

InverseErfc

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Notations

Traditional name

Inverse complementary error function

Traditional notation

$$\operatorname{erfc}^{-1}(z)$$

Mathematica StandardForm notation

`InverseErfc[z]`

Primary definition

06.31.02.0001.01
$$z = \operatorname{erfc}(w) /; w = \operatorname{erfc}^{-1}(z)$$

Specific values

Values at fixed points

06.31.03.0001.01
$$\operatorname{erfc}^{-1}(0) = \infty$$

06.31.03.0002.01
$$\operatorname{erfc}^{-1}(1) = 0$$

General characteristics

Domain and analyticity

$\operatorname{erfc}^{-1}(z)$ is an analytical function of z which is defined in the whole complex z -plane.

06.31.04.0001.01
$$z \rightarrow \operatorname{erfc}^{-1}(z) : \mathbb{C} \rightarrow \mathbb{C}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

06.31.06.0003.01

$$\begin{aligned} \operatorname{erfc}^{-1}(z) \propto \operatorname{erfc}^{-1}(z_0) - \frac{\sqrt{\pi}}{2} e^{\operatorname{erfc}^{-1}(z_0)^2} (z - z_0) + \frac{\pi}{4} e^{2\operatorname{erfc}^{-1}(z_0)^2} \operatorname{erfc}^{-1}(z_0) (z - z_0)^2 - \frac{\pi^{3/2}}{24} e^{3\operatorname{erfc}^{-1}(z_0)^2} \left(4\operatorname{erfc}^{-1}(z_0)^2 + 1\right) (z - z_0)^3 + \\ \frac{\pi^2}{96} e^{4\operatorname{erfc}^{-1}(z_0)^2} \operatorname{erfc}^{-1}(z_0) \left(12\operatorname{erfc}^{-1}(z_0)^2 + 7\right) (z - z_0)^4 - \frac{\pi^{5/2}}{960} e^{5\operatorname{erfc}^{-1}(z_0)^2} \left(8\operatorname{erfc}^{-1}(z_0)^2 + 7\right) \left(12\operatorname{erfc}^{-1}(z_0)^2 + 1\right) (z - z_0)^5 + \\ \frac{\pi^3}{5760} e^{6\operatorname{erfc}^{-1}(z_0)^2} \operatorname{erfc}^{-1}(z_0) \left(480\operatorname{erfc}^{-1}(z_0)^4 + 652\operatorname{erfc}^{-1}(z_0)^2 + 127\right) (z - z_0)^6 - \\ \frac{\pi^{7/2}}{80640} e^{7\operatorname{erfc}^{-1}(z_0)^2} \left(5760\operatorname{erfc}^{-1}(z_0)^6 + 10224\operatorname{erfc}^{-1}(z_0)^4 + 3480\operatorname{erfc}^{-1}(z_0)^2 + 127\right) (z - z_0)^7 + \\ \frac{\pi^4}{645120} e^{8\operatorname{erfc}^{-1}(z_0)^2} \operatorname{erfc}^{-1}(z_0) \left(40320\operatorname{erfc}^{-1}(z_0)^6 + 88848\operatorname{erfc}^{-1}(z_0)^4 + 44808\operatorname{erfc}^{-1}(z_0)^2 + 4369\right) (z - z_0)^8 - \\ \frac{\pi^{9/2}}{11612160} e^{9\operatorname{erfc}^{-1}(z_0)^2} \left(645120\operatorname{erfc}^{-1}(z_0)^8 + 1703808\operatorname{erfc}^{-1}(z_0)^6 + 1161168\operatorname{erfc}^{-1}(z_0)^4 + 204328\operatorname{erfc}^{-1}(z_0)^2 + 4369\right) (z - z_0)^9 + \\ \frac{\pi^5}{116121600} e^{10\operatorname{erfc}^{-1}(z_0)^2} \operatorname{erfc}^{-1}(z_0) \left(5806080\operatorname{erfc}^{-1}(z_0)^8 + 17914752\operatorname{erfc}^{-1}(z_0)^6 + 15561936\operatorname{erfc}^{-1}(z_0)^4 + 4161288\operatorname{erfc}^{-1}(z_0)^2 + 243649\right) (z - z_0)^{10} + \dots /; (z \rightarrow z_0) \end{aligned}$$

06.31.06.0004.01

$$\operatorname{erfc}^{-1}(z) \propto \operatorname{erfc}^{-1}(z_0) (1 + O(z - z_0))$$

Expansions at $z = 1$

06.31.06.0001.02

$$\operatorname{erfc}^{-1}(z) = -\frac{\sqrt{\pi}}{2} \left((z - 1) + \frac{\pi}{12} (z - 1)^3 + \frac{7\pi^2}{480} (z - 1)^5 + \dots \right) /; (z \rightarrow 1)$$

06.31.06.0005.01

$$\operatorname{erfc}^{-1}(z) = -\frac{\sqrt{\pi}}{2} \left((z - 1) + \frac{\pi}{12} (z - 1)^3 + \frac{7\pi^2}{480} (z - 1)^5 \right) + O((z - 1)^7)$$

06.31.06.0006.01

$$\operatorname{erfc}^{-1}(z) \propto -\frac{1}{2} \sqrt{\pi} (z - 1) \left(1 + \frac{\pi}{12} (z - 1)^2 + \frac{7\pi^2}{480} (z - 1)^4 + \frac{127\pi^3}{40320} (z - 1)^6 + \frac{4369\pi^4}{5806080} (z - 1)^8 \right) + O((z - 1)^{11})$$

06.31.06.0002.01

$$\operatorname{erfc}^{-1}(z) = -\sum_{k=0}^{\infty} \frac{c_k}{2k+1} \left(\frac{\sqrt{\pi}}{2} (z-1) \right)^{2k+1} /; c_0 = 1 \wedge c_k = \sum_{m=0}^{k-1} \frac{c_m c_{k-1-m}}{(m+1)(2m+1)}$$

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06.31.06.0007.01

$$\operatorname{erfc}^{-1}(z) = -\frac{\sqrt{\pi}}{2} (z-1) + O((z-1)^3)$$

Asymptotic series expansions

06.31.06.0008.01

$$\operatorname{erfc}^{-1}(z) \propto \frac{1}{\sqrt{2}} \sqrt{\log\left(\frac{2}{\pi z^2}\right) - \log\left(\log\left(\frac{2}{\pi z^2}\right)\right)} /; (z \rightarrow 0)$$

Differential equations

Ordinary nonlinear differential equations

06.31.13.0001.01

$$w''(z) - 2w(z)w'(z)^2 = 0 /; w(z) = \operatorname{erfc}^{-1}(z)$$

Differentiation

Low-order differentiation

06.31.20.0001.01

$$\frac{\partial \operatorname{erfc}^{-1}(z)}{\partial z} = -\frac{\sqrt{\pi}}{2} e^{\operatorname{erfc}^{-1}(z)^2}$$

06.31.20.0002.01

$$\frac{\partial^2 \operatorname{erfc}^{-1}(z)}{\partial z^2} = -\frac{\pi}{2} e^{2\operatorname{erfc}^{-1}(z)^2} \operatorname{erfc}^{-1}(z)$$

Symbolic differentiation

06.31.20.0003.01

$$\begin{aligned} \frac{\partial^n \operatorname{erfc}^{-1}(z)}{\partial z^n} &= \operatorname{erfc}^{-1}(z) \delta_n - \frac{\pi^{n/2}}{2^n} e^{n \operatorname{erfc}^{-1}(z)^2} \sum_{j_2=0}^n \dots \sum_{j_n=0}^n \delta_{\sum_{i=2}^n (i-1) j_i, n-1} (-1)^{\sum_{i=2}^n j_i} \left(n + \sum_{i=2}^n j_i - 1 \right)! \\ &\quad \prod_{i=2}^n \frac{1}{j_i!} \left(\frac{(-1)^{i-1} 2^{i-1} e^{\operatorname{erfc}^{-1}(z)^2} \sqrt{\pi} \operatorname{erfc}^{-1}(z)^{1-i}}{i!} \right)^{j_i} {}_2F_2\left(\frac{1}{2}, 1 - \frac{i}{2}, \frac{3-i}{2}; -\operatorname{erfc}^{-1}(z)^2\right)^{j_i} /; n \in \mathbb{N} \end{aligned}$$

Integration

Indefinite integration

Involving only one direct function

06.31.21.0001.01

$$\int \operatorname{erfc}^{-1}(z) dz = \frac{1}{\sqrt{\pi}} e^{-\operatorname{erfc}^{-1}(z)^2}$$

Representations through more general functions

Through other functions

06.31.26.0001.01

$$\operatorname{erfc}^{-1}(z) = \operatorname{erf}^{-1}(\infty, -z)$$

Representations through equivalent functions

With inverse function

06.31.27.0001.01

$$\operatorname{erfc}(\operatorname{erfc}^{-1}(z)) = z$$

06.31.27.0003.01

$$\operatorname{erfc}(\operatorname{erf}^{-1}(1-z)) = z$$

06.31.27.0004.01

$$\operatorname{erfc}(\operatorname{erf}^{-1}(\infty, -z)) = z$$

With related functions

06.31.27.0002.01

$$\operatorname{erfc}^{-1}(z) = \operatorname{erf}^{-1}(1-z)$$

Zeros

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$$\operatorname{erfc}^{-1}(0) = \infty$$

06.31.30.0002.01

$$\operatorname{erfc}^{-1}(z) = 0 \text{ /; } z = 1$$

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