InverseGammaRegularized3

Notations

Traditional name

Inverse of the generalized regularized incomplete gamma function

Traditional notation

$Q^{-1}(a, z_1, z_2)$

Mathematica StandardForm notation

`InverseGammaRegularized[`a`,` z_1`,` z_2`]`

Primary definition

06.13.02.0001.01

$z_2 == Q(a, z_1, w) /; w == Q^{-1}(a, z_1, z_2)$

Specific values

Specialized values

06.13.03.0001.01

$Q^{-1}(a, \infty, z) == Q^{-1}(a, -z)$

General characteristics

Domain and analyticity

$Q^{-1}(a, z_1, z_2)$ is an analytical function of a, z_1, z_2 which is defined in $\mathbb{C}^3$. For fixed noninteger a, it has one infinitely long branch cut.

06.13.04.0001.01

$(a * z_1 * z_2) \longrightarrow Q^{-1}(a, z_1, z_2) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Differential equations

Ordinary nonlinear differential equations

06.13.13.0001.01

$$w(z_2)\, w''(z_2) - w'(z_2)^2\, (-a + w(z_2) + 1) == 0 \;/;\; w(z_2) == Q^{-1}(a, z_1, z_2)$$

Differentiation

Low-order differentiation

With respect to *a*

06.13.20.0001.01

$$\frac{\partial Q^{-1}(a, z_1, z_2)}{\partial a} == e^w\, w^{1-a} \left(\frac{1}{a^2} \left(w^a\, {}_2F_2(a, a; a+1, a+1; -w) - z_1^a\, {}_2F_2(a, a; a+1, a+1; -z_1)\right) + \Gamma(a, w, 0) \log(w) + \Gamma(a, 0, z_1) \log(z_1) + \Gamma(a, z_1, w)\, \psi(a) \right) /;\; w == Q^{-1}(a, z_1, z_2)$$

06.13.20.0002.01

$$\frac{\partial^2 Q^{-1}(a, z_1, z_2)}{\partial a^2} == \frac{1}{a\, \Gamma(1-a)} \Big(e^w\, w^{1-2a} \Big((a - w - 1)\, (-a)\, e^w\, \Gamma(1-a)\, \Gamma(a)^2\, (\log(w) - \log(z_1))^2 + \Gamma(a)\, (a\, e^w\, \pi \csc(a\pi) - a\, \Gamma(1-a)\, ((\log(z_1) - \log(w))\, w^a + e^w\, \Gamma(a) + 2\, e^w\, (1 - a + w)\, \Gamma(a, w)\, (\log(w) - \psi(a)) - 2\, e^w\, (1 - a + w)\, \Gamma(a, z_1)\, (\log(z_1) - \psi(a))))\, (\log(w) - \log(z_1)) - a\, e^w\, (a - w - 1)\, \Gamma(1-a)\, \Gamma(a, w)^2\, (\log(w) - \psi(a))^2 - \Gamma(a, w) \Big(a\, e^w\, \pi \csc(a\pi)\, (\log(w) - \psi(a)) + a\, \Gamma(1-a) \Big(\left(\log^2(w) - 2\, \psi(a) \log(w) + \psi(a)^2 + \psi^{(1)}(a)\right) w^a - e^w\, \Gamma(a)\, (\log(w) - \psi(a)) + 2\, e^w\, (1 - a + w)\, \Gamma(a, z_1)\, (\log(w) - \psi(a))\, (\log(z_1) - \psi(a))\Big)\Big) + \Gamma(a, z_1) \Big(a\, e^w\, \pi \csc(a\pi)\, (\log(z_1) - \psi(a)) + a\, \Gamma(1-a) \Big(\Big(\psi(a)^2 - 2 \log(w)\, \psi(a) + (2 \log(w) - \log(z_1)) \log(z_1) + \psi^{(1)}(a)\Big) w^a + e^w\, (1 - a + w)\, \Gamma(a, z_1)\, (\log(z_1) - \psi(a))^2 - e^w\, \Gamma(a)\, (\log(z_1) - \psi(a))\Big)\Big)\Big)\Big) + \frac{1}{a^4} \Big(e^w\, w \Big(e^w\, (1 - a + w)\, {}_2F_2(a, a; a+1, a+1; -z_1)\, z_1^a\, (2\, {}_2F_2(a, a; a+1, a+1; -w)\, w^a + {}_2F_2(a, a; a+1, a+1; -z_1)\, z_1^a)\, w^{-2a} + 2\, a\, {}_3F_3(a, a, a; a+1, a+1, a+1; -z_1)\, z_1^a\, w^{-a} + e^w\, (1 - a + w)\, {}_2F_2(a, a; a+1, a+1; -w)^2 - 2\, a\, {}_3F_3(a, a, a; a+1, a+1, a+1; -w) + \frac{1}{\Gamma(1-a)} \Big(w^{-2a}\, {}_2F_2(a, a; a+1, a+1; -z_1)\, (a\, (a\, e^w\, \pi \csc(a\pi) - \Gamma(1-a)\, (a\, e^w\, \Gamma(a) + 2\, a\, (\log(z_1)\, w^a - e^w\, \Gamma(a, z_1) \log(z_1)\, w + e^w\, \Gamma(a, z_1)\, \psi(a)\, w - w^a \log(w) - e^w\, (1 - a + w)\, \Gamma(a)\, (\log(w) - \log(z_1)) + a\, e^w\, \Gamma(a, z_1) \log(z_1) - e^w\, \Gamma(a, z_1) \log(z_1) + e^w\, (1 - a + w)\, \Gamma(a, w)\, (\log(w) - \psi(a)) - a\, e^w\, \Gamma(a, z_1)\, \psi(a) + e^w\, \Gamma(a, z_1)\, \psi(a)))) - 2\, e^w\, w^a\, (1 - a + w)\, \Gamma(1-a)\, {}_2F_2(a, a; a+1, a+1; -w))\, z_1^a\Big) + \frac{1}{\Gamma(1-a)} (e^w\, w^{-a}\, {}_2F_2(a, a; a+1, a+1; -w)\, (2\, (a - w - 1)\, \Gamma(1-a)\, {}_2F_2(a, a; a+1, a+1; -z_1)\, z_1^a + a\, (a\, \Gamma(1-a)\, (\Gamma(a) + 2\, (a - w - 1)\, (\Gamma(a)\, (\log(w) - \log(z_1)) + \Gamma(a, z_1)\, (\log(z_1) - \psi(a)) + \Gamma(a, w)\, (\psi(a) - \log(w)))) - a\, \pi \csc(a\pi))))\Big)\Big) /;\; w == Q^{-1}(a, z_1, z_2)$$

With respect to z_1

06.13.20.0003.01

$$\frac{\partial Q^{-1}(a, z_1, z_2)}{\partial z_1} = e^{Q^{-1}(a,z_1,z_2)-z_1} \left(\frac{Q^{-1}(a, z_1, z_2)}{z_1}\right)^{1-a}$$

06.13.20.0004.01

$$\frac{\partial^2 Q^{-1}(a, z_1, z_2)}{\partial z_1^2} = \frac{1}{z_1} e^{w-2 z_1} \left(\frac{w}{z_1}\right)^{1-2a} \left(e^w w - (a-1)\left(e^w - e^{z_1}\left(\frac{w}{z_1}\right)^a\right) - e^{z_1} z_1 \left(\frac{w}{z_1}\right)^a\right) /; w = Q^{-1}(a, z_1, z_2)$$

With respect to z_2

06.13.20.0005.01

$$\frac{\partial Q^{-1}(a, z_1, z_2)}{\partial z_2} = e^{Q^{-1}(a,z_1,z_2)} \Gamma(a) Q^{-1}(a, z_1, z_2)^{1-a}$$

06.13.20.0006.01

$$\frac{\partial^2 Q^{-1}(a, z_1, z_2)}{\partial z_2^2} = e^{2w} \Gamma(a)^2 w^{1-2a} (1-a+w) /; w = Q^{-1}(a, z_1, z_2)$$

Symbolic differentiation

With respect to z_2

06.13.20.0007.01

$$\frac{\partial^n Q^{-1}(a, z_1, z_2)}{\partial z_2^n} = w\,\delta_n + \left(\frac{\Gamma(a)\,e^w}{w^{a-1}}\right)^n \sum_{j_2=0}^{n} \ldots \sum_{j_n=0}^{n} \delta_{\sum_{i=2}^{n}(i-1) j_i, n-1} (-1)^{\sum_{i=2}^{n} j_i} \left(n + \sum_{i=2}^{n} j_i - 1\right)! \prod_{i=2}^{n} \frac{1}{j_i!} \left(\frac{\Gamma(a+1)\,e^w w^{1-a}}{i!}\right)^{j_i}$$
$$\left(a\,w^{-i} \sum_{k=0}^{i} (-1)^{i-k} \binom{i}{k} (1-a-k)_{i-1} Q(a+k, w) + Q(a, z_1)\,\delta_i\right)^{j_i} /; w = Q^{-1}(a, z_1, z_2) \bigwedge n \in \mathbb{N}$$

Integration

Indefinite integration

Involving one direct function with respect to z_2

06.13.21.0001.01

$$\int Q^{-1}(a, z_1, z_2)\, dz_2 = -a\, Q\left(a+1, Q^{-1}(a, z_1, z_2)\right)$$

Representations through equivalent functions

With inverse function

06.13.27.0001.01

$$Q\left(a, z_1, Q^{-1}(a, z_1, z_2)\right) = z_2$$

06.13.27.0002.01

$$\Gamma\left(a, z_1, Q^{-1}(a, z_1, z_2)\right) = \Gamma(a)\, z_2$$

06.13.27.0004.01

$$Q\left(a, Q^{-1}(a, z_1, z_2)\right) = Q(a, z_1) - z_2$$

06.13.27.0005.01

$$\Gamma\left(a, Q^{-1}(a, z_1, z_2)\right) = \Gamma(a, z_1) - z_2\, \Gamma(a)$$

With related functions

06.13.27.0003.01

$$Q^{-1}(a, z_1, z_2) == Q^{-1}(a, Q(a, z_1) - z_2)$$

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