Hypergeometric4F3

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Notations

Traditional name

Generalized hypergeometric function ${}_4F_3$

Traditional notation

${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$

Mathematica StandardForm notation

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HypergeometricPFQ[{a1, a2, a3, a4}, {b1, b2, b3}, z]
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Primary definition

07.28.02.0001.01

$$ {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) == \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k z^k}{(b_1)_k (b_2)_k (b_3)_k k!} /; |z| < 1 \bigvee |z| == 1 \bigwedge \operatorname{Re}\left(\sum_{j=1}^{3} b_j - \sum_{j=1}^{4} a_j\right) > 0 $$

For $a_i == -n$, $b_j == -m$ /; $m \geq n$ being nonpositive integers and $\nexists_{a_k} (a_k > -n \bigwedge a_k \in \mathbb{N}) \bigwedge \nexists_{b_k} (b_k > -m \bigwedge b_k \in \mathbb{N})$ the function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ cannot be uniquely defined by a limiting procedure based on the above definition because the two variables a_i, b_j can approach nonpositive integers $-n$, $-m$; $m \geq n$ at different speeds. For the above conditions we define:

07.28.02.0002.01

$$ {}_4F_3\left(a_1, \ldots, a_i, \ldots, a_4; b_1, \ldots, b_j, \ldots, b_3; z\right) == \sum_{k=0}^{n} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k z^k}{(b_1)_k (b_2)_k (b_3)_k k!} /; a_i == -n \bigwedge b_j == -m \bigwedge m \in \mathbb{N} \bigwedge n \in \mathbb{N} \bigwedge m \geq n $$

Specific values

Values at $z == 0$

07.28.03.0001.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; 0) == 1$$

Values at $z = 1$

For fixed $a_1, a_2, a_3, a_4, b_1, b_2, b_3$

07.28.03.0002.01

$$ {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; 1) = \frac{\Gamma(\psi_3)\prod_{k=1}^{3}\Gamma(b_k)}{\prod_{k=3}^{4}\Gamma(a_k)} \sum_{k=0}^{\infty} \frac{(\psi_3)_k \, \mathcal{E}_k^{(3)}(\{a_1, a_2, a_3, a_4\}, \{b_1, b_2, b_3\})}{\Gamma(k + a_1 + \psi_3)\,\Gamma(k + a_2 + \psi_3)} /; $$

$$ \psi_3 = \sum_{j=1}^{3} b_j - \sum_{j=1}^{4} a_j \bigwedge \operatorname{Re}(\psi_3) > 0 \bigwedge \operatorname{Re}(a_3) > 0 \bigwedge \operatorname{Re}(a_4) > 0 $$

For fixed $a_1, a_2, a_3, a_4, b_2, b_3$

07.28.03.0003.01

$$ {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; 1) = (-1)^{-b_1-1}\,\Gamma(b_2)\,\Gamma(b_3)\sqrt{\Gamma(1-a_1)}\,\sqrt{\Gamma(1-a_2)} \sqrt{\Gamma(1-a_3)}\,\sqrt{\Gamma(1-a_4)}\,\sqrt{\Gamma(a_1-b_1+1)}\,\sqrt{\Gamma(a_2-b_1+1)}\,\sqrt{\Gamma(a_3-b_1+1)}\,\sqrt{\Gamma(a_4-b_1+1)} \begin{Bmatrix} \frac{b_3-a_1-a_4-1}{2} & \frac{b_2-a_1-a_3-1}{2} & \frac{a_1+a_2-b_1-1}{2} \\ \frac{b_3-a_2-a_3-1}{2} & \frac{b_2-a_2-a_4-1}{2} & \frac{a_3+a_4-b_1-1}{2} \end{Bmatrix} \Big/ \Big(\Gamma(1-b_1)\sqrt{\Gamma(b_3-a_1)}\,\sqrt{\Gamma(b_2-a_2)}\,\sqrt{\Gamma(b_2-a_3)}\,\sqrt{\Gamma(b_2-a_4)} \sqrt{\Gamma(b_2-a_1)}\,\sqrt{\Gamma(b_3-a_2)}\,\sqrt{\Gamma(b_3-a_3)}\,\sqrt{\Gamma(b_3-a_4)}\Big) /; a_1 + a_2 + a_3 + a_4 - b_1 - b_2 - b_3 = -1 $$

For fixed a_1, a_2, a_3, a_4

07.28.03.0004.01

$$ {}_4F_3(a, b, c, d; a+1, b+1, c-1; 1) = \frac{a\,b\,\Gamma(1-d)}{(c-1)\,(b-a)} \left(\frac{(c-a-1)\,\Gamma(a)}{\Gamma(a-d+1)} - \frac{(c-b-1)\,\Gamma(b)}{\Gamma(b-d+1)} \right) /; \operatorname{Re}(d) < 1 $$

For fixed a_1, a_2, a_3

07.28.03.0005.01

$$ {}_4F_3\left(a, b, c, \frac{a}{2}+1; \frac{a}{2}, a-b+1, a-c+1; 1\right) = \frac{\Gamma\left(\frac{a+1}{2}\right)\Gamma(a-b+1)\,\Gamma(a-c+1)\,\Gamma\left(\frac{a+1}{2}-b-c\right)}{\Gamma(a+1)\,\Gamma\left(\frac{a+1}{2}-b\right)\Gamma\left(\frac{a+1}{2}-c\right)\Gamma(a-b-c+1)} /; \operatorname{Re}(a-2b-2c) > -1 $$

07.28.03.0006.01

$$ {}_4F_3(a, b, c, 1; a+1, b+1, c+1; 1) = -a\,b\,c \left(\frac{\psi(a)}{(b-a)\,(c-a)} + \frac{\psi(b)}{(a-b)\,(c-b)} + \frac{\psi(c)}{(a-c)\,(b-c)} \right) /; a \neq b \bigwedge c \neq a \bigwedge b \neq c $$

07.28.03.0007.01

$$ {}_4F_3\left(a, b, c, \frac{a}{2}+1; \frac{a}{2}, a-b+1, a-c+1; 1\right) = \frac{\Gamma\left(\frac{a+1}{2}\right)\Gamma(a-b+1)\,\Gamma(a-c+1)\,\Gamma\left(\frac{a+1}{2}-b-c\right)}{\Gamma(a+1)\,\Gamma\left(\frac{a+1}{2}-b\right)\Gamma\left(\frac{a+1}{2}-c\right)\Gamma(a-b-c+1)} /; \operatorname{Re}(a-2b-2c) > -1 $$

For fixed a_2, a_3, a_4

07.28.03.0008.01

$$ {}_4F_3(1, b, c, d; b+1, c+1, d+1; 1) = -b\,c\,d \left(\frac{\psi(b)}{(c-b)\,(d-b)} + \frac{\psi(c)}{(b-c)\,(d-c)} + \frac{\psi(d)}{(b-d)\,(c-d)} \right) /; b \neq c \bigwedge b \neq d \bigwedge c \neq d $$

07.28.03.0009.01

$$ {}_4F_3(1, b, c, d; 3-b, 3-c, 3-d; 1) = \frac{1}{2\,(b-1)\,(c-1)\,(d-1)} \left(\frac{\Gamma(3-b)\,\Gamma(3-c)\,\Gamma(3-d)\,\Gamma(4-b-c-d)}{\Gamma(3-b-c)\,\Gamma(3-b-d)\,\Gamma(3-c-d)} - (2-b)\,(2-c)\,(2-d) \right) /; \operatorname{Re}(b+c+d) < 4 $$

07.28.03.0010.01

$$ {}_4F_3(1, b, c, d; 2, b-c+2, b-d+2; 1) = \frac{(b-c+1)(b-d+1)}{(b-1)(c-1)(d-1)} \left(\frac{\Gamma\left(\frac{b+1}{2}\right)\Gamma(b-c+1)\Gamma(b-d+1)\Gamma\left(\frac{b+5}{2}-c-d\right)}{\Gamma(b)\Gamma\left(\frac{b+3}{2}-c\right)\Gamma\left(\frac{b+3}{2}-d\right)\Gamma(b-c-d+2)} - 1 \right) /; \operatorname{Re}(b-2c-2d) > -5 $$

07.28.03.0011.01

$$ {}_4F_3(-n, b, c, d; b-k, c-l, d-m; 1) = 0 /; n \in \mathbb{N} \land m \in \mathbb{N} \land k \in \mathbb{N} \land l \in \mathbb{N} \land 0 \le k+l+m < n $$

07.28.03.0012.01

$$ {}_4F_3(-n, b, c, d; b-k, c-l, d-m; 1) = \frac{n!}{(1-b)_k (1-c)_l (1-d)_m} /; n \in \mathbb{N} \land m \in \mathbb{N} \land k \in \mathbb{N} \land l \in \mathbb{N} \land k+l+m = n $$

07.28.03.0013.01

$$ {}_4F_3(-n, b, c, d; b+1, c-l, d-m; 1) = \frac{n! (b-c+1)_l (a-d+1)_m}{(b+1)_n (1-c)_l (1-d)_m} /; n \in \mathbb{N} \land m \in \mathbb{N} \land l \in \mathbb{N} \land 0 \le l+m \le n $$

07.28.03.0014.01

$$ {}_4F_3\left(-n, b, c, d; -b-n, b+c+\frac{1}{2}, b+d+\frac{1}{2}; 1\right) = \frac{(-1)^n \left(b+\frac{1}{2}\right)_n \left(b+c+d-n+\frac{1}{2}\right)_{2n}}{\left(b+c+\frac{1}{2}\right)_n \left(b+d+\frac{1}{2}\right)_n \left(\frac{1}{2}-b-c-d\right)_n} {}_3F_2\left(-n, 2c, 2d; -2b-2n, b+c+d+\frac{1}{2}; 1\right) /; n \in \mathbb{N} $$

07.28.03.0015.01

$$ {}_4F_3\left(-n, b, c, d; \frac{b-n}{2}, \frac{b-n+1}{2}, c+d+\frac{1}{2}; 1\right) = \frac{(2d-b+1)_n}{(1-b)_n} {}_3F_2\left(-n, 2d, \frac{1}{2}+d-c; c+d+\frac{1}{2}, 2d-b+1; 1\right) /; n \in \mathbb{N} $$

07.28.03.0016.01

$$ {}_4F_3\left(-n, b, c, d; \frac{b-n}{2}, \frac{b-n+1}{2}, c+d+\frac{1}{2}; 1\right) = {}_3F_2\left(-n, 2c, 2d; b-n, c+d+\frac{1}{2}; 1\right) /; n \in \mathbb{N} $$

07.28.03.0017.01

$$ {}_4F_3\left(-n, b, c, d; \frac{b-n}{2}, \frac{b-n+1}{2}, c+d+\frac{1}{2}; 1\right) = \frac{\left(c+d-b+\frac{1}{2}\right)_n}{(1-b)_n} {}_3F_2\left(-n, c-d+\frac{1}{2}, \frac{1}{2}-c+d; c+d+\frac{1}{2}, b-c-d-n+\frac{1}{2}; 1\right) /; n \in \mathbb{N} $$

For fixed a_2, a_3, b_1

07.28.03.0018.01

$$ {}_4F_3\left(-n, b, c, \frac{b}{2}+1; e, \frac{b}{2}, b-c+1; 1\right) = \frac{e+n}{e} {}_3F_2(-n, b+1, c+1; e+1, b-c+1; 1) /; n \in \mathbb{N}^+ $$

For fixed a_3, a_4, b_1

07.28.03.0019.01

$$ {}_4F_3\left(-\frac{n}{2}, \frac{1-n}{2}, c, d; e, e+\frac{1}{2}, c+d-2e-n+1; 1\right) = \frac{(c)_n}{(2e)_n} {}_3F_2(-n, c-2e-n+1, 2e-c-d; 1-c-n, c+d-2e-n+1; 1) /; n \in \mathbb{N} $$

07.28.03.0020.01

$$ {}_4F_3\left(-\frac{n}{2}, \frac{1-n}{2}, c, d; e, 1-e-n, c+d+\frac{1}{2}; 1\right) = \frac{(e-c)_n}{(e)_n}\, {}_3F_2(-n, 2c, c+d; 2c+2d, c-e-n+1; 1) \,/;\, n \in \mathbb{N} $$

For fixed a_1, a_2

07.28.03.0021.01

$$ {}_4F_3(a, b, a, a; a+1, a+1, a+1; 1) = \frac{a^3}{2}\, \mathrm{B}(a, 1-b)\left((\psi(a)-\psi(a-b+1))^2 + \psi^{(1)}(a) - \psi^{(1)}(a-b+1)\right) /;\, \mathrm{Re}(b) < 3 $$

07.28.03.0022.01

$$ {}_4F_3\left(a, 2a+1, a+\frac{3}{2}, b; a+\frac{1}{2}, a+2, 2a-b+2; 1\right) = \frac{\sqrt{\pi}\, \Gamma(a+2)\, \Gamma(1-b)\, \Gamma(2a-b+2)}{2^{2a+1}\, \Gamma\left(a+\frac{3}{2}\right) \Gamma(a-b+1)\, \Gamma(a-b+2)} /;\, \mathrm{Re}(b) < 0 $$

07.28.03.0023.01

$$ {}_4F_3\left(a, b, a+\frac{1}{2}, b+\frac{1}{2}; \frac{1}{2}, a-b+\frac{1}{2}, a-b+1; 1\right) = \frac{2^{-2a-1}\, \Gamma(2a-2b+1)}{\Gamma(a-2b+1)} \left(\frac{\Gamma\left(\frac{1}{2}-2b\right)}{\Gamma\left(a-2b+\frac{1}{2}\right)} + \frac{\sqrt{\pi}}{\Gamma\left(a+\frac{1}{2}\right)}\right) /;\, \mathrm{Re}(b) < \frac{1}{4} $$

07.28.03.0024.01

$$ {}_4F_3\left(a, b, a+\frac{1}{2}, b+\frac{1}{2}; \frac{3}{2}, a-b+\frac{3}{2}, a-b+1; 1\right) = \frac{2^{-2a}\, \Gamma(2a-2b+2)}{(2a-1)(2b-1)\, \Gamma\left(a-2b+\frac{3}{2}\right)} \left(\frac{\Gamma\left(\frac{3}{2}-2b\right)}{\Gamma(a-2b+1)} - \frac{\sqrt{\pi}}{\Gamma(a)}\right) /;\, \mathrm{Re}(b) < \frac{3}{4} $$

For fixed a_2, a_3

07.28.03.0025.01

$$ {}_4F_3(1, b, c, b; b+1, c+1, b+1; 1) = \frac{b^2 c}{(b-c)^2} (\psi(b) - \psi(c)) - \frac{b^2 c}{b-c} \psi^{(1)}(b) \,/;\, b \neq c $$

07.28.03.0026.01

$$ {}_4F_3\left(-n, b, c, \frac{b}{2}+1; \frac{b}{2}, b+n+1, b-c+1; 1\right) = \frac{(b+1)_n \left(\frac{b+1}{2}-c\right)_n}{\left(\frac{b+1}{2}\right)_n (b-c+1)_n} /;\, n \in \mathbb{N} $$

07.28.03.0027.01

$$ {}_4F_3\left(-n, b, c, \frac{b}{2}+1; \frac{b}{2}, b-c+1, 2c-n+2; 1\right) = \frac{(b-2c-1)_n\, (-c-1)_n\, (b-2c+2n-1)}{(b-c+1)_n\, (-2c-1)_n\, (b-2c-1)} /;\, n \in \mathbb{N} $$

07.28.03.0028.01

$$ {}_4F_3\left(-n, b, c, \frac{b}{2}+1; \frac{b}{2}, b-c+1, 2c-n+2; 1\right) = $$
$$ \frac{(b-2c-1)_n}{(-2c-1)_n}\, {}_3F_2\left(-n, \frac{b+1}{2}, b-2c+n-1; b-c+1, \frac{b-1}{2}-c; 1\right) /;\, n \in \mathbb{N} $$

07.28.03.0029.01

$$ {}_4F_3\left(-n, b, c, b+\frac{1}{2}; 2b+1, \frac{c-n}{2}, \frac{c-n+1}{2}; 1\right) = \frac{(2b-c+1)_n}{(1-c)_n} /;\, n \in \mathbb{N} $$

07.28.03.0030.01

$$ {}_4F_3\left(-n, b, c, b+\frac{1}{2}; 2b+1, \frac{c-n+1}{2}, \frac{c-n}{2}+1; 1\right) = \frac{(2b-c+1)_n\, (2b-c-n)\, (c-n)}{(1-c)_n\, (2b-c+n)\, (c+n)} /;\, n \in \mathbb{N} $$

07.28.03.0031.01

$${}_4F_3\left(-n, b, c, \frac{1}{2}-b-c-n; -b-n, 1-c-n, b+c+\frac{1}{2}; 1\right) = \frac{(2b+1)_n (2c)_n (b+c)_n}{(b+1)_n (c)_n (2b+2c)_n} /; n \in \mathbb{N}$$

07.28.03.0032.01

$${}_4F_3\left(-n, b, c, \frac{1}{2}-b-c-n; 1-b-n, 1-c-n, b+c+\frac{1}{2}; 1\right) = \frac{(2b)_n (2c)_n (b+c)_n}{(b)_n (c)_n (2b+2c)_n} /; n \in \mathbb{N}$$

07.28.03.0033.01

$${}_4F_3\left(-n, b, c, \frac{1}{2}-b-c-n; 1-b-n, 1-c-n, b+c-\frac{1}{2}; 1\right) = \frac{(2b)_n (2c)_n (b+c)_n}{(b)_n (c)_n (2b+2c-1)_n} /; n \in \mathbb{N}$$

07.28.03.0034.01

$${}_4F_3\left(-n, b, c, \frac{3}{2}-b-c-n; 1-b-n, 1-c-n, b+c+\frac{1}{2}; 1\right) = \frac{(2b)_n (2c)_n (b+c)_n (2b+2c-1)}{(b)_n (c)_n (2b+2c-1)_n (2b+2c+2n-1)} /; n \in \mathbb{N}$$

07.28.03.0035.01

$${}_4F_3\left(-n, b, c, \frac{3}{2}-b-c-n; 1-b-n, 2-c-n, b+c-\frac{1}{2}; 1\right) = \frac{(2b)_n (2c-1)_n (b+c-1)_n}{(b)_n (c-1)_n (2b+2c-2)_n} /; n \in \mathbb{N}$$

07.28.03.0036.01

$${}_4F_3\left(-n, b, c, \frac{5}{2}-b-c-n; 2-b-n, 2-c-n, b+c-\frac{1}{2}; 1\right) = \frac{(2b-1)_n (2c-1)_n (b+c-1)_n (2b+2c-3)}{(b-1)_n (c-1)_n (2b+2c-3)_n (2b+2c+2n-3)} /; n \in \mathbb{N}$$

For fixed a_3, a_4

07.28.03.0037.01

$${}_4F_3(1, 1, c, d; 2, c+1, c-d+2; 1) = \frac{c(c-d+1)}{(c-1)(d-1)} \left(\psi\left(\frac{c+1}{2}\right) - \psi(c) - \psi\left(\frac{c+3}{2}-d\right) + \psi(c-d+1)\right) /; \operatorname{Re}(c-2d) > -3$$

07.28.03.0038.01

$${}_4F_3\left(-\frac{n}{2}, \frac{1-n}{2}, c, d; \frac{1}{2}-m, c+1, d+1; 1\right) = \frac{n!}{(c-d)\left(\frac{1}{2}\right)_m} \left(\frac{c\left(d+\frac{1}{2}\right)_m}{(2d+1)_n} - \frac{d\left(c+\frac{1}{2}\right)_m}{(2c+1)_n}\right) /; m \in \mathbb{N} \bigwedge n \in \mathbb{N} \bigwedge \left\lfloor \frac{n}{2} \right\rfloor \leq m \leq n$$

07.28.03.0039.01

$${}_4F_3\left(-\frac{n}{2}, \frac{1-n}{2}, c, d; -n, \frac{c+d+1}{2}, \frac{c+d}{2}+1; 1\right) = \frac{(c)_{n+1} - (d)_{n+1}}{(c-d)(c+d+1)_n} /; n \in \mathbb{N}$$

For fixed a_2, b_2

07.28.03.0040.01

$${}_4F_3\left(-n, b, n+1, b+\frac{1}{2}; \frac{1}{2}, f, 2b-f+2; 1\right) = \frac{1}{2(b-f+1)} \left(\frac{(1-f)_{n+1}}{(2b-f+2)_n} - \frac{(f-2b-1)_{n+1}}{(f)_n}\right) /; n \in \mathbb{N}$$

07.28.03.0041.01

$${}_4F_3\left(-n, b, n+2, b+\frac{1}{2}; \frac{3}{2}, f, 2b-f+2; 1\right) = \frac{1}{2(n+1)(b-f+1)(1-2b)} \left(\frac{(1-f)_{n+2}}{(2b-f+2)_n} - \frac{(f-2b-1)_{n+2}}{(f)_n}\right) /; n \in \mathbb{N}$$

For fixed a_3, b_2

07.28.03.0042.01

$${}_4F_3\left(-\frac{n}{2}, \frac{1-n}{2}, c, -c; \frac{1}{2}, f, -f-n+1; 1\right) = \frac{(f-c)_n + (c+f)_n}{2\,(f)_n} \;/;\; n \in \mathbb{N}$$

For fixed a_4, b_1

07.28.03.0043.01

$${}_4F_3\left(-\frac{n}{3}, \frac{1-n}{3}, \frac{2-n}{3}, d; e, e+\frac{1}{3}, e+\frac{2}{3}; 1\right) = \frac{(3e-3d)_n}{(3e)_n}\, {}_3F_2\left(-n, d, 3d-3e+1; \frac{1}{2}(3d-3e-n+1), \frac{1}{2}(3d-3e-n)+1; \frac{3}{4}\right) \;/;\; n \in \mathbb{N}$$

For fixed a_4, b_2

07.28.03.0044.01

$${}_4F_3(-n, 1, 1, d; 2, f, d-f-n+1; 1) = \frac{(f-1)(d-f-n)}{(n+1)(d-1)}\left(-\psi(f-1)+\psi(d-f+1)+\psi(f+n)-\psi(d-f-n)\right) \;/;\; n \in \mathbb{N}$$

For fixed a_1

07.28.03.0045.01

$${}_4F_3\left(a, a+\frac{1}{4}, a+\frac{1}{2}, a+\frac{3}{4}; b_1, b_2, b_3; 1\right) = n! \prod_{k=1}^{n} \frac{2^{-2a+\frac{n}{2}-1}\cos\left(\frac{1}{4}\pi(4a+n)\right) + 2^{-4a+n-2}}{k-4a} \;/;\; \mathrm{Re}(a) > \frac{n}{4}$$

Vector b_1, b_2, b_3 includes those three components of vector $(1+n)/4$, $(2+n)/4$, $(3+n)/4$, $(4+n)/4$ that not equal to 1, $n = 0, 1, 2, 3$

07.28.03.0046.01

$${}_4F_3\left(a, a, a, \frac{a}{2}+1; 1, 1, \frac{a}{2}; 1\right) = \frac{2^{1-a}\sqrt{\pi}\,\Gamma\left(\frac{1}{2}(1-3a)\right)}{a\,\Gamma\left(\frac{a}{2}\right)\Gamma(1-a)\,\Gamma\left(\frac{1-a}{2}\right)\Gamma\left(\frac{1-a}{2}\right)} \;/;\; \mathrm{Re}(a) < \frac{1}{3}$$

For fixed a_2

07.28.03.0047.01

$${}_4F_3(1, b, b, b; b+1, b+1, b+1; 1) = -\frac{b^3}{2}\,\psi^{(2)}(b)$$

For fixed a_3

07.28.03.0048.01

$${}_4F_3\left(\frac{1}{2}, 1, a, 1-a; \frac{3}{2}, a+1, 2-a; 1\right) = \frac{\pi a(1-a)}{(1-2a)^2}\left(\cot(\pi a) - \frac{2}{\pi}(2\log(2)+\psi(1-a)+\gamma)\right) \;/;\; a \neq \frac{1}{2}$$

07.28.03.0049.01

$${}_4F_3(-n, 1, c, 2-c; n+3, c+1, 3-c; 1) = \frac{n+2}{2(n+1)(c-1)^2}\left(\frac{(n+1)!^2}{(3-c)_n\,(c+1)_n} - c(2-c)\right) \;/;\; n \in \mathbb{N}$$

Values at $z = -1$

For fixed a_1, a_2, a_3, a_4

07.28.03.0050.01

$${}_4F_3(a,\,b,\,c,\,d;\,a-b+1,\,a-c+1,\,a-d+1;\,-1)=\frac{\Gamma(a-b+1)\,\Gamma(a-c+1)}{\Gamma(a+1)\,\Gamma(a-b-c+1)}\,{}_3F_2\left(\frac{a}{2}-d+1,\,b,\,c;\,\frac{a}{2}+1,\,a-d+1;\,1\right)$$

07.28.03.0051.01

$${}_4F_3(a,\,b,\,c,\,d;\,a-b+1,\,a-c+1,\,a-d+1;\,-1)=\frac{\Gamma(a-b+1)\,\Gamma(a-c+1)}{\Gamma(a+1)\,\Gamma(a-b-c+1)}\,{}_3F_2\left(\frac{a}{2}-d+1,\,b,\,c;\,\frac{a}{2}+1,\,a-d+1;\,1\right)$$

For fixed a_1, a_2, a_3

07.28.03.0052.01

$${}_4F_3\left(a,\,b,\,c,\,\frac{a}{2}+1;\,\frac{a}{2},\,a-b+1,\,a-c+1;\,-1\right)=\frac{\Gamma(a-b+1)\,\Gamma(a-c+1)}{\Gamma(a+1)\,\Gamma(a-b-c+1)}\;/;\;\operatorname{Re}(a-2b-2c)>-1$$

For fixed a_2, a_3, a_4

07.28.03.0053.01

$${}_4F_3(1,\,b,\,c,\,d;\,b+1,\,c+1,\,d+1;\,-1)=\frac{b\,c\,d}{2}\left(\frac{\psi\left(\frac{b+1}{2}\right)-\psi\left(\frac{b}{2}\right)}{(c-b)\,(d-b)}+\frac{\psi\left(\frac{c+1}{2}\right)-\psi\left(\frac{c}{2}\right)}{(b-c)\,(d-c)}+\frac{\psi\left(\frac{d+1}{2}\right)-\psi\left(\frac{d}{2}\right)}{(b-d)\,(c-d)}\right)\;/;\;b\neq c\land b\neq d\land c\neq d$$

For fixed a_1, a_2

07.28.03.0054.01

$${}_4F_3\left(a,\,b,\,2a+1,\,a+\frac{3}{2};\,a+\frac{1}{2},\,a+2,\,2a-b+2;\,-1\right)=\frac{\Gamma(2a-b+2)\,\Gamma(a+2)}{\Gamma(a-b+2)\,\Gamma(2a+2)}$$

For fixed a_2, a_3

07.28.03.0055.01

$${}_4F_3(1,\,b,\,c,\,b;\,b+1,\,c+1,\,b+1;\,-1)=$$
$$\frac{b^2\,c}{2\,(b-c)^2}\left(\psi\left(\frac{c+1}{2}\right)-\psi\left(\frac{b+1}{2}\right)+\psi\left(\frac{b}{2}\right)-\psi\left(\frac{c}{2}\right)\right)+\frac{b^2\,c}{4\,(b-c)}\left(\psi^{(1)}\left(\frac{b+1}{2}\right)-\psi^{(1)}\left(\frac{b}{2}\right)\right)\;/;\;b\neq c$$

07.28.03.0056.01

$${}_4F_3\left(-n,\,b,\,c,\,\frac{b}{2}+1;\,\frac{b}{2},\,b-c+1,\,b+n+1;\,-1\right)=\frac{(b+1)_n}{(b-c+1)_n}\;/;\;n\in\mathbb{N}$$

For fixed a_3, a_4

07.28.03.0057.01

$${}_4F_3\left(1,\,\frac{3}{2},\,c,\,d;\,\frac{1}{2},\,2-c,\,2-d;\,-1\right)=\frac{\Gamma(2-c)\,\Gamma(2-d)}{\Gamma(2-c-d)}$$

For fixed a_2

07.28.03.0058.01

$${}_4F_3(1,\,b,\,b,\,b;\,b+1,\,b+1,\,b+1;\,-1)=\frac{b^3}{16}\left(\psi^{(2)}\left(\frac{b+1}{2}\right)-\psi^{(2)}\left(\frac{b}{2}\right)\right)$$

For fixed a_3

07.28.03.0059.01

$${}_4F_3\left(1,\,\frac{3}{2},\,c,\,1-c;\,\frac{1}{2},\,c+1,\,2-c;\,-1\right)=\frac{\pi\,c\,(1-c)}{\sin(\pi\,c)}$$

07.28.03.0060.01

$${}_4F_3\left(\frac{1}{2}, 1, c, 1-c; \frac{3}{2}, c+1, 2-c; -1\right) = \frac{\pi c (1-c) (\csc(\pi c) - 1)}{(2c-1)^2}$$

07.28.03.0061.01

$${}_4F_3(-n, 1, c, 2-c; n+3, c+1, 3-c; -1) = \frac{(n+2) c (2-c)}{2 (n+1) (c-1)^2} \left(1 - \frac{(n+1)!^2}{(c)_{n+1} (2-c)_{n+1}} \sum_{k=0}^{n+1} \frac{(1-c)_k (c-1)_k 2^{2k}}{(2k)!}\right) /; n \in \mathbb{N}$$

07.28.03.0062.01

$${}_4F_3\left(-2n-1, \frac{1}{2}-n, c, -c-2n-1; -n-\frac{1}{2}, c+1, -c-2n; -1\right) = 0 /; n \in \mathbb{N}$$

Specialized values

For fixed $a_1, a_2, a_3, a_4, b_1, b_2, z$

07.28.03.0063.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, a_4; z) = {}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$$

For fixed $a_1, a_2, a_3, a_4, b_1, z$

07.28.03.0064.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, a_3, a_4; z) = {}_2F_1(a_1, a_2; b_1; z)$$

For fixed $a_1, a_2, a_3, a_4, b_3, z$

07.28.03.0065.01

$${}_4F_3(a, b, c, d; a+1, b+1, g; z) = \frac{1}{g (a-b)} (b (a-g) {}_3F_2(a, c, d; a+1, g+1; z) - a (b-g) {}_3F_2(b, c, d; b+1, g+1; z))$$

For fixed a_1, a_2, a_3, a_4, z

07.28.03.0066.01

$${}_4F_3(a, b, c, d; a+1, b+1, c+1; z) = \frac{b c}{(b-a) (c-a)} {}_2F_1(a, d; a+1; z) + \frac{a c}{(a-b) (c-b)} {}_2F_1(b, d; b+1; z) + \frac{a b}{(a-c) (b-c)} {}_2F_1(c, d; c+1; z)$$

For fixed a_1, a_2, a_3, z

07.28.03.0067.01

$${}_4F_3\left(a, b, c, a+b-c; \frac{a+b}{2}, \frac{a+b+1}{2}, a+b; \frac{z^2}{4 (z-1)}\right) = (1-z)^a {}_2F_1(a, c; a+b; z) {}_2F_1(a, a+b-c; a+b; z) /; z \notin (1, \infty)$$

07.28.03.0068.01

$${}_4F_3\left(a, b, c, a+b-c; \frac{a+b}{2}, \frac{a+b+1}{2}, a+b; z\right) = \left(1 - 2\left(z + \sqrt{z-1} \sqrt{z}\right)\right)^a {}_2F_1\left(a, c; a+b; 2\left(z + \sqrt{z-1} \sqrt{z}\right)\right) {}_2F_1\left(a, a+b-c; a+b; 2\left(z + \sqrt{z-1} \sqrt{z}\right)\right)$$

07.28.03.0069.01

$${}_4F_3\left(a, b, c, \frac{a}{2}+1; \frac{a}{2}, a-b+1, a-c+1; z\right) = (z+1) (1-z)^{-a-1} {}_3F_2\left(\frac{a}{2}+1, \frac{a+1}{2}, a-b-c+1; a-b+1, a-c+1; -\frac{4z}{(1-z)^2}\right)$$

For fixed a_1, a_2, b_2, z

07.28.03.0070.01

$${}_4F_3\left(a,\, b,\, \frac{a+b}{2},\, \frac{a+b+1}{2};\, a+b,\, f,\, a+b-f+1;\, z\right) = {}_2F_1\left(a,\, b;\, f;\, \frac{1-\sqrt{1-z}}{2}\right) {}_2F_1\left(a,\, b;\, a+b-f+1;\, \frac{1-\sqrt{1-z}}{2}\right)$$

07.28.03.0071.01

$${}_4F_3\left(a,\, b,\, a+\frac{1}{2},\, b+\frac{1}{2};\, \frac{1}{2},\, f,\, f+\frac{1}{2};\, z\right) = \frac{1}{2}\left({}_2F_1\left(2a,\, 2b;\, 2f;\, -\sqrt{z}\right) + {}_2F_1\left(2a,\, 2b;\, 2f;\, \sqrt{z}\right)\right)$$

07.28.03.0072.01

$${}_4F_3\left(a,\, b,\, a+\frac{1}{2},\, b+\frac{1}{2};\, \frac{3}{2},\, f,\, f+\frac{1}{2};\, z\right) = \frac{2f-1}{2(2a-1)(2b-1)\sqrt{z}}\left({}_2F_1\left(2a-1,\, 2b-1;\, 2f-1;\, \sqrt{z}\right) - {}_2F_1\left(2a-1,\, 2b-1;\, 2f-1;\, -\sqrt{z}\right)\right)$$

For fixed a_1, a_2, z

07.28.03.0073.01

$${}_4F_3\left(a,\, b,\, \frac{2a}{3}+1,\, a-b+\frac{1}{2};\, \frac{2a}{3},\, 2b,\, 2a-2b+1;\, z\right) = \frac{2^{2a-1}(z+8)}{(4-z)^{a+1}}\, {}_3F_2\left(\frac{a+1}{3},\, \frac{a+2}{3},\, \frac{a}{3}+1;\, b+\frac{1}{2},\, a-b+1;\, \frac{27z^2}{(4-z)^3}\right)$$

For fixed a_1, b_1, z

07.28.03.0074.01

$${}_4F_3\left(a,\, a+\frac{1}{4},\, a+\frac{1}{2},\, a+\frac{3}{4};\, e,\, 2a+\frac{1}{2},\, 2a-e+\frac{3}{2};\, z\right) = {}_2F_1\left(a,\, a+\frac{1}{2};\, e;\, \frac{1-\sqrt{1-z}}{2}\right) {}_2F_1\left(a,\, a+\frac{1}{2};\, 2a-e+\frac{3}{2};\, \frac{1-\sqrt{1-z}}{2}\right)$$

For fixed a_2, b_2, z

07.28.03.0075.01

$${}_4F_3\left(-n,\, a,\, a+\frac{1}{2},\, 2a+n;\, 2a,\, f,\, 2a-f+1;\, z\right) = \frac{(-1)^n\, n!^2}{(f)_n\,(2a-f+1)_n}\, P_n^{(f-1,2a-f)}\left(\sqrt{1-z}\right) P_n^{(f-1,2a-f)}\left(-\sqrt{1-z}\right) /;\, n \in \mathbb{N}$$

For fixed a_1, z

07.28.03.0076.01

$${}_4F_3\left(a,\, 1-a,\, \frac{3}{2}-a,\, a-\frac{1}{2};\, \frac{1}{2},\, 1,\, 1;\, z\right) = P_{1-2a}\left(\sqrt{1-z}-\sqrt{-z}\right) P_{1-2a}\left(\sqrt{1-z}+\sqrt{-z}\right)$$

07.28.03.0077.01

$${}_4F_3\left(a,\, 1-a,\, a+\frac{1}{2},\, \frac{1}{2}-a;\, \frac{1}{2},\, 1,\, 1;\, z\right) = P_{2a-1}\left(\sqrt{1-z}-\sqrt{-z}\right) P_{2a-1}\left(\sqrt{1-z}+\sqrt{-z}\right)$$

For fixed z and integer parameters

07.28.03.0078.01

$${}_4F_3\left(-n,\, -n,\, -n,\, \frac{1}{2}-n;\, \frac{1}{2},\, -2n,\, \frac{1}{2}-2n;\, z\right) = \frac{(2n)!\, n!^2\, 2^{2n}\, z^n}{(4n)!} /;\; P_{2n}\left(\frac{\sqrt{1-z}+1}{\sqrt{-z}}\right) P_{2n}\left(\frac{\sqrt{1-z}-1}{\sqrt{-z}}\right) /;\, n \in \mathbb{N}$$

07.28.03.0079.01

$${}_4F_3\left(-n,\, -n,\, -n,\, \frac{1}{2}-n;\, \frac{3}{2},\, -2n,\, -2n-\frac{1}{2};\, z\right) = \frac{(2n)!\, n!^2\, 2^{2n}\, z^n}{(4n+1)!}\, P_{2n+1}\left(\frac{\sqrt{1-z}+1}{\sqrt{-z}}\right) P_{2n+1}\left(\frac{\sqrt{1-z}-1}{\sqrt{-z}}\right) /;\, n \in \mathbb{N}$$

For fixed *z* and all parameters rational

07.28.03.0187.01

$$ {}_4F_3\left(\frac{3}{8},\frac{5}{8},\frac{7}{8},\frac{9}{8};\frac{5}{6},\frac{7}{6},\frac{9}{6};z\right)=\frac{4\sqrt[4]{2}}{3\sqrt{3}}\left(\sqrt{\frac{\sqrt[3]{z+\sqrt{z^2-z^3}}}{z}+\frac{1}{\sqrt[3]{z+\sqrt{z^2-z^3}}}}-\sqrt{-\frac{\sqrt[3]{z+\sqrt{z^2-z^3}}}{z}+\frac{2\sqrt{2}}{z\sqrt{\frac{\sqrt[3]{z+\sqrt{z^2-z^3}}}{z}+\frac{1}{\sqrt[3]{z+\sqrt{z^2-z^3}}}}}-\frac{1}{\sqrt[3]{z+\sqrt{z^2-z^3}}}}\right)^{3/2} /;\ \mathrm{Re}(z)\geq 0 $$

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For fixed *z*

07.28.03.0080.01

$$ {}_4F_3\left(\frac{1}{4},\frac{1}{2},\frac{3}{4},1;\frac{5}{4},\frac{3}{2},\frac{7}{4};z\right)=\frac{3}{4z^{3/4}}\left(-4\tanh^{-1}\left(\sqrt{z}\right)\sqrt[4]{z}+2\left(\sqrt{z}-1\right)\tan^{-1}\left(\sqrt[4]{z}\right)-\left(\sqrt{z}+1\right)\left(\log\left(1-\sqrt[4]{z}\right)-\log\left(\sqrt[4]{z}+1\right)\right)\right) $$

07.28.03.0081.01

$$ {}_4F_3\left(\frac{3}{4},1,1,\frac{5}{4};\frac{7}{4},2,\frac{9}{4};z\right)= -\frac{15}{2z^{5/4}}\left(4\sqrt[4]{z}-2\sqrt[4]{z}\log(1-z)+2\left(\sqrt{z}-1\right)\tan^{-1}\left(\sqrt[4]{z}\right)+\left(\sqrt{z}+1\right)\left(\log\left(1-\sqrt[4]{z}\right)-\log\left(\sqrt[4]{z}+1\right)\right)\right) $$

07.28.03.0188.01

$$ {}_4F_3\left(1,1,1,1;\frac{3}{2},2,2;z\right)=\frac{1}{3z}\left(2i\sin^{-1}\left(\sqrt{z}\right)^3+6\log\left(2\left(z+i\sqrt{(1-z)z}\right)\right)\sin^{-1}\left(\sqrt{z}\right)^2+ 6i\,\mathrm{Li}_2\left(1-2i\sqrt{1-z}\sqrt{z}-2z\right)\sin^{-1}\left(\sqrt{z}\right)+3\,\mathrm{Li}_3\left(1-2i\sqrt{1-z}\sqrt{z}-2z\right)-3\zeta(3)\right) $$

07.28.03.0189.01

$$ {}_4F_3\left(1,1,1,1;\frac{3}{2},2,2;z\right)= \frac{i}{3z}\left(2\sin^{-1}\left(\sqrt{z}\right)^3-6i\log\left(1-e^{-2i\sin^{-1}\left(\sqrt{z}\right)}\right)\sin^{-1}\left(\sqrt{z}\right)^2+6\,\mathrm{Li}_2\left(e^{-2i\sin^{-1}\left(\sqrt{z}\right)}\right)\sin^{-1}\left(\sqrt{z}\right)-3i\,\mathrm{Li}_3\left(e^{-2i\sin^{-1}\left(\sqrt{z}\right)}\right)+3i\zeta(3)\right) $$

Brychkov Yu.A. (2006)

07.28.03.0082.01

$$ {}_4F_3\left(1,1,3,3;2,\frac{7}{2},4;z\right)=\frac{5}{16z}\left(13+\frac{15}{z}-6(2z+1)\sqrt{1-z}\sin^{-1}\left(\sqrt{z}\right)z^{-3/2}-\frac{9}{z^2}\sin^{-1}\left(\sqrt{z}\right)^2\right) $$

07.28.03.0083.01

$$ {}_4F_3\left(\frac{4}{3}, \frac{5}{3}, 2, 2; \frac{7}{3}, \frac{8}{3}, 3; z\right) = \frac{10}{3 z^{8/3}} \left(\left(4 \sqrt[3]{z} - z^{2/3} + 6\right) z^{2/3} \log(1-z) - 2\sqrt{3} \left(\sqrt[3]{z} + 4\right) z \tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{z}}{\sqrt[3]{z}+2}\right) + 3\left(\sqrt[3]{z} - 4\right) z \log\left(1 - \sqrt[3]{z}\right)\right) $$

07.28.03.0084.01

$$ {}_4F_3(2, 2, 2, 2; 1, 1, 1; z) = -\frac{z^3 + 11 z^2 + 11 z + 1}{(z-1)^5} $$

Values at fixed points

For $z = 1$ and integer parameters

07.28.03.0085.01

$$ {}_4F_3(-n, 1, 1, n+3; 2, 2, m; 1) = \frac{(m-1)}{(n+1)(n+2)} (2\psi(n+2) - \psi(m-1) + \gamma) \;/;\; m-1 \in \mathbb{N}^+ \bigwedge n-1 \in \mathbb{N}^+ \bigwedge m \leq n+3 $$

07.28.03.0086.01

$$ {}_4F_3(-n, 1, 1, 2m+n-1; 2, m, m; 1) = \frac{2(m-1)^2}{(n+1)(2m+n-2)} (\psi(m+n) - \psi(m-1)) \;/;\; m-1 \in \mathbb{N}^+ \bigwedge n \in \mathbb{N} $$

07.28.03.0087.01

$$ {}_4F_3\left(-n, -\frac{3}{4}, -\frac{1}{4}, \frac{5}{8}; -\frac{3}{8}, \frac{1}{2}, \frac{1}{2} - n; 1\right) = -\frac{n!}{2(2n)!} \left(n + \frac{1}{2}\right)_{n-1} \;/;\; n \in \mathbb{N} $$

07.28.03.0088.01

$$ {}_4F_3\left(-n, -\frac{1}{4}, \frac{1}{4}, \frac{9}{8}; \frac{1}{8}, \frac{3}{2}, \frac{1}{2} - n; 1\right) = \frac{n!}{(2n+1)!} \left(n + \frac{1}{2}\right)_n \;/;\; n \in \mathbb{N} $$

07.28.03.0089.01

$$ {}_4F_3\left(-n, \frac{1}{2}, 1, 1; \frac{1}{2}, \frac{3}{2}, \frac{3}{2} - n; 1\right) = \frac{1-2n}{1+2n} \;/;\; n \in \mathbb{N} $$

07.28.03.0090.01

$$ {}_4F_3\left(-\frac{n}{2}, \frac{1-n}{2}, \frac{1-n}{2}, \frac{1-n}{2}; \frac{1}{2}, -\frac{n+1}{2}, -\frac{n+1}{2}; 1\right) = \frac{2^{n-3}(n+4)}{n+1} \;/;\; n-2 \in \mathbb{N}^+ $$

Values at $z = 1$

07.28.03.0091.01

$$ {}_4F_3\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}; \frac{5}{4}, \frac{5}{4}, \frac{5}{4}; 1\right) = \frac{\sqrt{\pi}}{128\,\Gamma\left(\frac{3}{4}\right)} \left(16 C + \pi^2\right) \Gamma\left(\frac{1}{4}\right) $$

07.28.03.0092.01

$$ {}_4F_3\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 1\right) = \frac{3\log(2)}{2} $$

07.28.03.0093.01

$$ {}_4F_3\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1; \frac{5}{4}, \frac{7}{4}, \frac{5}{2}; 1\right) = \frac{3}{10} (8\log(2) - \pi + 1) $$

07.28.03.0094.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1; \frac{3}{2}, \frac{7}{4}, \frac{9}{4}; 1\right) = \frac{5}{8}(4\log(2) - \pi + 2)$$

07.28.03.0095.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1; \frac{3}{2}, \frac{9}{4}, \frac{11}{4}; 1\right) = \frac{7}{12}(6\log(2) - 3\pi + 7)$$

07.28.03.0096.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1; \frac{7}{4}, \frac{9}{4}, \frac{5}{2}; 1\right) = \log(8) - \frac{9\pi}{4} + 6$$

07.28.03.0097.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1; \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{4}(10 - 3\pi)$$

07.28.03.0098.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, 1, 1; \frac{5}{4}, \frac{3}{2}, 2; 1\right) = \frac{\pi}{3}$$

07.28.03.0099.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, 1, 1; \frac{5}{4}, 2, \frac{5}{2}; 1\right) = \frac{1}{5}(3 - 6\log(2) + 2\pi)$$

07.28.03.0100.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, 1, 1; \frac{3}{2}, 2, \frac{9}{4}; 1\right) = \frac{10}{3}(\log(2) + 1)$$

07.28.03.0101.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, 1, 1; 2, \frac{9}{4}, \frac{5}{2}; 1\right) = -14\log(2) - 2\pi + 17$$

07.28.03.0102.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{7}{4}; \frac{5}{4}, \frac{3}{2}, \frac{11}{4}; 1\right) = \frac{7}{90}(\log(512) + 3\pi - 2)$$

07.28.03.0103.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{7}{4}; \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \log(128) - \frac{28}{3} + \frac{7\pi}{4}$$

07.28.03.0104.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, 1; \frac{5}{4}, \frac{7}{4}, 2; 1\right) = \pi - \log(8)$$

07.28.03.0105.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, 1; \frac{7}{4}, 2, \frac{9}{4}; 1\right) = \frac{5}{4}(6 - 4\log(8) + \pi)$$

07.28.03.0106.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, 1; 2, \frac{9}{4}, \frac{11}{4}; 1\right) = \frac{35}{18}(13 - 6\log(8))$$

07.28.03.0107.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, \frac{3}{2}; \frac{5}{4}, \frac{7}{4}, \frac{5}{2}; 1\right) = \frac{3}{10}(2\pi - \log(2) - 2)$$

07.28.03.0108.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, \frac{3}{2}; \frac{7}{4}, \frac{9}{4}, \frac{5}{2}; 1\right) = \frac{3}{8}(\log(16) + 7\pi - 22)$$

07.28.03.0109.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, \frac{3}{2}; \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{4}(\log(64) + 3\pi - 13)$$

07.28.03.0110.01

$${}_4F_3\left(\frac{1}{4}, 1, 1, \frac{3}{2}; \frac{5}{4}, 2, \frac{5}{2}; 1\right) = \frac{1}{5}(12\log(2) + \pi - 6)$$

07.28.03.0111.01

$${}_4F_3\left(\frac{1}{4}, 1, 1, \frac{3}{2}; 2, \frac{9}{4}, \frac{5}{2}; 1\right) = 6(3\log(2) - 4) + 4\pi$$

07.28.03.0112.01

$${}_4F_3\left(\frac{1}{4}, 1, 1, \frac{7}{4}; \frac{5}{4}, 2, \frac{11}{4}; 1\right) = \frac{7}{27}(9\log(2) - 2)$$

07.28.03.0113.01

$${}_4F_3\left(\frac{1}{4}, 1, \frac{3}{2}, \frac{7}{4}; \frac{5}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{30}(9\log(2) - 3\pi + 8)$$

07.28.03.0114.01

$${}_4F_3\left(\frac{1}{3}, \frac{1}{2}, 1, 1; \frac{4}{3}, \frac{3}{2}, 2; 1\right) = \log\left(\frac{9\sqrt[4]{3}}{16}\right) + \frac{\sqrt{3}\,\pi}{4}$$

07.28.03.0115.01

$${}_4F_3\left(\frac{1}{3}, \frac{2}{3}, 1, 1; \frac{4}{3}, \frac{5}{3}, 2; 1\right) = \frac{1}{2}\left(\sqrt{3}\,\pi - 3\log(3)\right)$$

07.28.03.0116.01

$${}_4F_3\left(\frac{1}{3}, \frac{2}{3}, 1, 1; \frac{5}{3}, 2, \frac{7}{3}; 1\right) = 4 - \log(729) + \frac{2\pi}{\sqrt{3}}$$

07.28.03.0117.01

$${}_4F_3\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{7}{3}; \frac{4}{3}, \frac{5}{3}, \frac{10}{3}; 1\right) = \frac{7}{180}\left(8\sqrt{3}\,\pi - 15\right)$$

07.28.03.0118.01

$${}_4F_3\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; 1\right) = \frac{\pi}{48}\left(3\log^2(4) + \pi^2\right)$$

07.28.03.0119.01

$${}_4F_3\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; 1\right) = \frac{7}{8}\zeta(3)$$

07.28.03.0120.01

$${}_4F_3\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{5}{2}, \frac{5}{2}, \frac{5}{2}; 1\right) = \frac{27}{2} - \frac{81\pi^2}{64}$$

07.28.03.0121.01

$${}_4F_3\left(\frac{1}{2},\frac{1}{2},1,1;2,\frac{5}{2},\frac{5}{2};1\right)=\frac{9}{2}(3-4\log(2))$$

07.28.03.0122.01

$${}_4F_3\left(\frac{1}{2},\frac{2}{3},1,1;\frac{3}{2},\frac{5}{3},2;1\right)=8\log(2)-9\log(3)+\sqrt{3}\,\pi$$

07.28.03.0123.01

$${}_4F_3\left(\frac{1}{2},\frac{3}{4},1,1;\frac{3}{2},\frac{7}{4},2;1\right)=3(\pi-4\log(2))$$

07.28.03.0124.01

$${}_4F_3\left(\frac{1}{2},\frac{3}{4},1,1;\frac{7}{4},2,\frac{5}{2};1\right)=-30\log(2)+6\pi+3$$

07.28.03.0125.01

$${}_4F_3\left(\frac{1}{2},\frac{3}{4},1,1;2,\frac{5}{2},\frac{11}{4};1\right)=\frac{7}{5}(11-42\log(2)+6\pi)$$

07.28.03.0126.01

$${}_4F_3\left(\frac{1}{2},\frac{3}{4},1,\frac{5}{4};\frac{3}{2},\frac{7}{4},\frac{9}{4};1\right)=\frac{5}{2}(\pi-\log(2)-2)$$

07.28.03.0127.01

$${}_4F_3\left(\frac{1}{2},\frac{3}{4},1,\frac{5}{4};\frac{7}{4},\frac{9}{4},\frac{5}{2};1\right)=\frac{15}{2}(\pi-3)$$

07.28.03.0128.01

$${}_4F_3\left(\frac{1}{2},\frac{3}{4},1,\frac{5}{4};\frac{9}{4},\frac{5}{2},\frac{11}{4};1\right)=21(\log(2)-3)+\frac{63\pi}{4}$$

07.28.03.0129.01

$${}_4F_3\left(\frac{1}{2},1,1,1;2,2,\frac{5}{2};1\right)=6-\frac{\pi^2}{2}$$

07.28.03.0130.01

$${}_4F_3\left(\frac{1}{2},1,1,\frac{5}{4};\frac{3}{2},2,\frac{9}{4};1\right)=\frac{5}{3}(8\log(2)+\pi-8)$$

07.28.03.0131.01

$${}_4F_3\left(\frac{1}{2},1,1,\frac{5}{4};2,\frac{9}{4},\frac{5}{2};1\right)=50\log(2)+10\pi-65$$

07.28.03.0132.01

$${}_4F_3\left(\frac{1}{2},1,1,\frac{7}{4};\frac{3}{2},2,\frac{11}{4};1\right)=\frac{7}{45}(36\log(2)-3\pi-8)$$

07.28.03.0133.01

$${}_4F_3\left(\frac{1}{2},1,\frac{5}{4},\frac{7}{4};\frac{3}{2},\frac{9}{4},\frac{11}{4};1\right)=\frac{7}{6}(8-\log(2)-2\pi)$$

07.28.03.0134.01

$${}_4F_3\left(\frac{1}{2},1,\frac{3}{2},\frac{9}{4};\frac{5}{4},2,3;1\right)=\frac{8}{5}$$

07.28.03.0135.01

$${}_4F_3\left(\frac{2}{3}, 1, 1, \frac{4}{3}; \frac{5}{3}, 2, \frac{7}{3}; 1\right) = 12 (\log(3) - 1)$$

07.28.03.0136.01

$${}_4F_3\left(\frac{2}{3}, 1, 1, \frac{4}{3}; \frac{5}{3}, 2, \frac{7}{3}; 1\right) = 12 (\log(3) - 1)$$

07.28.03.0137.01

$${}_4F_3\left(\frac{3}{4}, 1, 1, \frac{5}{4}; \frac{7}{4}, 2, \frac{9}{4}; 1\right) = 15 (3 \log(2) - 2)$$

07.28.03.0138.01

$${}_4F_3\left(\frac{3}{4}, 1, 1, \frac{3}{2}; \frac{7}{4}, 2, \frac{5}{2}; 1\right) = 3 (8 \log(2) - \pi - 2)$$

07.28.03.0139.01

$${}_4F_3\left(\frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}; \frac{7}{4}, \frac{9}{4}, \frac{5}{2}; 1\right) = \frac{15}{2} (4 - \log(2) - \pi)$$

07.28.03.0140.01

$${}_4F_3\left(\frac{5}{6}, 1, 1, \frac{7}{6}; \frac{11}{6}, 2, \frac{13}{6}; 1\right) = \frac{35}{2} (\log(432) - 6)$$

07.28.03.0141.01

$${}_4F_3\left(1, 1, \frac{5}{4}, \frac{3}{2}; 2, \frac{9}{4}, \frac{5}{2}; 1\right) = 15 (-\log(16) - \pi + 6)$$

07.28.03.0142.01

$${}_4F_3\left(1, 1, \frac{5}{4}, \frac{7}{4}; 2, \frac{9}{4}, \frac{11}{4}; 1\right) = \frac{35}{9} (16 - 9 \log(2) - 3 \pi)$$

07.28.03.0143.01

$${}_4F_3\left(1, 1, \frac{4}{3}, \frac{3}{2}; 2, \frac{7}{3}, \frac{5}{2}; 1\right) = 6 \left(8 \log(2) - 9 \log(3) - \sqrt{3} \pi + 10\right)$$

07.28.03.0144.01

$${}_4F_3\left(1, 1, \frac{4}{3}, \frac{5}{3}; \frac{7}{3}, \frac{8}{3}, 3; 1\right) = 110 - 20 \sqrt{3} \pi$$

07.28.03.0145.01

$${}_4F_3\left(1, 1, \frac{3}{2}, \frac{3}{2}; 2, 2, 2; 1\right) = 16 \left(\log(2) - \frac{2 C}{\pi}\right)$$

07.28.03.0146.01

$${}_4F_3\left(1, 1, \frac{3}{2}, \frac{5}{3}; 2, \frac{5}{2}, \frac{8}{3}; 1\right) = \frac{15}{4} \left(7 - 16 \log(2) + 9 \log(3) - \sqrt{3} \pi\right)$$

07.28.03.0147.01

$${}_4F_3\left(1, 1, \frac{3}{2}, \frac{7}{4}; 2, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{70}{3} - 7 \pi$$

07.28.03.0148.01

$${}_4F_3(1, 1, 1, 1; 3, 3, 3; 1) = 80 - 8 \pi^2$$

07.28.03.0149.01

$${}_4F_3(1, 1, 1, 1; 5, 5, 5; 1) = \frac{32}{27}\left(630\,\pi^2 - 6217\right)$$

07.28.03.0150.01

$${}_4F_3\left(1, 1, \frac{5}{2}, \frac{5}{2}; 2, 3, 3; 1\right) = \frac{32}{9}\left(1 - \frac{2\,(2\,C+1)}{\pi} + 2\log(2)\right)$$

07.28.03.0151.01

$${}_4F_3\left(1, 1, \frac{7}{2}, \frac{7}{2}; 2, 4, 4; 1\right) = \frac{4}{25}\left(27 - \frac{4\,(18\,C+13)}{\pi} + 36\log(2)\right)$$

For $z = -1$ and integer parameters

07.28.03.0152.01

$${}_4F_3(-n, 2, 2, 2; 1, 1, 1; -1) = 2^{n-3}\,(n+2)\left(n^2 + 7\,n + 4\right) /;\, n \in \mathbb{N}$$

Values at $z = -1$

07.28.03.0153.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -1\right) = \frac{3\,\pi}{4}\left(\sqrt{2} - 1\right)$$

07.28.03.0154.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1; \frac{5}{4}, \frac{7}{4}, \frac{5}{2}; -1\right) = \frac{3}{20}\left(\pi\left(8\sqrt{2} - 7\right) - 4\sqrt{2}\,\log\left(1+\sqrt{2}\right) - 2\right)$$

07.28.03.0155.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, 1, 1; \frac{5}{4}, \frac{3}{2}, 2; -1\right) = \frac{1}{6}\left(\pi\left(-3 + 2\sqrt{2}\right) + \log(4) + 4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)$$

07.28.03.0156.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, 1; \frac{5}{4}, \frac{7}{4}, 2; -1\right) = \log(2) + 2\sqrt{2}\,\log\left(1+\sqrt{2}\right) - \frac{\pi}{\sqrt{2}}$$

07.28.03.0157.01

$${}_4F_3\left(\frac{1}{4}, \frac{1}{2}, 1, 1; \frac{5}{4}, 2, \frac{5}{2}; -1\right) = \frac{1}{5}\left(\pi\left(2\sqrt{2} - 3\right) + \log(32) + 4\sqrt{2}\,\log\left(1+\sqrt{2}\right) - 3\right)$$

07.28.03.0158.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, \frac{3}{2}; \frac{5}{4}, \frac{7}{4}, \frac{5}{2}; -1\right) = \frac{3}{20}\left(8\sqrt{2}\,\log\left(1+\sqrt{2}\right) - \sqrt{2}\,\pi - \pi + 4\right)$$

07.28.03.0159.01

$${}_4F_3\left(\frac{1}{4}, \frac{3}{4}, 1, \frac{3}{2}; \frac{5}{4}, \frac{7}{4}, \frac{5}{2}; -1\right) = \frac{3}{20}\left(8\sqrt{2}\,\log\left(1+\sqrt{2}\right) - \sqrt{2}\,\pi - \pi + 4\right)$$

07.28.03.0160.01

$${}_4F_3\left(\frac{1}{4}, 1, \frac{3}{2}, \frac{7}{4}; \frac{5}{4}, \frac{5}{2}, \frac{11}{4}; -1\right) = \frac{7}{60}\left(\pi\left(9 - 6\sqrt{2}\right) + 18\sqrt{2}\,\log\left(1+\sqrt{2}\right) - 16\right)$$

07.28.03.0161.01

$${}_4F_3\left(\frac{1}{3}, \frac{1}{2}, 1, 1; \frac{4}{3}, \frac{3}{2}, 2; -1\right) = \frac{\pi}{2}\left(\sqrt{3} - 2\right) + \log(4)$$

07.28.03.0162.01

$${}_4F_3\left(\frac{1}{3}, \frac{2}{3}, 1, 1; \frac{4}{3}, \frac{5}{3}, 2; -1\right) = \log(16) - \frac{\pi}{\sqrt{3}}$$

07.28.03.0163.01

$${}_4F_3\left(\frac{1}{3}, \frac{2}{3}, 1, 1; \frac{5}{3}, 2, \frac{7}{3}; -1\right) = \frac{4}{9}\left(24\log(2) - \sqrt{3}\,\pi - 9\right)$$

07.28.03.0164.01

$${}_4F_3\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{7}{3}; \frac{4}{3}, \frac{5}{3}, \frac{10}{3}; -1\right) = \frac{7}{60}(16\log(2) - 3)$$

07.28.03.0165.01

$${}_4F_3\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{5}{2}, \frac{5}{2}, \frac{5}{2}; -1\right) = \frac{27}{128}\left(-64 + 12\pi + \pi^3\right)$$

07.28.03.0166.01

$${}_4F_3\left(\frac{1}{2}, \frac{2}{3}, 1, 1; \frac{3}{2}, \frac{5}{3}, 2; -1\right) = \log(256) + 2\left(1 - \sqrt{3}\right)\pi$$

07.28.03.0167.01

$${}_4F_3\left(\frac{1}{2}, \frac{3}{4}, 1, 1; \frac{3}{2}, \frac{7}{4}, 2; -1\right) = \frac{3}{2}\left(\log(4) + 4\sqrt{2}\,\log\left(1 + \sqrt{2}\right) - 2\sqrt{2}\,\pi + \pi\right)$$

07.28.03.0168.01

$${}_4F_3\left(\frac{1}{2}, \frac{3}{4}, 1, 1; \frac{7}{4}, 2, \frac{5}{2}; -1\right) = \pi\left(3 - 6\sqrt{2}\right) + \log(512) + 12\sqrt{2}\,\log\left(1 + \sqrt{2}\right) - 3$$

07.28.03.0169.01

$${}_4F_3\left(\frac{1}{2}, 1, 1, \frac{5}{4}; \frac{3}{2}, 2, \frac{9}{4}; -1\right) = \frac{5}{6}\left(\pi\left(1 - 2\sqrt{2}\right) - 6\log(2) + 4\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + 16\right)$$

07.28.03.0170.01

$${}_4F_3\left(\frac{1}{2}, 1, 1, \frac{7}{4}; \frac{3}{2}, 2, \frac{11}{4}; -1\right) = \frac{7}{90}\left(\pi\left(9 - 6\sqrt{2}\right) - 15\log(4) + 12\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + 16\right)$$

07.28.03.0171.01

$${}_4F_3\left(\frac{1}{2}, 1, 1, \frac{5}{2}; \frac{3}{2}, 3, \frac{7}{2}; -1\right) = \frac{10}{9}(5 - \log(64))$$

07.28.03.0172.01

$${}_4F_3\left(\frac{2}{3}, 1, 1, \frac{4}{3}; \frac{5}{3}, 2, \frac{7}{3}; -1\right) = 12 - 16\log(2)$$

07.28.03.0173.01

$${}_4F_3\left(\frac{3}{4}, 1, 1, \frac{5}{4}; \frac{7}{4}, 2, \frac{9}{4}; -1\right) = 30 - 15\log(2) - \frac{30}{\sqrt{2}}\log\left(1 + \sqrt{2}\right)$$

07.28.03.0174.01

$${}_4F_3\left(\frac{3}{4}, 1, 1, \frac{3}{2}; \frac{7}{4}, 2, \frac{5}{2}; -1\right) = \frac{3}{2}\left(\pi\left(-1 + 2\sqrt{2}\right) - 3\log(4) - 4\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + 4\right)$$

07.28.03.0175.01

$${}_4F_3\left(\frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}; \frac{7}{4}, \frac{9}{4}, \frac{5}{2}; -1\right) = \frac{15}{4}\left(\pi\left(-1 + 2\sqrt{2}\right) + 2\sqrt{2}\,\log\left(1 + \sqrt{2}\right) - 8\right)$$

07.28.03.0176.01

$${}_4F_3\left(\frac{5}{6}, 1, 1, \frac{7}{6}; \frac{11}{6}, 2, \frac{13}{6}; -1\right) = 35\left(3 - \log(2) - \sqrt{3}\log\left(2 + \sqrt{3}\right)\right)$$

07.28.03.0177.01

$${}_4F_3(1, 1, 1, 1; 3, 3, 3; -1) = 96\log(2) + 12\zeta(3) - 80$$

07.28.03.0178.01

$${}_4F_3\left(1, 1, \frac{5}{4}, \frac{3}{2}; 2, \frac{9}{4}, \frac{5}{2}; -1\right) = \frac{15}{2}\left(\pi\left(2\sqrt{2} - 1\right) + \log(4) + 4\sqrt{2}\log\left(1 + \sqrt{2}\right) - 12\right)$$

07.28.03.0179.01

$${}_4F_3\left(1, 1, \frac{5}{4}, \frac{7}{4}; 2, \frac{9}{4}, \frac{11}{4}; -1\right) = \frac{35}{18}\left(\log(64) + 12\sqrt{2}\log\left(1 + \sqrt{2}\right) + 3\sqrt{2}\pi - 32\right)$$

07.28.03.0180.01

$${}_4F_3\left(1, 1, \frac{4}{3}, \frac{3}{2}; 2, \frac{7}{3}, \frac{5}{2}; -1\right) = 12\left(\pi\left(\sqrt{3} - 1\right) + \log(16) - 5\right)$$

07.28.03.0181.01

$${}_4F_3\left(1, 1, \frac{4}{3}, \frac{5}{3}; \frac{7}{3}, \frac{8}{3}, 3; -1\right) = 160\log(2) - 110$$

07.28.03.0182.01

$${}_4F_3\left(1, 1, \frac{3}{2}, \frac{5}{3}; 2, \frac{5}{2}, \frac{8}{3}; -1\right) = \frac{15}{4}\left(2\pi\left(2 - \sqrt{3}\right) + 8\log(2) - 7\right)$$

07.28.03.0183.01

$${}_4F_3\left(1, 1, \frac{3}{2}, \frac{7}{4}; 2, \frac{5}{2}, \frac{11}{4}; -1\right) = \frac{7}{6}\left(\pi\left(9 - 6\sqrt{2}\right) + \log(64) + 12\sqrt{2}\log\left(1 + \sqrt{2}\right) - 20\right)$$

Values at $z = \frac{1}{2}$

07.28.03.0184.01

$${}_4F_3\left(1, 1, 1, 1; 2, 2, 2; \frac{1}{2}\right) = \frac{1}{12}\left(4\log^3(2) - \pi^2\log(4) + 21\zeta(3)\right)$$

Values at $z = \frac{3-\sqrt{5}}{2}$

07.28.03.0185.01

$${}_4F_3\left(1, 1, 1, 1; 2, 2, 2; \frac{3 - \sqrt{5}}{2}\right) = \frac{2}{3 - \sqrt{5}}\left(\frac{1}{15}\pi^2\log\left(\frac{3 - \sqrt{5}}{2}\right) - \frac{1}{12}\log^3\left(\frac{3 - \sqrt{5}}{2}\right) + \frac{4}{5}\zeta(3)\right)$$

For $z = -27$ and integer parameters

07.28.03.0186.01

$${}_4F_3\left(-\frac{n}{2}, \frac{1 - n}{2}, \frac{1}{3} - n, \frac{22 - 9n}{21}; \frac{5}{6}, \frac{4}{3}, \frac{1 - 9n}{21}; -27\right) = \frac{(-8)^n}{1 - 9n} \;/;\; n \in \mathbb{N}$$

General characteristics

Some abbreviations

07.28.04.0001.01

$$\mathcal{NT}(\{a_1, \ldots, a_p\}) = \neg(-a_1 \in \mathbb{N} \vee \ldots \vee -a_p \in \mathbb{N})$$

Domain and analyticity

${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ is an analytical function of $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and z which is defined in $\mathbb{C}^8$. If parameters $a_k$include negative integers, the function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ degenerates to a polynomial in z.

07.28.04.0002.01

$$(\{a_1 * a_2 * a_3 * a_4\} * \{b_1 * b_2 * b_3\} * z) \longrightarrow {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) :: (\{\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}\} \otimes \{\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}\} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.28.04.0003.02

$${}_4F_3(\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}; \overline{b_1}, \overline{b_2}, \overline{b_3}; \overline{z}) = \overline{{}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)} \;/;\; z \notin (1, \infty)$$

Permutation symmetry

07.28.04.0004.01

$${}_4F_3(a_1, a_2, \ldots, a_k, \ldots, a_j, \ldots, a_4; b_1, \ldots, b_3; z) = {}_4F_3(a_1, a_2, \ldots, a_j, \ldots, a_k, \ldots, a_4; b_1, \ldots, b_3; z) \;/;\; a_k \neq a_j \wedge k \neq j$$

07.28.04.0005.01

$${}_4F_3(a_1, \ldots, a_4; b_1, b_2, \ldots, b_k, \ldots, b_j, \ldots, b_3; z) = {}_4F_3(a_1, \ldots, a_4; b_1, b_2, \ldots, b_j, \ldots, b_k, \ldots, b_3; z) \;/;\; b_k \neq b_j \wedge k \neq j$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed a_l, b_j in nonpolynomial cases (when $\neg(-a_1 \in \mathbb{N} \vee \ldots \vee -a_4 \in \mathbb{N})$), the function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ does not have poles and essential singularities.

07.28.04.0006.01

$$\mathcal{S}ing_z({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) = \{\} \;/;\; \mathcal{NT}(\{a_1, a_2, a_3, a_4\})$$

If parameters a_k include r negative integers α_k, the function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ is a polynomial and has pole of order $\min(-\alpha_1, \ldots, -\alpha_r\}$ at $z = \tilde{\infty}$.

07.28.04.0007.01

$$\mathcal{S}ing_z({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) = \{\{\tilde{\infty}, -\alpha\}\} \;/;\; \neg(\mathcal{NT}(\{a_1, a_2, a_3, a_4\})) \bigwedge \alpha = \min(-a_{s_1}, \ldots, -a_{s_r}\} \bigwedge -a_{s_k} \in \mathbb{N}^+$$

With respect to a_l

The function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ as a function of a_l, $1 \leq l \leq 4$, has only one singular point at $a_l = \tilde{\infty}$. It is an essential singular point.

07.28.04.0008.01

$$\mathcal{S}ing_{a_l}({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) == \{\{\tilde{\infty}, \infty\}\} /; 1 \leq l \leq 4$$

With respect to b_j

The function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ as a function of b_j, $1 \leq j \leq 3$, has an infinite set of singular points:

a) $b_j == -k /; k \in \mathbb{N}$, are the simple poles with residues $\frac{(-1)^k}{k!}\, {}_4\tilde{F}_3(a_1, a_2, a_3, a_4; b_1, \ldots, b_{j-1}, -k, b_{j+1}, \ldots, b_3; z)$;

b) $b_j == \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

07.28.04.0009.01

$$\mathcal{S}ing_{b_j}({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) == \{\{\{-k, 1\} /; k \in \mathbb{N}\}, \{\tilde{\infty}, \infty\}\} /; 1 \leq j \leq 3$$

07.28.04.0010.01

$$\operatorname{res}_{b_j}({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z))(-k) == \frac{(-1)^k}{k!}\, {}_4\tilde{F}_3(a_1, a_2, a_3, a_4; b_1, \ldots, b_{j-1}, -k, b_{j+1}, \ldots, b_3; z) /; k \in \mathbb{N} \land 1 \leq j \leq 3$$

Branch points

With respect to z

For all a_k, not being negative integer, the function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ has two branch points: $z == 1$, $z == \tilde{\infty}$.

07.28.04.0011.01

$$\mathcal{BP}_z({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) == \{1, \tilde{\infty}\} /; \mathcal{NT}(\{a_1, a_2, a_3, a_4\})$$

07.28.04.0012.01

$$\mathcal{R}_z({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z), 1) == \log /; \psi_3 \in \mathbb{Z} \bigvee \psi_3 \notin \mathbb{Q} \bigwedge \psi_3 == \sum_{j=1}^{3} b_j - \sum_{j=1}^{4} a_j \bigwedge \mathcal{NT}(\{a_1, a_2, a_3, a_4\})$$

07.28.04.0013.01

$$\mathcal{R}_z({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z), 1) == s /; \\ \psi_3 == \sum_{j=1}^{3} b_j - \sum_{j=1}^{4} a_j == \frac{r}{s} \bigwedge r \in \mathbb{Z} \bigwedge s - 1 \in \mathbb{N}^+ \bigwedge \gcd(r, s) == 1 \bigwedge \mathcal{NT}(\{a_1, a_2, a_3, a_4\})$$

07.28.04.0014.01

$$\mathcal{R}_z({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z), \tilde{\infty}) == \log /; \exists_{a_i, a_j} (a_i - a_j \in \mathbb{Z} \land 1 \leq i \leq 4 \land 1 \leq j \leq 4 \land i \neq j) \bigwedge (a_1 \notin \mathbb{Q} \lor \ldots \lor a_4 \notin \mathbb{Q}$$

07.28.04.0015.01

$$\mathcal{R}_z({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z), \tilde{\infty}) == \operatorname{lcm}(s_1, s_2, s_3, s_4) /; \\ a_l == \frac{r_l}{s_l} \bigwedge \{r_l, s_l\} \in \mathbb{Z} \bigwedge s_l > 1 \bigwedge \gcd(r_l, s_l) == 1 \bigwedge 1 \leq l \leq 4 \bigwedge \mathcal{NT}(\{s_1, s_2, s_3, s_4\})$$

With respect to a_l

The function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ as a function of a_l, $1 \leq l \leq 4$, does not have branch points.

07.28.04.0016.01

$$\mathcal{BP}_{a_l}({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) == \{\} /; 1 \leq l \leq 4$$

With respect to b_j

The function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ as a function of b_j, $1 \leq j \leq 3$, does not have branch points.

07.28.04.0017.01

$$\mathcal{BP}_{b_j}({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) = \{\} /; 1 \leq j \leq 3$$

Branch cuts

With respect to z

For all a_k, not being negative integer, the function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ is a single-valued function on the z-plane cut along the interval $(1, \infty)$, where it is continuous from below.

07.28.04.0018.01

$$\mathcal{BC}_z({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) = \{\{(1, \infty), i\}\} /; \mathcal{NT}(\{a_1, a_2, a_3, a_4\})$$

07.28.04.0019.01

$$\lim_{\epsilon \to +0} {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; x - i\,\epsilon) = {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; x) /; x > 1$$

07.28.04.0023.01

$$\lim_{\epsilon \to +0} {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; x + i\,\epsilon) = \frac{\prod_{k=1}^{3} \Gamma(b_k)}{\prod_{k=1}^{4} \Gamma(a_k)} G_{4,4}^{4,1}\left(\frac{e^{\pi i}}{x} \,\middle|\, \begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix} \right) /; x > 1$$

07.28.04.0020.01

$$\lim_{\epsilon \to +0} {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; x + i\,\epsilon) = \frac{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)\Gamma(a_2 - a_1)\Gamma(a_3 - a_1)\Gamma(a_4 - a_1)}{\Gamma(a_2)\Gamma(a_3)\Gamma(a_4)\Gamma(b_1 - a_1)\Gamma(b_2 - a_1)\Gamma(b_3 - a_1)}$$
$$\left(-\frac{1}{x}\right)^{a_1} {}_4F_3\left(a_1, a_1 - b_1 + 1, a_1 - b_2 + 1, a_1 - b_3 + 1; a_1 - a_2 + 1, a_1 - a_3 + 1, a_1 - a_4 + 1; \frac{1}{x}\right) +$$
$$\frac{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)\Gamma(a_1 - a_2)\Gamma(a_3 - a_2)\Gamma(a_4 - a_2)}{\Gamma(a_1)\Gamma(a_3)\Gamma(a_4)\Gamma(b_1 - a_2)\Gamma(b_2 - a_2)\Gamma(b_3 - a_2)} \left(-\frac{1}{x}\right)^{a_2}$$
$${}_4F_3\left(a_2, a_2 - b_1 + 1, a_2 - b_2 + 1, a_2 - b_3 + 1; -a_1 + a_2 + 1, a_2 - a_3 + 1, a_2 - a_4 + 1; \frac{1}{x}\right) +$$
$$\frac{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)\Gamma(a_1 - a_3)\Gamma(a_2 - a_3)\Gamma(a_4 - a_3)}{\Gamma(a_1)\Gamma(a_2)\Gamma(a_4)\Gamma(b_1 - a_3)\Gamma(b_2 - a_3)\Gamma(b_3 - a_3)} \left(-\frac{1}{x}\right)^{a_3}$$
$${}_4F_3\left(a_3, a_3 - b_1 + 1, a_3 - b_2 + 1, a_3 - b_3 + 1; -a_1 + a_3 + 1, -a_2 + a_3 + 1, a_3 - a_4 + 1; \frac{1}{x}\right) +$$
$$\frac{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)\Gamma(a_1 - a_4)\Gamma(a_2 - a_4)\Gamma(a_3 - a_4)}{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)\Gamma(b_1 - a_4)\Gamma(b_2 - a_4)\Gamma(b_3 - a_4)} \left(-\frac{1}{x}\right)^{a_4}$$
$${}_4F_3\left(a_4, a_4 - b_1 + 1, a_4 - b_2 + 1, a_4 - b_3 + 1; -a_1 + a_4 + 1, -a_2 + a_4 + 1, -a_3 + a_4 + 1; \frac{1}{x}\right) /;$$
$$\forall_{\{j,k\},\{j,k\} \in \mathbb{Z} \land j \neq k \land 1 \leq j \leq 4 \land 1 \leq k \leq 4} \left(a_j - a_k \notin \mathbb{Z}\right) \land x > 1$$

With respect to a_l

The function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ as a function of a_l, $1 \leq l \leq 4$, does not have branch cuts.

07.28.04.0021.01

$$\mathcal{BC}_{a_l}({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) = \{\} /; 1 \leq l \leq 4$$

With respect to b_j

The function ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$ as a function of b_j, $1 \leq j \leq 3$, does not have branch cuts.

07.28.04.0022.01

$$\mathcal{BC}_{b_j}({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)) = \{\} \;/;\; 1 \leq j \leq 3$$

Limit representations

07.28.09.0001.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \lim_{p\to\infty} {}_5F_4(a_1, a_2, a_3, a_4, p\, z; b_1, b_2, b_3, p; 1) \;/;$$
$$\operatorname{Re}(p\,(1-z) - a_1 - a_2 - a_3 - a_4 + b_1 + b_2 + b_3) > 0$$

07.28.09.0002.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \lim_{p\to\infty} {}_4F_4(a_1, a_2, a_3, a_4; b_1, b_2, b_3, p; p\, z)$$

Continued fraction representations

07.28.10.0001.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = 1 + (a_1\, a_2\, a_3\, a_4\, z / (b_1\, b_2\, b_3)) \Bigg/ \left(1 + -\frac{z\,(1+a_1)\,(1+a_2)\,(1+a_3)\,(1+a_4)}{2\,(1+b_1)\,(1+b_2)\,(1+b_3)} \Bigg/ \left(1 + \frac{z\,(1+a_1)\,(1+a_2)\,(1+a_3)\,(1+a_4)}{2\,(1+b_1)\,(1+b_2)\,(1+b_3)} + \frac{-\frac{z\,(2+a_1)\,(2+a_2)\,(2+a_3)\,(2+a_4)}{3\,(2+b_1)\,(2+b_2)\,(2+b_3)}}{1 + \frac{z\,(2+a_1)\,(2+a_2)\,(2+a_3)\,(2+a_4)}{3\,(2+b_1)\,(2+b_2)\,(2+b_3)} + \ldots}\right)\right)$$

07.28.10.0002.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = 1 + \frac{a_1\, a_2\, a_3\, a_4\, z}{b_1\, b_2\, b_3 \left(1 + \operatorname{K}_k\left(-\frac{(k+a_1)\,(k+a_2)\,(k+a_3)\,(k+a_4)\,z}{(k+1)\,(k+b_1)\,(k+b_2)\,(k+b_3)}, 1 + \frac{(k+a_1)\,(k+a_2)\,(k+a_3)\,(k+a_4)\,z}{(k+1)\,(k+b_1)\,(k+b_2)\,(k+b_3)}\right)_1^\infty\right)}$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

Representation of fundamental system solutions near zero

07.28.13.0002.01

$$(1-z)\,z^3\,w^{(4)}(z)+(-z\,(a_1+a_2+a_3+a_4+6)+b_1+b_2+b_3+3)\,z^2\,w^{(3)}(z)+(-z\,(a_2\,a_1+a_3\,a_1+a_4\,a_1+3\,a_1+3\,a_2+a_2\,a_3+3\,a_3+a_2\,a_4+a_3\,a_4+3\,a_4+7)+b_1+b_1\,b_2+b_2+b_1\,b_3+b_2\,b_3+b_3+1)\,z\,w''(z)+(b_1\,b_2\,b_3-(a_2\,a_1+a_2\,a_3\,a_1+a_3\,a_1+a_2\,a_4\,a_1+a_3\,a_4\,a_1+a_4\,a_1+a_1+a_2+a_2\,a_3+a_3+a_2\,a_4+a_2\,a_3\,a_4+a_3\,a_4+a_4+1)\,z)\,w'(z)-a_1\,a_2\,a_3\,a_4\,w(z)=0\;/;$$
$$w(z)=c_1\,{}_4\tilde{F}_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;z)+c_2\left(G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_1,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_3,1-b_1,1-b_2\end{matrix}\right)\right)+c_3\left(G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_3,1-b_2\end{matrix}\right)+G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_3,1-b_1\end{matrix}\right)\right)+c_4\,G_{4,4}^{4,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)$$

07.28.13.0003.01

$$W_z\Bigg({}_4\tilde{F}_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;z),\;G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_1,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_3,1-b_1,1-b_2\end{matrix}\right),\;G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_3,1-b_2\end{matrix}\right)+G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_3,1-b_1\end{matrix}\right),\;G_{4,4}^{4,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)\Bigg)=(1-z)^{-a_1-a_2-a_3-a_4+b_1+b_2+b_3-3}\,(-z)^{-b_1-b_2-b_3}\,z^{-b_1-b_2-b_3-3}\,\Big(-(-z)^{b_2+b_3}\,(\csc(\pi\,(b_1-b_3))\sin(\pi\,(b_1-b_2))+\csc(\pi\,(b_1-b_2))\sin(\pi\,(b_1-b_3)))\,z^{b_1}+(-z)^{b_1+b_3}\,(\csc(\pi\,(b_2-b_3))\sin(\pi\,(b_1-b_2))+\csc(\pi\,(b_1-b_2))\sin(\pi\,(b_2-b_3)))\,z^{b_2}-(-z)^{b_1+b_2}\,(\csc(\pi\,(b_2-b_3))\sin(\pi\,(b_1-b_3))+\csc(\pi\,(b_1-b_3))\sin(\pi\,(b_2-b_3)))\,z^{b_3}-2\left((-z)^{b_2+b_3}\,z^{b_1}+(-z)^{b_1+b_3}\,z^{b_2}+(-z)^{b_1+b_2}\,z^{b_3}\right)\Big)\,\Gamma(a_1-b_1+1)\,\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(a_4-b_1+1)\,\Gamma(a_1-b_2+1)\,\Gamma(a_2-b_2+1)\,\Gamma(a_3-b_2+1)\,\Gamma(a_4-b_2+1)\,\Gamma(a_1-b_3+1)\,\Gamma(a_2-b_3+1)\,\Gamma(a_3-b_3+1)\,\Gamma(a_4-b_3+1)$$

07.28.13.0004.01

$$(1-z)\,z^3\,w^{(4)}(z)+(-z\,(a_1+a_2+a_3+a_4+6)+b_1+b_2+b_3+3)\,z^2\,w^{(3)}(z)+(-z\,(a_2\,a_1+a_3\,a_1+a_4\,a_1+3\,a_1+3\,a_2+a_2\,a_3+3\,a_3+a_2\,a_4+a_3\,a_4+3\,a_4+7)+b_1+b_1\,b_2+b_2+b_1\,b_3+b_2\,b_3+b_3+1)\,z\,w''(z)+(b_1\,b_2\,b_3-(a_2\,a_1+a_2\,a_3\,a_1+a_3\,a_1+a_2\,a_4\,a_1+a_3\,a_4\,a_1+a_4\,a_1+a_1+a_2+a_2\,a_3+a_3+a_2\,a_4+a_2\,a_3\,a_4+a_3\,a_4+a_4+1)\,z)\,w'(z)-a_1\,a_2\,a_3\,a_4\,w(z)=0\;/;\;w(z)=c_1\,{}_4\tilde{F}_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;z)+c_2\,z^{1-b_1}\,{}_4\tilde{F}_3(a_1-b_1+1,a_2-b_1+1,a_3-b_1+1,a_4-b_1+1;2-b_1,-b_1+b_2+1,-b_1+b_3+1;z)+c_3\,z^{1-b_2}\,{}_4\tilde{F}_3(a_1-b_2+1,a_2-b_2+1,a_3-b_2+1,a_4-b_2+1;2-b_2,b_1-b_2+1,-b_2+b_3+1;z)+c_4\,z^{1-b_3}\,{}_4\tilde{F}_3(a_1-b_3+1,a_2-b_3+1,a_3-b_3+1,a_4-b_3+1;2-b_3,b_1-b_3+1,b_2-b_3+1;z)\bigwedge b_1\notin\mathbb{Z}\bigwedge b_2\notin\mathbb{Z}\land b_3\notin\mathbb{Z}\bigwedge b_1-b_2\notin\mathbb{Z}\bigwedge b_1-b_3\notin\mathbb{Z}\bigwedge b_2-b_3\notin\mathbb{Z}$$

07.28.13.0005.01

$$W_z\big({}_4\tilde{F}_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z),\; z^{1-b_1}\, {}_4\tilde{F}_3(a_1 - b_1 + 1, a_2 - b_1 + 1, a_3 - b_1 + 1, a_4 - b_1 + 1; 2 - b_1, -b_1 + b_2 + 1, -b_1 + b_3 + 1; z),\; z^{1-b_2}\, {}_4\tilde{F}_3(a_1 - b_2 + 1, a_2 - b_2 + 1, a_3 - b_2 + 1, a_4 - b_2 + 1; 2 - b_2, b_1 - b_2 + 1, -b_2 + b_3 + 1; z),\; z^{1-b_3}\, {}_4\tilde{F}_3(a_1 - b_3 + 1, a_2 - b_3 + 1, a_3 - b_3 + 1, a_4 - b_3 + 1; 2 - b_3, b_1 - b_3 + 1, b_2 - b_3 + 1; z)\big) = \frac{\sin(\pi b_1)\sin(\pi(b_1 - b_2))\sin(\pi b_2)\sin(\pi(b_1 - b_3))\sin(\pi(b_2 - b_3))\sin(\pi b_3)}{\pi^6}(1 - z)^{-a_1-a_2-a_3-a_4+b_1+b_2+b_3-3} z^{-b_1-b_2-b_3-3}$$

07.28.13.0001.01

$$(1 - z) z^3 w^{(4)}(z) + (-z(a_1 + a_2 + a_3 + a_4 + 6) + b_1 + b_2 + b_3 + 3) z^2 w^{(3)}(z) + (-z(a_2 a_1 + a_3 a_1 + a_4 a_1 + 3 a_1 + 3 a_2 + a_2 a_3 + 3 a_3 + a_2 a_4 + a_3 a_4 + 3 a_4 + 7) + b_1 + b_1 b_2 + b_2 + b_1 b_3 + b_2 b_3 + b_3 + 1) z w''(z) + (b_1 b_2 b_3 - (a_2 a_1 + a_2 a_3 a_1 + a_3 a_1 + a_2 a_4 a_1 + a_3 a_4 a_1 + a_4 a_1 + a_1 + a_2 + a_2 a_3 + a_3 + a_2 a_4 + a_2 a_3 a_4 + a_3 a_4 + a_4 + 1) z) w'(z) - a_1 a_2 a_3 a_4 w(z) == 0 \;/;\; w(z) == c_1\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) + c_2 z^{1-b_1}\, {}_4F_3(a_1 - b_1 + 1, a_2 - b_1 + 1, a_3 - b_1 + 1, a_4 - b_1 + 1; 2 - b_1, -b_1 + b_2 + 1, -b_1 + b_3 + 1; z) + c_3 z^{1-b_2}\, {}_4F_3(a_1 - b_2 + 1, a_2 - b_2 + 1, a_3 - b_2 + 1, a_4 - b_2 + 1; 2 - b_2, b_1 - b_2 + 1, -b_2 + b_3 + 1; z) + c_4 z^{1-b_3}\, {}_4F_3(a_1 - b_3 + 1, a_2 - b_3 + 1, a_3 - b_3 + 1, a_4 - b_3 + 1; 2 - b_3, b_1 - b_3 + 1, b_2 - b_3 + 1; z) \bigwedge b_1 \notin \mathbb{Z} \bigwedge b_2 \notin \mathbb{Z} \wedge b_3 \notin \mathbb{Z} \bigwedge b_1 - b_2 \notin \mathbb{Z} \bigwedge b_1 - b_3 \notin \mathbb{Z} \bigwedge b_2 - b_3 \notin \mathbb{Z}$$

07.28.13.0006.01

$$W_z\big({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z),\; z^{1-b_1}\, {}_4F_3(a_1 - b_1 + 1, a_2 - b_1 + 1, a_3 - b_1 + 1, a_4 - b_1 + 1; 2 - b_1, -b_1 + b_2 + 1, -b_1 + b_3 + 1; z),\; z^{1-b_2}\, {}_4F_3(a_1 - b_2 + 1, a_2 - b_2 + 1, a_3 - b_2 + 1, a_4 - b_2 + 1; 2 - b_2, b_1 - b_2 + 1, -b_2 + b_3 + 1; z),\; z^{1-b_3}\, {}_4F_3(a_1 - b_3 + 1, a_2 - b_3 + 1, a_3 - b_3 + 1, a_4 - b_3 + 1; 2 - b_3, b_1 - b_3 + 1, b_2 - b_3 + 1; z)\big) = -(b_1 - 1)(b_2 - 1)(b_3 - 1)(b_1 - b_2)(b_1 - b_3)(b_2 - b_3)\,(1 - z)^{-a_1-a_2-a_3-a_4+b_1+b_2+b_3-3} z^{-b_1-b_2-b_3-3}$$

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$$w^{(4)}(z)+\left(\frac{(a_1+a_2+a_3+a_4+6)\,g'(z)}{g(z)-1}-\frac{(b_1+b_2+b_3+3)\,g'(z)}{(g(z)-1)\,g(z)}-\frac{6\,g''(z)}{g'(z)}\right)w^{(3)}(z)+\left(\frac{(a_1\,a_2+a_3\,a_2+a_4\,a_2+a_1\,a_3+a_1\,a_4+a_3\,a_4+3\,(a_1+a_2+a_3+a_4)+7)\,g'(z)^2}{(g(z)-1)\,g(z)}-\frac{((b_2+1)\,(b_3+1)+b_1\,(b_2+b_3+1))\,g'(z)^2}{(g(z)-1)\,g(z)^2}+\frac{15\,g''(z)^2}{g'(z)^2}+\frac{3\,(b_1+b_2+b_3+3)\,g''(z)}{(g(z)-1)\,g(z)}-\frac{3\,(a_1+a_2+a_3+a_4+6)\,g''(z)}{g(z)-1}-\frac{4\,g^{(3)}(z)}{g'(z)}\right)w''(z)+\left(\frac{b_1\,b_2\,b_3\,g'(z)^3}{(1-g(z))\,g(z)^3}+\frac{3\,(a_1+a_2+a_3+a_4+6)\,g''(z)^2}{(g(z)-1)\,g'(z)}+\frac{1}{(g(z)-1)\,g(z)^2}\,g'(z)\left((a_2\,a_1+a_2\,a_3\,a_1+a_3\,a_1+a_2\,a_4\,a_1+a_3\,a_4\,a_1+a_4\,a_1+a_1+a_2+a_2\,a_3+a_3+a_2\,a_4+a_2\,a_3\,a_4+a_3\,a_4+a_4+1)\,g'(z)^2+((b_2+1)\,(b_3+1)+b_1\,(b_2+b_3+1))\,g''(z)\right)+\frac{10\,g''(z)\,g^{(3)}(z)}{g'(z)^2}+\frac{1}{(g(z)-1)\,g(z)}\left((-a_1\,a_2-a_3\,a_2-a_4\,a_2-a_1\,a_3-a_1\,a_4-a_3\,a_4-3\,(a_1+a_2+a_3+a_4)-7)\,g'(z)\,g''(z)+(b_1+b_2+b_3+3)\,g^{(3)}(z)\right)-\frac{(a_1+a_2+a_3+a_4+6)\,g^{(3)}(z)}{g(z)-1}-\frac{3\,(b_1+b_2+b_3+3)\,g''(z)^2}{(g(z)-1)\,g(z)\,g'(z)}-\frac{15\,g''(z)^3}{g'(z)^3}-\frac{g^{(4)}(z)}{g'(z)}\right)w'(z)+\frac{a_1\,a_2\,a_3\,a_4\,g'(z)^4}{(g(z)-1)\,g(z)^3}\,w(z)=0\;/;\;w(z)=c_1\,{}_4\tilde{F}_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;g(z))+c_2\left(G_{4,4}^{2,4}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_1,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_3,1-b_1,1-b_2\end{matrix}\right)\right)+c_3\left(G_{4,4}^{3,4}\!\left(-g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{3,4}\!\left(-g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_3,1-b_2\end{matrix}\right)+G_{4,4}^{3,4}\!\left(-g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_3,1-b_1\end{matrix}\right)\right)+c_4\,G_{4,4}^{4,4}\!\left(g(z)\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)$$

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$$w^{(4)}(z)+\left(\frac{(a_1+a_2+a_3+a_4+6)\,g'(z)}{g(z)-1}-\frac{(b_1+b_2+b_3+3)\,g'(z)}{(g(z)-1)\,g(z)}-\frac{4\,h'(z)}{h(z)}-\frac{6\,g''(z)}{g'(z)}\right)w^{(3)}(z)+\left(\frac{(a_1\,a_2+a_3\,a_2+a_4\,a_2+a_1\,a_3+a_1\,a_4+a_3\,a_4+3\,(a_1+a_2+a_3+a_4)+7)\,g'(z)^2}{(g(z)-1)\,g(z)}-\frac{((b_2+1)\,(b_3+1)+b_1\,(b_2+b_3+1))\,g'(z)^2}{(g(z)-1)\,g(z)^2}+\frac{3\,(b_1+b_2+b_3+3)\,h'(z)\,g'(z)}{(g(z)-1)\,g(z)\,h(z)}-\frac{3\,(a_1+a_2+a_3+a_4+6)\,h'(z)\,g'(z)}{(g(z)-1)\,h(z)}+\frac{12\,h'(z)^2}{h(z)^2}+\frac{15\,g''(z)^2}{g'(z)^2}+\frac{3\,(b_1+b_2+b_3+3)\,g''(z)}{(g(z)-1)\,g(z)}+\frac{18\,h'(z)\,g''(z)}{h(z)\,g'(z)}-\frac{3\,(a_1+a_2+a_3+a_4+6)\,g''(z)}{g(z)-1}-\frac{6\,h''(z)}{h(z)}-\frac{4\,g^{(3)}(z)}{g'(z)}\right)w''(z)+$$

$$\Bigg(\frac{b_1 b_2 b_3 g'(z)^3}{(1-g(z))\, g(z)^3} + \frac{2\,((b_2+1)(b_3+1)+b_1 (b_2+b_3+1))\, h'(z)\, g'(z)^2}{(g(z)-1)\, g(z)^2\, h(z)} + \frac{6\,(a_1+a_2+a_3+a_4+6)\, h'(z)^2\, g'(z)}{(g(z)-1)\, h(z)^2} -$$

$$\frac{6\,(b_1+b_2+b_3+3)\, h'(z)^2\, g'(z)}{(g(z)-1)\, g(z)\, h(z)^2} + \frac{3\,(a_1+a_2+a_3+a_4+6)\, g''(z)^2}{(g(z)-1)\, g'(z)} +$$

$$\frac{1}{(g(z)-1)\, g(z)^2}\, g'(z) \big((a_2 a_1 + a_2 a_3 a_1 + a_3 a_1 + a_2 a_4 a_1 + a_3 a_4 a_1 + a_4 a_1 + a_1 + a_2 + a_2 a_3 + a_3 +$$

$$a_2 a_4 + a_2 a_3 a_4 + a_3 a_4 + a_4 + 1)\, g'(z)^2 + ((b_2+1)(b_3+1) + b_1 (b_2+b_3+1))\, g''(z)\big) +$$

$$\frac{24\, h'(z)\, h''(z)}{h(z)^2} + \frac{3\,(a_1+a_2+a_3+a_4+6)\,(2\, h'(z)\, g''(z) - g'(z)\, h''(z))}{(g(z)-1)\, h(z)} + \frac{1}{(g(z)-1)\, g(z)\, h(z)}$$

$$\big(3\,(b_1+b_2+b_3+3)\,(g'(z)\, h''(z) - 2\, h'(z)\, g''(z)) - 2\,(a_1 a_2 + a_3 a_2 + a_4 a_2 + a_1 a_3 + a_1 a_4 +$$

$$a_3 a_4 + 3\,(a_1+a_2+a_3+a_4) + 7)\, g'(z)^2\, h'(z)\big) + \frac{10\, g''(z)\, g^{(3)}(z)}{g'(z)^2} + \frac{1}{(g(z)-1)\, g(z)}$$

$$\big((b_1+b_2+b_3+3)\, g^{(3)}(z) - (a_1 a_2 + a_3 a_2 + a_4 a_2 + a_1 a_3 + a_1 a_4 + a_3 a_4 + 3\,(a_1+a_2+a_3+a_4) + 7)\, g'(z)\, g''(z)\big) +$$

$$\frac{18\, g''(z)\, h''(z) + 8\, h'(z)\, g^{(3)}(z)}{h(z)\, g'(z)} - \frac{(a_1+a_2+a_3+a_4+6)\, g^{(3)}(z)}{g(z)-1} - \frac{4\, h^{(3)}(z)}{h(z)} - \frac{24\, h'(z)^3}{h(z)^3} -$$

$$\frac{3\,(b_1+b_2+b_3+3)\, g''(z)^2}{(g(z)-1)\, g(z)\, g'(z)} - \frac{36\, h'(z)^2\, g''(z)}{h(z)^2\, g'(z)} - \frac{30\, h'(z)\, g''(z)^2}{h(z)\, g'(z)^2} - \frac{15\, g''(z)^3}{g'(z)^3} - \frac{g^{(4)}(z)}{g'(z)}\Bigg)\, w'(z) +$$

$$\Bigg(\frac{a_1 a_2 a_3 a_4\, g'(z)^4}{(g(z)-1)\, g(z)^3} + \frac{b_1 b_2 b_3\, h'(z)\, g'(z)^3}{(g(z)-1)\, g(z)^3\, h(z)} - \frac{2\,((b_2+1)(b_3+1) + b_1 (b_2+b_3+1))\, h'(z)^2\, g'(z)^2}{(g(z)-1)\, g(z)^2\, h(z)^2} +$$

$$\frac{6\,(b_1+b_2+b_3+3)\, h'(z)^3\, g'(z)}{(g(z)-1)\, g(z)\, h(z)^3} + \frac{6\,(a_1+a_2+a_3+a_4+6)\, h'(z)\, h''(z)\, g'(z)}{(g(z)-1)\, h(z)^2} - \frac{6\,(a_1+a_2+a_3+a_4+6)\, h'(z)^3\, g'(z)}{(g(z)-1)\, h(z)^3} +$$

$$\frac{24\, h'(z)^4}{h(z)^4} + \frac{15\, h'(z)\, g''(z)^3}{h(z)\, g'(z)^3} + \frac{30\, h'(z)^2\, g''(z)^2}{h(z)^2\, g'(z)^2} + \frac{3\,(b_1+b_2+b_3+3)\, h'(z)\, g''(z)^2}{(g(z)-1)\, g(z)\, h(z)\, g'(z)} + \frac{36\, h'(z)^3\, g''(z)}{h(z)^3\, g'(z)} +$$

$$\frac{1}{(g(z)-1)\, g(z)\, h(z)^2}\, (2\, h'(z)) \big(h'(z) \big((a_1 a_2 + a_3 a_2 + a_4 a_2 + a_1 a_3 + a_1 a_4 + a_3 a_4 + 3\,(a_1+a_2+a_3+a_4) + 7)\, g'(z)^2 +$$

$$3\,(b_1+b_2+b_3+3)\, g''(z)\big) - 3\,(b_1+b_2+b_3+3)\, g'(z)\, h''(z)\big) +$$

$$\frac{1}{(g(z)-1)\, g(z)^2\, h(z)}\, g'(z) \big(((b_2+1)(b_3+1) + b_1 (b_2+b_3+1))\,(g'(z)\, h''(z) - h'(z)\, g''(z)) - h'(z)\,(a_2 a_1 + a_2 a_3 a_1 +$$

$$a_3 a_1 + a_2 a_4 a_1 + a_3 a_4 a_1 + a_4 a_1 + a_1 + a_2 + a_2 a_3 + a_3 + a_2 a_4 + a_2 a_3 a_4 + a_3 a_4 + a_4 + 1)\, g'(z)^2\big) +$$

$$\frac{6\, h''(z)^2 + 8\, h'(z)\, h^{(3)}(z)}{h(z)^2} + \frac{(a_1+a_2+a_3+a_4+6)\big(3\, g''(z)\, h''(z) + h'(z)\, g^{(3)}(z) - g'(z)\, h^{(3)}(z)\big)}{(g(z)-1)\, h(z)} -$$

$$\frac{1}{(g(z)-1)\, g(z)\, h(z)} \big((b_1+b_2+b_3+3)\big(3\, g''(z)\, h''(z) + h'(z)\, g^{(3)}(z) - g'(z)\, h^{(3)}(z)\big) -$$

$$(a_4 a_3 + 3\, a_3 + 3\, a_4 + a_2 (a_3+a_4+3) + a_1 (a_2+a_3+a_4+3) + 7)\, g'(z)\,(h'(z)\, g''(z) - g'(z)\, h''(z))\big) +$$

$$\frac{4\, h''(z)\, g^{(3)}(z) + 6\, g''(z)\, h^{(3)}(z) + h'(z)\, g^{(4)}(z)}{h(z)\, g'(z)} - \frac{h^{(4)}(z)}{h(z)} - \frac{6\,(a_1+a_2+a_3+a_4+6)\, h'(z)^2\, g''(z)}{(g(z)-1)\, h(z)^2} -$$

$$\frac{36\,h'(z)^2\,h''(z)}{h(z)^3}-\frac{3\,(a_1+a_2+a_3+a_4+6)\,h'(z)\,g''(z)^2}{(g(z)-1)\,h(z)\,g'(z)}-$$
$$\left.\frac{4\,h'(z)\left(9\,g''(z)\,h''(z)+2\,h'(z)\,g^{(3)}(z)\right)}{h(z)^2\,g'(z)}-\frac{5\,g''(z)\left(3\,g''(z)\,h''(z)+2\,h'(z)\,g^{(3)}(z)\right)}{h(z)\,g'(z)^2}\right)w(z)=0\;/;$$

$$w(z)=c_1\,h(z)\,{}_4\tilde{F}_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;g(z))\;+\;c_2\,h(z)\left(G_{4,4}^{2,4}\left(g(z)\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{array}\right.\right)+\right.$$
$$\left.G_{4,4}^{2,4}\left(g(z)\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_1,1-b_3\end{array}\right.\right)+G_{4,4}^{2,4}\left(g(z)\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_3,1-b_1,1-b_2\end{array}\right.\right)\right)+$$
$$c_3\,h(z)\left(G_{4,4}^{3,4}\left(-g(z)\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{array}\right.\right)+G_{4,4}^{3,4}\left(-g(z)\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_3,1-b_2\end{array}\right.\right)+\right.$$
$$\left.G_{4,4}^{3,4}\left(-g(z)\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_3,1-b_1\end{array}\right.\right)\right)+c_4\,h(z)\,G_{4,4}^{4,4}\left(g(z)\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{array}\right.\right)$$

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$$w^{(4)}(z)+\frac{a\,r\,(a_1+a_2+a_3+a_4)\,z^r+3\,r-2\,(2\,s-3)\,(a\,z^r-1)-r\,(b_1+b_2+b_3)}{z\,(a\,z^r-1)}\,w^{(3)}(z)+$$
$$\frac{1}{z^2\,(a\,z^r-1)}\Big(a\,r\,a_2\,(-3\,s+r\,(a_3+a_4)+3)\,z^r+a\,r\,a_1\,(-3\,s+r\,(a_2+a_3+a_4)+3)\,z^r-$$
$$3\,r^2-9\,r\,(s-1)+(6\,(s-2)\,s+7)\,(a\,z^r-1)+r\,(-3\,a\,(s-1)\,a_4\,z^r-a\,a_3\,(3\,s-r\,a_4-3)\,z^r+$$
$$(2\,r+3\,s-3)\,b_3+b_2\,(-b_3\,r+2\,r+3\,s-3)-b_1\,((b_2+b_3)\,r-2\,r-3\,s+3))\Big)\,w''(z)+$$
$$\frac{1}{z^3\,(a\,z^r-1)}\Big(a\,r\,a_2\,(3\,(s-1)\,s+r\,(1-2\,s)\,a_4+r\,a_3\,(-2\,s+r\,a_4+1)+1)\,z^r+$$
$$a\,r\,a_1\,(3\,(s-1)\,s+r\,(1-2\,s)\,a_4+r\,a_3\,(-2\,s+r\,a_4+1)+r\,a_2\,(-2\,s+r\,(a_3+a_4)+1)+1)\,z^r+$$
$$r^3+r^2\,(6\,s-3)+r\,(9\,(s-1)\,s+3)-(2\,s\,(s\,(2\,s-3)+2)-1)\,(a\,z^r-1)-$$
$$r\,\Big(-a\,(3\,(s-1)\,s+1)\,a_4\,z^r-a\,a_3\,(3\,(s-1)\,s+r\,(1-2\,s)\,a_4+1)\,z^r+$$
$$\left(r^2+(4\,s-2)\,r+3\,(s-1)\,s+1\right)b_3+b_2\left((r-1)^2+3\,s^2+(4\,r-3)\,s-r\,(r+2\,s-1)\,b_3\right)+$$
$$b_1\left((r-1)^2+3\,s^2+(4\,r-3)\,s-r\,(r+2\,s-1)\,b_3+r\,b_2\,(b_3\,r-r-2\,s+1)\right)\Big)\Big)\,w'(z)+$$
$$\frac{a\,z^r\,(s-r\,a_1)\,(s-r\,a_2)\,(s-r\,a_3)\,(s-r\,a_4)-s\,(-b_1\,r+r+s)\,(-b_2\,r+r+s)\,(-b_3\,r+r+s)}{z^4\,(a\,z^r-1)}\,w(z)=0\;/;$$

$$w(z)=c_1\,z^s\,{}_4\tilde{F}_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;a\,z^r)\;+c_2\,z^s\left(G_{4,4}^{2,4}\left(a\,z^r\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{array}\right.\right)+\right.$$
$$\left.G_{4,4}^{2,4}\left(a\,z^r\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_1,1-b_3\end{array}\right.\right)+G_{4,4}^{2,4}\left(a\,z^r\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_3,1-b_1,1-b_2\end{array}\right.\right)\right)+$$
$$c_3\,z^s\left(G_{4,4}^{3,4}\left(-a\,z^r\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{array}\right.\right)+G_{4,4}^{3,4}\left(-a\,z^r\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_3,1-b_2\end{array}\right.\right)+\right.$$
$$\left.G_{4,4}^{3,4}\left(-a\,z^r\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_3,1-b_1\end{array}\right.\right)\right)+c_4\,z^s\,G_{4,4}^{4,4}\left(a\,z^r\left|\begin{array}{c}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{array}\right.\right)$$

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$$w^{(4)}(z)+\frac{a\log(r)\,(a_1+a_2+a_3+a_4)\,r^z+3\log(r)-4\,(a\,r^z-1)\log(s)-\log(r)\,(b_1+b_2+b_3)}{a\,r^z-1}\,w^{(3)}(z)+$$
$$\frac{1}{a\,r^z-1}\Big(a\log(r)\,a_2\,(\log(r)\,(a_3+a_4)-3\log(s))\,r^z+a\log(r)\,a_1\,(\log(r)\,(a_2+a_3+a_4)-3\log(s))\,r^z-$$
$$3\left(\log^2(r)+3\log(s)\log(r)-2\,(a\,r^z-1)\log^2(s)\right)+\log(r)\,(-3\,a\log(s)\,a_4\,r^z-a\,a_3\,(3\log(s)-\log(r)\,a_4)\,r^z+$$
$$(2\log(r)+3\log(s))\,b_3+b_2\,(-b_3\log(r)+2\log(r)+3\log(s))-b_1\,((b_2+b_3)\log(r)-2\log(r)-3\log(s)))\Big)\,w''(z)+$$
$$\frac{1}{a\,r^z-1}\Big(-4\,a\log^3(s)\,r^z+a\log(r)\,a_2\,(\log(r)\,a_3\,(\log(r)\,a_4-2\log(s))+\log(s)\,(3\log(s)-2\log(r)\,a_4))\,r^z+$$
$$a\log(r)\,a_1\left(3\log^2(s)+\log(r)\,(-2\log(s)\,a_4+a_3\,(\log(r)\,a_4-2\log(s))+a_2\,(\log(r)\,(a_3+a_4)-2\log(s)))\right)r^z+$$
$$(\log(r)+\log(s))^2\,(\log(r)+4\log(s))-\log(r)\Big(-3\,a\log^2(s)\,a_4\,r^z-a\log(s)\,a_3\,(3\log(s)-2\log(r)\,a_4)\,r^z+$$
$$(\log(r)+\log(s))\,(\log(r)+3\log(s))\,b_3+b_2\,((\log(r)+\log(s))\,(\log(r)+3\log(s))-\log(r)\,(\log(r)+2\log(s))\,b_3)+$$
$$b_1\,((\log(r)+\log(s))\,(\log(r)+3\log(s))+\log(r)\,(b_2\,(b_3\log(r)-\log(r)-2\log(s))-(\log(r)+2\log(s))\,b_3))\Big)\Big)$$
$$w'(z)+\frac{1}{a\,r^z-1}\,(a\,r^z\,(\log(s)-\log(r)\,a_1)\,(\log(s)-\log(r)\,a_2)\,(\log(s)-\log(r)\,a_3)\,(\log(s)-\log(r)\,a_4)-$$
$$\log(s)\,(-b_1\log(r)+\log(r)+\log(s))\,(-b_2\log(r)+\log(r)+\log(s))\,(-b_3\log(r)+\log(r)+\log(s)))\,w(z)=0\;/;$$
$$w(z)=c_1\,s^z\,{}_4\tilde{F}_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;a\,z^r)+c_2\,s^z\left(G_{4,4}^{2,4}\!\left(a\,r^z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+\right.$$
$$\left.G_{4,4}^{2,4}\!\left(a\,r^z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_1,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(a\,r^z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_3,1-b_1,1-b_2\end{matrix}\right)\right)+$$
$$c_3\,s^z\left(G_{4,4}^{3,4}\!\left(-a\,r^z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{3,4}\!\left(-a\,r^z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_3,1-b_2\end{matrix}\right)+\right.$$
$$\left.G_{4,4}^{3,4}\!\left(-a\,r^z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_3,1-b_1\end{matrix}\right)\right)+c_4\,s^z\,G_{4,4}^{4,4}\!\left(a\,r^z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)$$

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$$W_z\Big({}_4\tilde{F}_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;z),$$
$$G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_1,1-b_3\end{matrix}\right)+G_{4,4}^{2,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_3,1-b_1,1-b_2\end{matrix}\right),$$
$$G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)+G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_3,1-b_2\end{matrix}\right)+$$
$$G_{4,4}^{3,4}\!\left(-z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_2,1-b_3,1-b_1\end{matrix}\right),G_{4,4}^{4,4}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)\Big)=(1-z)^{-a_1-a_2-a_3-a_4+b_1+b_2+b_3-3}$$
$$(-z)^{-b_1-b_2-b_3}\,z^{-b_1-b_2-b_3-3}\left(-(-z)^{b_2+b_3}\,(\csc(\pi\,(b_1-b_3))\sin(\pi\,(b_1-b_2))+\csc(\pi\,(b_1-b_2))\sin(\pi\,(b_1-b_3)))\,z^{b_1}+\right.$$
$$(-z)^{b_1+b_3}\,(\csc(\pi\,(b_2-b_3))\sin(\pi\,(b_1-b_2))+\csc(\pi\,(b_1-b_2))\sin(\pi\,(b_2-b_3)))\,z^{b_2}-(-z)^{b_1+b_2}$$
$$\left.(\csc(\pi\,(b_2-b_3))\sin(\pi\,(b_1-b_3))+\csc(\pi\,(b_1-b_3))\sin(\pi\,(b_2-b_3)))\,z^{b_3}-2\left((-z)^{b_2+b_3}\,z^{b_1}+(-z)^{b_1+b_3}\,z^{b_2}+(-z)^{b_1+b_2}\,z^{b_3}\right)\right)$$
$$\Gamma(a_1-b_1+1)\,\Gamma(a_2-b_1+1)\,\Gamma(a_3-b_1+1)\,\Gamma(a_4-b_1+1)\,\Gamma(a_1-b_2+1)\,\Gamma(a_2-b_2+1)\,\Gamma(a_3-b_2+1)$$
$$\Gamma(a_4-b_2+1)\,\Gamma(a_1-b_3+1)\,\Gamma(a_2-b_3+1)\,\Gamma(a_3-b_3+1)\,\Gamma(a_4-b_3+1)$$

Representation of fundamental system solutions near unit

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$$(1-z)\,z^3\,w^{(4)}(z) + (-z\,(a_1+a_2+a_3+a_4+6)+b_1+b_2+b_3+3)\,z^2\,w^{(3)}(z) + (-z\,(a_2\,a_1+a_3\,a_1+a_4\,a_1+3\,a_1+3\,a_2+a_2\,a_3+3\,a_3+a_2\,a_4+a_3\,a_4+3\,a_4+7)+b_1+b_1\,b_2+b_2+b_1\,b_3+b_2\,b_3+b_3+1)\,z\,w''(z) + (b_1\,b_2\,b_3-(a_2\,a_1+a_2\,a_3\,a_1+a_3\,a_1+a_2\,a_4\,a_1+a_3\,a_4\,a_1+a_4\,a_1+a_1+a_2+a_2\,a_3+a_3+a_2\,a_4+a_2\,a_3\,a_4+a_3\,a_4+a_4+1)\,z)\,w'(z) - a_1\,a_2\,a_3\,a_4\,w(z) == 0\ /;$$

$$\left(w(z) == c_1\,G_{4,4}^{4,0}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right) + c_2\,G_{6,6}^{2,6}\!\left(z\,\middle|\,\begin{matrix}0,b_1,1-a_1,1-a_2,1-a_3,1-a_4\\0,b_1,0,1-b_1,1-b_2,1-b_3\end{matrix}\right) + c_3\,G_{6,6}^{2,6}\!\left(z\,\middle|\,\begin{matrix}0,b_2,1-a_1,1-a_2,1-a_3,1-a_4\\0,b_2,0,1-b_1,1-b_2,1-b_3\end{matrix}\right) + c_4\,G_{6,6}^{2,6}\!\left(z\,\middle|\,\begin{matrix}0,b_3,1-a_1,1-a_2,1-a_3,1-a_4\\0,b_3,0,1-b_1,1-b_2,1-b_3\end{matrix}\right) \bigwedge |z|<1 \bigwedge b_1+b_2+b_3-a_1-a_2-a_3-a_4 \notin \mathbb{Z} \bigwedge b_1 \notin \mathbb{Z} \bigwedge b_2 \notin \mathbb{Z} \bigwedge b_3 \notin \mathbb{Z} \bigwedge b_1-b_2 \notin \mathbb{Z} \bigwedge b_1-b_3 \notin \mathbb{Z} \bigwedge b_2-b_3 \notin \mathbb{Z}\right)$$

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$$W_z\!\left(G_{4,4}^{4,0}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right), G_{6,6}^{2,6}\!\left(z\,\middle|\,\begin{matrix}0,b_1,1-a_1,1-a_2,1-a_3,1-a_4\\0,b_1,0,1-b_1,1-b_2,1-b_3\end{matrix}\right), G_{6,6}^{2,6}\!\left(z\,\middle|\,\begin{matrix}0,b_2,1-a_1,1-a_2,1-a_3,1-a_4\\0,b_2,0,1-b_1,1-b_2,1-b_3\end{matrix}\right), G_{6,6}^{2,6}\!\left(z\,\middle|\,\begin{matrix}0,b_3,1-a_1,1-a_2,1-a_3,1-a_4\\0,b_3,0,1-b_1,1-b_2,1-b_3\end{matrix}\right)\right) ==$$

$$-G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-2,-a_2-2,-a_3-2,-a_4-2,b_3-3\\0,b_3-3,-b_1-2,-b_2-2,-b_3-2\end{matrix}\right)\Bigg(G_{4,4}^{4,0}\!\left(z\,\middle|\,\begin{matrix}-a_1,-a_2,-a_3,-a_4\\0,-b_1,-b_2,-b_3\end{matrix}\right)$$
$$\Bigg(G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1,b_2-2\\0,b_2-2,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right) G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4,b_1\\0,b_1,1-b_1,1-b_2,1-b_3\end{matrix}\right) -$$
$$G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1,b_1-2\\0,b_1-2,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right) G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4,b_2\\0,b_2,1-b_1,1-b_2,1-b_3\end{matrix}\right)\Bigg) +$$
$$G_{4,4}^{4,0}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)\Bigg(G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1,b_1-2\\0,b_1-2,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right)$$
$$G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1,-a_2,-a_3,-a_4,b_2-1\\0,b_2-1,-b_1,-b_2,-b_3\end{matrix}\right) - G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1,b_2-2\\0,b_2-2,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right)$$
$$G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1,-a_2,-a_3,-a_4,b_1-1\\0,b_1-1,-b_1,-b_2,-b_3\end{matrix}\right)\Bigg) + G_{4,4}^{4,0}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1\\0,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right)$$
$$\Bigg(G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4,b_2\\0,b_2,1-b_1,1-b_2,1-b_3\end{matrix}\right) G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1,-a_2,-a_3,-a_4,b_1-1\\0,b_1-1,-b_1,-b_2,-b_3\end{matrix}\right) -$$
$$G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4,b_1\\0,b_1,1-b_1,1-b_2,1-b_3\end{matrix}\right) G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1,-a_2,-a_3,-a_4,b_2-1\\0,b_2-1,-b_1,-b_2,-b_3\end{matrix}\right)\Bigg)\Bigg) +$$
$$G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-2,-a_2-2,-a_3-2,-a_4-2,b_2-3\\0,b_2-3,-b_1-2,-b_2-2,-b_3-2\end{matrix}\right)\Bigg(G_{4,4}^{4,0}\!\left(z\,\middle|\,\begin{matrix}-a_1,-a_2,-a_3,-a_4\\0,-b_1,-b_2,-b_3\end{matrix}\right)$$
$$\Bigg(G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1,b_3-2\\0,b_3-2,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right) G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4,b_1\\0,b_1,1-b_1,1-b_2,1-b_3\end{matrix}\right) -$$
$$G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1,b_1-2\\0,b_1-2,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right) G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4,b_3\\0,b_3,1-b_1,1-b_2,1-b_3\end{matrix}\right)\Bigg) +$$
$$G_{4,4}^{4,0}\!\left(z\,\middle|\,\begin{matrix}1-a_1,1-a_2,1-a_3,1-a_4\\0,1-b_1,1-b_2,1-b_3\end{matrix}\right)\Bigg(G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1,b_1-2\\0,b_1-2,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right)$$
$$G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1,-a_2,-a_3,-a_4,b_3-1\\0,b_3-1,-b_1,-b_2,-b_3\end{matrix}\right) - G_{5,5}^{2,5}\!\left(z\,\middle|\,\begin{matrix}-a_1-1,-a_2-1,-a_3-1,-a_4-1,b_3-2\\0,b_3-2,-b_1-1,-b_2-1,-b_3-1\end{matrix}\right)$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_1-1\\0,\,b_1-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)\bigg)\bigg)+G_{4,4}^{4,0}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1\\0,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)$$

$$\bigg(G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_3\\0,\,b_3,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_1-1\\0,\,b_1-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)-$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_1\\0,\,b_1,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_3-1\\0,\,b_3-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)\bigg)\bigg)\bigg)-$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1-2,\,-a_2-2,\,-a_3-2,\,-a_4-2,\,b_1-3\\0,\,b_1-3,\,-b_1-2,\,-b_2-2,\,-b_3-2\end{matrix}\right.\right)\bigg(G_{4,4}^{4,0}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4\\0,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)$$

$$\bigg(G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1,\,b_3-2\\0,\,b_3-2,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_2\\0,\,b_2,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)-$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1,\,b_2-2\\0,\,b_2-2,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_3\\0,\,b_3,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)\bigg)\bigg)+$$

$$G_{4,4}^{4,0}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4\\0,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)\bigg(G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1,\,b_2-2\\0,\,b_2-2,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_3-1\\0,\,b_3-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)-G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1,\,b_3-2\\0,\,b_3-2,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_2-1\\0,\,b_2-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)\bigg)+G_{4,4}^{4,0}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1\\0,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)$$

$$\bigg(G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_3\\0,\,b_3,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_2-1\\0,\,b_2-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)-$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_2\\0,\,b_2,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_3-1\\0,\,b_3-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)\bigg)\bigg)\bigg)+$$

$$G_{4,4}^{4,0}\!\left(z\left|\begin{matrix}-a_1-2,\,-a_2-2,\,-a_3-2,\,-a_4-2\\0,\,-b_1-2,\,-b_2-2,\,-b_3-2\end{matrix}\right.\right)\bigg(G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1,\,b_3-2\\0,\,b_3-2,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)$$

$$\bigg(G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_2\\0,\,b_2,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_1-1\\0,\,b_1-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)-$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_1\\0,\,b_1,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_2-1\\0,\,b_2-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)\bigg)\bigg)+$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1,\,b_2-2\\0,\,b_2-2,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)\bigg(G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_1\\0,\,b_1,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_3-1\\0,\,b_3-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)-G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_3\\0,\,b_3,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_1-1\\0,\,b_1-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)\bigg)+G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1-1,\,-a_2-1,\,-a_3-1,\,-a_4-1,\,b_1-2\\0,\,b_1-2,\,-b_1-1,\,-b_2-1,\,-b_3-1\end{matrix}\right.\right)$$

$$\bigg(G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_3\\0,\,b_3,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_2-1\\0,\,b_2-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)-$$

$$G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}1-a_1,\,1-a_2,\,1-a_3,\,1-a_4,\,b_2\\0,\,b_2,\,1-b_1,\,1-b_2,\,1-b_3\end{matrix}\right.\right)G_{5,5}^{2,5}\!\left(z\left|\begin{matrix}-a_1,\,-a_2,\,-a_3,\,-a_4,\,b_3-1\\0,\,b_3-1,\,-b_1,\,-b_2,\,-b_3\end{matrix}\right.\right)\bigg)\bigg)\bigg)$$

Representation of fundamental system solutions near infinity

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$$(1-z)\,z^3\,w^{(4)}(z) + (-z\,(a_1+a_2+a_3+a_4+6)+b_1+b_2+b_3+3)\,z^2\,w^{(3)}(z) + (-z\,(a_2\,a_1+a_3\,a_1+a_4\,a_1+3\,a_1+3\,a_2+a_2\,a_3+3\,a_3+a_2\,a_4+a_3\,a_4+3\,a_4+7)+b_1+b_1\,b_2+b_2+b_1\,b_3+b_2\,b_3+b_3+1)\,z\,w''(z) + (b_1\,b_2\,b_3-(a_2\,a_1+a_2\,a_3\,a_1+a_3\,a_1+a_2\,a_4\,a_1+a_3\,a_4\,a_1+a_4\,a_1+a_1+a_2+a_2\,a_3+a_3+a_2\,a_4+a_2\,a_3\,a_4+a_3\,a_4+a_4+1)\,z)\,w'(z) - a_1\,a_2\,a_3\,a_4\,w(z) = 0 \;/;$$

$$w(z) = c_1\,z^{-a_1}\,{}_4\tilde{F}_3\!\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right) + c_2\left(G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_2, b_1, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_3, b_1, b_2\\ a_1, a_2, a_3, a_4\end{matrix}\right)\right) + c_3\left(G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_3, b_2\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_2, b_3, b_1\\ a_1, a_2, a_3, a_4\end{matrix}\right)\right) + c_4\,G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right)$$

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$$W_z\Bigg(z^{-a_1}\,{}_4\tilde{F}_3\!\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right),$$
$$G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_2, b_1, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_3, b_1, b_2\\ a_1, a_2, a_3, a_4\end{matrix}\right),$$
$$G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_3, b_2\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_2, b_3, b_1\\ a_1, a_2, a_3, a_4\end{matrix}\right), G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right)\Bigg) =$$
$$-\Bigg(6\,G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right)z^2 + 6\,G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_3-1, b_2-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right)z^2 +$$
$$6\,G_{4,4}^{4,3}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_2-1, b_3-1, b_1-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right)z^2 - 6\,G_{5,5}^{4,4}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_1-2, b_2-2, b_3-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right)z -$$
$$6\,G_{5,5}^{4,4}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_1-2, b_3-2, b_2-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right)z -$$
$$6\,G_{5,5}^{4,4}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_2-2, b_3-2, b_1-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right)z + G_{5,5}^{4,4}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-3, -2, b_1-3, b_2-3, b_3-3\\ a_1-3, a_2-3, a_3-3, a_4-3, 0\end{matrix}\right) +$$
$$G_{5,5}^{4,4}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-3, -2, b_1-3, b_3-3, b_2-3\\ a_1-3, a_2-3, a_3-3, a_4-3, 0\end{matrix}\right) + G_{5,5}^{4,4}\!\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-3, -2, b_2-3, b_3-3, b_1-3\\ a_1-3, a_2-3, a_3-3, a_4-3, 0\end{matrix}\right)\Bigg)$$
$$\Bigg(\Bigg(2\,z\,G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + G_{5,5}^{4,5}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_1-2, b_2-2, b_3-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right)\Bigg)\Bigg)$$
$$\Bigg(-{}_4\tilde{F}_3\!\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right)$$
$$\Bigg(G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_2-1, b_1-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) +$$
$$G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_3-1, b_1-1, b_2-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right)\Bigg) -$$
$$\Bigg(G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_2, b_1, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_3, b_1, b_2\\ a_1, a_2, a_3, a_4\end{matrix}\right)\Bigg)a_1$$
$$\Bigg(-z\,{}_4\tilde{F}_3\!\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right) -$$
$${}_4\tilde{F}_3\!\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$
$$(a_1-b_1+1)\,(a_1-b_2+1)\,(a_1-b_3+1)\Bigg)\Bigg) -$$

$$\left(2\,z\,G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_1-1,\,b_2-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)+2\,z\,G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_2-1,\,b_1-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)+\right.$$
$$2\,z\,G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_3-1,\,b_1-1,\,b_2-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)+G_{5,5}^{4,3}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2,\,-1,\,b_1-2,\,b_2-2,\,b_3-2\\a_1-2,\,a_2-2,\,a_3-2,\,a_4-2,\,0\end{matrix}\right)+$$
$$\left.G_{5,5}^{4,3}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2,\,-1,\,b_2-2,\,b_1-2,\,b_3-2\\a_1-2,\,a_2-2,\,a_3-2,\,a_4-2,\,0\end{matrix}\right)+G_{5,5}^{4,3}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2,\,-1,\,b_3-2,\,b_1-2,\,b_2-2\\a_1-2,\,a_2-2,\,a_3-2,\,a_4-2,\,0\end{matrix}\right)\right)$$
$$\left(-{}_4\tilde{F}_3\!\left(a_1,\,a_1-b_1+1,\,a_1-b_2+1,\,a_1-b_3+1;\,a_1-a_2+1,\,a_1-a_3+1,\,a_1-a_4+1;\,\frac{1}{z}\right)\right.$$
$$G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_1-1,\,b_2-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)-G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1,\,b_1,\,b_2,\,b_3\\a_1,\,a_2,\,a_3,\,a_4\end{matrix}\right)a_1$$
$$\left(-z\,{}_4\tilde{F}_3\!\left(a_1,\,a_1-b_1+1,\,a_1-b_2+1,\,a_1-b_3+1;\,a_1-a_2+1,\,a_1-a_3+1,\,a_1-a_4+1;\,\frac{1}{z}\right)-\right.$$
$${}_4\tilde{F}_3\!\left(a_1+1,\,a_1-b_1+2,\,a_1-b_2+2,\,a_1-b_3+2;\,a_1-a_2+2,\,a_1-a_3+2,\,a_1-a_4+2;\,\frac{1}{z}\right)$$
$$\left.\left.(a_1-b_1+1)\,(a_1-b_2+1)\,(a_1-b_3+1)\right)\right)+$$
$$\left(-G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1,\,b_1,\,b_2,\,b_3\\a_1,\,a_2,\,a_3,\,a_4\end{matrix}\right)G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_1-1,\,b_2-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)-G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1,\,b_2,\,b_1,\,b_3\\a_1,\,a_2,\,a_3,\,a_4\end{matrix}\right)\right.$$
$$G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_1-1,\,b_2-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)-G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1,\,b_3,\,b_1,\,b_2\\a_1,\,a_2,\,a_3,\,a_4\end{matrix}\right)G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_1-1,\,b_2-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)+$$
$$G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_1-1,\,b_2-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1,\,b_1,\,b_2,\,b_3\\a_1,\,a_2,\,a_3,\,a_4\end{matrix}\right)+G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_2-1,\,b_1-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)$$
$$\left.G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1,\,b_1,\,b_2,\,b_3\\a_1,\,a_2,\,a_3,\,a_4\end{matrix}\right)+G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_3-1,\,b_1-1,\,b_2-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)G_{4,4}^{4,4}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}1,\,b_1,\,b_2,\,b_3\\a_1,\,a_2,\,a_3,\,a_4\end{matrix}\right)\right)a_1$$
$$\left({}_4\tilde{F}_3\!\left(a_1,\,a_1-b_1+1,\,a_1-b_2+1,\,a_1-b_3+1;\,a_1-a_2+1,\,a_1-a_3+1,\,a_1-a_4+1;\,\frac{1}{z}\right)(a_1+1)\,z^2+\right.$$
$$2\,{}_4\tilde{F}_3\!\left(a_1+1,\,a_1-b_1+2,\,a_1-b_2+2,\,a_1-b_3+2;\,a_1-a_2+2,\,a_1-a_3+2,\,a_1-a_4+2;\,\frac{1}{z}\right)$$
$$(a_1-b_1+1)\,(a_1-b_2+1)\,(a_1-b_3+1)\,z+2\,{}_4\tilde{F}_3\!\left(a_1+1,\,a_1-b_1+2,\,a_1-b_2+2,\,a_1-b_3+2;\right.$$
$$\left.a_1-a_2+2,\,a_1-a_3+2,\,a_1-a_4+2;\,\frac{1}{z}\right)a_1\,(a_1-b_1+1)\,(a_1-b_2+1)\,(a_1-b_3+1)\,z+$$
$${}_4\tilde{F}_3\!\left(a_1+2,\,a_1-b_1+3,\,a_1-b_2+3,\,a_1-b_3+3;\,a_1-a_2+3,\,a_1-a_3+3,\,a_1-a_4+3;\,\frac{1}{z}\right)(a_1+1)$$
$$\left.\left.(a_1-b_1+1)\,(a_1-b_1+2)\,(a_1-b_2+1)\,(a_1-b_2+2)\,(a_1-b_3+1)\,(a_1-b_3+2)\right)\right)z^{-a_1-12}-$$
$$\left(6\,G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_1-1,\,b_2-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)z^2+6\,G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_2-1,\,b_1-1,\,b_3-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)z^2+\right.$$
$$6\,G_{4,4}^{4,2}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1,\,b_3-1,\,b_1-1,\,b_2-1\\a_1-1,\,a_2-1,\,a_3-1,\,a_4-1\end{matrix}\right)z^2+$$
$$6\,G_{5,5}^{4,3}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2,\,-1,\,b_1-2,\,b_2-2,\,b_3-2\\a_1-2,\,a_2-2,\,a_3-2,\,a_4-2,\,0\end{matrix}\right)z+$$
$$6\,G_{5,5}^{4,3}\!\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2,\,-1,\,b_2-2,\,b_1-2,\,b_3-2\\a_1-2,\,a_2-2,\,a_3-2,\,a_4-2,\,0\end{matrix}\right)z+$$

$$6\, G_{5,5}^{4,3}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -2, -1, b_3-2, b_1-2, b_2-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right) z + G_{5,5}^{4,3}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -3, -2, b_1-3, b_2-3, b_3-3 \\ a_1-3, a_2-3, a_3-3, a_4-3, 0 \end{matrix}\right) +$$

$$G_{5,5}^{4,3}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -3, -2, b_2-3, b_1-3, b_3-3 \\ a_1-3, a_2-3, a_3-3, a_4-3, 0 \end{matrix}\right) + G_{5,5}^{4,3}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -3, -2, b_3-3, b_1-3, b_2-3 \\ a_1-3, a_2-3, a_3-3, a_4-3, 0 \end{matrix}\right)\Bigg)\Bigg)$$

$$\Bigg(\Bigg(2\, z\, G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{5,5}^{4,5}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -2, -1, b_1-2, b_2-2, b_3-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right)\Bigg)\Bigg)$$

$$\Bigg({}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right)$$

$$\Bigg(G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_3-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) +$$

$$G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_2-1, b_3-1, b_1-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right)\Bigg) -$$

$$\Bigg(G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_1, b_3, b_2 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_2, b_3, b_1 \\ a_1, a_2, a_3, a_4 \end{matrix}\right)\Bigg) a_1$$

$$\Bigg(-z\, {}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right) -$$

$${}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$

$$(a_1-b_1+1)\,(a_1-b_2+1)\,(a_1-b_3+1)\Bigg)\Bigg) -$$

$$\Bigg(-2\, z\, G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) - 2\, z\, G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_3-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) -$$

$$2\, z\, G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_2-1, b_3-1, b_1-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{5,5}^{4,4}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -2, -1, b_1-2, b_2-2, b_3-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right) +$$

$$G_{5,5}^{4,4}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -2, -1, b_1-2, b_3-2, b_2-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right) + G_{5,5}^{4,4}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -2, -1, b_2-2, b_3-2, b_1-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right)\Bigg)$$

$$\Bigg(-{}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right)$$

$$G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) - G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) a_1$$

$$\Bigg(-z\, {}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right) -$$

$${}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$

$$(a_1-b_1+1)\,(a_1-b_2+1)\,(a_1-b_3+1)\Bigg)\Bigg) -$$

$$\Bigg(G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_1, b_3, b_2 \\ a_1, a_2, a_3, a_4 \end{matrix}\right)$$

$$G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_2, b_3, b_1 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) +$$

$$G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_1-1, b_3-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right)$$

$$G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix} -1, b_2-1, b_3-1, b_1-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right)\Bigg) a_1$$

$$\Bigg({}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right)(a_1+1)\,z^2 +$$
$$2\,{}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$
$$(a_1-b_1+1)(a_1-b_2+1)(a_1-b_3+1)\,z + 2\,{}_4\tilde{F}_3\Big(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2;$$
$$a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\Big)\,a_1\,(a_1-b_1+1)(a_1-b_2+1)(a_1-b_3+1)\,z +$$
$${}_4\tilde{F}_3\left(a_1+2, a_1-b_1+3, a_1-b_2+3, a_1-b_3+3; a_1-a_2+3, a_1-a_3+3, a_1-a_4+3; \frac{1}{z}\right)(a_1+1)$$
$$(a_1-b_1+1)(a_1-b_1+2)(a_1-b_2+1)(a_1-b_2+2)(a_1-b_3+1)(a_1-b_3+2)\Bigg)\Bigg)\,z^{-a_1-12} -$$
$$\Bigg(-\Bigg(2\,z\,G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + 2\,z\,G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_2-1, b_1-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) +$$
$$2\,z\,G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_3-1, b_1-1, b_2-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + G_{5,5}^{4,3}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_1-2, b_2-2, b_3-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right) +$$
$$G_{5,5}^{4,3}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_2-2, b_1-2, b_3-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right) + G_{5,5}^{4,3}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_3-2, b_1-2, b_2-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right)\Bigg)$$
$$\Bigg(G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_3, b_2\\ a_1, a_2, a_3, a_4\end{matrix}\right)$$
$$G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}1, b_2, b_3, b_1\\ a_1, a_2, a_3, a_4\end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) +$$
$$G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_3-1, b_2-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right)$$
$$G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_2-1, b_3-1, b_1-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right)\Bigg) -$$
$$\Bigg(2\,z\,G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + 2\,z\,G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_3-1, b_2-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) +$$
$$2\,z\,G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_2-1, b_3-1, b_1-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) - G_{5,5}^{4,4}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_1-2, b_2-2, b_3-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right) -$$
$$G_{5,5}^{4,4}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_1-2, b_3-2, b_2-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right) - G_{5,5}^{4,4}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-2, -1, b_2-2, b_3-2, b_1-2\\ a_1-2, a_2-2, a_3-2, a_4-2, 0\end{matrix}\right)\Bigg)$$
$$\Bigg(G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_2, b_1, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right)$$
$$G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_3, b_1, b_2\\ a_1, a_2, a_3, a_4\end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) -$$
$$G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) - G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_2-1, b_1-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right)$$
$$G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) - G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_3-1, b_1-1, b_2-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) G_{4,4}^{4,4}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right)\Bigg) +$$
$$\Bigg(\Bigg(G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_1, b_2, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_2, b_1, b_3\\ a_1, a_2, a_3, a_4\end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z}\,\middle|\,\begin{matrix}1, b_3, b_1, b_2\\ a_1, a_2, a_3, a_4\end{matrix}\right)\Bigg)$$
$$\Bigg(G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_2-1, b_3-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}-1, b_1-1, b_3-1, b_2-1\\ a_1-1, a_2-1, a_3-1, a_4-1\end{matrix}\right) +$$

$$G_{4,4}^{4,3}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_2-1, b_3-1, b_1-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right)\Bigg) + \Bigg(G_{4,4}^{4,2}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) +$$

$$G_{4,4}^{4,2}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_2-1, b_1-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_3-1, b_1-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right)\Bigg)\Bigg)$$

$$\left(G_{4,4}^{4,3}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} 1, b_1, b_3, b_2 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} 1, b_2, b_3, b_1 \\ a_1, a_2, a_3, a_4 \end{matrix}\right)\right)\Bigg)\Bigg)$$

$$\left(2\,z\,G_{4,4}^{4,4}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{5,5}^{4,5}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -2, -1, b_1-2, b_2-2, b_3-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right)\right)\Bigg)\Bigg)$$

$$a_1\Bigg(-6\,{}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$

$$(a_1-b_1+1)\,(a_1-b_2+1)\,(a_1-b_3+1)\,z^2 -$$

$$3\,{}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$

$$a_1\,(a_1+1)\,(a_1-b_1+1)\,(a_1-b_2+1)\,(a_1-b_3+1)\,z^2 - 3\,a_1\,(a_1-b_1+1)\,(a_1-b_2+1)$$

$$\Bigg(2\,z\,{}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right) +$$

$${}_4\tilde{F}_3\left(a_1+2, a_1-b_1+3, a_1-b_2+3, a_1-b_3+3; a_1-a_2+3, a_1-a_3+3, a_1-a_4+3; \frac{1}{z}\right)$$

$$(a_1+1)\,(a_1-b_1+2)\,(a_1-b_2+2)\,(a_1-b_3+2)\Bigg)\,(a_1-b_3+1)\,z -$$

$$6\,{}_4\tilde{F}_3\left(a_1+2, a_1-b_1+3, a_1-b_2+3, a_1-b_3+3; a_1-a_2+3, a_1-a_3+3, a_1-a_4+3; \frac{1}{z}\right)(a_1+1)$$

$$(a_1-b_1+1)\,(a_1-b_1+2)\,(a_1-b_2+1)\,(a_1-b_2+2)\,(a_1-b_3+1)\,(a_1-b_3+2)\,z -$$

$$z^3\,{}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right)(a_1+1)\,(a_1+2) -$$

$${}_4\tilde{F}_3\left(a_1+3, a_1-b_1+4, a_1-b_2+4, a_1-b_3+4; a_1-a_2+4, a_1-a_3+4, a_1-a_4+4; \frac{1}{z}\right)$$

$$(a_1+1)\,(a_1+2)\,(a_1-b_1+1)\,(a_1-b_1+2)\,(a_1-b_1+3)\,(a_1-b_2+1)$$

$$(a_1-b_2+2)\,(a_1-b_2+3)\,(a_1-b_3+1)\,(a_1-b_3+2)\,(a_1-b_3+3)\Bigg)\,z^{-a_1-12} -$$

$$z^{-a_1-12}\left(6\,G_{4,4}^{4,4}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) z^2 + 6\,G_{5,5}^{4,5}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -2, -1, b_1-2, b_2-2, b_3-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right) z + \right.$$

$$\left. G_{5,5}^{4,5}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -3, -2, b_1-3, b_2-3, b_3-3 \\ a_1-3, a_2-3, a_3-3, a_4-3, 0 \end{matrix}\right)\right)$$

$$\Bigg(\Bigg(-2\,z\,G_{4,4}^{4,3}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) - 2\,z\,G_{4,4}^{4,3}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_1-1, b_3-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) -$$

$$2\,z\,G_{4,4}^{4,3}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_2-1, b_3-1, b_1-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{5,5}^{4,4}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} -2, -1, b_1-2, b_2-2, b_3-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right) +$$

$$G_{5,5}^{4,4}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} -2, -1, b_1-2, b_3-2, b_2-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right) + G_{5,5}^{4,4}\left(-\frac{1}{z} \,\middle|\, \begin{matrix} -2, -1, b_2-2, b_3-2, b_1-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right)\Bigg)\Bigg)$$

$$\Bigg(-{}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right)$$

$$\Bigg(G_{4,4}^{4,2}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z} \,\middle|\, \begin{matrix} -1, b_2-1, b_1-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) +$$

$$G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} -1, b_3-1, b_1-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right)\Bigg)\Bigg) -$$

$$\left(G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} 1, b_2, b_1, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} 1, b_3, b_1, b_2 \\ a_1, a_2, a_3, a_4 \end{matrix}\right)\right) a_1$$

$$\Bigg(-z\, {}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right) -$$

$${}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$

$$(a_1-b_1+1)(a_1-b_2+1)(a_1-b_3+1)\Bigg)\Bigg) -$$

$$\Bigg(2\, z\, G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + 2\, z\, G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} -1, b_2-1, b_1-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) +$$

$$2\, z\, G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} -1, b_3-1, b_1-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{5,5}^{4,3}\left(\frac{1}{z} \middle| \begin{matrix} -2, -1, b_1-2, b_2-2, b_3-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right) +$$

$$G_{5,5}^{4,3}\left(\frac{1}{z} \middle| \begin{matrix} -2, -1, b_2-2, b_1-2, b_3-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right) + G_{5,5}^{4,3}\left(\frac{1}{z} \middle| \begin{matrix} -2, -1, b_3-2, b_1-2, b_2-2 \\ a_1-2, a_2-2, a_3-2, a_4-2, 0 \end{matrix}\right)\Bigg)$$

$$\Bigg({}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right)$$

$$\Bigg(G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} -1, b_1-1, b_3-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) +$$

$$G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} -1, b_2-1, b_3-1, b_1-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right)\Bigg) -$$

$$\left(G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} 1, b_1, b_3, b_2 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} 1, b_2, b_3, b_1 \\ a_1, a_2, a_3, a_4 \end{matrix}\right)\right) a_1$$

$$\Bigg(-z\, {}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right) -$$

$${}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$

$$(a_1-b_1+1)(a_1-b_2+1)(a_1-b_3+1)\Bigg)\Bigg) +$$

$$\Bigg(\Bigg(G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} 1, b_2, b_1, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} 1, b_3, b_1, b_2 \\ a_1, a_2, a_3, a_4 \end{matrix}\right)\Bigg)$$

$$\Bigg(G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} -1, b_1-1, b_3-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) +$$

$$G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} -1, b_2-1, b_3-1, b_1-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right)\Bigg) + \Bigg(G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} -1, b_1-1, b_2-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) +$$

$$G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} -1, b_2-1, b_1-1, b_3-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right) + G_{4,4}^{4,2}\left(\frac{1}{z} \middle| \begin{matrix} -1, b_3-1, b_1-1, b_2-1 \\ a_1-1, a_2-1, a_3-1, a_4-1 \end{matrix}\right)\Bigg)$$

$$\left(G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} 1, b_1, b_2, b_3 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} 1, b_1, b_3, b_2 \\ a_1, a_2, a_3, a_4 \end{matrix}\right) + G_{4,4}^{4,3}\left(-\frac{1}{z} \middle| \begin{matrix} 1, b_2, b_3, b_1 \\ a_1, a_2, a_3, a_4 \end{matrix}\right)\right)\Bigg) a_1$$

$$\Bigg({}_4\tilde{F}_3\left(a_1, a_1-b_1+1, a_1-b_2+1, a_1-b_3+1; a_1-a_2+1, a_1-a_3+1, a_1-a_4+1; \frac{1}{z}\right)(a_1+1)\, z^2 +$$

$$2\, {}_4\tilde{F}_3\left(a_1+1, a_1-b_1+2, a_1-b_2+2, a_1-b_3+2; a_1-a_2+2, a_1-a_3+2, a_1-a_4+2; \frac{1}{z}\right)$$

$$(a_1-b_1+1)(a_1-b_2+1)(a_1-b_3+1)\,z+2\,{}_4\tilde{F}_3\Big(a_1+1,\,a_1-b_1+2,\,a_1-b_2+2,\,a_1-b_3+2;$$
$$a_1-a_2+2,\,a_1-a_3+2,\,a_1-a_4+2;\,\frac{1}{z}\Big)\,a_1\,(a_1-b_1+1)(a_1-b_2+1)(a_1-b_3+1)\,z+$$
$${}_4\tilde{F}_3\Big(a_1+2,\,a_1-b_1+3,\,a_1-b_2+3,\,a_1-b_3+3;\,a_1-a_2+3,\,a_1-a_3+3,\,a_1-a_4+3;\,\frac{1}{z}\Big)$$
$$(a_1+1)(a_1-b_1+1)(a_1-b_1+2)(a_1-b_2+1)(a_1-b_2+2)(a_1-b_3+1)(a_1-b_3+2)\Big)\Big)$$

07.28.13.0016.01

$$(1-z)\,z^3\,w^{(4)}(z)\;+(-z\,(a_1+a_2+a_3+a_4+6)+b_1+b_2+b_3+3)\,z^2\,w^{(3)}(z)\;+$$
$$(-z\,(a_2\,a_1+a_3\,a_1+a_4\,a_1+3\,a_1+3\,a_2+a_2\,a_3+3\,a_3+a_2\,a_4+a_3\,a_4+3\,a_4+7)+b_1+b_1\,b_2+b_2+b_1\,b_3+b_2\,b_3+b_3+1)$$
$$z\,w''(z)+$$
$$(b_1\,b_2\,b_3-(a_2\,a_1+a_2\,a_3\,a_1+a_3\,a_1+a_2\,a_4\,a_1+a_3\,a_4\,a_1+a_4\,a_1+a_1+a_2+a_2\,a_3+a_3+a_2\,a_4+a_2\,a_3\,a_4+a_3\,a_4+a_4+1)\,z)$$
$$w'(z)-a_1\,a_2\,a_3\,a_4\,w(z)=0\,/;$$
$$w(z)=c_1\,z^{-a_1}\,{}_4\tilde{F}_3\Big(a_1,\,a_1-b_1+1,\,a_1-b_2+1,\,a_1-b_3+1;\,a_1-a_2+1,\,a_1-a_3+1,\,a_1-a_4+1;\,\frac{1}{z}\Big)+$$
$$c_2\,z^{-a_2}\,{}_4\tilde{F}_3\Big(a_2,\,a_2-b_1+1,\,a_2-b_2+1,\,a_2-b_3+1;\,-a_1+a_2+1,\,a_2-a_3+1,\,a_2-a_4+1;\,\frac{1}{z}\Big)+$$
$$c_3\,z^{-a_3}\,{}_4\tilde{F}_3\Big(a_3,\,a_3-b_1+1,\,a_3-b_2+1,\,a_3-b_3+1;\,-a_1+a_3+1,\,-a_2+a_3+1,\,a_3-a_4+1;\,\frac{1}{z}\Big)+$$
$$c_4\,z^{-a_4}\,{}_4\tilde{F}_3\Big(a_4,\,a_4-b_1+1,\,a_4-b_2+1,\,a_4-b_3+1;\,-a_1+a_4+1,\,-a_2+a_4+1,\,-a_3+a_4+1;\,\frac{1}{z}\Big)\bigwedge$$
$$a_1-a_2\notin\mathbb{Z}\bigwedge a_1-a_3\notin\mathbb{Z}\bigwedge a_1-a_4\notin\mathbb{Z}\bigwedge a_2-a_3\notin\mathbb{Z}\bigwedge a_2-a_4\notin\mathbb{Z}\bigwedge a_3-a_4\notin\mathbb{Z}$$

07.28.13.0017.01

$$(1-z)\,z^3\,w^{(4)}(z)\;+(-z\,(a_1+a_2+a_3+a_4+6)+b_1+b_2+b_3+3)\,z^2\,w^{(3)}(z)\;+$$
$$(-z\,(a_2\,a_1+a_3\,a_1+a_4\,a_1+3\,a_1+3\,a_2+a_2\,a_3+3\,a_3+a_2\,a_4+a_3\,a_4+3\,a_4+7)+b_1+b_1\,b_2+b_2+b_1\,b_3+b_2\,b_3+b_3+1)$$
$$z\,w''(z)+$$
$$(b_1\,b_2\,b_3-(a_2\,a_1+a_2\,a_3\,a_1+a_3\,a_1+a_2\,a_4\,a_1+a_3\,a_4\,a_1+a_4\,a_1+a_1+a_2+a_2\,a_3+a_3+a_2\,a_4+a_2\,a_3\,a_4+a_3\,a_4+a_4+1)\,z)$$
$$w'(z)-a_1\,a_2\,a_3\,a_4\,w(z)=0\,/;$$
$$w(z)=c_1\,z^{-a_1}\,{}_4F_3\Big(a_1,\,a_1-b_1+1,\,a_1-b_2+1,\,a_1-b_3+1;\,a_1-a_2+1,\,a_1-a_3+1,\,a_1-a_4+1;\,\frac{1}{z}\Big)+$$
$$c_2\,z^{-a_2}\,{}_4F_3\Big(a_2,\,a_2-b_1+1,\,a_2-b_2+1,\,a_2-b_3+1;\,-a_1+a_2+1,\,a_2-a_3+1,\,a_2-a_4+1;\,\frac{1}{z}\Big)+$$
$$c_3\,z^{-a_3}\,{}_4F_3\Big(a_3,\,a_3-b_1+1,\,a_3-b_2+1,\,a_3-b_3+1;\,-a_1+a_3+1,\,-a_2+a_3+1,\,a_3-a_4+1;\,\frac{1}{z}\Big)+$$
$$c_4\,z^{-a_4}\,{}_4F_3\Big(a_4,\,a_4-b_1+1,\,a_4-b_2+1,\,a_4-b_3+1;\,-a_1+a_4+1,\,-a_2+a_4+1,\,-a_3+a_4+1;\,\frac{1}{z}\Big)\bigwedge$$
$$a_1-a_2\notin\mathbb{Z}\bigwedge a_1-a_3\notin\mathbb{Z}\bigwedge a_1-a_4\notin\mathbb{Z}\bigwedge a_2-a_3\notin\mathbb{Z}\bigwedge a_2-a_4\notin\mathbb{Z}\bigwedge a_3-a_4\notin\mathbb{Z}$$

Transformations

Products, sums, and powers of the direct function

Products of the direct function

07.28.16.0001.01

$$ {}_4F_3(\alpha_1, \alpha_2, \alpha_3, \alpha_4; \beta_1, \beta_2, \beta_3; d\, z) = \sum_{k=0}^{\infty} c_k\, z^k \;/;$$

$$ c_k = \frac{d^k \prod_{j=1}^4 (\alpha_j)_k}{k! \prod_{j=1}^3 (\beta_j)_k}\, {}_8F_7\!\left(-k, 1-k-\beta_1, 1-k-\beta_2, 1-k-\beta_3, a_1, a_2, a_3, a_4; 1-k-\alpha_1, 1-k-\alpha_2, 1-k-\alpha_3, 1-k-\alpha_4, b_1, b_2, b_3; \frac{c}{d}\right) \bigvee c_k = \frac{c^k \prod_{j=1}^4 (a_j)_k}{k! \prod_{j=1}^3 (b_j)_k}\, {}_8F_7\!\left(-k, 1-k-b_1, 1-k-b_2, 1-k-b_3, \alpha_1, \alpha_2, \alpha_3, \alpha_4; 1-k-a_1, 1-k-a_2, 1-k-a_3, 1-k-a_4, \beta_1, \beta_2, \beta_3; \frac{d}{c}\right)$$

07.28.16.0002.01

$$ {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; c\, z)\, {}_4F_3(\alpha_1, \alpha_2, \alpha_3, \alpha_4; \beta_1, \beta_2, \beta_3; d\, z) = \sum_{k=0}^{\infty} \sum_{m=0}^{k} \frac{\left(\prod_{j=1}^4 (a_j)_m\, c^m\right) \left(\prod_{j=1}^4 (\alpha_j)_{k-m}\, d^{k-m}\right) z^k}{\left(\prod_{j=1}^3 (b_j)_m\, m!\right) \prod_{j=1}^3 (\beta_j)_{k-m}\, (k-m)!}$$

07.28.16.0003.01

$$ {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; c\, z)\, {}_4F_3(\alpha_1, \alpha_2, \alpha_3, \alpha_4; \beta_1, \beta_2, \beta_3; d\, z) = F_{0:3;3}^{0:4;4}\!\left(\begin{matrix} : a_1, a_2, a_3, a_4; \alpha_1, \alpha_2, \alpha_3, \alpha_4; \\ : b_1, b_2, b_3; \beta_1, \beta_2, \beta_3; \end{matrix}\; c\, z, d\, z\right)$$

Identities

Recurrence identities

Consecutive neighbors

07.28.17.0001.01

$$ {}_4F_3(a, a_2, a_3, a_4; b_1, b_2, b_3; z) = (B_1 + C_1 z)\, {}_4F_3(a+1, a_2, a_3, a_4; b_1, b_2, b_3; z) + (B_2 + C_2 z)\, {}_4F_3(a+2, a_2, a_3, a_4; b_1, b_2, b_3; z) + (B_3 + C_3 z)\, {}_4F_3(a+3, a_2, a_3, a_4; b_1, b_2, b_3; z) + (B_4 + C_4 z)\, {}_4F_3(a+4, a_2, a_3, a_4; b_1, b_2, b_3; z) \;/;$$

$$ B_1 = \left((a+1)\left(\left(4a^2 + 11a + 8\right) - (3a+4)(b_1+b_2+b_3) + 2(b_1 b_2 + b_3 b_2 + b_1 b_3)\right) - b_1 b_2 b_3\right) \Big/ ((a-b_1+1)(a-b_2+1)(a-b_3+1)) \bigwedge C_1 = \frac{(-a+a_2-1)(-a+a_3-1)(-a+a_4-1)}{(a-b_1+1)(a-b_2+1)(a-b_3+1)} \bigwedge$$

$$ B_2 = \left((a+1)\left(-6a^2 - 21a - b_1 b_2 - b_1 b_3 - b_2 b_3 + (3a+5)(b_1+b_2+b_3) - 19\right)\right) \Big/ ((a-b_1+1)(a-b_2+1)(a-b_3+1)) \bigwedge$$

$$ C_2 = \left((a+1)\left(3a^2 + 9a + a_2 a_3 + a_2 a_4 + a_3 a_4 - (2a+3)(a_2+a_3+a_4) + 7\right)\right) \Big/ ((a-b_1+1)(a-b_2+1)(a-b_3+1)) \bigwedge$$

$$ B_3 = \frac{(a+1)(a+2)(4a - b_1 - b_2 - b_3 + 9)}{(a-b_1+1)(a-b_2+1)(a-b_3+1)} \bigwedge C_3 = \frac{(-3a + a_2 + a_3 + a_4 - 6)(a+1)(a+2)}{(a-b_1+1)(a-b_2+1)(a-b_3+1)} \bigwedge$$

$$ B_4 = -\frac{(a+1)(a+2)(a+3)}{(a-b_1+1)(a-b_2+1)(a-b_3+1)} \bigwedge C_4 = \frac{(a+1)(a+2)(a+3)}{(a-b_1+1)(a-b_2+1)(a-b_3+1)}$$

07.28.17.0002.01

$$ {}_4F_3(a, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{B_1 + C_1 z}{z-1} {}_4F_3(a-1, a_2, a_3, a_4; b_1, b_2, b_3; z) + \frac{B_2 + C_2 z}{z-1} {}_4F_3(a-2, a_2, a_3, a_4; b_1, b_2, b_3; z) + \frac{B_3 + C_3 z}{z-1} {}_4F_3(a-3, a_2, a_3, a_4; b_1, b_2, b_3; z) + \frac{B_4}{z-1} {}_4F_3(a-4, a_2, a_3, a_4; b_1, b_2, b_3; z) /; B_1 = \frac{7 - 4a + b_1 + b_2 + b_3}{a-1} \bigwedge C_1 = \frac{6 - 3a + a_2 + a_3 + a_4}{1-a} \bigwedge B_2 = \frac{1}{(a-1)(a-2)} \left(6a^2 - 27a + 31 + b_1 b_2 + b_1 b_3 + b_2 b_3 + (7-3a)(b_1 + b_2 + b_3)\right) \bigwedge C_2 = \frac{1}{(a-1)(a-2)} \left((2a-5)(a_2 + a_3 + a_4) - 19 - 3a^2 + 15a - a_2 a_3 - a_2 a_4 - a_3 a_4\right) \bigwedge B_3 = \frac{1}{(a-1)(a-2)(a-3)} \left((a-3)\left((3a-8)(b_1 + b_2 + b_3) - 2(b_1 b_2 + b_3 b_2 + b_1 b_3) - 28 - 4a^2 + 21a\right) + b_1 b_2 b_3\right) \bigwedge C_3 = \frac{(a - a_2 - 3)(a - a_3 - 3)(a - a_4 - 3)}{(a-1)(a-2)(a-3)} \bigwedge B_4 = \frac{(a - b_1 - 3)(a - b_2 - 3)(a - b_3 - 3)}{(a-1)(a-2)(a-3)} $$

07.28.17.0003.01

$$ {}_4F_3(a_1, a_2, a_3, a_4; b, b_2, b_3; z) = \frac{B_1 + C_1 z}{z-1} {}_4F_3(a_1, a_2, a_3, a_4; b+1, b_2, b_3; z) + \frac{B_2 + C_2 z}{z-1} {}_4F_3(a_1, a_2, a_3, a_4; b+2, b_2, b_3; z) + \frac{B_3 + C_3 z}{z-1} {}_4F_3(a_1, a_2, a_3, a_4; b+3, b_2, b_3; z) + \frac{C_4 z}{z-1} {}_4F_3(a_1, a_2, a_3, a_4; b+4, b_2, b_3; z) /; B_1 = \frac{b_2 + b_3 - 3b - 5}{b} \bigwedge C_1 = \frac{4b - a_1 - a_2 - a_3 - a_4 + 6}{b} \bigwedge B_2 = \frac{b_2 b_3 + (b+2)(3b - 2b_2 - 2b_3 + 7)}{b(b+1)} \bigwedge C_2 = \frac{1}{b(b+1)} \left(3(b+2)(a_1 + a_2 + a_3 + a_4) - 6b^2 - 24b - a_1 a_2 - a_1 a_3 - a_2 a_3 - a_1 a_4 - a_2 a_4 - a_3 a_4 - 25\right) \bigwedge B_3 = -\frac{(b - b_2 + 3)(b - b_3 + 3)}{b(b+1)} \bigwedge C_3 = \frac{1}{b(b+1)(b+2)} \left((2b+5)\left(2b^2 + 10b + a_1 a_2 + a_1 a_3 + a_2 a_3 + a_1 a_4 + a_2 a_4 + a_3 a_4 + 13\right) - a_1 a_2 a_3 - a_1 a_4 a_3 - a_2 a_4 a_3 - a_1 a_2 a_4 - \left(3b^2 + 15b + 19\right)(a_1 + a_2 + a_3 + a_4)\right) \bigwedge C_4 = -\frac{(b - a_1 + 3)(b - a_2 + 3)(b - a_3 + 3)(b - a_4 + 3)}{b(b+1)(b+2)(b+3)} $$

07.28.17.0004.01

$$ {}_4F_3(a_1, a_2, a_3, a_4; b, b_2, b_3; z) = \frac{B_1 + C_1 z}{z} {}_4F_3(a_1, a_2, a_3, a_4; b-1, b_2, b_3; z) + \frac{B_2 + C_2 z}{z} {}_4F_3(a_1, a_2, a_3, a_4; b-2, b_2, b_3; z) + \frac{B_3 + C_3 z}{z} {}_4F_3(a_1, a_2, a_3, a_4; b-3, b_2, b_3; z) + \frac{B_4 + C_4 z}{z} {}_4F_3(a_1, a_2, a_3, a_4; b-4, b_2, b_3; z) /; $$

$$ B_1 = -\frac{(b-1)(b-2)(b-b_2-1)(b-b_3-1)}{(b-a_1-1)(b-a_2-1)(b-a_3-1)(b-a_4-1)} \bigwedge $$

$$ C_1 = \left((b-1)\left(\left(-3b^2+9b-7\right)(a_1+a_2+a_3+a_4) + (2b-3)\left(2b^2-6b+a_1a_2+a_1a_3+a_2a_3+a_1a_4+a_2a_4+a_3a_4+5\right) - a_1a_2a_3 - a_1a_4a_3 - a_2a_4a_3 - a_1a_2a_4\right)\right) \Big/ ((b-a_1-1)(b-a_2-1)(b-a_3-1)(b-a_4-1)) \bigwedge $$

$$ B_2 = \frac{(b-1)(b-2)((3b-2b_2-2b_3-5)(b-2)+b_2b_3)}{(b-a_1-1)(b-a_2-1)(b-a_3-1)(b-a_4-1)} \bigwedge C_2 = \left((b-1)(b-2)\left(3(b-2)(a_1+a_2+a_3+a_4) - 6b^2 + 24b - a_1a_2 - a_1a_3 - a_2a_3 - a_1a_4 - a_2a_4 - a_3a_4 - 25\right)\right) \Big/ ((b-a_1-1)(b-a_2-1)(b-a_3-1)(b-a_4-1)) \bigwedge $$

$$ B_3 = \frac{(b-1)(b-2)(b-3)(-3b+b_2+b_3+7)}{(b-a_1-1)(b-a_2-1)(b-a_3-1)(b-a_4-1)} \bigwedge C_3 = \frac{(b-1)(b-2)(b-3)(4b-a_1-a_2-a_3-a_4-10)}{(b-a_1-1)(b-a_2-1)(b-a_3-1)(b-a_4-1)} \bigwedge $$

$$ B_4 = \frac{(b-1)(b-2)(b-3)(b-4)}{(b-a_1-1)(b-a_2-1)(b-a_3-1)(b-a_4-1)} \bigwedge $$

$$ C_4 = -\frac{(b-1)(b-2)(b-3)(b-4)}{(b-a_1-1)(b-a_2-1)(b-a_3-1)(b-a_4-1)} $$

Distant neighbors with respect to *q*

07.28.17.0005.01

$$ {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{\prod_{j=1}^{3}\Gamma(b_j)}{\Gamma(a_3)\Gamma(a_4)} \sum_{k=0}^{\infty} \mathcal{E}_k^{(3)}(\{a_1, a_2, a_3, a_4\}, \{b_1, b_2, b_3\}) \, {}_2\tilde{F}_1(a_1, a_2; a_1+a_2+\psi_3+k; z) /; \psi_3 = \sum_{j=1}^{3} b_j - \sum_{j=1}^{4} a_j $$

Functional identities

Relations between contiguous functions

07.28.17.0006.01

$$ b\,{}_4F_3(a, b+1, a_3, a_4; b_1, b_2, b_3; z) - a\,{}_4F_3(a+1, b, a_3, a_4; b_1, b_2, b_3; z) + (a-b)\,{}_4F_3(a, b, a_3, a_4; b_1, b_2, b_3; z) = 0 $$

07.28.17.0007.01

$$ c\,{}_4F_3(a, a_2, a_3, a_4; c, b_2, b_3; z) - a\,{}_4F_3(a+1, a_2, a_3, a_4; c+1, b_2, b_3; z) + (a-c)\,{}_4F_3(a, a_2, a_3, a_4; c+1, b_2, b_3; z) = 0 $$

07.28.17.0008.01

$$ d\,{}_4F_3(a_1, a_2, a_3, a_4; c+1, d, b_3; z) - c\,{}_4F_3(a_1, a_2, a_3, a_4; c, d+1, b_3; z) + (c-d)\,{}_4F_3(a_1, a_2, a_3, a_4; c+1, d+1, b_3; z) = 0 $$

07.28.17.0009.01

$$ (a-b)\,c\,{}_4F_3(a, b, a_3, a_4; c, b_2, b_3; z) - a(c-b)\,{}_4F_3(a+1, b, a_3, a_4; c+1, b_2, b_3; z) + (c-a)\,b\,{}_4F_3(a, b+1, a_3, a_4; c+1, b_2, b_3; z) = 0 $$

07.28.17.0010.01

$$ a(c-d)\,{}_4F_3(a+1, a_2, a_3, a_4; c+1, d+1, b_3; z) - d(c-a)\,{}_4F_3(a, a_2, a_3, a_4; c+1, d, b_3; z) + (d-a)\,c\,{}_4F_3(a, a_2, a_3, a_4; c, d+1, b_3; z) = 0 $$

07.28.17.0011.01

$$\left(\prod_{j=2}^{4} a_j\right) z\, {}_4F_3(a+1, a_2+1, a_3+1, a_4+1; b_1+1, b_2+1, b_3+1; z) + \left(\prod_{k=1}^{3} b_k\right) ({}_4F_3(a, a_2, a_3, a_4; b_1, b_2, b_3; z) - {}_4F_3(a+1, a_2, a_3, a_4; b_1, b_2, b_3; z)) = 0$$

07.28.17.0012.01

$$c\,(c+1)\, b_2\, b_3\, ({}_4F_3(a_1, a_2, a_3, a_4; c, b_2, b_3; z) - {}_4F_3(a_1, a_2, a_3, a_4; c+1, b_2, b_3; z)) - z\left(\prod_{j=1}^{4} a_j\right) {}_4F_3(a_1+1, a_2+1, a_3+1, a_4+1; c+2, b_2+1, b_3+1; z) = 0$$

07.28.17.0013.01

$$\left(\prod_{k=1}^{3} b_k\right) ({}_4F_3(a, b+1, a_3, a_4; b_1, b_2, b_3; z) - {}_4F_3(a+1, b, a_3, a_4; b_1, b_2, b_3; z)) + a_3\, a_4 (b-a)\, z\, {}_4F_3(a+1, b+1, a_3+1, a_4+1; b_1+1, b_2+1, b_3+1; z) = 0$$

07.28.17.0014.01

$$\left(\prod_{j=2}^{4} a_j\right) (c-a)\, z\, {}_4F_3(a+1, a_2+1, a_3+1, a_4+1; c+2, b_2+1, b_3+1; z) + b_2\, b_3 (c+1)\, c\, ({}_4F_3(a, a_2, a_3, a_4; c, b_2, b_3; z) - {}_4F_3(a+1, a_2, a_3, a_4; c+1, b_2, b_3; z)) = 0$$

07.28.17.0015.01

$$a_3\, a_4\, a\, z\, {}_4F_3(a+1, b+1, a_3+1, a_4+1; c+1, b_2+1, b_3+1; z) + b_2\, b_3 (c\, {}_4F_3(a, b, a_3, a_4; c, b_2, b_3; z) - a\, {}_4F_3(a+1, b+1, a_3, a_4; c+1, b_2, b_3; z) - (c-a)\, {}_4F_3(a, b+1, a_3, a_4; c+1, b_2, b_3; z)) = 0$$

07.28.17.0016.01

$${}_4F_3(a+1, b+1, a_3, a_4; c+1, d+1, e+1; z) - \frac{d\, e\, (a-c)\, (b-c)}{a\, b\, (d-c)\, (e-c)}\, {}_4F_3(a, b, a_3, a_4; c+1, d, e; z) - \frac{c\, e\, (a-d)\, (b-d)}{a\, b\, (c-d)\, (e-d)}\, {}_4F_3(a, b, a_3, a_4; c, d+1, e; z) - \frac{c\, d\, (a-e)\, (b-e)}{a\, b\, (c-e)\, (d-e)}\, {}_4F_3(a, b, a_3, a_4; c, d, e+1; z) = 0$$

07.28.17.0017.01

$${}_4F_3(a, b, c, a_4; d, e, b_3; z) - \frac{a\, b\, (d-c)\, (e-c)}{d\, e\, (a-c)\, (b-c)}\, {}_4F_3(a+1, b+1, c, a_4; d+1, e+1, b_3; z) - \frac{a\, c\, (d-b)\, (e-b)}{d\, e\, (a-b)\, (c-b)}\, {}_4F_3(a+1, b, c+1, a_4; d+1, e+1, b_3; z) - \frac{b\, c\, (d-a)\, (e-a)}{d\, e\, (b-a)\, (c-a)}\, {}_4F_3(a, b+1, c+1, a_4; d+1, e+1, b_3; z) = 0$$

07.28.17.0018.01

$$\left(a + z\sum_{j=1}^{3} (a_{j+1} - b_j)\right) {}_4F_3(a, a_2, a_3, a_4; b_1, b_2, b_3; z) + z\sum_{j=1}^{3} \frac{(b_j - a) \prod_{k=1}^{3} (b_j - a_{k+1})}{b_j \prod_{k=1, k\neq j}^{3} (b_j - b_k)}\, {}_4F_3(a, a_2, a_3, a_4; b_1, \ldots, b_{j-1}, b_j+1, b_{j+1}, \ldots, b_3; z) = a\, (1-z)\, {}_4F_3(a+1, a_2, a_3, a_4; b_1, b_2, b_3; z)$$

Relations of special kind

07.28.17.0019.01

$${}_4F_3(a_1, a_2, a_3, a_4; -c, c+1, b_3; z) + {}_4F_3(a_1, a_2, a_3, a_4; c, 1-c, b_3; z) = 2\, {}_4F_3(a_1, a_2, a_3, a_4; c+1, 1-c, b_3; z)$$

07.28.17.0020.01

$${}_4F_3(a, a_2, a_3, a_4; -a, a+1, b_3; z) - 2\, {}_4F_3(a, a_2, a_3, a_4; 1-a, a+1, b_3; z) = -{}_3F_2(a_2, a_3, a_4; 1-a, b_3; z)$$

07.28.17.0021.01

$${}_4F_3(-a, a_2, a_3, a_4; 1-a, b_2, b_3; z) + {}_4F_3(a, a_2, a_3, a_4; a+1, b_2, b_3; z) = 2\, {}_5F_4(a, -a, a_2, a_3, a_4; a+1, 1-a, b_2, b_3; z)$$

07.28.17.0022.01

$${}_4F_3(-a, a+1, a_3, a_4; b_1, b_2, b_3; z) + {}_4F_3(a, 1-a, a_3, a_4; b_1, b_2, b_3; z) = 2\, {}_4F_3(a, -a, a_3, a_4; b_1, b_2, b_3; z)$$

Division on even and odd parts and generalization

07.28.17.0023.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = A^+(z) + A^-(z) /;$$
$$A^+(z) = \frac{1}{2}\left({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) + {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; -z)\right) \bigwedge$$
$$A^-(z) = \frac{1}{2}\left({}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) - {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; -z)\right)$$

07.28.17.0024.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = A^+(z) + A^-(z) /;$$
$$A^+(z) = {}_8F_7\left(\frac{a_1}{2}, \frac{a_2}{2}, \frac{a_3}{2}, \frac{a_4}{2}, \frac{a_1+1}{2}, \frac{a_2+1}{2}, \frac{a_3+1}{2}, \frac{a_4+1}{2}; \frac{1}{2}, \frac{b_1}{2}, \frac{b_2}{2}, \frac{b_3}{2}, \frac{b_1+1}{2}, \frac{b_2+1}{2}, \frac{b_3+1}{2}; z^2\right) \bigwedge$$
$$A^-(z) = \frac{z \prod_{j=1}^4 a_j}{\prod_{j=1}^3 b_j}\, {}_8F_7\left(\frac{a_1+1}{2}, \frac{a_2+1}{2}, \frac{a_3+1}{2}, \frac{a_4+1}{2}, \frac{a_1+2}{2}, \frac{a_2+2}{2}, \frac{a_3+2}{2}, \frac{a_4+2}{2}; \frac{3}{2}, \frac{b_1+1}{2}, \frac{b_2+1}{2}, \frac{b_3+1}{2}, \frac{b_1+2}{2}, \frac{b_2+2}{2}, \frac{b_3+2}{2}; z^2\right)$$

07.28.17.0025.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{n-1} \frac{z^k \prod_{j=1}^4 (a_j)_k}{k! \prod_{j=1}^3 (b_j)_k}\, {}_{4n+1}F_{4n}\left(1, \frac{a_1+k}{n}, \ldots, \frac{a_1+k+n-1}{n}, \ldots, \frac{a_4+k}{n}, \ldots, \frac{a_4+k+n-1}{n}; \frac{k+1}{n}, \ldots, \frac{k+n}{n}, \frac{k+b_1}{n}, \ldots, \frac{k+n+b_1-1}{n}, \frac{k+b_2}{n}, \ldots, \frac{k+n+b_2-1}{n}, \frac{k+b_3}{n}, \ldots, \frac{k+n+b_3-1}{n}; z^n\right)$$

Major general cases

07.28.17.0026.01

$$ {}_4F_3(a_1,\,a_2,\,a_3,\,a_4;\,b_1,\,b_2,\,b_3;\,z) = \frac{\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)} \left(\frac{\Gamma(a_1)\,\Gamma(a_2-a_1)\,\Gamma(a_3-a_1)\,\Gamma(a_4-a_1)}{\Gamma(b_1-a_1)\,\Gamma(b_2-a_1)\,\Gamma(b_3-a_1)} (-z)^{-a_1}\, {}_4F_3\!\left(a_1,\,a_1-b_1+1,\,a_1-b_2+1,\,a_1-b_3+1;\,a_1-a_2+1,\,a_1-a_3+1,\,a_1-a_4+1;\,\frac{1}{z}\right) + \frac{\Gamma(a_2)\,\Gamma(a_1-a_2)\,\Gamma(a_3-a_2)\,\Gamma(a_4-a_2)}{\Gamma(b_1-a_2)\,\Gamma(b_2-a_2)\,\Gamma(b_3-a_2)} (-z)^{-a_2}\, {}_4F_3\!\left(a_2,\,a_2-b_1+1,\,a_2-b_2+1,\,a_2-b_3+1;\,-a_1+a_2+1,\,a_2-a_3+1,\,a_2-a_4+1;\,\frac{1}{z}\right) + \frac{\Gamma(a_3)\,\Gamma(a_1-a_3)\,\Gamma(a_2-a_3)\,\Gamma(a_4-a_3)}{\Gamma(b_1-a_3)\,\Gamma(b_2-a_3)\,\Gamma(b_3-a_3)} (-z)^{-a_3}\, {}_4F_3\!\left(a_3,\,a_3-b_1+1,\,a_3-b_2+1,\,a_3-b_3+1;\,-a_1+a_3+1,\,-a_2+a_3+1,\,a_3-a_4+1;\,\frac{1}{z}\right) + \frac{\Gamma(a_4)\,\Gamma(a_1-a_4)\,\Gamma(a_2-a_4)\,\Gamma(a_3-a_4)}{\Gamma(b_1-a_4)\,\Gamma(b_2-a_4)\,\Gamma(b_3-a_4)} (-z)^{-a_4}\, {}_4F_3\!\left(a_4,\,a_4-b_1+1,\,a_4-b_2+1,\,a_4-b_3+1;\,-a_1+a_4+1,\,-a_2+a_4+1,\,-a_3+a_4+1;\,\frac{1}{z}\right)\right) /;\, a_1-a_2\notin\mathbb{Z}\land a_1-a_3\notin\mathbb{Z}\land a_1-a_4\notin\mathbb{Z}\land a_2-a_3\notin\mathbb{Z}\land a_2-a_4\notin\mathbb{Z}\land a_3-a_4\notin\mathbb{Z}\land z\notin(0,\,1) $$

07.28.17.0027.01

$$ {}_4F_3(a_1,\,a_2,\,a_3,\,a_4;\,b_1,\,b_2,\,b_3;\,w\,z) = (1-z)^{-a_1} \sum_{k=0}^{\infty} \frac{(a_1)_k}{k!}\, {}_4F_3(-k,\,a_2,\,a_3,\,a_4;\,b_1,\,b_2,\,b_3;\,w) \left(\frac{z}{z-1}\right)^k $$

Functional identities for $z = 1$

07.28.17.0028.01

$$ {}_4F_3(-n,\,b,\,c,\,d;\,b-n+1,\,c-n+1,\,d+n-1;\,1) = \frac{n\,(n-1)\,(n-b+d-1)\,(d-c+n-1)}{(b-n+1)\,(c-n+1)\,(d+n-1)\,(d+n)}\, {}_4F_3(2-n,\,b+1,\,c+1,\,d;\,b-n+2,\,c-n+2,\,d+n+1;\,1) /;\, n\in\mathbb{N}^+ $$

07.28.17.0029.01

$$ {}_4F_3\!\left(-n,\,b,\,c,\,\frac{b}{2}+1;\,e,\,\frac{b}{2},\,b-c+1;\,1\right) = \frac{(e-2\,c-1)_n}{(e)_n}\, {}_4F_3\!\left(-n,\,b-2\,c-1,\,\frac{b+1}{2}-c,\,-c-1;\,\frac{b-1}{2}-c,\,b-c+1,\,e-2\,c-1;\,1\right) /;\, n\in\mathbb{N}^+ $$

07.28.17.0030.01

$$ {}_4F_3(-n,\,b,\,c,\,1-b;\,e,\,-c-n+1,\,2\,c-e+1;\,1) = \frac{\left(\frac{b+e-1}{2}\right)_n \left(\frac{e-b}{2}\right)_n (2\,c)_n}{(c)_n \left(c+\frac{1}{2}\right)_n (e)_n}\, {}_4F_3\!\left(-n,\,c-\frac{b+e}{2}+1,\,c+\frac{b-e+1}{2},\,1-e-n;\,\frac{3-b-e}{2}-n,\,\frac{b-e}{2}-n+1,\,2\,c-e+1;\,1\right) /;\, n\in\mathbb{N} $$

07.28.17.0031.01

$$ {}_4F_3(-n,\,-n,\,-n,\,-n;\,2,\,2,\,2;\,1) = \frac{1}{2\,(n+1)^3}\, {}_4F_3(-n-1,\,-n-1,\,-n-1,\,-n-1;\,1,\,1,\,1;\,1) /;\, n\in\mathbb{N} $$

Differentiation

Low-order differentiation

With respect to a_1

07.28.20.0001.01

$$ {}_4F_3^{(\{1,0,0,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k \psi(k+a_1) z^k}{(b_1)_k (b_2)_k (b_3)_k k!} - \psi(a_1)\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) /; |z| < 1 $$

07.28.20.0002.01

$$ {}_4F_3^{(\{1,0,0,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{z \prod_{j=2}^{4} a_j}{\prod_{j=1}^{3} b_j} F_{4\times 0\times 1}^{4\times 1\times 2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1, a_4+1; 1; 1, a_1; \\ 2, b_1+1, b_2+1, b_3+1;; a_1+1; \end{matrix} z, z \right) $$

With respect to a_2

07.28.20.0003.01

$$ {}_4F_3^{(\{0,1,0,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k \psi(k+a_2) z^k}{(b_1)_k (b_2)_k (b_3)_k k!} - \psi(a_2)\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) /; |z| < 1 $$

07.28.20.0004.01

$$ {}_4F_3^{(\{0,1,0,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{z\, a_1 a_3 a_4}{\prod_{j=1}^{3} b_j} F_{4\times 0\times 1}^{4\times 1\times 2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1, a_4+1; 1; 1, a_2; \\ 2, b_1+1, b_2+1, b_3+1;; a_2+1; \end{matrix} z, z \right) $$

With respect to a_3

07.28.20.0005.01

$$ {}_4F_3^{(\{0,0,1,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k \psi(k+a_3) z^k}{(b_1)_k (b_2)_k (b_3)_k k!} - \psi(a_3)\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) /; |z| < 1 $$

07.28.20.0006.01

$$ {}_4F_3^{(\{0,0,1,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{z\, a_1 a_2 a_4}{\prod_{j=1}^{3} b_j} F_{4\times 0\times 1}^{4\times 1\times 2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1, a_4+1; 1; 1, a_3; \\ 2, b_1+1, b_2+1, b_3+1;; a_3+1; \end{matrix} z, z \right) $$

With respect to a_4

07.28.20.0007.01

$$ {}_4F_3^{(\{0,0,0,1\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k \psi(k+a_4) z^k}{(b_1)_k (b_2)_k (b_3)_k k!} - \psi(a_4)\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) /; |z| < 1 $$

07.28.20.0008.01

$$ {}_4F_3^{(\{0,0,0,1\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{z\, a_1 a_2 a_3}{\prod_{j=1}^{3} b_j} F_{4\times 0\times 1}^{4\times 1\times 2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1, a_4+1; 1; 1, a_4; \\ 2, b_1+1, b_2+1, b_3+1;; a_4+1; \end{matrix} z, z \right) $$

With respect to b_1

07.28.20.0009.01

$${}_4F_3^{(\{0,0,0,0\},\{1,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \psi(b_1)\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) - \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k \psi(k+b_1) z^k}{(b_1)_k (b_2)_k (b_3)_k k!} /; |z| < 1$$

07.28.20.0010.01

$${}_4F_3^{(\{0,0,0,0\},\{1,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = -\frac{z \prod_{j=1}^{4} a_j}{b_1 \prod_{j=1}^{3} b_j} F_{4\times 0\times 1}^{4\times 1\times 2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1, a_4+1; 1; 1, b_1; \\ 2, b_1+1, b_2+1, b_3+1;; b_1+1; \end{matrix} z, z \right)$$

With respect to b_2

07.28.20.0011.01

$${}_4F_3^{(\{0,0,0,0\},\{0,1,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \psi(b_2)\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) - \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k \psi(k+b_2) z^k}{(b_1)_k (b_2)_k (b_3)_k k!} /; |z| < 1$$

07.28.20.0012.01

$${}_4F_3^{(\{0,0,0,0\},\{0,1,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = -\frac{z \prod_{j=1}^{4} a_j}{b_2 \prod_{j=1}^{3} b_j} F_{4\times 0\times 1}^{4\times 1\times 2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1, a_4+1; 1; 1, b_2; \\ 2, b_1+1, b_2+1, b_3+1;; b_2+1; \end{matrix} z, z \right)$$

With respect to b_3

07.28.20.0013.01

$${}_4F_3^{(\{0,0,0,0\},\{0,0,1\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \psi(b_3)\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) - \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k \psi(k+b_3) z^k}{(b_1)_k (b_2)_k (b_3)_k k!} /; |z| < 1$$

07.28.20.0014.01

$${}_4F_3^{(\{0,0,0,0\},\{0,0,1\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = -\frac{z \prod_{j=1}^{4} a_j}{b_3 \prod_{j=1}^{3} b_j} F_{4\times 0\times 1}^{4\times 1\times 2}\left(\begin{matrix} a_1+1, a_2+1, a_3+1, a_4+1; 1; 1, b_3; \\ 2, b_1+1, b_2+1, b_3+1;; b_3+1; \end{matrix} z, z \right)$$

With respect to element of parameters ||| With respect to element of parameters

07.28.20.0015.01

$$\frac{\partial\, {}_4F_3(a, a_2, a_3, a_4; a+1, b_2, b_3; z)}{\partial a} = \frac{z \prod_{j=2}^{4} a_j}{(a+1)^2 b_2 b_3}\, {}_5F_4(a+1, a+1, a_2+1, a_3+1, a_4+1; a+2, a+2, b_2+1, b_3+1; z)$$

07.28.20.0016.01

$$\frac{\partial\, {}_4F_3(a+1, a_2, a_3, a_4; a, b_2, b_3; z)}{\partial a} = -\frac{z \prod_{j=2}^{4} a_j}{a^2 b_2 b_3}\, {}_3F_2(a_2+1, a_3+1, a_4+1; b_2+1, b_3+1; z)$$

With respect to z

07.28.20.0017.01

$$\frac{\partial\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)}{\partial z} = \frac{a_1 a_2 a_3 a_4}{b_1 b_2 b_3}\, {}_4F_3(a_1+1, a_2+1, a_3+1, a_4+1; b_1+1, b_2+1, b_3+1; z)$$

07.28.20.0018.01

$$\frac{\partial^2 {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)}{\partial z^2} = \frac{a_1 (a_1+1) a_2 (a_2+1) a_3 (a_3+1) a_4 (a_4+1)}{b_1 (b_1+1) b_2 (b_2+1) b_3 (b_3+1)} {}_4F_3(a_1+2, a_2+2, a_3+2, a_4+2; b_1+2, b_2+2, b_3+2; z)$$

Symbolic differentiation

With respect to a_1

07.28.20.0019.01

$${}_4F_3^{(\{1,0,0,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_2)_k (a_3)_k (a_4)_k}{(b_1)_k (b_2)_k (b_3)_k \, k!} \frac{\partial^n (a_1)_k}{\partial a_1^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+$$

With respect to a_2

07.28.20.0020.01

$${}_4F_3^{(\{0,n,0,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_3)_k (a_4)_k}{(b_1)_k (b_2)_k (b_3)_k \, k!} \frac{\partial^n (a_2)_k}{\partial a_2^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+$$

With respect to a_3

07.28.20.0021.01

$${}_4F_3^{(\{0,0,n,0\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_4)_k}{(b_1)_k (b_2)_k (b_3)_k \, k!} \frac{\partial^n (a_3)_k}{\partial a_3^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+$$

With respect to a_4

07.28.20.0022.01

$${}_4F_3^{(\{0,0,0,n\},\{0,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k}{(b_1)_k (b_2)_k (b_3)_k \, k!} \frac{\partial^n (a_4)_k}{\partial a_4^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+$$

With respect to b_1

07.28.20.0023.01

$${}_4F_3^{(\{0,0,0,0\},\{n,0,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k}{(b_2)_k (b_3)_k \, k!} \frac{\partial^n \frac{1}{(b_1)_k}}{\partial b_1^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+$$

With respect to b_2

07.28.20.0024.01

$${}_4F_3^{(\{0,0,0,0\},\{0,n,0\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k}{(b_1)_k (b_3)_k \, k!} \frac{\partial^n \frac{1}{(b_2)_k}}{\partial b_2^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+$$

With respect to b_3

07.28.20.0025.01

$${}_4F_3^{(\{0,0,0,0\},\{0,0,n\},0)}(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k}{(b_1)_k (b_2)_k \, k!} \frac{\partial^n \frac{1}{(b_3)_k}}{\partial b_3^n} z^k \;/;\; |z| < 1 \bigwedge n \in \mathbb{N}^+$$

With respect to element of parameters ||| With respect to element of parameters

07.28.20.0036.01

$$\frac{\partial^n {}_pF_q(a, a_2, a_3, a_4; a+1, b_2, b_3; z)}{\partial a^n} = \frac{(-1)^{n-1} n! \, z \, a_2 \, a_3 \, a_4}{(a+1)^{n+1} b_2 \, b_3} {}_{n+4}F_{n+3}(a+1, \ldots, a+1, a_2+1, a_3+1, a_4+1; a+2, \ldots, a+2, b_2+1, b_3+1; z) /; n \in \mathbb{N}^+$$

07.28.20.0037.01

$$\frac{\partial^n {}_4F_3(a+1, a_2, a_3, a_4; a, b_2, b_3; z)}{\partial a^n} = \frac{(-1)^n n!}{a^{n+1}} \left(a \, {}_3F_2(a_2, a_3, a_4; b_2, b_3; z) + \frac{z \, a_2 \, a_3 \, a_4}{b_2 \, b_3} {}_3F_2(a_2+1, a_3+1, a_4+1; b_2+1, b_3+1; z) \right) + \frac{(-1)^{n-1} n!}{a^n} {}_3F_2(a_2, a_3, a_4; b_2, b_3; z) /; n \in \mathbb{N}^+$$

With respect to z

07.28.20.0026.01

$$\frac{\partial^n {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)}{\partial z^n} = \frac{\prod_{j=1}^4 (a_j)_n}{\prod_{j=1}^3 (b_j)_n} {}_4F_3(n+a_1, n+a_2, n+a_3, n+a_4; n+b_1, n+b_2, n+b_3; z) /; n \in \mathbb{N}^+$$

07.28.20.0027.01

$$\frac{\partial^n {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)}{\partial z^n} = z^{-n} \prod_{j=1}^{3} \Gamma(b_j) \, {}_5\tilde{F}_4(1, a_1, a_2, a_3, a_4; 1-n, b_1, b_2, b_3; z) /; n \in \mathbb{N}^+$$

07.28.20.0028.01

$$\frac{\partial^n (z^\alpha \, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z))}{\partial z^n} = (-1)^n (-\alpha)_n \, z^{\alpha-n} \, {}_5F_4(\alpha+1, a_1, a_2, a_3, a_4; 1-n+\alpha, b_1, b_2, b_3; z) /; n \in \mathbb{N}^+$$

07.28.20.0029.01

$$\frac{\partial^n \left(z^{a+n-1} \, {}_4F_3(a, a_2, a_3, a_4; b_1, b_2, b_3; z)\right)}{\partial z^n} = (a)_n \, z^{a-1} \, {}_4F_3(a+n, a_2, a_3, a_4; b_1, b_2, b_3; z) /; n \in \mathbb{N}^+$$

07.28.20.0030.01

$$\frac{\partial^n \left(z^{c-1} \, {}_4F_3(a_1, a_2, a_3, a_4; c, b_2, b_3; z)\right)}{\partial z^n} = (c-n)_n \, z^{c-n-1} \, {}_4F_3(a_1, a_2, a_3, a_4; c-n, b_2, b_3; z) /; n \in \mathbb{N}^+$$

07.28.20.0031.01

$$\frac{\partial^n \left(z^n \, {}_4F_3\left(-n, a_2, a_3, a_4; \frac{1}{2}, b_2, b_3; z\right)\right)}{\partial z^n} = n! \, {}_5F_4\left(-n, n+1, a_2, a_3, a_4; \frac{1}{2}, 1, b_2, b_3; z\right) /; n \in \mathbb{N}^+$$

07.28.20.0032.01

$$\frac{\partial^n (z^\alpha \, {}_4F_3(-n, a_2, a_3, a_4; b_1, b_2, b_3; z))}{\partial z^n} = (-1)^n (-\alpha)_n \, z^{\alpha-n} \, {}_5F_4(-n, \alpha+1, a_2, a_3, a_4; 1-n+\alpha, b_1, b_2, b_3; z) /; n \in \mathbb{N}^+$$

07.28.20.0033.01

$$\frac{\partial^n \left(z^\alpha\, {}_4F_3\left(-\frac{n}{4}, \frac{1-n}{4}, \frac{2-n}{4}, \frac{3-n}{4}; b_1, b_2, b_3; z^m\right)\right)}{\partial z^n} = (-1)^n\, (-\alpha)_n\, z^{\alpha-n}\, {}_{m+4}F_{m+3}\left(-\frac{n}{4}, \frac{1-n}{4}, \frac{2-n}{4}, \frac{3-n}{4}, \frac{\alpha+1}{m}, \frac{\alpha+2}{m}, \ldots, \frac{\alpha+m}{m}; \frac{1-n+\alpha}{m}, \frac{2-n+\alpha}{m}, \ldots, \frac{m-n+\alpha}{m}, b_1, b_2, b_3; z^m\right) /; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+$$

07.28.20.0034.01

$$\frac{\partial^n \left(e^{-z}\, {}_4F_3(-n, a_2, a_3, a_4; b_1, b_2, b_3; z)\right)}{\partial z^n} = (-1)^n\, e^{-z} \sum_{k=0}^{n} \frac{(-n)_k\, z^k}{k! \prod_{j=1}^{3} (b_j)_k}\, {}_5F_3(-n, k-n, k+a_2, k+a_3, k+a_4; k+b_1, k+b_2, k+b_3; z) /; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to z

07.28.20.0035.01

$$\frac{\partial^\alpha\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)}{\partial z^\alpha} = z^{-\alpha} \prod_{j=1}^{3} \Gamma(b_j)\, {}_5\tilde{F}_4(1, a_1, a_2, a_3, a_4; 1-\alpha, b_1, b_2, b_3; z)$$

Integration

Indefinite integration

Involving only one direct function

07.28.21.0001.01

$$\int {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)\, dz = \frac{(b_1-1)(b_2-1)(b_3-1)}{(a_1-1)(a_2-1)(a_3-1)(a_4-1)}\, {}_4F_3(a_1-1, a_2-1, a_3-1, a_4-1; b_1-1, b_2-1, b_3-1; z)$$

Involving one direct function and elementary functions

Involving power function

07.28.21.0002.01

$$\int z^{\alpha-1}\, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)\, dz = \frac{z^\alpha}{\alpha}\, {}_5F_4(\alpha, a_1, a_2, a_3, a_4; \alpha+1, b_1, b_2, b_3; z)$$

Definite integration

For the direct function itself

07.28.21.0003.01

$$\int_0^\infty t^{\alpha-1}\,{}_4F_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;-t)\,dt = (\Gamma(\alpha)\,\Gamma(a_1-\alpha)\,\Gamma(a_2-\alpha)\,\Gamma(a_3-\alpha)\,\Gamma(a_4-\alpha)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3))/(\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)\,\Gamma(b_1-\alpha)\,\Gamma(b_2-\alpha)\,\Gamma(b_3-\alpha))\;/;\; 0<\mathrm{Re}(\alpha)<\min(\mathrm{Re}(a_1),\mathrm{Re}(a_2),\mathrm{Re}(a_3),\mathrm{Re}(a_4))$$

Involving the direct function

07.28.21.0004.01

$$\int_0^\infty t^{\alpha-1}\,e^{-ct}\,{}_4F_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;-t)\,dt = (\Gamma(\alpha)\,\Gamma(a_1-\alpha)\,\Gamma(a_2-\alpha)\,\Gamma(a_3-\alpha)\,\Gamma(a_4-\alpha)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3))/(\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)\,\Gamma(b_1-\alpha)\,\Gamma(b_2-\alpha)\,\Gamma(b_3-\alpha))\,{}_4F_4(\alpha,\alpha-b_1+1,\alpha-b_2+1,\alpha-b_3+1;\alpha-a_1+1,\alpha-a_2+1,\alpha-a_3+1,\alpha-a_4+1;c)+\frac{\Gamma(\alpha-a_1)\,\Gamma(a_2-a_1)\,\Gamma(a_3-a_1)\,\Gamma(a_4-a_1)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)\,\Gamma(b_1-a_1)\,\Gamma(b_2-a_1)\,\Gamma(b_3-a_1)}\,c^{a_1-\alpha}\,{}_4F_4(a_1,a_1-b_1+1,a_1-b_2+1,a_1-b_3+1;-\alpha+a_1+1,a_1-a_2+1,a_1-a_3+1,a_1-a_4+1;c)+\frac{\Gamma(\alpha-a_2)\,\Gamma(a_1-a_2)\,\Gamma(a_3-a_2)\,\Gamma(a_4-a_2)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_3)\,\Gamma(a_4)\,\Gamma(b_1-a_2)\,\Gamma(b_2-a_2)\,\Gamma(b_3-a_2)}\,c^{a_2-\alpha}\,{}_4F_4(a_2,a_2-b_1+1,a_2-b_2+1,a_2-b_3+1;-\alpha+a_2+1,-a_1+a_2+1,a_2-a_3+1,a_2-a_4+1;c)+\frac{\Gamma(\alpha-a_3)\,\Gamma(a_1-a_3)\,\Gamma(a_2-a_3)\,\Gamma(a_4-a_3)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_4)\,\Gamma(b_1-a_3)\,\Gamma(b_2-a_3)\,\Gamma(b_3-a_3)}\,c^{a_3-\alpha}\,{}_4F_4(a_3,a_3-b_1+1,a_3-b_2+1,a_3-b_3+1;-\alpha+a_3+1,-a_1+a_3+1,-a_2+a_3+1,a_3-a_4+1;c)+\Gamma(\alpha-a_4)\,\Gamma(a_1-a_4)\,\Gamma(a_2-a_4)\,\Gamma(a_3-a_4)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)/(\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(b_1-a_4)\,\Gamma(b_2-a_4)\,\Gamma(b_3-a_4))\,c^{a_4-\alpha}\,{}_4F_4(a_4,a_4-b_1+1,a_4-b_2+1,a_4-b_3+1;-\alpha+a_4+1,-a_1+a_4+1,-a_2+a_4+1,-a_3+a_4+1;c)\;/;\;\mathrm{Re}(\alpha)>0\land\mathrm{Re}(c)>0$$

Integral transforms

Laplace transforms

07.28.22.0001.01

$$\mathcal{L}_t[{}_4F_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;-t)](z) = \frac{\pi\csc(\pi a_1)\,\Gamma(a_2-a_1)\,\Gamma(a_3-a_1)\,\Gamma(a_4-a_1)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)\,\Gamma(b_1-a_1)\,\Gamma(b_2-a_1)\,\Gamma(b_3-a_1)}\,z^{a_1-1}\,{}_3F_3(a_1-b_1+1,a_1-b_2+1,a_1-b_3+1;a_1-a_2+1,a_1-a_3+1,a_1-a_4+1;z)+\frac{\pi\csc(\pi a_2)\,\Gamma(a_1-a_2)\,\Gamma(a_3-a_2)\,\Gamma(a_4-a_2)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)\,\Gamma(b_1-a_2)\,\Gamma(b_2-a_2)\,\Gamma(b_3-a_2)}\,z^{a_2-1}\,{}_3F_3(a_2-b_1+1,a_2-b_2+1,a_2-b_3+1;-a_1+a_2+1,a_2-a_3+1,a_2-a_4+1;z)+\frac{\pi\csc(\pi a_3)\,\Gamma(a_1-a_3)\,\Gamma(a_2-a_3)\,\Gamma(a_4-a_3)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)\,\Gamma(b_1-a_3)\,\Gamma(b_2-a_3)\,\Gamma(b_3-a_3)}\,z^{a_3-1}\,{}_3F_3(a_3-b_1+1,a_3-b_2+1,a_3-b_3+1;-a_1+a_3+1,-a_2+a_3+1,a_3-a_4+1;z)+\frac{\pi\csc(\pi a_4)\,\Gamma(a_1-a_4)\,\Gamma(a_2-a_4)\,\Gamma(a_3-a_4)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)\,\Gamma(b_1-a_4)\,\Gamma(b_2-a_4)\,\Gamma(b_3-a_4)}\,z^{a_4-1}\,{}_3F_3(a_4-b_1+1,a_4-b_2+1,a_4-b_3+1;-a_1+a_4+1,-a_2+a_4+1,-a_3+a_4+1;z)+\frac{(b_1-1)\,(b_2-1)\,(b_3-1)}{(a_1-1)\,(a_2-1)\,(a_3-1)\,(a_4-1)}\,{}_4F_4(1,2-b_1,2-b_2,2-b_3;2-a_1,2-a_2,2-a_3,2-a_4;z)\;/;\;\mathrm{Re}(z)>0$$

Summation

Infinite summation

07.28.23.0001.01

$$\sum_{k=0}^{\infty} \frac{(a_1)_k}{k!} \, {}_4F_3(-k, a_2, a_3, a_4; b_1, b_2, b_3; w) \, z^k = \left(\frac{1}{1-z}\right)^{a_1} {}_4F_3\left(a_1, a_2, a_3, a_4; b_1, b_2, b_3; \frac{z\,w}{z-1}\right)$$

Operations

Limit operation

07.28.25.0001.01

$$\lim_{z \to 1} (1-z)^{-\psi_3} \, {}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{\Gamma(-\psi_3)\,\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)} \;/;\; \psi_3 = b_1 + b_2 + b_3 - a_1 - a_2 - a_3 - a_4 \bigwedge \operatorname{Re}(\psi_3) < 0$$

07.28.25.0002.01

$$\lim_{b_1 \to -n} \frac{{}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)}{\Gamma(b_1)} =$$

$$z^{n+1} \prod_{j=1}^{4} \left(a_j\right)_{n+1} \, {}_4\tilde{F}_3(n + a_1 + 1, n + a_2 + 1, n + a_3 + 1, n + a_4 + 1; n + 2, n + b_2 + 1, n + b_3 + 1; z) \;/;\; n \in \mathbb{N}$$

07.28.25.0003.01

$$\lim_{a \to \infty} {}_4F_3\left(a, a_2, a_3, a_4; b_1, b_2, b_3; \frac{z}{a}\right) = {}_3F_3(a_2, a_3, a_4; b_1, b_2, b_3; z)$$

07.28.25.0004.01

$$\lim_{b \to \infty} \lim_{a \to \infty} {}_4F_3\left(a, b, a_3, a_4; b_1, b_2, b_3; \frac{z}{a\,b}\right) = {}_2F_3(a_1, a_2; b_1, b_2, b_3; z)$$

07.28.25.0005.01

$$\lim_{a \to \infty} {}_4F_3\left(a, a_2, a_3, a_4; \frac{a}{z}, b_2, b_3; 1\right) = {}_3F_2(a_2, a_3, a_4; b_2, b_3; z) \;/;\; \operatorname{Re}\left(\frac{a\,(1-z)}{z} - a_2 - a_3 - a_4 + b_2 + b_3\right) > 0$$

07.28.25.0006.01

$$\lim_{a \to n} \frac{1}{a^3\,\Gamma(1-a)} \, {}_4F_3(a, a, a, a; a+1, a+1, a+1; 1) = (-1)^{n-1} \, S_n^{(3)} \;/;\; n \in \mathbb{N}$$

07.28.25.0007.01

$$\lim_{z \to 1} {}_4F_3(1-m, 2, 2, 2; 1, 1, 1; z) = (-1)^{m-1} \, \Gamma(m) \, \mathcal{S}_4^{(m)} \;/;\; m \in \mathbb{N}^+$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

07.28.26.0001.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3) \, {}_4\tilde{F}_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$$

Involving $_pF_q$

07.28.26.0002.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z) \,/;\, p = 4 \land q = 3$$

07.28.26.0003.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = {}_5F_4(a_1, a_2, a_3, a_4, a_5; b_1, b_2, b_3, a_5; z)$$

Through Meijer *G*

Classical cases for the direct function itself

07.28.26.0004.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{\Gamma(b_1)\,\Gamma(b_2)\,\Gamma(b_3)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(a_3)\,\Gamma(a_4)}\, G_{4,4}^{1,4}\!\left(-z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3, 1-a_4 \\ 0, 1-b_1, 1-b_2, 1-b_3 \end{matrix}\right)$$

07.28.26.0005.01

$${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) = \frac{\prod_{k=1}^{3}\Gamma(b_k)}{\pi \sin(\psi_3\,\pi)\prod_{k=1}^{4}\Gamma(a_k)} \sum_{j=1}^{3} \frac{\prod_{k=1}^{4}\sin\left(\pi\left(b_j - a_k\right)\right)}{\prod_{\substack{k=1 \\ k\neq j}}^{3}\sin\left(\pi\left(b_j - b_k\right)\right)}\, G_{4,4}^{2,4}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3, 1-a_4 \\ 0, 1-b_j, 1-b_1, \ldots, 1-b_{j-1}, 1-b_{j+1}, \ldots, 1-b_q \end{matrix}\right) -$$

$$\frac{\pi\prod_{k=1}^{3}\Gamma(b_k)}{\sin(\psi_3\,\pi)\prod_{k=1}^{4}\Gamma(a_k)} \left((1-z)^{\psi_3}\,(z-1)^{-\psi_3}\, G_{4,4}^{0,4}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3, 1-a_4 \\ 0, 1-b_1, 1-b_2, 1-b_3 \end{matrix}\right) + \right.$$

$$\left. G_{4,4}^{4,0}\!\left(z \,\middle|\, \begin{matrix} 1-a_1, 1-a_2, 1-a_3, 1-a_4 \\ 0, 1-b_1, 1-b_2, 1-b_3 \end{matrix}\right)\right) /;\, \psi_3 = \sum_{j=1}^{3} b_j - \sum_{j=1}^{4} a_j \bigwedge z \notin (-1, 0) \bigwedge \psi_3 \notin \mathbb{Z}$$

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