# InverseGammaRegularized



## Notations

### Traditional name

Inverse of the regularized incomplete gamma function

### Traditional notation

$Q^{-1}(a, z)$

### *Mathematica* StandardForm notation

`InverseGammaRegularized[a, z]`

## Primary definition

06.12.02.0001.01

$z == Q(a, w) /; w == Q^{-1}(a, z)$

## Specific values

### Specialized values

06.12.03.0001.01

$Q^{-1}(a, 0) == \infty /; a > 0$

06.12.03.0002.01

$Q^{-1}(a, 1) == 0 /; a > 0$

## General characteristics

### Domain and analyticity

$Q^{-1}(a, z)$ is an analytical function of $a$ and $z$ which is defined in $\mathbb{C}^2$.

06.12.04.0001.01

$(a * z) \longrightarrow Q^{-1}(a, z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

### Symmetries and periodicities

**Symmetry**

No symmetry

**Periodicity**

No periodicity

# Series representations

## Generalized power series

**Expansions at generic point $a == a_0$**

### For the function itself

06.12.06.0003.01

$$Q^{-1}(a, z) \propto (a - a_0)^2 + Q^{-1}(a_0, z) + w^{1-a_0} e^w \left((z-1)\,\Gamma(a_0)\log(w) + (\Gamma(a_0) - \Gamma(a_0, w))\,\psi(a_0) + \frac{w^{a_0}}{a_0^2}\,{}_2F_2(a_0, a_0; a_0+1, a_0+1; -w)\right)(a - a_0) + \frac{1}{2}\Big(2 e^{2w}(w - a_0 + 1)\,\Gamma(a_0)^2 w^{1-a_0}(\Gamma(a_0)((z-1)\log(w) + \psi(a_0)) - \Gamma(a_0, w)\,\psi(a_0))\,{}_2\tilde{F}_2(a_0, a_0; a_0+1, a_0+1; -w) + w^{1-2a_0} e^w \Big(e^w (w - a_0 + 1)\,\Gamma(a_0)^2((z-1)\log(w) + \psi(a_0))^2 + \Gamma(a_0, w)\Big(\big(\log^2(w) - \psi^{(1)}(a_0)\big) w^{a_0} + (e^w (w - a_0 + 1)\,\Gamma(a_0, w) - w^{a_0})\,\psi(a_0)^2 + 2 e^w\,\Gamma(a_0)\,\psi(a_0)((z-1)\log(w) + \psi(a_0))\,a_0\Big) + \Gamma(a_0)\Big(\big((1-2z)\log^2(w) + \psi^{(1)}(a_0)\big) w^{a_0} + (w^{a_0} - 2 e^w (w+1)\,\Gamma(a_0, w))\,\psi(\mathrm{aa}_0)^2 + 2(z-1)(w^{a_0} - e^w (w+1)\,\Gamma(a_0, w))\log(w)\,\psi(a_0)\Big)\Big) + e^w\,\Gamma(a_0)^3 w \Big(e^w (w - a_0 + 1)\,\Gamma(a_0)\,{}_2\tilde{F}_2(a_0, a_0; a_0+1, a_0+1; -w)^2 - 2\,{}_3\tilde{F}_3(a_0, a_0, a_0; a_0+1, a_0+1, a_0+1; -w)\Big)\Big)(a - a_0)^2 + \ldots \;/;\; (a \to a_0) \bigwedge w == Q^{-1}(a_0, z)$$

06.12.06.0004.01

$$Q^{-1}(a, z) \propto Q^{-1}(a_0, z)\,(1 + O(a - a_0))$$

**Expansions at generic point $z == z_0$**

### For the function itself

06.12.06.0005.01

$$Q^{-1}(a, z) \propto Q^{-1}(a, z_0) - e^{w}\,\Gamma(a)\,w^{1-a}\,(z - z_0) + \frac{1}{2}\,e^{2w}\,(-a + w + 1)\,\Gamma(a)^2\;w^{1-2a}(z - z_0)^2 - \frac{1}{6}\,e^{3w}\left(2a^2 - (4w + 3)\,a + 2w\,(w + 2) + 1\right)\Gamma(a)^3\;w^{1-3a}(z - z_0)^3 + \frac{1}{24}\,e^{4w}\left(-6a^3 + (18w + 11)\,a^2 - (w\,(18w + 29) + 6)\,a + w\,(6w\,(w + 3) + 11) + 1\right)\Gamma(a)^4\;w^{1-4a}(z - z_0)^4 - \frac{1}{120}\,e^{5w}\left(24a^4 - 2\,(48w + 25)\,a^3 + (4w\,(36w + 49) + 35)\,a^2 - 2\,(w\,(w\,(48w + 121) + 63) + 5)\,a + 2w\,(w + 1)\,(12w\,(w + 3) + 13) + 1\right)\Gamma(a)^5\,w^{1-5a}(z - z_0)^5 + \frac{1}{720}\,e^{6w}\left(-120a^5 + (600w + 274)\,a^4 - 3\left(400w^2 + 474w + 75\right)a^3 + \left(3w\left(400w^2 + 874w + 399\right) + 85\right)a^2 - (2w\,(w\,(w\,(300w + 1037) + 923) + 216) + 15)\,a + w\,(2w\,(w\,(60w\,(w + 5) + 437) + 212) + 57) + 1\right)\Gamma(a)^6\,w^{1-6a}(z - z_0)^6 - \frac{1}{5040}\,e^{7w}\left(720w^6 - 4320\,(a - 1)\,w^5 + 36\,(a - 1)\,(300a - 229)\,w^4 - 8\,(a - 1)\,(18a\,(100a - 129) + 755)\,w^3 + 6\,(a - 1)\,(2a - 1)\,(36a\,(25a - 27) + 269)\,w^2 - 24a\,(a\,(6a\,(a\,(30a - 79) + 79) - 229) + 54)\,w + 120w + a\,(a\,(a\,(4a\,(9a\,(20a - 49) + 406) - 735) + 175) - 21) + 1\right)\Gamma(a)^7\,w^{1-7a}(z - z_0)^7 + \frac{1}{40\,320}\,e^{8w}\left(-5040a^7 + 13\,068a^6 - 13\,132a^5 + 6769a^4 - 1960a^3 + 322a^2 - 28a + 5040w^7 - 35\,280\,(a - 1)\,w^6 + 36\,(a - 1)\,(2940a - 2323)\,w^5 - 20\,(a - 1)\,(9a\,(980a - 1343) + 4175)\,w^4 + 2\,(a - 1)\,(2a\,(90a\,(490a - 853) + 45\,001) - 17\,729)\,w^3 - 6\,(a - 1)\,(2a - 1)\,(a\,(60a\,(147a - 206) + 5891) - 947)\,w^2 + (a - 1)\,(2a - 1)\,(3a - 1)\,(a\,(10a\,(588a - 599) + 2097) - 247)\,w + 1\right)\Gamma(a)^8\,w^{1-8a}(z - z_0)^8 + \ldots\;/;\;(z \to z_0) \bigwedge w = Q^{-1}(a, z_0)$$

06.12.06.0006.01

$$Q^{-1}(a, z) \propto Q^{-1}(a, z_0)\,(1 + O(z - z_0))$$

## Expansions at $z == 1$

06.12.06.0001.01

$$Q^{-1}(a, z) \propto (-(z - 1)\,\Gamma(a + 1))^{1/a} + \frac{\left((-(z - 1)\,\Gamma(a + 1))^{1/a}\right)^2}{a + 1} + \frac{(3a + 5)\left((-(z - 1)\,\Gamma(a + 1))^{1/a}\right)^3}{2\,(a + 1)^2\,(a + 2)} + O\!\left((z - 1)^{4/a}\right)$$

06.12.06.0007.01

$$Q^{-1}(a, z) \propto w + \frac{w^2}{a+1} + \frac{(3a+5)w^3}{2(a+1)^2(a+2)} + \frac{(a(8a+33)+31)w^4}{3(a+1)^3(a+2)(a+3)} + \frac{(a(a(125a+1179)+3971)+5661)+2888)w^5}{24(a+1)^4(a+2)^2(a+3)(a+4)} + \frac{(a(a(a(108a+1471)+7575)+18375)+20997)+9074)w^6}{10(a+1)^5(a+2)^2(a+3)(a+4)(a+5)} + \frac{1}{720(a+1)^6(a+2)^3(a+3)^2(a+4)(a+5)(a+6)} (a(a(a(a(a(a(16807a+398516)+3987861)+21989226)+73069137)+149847504)+185250179)+126276754)+36360816)w^7 + \frac{1}{315(a+1)^7(a+2)^3(a+3)^2(a+4)(a+5)(a+6)(a+7)} (a(a(a(a(a(a(a(16384a+486927)+6181022)+43936962)+192606624)+539832153)+967463528)+1069554738)+662420842)+175331220) w^8 + \frac{1}{4480(a+1)^8(a+2)^4(a+3)^2(a+4)^2(a+5)(a+6)(a+7)(a+8)} (a(a(a(a(a(a(a(a(a(531441a+22669360)+429446950)+4774846502)+34688913336)+173423227630)+611682241930)+1533600689586)+2712896385015)+3302979236810)+2628223795120)+1227828808512)+254843639808)w^9 + \frac{1}{4536(a+1)^9(a+2)^4(a+3)^3(a+4)^2(a+5)(a+6)(a+7)(a+8)(a+9)} (a(a(a(a(a(a(a(a(a(a(2a(625000a+33456213)+1614236211)+23253261756)+223389222954)+1513727910042)+7460377417652)+27165941822238)+73445003464698)+146718794936892)+213209054640489)+218591764868766)+149504225861948)+61077979985160)+11250058301568)w^{10} + O\left((z-1)^{11/a}\right) /; w = (-(z-1)\Gamma(a+1))^{1/a}$$

### Asymptotic series expansions

#### Expansions at $z == 0$

06.12.06.0002.01

$$Q^{-1}(a, z) \propto (1-a)\, W_{-1}\left(-\frac{z^{\frac{1}{a-1}}\,\Gamma(a)^{\frac{1}{a-1}}}{a-1}\right) /; (z \to 0)$$

## Differential equations

### Ordinary nonlinear differential equations

06.12.13.0001.01

$$w(z)\, w''(z) - w'(z)^2\,(w(z)+1-a) == 0 /; w(z) == Q^{-1}(a, z)$$

## Differentiation

### Low-order differentiation

#### With respect to $a$

06.12.20.0001.01

$$\frac{\partial Q^{-1}(a,z)}{\partial a} = e^w w^{1-a} \left(\Gamma(a)^2 \, {}_2\tilde{F}_2(a,a;a+1,a+1;-w)\, w^a + (z-1)\,\Gamma(a)\log(w) + (\Gamma(a)-\Gamma(a,w))\,\psi(a)\right) /;\ w = Q^{-1}(a,z)$$

06.12.20.0002.01

$$\frac{\partial^2 Q^{-1}(a,z)}{\partial a^2} = 2\, e^{2w}\,(1-a+w)\,\Gamma(a)^2\, {}_2\tilde{F}_2(a,a;a+1,a+1;-w)\,(\Gamma(a)\,((z-1)\log(w)+\psi(a)) - \Gamma(a,w)\,\psi(a))\, w^{1-a} + e^w \Big(e^w\,(1-a+w)\,\Gamma(a)^2\,((z-1)\log(w)+\psi(a))^2 + \Gamma(a,w)\left(\left(\log^2(w)-\psi^{(1)}(a)\right) w^a + (e^w\,(1-a+w)\,\Gamma(a,w) - w^a)\,\psi(a)^2 + 2\,a\,e^w\,\Gamma(a)\,\psi(a)\,((z-1)\log(w)+\psi(a))\right) + \Gamma(a)\left(\left((1-2z)\log^2(w)+\psi^{(1)}(a)\right) w^a + (w^a - 2\,e^w\,(w+1)\,\Gamma(a,w))\,\psi(a)^2 + 2\,(z-1)\,(w^a - e^w\,(w+1)\,\Gamma(a,w))\log(w)\,\psi(a)\right)\Big)\, w^{1-2a} + e^w\,\Gamma(a)^3\, w \left(e^w\,(1-a+w)\,\Gamma(a)\, {}_2\tilde{F}_2(a,a;a+1,a+1;-w)^2 - 2\, {}_3\tilde{F}_3(a,a,a;a+1,a+1,a+1;-w)\right) /;\ w = Q^{-1}(a,z)$$

## With respect to z

06.12.20.0003.01

$$\frac{\partial Q^{-1}(a,z)}{\partial z} = -e^{Q^{-1}(a,z)}\, Q^{-1}(a,z)^{1-a}\,\Gamma(a)$$

06.12.20.0004.01

$$\frac{\partial^2 Q^{-1}(a,z)}{\partial z^2} = e^{2w}\, w^{1-2a}\,(1-a+w)\,\Gamma(a)^2 /;\ w = Q^{-1}(a,z)$$

06.12.20.0005.01

$$\frac{\partial^3 Q^{-1}(a,z)}{\partial z^3} = -e^{3w}\, w^{1-3a}\left(2a^2 - (4w+3)\,a + 2w\,(w+2) + 1\right)\Gamma(a)^3 /;\ w = Q^{-1}(a,z)$$

06.12.20.0006.01

$$\frac{\partial^4 Q^{-1}(a,z)}{\partial z^4} = e^{4w}\, w^{1-4a}\left(-6a^3 + (18w+11)\,a^2 - (w\,(18w+29)+6)\,a + w\,(6w\,(w+3)+11)+1\right)\Gamma(a)^4 /;\ w = Q^{-1}(a,z)$$

06.12.20.0007.01

$$\frac{\partial^5 Q^{-1}(a,z)}{\partial z^5} = -e^{5w}\, w^{1-5a}\left(24a^4 - 2\,(48w+25)\,a^3 + (4w\,(36w+49)+35)\,a^2 - 2\,(w\,(w\,(48w+121)+63)+5)\,a + 2w\,(w+1)\,(12w\,(w+3)+13)+1\right)\Gamma(a)^5 /;\ w = Q^{-1}(a,z)$$

06.12.20.0008.01

$$\frac{\partial^6 Q^{-1}(a,z)}{\partial z^6} = e^{6w}\, w^{1-6a}\left(-120a^5 + (600w+274)\,a^4 - 3\left(400w^2+474w+75\right)a^3 + \left(3w\left(400w^2+874w+399\right)+85\right)a^2 - (2w\,(w\,(w\,(300w+1037)+923)+216)+15)\,a + w\,(2w\,(w\,(60w\,(w+5)+437)+212)+57)+1\right)\Gamma(a)^6 /;\ w = Q^{-1}(a,z)$$

06.12.20.0009.01

$$\frac{\partial^7 Q^{-1}(a,z)}{\partial z^7} = -e^{7w}\, w^{1-7a}\left(720w^6 - 4320\,(a-1)\,w^5 + 36\,(a-1)\,(300a-229)\,w^4 - 8\,(a-1)\,(18a\,(100a-129)+755)\,w^3 + 6\,(a-1)\,(2a-1)\,(36a\,(25a-27)+269)\,w^2 - 24a\,(a\,(6a\,(a\,(30a-79)+79)-229)+54)\,w + 120w + a\,(a\,(a\,(4a\,(9a\,(20a-49)+406)-735)+175)-21)+1\right)\Gamma(a)^7 /;\ w = Q^{-1}(a,z)$$

06.12.20.0010.01

$$\frac{\partial^8 Q^{-1}(a,z)}{\partial z^8} = e^{8w} w^{1-8a} \big(5040\, w^7 - 35\,280\,(a-1)\, w^6 + 36\,(a-1)\,(2940\, a - 2323)\, w^5 - 20\,(a-1)\,(9\, a\,(980\, a - 1343) + 4175)\, w^4 + 2\,(a-1)\,(2\, a\,(90\, a\,(490\, a - 853) + 45\,001) - 17\,729)\, w^3 - 6\,(a-1)\,(2\, a-1)\,(a\,(60\, a\,(147\, a - 206) + 5891) - 947)\, w^2 + a\,(a\,(a\,(4\, a\,(45\, a\,(196\, a - 559) + 28\,438) - 66\,369) + 21\,289) - 3579)\, w + 247\, w - a\,(a\,(a\,(a\,(4\, a\,(9\, a\,(140\, a - 363) + 3283) - 6769) + 1960) - 322) + 28) + 1\big)\, \Gamma(a)^8 \;/;\; w = Q^{-1}(a,z)$$

06.12.20.0011.01

$$\frac{\partial^9 Q^{-1}(a,z)}{\partial z^9} = -e^{9w} w^{1-9a} \big(40\,320\, w^8 - 322\,560\,(a-1)\, w^7 + 144\,(a-1)\,(7840\, a - 6361)\, w^6 - 8\,(a-1)\,(36\, a\,(7840\, a - 11\,243) + 146\,221)\, w^5 + 4\,(a-1)\,(a\,(180\, a\,(3920\, a - 7323) + 829\,183) - 175\,291)\, w^4 - 8\,(a-1)\,(2\, a-1)\,(4\, a\,(45\, a\,(784\, a - 1265) + 31\,337) - 23\,411)\, w^3 + 2\,(a-1)\,(2\, a-1)\,(a\,(4\, a\,(45\, a\,(1568\, a - 2619) + 75\,437) - 87\,099) + 9511)\, w^2 - 2\, a\,(a\,(a\,(4\, a\,(a\,(36\, a\,(1120\, a - 3403) + 152\,039) - 101\,576) + 159\,365) - 36\,893) + 4679)\, w + 502\, w + a\,(a\,(a\,(a\,(4\, a\,(a\,(36\, a\,(280\, a - 761) + 29\,531) - 16\,821) + 22\,449) - 4536) + 546) - 36) + 1\big)\, \Gamma(a)^9 \;/;\; w = Q^{-1}(a,z)$$

06.12.20.0012.01

$$\frac{\partial^{10} Q^{-1}(a,z)}{\partial z^{10}} = e^{10w} w^{1-10a} \big(362\,880\, w^9 - 3\,265\,920\,(a-1)\, w^8 + 144\,(a-1)\,(90\,720\, a - 75\,169)\, w^7 - 72\,(a-1)\,(14\, a\,(30\,240\, a - 44\,929) + 235\,275)\, w^6 + 12\,(a-1)\,(63\, a\,(4\, a\,(15\,120\, a - 29\,809) + 79\,109) - 1\,110\,757)\, w^5 - 4\,(a-1)\,(a\,(45\, a\,(28\, a\,(9072\, a - 20\,737) + 503\,655) - 8\,822\,594) + 1\,297\,853)\, w^4 + 10\,(a-1)\,(2\, a-1)\,(a\,(36\, a\,(7\, a\,(6048\, a - 11\,665) + 60\,397) - 726\,451) + 92\,035)\, w^3 - 12\,(a-1)\,(2\, a-1)\,(3\, a-1)\,(a\,(a\,(42\, a\,(4320\, a - 6769) + 173\,515) - 48\,309) + 5132)\, w^2 + a\,(a\,(a\,(a\,(4\, a\,(9\, a\,(28\, a\,(3240\, a - 10\,369) + 388\,057) - 2\,587\,913) + 4\,695\,827) - 1\,342\,358) + 236\,708) - 23\,558)\, w + 1013\, w - a\,(a\,(a\,(a\,(a\,(4\, a\,(9\, a\,(4\, a\,(2520\, a - 7129) + 32\,575) - 180\,920) + 269\,325) - 63\,273) + 9450) - 870) + 45) + 1\big)\, \Gamma(a)^{10} \;/;\; w = Q^{-1}(a,z)$$

## Symbolic differentiation

### With respect to *z*

06.12.20.0013.01

$$\frac{\partial^n Q^{-1}(a,z)}{\partial z^n} == w\, \delta_n + \left(-\frac{\Gamma(a)\, e^w}{w^{a-1}}\right)^n \sum_{j_2=0}^{n} \cdots \sum_{j_n=0}^{n} \delta_{\sum_{i=2}^{n} (i-1)\, j_i,\, n-1}\, (-1)^{\sum_{i=2}^{n} j_i} \left(n + \sum_{i=2}^{n} j_i - 1\right)! \prod_{i=2}^{n} \frac{1}{j_i\,!} \left(\frac{\Gamma(a+1)\, e^w\, w^{-a-i+1}}{i\,!}\right)^{j_i} \left(\sum_{k=0}^{i} (-1)^{i-k} \binom{i}{k} (-a-k+1)_{i-1}\, Q(a+k, w)\right)^{j_i} \;/;\; w == Q^{-1}(a,z) \bigwedge n \in \mathbb{N}$$

# Integration

## Indefinite integration

### Involving only one direct function

06.12.21.0001.01

$$\int Q^{-1}(a,z)\, dz == a\, Q\big(a+1,\, Q^{-1}(a,z)\big)$$

## Representations through equivalent functions

### With inverse function

06.12.27.0001.01

$$Q\left(a, Q^{-1}(a, z)\right) == z$$

06.12.27.0002.01

$$\Gamma\left(a, Q^{-1}(a, z)\right) == \Gamma(a)\, z$$

06.12.27.0003.01

$$Q^{-1}(a, Q(a, z_1) - z_2) == Q^{-1}(a, z_1, z_2)$$

## Copyright