Hypergeometric6F5

Notations

Traditional name

Generalized hypergeometric function ${}_6F_5$

Traditional notation

${}_6F_5(a_1, a_2, a_3, a_4, a_5, a_6; b_1, b_2, b_3, b_4, b_5; z)$

Mathematica StandardForm notation

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HypergeometricPFQ[{a1, a2, a3, a4, a5, a6}, {b1, b2, b3, b4, b5}, z]
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Primary definition

07.30.02.0001.01

$${}_6F_5(a_1, a_2, a_3, a_4, a_5, a_6; b_1, b_2, b_3, b_4, b_5; z) == \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k (a_5)_k (a_6)_k z^k}{(b_1)_k (b_2)_k (b_3)_k (b_4)_k (b_5)_k k!} /;$$

$$|z| < 1 \bigvee |z| == 1 \bigwedge \mathrm{Re}\left(\sum_{j=1}^{5} b_j - \sum_{j=1}^{6} a_j\right) > 0$$

For $a_i == -n$, $b_j == -m$ /; $m \geq n$ being nonpositive integers and $\nexists_{a_k} (a_k > -n \wedge a_k \in \mathbb{N}) \bigwedge \nexists_{b_k} (b_k > -m \wedge b_k \in \mathbb{N})$ the function ${}_6F_5(a_1, a_2, a_3, a_4, a_5, a_6; b_1, b_2, b_3, b_4, b_5; z)$ cannot be uniquely defined by a limiting procedure based on the above definition because the two variables a_i, b_j can approach nonpositive integers $-n$, $-m$; $m \geq n$ at different speeds. For the above conditions we define:

07.30.02.0002.01

$${}_6F_5\left(a_1, \ldots, a_i, \ldots, a_6; b_1, \ldots, b_j, \ldots, b_5; z\right) == \sum_{k=0}^{n} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k (a_5)_k (a_6)_k z^k}{(b_1)_k (b_2)_k (b_3)_k (b_4)_k (b_5)_k k!} /;$$

$$a_i == -n \bigwedge b_j == -m \bigwedge m \in \mathbb{N} \bigwedge n \in \mathbb{N} \bigwedge m \geq n$$

Specific values

Values at $z == 0$

07.30.03.0001.01

$${}_6F_5(a_1, a_2, a_3, a_4, a_5, a_6; b_1, b_2, b_3, b_4, b_5; 0) == 1$$

Values at $z = 1$

For fixed a_3, a_4, b_1

07.30.03.0002.01

$${}_6F_5\left(-n, 1-\frac{n}{3}, c, d, 1-2c-2d-n, \frac{1}{2}-c-d-n; -\frac{n}{3}, 1-2c-n, 1-2d-n, 1-c-d-n, c+d+\frac{1}{2}; 1\right) = 0 /; n \in \mathbb{N}^+ \bigwedge \left\lfloor \frac{n+1}{3} \right\rfloor = \left\lfloor \frac{n+2}{3} \right\rfloor$$

For fixed a_3, a_4

07.30.03.0003.01

$${}_6F_5(-n, 1, c, d, 2-c, 2-d; n+2, c+1, 3-c, d+1, 3-d; 1) = \frac{(n+2)(2-c)c(2-d)d}{2(n+1)(c-1)^2(d-1)^2} + \frac{(n+2)!\,n!}{2(d-c)(c+d-2)}\left(\frac{c(2-c)}{(d-1)^2(d+1)_n(3-d)_n} - \frac{d(2-d)}{(c-1)^2(c+1)_n(3-c)_n}\right) /; c \neq d \bigwedge c \neq 1 \bigwedge d \neq 1$$

07.30.03.0004.01

$${}_6F_5\left(-n, 1-\frac{n}{3}, c, d, 1-2c-2d-n, \frac{1}{2}-c-d-n; -\frac{n}{3}, 1-2c-n, 1-2d-n, 1-c-d-n, c+d+\frac{1}{2}; 1\right) = 0 /; n \in \mathbb{N}^+ \bigwedge \left\lfloor \frac{n+1}{3} \right\rfloor = \left\lfloor \frac{n+2}{3} \right\rfloor$$

For fixed a_3

07.30.03.0005.01

$${}_6F_5(2, 2, c, c, 2-c, 2-c; 1, c+1, c+1, 3-c, 3-c; 1) = \frac{\pi c^2 (2-c)^2}{4(1-c)}\left(\frac{\pi(1-c)}{\sin^2(\pi c)} + \cot(\pi c)\right)$$

Values at $z = -1$

For fixed a_1

07.30.03.0006.01

$${}_6F_5\left(a, a+\frac{1}{3}, a+\frac{2}{3}, 3a-\frac{1}{3}, 3a, 3a+\frac{1}{3}; \frac{1}{3}, \frac{2}{3}, 2a, 2a+\frac{1}{3}, 2a+\frac{2}{3}; -1\right) = \frac{\sqrt{\pi}\,\Gamma(6a)}{3}\left(\frac{4\,3^{-\frac{9a}{2}}\sqrt{\pi}\cos\left(\frac{9a-1}{6}\pi\right)}{\Gamma\left(\frac{2}{3}\right)\Gamma(3a)\Gamma\left(3a+\frac{1}{3}\right)} + \frac{2^{1-9a}}{\Gamma\left(\frac{3a+1}{2}\right)\Gamma\left(\frac{9a}{2}\right)}\right)$$

For fixed a_3

07.30.03.0007.01

$${}_6F_5\left(\frac{1}{2}, 1, c, c, 1-c, 1-c; \frac{3}{2}, c+1, c+1, 2-c, 2-c; -1\right) = \frac{\pi(c-1)^2 c^2 \csc^2(c\pi)}{(1-2c)^4}(2-(2c-1)\pi\cos(c\pi)-2\cos(2c\pi)-4\sin(c\pi)) /; c \neq \frac{1}{2}$$

07.30.03.0008.01

$$ {}_6F_5\left(1, \frac{3}{2}, c, c, 1-c, 1-c; \frac{1}{2}, c+1, c+1, 2-c, 2-c; -1\right) = \frac{(c-1)^2 c^2 \pi^2 \cot(c\pi)\csc(c\pi)}{1-2c} /; c \neq \frac{1}{2} $$

07.30.03.0009.01

$$ {}_6F_5(2, 2, c, c, 2-c, 2-c; 1, c+1, c+1, 3-c, 3-c; -1) = \frac{(c-2)^2 c^2 \pi \csc^2(c\pi)\,((c-1)\pi\cos(c\pi) - \sin(c\pi))}{4(c-1)} /; c \neq 1 $$

Specialized values

For fixed a_1, a_2, b_3, z

07.30.03.0010.01

$$ {}_6F_5\left(a, b, a+\frac{1}{3}, a+\frac{2}{3}, b+\frac{1}{3}, b+\frac{2}{3}; \frac{1}{3}, \frac{2}{3}, g, g+\frac{1}{3}, g+\frac{2}{3}; z\right) = \frac{1}{3}\sum_{k=0}^{2} {}_2F_1\left(3a, 3b; 3g; e^{\frac{2\pi i k}{3}} \sqrt[3]{z}\right) $$

Values at fixed points

Values at $z = 1$

07.30.03.0011.01

$$ {}_6F_5\left(\frac{1}{2}, 1, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{9}{4}; \frac{5}{4}, 2, 2, 2, 3; 1\right) = \frac{32}{5}\left(1 - \frac{8}{\pi^2}\right) $$

Values at $z = -1$

07.30.03.0012.01

$$ {}_6F_5\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; -1\right) = \frac{5\pi^5}{1536} $$

07.30.03.0013.01

$$ {}_6F_5\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}; \frac{1}{4}, 1, 1, 1, 1; -1\right) = \frac{2}{\Gamma\left(\frac{3}{4}\right)^4} $$

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