

InverseBetaRegularized4

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Notations

Traditional name

Inverse of the generalized regularized incomplete beta function

Traditional notation

$$I_{(z_1, z_2)}^{-1}(a, b)$$

Mathematica StandardForm notation

`InverseBetaRegularized[z1, z2, a, b]`

Primary definition

06.24.02.0001.01

$$z_2 = I_{(z_1, w)}(a, b) /; w = I_{(z_1, z_2)}^{-1}(a, b)$$

Specific values

Specialized values

06.24.03.0001.01

$$I_{(0, z_2)}^{-1}(a, b) = I_{z_2}^{-1}(a, b)$$

06.24.03.0002.01

$$I_{(z_1, 0)}^{-1}(a, b) = z_1$$

General characteristics

Domain and analyticity

$I_{(z_1, z_2)}^{-1}(a, b)$ is an analytical function of z_1, z_2, a, b which is defined in \mathbb{C}^4 .

06.24.04.0001.01

$$(z_1 * z_2 * a * b) \rightarrow I_{(z_1, z_2)}^{-1}(a, b) :: (\mathbb{C}^4) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Differential equations

Ordinary nonlinear differential equations

06.24.13.0001.01

$$w(z_2) (1 - w(z_2)) w''(z_2) - (1 - a + (a + b - 2) w(z_2)) w'(z_2)^2 = 0 /; w(z_2) = I_{(z_1, z_2)}^{-1}(a, b)$$

Differentiation

Low-order differentiation

With respect to z_1

06.24.20.0001.01

$$\frac{\partial I_{(z_1, z_2)}^{-1}(a, b)}{\partial z_1} = (1 - w)^{1-b} w^{1-a} (1 - z_1)^{b-1} z_1^{a-1} /; w = I_{(z_1, z_2)}^{-1}(a, b)$$

06.24.20.0002.01

$$\frac{\partial^2 I_{(z_1, z_2)}^{-1}(a, b)}{\partial z_1^2} = (1 - w)^{1-2b} w^{1-2a} (1 - z_1)^{b-2} z_1^{a-2} ((a - 1)(1 - w)^b w^a - (a + b - 2)(1 - w)^b z_1 w^a + (a(w - 1) + (b - 2)w + 1)(1 - z_1)^b z_1^a) /; w = I_{(z_1, z_2)}^{-1}(a, b)$$

With respect to z_2

06.24.20.0003.01

$$\frac{\partial I_{(z_1, z_2)}^{-1}(a, b)}{\partial z_2} = (1 - w)^{1-b} w^{1-a} B(a, b) /; w = I_{(z_1, z_2)}^{-1}(a, b)$$

06.24.20.0004.01

$$\frac{\partial^2 I_{(z_1, z_2)}^{-1}(a, b)}{\partial z_2^2} = (1 - w)^{1-2b} w^{1-2a} ((w - 1)a + (b - 2)w + 1) B(a, b)^2 /; w = I_{(z_1, z_2)}^{-1}(a, b)$$

With respect to a

06.24.20.0005.01

$$\frac{\partial I_{(z_1, z_2)}^{-1}(a, b)}{\partial a} = (1 - w)^{1-b} w^{1-a} (\Gamma(a)^2 w^a {}_3\tilde{F}_2(a, a, 1 - b; a + 1, a + 1; w) - \Gamma(a)^2 z_1^a {}_3\tilde{F}_2(a, a, 1 - b; a + 1, a + 1; z_1) + B(a, b) (\log(z_1) I_{z_1}(a, b) - \log(w) I_w(a, b) + I_{(z_1, w)}(a, b) (\psi(a) - \psi(a + b)))) /; w = I_{(z_1, z_2)}^{-1}(a, b)$$

With respect to b

06.24.20.0006.01

$$\frac{\partial I_{(z_1, z_2)}^{-1}(a, b)}{\partial b} = B(a, b) (1-w)^{1-b} w^{1-a} \left(\frac{1}{b^2 B(a, b)} \left({}_3F_2(1-a, b, b; b+1, b+1; 1-z_1) (1-z_1)^b - {}_3F_2(1-a, b, b; b+1, b+1; 1-w) (1-w)^b \right) + (\psi(b) - \psi(a+b)) z_2 - \log(1-w) (I_{z_1}(a, b) + z_2 - 1) - I_{1-z_1}(b, a) \log(1-z_1) \right) /; w = I_{(z_1, z_2)}^{-1}(a, b)$$

Integration

Indefinite integration

Involving one direct function with respect to z_2

06.24.21.0001.01

$$\int I_{(z_1, z_2)}^{-1}(a, b) dz_2 = \frac{1}{(a+1) B(a, b)} {}_2F_1(a+1, 1-b; a+2; I_{(z_1, z_2)}^{-1}(a, b)) I_{(z_1, z_2)}^{-1}(a, b)^{a+1}$$

Representations through equivalent functions

With inverse function

06.24.27.0001.01

$$I_{(z_1, I_{(z_1, z_2)}^{-1}(a, b))}^{-1}(a, b) = z_2$$

06.24.27.0002.01

$$B_{(z_1, I_{(z_1, z_2)}^{-1}(a, b))}^{-1}(a, b) = B(a, b) z_2$$

With related functions

06.24.27.0003.01

$$I_{(z_1, z_2)}^{-1}(a, b) = I_{z_1(a, b) + z_2}^{-1}(a, b)$$

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