left(a,\,b;\,a+b-\frac{1}{2};\,-z\right)=\frac{2^{2a+2b-2}\,\Gamma\!\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}1-2a,\,1-2b,\,1-a-b\\ 0,\,\frac{1}{2}-a-b,\,2(1-a-b)\end{array}\right)$$

07.23.26.0144.01

$${}_2F_1\!\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right){}_2\tilde{F}_1\!\left(a,\,b;\,a+b+\frac{1}{2};\,-z\right)=\frac{2^{2a+2b-1}\,\Gamma\!\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}1-2a,\,1-2b,\,1-a-b\\ 0,\,\frac{1}{2}-a-b,\,1-2a-2b\end{array}\right)$$

07.23.26.0145.01

$$ {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(a+1,\, b;\, a+b+\frac{1}{2};\, -z\right) = \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2b)\,\Gamma(2a+1)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} -2a,\, 1-2b,\, 1-a-b \\ 0,\, \frac{1}{2}-a-b,\, 1-2a-2b \end{matrix}\right) $$

07.23.26.0146.01

$$ {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(a,\, b+1;\, a+b+\frac{1}{2};\, -z\right) = \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b+1)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 1-2a,\, -2b,\, 1-a-b \\ 0,\, \frac{1}{2}-a-b,\, 1-2a-2b \end{matrix}\right) $$

07.23.26.0147.01

$$ {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(a+1,\, b+1;\, a+b+\frac{3}{2};\, -z\right) = \frac{2^{2a+2b}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b+1)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} -2a,\, -2b,\, -a-b \\ 0,\, -a-b-\frac{1}{2},\, -2(a+b) \end{matrix}\right) $$

07.23.26.0148.01

$$ {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(\frac{1}{2}-a,\, \frac{1}{2}-b;\, \frac{3}{2}-a-b;\, -z\right) = \frac{\cos(\pi(a-b))\,\Gamma\left(a+b+\frac{1}{2}\right)}{\pi^{3/2}} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} \frac{1}{2},\, a-b+\frac{1}{2},\, \frac{1}{2}-a+b \\ 0,\, \frac{1}{2}-a-b,\, a+b-\frac{1}{2} \end{matrix}\right) $$

07.23.26.0149.01

$$ (z+1)\, {}_2F_1\left(a,\, b;\, a+b-\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(a,\, b;\, a+b-\frac{1}{2};\, -z\right) = \frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 2-2a,\, 2-2b,\, 2-a-b \\ 0,\, \frac{3}{2}-a-b,\, 3-2a-2b \end{matrix}\right) $$

Classical cases involving algebraic functions and ${}_2\tilde{F}_1$ with linear arguments

07.23.26.0150.01

$$ \frac{1}{\sqrt{z+1}}\, {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(a-\frac{1}{2},\, b-\frac{1}{2};\, a+b-\frac{1}{2};\, -z\right) = \frac{4^{a+b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 1-2a,\, 1-2b,\, 1-a-b \\ 0,\, \frac{1}{2}-a-b,\, 2(1-a-b) \end{matrix}\right) $$

07.23.26.0151.01

$$ \frac{1}{\sqrt{z+1}}\, {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(a-\frac{1}{2},\, b+\frac{1}{2};\, a+b+\frac{1}{2};\, -z\right) = $$
$$ \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b+1)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 1-2a,\, -2b,\, 1-a-b \\ 0,\, \frac{1}{2}-a-b,\, 1-2a-2b \end{matrix}\right) $$

07.23.26.0152.01

$$ \frac{1}{\sqrt{z+1}}\, {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(a+\frac{1}{2},\, b-\frac{1}{2};\, a+b+\frac{1}{2};\, -z\right) = $$
$$ \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} -2a,\, 1-2b,\, 1-a-b \\ 0,\, \frac{1}{2}-a-b,\, 1-2a-2b \end{matrix}\right) $$

07.23.26.0153.01

$$ \frac{1}{\sqrt{z+1}}\, {}_2F_1\left(a,\, b;\, a+b+\frac{1}{2};\, -z\right) {}_2\tilde{F}_1\left(a+\frac{1}{2},\, b+\frac{1}{2};\, a+b+\frac{3}{2};\, -z\right) = $$
$$ \frac{2^{2a+2b}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a+1)\,\Gamma(2b+1)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} -2a,\, -2b,\, -a-b \\ 0,\, -a-b-\frac{1}{2},\, -2(a+b) \end{matrix}\right) $$

07.23.26.0154.01

$$\sqrt{z+1}\ {}_2F_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(a-\frac{1}{2},b-\frac{1}{2};a+b-\frac{1}{2};-z\right)= \frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{l}2-2a,2-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.23.26.0155.01

$$\sqrt{z+1}\ {}_2F_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(a-\frac{1}{2},b-\frac{1}{2};a+b-\frac{1}{2};-z\right)= \frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{l}2-2a,2-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.23.26.0156.01

$$\sqrt{z+1}\ {}_2F_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(a-\frac{1}{2},b+\frac{1}{2};a+b-\frac{1}{2};-z\right)= \frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a-1)\,\Gamma(2b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{l}2-2a,1-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.23.26.0157.01

$$\sqrt{z+1}\ {}_2F_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(a+\frac{1}{2},b-\frac{1}{2};a+b-\frac{1}{2};-z\right)= \frac{2^{2a+2b-3}\,\Gamma\left(a+b-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b-1)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{l}1-2a,2-2b,2-a-b\\0,\frac{3}{2}-a-b,3-2a-2b\end{array}\right)$$

07.23.26.0158.01

$$\sqrt{z+1}\ {}_2F_1\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\left(a+\frac{1}{2},b+\frac{1}{2};a+b+\frac{1}{2};-z\right)= \frac{2^{2a+2b-1}\,\Gamma\left(a+b+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2b)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{l}1-2a,1-2b,1-a-b\\0,\frac{1}{2}-a-b,1-2a-2b\end{array}\right)$$

07.23.26.0159.01

$$\sqrt{z+1}\ {}_2F_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(1-a,1-b;-a-b+\frac{5}{2};-z\right)= \frac{1}{\pi^{3/2}}\cos((b-a)\pi)\,\Gamma\left(a+b-\frac{1}{2}\right)G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{l}\frac{1}{2},\frac{1}{2}-a+b,a-b+\frac{1}{2}\\0,a+b-\frac{3}{2},\frac{3}{2}-a-b\end{array}\right)$$

07.23.26.0160.01

$$\sqrt{z+1}\ {}_2F_1\left(a,b;a+b+\frac{1}{2};-z\right){}_2\tilde{F}_1\left(1-a,1-b;-a-b+\frac{3}{2};-z\right)= \frac{\cos((b-a)\pi)\,\Gamma\left(a+b+\frac{1}{2}\right)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{l}\frac{1}{2},\frac{1}{2}-a+b,a-b+\frac{1}{2}\\0,a+b-\frac{1}{2},\frac{1}{2}-a-b\end{array}\right)$$

07.23.26.0161.01

$$(z+1)\,{}_2F_1\left(a,b;a+b-\frac{1}{2};-z\right){}_2\tilde{F}_1\left(\frac{3}{2}-a,\frac{3}{2}-b;\frac{5}{2}-a-b;-z\right)= \frac{\cos((a-b)\pi)\,\Gamma\left(a+b-\frac{1}{2}\right)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{l}\frac{1}{2},a-b+\frac{1}{2},b-a+\frac{1}{2}\\0,\frac{3}{2}-a-b,a+b-\frac{3}{2}\end{array}\right)$$

Classical cases involving ${}_2\tilde{F}_1$ with algebraic arguments

07.23.26.0162.01

$$ {}_2F_1\left(a,b;c;-2\left(z+\sqrt{z+1}\,\sqrt{z}\right)\right){}_2\tilde{F}_1\left(a,b;c;-2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{array}{c}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{array}\right) $$

07.23.26.0163.01

$$ {}_2F_1\left(a,b;c;-2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right){}_2\tilde{F}_1\left(a,b;c;-2\left(z+\sqrt{z+1}\,\sqrt{z}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{array}{c}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{array}\right) $$

07.23.26.0164.01

$$ {}_2F_1\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(a,b;a+b-c+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{a+b-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{array}{c}1-a,1-b,1-\frac{a+b}{2},\frac{1-a-b}{2}\\0,1-a-b,1-c,c-a-b\end{array}\right) $$

07.23.26.0165.01

$$ {}_2F_1\left(a,b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right){}_2\tilde{F}_1\left(a,b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}1,\frac{c}{2},\frac{c+1}{2},c\\a,b,c-a,c-b\end{array}\right)/;\,z\notin(-1,0) $$

07.23.26.0166.01

$$ {}_2F_1\left(a,b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right){}_2\tilde{F}_1\left(a,b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}1,\frac{c}{2},\frac{c+1}{2},c\\a,b,c-a,c-b\end{array}\right)/;\,z\notin(-1,0) $$

07.23.26.0167.01

$$ {}_2F_1\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(a,a+\frac{1}{2};2a-c+\frac{3}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{4a-\frac{3}{2}}\,\Gamma(c)}{\pi\,\Gamma(2a)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{array}{c}\frac{1}{4}-a,\frac{1}{2}-a,\frac{3}{4}-a,1-a\\0,1-c,\frac{1}{2}-2a,\frac{1}{2}-2a+c\end{array}\right) $$

07.23.26.0168.01

$$ {}_2F_1\left(a,1-a;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(a,1-a;2-c;\frac{1-\sqrt{1+z}}{2}\right)=\frac{\sin(a\pi)\,\Gamma(c)}{\pi^{3/2}}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},1-a,a\\0,1-c,c-1\end{array}\right) $$

07.23.26.0169.01

$$ {}_2F_1\left(a,b;\frac{a+b+1}{2};\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(a,b;\frac{a+b+1}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{a+b-1}\,\Gamma\left(\frac{a+b+1}{2}\right)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}1-a,1-b,1-\frac{a+b}{2}\\0,\frac{1-a-b}{2},1-a-b\end{array}\right) $$

07.23.26.0170.01

$$ {}_2F_1\left(a,b;\frac{1+a+b}{2};\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(1-a,1-b;\frac{3-a-b}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{\cos\left(\frac{1}{2}(a-b)\pi\right)\Gamma\left(\frac{1}{2}(a+b+1)\right)}{\pi^{3/2}}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{a-b+1}{2},\frac{b-a+1}{2}\\0,\frac{1-a-b}{2},\frac{a+b-1}{2}\end{array}\right) $$

07.23.26.0171.01

$${}_2F_1\left(a,\, b;\, c;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(a,\, b;\, a+b-c+1;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{a+b-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\, G_{4,4}^{4,1}\left(z \,\middle|\, \begin{array}{c} 1,\, c,\, a+b,\, a+b-c+1 \\ a,\, b,\, \frac{a+b}{2},\, \frac{a+b+1}{2} \end{array}\right) /;\, z \notin (-1,\, 0)$$

07.23.26.0172.01

$${}_2F_1\left(a,\, a+\frac{1}{2};\, c;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, 2a-c+\frac{3}{2};\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{4a-\frac{3}{2}}\,\Gamma(c)}{\pi\,\Gamma(2a)}\, G_{4,4}^{4,1}\left(z \,\middle|\, \begin{array}{c} 1,\, c,\, 2a+\frac{1}{2},\, 2a-c+\frac{3}{2} \\ a,\, a+\frac{1}{4},\, a+\frac{1}{2},\, a+\frac{3}{4} \end{array}\right) /;\, z \notin (-1,\, 0)$$

07.23.26.0173.01

$${}_2F_1\left(a,\, 1-a;\, c;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(a,\, 1-a;\, 2-c;\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\sin(a\pi)\,\Gamma(c)}{\pi^{3/2}}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 1,\, c,\, 2-c \\ a,\, 1-a,\, \frac{1}{2} \end{array}\right) /;\, z \notin (-1,\, 0)$$

07.23.26.0174.01

$${}_2F_1\left(a,\, b;\, \frac{a+b+1}{2};\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(a,\, b;\, \frac{a+b+1}{2};\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{2^{a+b-1}\,\Gamma\left(\frac{a+b+1}{2}\right)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 1,\, \frac{1+a+b}{2},\, a+b \\ b,\, a,\, \frac{a+b}{2} \end{array}\right) /;\, z \notin (-1,\, 0)$$

07.23.26.0175.01

$${}_2F_1\left(a,\, b;\, \frac{a+b+1}{2};\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(1-a,\, 1-b;\, \frac{3-a-b}{2};\, \frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\cos\left(\frac{1}{2}(a-b)\pi\right)\Gamma\left(\frac{1}{2}(a+b+1)\right)}{\pi^{3/2}}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 1,\, \frac{1+a+b}{2},\, \frac{3-a-b}{2} \\ \frac{1}{2},\, \frac{1+a-b}{2},\, \frac{1-a+b}{2} \end{array}\right) /;\, z \notin (-1,\, 0)$$

Classical cases involving algebraic functions and ${}_2\tilde{F}_1$ with algebraic arguments

07.23.26.0176.01

$$(z+1)^{-2a}\, {}_2F_1\left(a,\, a+\frac{1}{2};\, c;\, \frac{1}{z+1}\right){}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, \frac{1}{z+1}\right) = \frac{4^{c-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2c-2a-1)}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{array}{c} 1-2a,\, c-2a,\, 2c-2a-1 \\ 0,\, 2c-4a-1,\, c-2a-\frac{1}{2} \end{array}\right) /;\, z \notin (-1,\, 0)$$

07.23.26.0177.01

$$(z+1)^{-2a}\, {}_2F_1\left(a,\, a+\frac{1}{2};\, c;\, \frac{z}{z+1}\right){}_2\tilde{F}_1\left(a,\, a+\frac{1}{2};\, c;\, \frac{z}{z+1}\right) = \frac{4^{c-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(2a)\,\Gamma(2c-2a-1)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{array}{c} 1-2a,\, 2a-2c+2,\, \frac{3}{2}-c \\ 0,\, 1-c,\, 2-2c \end{array}\right) /;\, z \notin (-\infty,\, -1)$$

07.23.26.0178.01

$$\left(\sqrt{z+1}+1\right)^{2-2c}\, {}_2F_1\left(a,1-a;c;\frac{1-\sqrt{1+z}}{2}\right) {}_2\tilde{F}_1\left(a,1-a;c;\frac{1-\sqrt{1+z}}{2}\right) = \frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} a-c+1,\, 2-a-c,\, \frac{3}{2}-c \\ 0,\, 1-c,\, 2-2c \end{matrix}\right)$$

07.23.26.0179.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2-2c}\, {}_2F_1\left(a,1-a;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) {}_2\tilde{F}_1\left(a,1-a;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 2-c,\, 1,\, c \\ a,\, 1-a,\, \frac{1}{2} \end{matrix}\right) /;\, z \notin (-1,0)$$

07.23.26.0180.01

$$\left(\sqrt{z+1}+1\right)^{-2a}\, {}_2F_1\left(a,b;a-b+1;\frac{\sqrt{z+1}-1}{\sqrt{z+1}+1}\right) {}_2\tilde{F}_1\left(a,b;a-b+1;\frac{\sqrt{z+1}-1}{\sqrt{z+1}+1}\right) = \frac{4^{-b}\,\Gamma(a-b+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\, G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 1-a,\, 2b-a,\, \frac{1}{2}-a+b \\ 0,\, b-a,\, 2b-2a \end{matrix}\right)$$

07.23.26.0181.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{-2a}\, {}_2F_1\left(a,b;a-b+1;\frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right) {}_2\tilde{F}_1\left(a,b;a-b+1;\frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z+1}+\sqrt{z}}\right) = \frac{4^{-b}\,\Gamma(a-b+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\, G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 1-a,\, 1-b,\, a-2b+1 \\ 0,\, 1-2b,\, \frac{1}{2}-b \end{matrix}\right) /;\, z \notin (-1,0)$$

07.23.26.0182.01

$$\left(2z+2\sqrt{z+1}\,\sqrt{z}+1\right)^{c-a-b}\, {}_2F_1\left(a,b;c;-2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right) {}_2\tilde{F}_1\left(c-a,c-b;c;-2\left(z+\sqrt{z}\,\sqrt{z+1}\right)\right) = \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{matrix} 1-a,\, 1-b,\, a-c+1,\, b-c+1 \\ 0,\, \frac{1-c}{2},\, 1-\frac{c}{2},\, 1-c \end{matrix}\right)$$

07.23.26.0183.01

$$\left(2z-2\sqrt{z+1}\,\sqrt{z}+1\right)^{c-a-b}\, {}_2F_1\left(a,b;c;-2\left(z+\sqrt{z}\,\sqrt{z+1}\right)\right) {}_2\tilde{F}_1\left(c-a,c-b;c;-2\left(z-\sqrt{z}\,\sqrt{z+1}\right)\right) = \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{matrix} 1-a,\, 1-b,\, a-c+1,\, b-c+1 \\ 0,\, \frac{1-c}{2},\, 1-\frac{c}{2},\, 1-c \end{matrix}\right)$$

07.23.26.0184.01

$$\left(2z+2\sqrt{z+1}\,\sqrt{z}+1\right)^{a+b-c}\, {}_2F_1\left(a,b;c;-2\left(z+\sqrt{z+1}\,\sqrt{z}\right)\right) {}_2\tilde{F}_1\left(c-a,c-b;c;2\sqrt{z}\,\sqrt{z+1}-2z\right) = \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{matrix} 1-a,\, 1-b,\, a-c+1,\, b-c+1 \\ 0,\, \frac{1-c}{2},\, 1-\frac{c}{2},\, 1-c \end{matrix}\right)$$

07.23.26.0185.01

$$\left(2z-2\sqrt{z+1}\,\sqrt{z}+1\right)^{a+b-c}\, {}_2F_1\left(a,b;c;2\sqrt{z}\,\sqrt{z+1}-2z\right) {}_2\tilde{F}_1\left(c-a,c-b;c;-2\left(z+\sqrt{z+1}\,\sqrt{z}\right)\right) = \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\, G_{4,4}^{1,4}\left(z \,\middle|\, \begin{matrix} a-c+1,\, b-c+1,\, 1-a,\, 1-b \\ 0,\, \frac{1-c}{2},\, 1-\frac{c}{2},\, 1-c \end{matrix}\right)$$

07.23.26.0186.01

$$\left(z+2\sqrt{z+1}+2\right)^{c-a-b}\,{}_2F_1\!\left(a,b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right)\,{}_2\tilde{F}_1\!\left(c-a,c-b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}c-a-b+1,\ \frac{3c}{2}-a-b,\ \frac{3c+1}{2}-a-b,\ 2c-a-b\\ c-a,\ c-b,\ 2c-2a-b,\ 2c-2b-a\end{array}\right)/;\ z\notin(-1,0)$$

07.23.26.0187.01

$$\left(z-2\sqrt{z+1}+2\right)^{c-a-b}\,{}_2F_1\!\left(a,b;c;-\frac{2\left(1+\sqrt{z+1}\right)}{z}\right)\,{}_2\tilde{F}_1\!\left(c-a,c-b;c;-\frac{2\left(1-\sqrt{z+1}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}c-a-b+1,\ \frac{3c}{2}-a-b,\ \frac{3c+1}{2}-a-b,\ 2c-a-b\\ c-a,\ c-b,\ 2c-2a-b,\ 2c-2b-a\end{array}\right)/;\ z\notin(-1,0)$$

07.23.26.0188.01

$$\left(z+2\sqrt{z+1}+2\right)^{a+b-c}\,{}_2F_1\!\left(a,b;c;-\frac{2\left(\sqrt{z+1}+1\right)}{z}\right)\,{}_2\tilde{F}_1\!\left(c-a,c-b;c;\frac{2\left(\sqrt{z+1}-1\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}a+b-c+1,\ a+b-\frac{c}{2},\ a+b+\frac{1-c}{2},\ a+b\\ a,\ b,\ 2a+b-c,\ a+2b-c\end{array}\right)/;\ z\notin(-1,0)$$

07.23.26.0189.01

$$\left(z-2\sqrt{z+1}+2\right)^{a+b-c}\,{}_2F_1\!\left(a,b;c;\frac{2\left(\sqrt{z+1}-1\right)}{z}\right)\,{}_2\tilde{F}_1\!\left(c-a,c-b;c;-\frac{2\left(\sqrt{z+1}+1\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\!\left(z\,\middle|\,\begin{array}{c}a+b-c+1,\ a+b-\frac{c}{2},\ a+b+\frac{1-c}{2},\ a+b\\ a,\ b,\ 2a+b-c,\ a+2b-c\end{array}\right)/;\ z\notin(-1,0)$$

07.23.26.0190.01

$$\left(1-2z-2\sqrt{z-1}\,\sqrt{z}\right)^{a}\,{}_2F_1\!\left(a,b;c;2\left(z+\sqrt{z-1}\,\sqrt{z}\right)\right)\,{}_2\tilde{F}_1\!\left(a,c-b;c;2\left(z+\sqrt{z-1}\,\sqrt{z}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(-z\,\middle|\,\begin{array}{c}1-a,\ a-c+1,\ 1-b,\ b-c+1\\ 0,\ 1-\frac{c}{2},\ \frac{1-c}{2},\ 1-c\end{array}\right)$$

07.23.26.0191.01

$$\left(1-2z-2\sqrt{z-1}\,\sqrt{z}\right)^{b}\,{}_2F_1\!\left(a,b;c;2\left(z+\sqrt{z-1}\,\sqrt{z}\right)\right)\,{}_2\tilde{F}_1\!\left(c-a,b;c;2\left(z+\sqrt{z-1}\,\sqrt{z}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(-z\,\middle|\,\begin{array}{c}1-a,\ 1-b,\ a-c+1,\ b-c+1\\ 0,\ 1-\frac{c}{2},\ \frac{1-c}{2},\ 1-c\end{array}\right)$$

07.23.26.0192.01

$$\left(z-2\left(\sqrt{1-z}+1\right)\right)^{a}\,{}_2F_1\!\left(a,b;c;\frac{2\left(1+\sqrt{1-z}\right)}{z}\right)\,{}_2\tilde{F}_1\!\left(a,c-b;c;\frac{2\left(1+\sqrt{1-z}\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,z^{a}\,G_{4,4}^{4,1}\!\left(-z\,\middle|\,\begin{array}{c}1,\ \frac{c}{2},\ \frac{c+1}{2},\ c\\ a,\ b,\ c-a,\ c-b\end{array}\right)$$

07.23.26.0193.01

$$\left(z-2\left(\sqrt{1-z}+1\right)\right)^{b}\,{}_2F_1\!\left(a,b;c;\frac{2\left(\sqrt{1-z}+1\right)}{z}\right){}_2\tilde{F}_1\!\left(c-a,b;c;\frac{2\left(\sqrt{1-z}+1\right)}{z}\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,z^{a}\,G_{4,4}^{4,1}\!\left(-z\,\middle|\,\begin{matrix}1,\frac{c}{2},\frac{c+1}{2},c\\ a,b,c-a,c-b\end{matrix}\right)$$

07.23.26.0194.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2F_1\!\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\!\left(a-c+1,b-c+1;a+b-c+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{a+b-c}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}1-a,1-b,\frac{1-a-b}{2},1-\frac{a+b}{2}\\ 0,1-a-b,1-c,c-a-b\end{matrix}\right)$$

07.23.26.0195.01

$$\left(\sqrt{z+1}+1\right)^{a+b-c}\,{}_2F_1\!\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\!\left(c-b,c-a;c-a-b+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{c-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}a-c+1,b-c+1,\frac{a+b+1}{2}-c,\frac{a+b}{2}-c+1\\ 0,a+b-2c+1,a+b-c,1-c\end{matrix}\right)$$

07.23.26.0196.01

$$\left(\sqrt{1+z}+1\right)^{a+b-2c+1}\,{}_2F_1\!\left(a,b;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\!\left(1-a,1-b;c-a-b+1;\frac{1-\sqrt{1+z}}{2}\right)=\frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}a-c+1,b-c+1,\frac{a+b+1}{2}-c,\frac{a+b}{2}-c+1\\ 0,a+b-2c+1,1-c,a+b-c\end{matrix}\right)$$

07.23.26.0197.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2F_1\!\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\!\left(a-c+1,a-c+\frac{3}{2};2a-c+\frac{3}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{4a-c-\frac{1}{2}}\,\Gamma(c)}{\pi\,\Gamma(2a)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}\frac{1}{4}-a,\frac{1}{2}-a,\frac{3}{4}-a,1-a\\ 0,c-2a-\frac{1}{2},\frac{1}{2}-2a,1-c\end{matrix}\right)$$

07.23.26.0198.01

$$\left(\sqrt{z+1}+1\right)^{2a-c+\frac{1}{2}}\,{}_2F_1\!\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\!\left(c-a-\frac{1}{2},c-a;c-2a+\frac{1}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{3c-2a-3}\,\Gamma(c)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}a-c+\frac{3}{4},a-c+1,a-c+\frac{5}{4},a-c+\frac{3}{2}\\ 0,1-c,2a-2c+\frac{3}{2},2a-c+\frac{1}{2}\end{matrix}\right)$$

07.23.26.0199.01

$$\left(\sqrt{1+z}+1\right)^{2a-2c+\frac{3}{2}}\,{}_2F_1\!\left(a,a+\frac{1}{2};c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\!\left(\frac{1}{2}-a,1-a;c-2a+\frac{1}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{4^{c-a-1}\,\Gamma(c)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{1,4}\!\left(z\,\middle|\,\begin{matrix}a-c+\frac{3}{4},a-c+1,a-c+\frac{5}{4},a-c+\frac{3}{2}\\ 0,1-c,2a-2c+\frac{3}{2},2a-c+\frac{1}{2}\end{matrix}\right)$$

07.23.26.0200.01

$$\left(\sqrt{z+1}+1\right)^{1-c}\,{}_2F_1\left(a,1-a;c;\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(2-a-c,a-c+1;2-c;\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{1-c}\sin(a\pi)\,\Gamma(c)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1-a,a\\0,c-1,1-c\end{matrix}\right)$$

07.23.26.0201.01

$$\left(\sqrt{z+1}+1\right)^{\frac{a+b-1}{2}}\,{}_2F_1\left(a,b;\frac{a+b+1}{2};\frac{1-\sqrt{1+z}}{2}\right){}_2\tilde{F}_1\left(\frac{a-b+1}{2},\frac{1-a+b}{2};\frac{3-a-b}{2};\frac{1-\sqrt{1+z}}{2}\right)=\frac{2^{\frac{a+b-1}{2}}}{\pi^{3/2}}\cos\left(\frac{1}{2}(a-b)\pi\right)\Gamma\left(\frac{1}{2}(a+b+1)\right)G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1-a+b}{2},\frac{a-b+1}{2}\\0,\frac{1-a-b}{2},\frac{a+b-1}{2}\end{matrix}\right)$$

07.23.26.0202.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c}\,{}_2F_1\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(a-c+1,b-c+1;a+b-c+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{a+b-c}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{matrix}\frac{3-c}{2},a+b+\frac{1-c}{2},\frac{c+1}{2},a+b+\frac{3-3c}{2}\\a+\frac{1-c}{2},b+\frac{1-c}{2},\frac{a+b-c+1}{2},\frac{a+b-c}{2}+1\end{matrix}\right)/;\,z\notin(-1,0)$$

07.23.26.0203.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{a+b-c}\,{}_2F_1\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(c-a,c-b;c-a-b+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{c-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{matrix}\frac{a+b-c}{2}+1,\frac{3c-a-b}{2},\frac{a+b+c}{2},\frac{c-a-b}{2}+1\\\frac{b-a+c}{2},\frac{a-b+c}{2},\frac{c}{2},\frac{c+1}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.23.26.0204.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{a+b-2c+1}\,{}_2F_1\left(a,b;c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(1-a,1-b;c-a-b+1;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{matrix}\frac{a+b+3}{2}-c,\frac{1-a-b}{2}+c,\frac{1+a+b}{2},\frac{3-a-b}{2}\\\frac{1}{2},1,\frac{1-a+b}{2},\frac{1+a-b}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.23.26.0205.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{1-c}\,{}_2F_1\left(a,a+\frac{1}{2};c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(a-c+1,a-c+\frac{3}{2};2a-c+\frac{3}{2};\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{4a-c-\frac{1}{2}}\,\Gamma(c)}{\pi\,\Gamma(2a)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{matrix}\frac{3-c}{2},2a-\frac{3c}{2}+2,2a-\frac{c}{2}+1,\frac{c+1}{2}\\a+\frac{1-c}{2},a+\frac{3-2c}{4},a-\frac{c}{2}+1,a+\frac{5-2c}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.23.26.0206.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2a-c+\frac{1}{2}}\,{}_2F_1\left(a,a+\frac{1}{2};c;\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(c-a-\frac{1}{2},c-a;c-2a+\frac{1}{2};\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{3c-2a-3}\,\Gamma(c)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{matrix}a+\frac{5-2c}{4},a+\frac{2c+1}{4},\frac{6c-1}{4}-a,\frac{2c+3}{4}-a\\\frac{2c-1}{4},\frac{c}{2},\frac{2c+1}{4},\frac{c+1}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.23.26.0207.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{2a-2c+\frac{3}{2}}\,{}_2F_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(\frac{1}{2}-a,\,1-a;\,c-2a+\frac{1}{2};\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{4^{c-a-1}\,\Gamma(c)}{\pi\,\Gamma(2c-2a-1)}\,G_{4,4}^{4,1}\left(z\,\middle|\,\begin{array}{c}a-c+\frac{7}{4},\,a+\frac{3}{4},\,c-a+\frac{1}{4},\,\frac{5}{4}-a\\ \frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,1\end{array}\right)\,/;\,z\notin(-1,\,0)$$

07.23.26.0208.01

$$\left(\sqrt{1+z}+1\right)^{1-c}\,{}_2F_1\left(a,\,1-a;\,c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(2-a-c,\,a-c+1;\,2-c;\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{1-c}\,\sin(a\,\pi)\,\Gamma(c)}{\pi^{3/2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\,1-a,\,a\\ 0,\,c-1,\,1-c\end{array}\right)\,/;\,z\notin(-1,\,0)$$

07.23.26.0209.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)^{\frac{a+b-1}{2}}\,{}_2F_1\left(a,\,b;\,\frac{a+b+1}{2};\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right){}_2\tilde{F}_1\left(\frac{b-a+1}{2},\,\frac{a-b+1}{2};\,\frac{3-a-b}{2};\,\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)=\frac{2^{\frac{a+b-1}{2}}}{\pi^{3/2}}\cos\left(\frac{1}{2}(a-b)\,\pi\right)\Gamma\left(\frac{1}{2}(a+b+1)\right)G_{3,3}^{3,1}\left(z\,\middle|\,\begin{array}{c}\frac{a+b+3}{4},\,\frac{3a+3b+1}{4},\,\frac{5-a-b}{4}\\ \frac{a+b+1}{4},\,\frac{3a-b+1}{4},\,\frac{3b-a+1}{4}\end{array}\right)\,/;\,z\notin(-1,\,0)$$

Generalized cases for the direct function itself

07.23.26.0210.01

$${}_2F_1(a,\,b;\,c;\,z)-{}_2F_1(a,\,b;\,c;\,-z)=\frac{2^{a+b-c}\,\pi\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\,G_{5,5}^{1,4}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-a}{2},\,\frac{1-b}{2},\,1-\frac{a}{2},\,1-\frac{b}{2},\,1\\ \frac{1}{2},\,0,\,1,\,\frac{1-c}{2},\,1-\frac{c}{2}\end{array}\right)\,/;\,|z|<1$$

07.23.26.0211.01

$${}_2F_1(a,\,b;\,c;\,-z)+{}_2F_1(a,\,b;\,c;\,z)=\frac{2^{a+b-c}\,\pi\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\,G_{5,5}^{1,4}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-a}{2},\,\frac{1-b}{2},\,1-\frac{a}{2},\,1-\frac{b}{2},\,\frac{1}{2}\\ 0,\,\frac{1}{2},\,\frac{1}{2},\,\frac{1-c}{2},\,1-\frac{c}{2}\end{array}\right)\,/;\,|z|<1$$

Generalized cases involving algebraic functions with quadratic arguments

07.23.26.0212.01

$$(z+1)^{-2a}\,{}_2F_1\left(a,\,b;\,2b;\,\frac{4z}{(z+1)^2}\right)=\frac{\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)}\,G_{2,2}^{1,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}1-a,\,b-a+\frac{1}{2}\\ 0,\,\frac{1}{2}-b\end{array}\right)$$

07.23.26.0213.01

$$(z+1)^{2(a-2b)}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{a-b}\,{}_2F_1\left(a,\,b;\,2b;\,\frac{4z}{(z+1)^2}\right)=\frac{\Gamma\left(a-b+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)}{\Gamma(2b-a)}\,G_{2,2}^{1,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}a-2b+1,\,a-b+\frac{1}{2}\\ 0,\,\frac{1}{2}-b\end{array}\right)$$

07.23.26.0214.01

$$(z+1)^{-2a}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{-a}\,{}_2F_1\left(a,\,b;\,2b;\,-\frac{4z}{(z-1)^2}\right)=\frac{\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)}\,G_{2,2}^{1,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}1-a,\,b-a+\frac{1}{2}\\ 0,\,\frac{1}{2}-b\end{array}\right)$$

07.23.26.0215.01

$$(z+1)^{2(a-2b)}\left(\frac{(z-1)^2}{(z+1)^2}\right)^{-b} {}_2F_1\left(a, b; 2b; -\frac{4z}{(z-1)^2}\right) = \frac{\Gamma\left(a-b+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)}{\Gamma(2b-a)} G_{2,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a-2b+1, a-b+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{matrix}\right)$$

Generalized cases involving algebraic functions with squares in arguments

07.23.26.0216.01

$$\left(z+\sqrt{z^2+1}\right)^{-b} {}_2F_1\left(a, b; b+1; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{a-1} b}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1-\frac{b}{2}, \frac{b+2}{2}, a+\frac{b}{2} \\ \frac{a+1}{2}, \frac{a}{2}, \frac{b}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0217.02

$$\left(z+\sqrt{z^2+1}\right)^{a-2c+2} {}_2F_1\left(a, 1; c; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{c-1}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{a}{2}-b+2, \frac{a}{2}+1, c-\frac{a}{2} \\ 1, \frac{1}{2}, \frac{a}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0218.01

$$\left(\sqrt{z^2+1}-z\right)^{2a} {}_2F_1\left(a, b; a+1; 2z^2-2z\sqrt{z^2+1}+1\right) = \frac{2^{-b} a}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1-a, 1, a-b+1 \\ \frac{1-b}{2}, 1-\frac{b}{2}, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0219.01

$$\left(\sqrt{z^2+1}-z\right)^{b+c-1} {}_2F_1\left(1, b; c; 2z^2-2z\sqrt{z^2+1}+1\right) = \frac{c-1}{2\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{3-b-c}{2}, \frac{c-b+1}{2}, \frac{b+c-1}{2} \\ 0, \frac{1}{2}, \frac{c-b-1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0220.01

$$\left(z+\sqrt{z^2+1}\right)^{1-a-c} {}_2F_1\left(a, 1; c; \frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right) = \frac{c-1}{2\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{3-a-c}{2}, \frac{1+c-a}{2}, \frac{a+c-1}{2} \\ 0, \frac{1}{2}, \frac{c-a-1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0221.01

$$\left(z+\sqrt{z^2+1}\right)^{-2b} {}_2F_1\left(a, b; b+1; \frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right) = \frac{2^{-a} b}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1-b, 1, b-a+1 \\ 0, \frac{1-a}{2}, 1-\frac{a}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0222.01

$$\left(\sqrt{z^2+1}-z\right)^{a} {}_2F_1\left(a, b; a+1; \frac{2z^2-2z\sqrt{z^2+1}+1}{2z^2-2z\sqrt{z^2+1}}\right) = \frac{2^{b-1} a}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1-\frac{a}{2}, \frac{a+2}{2}, \frac{a}{2}+b \\ \frac{b}{2}, \frac{b+1}{2}, \frac{a}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0223.01

$$\left(\sqrt{z^2+1}-z\right)^{2c-b-2} {}_2F_1\left(1, b; c; \frac{2z^2-2z\sqrt{z^2+1}+1}{2z^2-2z\sqrt{z^2+1}}\right) = \frac{c-1}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{b}{2}-c+2, \frac{b}{2}+1, c-\frac{b}{2} \\ \frac{1}{2}, 1, \frac{b}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

Classical cases involving sgn

07.23.26.0224.01

$$(z+1)^{-2b}\left((1-z)\operatorname{sgn}(1-|z|)\right)^{2a-2b} {}_2F_1\left(a, b; 2b; \frac{4z}{(z+1)^2}\right) = \frac{\Gamma\left(a-b+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)}{\Gamma(2b-a)} G_{2,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} a-2b+1, a-b+\frac{1}{2} \\ 0, \frac{1}{2}-b \end{matrix}\right)$$

07.23.26.0225.01

$$((1-z)\operatorname{sgn}(1-|z|))^{-2a}\,{}_2F_1\!\left(a,b;2b;-\frac{4z}{(z-1)^2}\right)=\frac{\Gamma\!\left(b+\frac{1}{2}\right)\Gamma\!\left(b-a+\frac{1}{2}\right)}{\Gamma(a)}\,G_{2,2}^{1,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-a,\,b-a+\frac{1}{2}\\ 0,\,\frac{1}{2}-b\end{array}\right)$$

07.23.26.0226.01

$$(z+1)^{2a-2b}\,((1-z)\operatorname{sgn}(1-|z|))^{-2b}\,{}_2F_1\!\left(a,b;2b;-\frac{4z}{(z-1)^2}\right)=\frac{\Gamma\!\left(a-b+\frac{1}{2}\right)\Gamma\!\left(b+\frac{1}{2}\right)}{\Gamma(2b-a)}\,G_{2,2}^{1,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}a-2b+1,\,a-b+\frac{1}{2}\\ 0,\,\frac{1}{2}-b\end{array}\right)$$

Generalized cases involving powers of ${}_2F_1$

07.23.26.0227.01

$${}_2F_1\!\left(a,b;\frac{a+b+1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)^2=\frac{2^{a+b-1}\,\Gamma\!\left(\frac{a+b+1}{2}\right)^2}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1,\,\frac{a+b+1}{2},\,a+b\\ a,\,b,\,\frac{a+b}{2}\end{array}\right)/;\operatorname{Re}(z)>0$$

07.23.26.0228.01

$$\left(z+\sqrt{z^2+1}\right)^{2-2c}\,{}_2F_1\!\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right)^2=\frac{\Gamma(c)^2}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}2-c,\,1,\,c\\ a,\,1-a,\,\frac{1}{2}\end{array}\right)/;\operatorname{Re}(z)>0$$

07.23.26.0229.01

$$\left(z+\sqrt{z^2+1}\right)^{-2a}\,{}_2F_1\!\left(a,b;a-b+1;\frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right)^2=$$
$$\frac{4^{-b}\,\Gamma(a-b+1)^2}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-a,\,1-b,\,a-2b+1\\ 0,\,1-2b,\,\frac{1}{2}-b\end{array}\right)/;\operatorname{Re}(z)>0$$

Generalized cases for products of ${}_2F_1$ with algebraic arguments

07.23.26.0230.01

$${}_2F_1\!\left(a,b;c;-2\left(z^2+\sqrt{z^2+1}\,z\right)\right){}_2F_1\!\left(a,b;c;-2\left(z^2-z\sqrt{z^2+1}\right)\right)=$$
$$\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-a,\,1-b,\,a-c+1,\,b-c+1\\ 0,\,\frac{1-c}{2},\,1-\frac{c}{2},\,1-c\end{array}\right)$$

07.23.26.0231.01

$${}_2F_1\!\left(a,b;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\!\left(a,b;a+b-c+1;\frac{z-\sqrt{z^2+1}}{2z}\right)=$$
$$\frac{2^{a+b-1}\,\Gamma(a+b-c+1)\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1,\,c,\,a+b,\,a+b-c+1\\ a,\,b,\,\frac{a+b}{2},\,\frac{a+b+1}{2}\end{array}\right)/;\operatorname{Re}(z)>0$$

07.23.26.0232.01

$$ {}_2F_1\left(a,a+\frac{1}{2};c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\left(a,a+\frac{1}{2};2a-c+\frac{3}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{2^{4a-\frac{3}{2}}\,\Gamma\left(2a-c+\frac{3}{2}\right)\Gamma(c)}{\pi\,\Gamma(2a)}G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,c,2a+\frac{1}{2},2a-c+\frac{3}{2}\\ a,a+\frac{1}{4},a+\frac{1}{2},a+\frac{3}{4}\end{matrix}\right)\text{/; }\operatorname{Re}(z)>0 $$

07.23.26.0233.01

$$ {}_2F_1\left(a,b;\frac{a+b+1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\left(1-a,1-b;\frac{3-a-b}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{(1-a-b)\cos\left(\frac{1}{2}(a-b)\pi\right)}{2\sqrt{\pi}\,\cos\left(\frac{1}{2}(a+b)\pi\right)}G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,\frac{a+b+1}{2},\frac{3-a-b}{2}\\ \frac{1}{2},\frac{1+a-b}{2},\frac{1-a+b}{2}\end{matrix}\right)\text{/; }\operatorname{Re}(z)>0 $$

07.23.26.0234.01

$$ {}_2F_1\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2F_1\left(a,1-a;2-c;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{(1-c)\sin(a\pi)}{\sqrt{\pi}\,\sin(c\pi)}G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,c,2-c\\ a,1-a,\frac{1}{2}\end{matrix}\right)\text{/; }\operatorname{Re}(z)>0 $$

Generalized cases involving products of ${}_2F_1$ with algebraic arguments

07.23.26.0235.01

$$ \left(2z^2+2\sqrt{z^2+1}\,z+1\right)^{c-a-b}{}_2F_1\left(a,b;c;-2\left(z^2-z\sqrt{z^2+1}\right)\right){}_2F_1\left(c-a,c-b;c;-2\left(z^2+z\sqrt{z^2+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\ 0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{matrix}\right) $$

07.23.26.0236.01

$$ \left(1+2z^2-2\sqrt{z^2+1}\,z\right)^{c-a-b}{}_2F_1\left(a,b;c;-2\left(z^2+z\sqrt{z^2+1}\right)\right){}_2F_1\left(c-a,c-b;c;-2\left(z^2-z\sqrt{z^2+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\ 0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{matrix}\right) $$

07.23.26.0237.01

$$ \left(1-2\left(z^2+z\sqrt{z^2-1}\right)\right)^{a}{}_2F_1\left(a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right){}_2F_1\left(a,c-b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,a-c+1,1-b,b-c+1\\ 0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)\text{/; }z\notin(-\infty,-1) $$

07.23.26.0238.01

$$ \left(1-2\left(z^2+z\sqrt{z^2-1}\right)\right)^{b}{}_2F_1\left(a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right){}_2F_1\left(c-a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)^2}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,a-c+1,1-b,b-c+1\\ 0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)\text{/; }z\notin(-\infty,-1) $$

07.23.26.0239.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c} {}_2F_1\left(a, b; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(a-c+1, b-c+1; a+b-c+1; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{a+b-c}\,\Gamma(a+b-c+1)\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{3-c}{2}, a+b+\frac{1-c}{2}, \frac{c+1}{2}, a+b+\frac{3-3c}{2} \\ a+\frac{1-c}{2}, b+\frac{1-c}{2}, \frac{a+b-c+1}{2}, \frac{a+b-c}{2}+1 \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0240.01

$$\left(z+\sqrt{z^2+1}\right)^{a+b-2c+1} {}_2F_1\left(a, b; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(1-a, 1-b; c-a-b+1; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{\Gamma(c)\,\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{a+b+3}{2}-c, \frac{1-a-b}{2}+c, \frac{a+b+1}{2}, \frac{3-a-b}{2} \\ \frac{1}{2}, 1, \frac{1-a+b}{2}, \frac{a-b+1}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0241.01

$$\left(z+\sqrt{z^2+1}\right)^{a+b-c} {}_2F_1\left(a, b; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(c-a, c-b; c-a-b+1; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{c-1}\,\Gamma(c)\,\Gamma(c-a-b+1)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{a+b-c}{2}+1, \frac{3c-a-b}{2}, \frac{a+b+c}{2}, \frac{c-a-b}{2}+1 \\ \frac{b-a+c}{2}, \frac{a-b+c}{2}, \frac{c}{2}, \frac{c+1}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0242.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c} {}_2F_1\left(a, a+\frac{1}{2}; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(a-c+1, a-c+\frac{3}{2}; 2a-c+\frac{3}{2}; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{4a-c-\frac{1}{2}}\,\Gamma\left(2a-c+\frac{3}{2}\right)\Gamma(c)}{\pi\,\Gamma(2a)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{3-c}{2}, 2a-\frac{3c}{2}+2, 2a-\frac{c}{2}+1, \frac{c+1}{2} \\ a+\frac{1-c}{2}, a+\frac{3-2c}{4}, a-\frac{c}{2}+1, a+\frac{5-2c}{4} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0243.01

$$\left(z+\sqrt{z^2+1}\right)^{2a-2c+\frac{3}{2}} {}_2F_1\left(a, a+\frac{1}{2}; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(\frac{1}{2}-a, 1-a; c-2a+\frac{1}{2}; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{4^{c-a-1}\,\Gamma(c)\,\Gamma\left(c-2a+\frac{1}{2}\right)}{\pi\,\Gamma(2c-2a-1)} G_{4,4}^{4,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} a-c+\frac{7}{4}, a+\frac{3}{4}, c-a+\frac{1}{4}, \frac{5}{4}-a \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.23.26.0244.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c} {}_2F_1\left(a, 1-a; c; \frac{z-\sqrt{z^2+1}}{2z}\right) {}_2F_1\left(2-a-c, a-c+1; 2-c; \frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{2^{1-c}\,(1-c)\csc(c\pi)\sin(a\pi)}{\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{3-c}{2}, \frac{5-3c}{2}, \frac{c+1}{2} \\ 1-\frac{c}{2}, \frac{3-c}{2}-a, a+\frac{1-c}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

Generalized cases involving ${}_2\tilde{F}_1$ with algebraic arguments

07.23.26.0245.01

$$ {}_2F_1\left(a,b;c;-2\left(z^2+z\sqrt{z^2+1}\right)\right){}_2\tilde{F}_1\left(a,b;c;-2\left(z^2-z\sqrt{z^2+1}\right)\right)= \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{array}\right) $$

07.23.26.0246.01

$$ {}_2F_1\left(a,b;c;-2\left(z^2-z\sqrt{z^2+1}\right)\right){}_2\tilde{F}_1\left(a,b;c;-2\left(z^2+z\sqrt{z^2+1}\right)\right)= \frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{array}\right) $$

07.23.26.0247.01

$$ {}_2F_1\left(a,b;\frac{a+b+1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\left(a,b;\frac{a+b+1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)= \frac{2^{a+b-1}\,\Gamma\left(\frac{a+b+1}{2}\right)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1,\frac{a+b+1}{2},a+b\\a,b,\frac{a+b}{2}\end{array}\right)\;/;\;\mathrm{Re}(z)>0 $$

07.23.26.0248.01

$$ {}_2F_1\left(a,b;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\left(a,b;a+b-c+1;\frac{z-\sqrt{z^2+1}}{2z}\right)= \frac{2^{a+b-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1,c,a+b,a+b-c+1\\a,b,\frac{a+b}{2},\frac{1}{2}(a+b+1)\end{array}\right)\;/;\;\mathrm{Re}(z)>0 $$

07.23.26.0249.01

$$ {}_2F_1\left(a,a+\frac{1}{2};c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\left(a,a+\frac{1}{2};2a-c+\frac{3}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)= \frac{2^{4a-\frac{3}{2}}\,\Gamma(c)}{\pi\,\Gamma(2a)}\,G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1,c,2a+\frac{1}{2},2a-c+\frac{3}{2}\\a,a+\frac{1}{4},a+\frac{1}{2},a+\frac{3}{4}\end{array}\right)\;/;\;\mathrm{Re}(z)>0 $$

07.23.26.0250.01

$$ {}_2F_1\left(a,b;\frac{a+b+1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\left(1-a,1-b;\frac{3-a-b}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)= \frac{\cos\left(\frac{1}{2}(a-b)\pi\right)\Gamma\left(\frac{1}{2}(a+b+1)\right)}{\pi^{3/2}}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1,\frac{a+b+1}{2},\frac{3-a-b}{2}\\\frac{1}{2},\frac{1+a-b}{2},\frac{1-a+b}{2}\end{array}\right)\;/;\;\mathrm{Re}(z)>0 $$

07.23.26.0251.01

$$ {}_2F_1\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\left(a,1-a;2-c;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{\sin(a\pi)\,\Gamma(c)}{\pi^{3/2}}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1,c,2-c\\a,1-a,\frac{1}{2}\end{array}\right)\;/;\;\mathrm{Re}(z)>0 $$

Generalized cases involving algebraic functions and ${}_2\tilde{F}_1$ with algebraic arguments

07.23.26.0252.01

$$\left(2z^2+2\sqrt{z^2+1}\,z+1\right)^{c-a-b}\,{}_2F_1\left(a,b;c;-2\left(z^2-z\sqrt{z^2+1}\right)\right){}_2\tilde{F}_1\left(c-a,c-b;c;-2\left(z^2+\sqrt{z^2+1}\,z\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{matrix}\right)$$

07.23.26.0253.01

$$\left(2z^2-2\sqrt{z^2+1}\,z+1\right)^{c-a-b}\,{}_2F_1\left(a,b;c;-2\left(z^2+z\sqrt{z^2+1}\right)\right){}_2\tilde{F}_1\left(c-a,c-b;c;-2\left(z^2-z\sqrt{z^2+1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{matrix}\right)$$

07.23.26.0254.01

$$\left(2z^2+2\sqrt{z^2+1}\,z+1\right)^{a+b-c}\,{}_2F_1\left(a,b;c;-2z\left(\sqrt{z^2+1}+z\right)\right){}_2\tilde{F}_1\left(c-a,c-b;c;2z\left(\sqrt{z^2+1}-z\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{matrix}\right)$$

07.23.26.0255.01

$$\left(2z^2-2\sqrt{z^2+1}\,z+1\right)^{a+b-c}\,{}_2F_1\left(a,b;c;2z\left(\sqrt{z^2+1}-z\right)\right){}_2\tilde{F}_1\left(c-a,c-b;c;-2z\left(\sqrt{z^2+1}+z\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,\frac{1-c}{2},1-\frac{c}{2},1-c\end{matrix}\right)$$

07.23.26.0256.01

$$\left(1-2z^2-2z\sqrt{z^2-1}\right)^{a}\,{}_2F_1\left(a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right){}_2\tilde{F}_1\left(a,c-b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,1+a-c,1+b-c\\0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)/;\ z\notin(-\infty,-1)$$

07.23.26.0257.01

$$\left(1-2z^2-2z\sqrt{z^2-1}\right)^{b}\,{}_2F_1\left(a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right){}_2\tilde{F}_1\left(c-a,b;c;2\left(z^2+z\sqrt{z^2-1}\right)\right)=\frac{2^{1-c}\sqrt{\pi}\,\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}G_{4,4}^{1,4}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}1-a,1-b,a-c+1,b-c+1\\0,1-\frac{c}{2},\frac{1-c}{2},1-c\end{matrix}\right)/;\ z\notin(-\infty,-1)$$

07.23.26.0258.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2F_1\left(a,b;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\left(a-c+1,b-c+1;a+b-c+1;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{2^{a+b-c}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\Gamma(b)}G_{4,4}^{4,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-c}{2},a+b+\frac{1-c}{2},\frac{c+1}{2},a+b+\frac{3-3c}{2}\\a+\frac{1-c}{2},b+\frac{1-c}{2},\frac{a+b-c+1}{2},\frac{a+b-c}{2}+1\end{matrix}\right)/;\ \mathrm{Re}(z)>0$$

07.23.26.0259.01

$$\left(z+\sqrt{z^2+1}\right)^{a+b-c}\,{}_2F_1\left(a,\,b;\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(c-a,\,c-b;\,c-a-b+1;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=\frac{2^{c-1}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{a+b-c}{2}+1,\,\frac{3c-a-b}{2},\,\frac{a+b+c}{2},\,\frac{c-a-b}{2}+1\\ \frac{b-a+c}{2},\,\frac{a-b+c}{2},\,\frac{c}{2},\,\frac{c+1}{2}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.23.26.0260.01

$$\left(z+\sqrt{z^2+1}\right)^{a+b-2c+1}\,{}_2F_1\left(a,\,b;\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(1-a,\,1-b;\,c-a-b+1;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=\frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(c-b)}\,G_{4,4}^{4,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{a+b+3}{2}-c,\,\frac{1-a-b}{2}+c,\,\frac{a+b+1}{2},\,\frac{3-a-b}{2}\\ \frac{1}{2},\,1,\,\frac{1-a+b}{2},\,\frac{a-b+1}{2}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.23.26.0261.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2F_1\left(a,\,b;\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(a-c+1,\,b-c+1;\,a+b-c+1;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=\frac{2^{a+b-c}\,\Gamma(c)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(b)}\,G_{4,4}^{4,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-c}{2},\,a+b+\frac{1-c}{2},\,a+b+\frac{3-3c}{2},\,\frac{c+1}{2}\\ a+\frac{1-c}{2},\,b+\frac{1-c}{2},\,\frac{a+b-c+1}{2},\,\frac{a+b-c}{2}+1\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.23.26.0262.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2F_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(a-c+1,\,a-c+\frac{3}{2};\,2\,a-c+\frac{3}{2};\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=\frac{2^{4a-c-\frac{1}{2}}\,\Gamma(c)}{\pi\,\Gamma(2\,a)}\,G_{4,4}^{4,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-c}{2},\,2\,a-\frac{3c}{2}+2,\,2\,a-\frac{c}{2}+1,\,\frac{c+1}{2}\\ a+\frac{1-c}{2},\,a+\frac{3-2c}{4},\,a-\frac{c}{2}+1,\,a+\frac{5-2c}{4}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.23.26.0263.01

$$\left(z+\sqrt{z^2+1}\right)^{2a-c+\frac{1}{2}}\,{}_2F_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(c-a-\frac{1}{2},\,c-a;\,c-2\,a+\frac{1}{2};\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=\frac{2^{3c-2a-3}\,\Gamma(c)}{\pi\,\Gamma(2\,c-2\,a-1)}\,G_{4,4}^{4,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}a+\frac{5-2c}{4},\,a+\frac{2c+1}{4},\,\frac{6c-1}{4}-a,\,\frac{2c+3}{4}-a\\ \frac{2c-1}{4},\,\frac{c}{2},\,\frac{2c+1}{4},\,\frac{c+1}{2}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.23.26.0264.01

$$\left(z+\sqrt{z^2+1}\right)^{2a-2c+\frac{3}{2}}\,{}_2F_1\left(a,\,a+\frac{1}{2};\,c;\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)\,{}_2\tilde{F}_1\left(\frac{1}{2}-a,\,1-a;\,c-2\,a+\frac{1}{2};\,\frac{z-\sqrt{z^2+1}}{2\,z}\right)=\frac{4^{c-a-1}\,\Gamma(c)}{\pi\,\Gamma(2\,c-2\,a-1)}\,G_{4,4}^{4,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}a-c+\frac{7}{4},\,a+\frac{3}{4},\,c-a+\frac{1}{4},\,\frac{5}{4}-a\\ \frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,1\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.23.26.0265.01

$$\left(z+\sqrt{z^2+1}\right)^{2-2c}\,{}_2F_1\!\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\!\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{\Gamma(c)}{\sqrt{\pi}\,\Gamma(c-a)\,\Gamma(a+c-1)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}2-c,1,c\\ a,1-a,\frac{1}{2}\end{array}\right)/;\ \mathrm{Re}(z)>0$$

07.23.26.0266.01

$$\left(z+\sqrt{z^2+1}\right)^{1-c}\,{}_2F_1\!\left(a,1-a;c;\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\!\left(2-a-c,a-c+1;2-c;\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{2^{1-c}\sin(a\pi)\,\Gamma(c)}{\pi^{3/2}}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{3-c}{2},\frac{5-3c}{2},\frac{c+1}{2}\\ 1-\frac{c}{2},\frac{3-c}{2}-a,a+\frac{1-c}{2}\end{array}\right)/;\ \mathrm{Re}(z)>0$$

07.23.26.0267.01

$$\left(z+\sqrt{z^2+1}\right)^{\frac{a+b-1}{2}}\,{}_2F_1\!\left(a,b;\frac{a+b+1}{2};\frac{z-\sqrt{z^2+1}}{2z}\right){}_2\tilde{F}_1\!\left(\frac{b-a+1}{2},\frac{a-b+1}{2};\frac{3-a-b}{2};\frac{z-\sqrt{z^2+1}}{2z}\right)=\frac{1}{\pi^{3/2}}\,2^{\frac{a+b-1}{2}}\cos\!\left(\frac{1}{2}(a-b)\pi\right)\Gamma\!\left(\frac{1}{2}(a+b+1)\right)G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{a+b+3}{4},\frac{3a+3b+1}{4},\frac{5-a-b}{4}\\ \frac{a+b+1}{4},\frac{3a-b+1}{4},\frac{3b-a+1}{4}\end{array}\right)/;\ \mathrm{Re}(z)>0$$

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$$\left(z+\sqrt{z^2+1}\right)^{-2a}\,{}_2F_1\!\left(a,b;a-b+1;\frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right){}_2\tilde{F}_1\!\left(a,b;a-b+1;\frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right)=\frac{4^{-b}\,\Gamma(a-b+1)}{\sqrt{\pi}\,\Gamma(a)\,\Gamma(a-2b+1)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{array}{c}1-a,1-b,a-2b+1\\ 0,1-2b,\frac{1}{2}-b\end{array}\right)/;\ \mathrm{Re}(z)>0$$

Through other functions

Involving some hypergeometric-type functions

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$${}_2F_1(a,b;c;z)=F_1(a;b,b_2;c;z,0)$$

07.23.26.0270.01

$${}_2F_1(a,b;c;z)=F_1(a;b,0;c;z,z_2)$$

07.23.26.0271.01

$${}_2F_1(a,b;c;z)=F_1(a;d,b-d;c;z,z)$$

07.23.26.0272.01

$${}_2F_1(a,b;c;z)=\frac{\Gamma(p-a+c)\,\Gamma(c)}{\Gamma(c+p)\,\Gamma(c-a)}\,F_1(a;b,p;c+p;z,1)\ /;\ \mathrm{Re}(c-a)>0$$

Representations through equivalent functions

With related functions

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$$ {}_2F_1(a, b; c; z) = \frac{\Gamma(1-a)\,\Gamma(c)}{\Gamma(c-a)} P_{-a}^{(c-1, a+b-c)}(1-2z) $$

Theorems

The Gauss and Riemann-Papperitz differential equations

The Gauss hypergeometric differential equation

$$ z(1-z)\,w''(z) + (c - (a+b+1)\,z)\,w'(z) - a\,b\,w(z) = 0 $$

has three regular singular points $z = 0, 1, \infty$ with corresponding pairs of indicial exponents $\{0, 1-c\}, \{0, c-a-b\}, \{a, b\}$. After fractional-linear transformations like $z \to \frac{A z + B}{C z + D}$ these three regular points can be moved into arbitrary points $z = \tilde{a},\ \tilde{b},\ \tilde{c}$ with corresponding pairs of indicial exponents $\{\alpha, \alpha'\},\ \{\beta, \beta'\},\ \{\gamma, \gamma'\}$ and the Gauss' equation will be transformed into the Riemann–Papperitz differential equation

$$ w''(z) + \left(\frac{1-\alpha-\alpha'}{z-\tilde{a}} + \frac{1-\beta-\beta'}{z-\tilde{b}} + \frac{1-\gamma-\gamma'}{z-\tilde{c}}\right) w'(z) + \left(\frac{\alpha\,\alpha'\,(\tilde{a}-\tilde{b})\,(\tilde{a}-\tilde{c})}{z-\tilde{a}} + \frac{\beta\,\beta'\,(\tilde{b}-\tilde{c})\,(\tilde{b}-\tilde{a})}{z-\tilde{b}} + \frac{\gamma\,\gamma'\,(\tilde{c}-\tilde{a})\,(\tilde{c}-\tilde{b})}{z-\tilde{c}}\right) \frac{w(z)}{(z-\tilde{a})\,(z-\tilde{b})\,(z-\tilde{c})} = 0 \;/;\quad \alpha + \alpha' + \beta + \beta' + \gamma + \gamma' = 1. $$

A solution of this equation may be expressed in the form

$$ \left(\frac{z-\tilde{a}}{z-\tilde{b}}\right)^{\alpha} \left(\frac{z-\tilde{c}}{z-\tilde{b}}\right)^{\gamma} {}_2F_1\left(\alpha+\beta+\gamma,\ \alpha+\beta'+\gamma;\ 1+\alpha-\alpha';\ \frac{(z-\tilde{a})\,(\tilde{c}-\tilde{b})}{(z-\tilde{b})\,(\tilde{c}-\tilde{a})}\right). $$

23 other similar solutions can be produced when the parameters are interchanged.

The conformal maps from triangles with circular edges onto the unit disk

Using rational functions drawn from among hypergeometric functions, one can construct conformal maps from triangles with circular edges onto the unit disk.

The eigenvalues and (non-normalized) eigenfunctions of the Schrödinger equation with the Pöschl-Teller potential

The eigenvalues and (non–normalized) eigenfunctions of the Schrödinger equation with the Pöschl–Teller potential (with singularities at $x = 0$ and $x = \pi/(2\alpha)$)

$$ -\frac{\partial^2 \psi_n(x)}{\partial x^2} + \alpha^2 \left(\frac{\kappa(\kappa+1)}{\sin^2(\alpha x)} + \frac{\lambda(\lambda+1)}{\cos^2(\alpha x)}\right) \psi_n(x) = \alpha^2 (\kappa + \lambda + 2n)\,\psi_n(x) \;/;\qquad \kappa, \lambda > 1,\ n \in \mathbb{N} $$

are given by $\psi_n(x) = \sin^{\kappa}(\alpha x) \cos^{\lambda}(\alpha x)\ {}_2F_1\left(-n, \kappa+\lambda+n; \kappa+\frac{1}{2}; \sin^2(\alpha x)\right)$.

Traffic flow modelling with cellular automaton rule 184

The average flow $F(t)$ in the cellular automaton rule 184 based model, starting from random initial conditions is

$$F(t) = 1 - \rho - \frac{(1-\rho)^{1+v+v\,t}\,\rho^t\,(1+v+t+v\,t)!}{(1+v+v\,t)\,(1+t)!\,(v+v\,t)!}\,{}_2F_1\left(2, -t\;;2+v+v\,t;1-\frac{1}{\rho}\right)$$

where ρ is the initial random density of cars and v their maximal velocity.

History

– Chu Shih–Chieh (1303)
– J. Wallis (1655) introduced the name "hypergeometric"
– L. Euler (1748, 1769, 1778, 1794)
– A. T. Vandermonde (1772)
– J. Fr. Pfaff (1797)
– C. F. Gauss (1812)
– E. E. Kummer (1832, 1836)
– B. Riemann (1856)
– H. A. Schwarz (1873);
– E. Goursat (1881)
– E. W. Barnes (1908)

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