InverseBetaRegularized

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Notations

Traditional name

Inverse of the regularized incomplete beta function

Traditional notation

$I_z^{-1}(a, b)$

Mathematica StandardForm notation

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InverseBetaRegularized[z, a, b]
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Primary definition

06.23.02.0001.01

$z == I_w(a, b) /; w == I_z^{-1}(a, b)$

Specific values

Specialized values

06.23.03.0001.01

$I_0^{-1}(a, b) == 0 /; a > 0$

06.23.03.0002.01

$I_1^{-1}(a, b) == 1 /; a > 0$

General characteristics

Domain and analyticity

$I_z^{-1}(a, b)$ is an analytical function of z, a, b which is defined in $\mathbb{C}^3$.

06.23.04.0001.01

$(z * a * b) \longrightarrow I_z^{-1}(a, b) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

06.23.06.0002.01

$$I_z^{-1}(a, b) \propto I_{z_0}^{-1}(a, b) + (1-w)^{1-b}\,\mathrm{B}(a, b)\,w^{1-a}\,(z-z_0) + \frac{1}{2}(1-w)^{1-2b}\,(-a+(a+b-2)\,w+1)\,\mathrm{B}(a, b)^2\,w^{1-2a}(z-z_0)^2 + \frac{1}{6}(1-w)^{1-3b}\left(2a^2(w-1)^2 + a((4b-7)w+3)(w-1) + (2b-3)w((b-2)w+2)+1\right)\mathrm{B}(a, b)^3\,w^{1-3a}(z-z_0)^3 + \frac{1}{24}(1-w)^{1-4b}\left(-6a^3+11a^2-6a+(a+b-2)(2a+2b-3)(3a+3b-4)w^3 - 3(a-1)(2a+2b-3)(3a+3b-4)w^2 + (a-1)(-11b+3a(6a+6b-11)+14)w+1\right)\mathrm{B}(a, b)^4\,w^{1-4a}(z-z_0)^4 + \frac{1}{120}(1-w)^{1-5b}\left(24a^4-50a^3+35a^2-10a+(a+b-2)(2a+2b-3)(3a+3b-4)(4a+4b-5)w^4 - 4(a-1)(2a+2b-3)(3a+3b-4)(4a+4b-5)w^3 + 2(a-1)\left(72a^3+6(24b-37)a^2+(b(72b-271)+226)a+b(121-49b)-75\right)w^2 - 2(a-1)(2a-1)(-13b+2a(12a+12b-19)+15)w+1\right)\mathrm{B}(a, b)^5\,w^{1-5a}(z-z_0)^5 + \frac{1}{720}(1-w)^{1-6b}\left((a+b-2)(2a+2b-3)(3a+3b-4)(4a+4b-5)(5a+5b-6)w^5 - 5(a-1)(2a+2b-3)(3a+3b-4)(4a+4b-5)(5a+5b-6)w^4 + 2(a-1)\left((600a-437)b^3+(12a(150a-287)+1585)b^2 + a(3a(600a-1859)+5717)b-1923b+a(2a(5a(60a-257)+2066)-2943)+780\right)w^3 - 2(a-1)\left(\left(600a^2-711a+212\right)b^2+(a(a(1200a-2681)+1973)-477)b+2(4a-3)(5a-3)(a(15a-29)+15)\right)w^2 + (a(a(a(10a(60a-197)+2547)-1622)+507)+3(a-1)(2a-1)(a(100a-87)+19)b)w - 62w - a(a(a(2a(60a-137)+225)-85)+15)+1\right)\mathrm{B}(a, b)^6\,w^{1-6a}(z-z_0)^6 + \ldots /;\ (z \to z_0) \bigwedge w = I_{z_0}^{-1}(a, b)$$

06.23.06.0003.01

$$I_z^{-1}(a, b) \propto I_{z_0}^{-1}(a, b)\,(1 + O(z-z_0))$$

Expansions at $z == 0$

06.23.06.0001.01

$$I_z^{-1}(a, b) \propto (a\,z\,\mathrm{B}(a, b))^{1/a} + \frac{b-1}{a+1}(a\,z\,\mathrm{B}(a, b))^{2/a} + \frac{(b-1)\left(a^2+3ba-a+5b-4\right)}{2(a+1)^2(a+2)}(a\,z\,\mathrm{B}(a, b))^{3/a} + O\left(z^{4/a}\right) /;\ a > 0$$

06.23.06.0004.01

$$I_z^{-1}(a, b) \propto w + \frac{b-1}{a+1} w^2 + \frac{(b-1)\left(a^2+3\,b\,a-a+5\,b-4\right)}{2\,(a+1)^2\,(a+2)} w^3 +$$
$$\frac{(b-1)\left(a^4+(6\,b-1)\,a^3+(b+2)\,(8\,b-5)\,a^2+\left(33\,b^2-30\,b+4\right)a+b\,(31\,b-47)+18\right)}{3\,(a+1)^3\,(a+2)\,(a+3)} w^4 +$$
$$\frac{1}{24\,(a+1)^4\,(a+2)^2\,(a+3)\,(a+4)}$$
$$\Big((b-1)\big((a\,(a\,(a\,(125\,a+1179)+3971)+5661)+2888)\,b^3+3\,(a\,(a\,(a\,(a\,(50\,a+367)+637)-831)-3111)-2104)\,b^2+$$
$$a\,(a\,(a\,(a\,(55\,a+269)-259)-2581)-1636)+4688)\,b+$$
$$a\,(a\,(a\,(a\,(6\,a+11)-119)-235)+529)+1016)-632)+8\,(581\,b-144)\big)\Big)\,w^5+$$
$$\frac{1}{10\,(a+1)^5\,(a+2)^2\,(a+3)\,(a+4)\,(a+5)}\Big((b-1)\big((a\,(a\,(a\,(a\,(108\,a+1471)+7575)+18\,375)+20\,997)+9074)\,b^4+$$
$$2\,(a\,(a\,(a\,(a\,(90\,a+1049)+3978)+3650)-9970)-22\,939)-12\,818)\,b^3+$$
$$(a\,(a\,(a\,(a\,(15\,a\,(7\,a+66)+1993)-5564)-20\,855)-5120)+35\,077)+27\,454)\,b^2+$$
$$a\,(a\,(a\,(a\,(5\,a\,(5\,a+34)-27)-2612)-2249)+9810)+11\,715)-10\,248)\,b-13\,196\,b+$$
$$a\,(a\,(a\,(a\,(a\,(a\,(2\,a+7)-57)-203)+379)+1378)-888)-3222)+684)+2400\big)\Big)$$
$$w^6+\frac{1}{\left(720\,(a+1)^6\,(a+2)^3\,(a+3)^2\,(a+4)\,(a+5)\,(a+6)\right)}$$
$$\Big((b-1)\big((a\,(a\,(a\,(a\,(a\,(a\,(16\,807\,a+398\,516)+3\,987\,861)+21\,989\,226)+73\,069\,137)+149\,847\,504)+185\,250\,179)+$$
$$126\,276\,754)+36\,360\,816)\,b^5+$$
$$5\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(7203\,a+157\,286)+1\,395\,565)+6\,350\,196)+14\,698\,401)+10\,053\,702)-$$
$$29\,625\,237)-78\,195\,392)-72\,983\,068)-25\,045\,728)\,b^4+$$
$$5\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(5831\,a+115\,196)+870\,096)+2\,877\,494)+1\,747\,434)-15\,228\,558)-$$
$$41\,061\,344)-22\,412\,726)+51\,845\,151)+82\,583\,314)+34\,873\,152)\,b^3+$$
$$5\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(3\,a\,(a\,(735\,a+12\,776)+75\,744)+325\,642)-1\,891\,930)-7\,786\,674)-4\,567\,008)+$$
$$22\,941\,654)+38\,501\,757)-4\,030\,870)-44\,955\,776)-24\,517\,200)\,b^2+$$
$$a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(2\,a\,(959\,a+13\,968)+104\,123)-283\,544)-2\,654\,803)-3\,434\,538)+12\,172\,733)+$$
$$32\,310\,972)-6\,773\,219)-77\,685\,178)-37\,606\,016)+56\,926\,752)\,b+$$
$$a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(2\,a\,(60\,a+661)+159)-45\,078)-140\,531)+252\,156)+1\,555\,497)+$$
$$388\,306)-5\,856\,333)-4\,942\,098)+9\,049\,664)+10\,065\,072)-4\,936\,896)+$$
$$1728\,(25\,163\,b-3600)\big)\Big)\,w^7+\frac{1}{\left(315\,(a+1)^7\,(a+2)^3\,(a+3)^2\,(a+4)\,(a+5)\,(a+6)\,(a+7)\right)}$$
$$\Big((b-1)\big((a\,(a\,(a\,(a\,(a\,(a\,(a\,(16\,384\,a+486\,927)+6\,181\,022)+43\,936\,962)+192\,606\,624)+539\,832\,153)+$$
$$967\,463\,528)+1\,069\,554\,738)+662\,420\,842)+175\,331\,220)\,b^6+$$
$$3\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(14\,336\,a+400\,147)+4\,653\,201)+29\,039\,428)+102\,900\,786)+189\,789\,789)+$$
$$68\,351\,521)-435\,090\,718)-910\,884\,912)-754\,751\,926)-236\,555\,412)\,b^5+$$
$$5\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(7\,a\,(1280\,a+33\,199)+2\,423\,517)+12\,611\,711)+30\,300\,322)-358\,935)-$$
$$175\,887\,971)-361\,456\,837)-119\,971\,710)+478\,191\,784)+632\,874\,386)+$$
$$241\,888\,188)\,b^4+5\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(7\,a\,(672\,a+15\,949)+143\,999)+3\,900\,755)+1\,843\,387)-$$
$$34\,547\,979)-98\,700\,213)-20\,690\,677)+296\,612\,281)+$$
$$387\,270\,256)-100\,517\,620)-455\,975\,838)-221\,996\,772)\,b^3+$$
$$(a\,(a\,(a\,(a\,(a\,(a\,(a\,(7\,a\,(a\,(928\,a+19\,681)+143\,843)+273\,681)-11\,263\,661)-58\,906\,876)-$$
$$40\,569\,269)+270\,353\,999)+517\,093\,087)-251\,110\,671)-$$
$$1\,231\,041\,382)-430\,118\,066)+868\,415\,428)+578\,179\,680)\,b^2+$$
$$a\,(a\,(a\,(a\,(a\,(a\,(a\,(7\,a\,(a\,(a\,(126\,a+2293)+11\,479)-23\,465)-362\,783)-4\,904\,714)+$$
$$14\,826\,616)+56\,621\,531)-6\,699\,043)-203\,593\,999)-$$

$$119\,162\,155) + 298\,948\,394) + 264\,298\,412) - 155\,359\,968)\,b +$$
$$a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(9\,(5\,a + 72) + 898) - 26\,720) - 126\,531) + 126\,356) + 1\,714\,400) + 1\,366\,440) -$$
$$8\,525\,338) - 12\,116\,446) + 19\,335\,546) + 34\,607\,770) -$$
$$20\,353\,436) - 42\,419\,568) + 8\,090\,496) - 432\,(374\,917\,b - 44\,100))))$$

$$w^8 + \frac{1}{\left(4480\,(a+1)^8\,(a+2)^4\,(a+3)^2\,(a+4)^2\,(a+5)\,(a+6)\,(a+7)\,(a+8)\right)}$$
$$\Big((b-1)$$
$$\Big((a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(531\,441\,a + 22\,669\,360) + 429\,446\,950) + 4\,774\,846\,502) + 34\,688\,913\,336) +$$
$$173\,423\,227\,630) + 611\,682\,241\,930) +$$
$$1\,533\,600\,689\,586) + 2\,712\,896\,385\,015) + 3\,302\,979\,236\,810) +$$
$$2\,628\,223\,795\,120) + 1\,227\,828\,808\,512) + 254\,843\,639\,808)\,b^7 +$$
$$7\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(236\,196\,a + 9\,660\,875) + 173\,459\,768) + 1\,797\,336\,682) + 11\,858\,117\,010) +$$
$$51\,584\,830\,832) + 146\,168\,793\,874) + 244\,242\,739\,526) +$$
$$123\,626\,364\,242) - 421\,547\,700\,219) - 1\,103\,344\,548\,242) -$$
$$1\,239\,263\,201\,008) - 710\,423\,542\,848) - 168\,900\,261\,888)\,b^6 +$$
$$7\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(2\,a\,(150\,903\,a + 5\,886\,164) + 198\,591\,305) + 1\,888\,576\,180) + 10\,980\,721\,902) +$$
$$38\,732\,558\,322) + 69\,805\,890\,852) - 16\,724\,195\,230) -$$
$$391\,881\,517\,772) - 841\,600\,887\,538) - 569\,779\,214\,349) +$$
$$674\,128\,142\,850) + 1\,622\,305\,374\,864) + 1\,224\,065\,844\,288) + 339\,278\,088\,192)\,b^5 +$$
$$35\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(2\,a\,(20\,412\,a + 753\,731) + 11\,797\,602) + 201\,016\,569) +$$
$$972\,868\,524) + 2\,280\,051\,830) - 1\,003\,529\,470) -$$
$$21\,662\,382\,244) - 52\,748\,505\,534) - 22\,384\,008\,300) +$$
$$126\,430\,631\,394) + 236\,524\,505\,707) + 75\,883\,727\,026) -$$
$$200\,081\,481\,552) - 230\,992\,970\,688) - 76\,448\,827\,392)\,b^4 +$$
$$7\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(78\,327\,a + 2\,709\,808) + 38\,607\,352) + 282\,216\,762) + 987\,537\,420) +$$
$$80\,359\,782) - 12\,567\,223\,562) - 41\,761\,843\,646) -$$
$$22\,491\,338\,675) + 163\,156\,798\,722) + 356\,264\,090\,898) +$$
$$28\,376\,157\,276) - 707\,108\,331\,568) - 699\,067\,619\,312) +$$
$$213\,553\,465\,312) + 637\,619\,092\,608) + 260\,688\,362\,496)\,b^3 +$$
$$7\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(16\,884\,a + 538\,859) + 6\,765\,568) + 38\,636\,240) +$$
$$48\,009\,906) - 599\,459\,724) - 3\,157\,483\,530) - 3\,468\,964\,314) +$$
$$16\,206\,674\,502) + 51\,883\,401\,009) + 10\,055\,959\,986) -$$
$$159\,569\,974\,902) - 189\,374\,967\,348) + 139\,645\,505\,968) +$$
$$357\,300\,754\,976) + 68\,121\,222\,976) - 202\,185\,020\,544) - 107\,556\,621\,312)\,b^2 +$$
$$a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(4\,a\,(3267\,a + 93\,802) + 3\,894\,867) + 12\,708\,504) -$$
$$68\,050\,574) - 682\,231\,254) - 1\,499\,040\,192) + 3\,876\,268\,114) +$$
$$22\,622\,916\,418) + 15\,858\,492\,678) - 91\,909\,279\,251) -$$
$$169\,674\,022\,930) + 113\,008\,548\,704) + 469\,673\,350\,224) +$$
$$95\,598\,525\,280) - 544\,065\,753\,088) - 305\,774\,479\,616) + 229\,945\,657\,344)\,b +$$
$$a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(4\,a\,(a\,(140\,a + 3453) + 25\,051) - 197\,089) - 6\,174\,816) - 25\,691\,102) +$$
$$24\,930\,050) + 435\,172\,344) + 728\,117\,266) - 2\,067\,637\,494) -$$
$$7\,135\,515\,526) + 1\,498\,242\,409) + 26\,065\,842\,646) +$$
$$13\,779\,022\,496) - 46\,150\,895\,904) - 41\,076\,704\,064) + 39\,777\,942\,144) +$$
$$44\,576\,037\,888) - 13\,391\,437\,824) + 221\,184\,(786\,269\,b - 78\,400))\Big)\,w^9 +$$

$$\frac{1}{\left(4536\,(a+1)^9\,(a+2)^4\,(a+3)^3\,(a+4)^2\,(a+5)\,(a+6)\,(a+7)\,(a+8)\,(a+9)\right)}$$
$$\Big((b-1)$$
$$\Big((a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(2\,a\,(625\,000\,a + 33\,456\,213) + 1\,614\,236\,211) + 23\,253\,261\,756) + 223\,389\,222\,954) +$$

$$
\begin{aligned}
&1\,513\,727\,910\,042) + 7\,460\,377\,417\,652) + \\
&27\,165\,941\,822\,238) + 73\,445\,003\,464\,698) + \\
&146\,718\,794\,936\,892) + 213\,209\,054\,640\,489) + 218\,591\,764\,868\,766) + \\
&149\,504\,225\,861\,948) + 61\,077\,979\,985\,160) + 11\,250\,058\,301\,568)\, b^8 + \\
&4\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(4\,a\,(281\,250\,a + 14\,574\,029) + 1\,351\,167\,633) + 18\,508\,290\,993) + \\
&166\,644\,795\,390) + 1\,035\,984\,438\,060) + 4\,529\,911\,782\,282) + \\
&13\,812\,363\,372\,194) + 27\,820\,849\,055\,892) + \\
&29\,741\,210\,038\,620) - 10\,189\,434\,004\,683) - 93\,686\,645\,350\,779) - \\
&159\,973\,862\,320\,482) - 144\,324\,960\,690\,196) - 70\,440\,758\,773\,512) - 14\,686\,204\,963\,968)\, \\
&b^7 + 14\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(3\,a\,(162\,500\,a + 8\,122\,723) + 539\,825\,449) + 6\,975\,061\,677) + \\
&58\,070\,902\,437) + 323\,073\,734\,052) + 1\,189\,038\,960\,234) + \\
&2\,634\,297\,002\,646) + 1\,918\,479\,329\,530) - \\
&7\,457\,711\,606\,907) - 26\,493\,465\,624\,255) - \\
&35\,284\,629\,824\,571) - 6\,795\,617\,848\,539) + 44\,927\,776\,666\,926) + \\
&66\,039\,283\,849\,372) + 40\,444\,333\,897\,848) + 9\,682\,198\,814\,592)\, b^6 + \\
&14\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(9\,a\,(a\,(45\,000\,a + 2\,159\,941) + 45\,416\,299) + 4\,924\,980\,139) + \\
&37\,125\,466\,299) + 176\,677\,516\,512) + 480\,937\,433\,610) + \\
&326\,502\,338\,718) - 2\,581\,867\,711\,554) - 9\,664\,461\,018\,083) - \\
&12\,246\,352\,312\,569) + 7\,874\,944\,477\,119) + \\
&45\,065\,480\,843\,967) + 49\,553\,472\,957\,270) - 3\,977\,669\,713\,332) - \\
&54\,621\,049\,672\,136) - 46\,004\,246\,797\,152) - 12\,885\,993\,451\,008)\, b^5 + \\
&7\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(3\,a\,(a\,(a\,(133\,625\,a + 6\,121\,388) + 120\,980\,565) + 1\,335\,838\,790) + \\
&26\,371\,647\,167) + 96\,918\,532\,344) + 103\,272\,232\,677) - \\
&721\,649\,562\,198) - 3\,507\,709\,254\,447) - \\
&5\,237\,418\,136\,488) + 5\,778\,225\,539\,597) + \\
&32\,072\,766\,140\,850) + 34\,312\,821\,152\,433) - \\
&28\,422\,118\,664\,316) - 92\,980\,252\,832\,145) - 52\,322\,271\,929\,982) + \\
&44\,443\,778\,307\,572) + 64\,309\,636\,625\,016) + 21\,624\,965\,732\,736)\, b^4 + \\
&14\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(3\,a\,(a\,(a\,(20\,025\,a + 868\,112) + 15\,893\,261) + 156\,207\,572) + \\
&831\,686\,655) + 4\,833\,775\,438) - 21\,643\,554\,933) - \\
&161\,575\,565\,736) - 337\,543\,911\,387) + 310\,216\,327\,308) + \\
&2\,708\,072\,355\,189) + 3\,619\,004\,651\,884) - \\
&4\,077\,292\,928\,181) - 15\,178\,250\,996\,346) - \\
&7\,452\,457\,860\,039) + 17\,930\,691\,686\,928) + 22\,385\,776\,830\,648) - \\
&1\,448\,304\,049\,376) - 13\,975\,598\,115\,120) - 5\,854\,010\,234\,112)\, b^3 + \\
&(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(147\,655\,a + 5\,982\,126) + 99\,086\,523) + 819\,322\,916) + \\
&2\,813\,474\,061) - 6\,667\,777\,764) - 98\,926\,229\,879) - \\
&315\,787\,117\,314) + 88\,399\,397\,505) + 3\,019\,366\,795\,982) + \\
&6\,004\,954\,176\,669) - 5\,155\,866\,183\,192) - 32\,148\,728\,798\,501) - \\
&21\,981\,803\,591\,352) + 58\,901\,032\,905\,903) + \\
&90\,942\,075\,741\,590) - 25\,985\,348\,388\,432) - 116\,066\,465\,941\,152) - \\
&33\,642\,942\,036\,336) + 50\,569\,494\,562\,080) + 27\,945\,269\,609\,472)\, b^2 + \\
&2\,a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(6849\,a + 254\,213) + 3\,647\,586) + 21\,978\,225) - \\
&9\,187\,262) - 891\,936\,810) - 4\,569\,660\,237) - 3\,105\,877\,906) + \\
&50\,034\,011\,493) + 162\,435\,141\,633) - 39\,938\,157\,440) - \\
&1\,037\,286\,954\,171) - 1\,251\,509\,376\,336) + 2\,421\,217\,082\,180) + \\
&5\,878\,948\,725\,435) - 1\,137\,521\,480\,196) - 11\,213\,166\,021\,464) - \\
&4\,043\,188\,991\,232) + 9\,956\,390\,624\,496) + 6\,348\,442\,039\,488) - 3\,399\,385\,708\,800)\, b + \\
&a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(a\,(18\,a\,(28\,a + 919) + 186\,103) + 367\,620) - 9\,574\,386) - \\
&83\,305\,942) - 168\,955\,152) + 993\,089\,874) + 5\,730\,579\,406) + \\
&3\,580\,326\,744) - 42\,123\,596\,859) - 93\,464\,155\,906) +
\end{aligned}
$$

$$100\,126\,221\,528) + 500\,786\,446\,416) + 93\,801\,596\,020) - 1\,269\,592\,245\,948) - 897\,596\,215\,932) + 1\,703\,029\,775\,000) + 1\,738\,412\,530\,608) - 1\,165\,993\,518\,240) - 1\,469\,876\,129\,280) + 320\,899\,968\,000) - 746\,496\,(7\,346\,593\,b - 635\,040)\Big)\,w^{10} + O\left(z^{11/a}\right) /; w = (a\,z\,\mathrm{B}(a,\,b))^{1/a} \bigwedge a > 0$$

Expansions at generic point $a == a_0$

For the function itself

06.23.06.0005.01

$$I_z^{-1}(a,\,b) \propto I_z^{-1}(a_0,\,b) - w\,(1-w)^{1-b}\left(w^{-a_0}\,\mathrm{B}_w(a_0,\,b)\,(\log(w) + \psi(b+a_0) - \psi(a_0)) - \frac{1}{a_0^2}\,{}_3F_2(a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1;\,w)\right)(a-a_0) + \frac{1}{2}\,w\,(1-w)^{1-2\,b}\left(-2\,\Gamma(a_0)^3\,{}_4\tilde{F}_3(a_0,\,a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1,\,a_0+1;\,w)\,(1-w)^b - w^{-a_0}\,\mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\,(\psi(a_0) - \psi(b+a_0))\,(\log(w) + \psi(b+a_0) - \psi(a_0))\,(1-w)^b - w^{-a_0}\,\mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\,(\log(w) + \psi(b+a_0) - \psi(a_0))\left((1-a_0)\,(1-w)^{1-b}\,w^{-a_0}\left(w^{a_0}\,\Gamma(a_0)^2\,{}_3\tilde{F}_2(a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1;\,w) - \mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\,(\log(w) + \psi(b+a_0) - \psi(a_0))\right) - \log(w)\right)(1-w)^b + (b-1)\,w^{1-a_0}\,\Gamma(a_0)^2\,{}_3\tilde{F}_2(a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1;\,w)\left(w^{a_0}\,\Gamma(a_0)^2\,{}_3\tilde{F}_2(a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1;\,w) - \mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\,(\log(w) + \psi(b+a_0) - \psi(a_0))\right) + (w-1)\,w^{-a_0}\,\Gamma(a_0)^2\left(a_0^2\,(b-1)\,w\,{}_3\tilde{F}_2(a_0+1,\,a_0+1,\,2-b;\,a_0+2,\,a_0+2;\,w) - {}_3\tilde{F}_2(a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1;\,w)\right)\left(w^{a_0}\,\Gamma(a_0)^2\,{}_3\tilde{F}_2(a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1;\,w) - \mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\,(\log(w) + \psi(b+a_0) - \psi(a_0))\right) + (1-b)\,w^{1-2\,a_0}\,\mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\,(\log(w) + \psi(b+a_0) - \psi(a_0))\left(w^{a_0}\,\Gamma(a_0)^2\,{}_3\tilde{F}_2(a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1;\,w) - \mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\,(\log(w) + \psi(b+a_0) - \psi(a_0))\right) + w^{-2\,a_0}\,\mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\left(w^{a_0}\left(\psi^{(1)}(a_0) - \psi^{(1)}(b+a_0)\right)(1-w)^b + (w-1)\,w^{a_0}\,\Gamma(a_0)^2\,{}_3\tilde{F}_2(a_0,\,a_0,\,1-b;\,a_0+1,\,a_0+1;\,w) - (w-1)\,\mathrm{B}(a_0,\,b)\,I_w(a_0,\,b)\,(\log(w) + \psi(b+a_0) - \psi(a_0))\right)\right)(a-a_0)^2 + \ldots /; (a \to a_0) \bigwedge w == I_z^{-1}(a_0,\,b)$$

06.23.06.0006.01

$$Q^{-1}(a,\,z) \propto Q^{-1}(a_0,\,z)\,(1 + O(a-a_0))$$

Expansions at generic point $b == b_0$

For the function itself

06.23.06.0007.01

$$I_z^{-1}(a, b) \propto I_z^{-1}(a, b_0) - w^{1-a}(1-w)^{1-b_0}$$
$$\left(\frac{(1-w)^{b_0}}{b_0^2}\,{}_3F_2(1-a, b_0, b_0; b_0+1, b_0+1; 1-w) - \mathrm{B}_{1-w}(b_0, a)\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))\right)(b-b_0) +$$
$$\frac{1}{2}\,w^{1-2a}\Big(-2\,(w-1)\,\Gamma(b_0)^3\,{}_4\tilde{F}_3(b_0, b_0, b_0, 1-a; b_0+1, b_0+1, b_0+1; 1-w)\,w^a + (1-w)^{1-b_0}\,\mathrm{B}_{1-w}(b_0, a)$$
$$(\psi(b_0) - \psi(a+b_0))\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))\,w^a - (1-w)^{2-b_0}\,\Gamma(b_0)^2\,{}_3\tilde{F}_2(b_0, b_0, 1-a; b_0+1, b_0+1; 1-w)$$
$$\Big((1-w)^{b_0}\,\Gamma(b_0)^2\,{}_3\tilde{F}_2(b_0, b_0, 1-a; b_0+1, b_0+1; 1-w) - \mathrm{B}_{1-w}(b_0, a)\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))\Big)\,w +$$
$$(1-a)\,(1-w)^{2-b_0}\,\Gamma(b_0)^2\,{}_3\tilde{F}_2(b_0, b_0, 1-a; b_0+1, b_0+1; 1-w)$$
$$\Big((1-w)^{b_0}\,\Gamma(b_0)^2\,{}_3\tilde{F}_2(b_0, b_0, 1-a; b_0+1, b_0+1; 1-w) - \mathrm{B}_{1-w}(b_0, a)\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))\Big) +$$
$$(a-1)\,(1-w)^{2-2\,b_0}\,\mathrm{B}_{1-w}(b_0, a)\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))$$
$$\Big((1-w)^{b_0}\,\Gamma(b_0)^2\,{}_3\tilde{F}_2(b_0, b_0, 1-a; b_0+1, b_0+1; 1-w) - \mathrm{B}_{1-w}(b_0, a)\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))\Big) +$$
$$(1-w)^{-2\,b_0}\,(w-1)\,\mathrm{B}_{1-w}(b_0, a)\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))$$
$$\Big((1-w)^{b_0}\log(1-w)\,w^a - (b_0-1)\,\mathrm{B}_{1-w}(b_0, a)\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))\,w +$$
$$(1-w)^{b_0}\,\Gamma(b_0)^2\,{}_3\tilde{F}_2(b_0, b_0, 1-a; b_0+1, b_0+1; 1-w)\,(b_0-1)\,w\Big) + (1-w)^{-2\,b_0}\,(w-1)\,\mathrm{B}_{1-w}(b_0, a)$$
$$\Big((1-w)^{b_0}\left(\psi^{(1)}(b_0) - \psi^{(1)}(a+b_0)\right)w^a - (1-w)^{b_0}\,\Gamma(b_0)^2\,{}_3\tilde{F}_2(b_0, b_0, 1-a; b_0+1, b_0+1; 1-w)\,w +$$
$$\mathrm{B}_{1-w}(b_0, a)\,(\log(1-w) - \psi(b_0) + \psi(a+b_0))\,w\Big)\,(b-b_0)^2 + \ldots\,/;\,(b \to b_0)\bigwedge w == I_z^{-1}(a, b_0)$$

06.23.06.0008.01

$$I_z^{-1}(a, b) \propto I_z^{-1}(a, b_0)\,(1 + O(b-b_0))$$

Differential equations

Ordinary nonlinear differential equations

06.23.13.0001.01

$$w(z)\,(1-w(z))\,w''(z) - (1-a+(a+b-2)\,w(z))\,w'(z)^2 == 0\,/;\,w(z) == I_z^{-1}(a, b)$$

Differentiation

Low-order differentiation

With respect to z

06.23.20.0001.01

$$\frac{\partial I_z^{-1}(a, b)}{\partial z} == (1-w)^{1-b}\,w^{1-a}\,\mathrm{B}(a, b)\,/;\,w == I_z^{-1}(a, b)$$

06.23.20.0002.01

$$\frac{\partial^2 I_z^{-1}(a, b)}{\partial z^2} == (1-w)^{1-2\,b}\,w^{1-2\,a}\,((w-1)\,a + (b-2)\,w + 1)\,\mathrm{B}(a, b)^2\,/;\,w == I_z^{-1}(a, b)$$

06.23.20.0007.01

$$\frac{\partial^3 I_z^{-1}(a, b)}{\partial z^3} == (1-w)^{1-3\,b}\,w^{1-3\,a}\left(2\,a^2\,(w-1)^2 + a\,((4\,b-7)\,w+3)\,(w-1) + \left(2\,b^2 - 7\,b + 6\right)w^2 + (4\,b-6)\,w + 1\right)\mathrm{B}(a, b)^3\,/;$$
$$w == I_z^{-1}(a, b)$$

06.23.20.0008.01

$$\frac{\partial^4 I_z^{-1}(a,b)}{\partial z^4} == (1-w)^{1-4b}\, w^{1-4a}\left(-6a^3+11a^2-6a+(a+b-2)(2a+2b-3)(3a+3b-4)w^3-\right.$$
$$\left.3(a-1)(2a+2b-3)(3a+3b-4)w^2+(a-1)(-11b+3a(6a+6b-11)+14)w+1\right)\mathrm{B}(a,b)^4 /; w == I_z^{-1}(a,b)$$

06.23.20.0009.01

$$\frac{\partial^5 I_z^{-1}(a,b)}{\partial z^5} == (1-w)^{1-5b}\, w^{1-5a}\left(24a^4-50a^3+35a^2-10a+\right.$$
$$(a+b-2)(2a+2b-3)(3a+3b-4)(4a+4b-5)w^4-4(a-1)(2a+2b-3)(3a+3b-4)(4a+4b-5)w^3+$$
$$2(a-1)\left(72a^3+6(24b-37)a^2+(b(72b-271)+226)a+b(121-49b)-75\right)w^2-$$
$$\left.2(a-1)(2a-1)(-13b+2a(12a+12b-19)+15)w+1\right)\mathrm{B}(a,b)^5 /; w == I_z^{-1}(a,b)$$

06.23.20.0010.01

$$\frac{\partial^6 I_z^{-1}(a,b)}{\partial z^6} ==$$
$$(1-w)^{1-6b}\, w^{1-6a}\left((a+b-2)(2a+2b-3)(3a+3b-4)(4a+4b-5)(5a+5b-6)w^5-5(a-1)(2a+2b-3)\right.$$
$$(3a+3b-4)(4a+4b-5)(5a+5b-6)w^4+2(a-1)\left((600a-437)b^3+(12a(150a-287)+1585)b^2+\right.$$
$$\left.a(3a(600a-1859)+5717)b-1923b+a(2a(5a(60a-257)+2066)-2943)+780\right)w^3-2(a-1)$$
$$\left.\left(\left(600a^2-711a+212\right)b^2+(a(a(1200a-2681)+1973)-477)b+2(4a-3)(5a-3)(a(15a-29)+15)\right)w^2+\right.$$
$$(a(a(a(10a(60a-197)+2547)-1622)+507)+3(a-1)(2a-1)(a(100a-87)+19)b)w-$$
$$\left.62w-a(a(a(2a(60a-137)+225)-85)+15)+1\right)\mathrm{B}(a,b)^6 /; w == I_z^{-1}(a,b)$$

06.23.20.0011.01

$$\frac{\partial^7 I_z^{-1}(a,b)}{\partial z^7} == (1-w)^{1-7b}\, w^{1-7a}\left((a+b-2)(2a+2b-3)(3a+3b-4)(4a+4b-5)(5a+5b-6)(6a+6b-7)w^6-\right.$$
$$6(a-1)(2a+2b-3)(3a+3b-4)(4a+4b-5)(5a+5b-6)(6a+6b-7)w^5+$$
$$3(a-1)\left(12(300a-229)b^4+24(3a(200a-387)+545)b^3+(4a(72a(75a-233)+17267)-23417)b^2+\right.$$
$$8a(a(3a(600a-2567)+12362)-8798)b+a(a(4a(45a(20a-109)+10727)-46967)+25680)+$$
$$40(467b-140)\Big)w^4-4(a-1)\left(2(18a(100a-129)+755)b^3+\right.$$
$$3(6a-5)\left(600a^2-906a+335\right)b^2+2a(a(18a(300a-1019)+23447)-13288)b+$$
$$\left.a(a(4a(45a(20a-89)+7196)-25961)+11700)+30(187b-70)\right)w^3+$$
$$3(a-1)(2a-1)\left(2(36a(25a-27)+269)b^2+a(18a(200a-403)+4963)b-1135b+\right.$$
$$\left.a(a(90a(20a-59)+6049)-3109)+602\right)w^2-$$
$$6(a(a(a(4a(9a(20a-69)+880)-2631)+1091)-237)+4(a-1)(2a-1)(3a-1)(6a(5a-4)+5)b)w-$$
$$\left.126w+a(a(a(4a(9a(20a-49)+406)-735)+175)-21)+1\right)\mathrm{B}(a,b)^7 /; w == I_z^{-1}(a,b)$$

06.23.20.0012.01

$$\frac{\partial^8 I_z^{-1}(a,\,b)}{\partial z^8} == (1-w)^{1-8\,b}\,w^{1-8\,a}\left(5040\,a^7\,(w-1)^7+36\,a^6\,((980\,b-1343)\,w+363)\,(w-1)^6+\right.$$
$$4\,a^5\,(w\,(49\,345\,w+9\,b\,(6\,(490\,b-1343)\,w+2795)-31\,721)+3283)\,(w-1)^5+$$
$$a^4\left(w\left((20\,b\,(9\,b\,(980\,b-4029)+49\,345)-444\,849)\,w^2+\right.\right.$$
$$\left.(4\,b\,(76\,770\,b-193\,853)+488\,639)\,w+113\,752\,b-134\,059\right)+6769\Big)\,(w-1)^4+$$
$$a^3\left(w\left(4\,(b\,(10\,b\,(18\,b\,(245\,b-1343)+49\,345)-444\,849)+149\,529)\,w^3+(b\,(24\,b\,(20\,145\,b-76\,367)+2\,311\,825)-\right.\right.$$
$$\left.970\,337)\,w^2+12\,(b\,(28\,841\,b-67\,855)+39\,994)\,w+66\,369\,b-74\,209\right)+1960\Big)\,(w-1)^3+$$
$$a^2\left(w\left(2\,(b\,(b\,(10\,b\,(27\,b\,(196\,b-1343)+98\,690)-1\,334\,547)+897\,174)-239\,998)\,w^4+\right.\right.$$
$$(b\,(b\,(4\,b\,(104\,535\,b-528\,698)+4\,003\,641)-3\,362\,801)+1\,057\,422)\,w^3+$$
$$(b\,(b\,(487\,084\,b-1\,717\,671)+2\,023\,805)-796\,438)\,w^2+$$
$$\left.7\,(b\,(27\,366\,b-60\,607)+33\,701)\,w+21\,289\,b-22\,899\right)+322\Big)\,(w-1)^2+$$
$$a\left(w\left(35\,280\,w^5\,b^6-36\,w^4\,(8058\,w-5263)\,b^5+4\,w^3\,(w\,(246\,725\,w-299\,597)+81\,310)\,b^4+\right.\right.$$
$$w^2\,(w\,((3\,026\,363-1\,779\,396\,w)\,w-1\,528\,798)+215\,462)\,b^3+$$
$$w\,(w\,(w\,(w\,(1\,794\,348\,w-3\,814\,591)+2\,701\,490)-712\,350)+52\,392)\,b^2-$$
$$(2\,w-1)\,(44\,(w-1)\,w\,(10\,909\,(w-1)\,w+2343)+3579)\,b+58\,278\,w+$$
$$\left.24\,w^2\,(w\,(3\,w\,(2958\,w-8375)+26\,194)-12\,206)-3747\right)+28\Big)\,(w-1)+$$
$$w\left((b-2)\,(2\,b-3)\,(3\,b-4)\,(4\,b-5)\,(5\,b-6)\,(6\,b-7)\,(7\,b-8)\,w^6+7\,(2\,b-3)\,(3\,b-4)\,(4\,b-5)\,(5\,b-6)\right.$$
$$(6\,b-7)\,(7\,b-8)\,w^5+3\,(b\,(b\,(b\,(12\,b\,(2323\,b-13\,648)+385\,871)-455\,570)+269\,432)-63\,840)\,w^4+$$
$$5\,(b\,(b\,(20\,b\,(835\,b-3671)+121\,767)-90\,240)+25\,200)\,w^3+(b\,(b\,(35\,458\,b-110\,691)+116\,022)-40\,824)$$
$$\left.w^2+3\,(b\,(1894\,b-3819)+1932)\,w+247\,b-254\right)+1\Big)\,\mathrm{B}(a,\,b)^8\;/;\;w==I_z^{-1}(a,\,b)$$

06.23.20.0013.01

$$\frac{\partial^9 I_z^{-1}(a,b)}{\partial z^9} == (1-w)^{1-9b}\, w^{1-9a} \left(40\,320\, a^8 (w-1)^8 + 144\, a^7 ((2240\, b - 3001)\, w + 761)(w-1)^7 + \right.$$
$$4\, a^6 (w (503\,543\, w + 36\, b (7 (1120\, b - 3001)\, w + 6806) - 304\,078) + 29\,531)(w-1)^6 +$$
$$4\, a^5 \left(w \left(3 (2\, b (42\, b (2240\, b - 9003) + 503\,543) - 444\,601)\, w^2 + \left(894\,780\, b^2 - 2\,223\,398\, b + 1\,379\,081\right) w + 304\,078\, b - \right.\right.$$
$$\left.\left. 354\,541\right) + 16\,821\right)(w-1)^5 + a^4 \left(w \left(3 (20\, b (b (84\, b (560\, b - 3001) + 503\,543) - 444\,601) + 2\,929\,523)\, w^3 + \right.\right.$$
$$4 (4\, b (5\, b (87\,876\, b - 327\,791) + 2\,034\,663) - 3\,362\,801)\, w^2 +$$
$$\left.\left. 4\, b (1\,149\,998\, b - 2\,677\,263)\, w + 6\,243\,754\, w + 812\,608\, b - 902\,404\right) + 22\,449\right)(w-1)^4 +$$
$$2\, a^3 \left(w \left(2 (b (10\, b (2\, b (63\, b (448\, b - 3001) + 503\,543) - 1\,333\,803) + 8\,788\,569) - 2\,305\,753)\, w^4 + \right.\right.$$
$$2 (b (2\, b (10\, b (101\,187\, b - 503\,543) + 9\,381\,899) - 15\,514\,171) + 4\,803\,163)\, w^3 +$$
$$2 (b (b (2\,149\,832\, b - 7\,502\,331) + 8\,744\,246) - 3\,403\,087)\, w^2 +$$
$$\left.\left. b (1\,550\,862\, b - 3\,414\,329)\, w + 1\,886\,147\, w + 159\,365\, b - 170\,705\right) + 2268\right)(w-1)^3 +$$
$$2\, a^2 \left(w \left(3 (b (b (2\, b (b (84\, b (1120\, b - 9003) + 2\,517\,715) - 4\,446\,010) + 8\,788\,569) - 4\,611\,506) + 1\,003\,962)\, w^5 + \right.\right.$$
$$2 (b (b (2\, b (b (686\,988\, b - 4\,275\,235) + 10\,625\,146) - 26\,365\,707) + 16\,331\,928) - 4\,040\,838)\, w^4 +$$
$$(b (b (2\, b (2\,147\,323\, b - 9\,988\,825) + 34\,922\,131) - 27\,180\,744) + 7\,945\,712)\, w^3 +$$
$$(b (8\, b (325\,283\, b - 1\,069\,553) + 9\,421\,073) - 3\,472\,373)\, w^2 +$$
$$\left.\left. (b (582\,067\, b - 1\,222\,192) + 644\,220)\, w + 36\,893\, b - 38\,531\right) + 273\right)(w-1)^2 +$$
$$2\, a \left(w \left(161\,280\, w^6 b^7 - 72\, w^5 (21\,007\, w - 14\,201)\, b^6 + 4\, w^4 (w (1\,510\,629\, w - 1\,909\,559) + 550\,969)\, b^5 - \right.\right.$$
$$2\, w^3 (3\, w (w (2\,223\,005\, w - 3\,956\,131) + 2\,135\,668) - 1\,004\,474)\, b^4 +$$
$$w^2 (w (2\, w (w (8\,788\,569\, w - 19\,640\,105) + 14\,933\,147) - 8\,786\,181) + 782\,324)\, b^3 +$$
$$w (w (w (14\,483\,203 - 6\, w (w (2\,305\,753\, w - 6\,084\,789) + 5\,811\,750)) - 2\,439\,444) + 115\,632)\, b^2 +$$
$$((w-1)\, w (4 (w-1)\, w (1\,505\,943 (w-1)\, w + 579\,505) + 233\,349) + 4679)\, b +$$
$$\left.\left. 3\, w (w (8\, w (w (3 (51\,693 - 15\,551\, w)\, w - 198\,974) + 123\,103) - 298\,293) + 39\,365) - 4805\right) + 18\right)(w-1) +$$
$$w \left((b-2)(2\, b-3)(3\, b-4)(4\, b-5)(5\, b-6)(6\, b-7)(7\, b-8)(8\, b-9)\, w^7 + \right.$$
$$8 (2\, b-3)(3\, b-4)(4\, b-5)(5\, b-6)(6\, b-7)(7\, b-8)(8\, b-9)\, w^6 +$$
$$4 (b (b (b (b (36\, b (6361\, b - 44\,383) + 4\,655\,389) - 7\,247\,965) + 6\,357\,811) - 2\,978\,556) + 582\,120)\, w^5 +$$
$$2 (b (b (b (4\, b (146\,221\, b - 798\,221) + 7\,009\,479) - 7\,731\,435) + 4\,282\,488) - 952\,560)\, w^4 +$$
$$2 (b (b (b (350\,582\, b - 1\,449\,159) + 2\,262\,903) - 1\,581\,351) + 417\,060)\, w^3 +$$
$$\left. 4 (b (b (46\,822\, b - 139\,287) + 139\,071) - 46\,620)\, w^2 + 2 (b (9511\, b - 18\,572) + 9075)\, w + 502\, b - 510\right) + 1\Big) \mathrm{B}(a,b)^9$$

06.23.20.0014.01

$$\frac{\partial^{10} I_z^{-1}(a,\,b)}{\partial z^{10}} = (1-w)^{1-10\,b}\,w^{1-10\,a}\left(362\,880\,a^9\,(w-1)^9 + 144\,a^8\,((22\,680\,b-29\,809)\,w+7129)\,(w-1)^8 + \right.$$
$$36\,a^7\,(w\,(9\,(69\,287\,w-39\,498)+4\,b\,(8\,(11\,340\,b-29\,809)\,w+72\,583))+32\,575)\,(w-1)^7 +$$
$$4\,a^6\left(w\left((63\,b\,(16\,b\,(7560\,b-29\,809)+623\,583)-17\,039\,657)\,w^2 + \right.\right.$$
$$\left.12\,(9\,b\,(102\,823\,b-252\,048)+1\,388\,255)\,w+3\,492\,513\,b-4\,035\,273\right)+180\,920\Big)\,(w-1)^6 +$$
$$a^5\left(w\left(3\,(4\,b\,(21\,b\,(32\,b\,(5670\,b-29\,809)+1\,870\,749)-34\,079\,314)+44\,149\,747)\,w^3 + \right.\right.$$
$$12\,(b\,(6\,b\,(1\,451\,590\,b-5\,341\,167)+39\,251\,737)-16\,004\,050)\,w^2 +$$
$$\left.8\,b\,(7\,885\,512\,b-18\,194\,527)\,w+84\,088\,070\,w+10\,351\,652\,b-11\,428\,952\right)+269\,325\Big)\,(w-1)^5 +$$
$$a^4\left(w\left(3\,(5\,b\,(4\,b\,(21\,b\,(8\,b\,(4536\,b-29\,809)+623\,583)-17\,039\,657)+44\,149\,747)-56\,964\,266)\,w^4 + \right.\right.$$
$$3\,(4\,b\,(5\,b\,(12\,b\,(208\,663\,b-1\,024\,245)+22\,592\,677)-92\,161\,897)+112\,639\,183)\,w^3 +$$
$$10\,(b\,(4\,b\,(3\,672\,855\,b-12\,703\,961)+58\,687\,749)-22\,626\,598)\,w^2 +$$
$$\left.2\,(b\,(24\,514\,482\,b-53\,697\,485)+29\,499\,368)\,w+4\,695\,827\,b-5\,012\,192\right)+63\,273\Big)\,(w-1)^4+2\,a^3$$
$$\left(w\left((b\,(b\,(2\,b\,(63\,b\,(32\,b\,(3780\,b-29\,809)+3\,117\,915)-340\,793\,140)+662\,246\,205)-341\,785\,596)+73\,222\,034)\right.\right.$$
$$w^5+6\,(2\,b\,(b\,(b\,(9\,b\,(629\,006\,b-3\,860\,985)+85\,198\,285)-104\,303\,544)+63\,762\,462)-31\,143\,509)$$
$$w^4+(b\,(b\,(90\,b\,(1\,084\,291\,b-4\,999\,408)+779\,370\,883)-600\,931\,908)+173\,992\,430)\,w^3 +$$
$$(b\,(b\,(54\,576\,790\,b-178\,611\,459)+195\,619\,251)-71\,679\,082)\,w^2 +$$
$$\left.9\,(2\,b\,(629\,193\,b-1\,318\,625)+1\,386\,739)\,w+671\,179\,b-699\,529\right)+4725\Big)\,(w-1)^3 +$$
$$2\,a^2\left(w\left(6\,531\,840\,w^6\,b^7-504\,w^5\,(119\,236\,w-75\,169)\,b^6+162\,w^4\,(3\,w\,(485\,009\,w-574\,376)+462\,805)\,b^5 + \right.\right.$$
$$2\,w^3\left(31\,487\,069-5\,w\left(51\,118\,971\,w^2-85\,578\,882\,w+43\,208\,773\right)\right)b^4 +$$
$$w^2\,(w\,(w\,(3\,w\,(220\,748\,735\,w-465\,780\,764)+997\,920\,529)-274\,187\,543)+22\,688\,575)\,b^3 -$$
$$w\,(2\,w-1)\,((w-1)\,w\,(256\,339\,197\,(w-1)\,w+64\,434\,970)+3\,118\,926)\,b^2 +$$
$$(w\,(w\,(2\,w\,(3\,w\,(w\,(36\,611\,017\,w-104\,365\,543)+111\,407\,420)-164\,520\,448)+73\,777\,745)-6\,297\,961)+$$
$$118\,354)\,b+w\,(w\,(w\,(90\,625\,419-2\,w\,(9\,w\,(2\,233\,106\,w-7\,073\,075)+77\,517\,068))-25\,808\,723)+$$
$$\left.3\,188\,170)-121\,399\right)+435\Big)\,(w-1)^2 +$$
$$a\left(w\left(3\,265\,920\,w^7\,b^8-144\,w^6\,(238\,472\,w-165\,889)\,b^7+36\,w^5\,(9\,w\,(485\,009\,w-633\,954)+1\,728\,562)\,b^6 + \right.\right.$$
$$4\,w^4\,(w\,(3\,(62\,986\,205-34\,079\,314\,w)\,w-107\,592\,458)+18\,283\,872)\,b^5 +$$
$$w^3\,(w\,(3\,w\,(w\,(220\,748\,735\,w-514\,347\,352)+414\,175\,476)-397\,516\,538)+40\,481\,788)\,b^4 -$$
$$2\,w^2\,(3\,w\,(w\,(4\,w\,(w\,(28\,482\,133\,w-78\,648\,203)+79\,849\,141)-144\,711\,723)+27\,874\,092)-5\,012\,780)$$
$$b^3+2\,w\,(w\,(w\,(2\,w\,(3\,w\,(w\,(36\,611\,017\,w-115\,300\,559)+138\,744\,960)-238\,063\,322)+130\,381\,127)-$$
$$14\,943\,243)+474\,606)\,b^2-2\,(2\,w-1)$$
$$(2\,(w-1)\,w\,(2\,(w-1)\,w\,(10\,048\,977\,(w-1)\,w+3\,055\,390)+452\,799)+11\,779)\,b+910\,360\,w+6\,w^2$$
$$\left.(2\,w\,(20\,w\,(w\,(3\,w\,(35\,638\,w-136\,073)+622\,787)-482\,555)+3\,982\,529)-1\,669\,717)-23\,918\right)+45\Big)$$
$$(w-1)+w\left((b-2)\,(2\,b-3)\,(3\,b-4)\,(4\,b-5)\,(5\,b-6)\,(6\,b-7)\,(7\,b-8)\,(8\,b-9)\,(9\,b-10)\,w^8 + \right.$$
$$9\,(2\,b-3)\,(3\,b-4)\,(4\,b-5)\,(5\,b-6)\,(6\,b-7)\,(7\,b-8)\,(8\,b-9)\,(9\,b-10)\,w^7 +$$
$$4\,(b\,(b\,(b\,(b\,(b\,(36\,(75\,169\,b-606\,559)+75\,659\,429)-145\,880\,941)+169\,008\,835)-117\,628\,052)+$$
$$45\,530\,760)-7\,560\,000)\,w^6 +$$
$$2\left(b\left(b\left(b\left(b\left(8\,469\,900\,b^2-55\,199\,176\,b+150\,610\,945\right)-220\,120\,778\right)+181\,671\,791\right)-80\,250\,240\right)+$$
$$14\,817\,600\Big)\,w^5 +$$
$$6\,(b\,(b\,(b\,(b\,(2\,221\,514\,b-11\,434\,409)+23\,705\,985)-24\,732\,729)+12\,978\,858)-2\,739\,240)\,w^4 +$$
$$2\,(b\,(b\,(b\,(2\,595\,706\,b-10\,221\,181)+15\,210\,663)-10\,136\,625)+2\,551\,500)\,w^3 +$$
$$2\,(b\,(b\,(460\,175\,b-1\,319\,488)+1\,268\,531)-409\,260)\,w^2 +$$
$$\left.12\left(5132\,b^2-9794\,b+4665\right)w+1013\,b-1022\right)+1\Big)\,\mathrm{B}(a,\,b)^{10}\,/;\,w=I_z^{-1}(a,\,b)$$

With respect to *a*

06.23.20.0003.01

$$\frac{\partial I_z^{-1}(a,\,b)}{\partial a} == (1-w)^{1-b}\,w^{1-a}\left(w^a\,\Gamma(a)^2\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w) - \mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\log(w)-\psi(a)+\psi(a+b))\right)/;$$
$$w == I_z^{-1}(a,\,b)$$

06.23.20.0004.01

$$\frac{\partial^2 I_z^{-1}(a,\,b)}{\partial a^2} == (1-w)^{1-2\,b}\,w\left((b-1)\,\Gamma(a)^2\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w)\right.$$
$$\left(w^a\,\Gamma(a)^2\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w) - \mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\log(w)-\psi(a)+\psi(a+b))\right)w^{1-a} +$$
$$\mathrm{B}(a,\,b)\,I_w(a,\,b)\left((w-1)\,\Gamma(a)^2\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w)\,w^a + (1-w)^b\left(\psi^{(1)}(a)-\psi^{(1)}(a+b)\right)w^a -\right.$$
$$\left.(w-1)\,\mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\log(w)-\psi(a)+\psi(a+b))\right)w^{-2\,a} -$$
$$(1-w)^b\,\mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\psi(a)-\psi(a+b))\,(\log(w)-\psi(a)+\psi(a+b))\,w^{-a} +$$
$$(w-1)\,\Gamma(a)^2\left(a^2\,(b-1)\,w\,{}_3\tilde{F}_2(a+1,\,a+1,\,2-b;\,a+2,\,a+2;\,w) - {}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w)\right)$$
$$\left(w^a\,\Gamma(a)^2\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w) - \mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\log(w)-\psi(a)+\psi(a+b))\right)w^{-a} -$$
$$(1-w)^b\,\mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\log(w)-\psi(a)+\psi(a+b))$$
$$\left((1-a)\,(1-w)^{1-b}\,w^{-a}\left(w^a\,\Gamma(a)^2\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w) - \mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\log(w)-\psi(a)+\psi(a+b))\right) -\right.$$
$$\left.\log(w)\right)w^{-a} + (1-b)\,\mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\log(w)-\psi(a)+\psi(a+b))$$
$$\left(w^a\,\Gamma(a)^2\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w) - \mathrm{B}(a,\,b)\,I_w(a,\,b)\,(\log(w)-\psi(a)+\psi(a+b))\right)w^{1-2\,a} -$$
$$\left.2\,(1-w)^b\,\Gamma(a)^3\,{}_4\tilde{F}_3(a,\,a,\,a,\,1-b;\,a+1,\,a+1,\,a+1;\,w)\right)/;\; w == I_z^{-1}(a,\,b)$$

With respect to *b*

06.23.20.0005.01

$$\frac{\partial I_z^{-1}(a,\,b)}{\partial b} == (1-w)^{-b}\,(w-1)\,w^{1-a}$$
$$\left((1-w)^b\,\Gamma(b)^2\,{}_3\tilde{F}_2(b,\,b,\,1-a;\,b+1,\,b+1;\,1-w) - \mathrm{B}_{1-w}(b,\,a)\,(\log(1-w)-\psi(b)+\psi(a+b))\right)/;\; w == I_z^{-1}(a,\,b)$$

06.23.20.0006.01

$$\frac{\partial^2 I_z^{-1}(a,\,b)}{\partial b^2} == w^{1-2\,a}\left(-2\,(w-1)\,\Gamma(b)^3\,{}_4\tilde{F}_3(b,\,b,\,b,\,1-a;\,b+1,\,b+1,\,b+1;\,1-w)\,w^a + (1-w)^{1-b}\,\mathrm{B}_{1-w}(b,\,a)\right.$$
$$(\psi(b)-\psi(a+b))\,(\log(1-w)-\psi(b)+\psi(a+b))\,w^a - (1-w)^{2-b}\,\Gamma(b)^2\,{}_3\tilde{F}_2(b,\,b,\,1-a;\,b+1,\,b+1;\,1-w)$$
$$\left((1-w)^b\,\Gamma(b)^2\,{}_3\tilde{F}_2(b,\,b,\,1-a;\,b+1,\,b+1;\,1-w) - \mathrm{B}_{1-w}(b,\,a)\,(\log(1-w)-\psi(b)+\psi(a+b))\right)w +$$
$$(1-a)\,(1-w)^{2-b}\,\Gamma(b)^2\,{}_3\tilde{F}_2(b,\,b,\,1-a;\,b+1,\,b+1;\,1-w)$$
$$\left((1-w)^b\,\Gamma(b)^2\,{}_3\tilde{F}_2(b,\,b,\,1-a;\,b+1,\,b+1;\,1-w) - \mathrm{B}_{1-w}(b,\,a)\,(\log(1-w)-\psi(b)+\psi(a+b))\right) +$$
$$(a-1)\,(1-w)^{2-2\,b}\,\mathrm{B}_{1-w}(b,\,a)\,(\log(1-w)-\psi(b)+\psi(a+b))$$
$$\left((1-w)^b\,\Gamma(b)^2\,{}_3\tilde{F}_2(b,\,b,\,1-a;\,b+1,\,b+1;\,1-w) - \mathrm{B}_{1-w}(b,\,a)\,(\log(1-w)-\psi(b)+\psi(a+b))\right) +$$
$$(1-w)^{-2\,b}\,(w-1)\,\mathrm{B}_{1-w}(b,\,a)\,(\log(1-w)-\psi(b)+\psi(a+b))\left((1-w)^b\log(1-w)\,w^a + (b-1)\,(1-w)^b\right.$$
$$\left.\Gamma(b)^2\,{}_3\tilde{F}_2(b,\,b,\,1-a;\,b+1,\,b+1;\,1-w)\,w - (b-1)\,\mathrm{B}_{1-w}(b,\,a)\,(\log(1-w)-\psi(b)+\psi(a+b))\,w\right) +$$
$$(1-w)^{-2\,b}\,(w-1)\,\mathrm{B}_{1-w}(b,\,a)\left((1-w)^b\left(\psi^{(1)}(b)-\psi^{(1)}(a+b)\right)w^a - (1-w)^b\,\Gamma(b)^2\right.$$
$$\left.\left.{}_3\tilde{F}_2(b,\,b,\,1-a;\,b+1,\,b+1;\,1-w)\,w + \mathrm{B}_{1-w}(b,\,a)\,(\log(1-w)-\psi(b)+\psi(a+b))\,w\right)\right)/;\; w == I_z^{-1}(a,\,b)$$

Integration

Indefinite integration

Involving only one direct function

06.23.21.0001.01

$$\int I_z^{-1}(a,\, b)\, dz == \frac{1}{(a+1)\,\mathrm{B}(a,\, b)}\, {}_2F_1\left(a+1,\, 1-b;\, a+2;\, I_z^{-1}(a,\, b)\right) I_z^{-1}(a,\, b)^{a+1}$$

Representations through equivalent functions

With inverse function

06.23.27.0001.01

$$I_{I_z^{-1}(a,b)}(a,\, b) == z$$

06.23.27.0002.01

$$\mathrm{B}_{I_z^{-1}(a,b)}(a,\, b) == \mathrm{B}(a,\, b)\, z$$

06.23.27.0003.01

$$I^{-1}_{I_{z_1}(a,b)+z_2}(a,\, b) == I^{-1}_{(z_1,z_2)}(a,\, b)$$

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