

InverseJacobiNS

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Notations

Traditional name

Inverse of the Jacobi elliptic function ns

Traditional notation

 $ns^{-1}(z | m)$

Mathematica StandardForm notation

InverseJacobiNS[z, m]

Primary definition

09.45.02.0001.01

 $z = ns(w | m) /; w = ns^{-1}(z | m)$

09.45.02.0002.01

$$ns^{-1}(z | m) = \int_z^\infty \frac{1}{\sqrt{t^2 - 1} \sqrt{t^2 - m}} dt /; z \in \mathbb{R} \wedge z^2 > 1 \wedge z^2 > m$$

Specific values

Specialized values

For fixed z

09.45.03.0001.01

 $ns^{-1}(z | 0) = \csc^{-1}(z)$

09.45.03.0002.01

$$ns^{-1}\left(z \left| \frac{1}{2} \right. \right) = \sqrt{2} F(\sin^{-1}(z) | 2) + \frac{i\pi^{3/2}}{2\Gamma\left(\frac{3}{4}\right)^2}$$

09.45.03.0003.01

 $ns^{-1}(z | 1) = \coth^{-1}(z)$

For fixed m

09.45.03.0004.01

 $ns^{-1}(-1 | m) = -K(m)$

09.45.03.0005.01

$$\operatorname{ns}^{-1}\left(-\frac{1}{2} \mid m\right) = K(m) - \frac{1}{\sqrt{m}} \left(F\left(\frac{\pi}{6} \mid \frac{1}{m}\right) + K\left(\frac{1}{m}\right) \right)$$

09.45.03.0006.01

$$\operatorname{ns}^{-1}(0 \mid m) = K(m) - \frac{1}{\sqrt{m}} K\left(\frac{1}{m}\right)$$

09.45.03.0007.01

$$\operatorname{ns}^{-1}\left(\frac{1}{2} \mid m\right) = \frac{1}{\sqrt{m}} \left(F\left(\frac{\pi}{6} \mid \frac{1}{m}\right) - K\left(\frac{1}{m}\right) \right) + K(m)$$

09.45.03.0008.01

$$\operatorname{ns}^{-1}(1 \mid m) = K(m)$$

09.45.03.0009.01

$$\operatorname{ns}^{-1}(i \mid m) = K(m) - \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) - F\left(i \sinh^{-1}(1) \mid \frac{1}{m}\right) \right)$$

09.45.03.0010.01

$$\operatorname{ns}^{-1}(-i \mid m) = K(m) - \frac{1}{\sqrt{m}} \left(K\left(\frac{1}{m}\right) + F\left(i \sinh^{-1}(1) \mid \frac{1}{m}\right) \right)$$

Values at infinities

09.45.03.0011.01

$$\operatorname{ns}^{-1}(z \mid \infty) = 0$$

09.45.03.0012.01

$$\operatorname{ns}^{-1}(z \mid -\infty) = 0$$

09.45.03.0013.01

$$\operatorname{ns}^{-1}(\infty \mid m) = 0$$

09.45.03.0014.01

$$\operatorname{ns}^{-1}(-\infty \mid m) = 2 K(m) - \frac{2}{\sqrt{m}} K\left(\frac{1}{m}\right)$$

General characteristics

Domain and analyticity

$\operatorname{ns}^{-1}(z \mid m)$ is an analytical function of z and m which is defined over \mathbb{C}^2 .

09.45.04.0001.01

$$(z * m) \rightarrow \operatorname{ns}^{-1}(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

09.45.04.0002.01

$$\operatorname{ns}^{-1}(\bar{z} \mid \bar{m}) = \overline{\operatorname{ns}^{-1}(z \mid m)}$$

Quasi-reflection symmetry

09.45.04.0003.01

$$\operatorname{ns}^{-1}(-z | m) = \operatorname{ns}^{-1}(z | m) - \frac{2}{\sqrt{m}} F\left(\sin^{-1}(z) \left| \frac{1}{m} \right.\right)$$

09.45.04.0004.01

$$\operatorname{ns}^{-1}(-z | m) = \operatorname{ns}^{-1}(z | m) - \frac{2}{\sqrt{m}} \operatorname{ns}^{-1}\left(\frac{1}{z} \left| \frac{1}{m} \right.\right)$$

Poles and essential singularities

With respect to m

The function $\operatorname{ns}^{-1}(z | m)$ does not have poles and essential singularities with respect to m .

09.45.04.0005.01

$$\operatorname{Sing}_m(\operatorname{ns}^{-1}(z | m)) = \{\}$$

With respect to z

The function $\operatorname{ns}^{-1}(z | m)$ does not have poles and essential singularities with respect to z .

09.45.04.0006.01

$$\operatorname{Sing}_z(\operatorname{ns}^{-1}(z | m)) = \{\}$$

Branch points

With respect to m

For fixed z , the function $\operatorname{ns}^{-1}(z | m)$ has two branch points: $m = z^2$, $m = \tilde{\infty}$.

09.45.04.0007.01

$$\mathcal{BP}_m(\operatorname{ns}^{-1}(z | m)) = \{z^2, \tilde{\infty}\}$$

09.45.04.0008.01

$$\mathcal{R}_m(\operatorname{ns}^{-1}(z | m), z^2) = 2$$

09.45.04.0009.01

$$\mathcal{R}_m(\operatorname{ns}^{-1}(z | m), \tilde{\infty}) = 2$$

With respect to z

For fixed m , the function $\operatorname{ns}^{-1}(z | m)$ has five branch points: $z = \pm 1$, $z = \pm \sqrt{m}$, $z = \tilde{\infty}$.

09.45.04.0010.01

$$\mathcal{BP}_z(\operatorname{ns}^{-1}(z | m)) = \{1, -1, \sqrt{m}, -\sqrt{m}, \tilde{\infty}\}$$

09.45.04.0011.01

$$\mathcal{R}_z(\operatorname{ns}^{-1}(z | m), 1) = 2$$

09.45.04.0012.01

$$\mathcal{R}_z(\operatorname{ns}^{-1}(z | m), -1) = 2$$

09.45.04.0013.01

$$\mathcal{R}_z(\text{ns}^{-1}(z | m), \sqrt{m}) = 2$$

09.45.04.0014.01

$$\mathcal{R}_z(\text{ns}^{-1}(z | m), -\sqrt{m}) = 2$$

09.45.04.0015.01

$$\mathcal{R}_z(\text{ns}^{-1}(z | m), \infty) = \log$$

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

Expansions at $z = 0$

09.45.06.0001.02

$$\text{ns}^{-1}(z | m) \propto \frac{1}{\sqrt{-m}} \left(\left(-\frac{1}{m} \right)^{-1/2} K(m) + i K\left(\frac{1}{m}\right) \right) + \frac{1}{\sqrt{m}} \left(z + \frac{1+m}{6m} z^3 + \frac{3+2m+3m^2}{40m^2} z^5 + \dots \right); (z \rightarrow 0)$$

09.45.06.0002.01

$$\text{ns}^{-1}(z | m) = \frac{1}{\sqrt{-m}} \left(\left(-\frac{1}{m} \right)^{-1/2} K(m) + i K\left(\frac{1}{m}\right) \right) + \sum_{k=0}^{\infty} \frac{m^{-k-\frac{1}{2}} \left(\frac{1}{2}\right)_k}{(2k+1)k!} {}_2F_1\left(\frac{1}{2}, -k; \frac{1}{2} - k; m\right) z^{2k+1}$$

09.45.06.0007.01

$$\text{ns}^{-1}(z | m) \propto \frac{1}{\sqrt{-m}} \left(K(m) \frac{1}{\sqrt{-\frac{1}{m}}} + i K\left(\frac{1}{m}\right) \right) + \frac{z}{\sqrt{m}} (1 + O(z^2))$$

Expansions at $m = 0$

09.45.06.0003.02

$$\text{ns}^{-1}(z | m) \propto \csc^{-1}(z) - \frac{1}{4z} \left(\sqrt{1 - \frac{1}{z^2}} - z \csc^{-1}(z) \right) m - \frac{3}{64z^3} \left(\sqrt{1 - \frac{1}{z^2}} (3z^2 + 2) - 3z^3 \csc^{-1}(z) \right) m^2 - \dots; (m \rightarrow 0)$$

09.45.06.0004.01

$$\text{ns}^{-1}(z | m) = \sum_{k=0}^{\infty} \frac{z^{-2k-1} \left(\frac{1}{2}\right)_k}{k!(2k+1)} {}_2F_1\left(\frac{1}{2}, k + \frac{1}{2}; k + \frac{3}{2}; \frac{1}{z^2}\right) m^k; |m| < 1$$

09.45.06.0008.01

$$\text{ns}^{-1}(z | m) = \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k^2}{(k!)^2} \left(\csc^{-1}(z) - \frac{1}{2} z \sqrt{1 - \frac{1}{z^2}} \sum_{j=1}^k \frac{(j-1)! z^{-2j}}{\left(\frac{1}{2}\right)_j} \right) m^k; |m| < 1$$

09.45.06.0005.01

$$\operatorname{ns}^{-1}(z | m) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{m^k z^{-2j-2k-1} \left(\frac{1}{2}\right)_j \left(\frac{1}{2}\right)_k}{(2j+2k+1) j! k!}$$

09.45.06.0006.01

$$\operatorname{ns}^{-1}(z | m) = \frac{1}{z} F_{1 \times 1 \times 1 \times 0}^{1 \times 1 \times 1} \left(\begin{matrix} \frac{1}{2}; \frac{1}{2}; \frac{1}{2}; \\ \frac{3}{2}; \end{matrix}; \frac{m}{z^2}, \frac{1}{z^2} \right)$$

09.45.06.0009.01

$$\operatorname{ns}^{-1}(z | m) \propto \operatorname{csc}^{-1}(z) (1 + O(m))$$

Integral representations

On the real axis

Of the direct function

09.45.07.0001.01

$$\operatorname{ns}^{-1}(z | m) = \int_z^{\infty} \frac{1}{\sqrt{t^2 - 1} \sqrt{t^2 - m}} dt /; z \in \mathbb{R} \wedge z^2 > 1 \wedge z^2 > m$$

09.45.07.0002.01

$$\begin{aligned} \operatorname{ns}^{-1}(z | m) &= \operatorname{ns}^{-1}(z_0 | m) - \frac{\sqrt{z^2 - m} \operatorname{cd}(\operatorname{ns}^{-1}(z | m) | m)}{\sqrt{z^2 - 1}} \int_{z_0}^z \frac{1}{\sqrt{t^2 - 1} \sqrt{t^2 - m}} dt /; \\ &\neg \exists_{\tau, (\tau \in \mathbb{R}, 0 < \tau < 1)} \left(\operatorname{Im}((\tau(z - z_0) + z_0)^2 - 1) = 0 \wedge \right. \\ &\quad \left. (\tau(z - z_0) + z_0)^2 - 1 < 0 \wedge \operatorname{Im}((\tau(z - z_0) + z_0)^2 - m) = 0 \wedge (\tau(z - z_0) + z_0)^2 - m < 0 \right) \end{aligned}$$

09.45.07.0003.01

$$\begin{aligned} \operatorname{ns}^{-1}(z | m) &= \frac{\sqrt{z^2 - m} \operatorname{cd}(\operatorname{ns}^{-1}(z | m) | m)}{\sqrt{z^2 - 1}} \int_z^{\infty} \frac{1}{\sqrt{t^2 - 1} \sqrt{t^2 - m}} dt /; \\ &\neg \exists_{\tau, (\tau \in \mathbb{R}, 0 < \tau < 1)} \left(\operatorname{Im} \left(\left(z + \tan \left(\frac{\pi \tau}{2} \right) \right)^2 - 1 \right) = 0 \wedge \left(z + \tan \left(\frac{\pi \tau}{2} \right) \right)^2 - 1 < 0 \wedge \right. \\ &\quad \left. \operatorname{Im} \left(\left(z + \tan \left(\frac{\pi \tau}{2} \right) \right)^2 - m \right) = 0 \wedge \left(z + \tan \left(\frac{\pi \tau}{2} \right) \right)^2 - m < 0 \right) \end{aligned}$$

Differential equations

Ordinary nonlinear differential equations

09.45.13.0001.01

$$w''(z) + (2z^2 - m - 1)z w'(z)^3 = 0 /; w(z) = \operatorname{ns}^{-1}(z | m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.45.16.0001.01

$$\operatorname{ns}^{-1}(-z | m) = \operatorname{ns}^{-1}(z | m) - \frac{2}{\sqrt{m}} F\left(\sin^{-1}(z) \left| \frac{1}{m} \right.\right)$$

09.45.16.0002.01

$$\operatorname{ns}^{-1}(-z | m) = \operatorname{ns}^{-1}(z | m) - \frac{2}{\sqrt{m}} \operatorname{ns}^{-1}\left(\frac{1}{z} \left| \frac{1}{m} \right.\right)$$

Identities

Functional identities

09.45.17.0001.01

$$(z_1^2 - z_2^2)^2 \operatorname{ns}(w(z_1) + w(z_2) | m)^4 + (-2 z_2^2 z_1^4 + (-2 z_2^4 + 4(m+1) z_2^2 - 2m) z_1^2 - 2m z_2^2) \operatorname{ns}(w(z_1) + w(z_2) | m)^2 + (m - z_1^2 z_2^2)^2 = 0 /;$$

$$w(z) = \operatorname{ns}^{-1}(z | m)$$

Differentiation

Low-order differentiation

With respect to z

09.45.20.0001.01

$$\frac{\partial \operatorname{ns}^{-1}(z | m)}{\partial z} = - \frac{1}{\operatorname{cs}(\operatorname{ns}^{-1}(z | m) | m) \operatorname{ds}(\operatorname{ns}^{-1}(z | m) | m)}$$

09.45.20.0011.01

$$\frac{\partial \operatorname{ns}^{-1}(z | m)}{\partial z} = \frac{\operatorname{cs}(\operatorname{ns}^{-1}(z | m) | m) \operatorname{ds}(\operatorname{ns}^{-1}(z | m) | m)}{(m - z^2)(z^2 - 1)}$$

09.45.20.0002.01

$$\frac{\partial \operatorname{ns}^{-1}(z | m)}{\partial z} = - \frac{1}{\sqrt{z^2 - 1} \sqrt{z^2 - m}} /; z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge z^2 > m$$

09.45.20.0003.02

$$\frac{\partial^2 \operatorname{ns}^{-1}(z | m)}{\partial z^2} = \frac{z(2z^2 - m - 1) \operatorname{cd}(\operatorname{ns}^{-1}(z | m) | m)}{(z^2 - 1)^2 (z^2 - m)}$$

09.45.20.0012.01

$$\frac{\partial^2 \operatorname{ns}^{-1}(z | m)}{\partial z^2} = - \frac{\sqrt{z^2 - m} \operatorname{cd}(\operatorname{ns}^{-1}(z | m) | m)}{\sqrt{z^2 - 1}} \frac{\partial}{\partial z} \frac{1}{\sqrt{z^2 - 1} \sqrt{z^2 - m}}$$

With respect to m

09.45.20.0004.01

$$\frac{\partial \operatorname{ns}^{-1}(z|m)}{\partial m} = \frac{1}{2(m-1)m} \left(\frac{m \operatorname{cd}(\operatorname{ns}^{-1}(z|m)|m)}{z} - E(\operatorname{am}(\operatorname{ns}^{-1}(z|m)|m)|m) + (1-m) \operatorname{ns}^{-1}(z|m) \right)$$

09.45.20.0005.01

$$\frac{\partial \operatorname{ns}^{-1}(z|m)}{\partial m} = \frac{1}{2m(m-1)} \left(\frac{\sqrt{z^2-1} z}{\sqrt{z^2-m}} + \sqrt{m} E\left(\frac{1}{m}\right) - E(m) - \sqrt{m} E\left(\sin^{-1}(z) \middle| \frac{1}{m}\right) + (1-m) K(m) \right) /;$$

$$z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge z^2 > m$$

09.45.20.0006.02

$$\frac{\partial^2 \operatorname{ns}^{-1}(z|m)}{\partial m^2} = \frac{1}{4(m-1)^2 m^2 z^2} \left(((4m-2) E(\operatorname{am}(\operatorname{ns}^{-1}(z|m)|m)|m) + (m-1) F(\operatorname{am}(\operatorname{ns}^{-1}(z|m)|m)|m)) z^2 + 3(m-1)^2 \operatorname{ns}^{-1}(z|m) z^2 + m \left(-3mz + \frac{(m-1)mz}{z^2-m} + z \right) \operatorname{cd}(\operatorname{ns}^{-1}(z|m)|m) \right)$$

09.45.20.0013.01

$$\frac{\partial^3 \operatorname{ns}^{-1}(z|m)}{\partial m^3} = \frac{1}{8(m-1)^3 m^3} \left((-23(m-1)m-8) E(\operatorname{am}(\operatorname{ns}^{-1}(z|m)|m)|m) - (m-1)(11m-7) F(\operatorname{am}(\operatorname{ns}^{-1}(z|m)|m)|m) + \frac{1}{(m-z^2)^3} \left(mz \left(\frac{1}{z^2} (m-z^2) ((3m(5m-4)+5) z^4 - m(5m(7m-6)+11) z^2 + m^2(m(23m-24)+9)) \operatorname{cd}(\operatorname{ns}^{-1}(z|m)|m) - (m-1) \sqrt{1-\frac{m}{z^2}} (m-z^2)^2 \operatorname{cn}(\operatorname{ns}^{-1}(z|m)|m) \right) - 15(m-1)^3 (m-z^2)^3 \operatorname{ns}^{-1}(z|m) \right) \right)$$

Symbolic differentiation

With respect to z

09.45.20.0014.01

$$\frac{\partial^n \operatorname{ns}^{-1}(z|m)}{\partial z^n} = \operatorname{ns}^{-1}(z|m) \delta_n + \frac{\sqrt{z^2-m} \operatorname{cd}(\operatorname{ns}^{-1}(z|m)|m)}{\sqrt{z^2-1}} \sum_{j=0}^{n-1} \frac{(-1)^{j-1} (1-n)_{2(n-j)-2}}{(n-j-1)! (2z)^{n-2j-1}} \sum_{k=0}^j \binom{j}{k} \binom{1}{2}_k \binom{1}{2}_{j-k} (z^2-1)^{-k-\frac{1}{2}} (z^2-m)^{k-j-\frac{1}{2}} /; n \in \mathbb{N}$$

09.45.20.0015.01

$$\frac{\partial^n \operatorname{ns}^{-1}(z|m)}{\partial z^n} = \operatorname{ns}^{-1}(z|m) \delta_n + \frac{\operatorname{cd}(\operatorname{ns}^{-1}(z|m)|m)}{z^2-1} \sum_{j=0}^{n-1} \frac{(-1)^{j-1} 2^{2j-n+1} z^{2j-n+1} (z^2-m)^{-j} (1-n)_{2(n-j)-2} \left(\frac{1}{2}\right)_j}{(n-j-1)!} {}_2F_1\left(\frac{1}{2}, -j; \frac{1}{2} - j; \frac{z^2-m}{z^2-1}\right) /; n \in \mathbb{N}$$

09.45.20.0016.01

$$\frac{\partial^n \operatorname{ns}^{-1}(z | m)}{\partial z^n} = \delta_n \operatorname{ns}^{-1}(z | m) - \frac{\sqrt{z^2 - m} \operatorname{cd}(\operatorname{ns}^{-1}(z | m) | m)}{\sqrt{z^2 - 1}} \frac{\partial^{n-1} \frac{1}{\sqrt{z^2 - 1} \sqrt{z^2 - m}}}{\partial z^{n-1}}; n \in \mathbb{N}^+$$

09.45.20.0007.02

$$\frac{\partial^n \operatorname{ns}^{-1}(z | m)}{\partial z^n} = \delta_n \operatorname{ns}^{-1}(z | m) - \frac{2^{n-1} \pi z^{n-1} (n-1)! \operatorname{cd}(\operatorname{ns}^{-1}(z | m) | m)}{z^2 - 1} \sum_{j=0}^{n-1} \frac{(z^2 - 1)^{-j} (z^2 - m)^{j-n+1}}{j! (n-j-1)! \Gamma\left(\frac{1}{2} - j\right) \Gamma\left(j - n + \frac{3}{2}\right)}$$

$${}_2F_1\left(\frac{1-j}{2}, -\frac{j}{2}; \frac{1}{2} - j; 1 - \frac{1}{z^2}\right) {}_2F_1\left(\frac{j-n+2}{2}, \frac{j-n+1}{2}; j-n + \frac{3}{2}; 1 - \frac{m}{z^2}\right); n \in \mathbb{N}$$

With respect to m

09.45.20.0008.02

$$\frac{\partial^n \operatorname{ns}^{-1}(z | m)}{\partial m^n} = \frac{(-1)^n \sqrt{\pi} z^{-2n-1}}{(2n+1) \Gamma\left(\frac{1}{2} - n\right)} F_1\left(n + \frac{1}{2}; \frac{1}{2}, n + \frac{1}{2}; n + \frac{3}{2}; \frac{1}{z^2}, \frac{m}{z^2}\right); |z| > 1 \wedge n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to z

09.45.20.0009.01

$$\frac{\partial^\alpha \operatorname{ns}^{-1}(z | m)}{\partial z^\alpha} = \frac{\sqrt{\pi} z^{1-\alpha}}{\sqrt{m}} \tilde{F}_{2 \times 0 \times 0}^{2 \times 1 \times 1}\left(\frac{1}{2}, 1; \frac{1}{2}, \frac{1}{2}; \frac{1}{2}; z^2, \frac{z^2}{m}\right) + \frac{z^{-\alpha}}{\Gamma(1-\alpha)} \left(K(m) - \frac{1}{\sqrt{m}} K\left(\frac{1}{m}\right)\right); -1 < z < 1 \wedge m < 0$$

With respect to m

09.45.20.0010.01

$$\frac{\partial^\alpha \operatorname{ns}^{-1}(z | m)}{\partial m^\alpha} = \frac{m^{-\alpha} \sqrt{\pi}}{2z} \tilde{F}_{1 \times 0 \times 1}^{1 \times 1 \times 2}\left(\frac{1}{2}, \frac{1}{2}; \frac{1}{2}, 1; \frac{1}{z^2}, \frac{m}{z^2}\right); (z > 1 \vee z < -1) \wedge z^2 > m$$

Integration

Indefinite integration

Involving only one direct function

09.45.21.0001.01

$$\int \operatorname{ns}^{-1}(z | m) dz = z \operatorname{ns}^{-1}(z | m) + \log(\operatorname{cs}(\operatorname{ns}^{-1}(z | m) | m) + \operatorname{ds}(\operatorname{ns}^{-1}(z | m) | m))$$

Involving only one direct function with respect to m

09.45.21.0002.01

$$\int \operatorname{ns}^{-1}(z | m) dm = 2 \left(-z - \sqrt{m} E\left(\frac{1}{m}\right) + E(m) + \sqrt{m} E\left(\sin^{-1}(z) \middle| \frac{1}{m}\right) + (m-1) K(m)\right); z > 1 \wedge m < 1$$

Representations through more general functions

Through hypergeometric functions of two variables

09.45.26.0001.01

$$\operatorname{ns}^{-1}(z | m) = \frac{1}{z} F_{1 \times 0 \times 0}^{1 \times 1 \times 1} \left(\begin{matrix} \frac{1}{2}; \frac{1}{2}; \frac{1}{2}; \\ \frac{3}{2}; \end{matrix} ; \frac{m}{z^2}, \frac{1}{z^2} \right)$$

Through other functions

Involving some hypergeometric-type functions

09.45.26.0002.01

$$\operatorname{ns}^{-1}(z | m) = \frac{1}{z} F_1 \left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}, \frac{3}{2}; \frac{1}{z^2}, \frac{m}{z^2} \right); (z > 1 \vee z < -1) \wedge z^2 > m$$

Representations through equivalent functions

With inverse function

09.45.27.0001.01

$$\operatorname{ns}(\operatorname{ns}^{-1}(z | m) | m) = z$$

With related functions

Involving cd^{-1}

09.45.27.0002.01

$$\operatorname{ns}^{-1}(z | m) = K(m) - \frac{1}{\sqrt{m}} \operatorname{cd}^{-1} \left(z \left| \frac{1}{m} \right. \right); -1 < z < 1 \wedge m < 0$$

Involving cn^{-1}

09.45.27.0003.01

$$\operatorname{ns}^{-1}(z | m) = \operatorname{cn}^{-1} \left(\frac{\sqrt{z^2 - 1}}{z} \left| m \right. \right); z > 1 \wedge m < 1$$

Involving cs^{-1}

09.45.27.0004.01

$$\operatorname{ns}^{-1}(iz | m) = i \operatorname{cs}^{-1}(-z | 1 - m)$$

Involving dc^{-1}

09.45.27.0005.01

$$\operatorname{ns}^{-1}(z | m) = K(m) - \frac{1}{\sqrt{m}} \operatorname{dc}^{-1} \left(\frac{1}{z} \left| \frac{1}{m} \right. \right); -1 < z < 1 \wedge m < 0$$

Involving dn^{-1}

09.45.27.0006.01

$$\operatorname{ns}^{-1}(z | m) = K(m) + \frac{i}{\sqrt{m}} \operatorname{dn}^{-1} \left(\frac{1}{z} \left| 1 - \frac{1}{m} \right. \right); z > -1 \wedge m > 1$$

Involving ds^{-1}

09.45.27.0007.01

$$ns^{-1}(z|m) = \frac{1}{\sqrt{1-m}} ds^{-1}\left(\frac{z}{\sqrt{1-m}} \middle| \frac{m}{m-1}\right); 0 < z < 1 \wedge m < 1$$

Involving nc^{-1}

09.45.27.0008.01

$$ns^{-1}(z|m) = K(m) + \frac{i}{\sqrt{1-m}} nc^{-1}\left(\frac{1}{z} \middle| \frac{1}{1-m}\right); z > 1 \wedge m \in \mathbb{R}$$

09.45.27.0009.01

$$ns^{-1}(z|m) = K(m) - \frac{1}{\sqrt{1-m}} nc^{-1}\left(z \middle| \frac{m}{m-1}\right); z > 1 \wedge m < 1$$

Involving nd^{-1}

09.45.27.0010.01

$$ns^{-1}(z|m) = K(m) + \frac{i}{\sqrt{m}} nd^{-1}\left(z \middle| 1 - \frac{1}{m}\right); -1 < z < 1 \wedge m < 0$$

Involving sc^{-1}

09.45.27.0011.01

$$ns^{-1}(z|m) = -i sc^{-1}\left(\frac{i}{z} \middle| 1-m\right); z > 1 \wedge m \in \mathbb{R}$$

Involving sd^{-1}

09.45.27.0012.01

$$ns^{-1}(z|m) = \frac{i}{\sqrt{1-m}} sd^{-1}\left(\frac{\sqrt{m-1}}{z} \middle| \frac{1}{1-m}\right); z > 0 \wedge m > 1$$

Involving sn^{-1}

09.45.27.0013.01

$$ns^{-1}(z|m) = sn^{-1}\left(\frac{1}{z} \middle| m\right); z \in \mathbb{R} \wedge m < 1$$

Involving elliptic integrals

09.45.27.0014.01

$$ns^{-1}(z|m) = F\left(\sin^{-1}\left(\frac{1}{z}\right) \middle| m\right); (z < -1 \vee z > 1) \wedge m < 1$$

09.45.27.0015.01

$$ns^{-1}(z|m) = \frac{1}{\sqrt{m}} F\left(\sin^{-1}(z) \middle| \frac{1}{m}\right) + \frac{1}{\sqrt{-m}} \left(\left(-\frac{1}{m}\right)^{-1/2} K(m) + i K\left(\frac{1}{m}\right)\right)$$

09.45.27.0017.01

$$\operatorname{ns}^{-1}(z \mid m) =$$

$$\operatorname{ns}^{-1}(z_0 \mid m) - \frac{\sqrt{z^2 - m} \operatorname{cd}(\operatorname{ns}^{-1}(z \mid m) \mid m)}{\sqrt{z^2 - 1}} \left(\frac{\sqrt{1 - z^2} \sqrt{\frac{m - z^2}{m}}}{\sqrt{z^2 - 1} \sqrt{z^2 - m}} F\left(\sin^{-1}(z) \mid \frac{1}{m}\right) - \frac{\sqrt{1 - z_0^2} \sqrt{\frac{m - z_0^2}{m}}}{\sqrt{z_0^2 - 1} \sqrt{z_0^2 - m}} F\left(\sin^{-1}(z_0) \mid \frac{1}{m}\right) \right) /;$$

$$\neg \exists_{\tau, (\tau \in \mathbb{R}, 0 < \tau < 1)} \left(\operatorname{Im}\left((\tau(z - z_0) + z_0)^2 - 1\right) = 0 \wedge (\tau(z - z_0) + z_0)^2 - 1 < 0 \wedge \operatorname{Im}\left((\tau(z - z_0) + z_0)^2 - m\right) = 0 \wedge (\tau(z - z_0) + z_0)^2 - m < 0 \right)$$

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$$\operatorname{ns}^{-1}(z \mid m) = \frac{z^2 \operatorname{cd}(\operatorname{ns}^{-1}(z \mid m) \mid m)}{z^2 - 1} \sqrt{1 - \frac{1}{z^2}} \sqrt{1 - \frac{m}{z^2}} F(\operatorname{csc}^{-1}(z) \mid m) /;$$

$$\neg \exists_{\tau, (\tau \in \mathbb{R}, 0 < \tau < 1)} \left(\operatorname{Im}\left(\left(z + \tan\left(\frac{\pi \tau}{2}\right)\right)^2 - 1\right) = 0 \wedge \left(z + \tan\left(\frac{\pi \tau}{2}\right)\right)^2 - 1 < 0 \wedge \operatorname{Im}\left(\left(z + \tan\left(\frac{\pi \tau}{2}\right)\right)^2 - m\right) = 0 \wedge \left(z + \tan\left(\frac{\pi \tau}{2}\right)\right)^2 - m < 0 \right)$$

Involving other related functions

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$$\operatorname{ns}^{-1}(z \mid m) = -\frac{\sqrt{z_2^2}}{z_2} \operatorname{elog}(z_1, z_2; a, b) /; \{a, b, z_1\} = \{-m - 1, m, z^2\} \wedge z_1^3 + a z_1^2 + b z_1 - z_2^2 = 0 \wedge z > 0 \wedge m < 1$$

History

- N. H. Abel (1826)
- A. G. Greenhill (1892)
- L. M. Milne-Thompson (1948)

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