JacobiAmplitude

Notations

Traditional name

Amplitude

Traditional notation

$\operatorname{am}(z \mid m)$

Mathematica StandardForm notation

`JacobiAmplitude[z, m]`

Primary definition

09.24.02.0001.01

$z = \operatorname{am}(w \mid m) /; w = F(z \mid m)$

Specific values

Specialized values

For fixed *z*

Case $m = 0$

09.24.03.0001.01

$\operatorname{am}(z \mid 0) = z$

Case $m = 1$

09.24.03.0002.01

$$\operatorname{am}(z \mid 1) = 2\tan^{-1}(e^z) - \frac{\pi}{2}$$

For fixed *m*

09.24.03.0003.01

$\operatorname{am}(0 \mid m) = 0$

Other

09.24.03.0004.01

$$\operatorname{am}(K(m) \mid m) == \frac{\pi}{2}$$

Derivatives with respect to *z*

For $z == 0$

09.24.03.0005.01

$$\operatorname{am}^{(1,0)}(0, m) == 1$$

09.24.03.0006.01

$$\operatorname{am}^{(3,0)}(0, m) == -m$$

09.24.03.0007.01

$$\operatorname{am}^{(5,0)}(0, m) == 4\, m + m^2$$

09.24.03.0008.01

$$\operatorname{am}^{(7,0)}(0, m) == -16\, m - 44\, m^2 - m^3$$

09.24.03.0009.01

$$\operatorname{am}^{(9,0)}(0, m) == 64\, m + 912\, m^2 + 408\, m^3 + m^4$$

09.24.03.0010.01

$$\operatorname{am}^{(11,0)}(0, m) == -m^5 - 3688\, m^4 - 30\,768\, m^3 - 15\,808\, m^2 - 256\, m$$

09.24.03.0011.01

$$\operatorname{am}^{(13,0)}(0, m) == m^6 + 33\,212\, m^5 + 870\,640\, m^4 + 1\,538\,560\, m^3 + 259\,328\, m^2 + 1024\, m$$

09.24.03.0012.01

$$\operatorname{am}^{(15,0)}(0, m) == -m^7 - 298\,932\, m^6 - 22\,945\,056\, m^5 - 106\,923\,008\, m^4 - 65\,008\,896\, m^3 - 4\,180\,992\, m^2 - 4096\, m$$

09.24.03.0013.01

$$\operatorname{am}^{(17,0)}(0, m) == m^8 + 2\,690\,416\, m^7 + 586\,629\,984\, m^6 + 6\,337\,665\,152\, m^5 + 9\,860\,488\,448\, m^4 + 2\,536\,974\,336\, m^3 + 67\,047\,424\, m^2 + 16\,384\, m$$

09.24.03.0014.01

$$\operatorname{am}^{(19,0)}(0, m) == -m^9 - 24\,213\,776\, m^8 - 14\,804\,306\,080\, m^7 - 345\,558\,617\,984\, m^6 - 1\,165\,333\,452\,544\, m^5 - 782\,931\,974\,144\, m^4 - 95\,153\,582\,080\, m^3 - 1\,073\,463\,296\, m^2 - 65\,536\, m$$

09.24.03.0015.01

$$\operatorname{am}^{(2k,0)}(0, m) == 0 \;/;\; k \in \mathbb{N}$$

Derivatives with respect to *m*

For $m == 0$

09.24.03.0016.01

$$\operatorname{am}^{(0,1)}(z, 0) == \frac{1}{8} (\sin(2\, z) - 2\, z)$$

09.24.03.0017.01

$$\operatorname{am}^{(0,2)}(z, 0) == \frac{1}{128} (-16 \cos(2\, z)\, z - 20\, z + 16 \sin(2\, z) + \sin(4\, z))$$

09.24.03.0018.01

$$\operatorname{am}^{(0,3)}(z,0)=\frac{1}{1024}\left(-312\cos(2z)\,z-24\cos(4z)\,z-264\,z-3\left(32z^2-83\right)\sin(2z)+24\sin(4z)+\sin(6z)\right)$$

09.24.03.0019.01

$$\operatorname{am}^{(0,4)}(z,0)=\frac{1}{16\,384}\left(32\left(32z^2-501\right)\cos(2z)\,z-2016\cos(4z)\,z-96\cos(6z)\,z-11\,256\,z-96\left(72z^2-121\right)\sin(2z)-48\left(16z^2-29\right)\sin(4z)+96\sin(6z)+3\sin(8z)\right)$$

09.24.03.0020.01

$$\operatorname{am}^{(0,5)}(z,0)=\frac{1}{65\,536}\left(160\left(184z^2-1653\right)\cos(2z)\,z+160\left(32z^2-267\right)\cos(4z)\,z-3480\cos(6z)\,z-120\cos(8z)\,z-165\,480\,z+5\left(512z^4-26\,496z^2+36\,213\right)\sin(2z)-24\,960\,(z-1)\,(z+1)\sin(4z)-60\left(24z^2-37\right)\sin(6z)+120\sin(8z)+3\sin(10z)\right)$$

09.24.03.0021.01

$$\operatorname{am}^{(0,6)}(z,0)=\frac{3}{1\,048\,576}\left(-16\left(512z^4-65\,600z^2+447\,465\right)\cos(2z)\,z+80\left(3968z^2-16\,983\right)\cos(4z)\,z+7680\left(3z^2-19\right)\cos(6z)\,z-8880\cos(8z)\,z-240\cos(10z)\,z-4\,133\,520\,z+80\left(1792z^4-48\,960z^2+58\,953\right)\sin(2z)+5\left(8192z^4-193\,920z^2+143\,865\right)\sin(4z)-320\left(306z^2-235\right)\sin(6z)-120\left(32z^2-45\right)\sin(8z)+240\sin(10z)+5\sin(12z)\right)$$

09.24.03.0022.01

$$\operatorname{am}^{(0,7)}(z,0)=\frac{1}{8\,388\,608}\left(-336\left(8448z^4-537\,680z^2+3\,090\,765\right)\cos(2z)\,z-168\left(8192z^4-434\,560z^2+1\,317\,345\right)\cos(4z)\,z+7560\left(1248z^2-3755\right)\cos(6z)\,z+3360\left(128z^2-693\right)\cos(8z)\,z-113\,400\cos(10z)\,z-2520\cos(12z)\,z-564\,490\,080\,z-7\left(16\,384z^6-4\,369\,920z^4+86\,129\,280z^2-94\,986\,225\right)\sin(2z)+2520\left(6144z^4-70\,672z^2+43\,513\right)\sin(4z)+1890\left(768z^4-12\,752z^2+6803\right)\sin(6z)-20\,160\left(84z^2-55\right)\sin(8z)-1260\left(40z^2-53\right)\sin(10z)+2520\sin(12z)+45\sin(14z)\right)$$

09.24.03.0023.01

$$\operatorname{am}^{(0,8)}(z,0)=\frac{1}{134\,217\,728}\left(64\left(16\,384z^6-8\,096\,256z^4+357\,215\,040z^2-1\,822\,886\,415\right)\cos(2z)\,z-1344\left(335\,872z^4-8\,302\,080z^2+20\,225\,565\right)\cos(4z)\,z-72\,576\left(768z^4-26\,960z^2+54\,545\right)\cos(6z)\,z+6720\left(24\,064z^2-58\,671\right)\cos(8z)\,z+53\,760\left(100z^2-489\right)\cos(10z)\,z-1\,068\,480\cos(12z)\,z-20\,160\cos(14z)\,z-60\,474\,299\,760\,z-448\left(77\,824z^6-9\,841\,920z^4+157\,888\,080z^2-163\,063\,575\right)\sin(2z)-224\left(131\,072z^6-13\,486\,080z^4+105\,953\,040z^2-57\,287\,025\right)\sin(4z)+120\,960\left(4224z^4-32\,100z^2+13\,573\right)\sin(6z)+1680\left(16\,384z^4-221\,376z^2+96\,087\right)\sin(8z)-80\,640\left(250z^2-147\right)\sin(10z)-10\,080\left(48z^2-61\right)\sin(12z)+20\,160\sin(14z)+315\sin(16z)\right)$$

09.24.03.0024.01

$$\mathrm{am}^{(0,9)}(z,0) = \frac{9}{268435456}\left(8\left(704512 z^6 - 159108096 z^4 + 5555289600 z^2 - 25939014255\right)\cos(2z)\,z + 32\left(262144 z^6 - 47394816 z^4 + 777023520 z^2 - 1623372345\right)\cos(4z)\,z - 1008\left(338688 z^4 - 5280000 z^2 + 8267935\right)\cos(6z)\,z - 672\left(32768 z^4 - 900480 z^2 + 1420635\right)\cos(8z)\,z + 840\left(44000 z^2 - 93147\right)\cos(10z)\,z + 10080\left(96 z^2 - 437\right)\cos(12z)\,z - 153720\cos(14z)\,z - 2520\cos(16z)\,z - 103483364040\,z + 2\left(65536 z^8 - 55164928 z^6 + 4684162560 z^4 - 65048634000 z^2 + 63806773905\right)\sin(2z) - 896\left(188416 z^6 - 8736000 z^4 + 53735895 z^2 - 26369730\right)\sin(4z) - 21\left(1327104 z^6 - 86400000 z^4 + 429920640 z^2 - 154280035\right)\sin(6z) + 13440\left(13312 z^4 - 78822 z^2 + 26199\right)\sin(8z) + 210\left(32000 z^4 - 377520 z^2 + 142083\right)\sin(10z) - 13440\left(261 z^2 - 142\right)\sin(12z) - 1260\left(56 z^2 - 69\right)\sin(14z) + 2520\sin(16z) + 35\sin(18z)\right)$$

09.24.03.0025.01

$$\mathrm{am}^{(0,10)}(z,0) = \frac{5}{4294967296}\left(-32\left(65536 z^8 - 88178688 z^6 + 13025263104 z^4 - 385499383920 z^2 + 1679120073855\right)\cos(2z)\,z + 1728\left(4456448 z^6 - 353282048 z^4 + 4437323520 z^2 - 8261810235\right)\cos(4z)\,z + 1296\left(1327104 z^6 - 146506752 z^4 + 1462728960 z^2 - 1908968285\right)\cos(6z)\,z - 36288\left(622592 z^4 - 7329280 z^2 + 8693825\right)\cos(8z)\,z - 30240\left(32000 z^4 - 746800 z^2 + 993189\right)\cos(10z)\,z + 45360\left(24192 z^2 - 46253\right)\cos(12z)\,z + 241920\left(98 z^2 - 423\right)\cos(14z)\,z - 3129840\cos(16z)\,z - 45360\cos(18z)\,z - 25942107537360\,z + 864\left(131072 z^8 - 48943104 z^6 + 3212948480 z^4 - 40049795940 z^2 + 37672070835\right)\sin(2z) + 9\left(33554432 z^8 - 9916383232 z^6 + 297656647680 z^4 - 1539489047040 z^2 + 700334220285\right)\sin(4z) - 27216\left(995328 z^6 - 28070400 z^4 + 105973040 z^2 - 33616995\right)\sin(6z) - 8064\left(262144 z^6 - 12948480 z^4 + 48613140 z^2 - 13355685\right)\sin(8z) + 18144\left(400000 z^4 - 2000900 z^2 + 561453\right)\sin(10z) + 11340\left(18432 z^4 - 197472 z^2 + 66917\right)\sin(12z) - 544320\left(154 z^2 - 79\right)\sin(14z) - 22680\left(64 z^2 - 77\right)\sin(16z) + 45360\sin(18z) + 567\sin(20z)\right)$$

For *m* == 1

09.24.03.0026.01

$$\mathrm{am}^{(0,1)}(z,1) = \frac{1}{4}\left(z\,\mathrm{sech}(z) - \sinh(z)\right)$$

09.24.03.0027.01

$$\mathrm{am}^{(0,2)}(z,1) = \frac{1}{32}\left(-4z\cosh(z) + 9\sinh(z) - z\,\mathrm{sech}(z)\left(2z\tanh(z) + 5\right)\right)$$

09.24.03.0028.01

$$\mathrm{am}^{(0,3)}(z,1) = \frac{1}{4096}\,\mathrm{sech}^3(z)\left(-96z^3 + 16\left(2z^2 + 75\right)\cosh(2z)\,z + 168\cosh(4z)\,z + 1032\,z + 3\left(64z^2 - 201\right)\sinh(2z) - 12\left(2z^2 + 25\right)\sinh(4z) + \sinh(6z)\right)$$

09.24.03.0029.01

$$\mathrm{am}^{(0,4)}(z,1) = \frac{1}{32768}\,\mathrm{sech}^4(z)\left(104z\left(20z^2 - 381\right)\cosh(z) - 4z\left(160z^2 + 4239\right)\cosh(3z) - 8z\left(4z^2 + 279\right)\cosh(5z) + 12z\cosh(7z) + 2\left(368z^4 - 900z^2 + 3711\right)\sinh(z) - \left(32z^4 + 1344z^2 - 11097\right)\sinh(3z) + 3\left(152z^2 + 1217\right)\sinh(5z) - 24\sinh(7z)\right)$$

09.24.03.0030.01

$$\operatorname{am}^{(0,5)}(z,\,1) == -\frac{1}{1\,048\,576}\operatorname{sech}^5(z)\Big(-14\,720\,z^5+127\,200\,z^3+8\left(1216\,z^4+9360\,z^2-457\,725\right)\cosh(2\,z)\,z-16\left(8\,z^4+3510\,z^2+78\,525\right)\cosh(4\,z)\,z-120\left(32\,z^2+1237\right)\cosh(6\,z)\,z+1740\cosh(8\,z)\,z-2\,555\,580\,z+40\left(1780\,z^4-3789\,z^2+30\,099\right)\sinh(2\,z)-10\left(256\,z^4+2304\,z^2-95\,697\right)\sinh(4\,z)+5\left(32\,z^4+6936\,z^2+46\,743\right)\sinh(6\,z)-45\left(8\,z^2+49\right)\sinh(8\,z)+3\sinh(10\,z)\Big)$$

09.24.03.0031.01

$$\operatorname{am}^{(0,6)}(z,\,1) == -\frac{1}{16\,777\,216}\operatorname{sech}^6(z)\Big(12\,288\cosh(5\,z)\,z^5+384\cosh(7\,z)\,z^5+2\,792\,640\cosh(5\,z)\,z^3+186\,720\cosh(7\,z)\,z^3-4320\cosh(9\,z)\,z^3+360\left(4144\,z^4-32\,816\,z^2+981\,117\right)\cosh(z)\,z-864\left(824\,z^4-310\,z^2-229\,865\right)\cosh(3\,z)\,z+57\,061\,260\cosh(5\,z)\,z+5\,960\,160\cosh(7\,z)\,z-108\,360\cosh(9\,z)\,z+180\cosh(11\,z)\,z+\left(430\,592\,z^6-3\,512\,160\,z^4+4\,667\,040\,z^2-48\,694\,590\right)\sinh(z)+\left(-60\,672\,z^6-3\,411\,360\,z^4+4\,689\,360\,z^2-87\,336\,540\right)\sinh(3\,z)+\left(256\,z^6+86\,880\,z^4-1\,469\,520\,z^2-48\,023\,100\right)\sinh(5\,z)+\left(-13\,920\,z^4-1\,455\,120\,z^2-9\,270\,135\right)\sinh(7\,z)+\left(36\,720\,z^2+110\,655\right)\sinh(9\,z)-360\sinh(11\,z)\Big)$$

09.24.03.0032.01

$$\operatorname{am}^{(0,7)}(z,\,1) == \frac{1}{536\,870\,912}\operatorname{sech}^7(z)\Big(-12\,056\,576\,z^7+197\,756\,160\,z^5-1\,013\,947\,200\,z^3+8\left(1\,349\,504\,z^6+13\,115\,424\,z^4-113\,929\,200\,z^2+6\,658\,689\,555\right)\cosh(2\,z)\,z-4\left(184\,832\,z^6+22\,732\,416\,z^4-98\,552\,160\,z^2-6\,182\,430\,345\right)\cosh(4\,z)\,z+4\left(256\,z^6+498\,624\,z^4+77\,437\,080\,z^2+1\,527\,136\,695\right)\cosh(6\,z)\,z+672\left(136\,z^4+25\,110\,z^2+835\,245\right)\cosh(8\,z)\,z-1260\left(936\,z^2+10\,951\right)\cosh(10\,z)\,z+56\,700\cosh(12\,z)\,z+34\,073\,202\,240\,z+7\left(11\,094\,016\,z^6-107\,246\,400\,z^4+91\,432\,800\,z^2-1\,880\,382\,285\right)\sinh(2\,z)-7\left(1\,816\,064\,z^6+51\,792\,000\,z^4-12\,252\,600\,z^2+1\,869\,500\,115\right)\sinh(4\,z)+21\left(2048\,z^6+310\,560\,z^4-16\,492\,800\,z^2-263\,197\,425\right)\sinh(6\,z)-28\left(64\,z^6+50\,160\,z^4+4\,828\,500\,z^2+31\,327\,965\right)\sinh(8\,z)+315\left(288\,z^4+18\,848\,z^2+39\,353\right)\sinh(10\,z)-315\left(40\,z^2+211\right)\sinh(12\,z)+45\sinh(14\,z)\Big)$$

09.24.03.0033.01

$$\operatorname{am}^{(0,8)}(z,\,1) == \frac{1}{2\,147\,483\,648}\operatorname{sech}^8(z)\Big(2016\,z\left(231\,136\,z^6-4\,250\,076\,z^4+17\,801\,835\,z^2-846\,574\,260\right)\cosh(z)-28\,z\left(12\,573\,952\,z^6+9\,650\,016\,z^4-138\,963\,600\,z^2+39\,183\,167\,835\right)\cosh(3\,z)+32\,z\left(806\,368\,z^6+47\,006\,064\,z^4-345\,628\,500\,z^2-13\,568\,817\,465\right)\cosh(5\,z)-32\,z\left(1408\,z^6+1\,082\,844\,z^4+146\,896\,050\,z^2+2\,933\,497\,035\right)\cosh(7\,z)-4\,z\left(256\,z^6+99\,456\,z^4+41\,380\,920\,z^2+1\,894\,043\,655\right)\cosh(9\,z)+84\,z\left(2592\,z^4+355\,960\,z^2+2\,790\,165\right)\cosh(11\,z)-3360\,z\left(25\,z^2+486\right)\cosh(13\,z)+1260\,z\cosh(15\,z)+2\left(66\,489\,088\,z^8-826\,836\,864\,z^6+5\,672\,582\,160\,z^4-3\,110\,860\,620\,z^2+92\,708\,054\,145\right)\sinh(z)-12\left(2\,588\,032\,z^8+115\,204\,544\,z^6-1\,401\,711\,080\,z^4+427\,711\,410\,z^2-30\,801\,845\,235\right)\sinh(3\,z)+4\left(278\,912\,z^8+67\,623\,360\,z^6+1\,340\,887\,800\,z^4+1\,630\,969\,200\,z^2+65\,465\,190\,945\right)\sinh(5\,z)-\left(512\,z^8+655\,872\,z^6+102\,063\,360\,z^4-7\,263\,574\,920\,z^2-89\,900\,762\,805\right)\sinh(7\,z)+21\left(3328\,z^6+274\,400\,z^4+81\,521\,280\,z^2+573\,855\,975\right)\sinh(9\,z)-2520\left(1584\,z^4+46\,153\,z^2+76\,823\right)\sinh(11\,z)+25\,200\left(25\,z^2+58\right)\sinh(13\,z)-2520\sinh(15\,z)\Big)$$

09.24.03.0034.01

$$\operatorname{am}^{(0,9)}(z,1) = -\frac{1}{68\,719\,476\,736}\operatorname{sech}^9(z)\left(-4\,787\,214\,336\,z^9 + 99\,787\,355\,136\,z^7 - 1\,257\,474\,347\,136\,z^5 + 4\,018\,881\,938\,400\,z^3 + 32\left(159\,058\,432\,z^8 + 761\,193\,216\,z^6 - 40\,569\,297\,552\,z^4 + 130\,346\,098\,155\,z^2 - 11\,253\,659\,686\,605\right)\cosh(2z)\,z - 4\left(169\,785\,344\,z^8 + 17\,477\,176\,320\,z^6 - 45\,079\,100\,640\,z^4 + 342\,233\,262\,000\,z^2 + 49\,191\,077\,513\,655\right)\cosh(4z)\,z + 72\left(186\,368\,z^8 + 76\,515\,840\,z^6 + 3\,015\,563\,040\,z^4 - 29\,481\,335\,100\,z^2 - 942\,604\,664\,715\right)\cosh(6z)\,z - 8\left(256\,z^8 + 1\,466\,496\,z^6 + 444\,742\,704\,z^4 + 76\,644\,470\,700\,z^2 + 1\,626\,311\,735\,055\right)\cosh(8z)\,z - 72\left(5632\,z^6 - 6\,626\,592\,z^4 + 82\,926\,900\,z^2 + 12\,803\,092\,155\right)\cosh(10z)\,z + 6804\left(14\,112\,z^4 + 825\,040\,z^2 + 5\,052\,795\right)\cosh(12z)\,z - 1\,890\,000\left(22\,z^2 + 183\right)\cosh(14z)\,z + 691\,740\cosh(16z)\,z - 219\,166\,390\,309\,860\,z + 63\left(582\,413\,312\,z^8 - 8\,089\,396\,736\,z^6 + 60\,126\,039\,840\,z^4 - 10\,868\,996\,160\,z^2 + 1\,133\,054\,482\,725\right)\sinh(2z) - 9\left(1\,197\,604\,864\,z^8 + 20\,758\,863\,104\,z^6 - 330\,406\,228\,320\,z^4 - 104\,637\,866\,760\,z^2 - 9\,019\,729\,066\,005\right)\sinh(4z) + 81\left(4\,957\,696\,z^8 + 556\,510\,976\,z^6 + 8\,539\,715\,520\,z^4 + 25\,553\,168\,760\,z^2 + 557\,722\,363\,275\right)\sinh(6z) - 18\left(8192\,z^8 + 11\,273\,472\,z^6 + 1\,306\,045\,440\,z^4 - 64\,966\,849\,920\,z^2 - 725\,821\,080\,075\right)\sinh(8z) + 9\left(512\,z^8 - 2\,109\,184\,z^6 - 350\,232\,960\,z^4 + 20\,508\,301\,800\,z^2 + 167\,613\,554\,115\right)\sinh(10z) - 189\left(20\,736\,z^6 + 5\,220\,000\,z^4 + 99\,037\,080\,z^2 + 141\,823\,235\right)\sinh(12z) + 1890\left(2000\,z^4 + 93\,624\,z^2 + 138\,363\right)\sinh(14z) - 2835\left(56\,z^2 + 275\right)\sinh(16z) + 315\sinh(18z)\right)$$

09.24.03.0035.01

$$\operatorname{am}^{(0,10)}(z,1) = -\frac{1}{1\,099\,511\,627\,776}\left(\operatorname{sech}^{10}(z)\left(60\,z\left(16\,817\,174\,528\,z^8 - 378\,611\,515\,392\,z^6 + 5\,047\,067\,174\,688\,z^4 - 12\,541\,133\,626\,560\,z^2 + 960\,755\,170\,934\,475\right)\cosh(z) - 60\,z\left(16\,540\,175\,360\,z^8 - 76\,926\,124\,800\,z^6 - 1\,472\,601\,799\,872\,z^4 + 2\,012\,536\,486\,800\,z^2 - 670\,184\,901\,344\,385\right)\cosh(3z) + 300\,z\left(499\,297\,792\,z^8 + 21\,757\,585\,152\,z^6 - 107\,571\,721\,824\,z^4 + 936\,659\,223\,360\,z^2 + 63\,704\,433\,212\,343\right)\cosh(5z) - 20\,z\left(150\,906\,880\,z^8 + 28\,037\,781\,504\,z^6 + 876\,500\,345\,952\,z^4 - 9\,984\,229\,229\,520\,z^2 - 291\,923\,536\,963\,905\right)\cosh(7z) + 20\,z\left(28\,672\,z^8 - 4\,995\,072\,z^6 - 684\,639\,648\,z^4 + 2\,103\,990\,139\,440\,z^2 + 50\,122\,724\,308\,335\right)\cosh(9z) + 20\,z\left(512\,z^8 - 13\,918\,464\,z^6 - 5\,450\,947\,488\,z^4 - 50\,866\,492\,320\,z^2 + 3\,115\,889\,546\,175\right)\cosh(11z) - 1080\,z\left(31\,104\,z^6 + 13\,035\,008\,z^4 + 483\,566\,440\,z^2 + 2\,523\,588\,165\right)\cosh(13z) + 18\,900\,z\left(4000\,z^4 + 369\,480\,z^2 + 1\,918\,833\right)\cosh(15z) - 37\,800\,z\left(196\,z^2 + 3369\right)\cosh(17z) + 56\,700\,z\cosh(19z) + \left(284\,316\,852\,224\,z^{10} - 4\,825\,025\,049\,600\,z^8 + 44\,507\,024\,432\,640\,z^6 - 281\,744\,300\,980\,800\,z^4 + 5\,478\,046\,459\,200\,z^2 - 5\,159\,604\,364\,407\,675\right)\sinh(z) - 2\left(44\,457\,926\,656\,z^{10} + 1\,703\,842\,398\,720\,z^8 - 30\,435\,995\,600\,640\,z^6 + 250\,898\,785\,426\,800\,z^4 + 52\,403\,872\,468\,500\,z^2 + 5\,512\,444\,108\,203\,375\right)\sinh(3z) + 20\left(359\,698\,432\,z^{10} + 68\,225\,961\,600\,z^8 + 624\,136\,423\,680\,z^6 - 13\,488\,971\,580\,000\,z^4 - 13\,746\,266\,210\,250\,z^2 - 456\,326\,879\,156\,127\right)\sinh(5z) - \left(80\,580\,608\,z^{10} + 52\,805\,813\,760\,z^8 + 3\,841\,539\,206\,400\,z^6 + 46\,922\,997\,492\,000\,z^4 + 248\,800\,859\,420\,400\,z^2 + 4\,202\,594\,639\,779\,725\right)\sinh(7z) + \left(4096\,z^{10} + 14\,077\,440\,z^8 + 48\,452\,947\,200\,z^6 + 3\,341\,533\,456\,800\,z^4 - 95\,964\,233\,576\,400\,z^2 - 1\,049\,340\,690\,712\,125\right)\sinh(9z) - 2520\left(448\,z^8 - 3\,496\,832\,z^6 - 256\,959\,000\,z^4 + 4\,048\,385\,535\,z^2 + 42\,037\,315\,020\right)\sinh(11z) + 5670\left(186\,624\,z^6 + 19\,053\,520\,z^4 + 278\,440\,340\,z^2 + 360\,007\,785\right)\sinh(13z) - 2835\left(400\,000\,z^4 + 7\,818\,800\,z^2 + 8\,605\,849\right)\sinh(15z) + 1\,587\,600\left(33\,z^2 + 67\right)\sinh(17z) - 113\,400\sinh(19z)\right)\right)$$

General characteristics

Domain and analyticity

$\text{am}(z \mid m)$ is an analytical meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.24.04.0001.01

$$(z * m) \longrightarrow \text{am}(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\text{am}(z \mid m)$ is an odd function with respect to z.

09.24.04.0002.01

$$\text{am}(-z \mid m) = -\text{am}(z \mid m)$$

Mirror symmetry

09.24.04.0003.01

$$\text{am}(\bar{z} \mid \bar{m}) = \overline{\text{am}(z \mid m)}$$

Periodicity

$\text{am}(z \mid m)$ is a periodic function with respect to z with period $2 \, i \, K(1-m)$ and a pseudo-periodic function with respect to z with period $2 \, K(m)$.

09.24.04.0011.01

$$\text{am}(z + 2 \, i \, K(1-m) \mid m) = \text{am}(z \mid m) \,/;\, m > 0$$

09.24.04.0012.01

$$\text{am}(z + 2 \, i \, s \, K(1-m) \mid m) = \text{am}(z \mid m) \,/;\, s \in \mathbb{Z} \land m > 0$$

09.24.04.0013.01

$$\text{am}(z \mid m) = \text{am}\left(z - 2 \left\lfloor \frac{i \, \text{Im}(z)}{2 \, K(1-m)} \right\rfloor K(1-m) \,\middle|\, m\right)$$

09.24.04.0014.01

$$\text{am}(z + 2 \, K(m) \mid m) = \text{am}(z \mid m) + \pi \,/;\, m < 1$$

09.24.04.0015.01

$$\text{am}(z + 2 \, r \, K(m) \mid m) = \text{am}(z \mid m) + r \, \pi \,/;\, r \in \mathbb{Z} \land m < 1$$

09.24.04.0016.01

$$\text{am}(z \mid m) = \text{am}\left(z + 2 \left\lfloor \frac{\text{Re}(z)}{2 \, K(m)} \right\rfloor K(m) \,\middle|\, m\right) - \pi \left\lfloor \frac{\text{Re}(z)}{2 \, K(m)} \right\rfloor \,/;\, m < 1$$

09.24.04.0004.01

$$\text{am}(z + 2 \, r \, K(m) + 2 \, i \, s \, K(1-m) \mid m) = \text{am}(z \mid m) + r \, \pi \,/;\, \{r, s\} \in \mathbb{Z} \bigwedge 0 < m < 1$$

Poles and essential singularities

With respect to *m*

The function $\text{am}(z \mid m)$ does not have poles and essential singularities with respect to m.

09.24.04.0009.01

$$\mathcal{Sing}_m(\text{am}(z \mid m)) = \{\}$$

With respect to *z*

The function $\operatorname{am}(z \mid m)$ does not have poles and essential singularities with respect to z.

09.24.04.0010.01

$$\mathcal{Sing}_z(\operatorname{am}(z \mid m)) == \{\}$$

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

09.24.06.0005.01

$$\operatorname{am}(z \mid m) \propto \operatorname{am}(z_0 \mid m) + \operatorname{dn}(z_0 \mid m)(z - z_0) - \frac{1}{2} m \operatorname{cn}(z_0 \mid m) \operatorname{sn}(z_0 \mid m)(z - z_0)^2 - \frac{1}{6} m \operatorname{dn}(z_0 \mid m)\left(\operatorname{cn}(z_0 \mid m)^2 - \operatorname{sn}(z_0 \mid m)^2\right)(z - z_0)^3 + \frac{1}{24} m \operatorname{cn}(z_0 \mid m) \operatorname{sn}(z_0 \mid m)\left(m \operatorname{cn}(z_0 \mid m)^2 + 4 \operatorname{dn}(z_0 \mid m)^2 - m \operatorname{sn}(z_0 \mid m)^2\right)(z - z_0)^4 + \frac{1}{120} m \operatorname{dn}(z_0 \mid m)\left(m \operatorname{cn}(z_0 \mid m)^4 + 4 \operatorname{dn}(z_0 \mid m)^2 \operatorname{cn}(z_0 \mid m)^2 + m \operatorname{sn}(z_0 \mid m)^4 - 2\left(7 m \operatorname{cn}(z_0 \mid m)^2 + 2 \operatorname{dn}(z_0 \mid m)^2\right) \operatorname{sn}(z_0 \mid m)^2\right)(z - z_0)^5 + \ldots + /; (z \to z_0)$$

09.24.06.0006.01

$$\operatorname{am}(z \mid m) \propto \operatorname{am}(z_0 \mid m)(1 + O(z - z_0))$$

Expansions at $z == 0$

09.24.06.0007.01

$$\operatorname{am}(z \mid m) \propto z - \frac{m z^3}{6} + \ldots /; (z \to 0)$$

09.24.06.0001.02

$$\operatorname{am}(z \mid m) \propto z - \frac{m z^3}{6} + \frac{1}{120}\left(4 m + m^2\right) z^5 + \frac{\left(-16 m - 44 m^2 - m^3\right) z^7}{5040} + \frac{\left(64 m + 912 m^2 + 408 m^3 + m^4\right) z^9}{362\,880} + O\left(z^{11}\right)$$

09.24.06.0008.01

$$\operatorname{am}(z \mid m) \propto z - \frac{m z^3}{6} + \frac{1}{120}\left(4 m + m^2\right) z^5 - \frac{\left(16 m + 44 m^2 + m^3\right) z^7}{5040} + \frac{\left(64 m + 912 m^2 + 408 m^3 + m^4\right) z^9}{362\,880} - \frac{\left(256 m + 15\,808 m^2 + 30\,768 m^3 + 3688 m^4 + m^5\right) z^{11}}{39\,916\,800} + \frac{\left(1024 m + 259\,328 m^2 + 1\,538\,560 m^3 + 870\,640 m^4 + 33\,212 m^5 + m^6\right) z^{13}}{6\,227\,020\,800} - \frac{\left(4096 m + 4\,180\,992 m^2 + 65\,008\,896 m^3 + 106\,923\,008 m^4 + 22\,945\,056 m^5 + 298\,932 m^6 + m^7\right) z^{15}}{1\,307\,674\,368\,000} + \frac{1}{355\,687\,428\,096\,000}\left(\left(m^8 + 2\,690\,416 m^7 + 586\,629\,984 m^6 + 6\,337\,665\,152 m^5 + 9\,860\,488\,448 m^4 + 2\,536\,974\,336 m^3 + 67\,047\,424 m^2 + 16\,384 m\right) z^{17}\right) + \frac{1}{121\,645\,100\,408\,832\,000}\left(-m^9 - 24\,213\,776 m^8 - 14\,804\,306\,080 m^7 - 345\,558\,617\,984 m^6 - 1\,165\,333\,452\,544 m^5 - 782\,931\,974\,144 m^4 - 95\,153\,582\,080 m^3 - 1\,073\,463\,296 m^2 - 65\,536 m\right) z^{19} + O\left(z^{11}\right)$$

09.24.06.0009.01

$$\operatorname{am}(z \mid m) \propto z\left(1 + O\left(z^2\right)\right)$$

Expansions at generic point $m == m_0$

For the function itself

09.24.06.0010.01

$$\operatorname{am}(z \mid m) \propto \operatorname{am}(z \mid m_0) + \frac{(E(\operatorname{am}(z \mid m_0) \mid m_0) + z (m_0 - 1)) \operatorname{dn}(z \mid m_0) - m_0 \operatorname{cn}(z \mid m_0) \operatorname{sn}(z \mid m_0)}{2 (m_0 - 1) m_0} (m - m_0) - \frac{1}{8 (m_0 - 1)^2 m_0^2}\Big(-(E(\operatorname{am}(z \mid m_0) \mid m_0) + z (m_0 - 1)) \sqrt{1 - m_0 \operatorname{sn}(z \mid m_0)^2} \operatorname{dn}(z \mid m_0)^2 + \Big(E(\operatorname{am}(z \mid m_0) \mid m_0)\left(m_0 \operatorname{cn}(z \mid m_0)^2 - 2 m_0 \operatorname{sn}(z \mid m_0)^2 + 3 m_0 - 1\right) + \Big(F(\operatorname{am}(z \mid m_0) \mid m_0) + 2 z (m_0 - 1) + z\left(\operatorname{cn}(z \mid m_0)^2 - 2 \operatorname{sn}(z \mid m_0)^2\right) m_0\Big)(m_0 - 1) + \operatorname{cn}(z \mid m_0) \operatorname{sn}(z \mid m_0) \sqrt{1 - m_0 \operatorname{sn}(z \mid m_0)^2}\, m_0\Big) \operatorname{dn}(z \mid m_0) + \operatorname{cn}(z \mid m_0) \operatorname{sn}(z \mid m_0)\left(-m_0 \operatorname{cn}(z \mid m_0)^2 + z^2 (m_0 - 1)^2 + E(\operatorname{am}(z \mid m_0) \mid m_0)(E(\operatorname{am}(z \mid m_0) \mid m_0) + 2 z (m_0 - 1)) + \operatorname{sn}(z \mid m_0)^2 m_0 - 2 m_0\right) m_0\Big) (m - m_0)^2 + \ldots /; (m \to m_0)$$

09.24.06.0011.01

$$\operatorname{am}(z \mid m) \propto \operatorname{am}(z \mid m_0)(1 + O(m - m_0))$$

Expansions at $m == 0$

09.24.06.0012.01

$$\operatorname{am}(z \mid m) \propto z + \frac{1}{8}(\sin(2 z) - 2 z) m + \frac{1}{256}(-16 \cos(2 z) z - 20 z + 16 \sin(2 z) + \sin(4 z)) m^2 + \ldots /; (m \to 0)$$

09.24.06.0013.01

$$\operatorname{am}(z \mid m) \propto z + \frac{1}{8} (\sin(2z) - 2z)\, m + \frac{1}{256} (-16\cos(2z)\, z - 20z + 16\sin(2z) + \sin(4z))\, m^2 + \frac{1}{6144} \left(-312\cos(2z)\, z - 24\cos(4z)\, z - 264z + \left(249 - 96z^2\right) \sin(2z) + 24\sin(4z) + \sin(6z)\right) m^3 + \frac{1}{393\,216} \left(32\left(32z^2 - 501\right)\cos(2z)\, z - 2016\cos(4z)\, z - 96\cos(6z)\, z - 11\,256z - 96\left(72z^2 - 121\right)\sin(2z) - 48\left(16z^2 - 29\right)\sin(4z) + 96\sin(6z) + 3\sin(8z)\right) m^4 + \frac{1}{7\,864\,320} \left(160\left(184z^2 - 1653\right)\cos(2z)\, z + 160\left(32z^2 - 267\right)\cos(4z)\, z - 3480\cos(6z)\, z - 120\cos(8z)\, z - 165\,480z + 5\left(512z^4 - 26\,496z^2 + 36\,213\right)\sin(2z) - 24\,960\left(z^2 - 1\right)\sin(4z) - 60\left(24z^2 - 37\right)\sin(6z) + 120\sin(8z) + 3\sin(10z)\right) m^5 + \frac{1}{251\,658\,240} \left(-16\left(512z^4 - 65\,600z^2 + 447\,465\right)\cos(2z)\, z + 80\left(3968z^2 - 16\,983\right)\cos(4z)\, z + 7680\left(3z^2 - 19\right)\cos(6z)\, z - 8880\cos(8z)\, z - 240\cos(10z)\, z - 4\,133\,520z + 80\left(1792z^4 - 48\,960z^2 + 58\,953\right)\sin(2z) + 5\left(8192z^4 - 193\,920z^2 + 143\,865\right)\sin(4z) - 320\left(306z^2 - 235\right)\sin(6z) - 120\left(32z^2 - 45\right)\sin(8z) + 240\sin(10z) + 5\sin(12z)\right) m^6 + \frac{1}{42\,278\,584\,320} \left(-336\left(8448z^4 - 537\,680z^2 + 3\,090\,765\right)\cos(2z)\, z - 168\left(8192z^4 - 434\,560z^2 + 1\,317\,345\right)\cos(4z)\, z + 7560\left(1248z^2 - 3755\right)\cos(6z)\, z + 3360\left(128z^2 - 693\right)\cos(8z)\, z - 113\,400\cos(10z)\, z - 2520\cos(12z)\, z - 564\,490\,080z - 7\left(16\,384z^6 - 4\,369\,920z^4 + 86\,129\,280z^2 - 94\,986\,225\right)\sin(2z) + 2520\left(6144z^4 - 70\,672z^2 + 43\,513\right)\sin(4z) + 1890\left(768z^4 - 12\,752z^2 + 6803\right)\sin(6z) - 20\,160\left(84z^2 - 55\right)\sin(8z) - 1260\left(40z^2 - 53\right)\sin(10z) + 2520\sin(12z) + 45\sin(14z)\right) m^7 + \frac{1}{5\,411\,658\,792\,960} \left(64\left(16\,384z^6 - 8\,096\,256z^4 + 357\,215\,040z^2 - 1\,822\,886\,415\right)\cos(2z)\, z - 1344\left(335\,872z^4 - 8\,302\,080z^2 + 20\,225\,565\right)\cos(4z)\, z - 72\,576\left(768z^4 - 26\,960z^2 + 54\,545\right)\cos(6z)\, z + 6720\left(24\,064z^2 - 58\,671\right)\cos(8z)\, z + 53\,760\left(100z^2 - 489\right)\cos(10z)\, z - 1\,068\,480\cos(12z)\, z - 20\,160\cos(14z)\, z - 60\,474\,299\,760z - 448\left(77\,824z^6 - 9\,841\,920z^4 + 157\,888\,080z^2 - 163\,063\,575\right)\sin(2z) - 224\left(131\,072z^6 - 13\,486\,080z^4 + 105\,953\,040z^2 - 57\,287\,025\right)\sin(4z) + 120\,960\left(4224z^4 - 32\,100z^2 + 13\,573\right)\sin(6z) + 1680\left(16\,384z^4 - 221\,376z^2 + 96\,087\right)\sin(8z) - 80\,640\left(250z^2 - 147\right)\sin(10z) - 10\,080\left(48z^2 - 61\right)\sin(12z) + 20\,160\sin(14z) + 315\sin(16z)\right) m^8 + \frac{1}{10\,823\,317\,585\,920} \left(8\left(704\,512z^6 - 159\,108\,096z^4 + 5\,555\,289\,600z^2 - 25\,939\,014\,255\right)\cos(2z)\, z + 32\left(262\,144z^6 - 47\,394\,816z^4 + 777\,023\,520z^2 - 1\,623\,372\,345\right)\cos(4z)\, z - 1008\left(338\,688z^4 - 5\,280\,000z^2 + 8\,267\,935\right)\cos(6z)\, z - 672\left(32\,768z^4 - 900\,480z^2 + 1\,420\,635\right)\cos(8z)\, z + 840\left(44\,000z^2 - 93\,147\right)\cos(10z)\, z + 10\,080\left(96z^2 - 437\right)\cos(12z)\, z - 153\,720\cos(14z)\, z - 2520\cos(16z)\, z - 103\,483\,364\,040z + 2\left(65\,536z^8 - 55\,164\,928z^6 + 4\,684\,162\,560z^4 - 65\,048\,634\,000z^2 + 63\,806\,773\,905\right)\sin(2z) - 896\left(188\,416z^6 - 8\,736\,000z^4 + 53\,735\,895z^2 - 26\,369\,730\right)\sin(4z) - 21\left(1\,327\,104z^6 - 86\,400\,000z^4 + 429\,920\,640z^2 - 154\,280\,035\right)\sin(6z) + 13\,440\left(13\,312z^4 - 78\,822z^2 + 26\,199\right)\sin(8z) + 210\left(32\,000z^4 - 377\,520z^2 + 142\,083\right)\sin(10z) - 13\,440\left(261z^2 - 142\right)\sin(12z) - 1260\left(56z^2 - 69\right)\sin(14z) + 2520\sin(16z) + 35\sin(18z)\right) m^9 + \frac{1}{3\,117\,115\,464\,744\,960} \left(-32\left(65\,536z^8 - 88\,178\,688z^6 + 13\,025\,263\,104z^4 - 385\,499\,383\,920z^2 + 1\,679\,120\,073\,855\right)\cos(2z)\, z + 1728\left(4\,456\,448z^6 - 353\,282\,048z^4 + 4\,437\,323\,520z^2 - 8\,261\,810\,235\right)\cos(4z)\, z + 1296\left(1\,327\,104z^6 - 146\,506\,752z^4 + 1\,462\,728\,960z^2 - 1\,908\,968\,285\right)\cos(6z)\, z - \right.$$

$36\,288\left(622\,592\,z^4-7\,329\,280\,z^2+8\,693\,825\right)\cos(8\,z)\,z-$
$30\,240\left(32\,000\,z^4-746\,800\,z^2+993\,189\right)\cos(10\,z)\,z+45\,360\left(24\,192\,z^2-46\,253\right)\cos(12\,z)\,z+$
$241\,920\left(98\,z^2-423\right)\cos(14\,z)\,z-3\,129\,840\cos(16\,z)\,z-45\,360\cos(18\,z)\,z-25\,942\,107\,537\,360\,z+$
$864\left(131\,072\,z^8-48\,943\,104\,z^6+3\,212\,948\,480\,z^4-40\,049\,795\,940\,z^2+37\,672\,070\,835\right)\sin(2\,z)+$
$9\left(33\,554\,432\,z^8-9\,916\,383\,232\,z^6+297\,656\,647\,680\,z^4-1\,539\,489\,047\,040\,z^2+700\,334\,220\,285\right)\sin(4\,z)-$
$27\,216\left(995\,328\,z^6-28\,070\,400\,z^4+105\,973\,040\,z^2-33\,616\,995\right)\sin(6\,z)-$
$8064\left(262\,144\,z^6-12\,948\,480\,z^4+48\,613\,140\,z^2-13\,355\,685\right)\sin(8\,z)+$
$18\,144\left(400\,000\,z^4-2\,000\,900\,z^2+561\,453\right)\sin(10\,z)+11\,340\left(18\,432\,z^4-197\,472\,z^2+66\,917\right)\sin(12\,z)-$
$544\,320\left(154\,z^2-79\right)\sin(14\,z)-22\,680\left(64\,z^2-77\right)\sin(16\,z)+45\,360\sin(18\,z)+567\sin(20\,z)\Big)\,m^{10}+O\left(m^{11}\right)$

09.24.06.0014.01

$$\operatorname{am}(z\mid m)\propto z(1+O[m])$$

Expansions at $m == 1$

09.24.06.0015.01

$$\operatorname{am}(z\mid m)\propto 2\tan^{-1}(e^z)-\frac{\pi}{2}+\frac{1}{4}(z\operatorname{sech}(z)-\sinh(z))\,(m-1)+\frac{1}{64}(-4\,z\cosh(z)+9\sinh(z)-z\operatorname{sech}(z)\,(2\,z\tanh(z)+5))\,(m-1)^2+\ldots\;/;\;(m\to 1)$$

09.24.06.0016.01

$$\operatorname{am}(z\mid m)\propto 2\tan^{-1}(e^z)-\frac{\pi}{2}+\frac{1}{4}(z\operatorname{sech}(z)-\sinh(z))\,(m-1)+$$
$$\frac{1}{64}(-4\,z\cosh(z)+9\sinh(z)-z\operatorname{sech}(z)\,(2\,z\tanh(z)+5))\,(m-1)^2+\frac{1}{24\,576}\operatorname{sech}^3(z)\Big(-96\,z^3+16\left(2\,z^2+75\right)\cosh(2\,z)\,z+$$
$$168\cosh(4\,z)\,z+1032\,z+3\left(64\,z^2-201\right)\sinh(2\,z)-12\left(2\,z^2+25\right)\sinh(4\,z)+\sinh(6\,z)\Big)\,(m-1)^3+$$
$$\frac{1}{786\,432}\operatorname{sech}^4(z)\Big(104\,z\left(20\,z^2-381\right)\cosh(z)-4\,z\left(160\,z^2+4239\right)\cosh(3\,z)-8\,z\left(4\,z^2+279\right)\cosh(5\,z)+12\,z\cosh(7\,z)+$$
$$2\left(368\,z^4-900\,z^2+3711\right)\sinh(z)-\left(32\,z^4+1344\,z^2-11\,097\right)\sinh(3\,z)+3\left(152\,z^2+1217\right)\sinh(5\,z)-24\sinh(7\,z)\Big)$$
$$(m-1)^4-\frac{1}{125\,829\,120}\operatorname{sech}^5(z)\Big(-14\,720\,z^5+127\,200\,z^3+8\left(1216\,z^4+9360\,z^2-457\,725\right)\cosh(2\,z)\,z-$$
$$16\left(8\,z^4+3510\,z^2+78\,525\right)\cosh(4\,z)\,z-120\left(32\,z^2+1237\right)\cosh(6\,z)\,z+1740\cosh(8\,z)\,z-$$
$$2\,555\,580\,z+40\left(1780\,z^4-3789\,z^2+30\,099\right)\sinh(2\,z)-10\left(256\,z^4+2304\,z^2-95\,697\right)\sinh(4\,z)+$$
$$5\left(32\,z^4+6936\,z^2+46\,743\right)\sinh(6\,z)-45\left(8\,z^2+49\right)\sinh(8\,z)+3\sinh(10\,z)\Big)\;(m-1)^5-$$
$$\frac{1}{12\,079\,595\,520}\operatorname{sech}^6(z)\Big(12\,288\cosh(5\,z)\,z^5+384\cosh(7\,z)\,z^5+2\,792\,640\cosh(5\,z)\,z^3+$$
$$186\,720\cosh(7\,z)\,z^3-4320\cosh(9\,z)\,z^3+360\left(4144\,z^4-32\,816\,z^2+981\,117\right)\cosh(z)\,z-$$
$$864\left(824\,z^4-310\,z^2-229\,865\right)\cosh(3\,z)\,z+57\,061\,260\cosh(5\,z)\,z+5\,960\,160\cosh(7\,z)\,z-$$
$$108\,360\cosh(9\,z)\,z+180\cosh(11\,z)\,z+\left(430\,592\,z^6-3\,512\,160\,z^4+4\,667\,040\,z^2-48\,694\,590\right)\sinh(z)+$$
$$\left(-60\,672\,z^6-3\,411\,360\,z^4+4\,689\,360\,z^2-87\,336\,540\right)\sinh(3\,z)+\left(256\,z^6+86\,880\,z^4-1\,469\,520\,z^2-48\,023\,100\right)$$
$$\sinh(5\,z)+\left(-13\,920\,z^4-1\,455\,120\,z^2-9\,270\,135\right)\sinh(7\,z)+\left(36\,720\,z^2+110\,655\right)\sinh(9\,z)-360\sinh(11\,z)\Big)$$
$$(m-1)^6+\frac{1}{2\,705\,829\,396\,480}\operatorname{sech}^7(z)\Big(-12\,056\,576\,z^7+197\,756\,160\,z^5-1\,013\,947\,200\,z^3+$$
$$8\left(1\,349\,504\,z^6+13\,115\,424\,z^4-113\,929\,200\,z^2+6\,658\,689\,555\right)\cosh(2\,z)\,z-$$
$$4\left(184\,832\,z^6+22\,732\,416\,z^4-98\,552\,160\,z^2-6\,182\,430\,345\right)\cosh(4\,z)\,z+$$
$$4\left(256\,z^6+498\,624\,z^4+77\,437\,080\,z^2+1\,527\,136\,695\right)\cosh(6\,z)\,z+$$

$$672\left(136\,z^4+25\,110\,z^2+835\,245\right)\cosh(8\,z)\,z-1260\left(936\,z^2+10\,951\right)\cosh(10\,z)\,z+56\,700\cosh(12\,z)\,z+$$
$$34\,073\,202\,240\,z+7\left(11\,094\,016\,z^6-107\,246\,400\,z^4+91\,432\,800\,z^2-1\,880\,382\,285\right)\sinh(2\,z)-$$
$$7\left(1\,816\,064\,z^6+51\,792\,000\,z^4-12\,252\,600\,z^2+1\,869\,500\,115\right)\sinh(4\,z)+$$
$$21\left(2048\,z^6+310\,560\,z^4-16\,492\,800\,z^2-263\,197\,425\right)\sinh(6\,z)-28\left(64\,z^6+50\,160\,z^4+4\,828\,500\,z^2+31\,327\,965\right)$$
$$\sinh(8\,z)+315\left(288\,z^4+18\,848\,z^2+39\,353\right)\sinh(10\,z)-315\left(40\,z^2+211\right)\sinh(12\,z)+45\sinh(14\,z)\big)(m-1)^7+$$

$$\frac{1}{86\,586\,540\,687\,360}\operatorname{sech}^8(z)\left(2016\,z\left(231\,136\,z^6-4\,250\,076\,z^4+17\,801\,835\,z^2-846\,574\,260\right)\cosh(z)-\right.$$
$$28\,z\left(12\,573\,952\,z^6+9\,650\,016\,z^4-138\,963\,600\,z^2+39\,183\,167\,835\right)\cosh(3\,z)+$$
$$32\,z\left(806\,368\,z^6+47\,006\,064\,z^4-345\,628\,500\,z^2-13\,568\,817\,465\right)\cosh(5\,z)-$$
$$32\,z\left(1408\,z^6+1\,082\,844\,z^4+146\,896\,050\,z^2+2\,933\,497\,035\right)\cosh(7\,z)-$$
$$4\,z\left(256\,z^6+99\,456\,z^4+41\,380\,920\,z^2+1\,894\,043\,655\right)\cosh(9\,z)+$$
$$84\,z\left(2592\,z^4+355\,960\,z^2+2\,790\,165\right)\cosh(11\,z)-3360\,z\left(25\,z^2+486\right)\cosh(13\,z)+1260\,z\cosh(15\,z)+$$
$$2\left(66\,489\,088\,z^8-826\,836\,864\,z^6+5\,672\,582\,160\,z^4-3\,110\,860\,620\,z^2+92\,708\,054\,145\right)\sinh(z)-$$
$$12\left(2\,588\,032\,z^8+115\,204\,544\,z^6-1\,401\,711\,080\,z^4+427\,711\,410\,z^2-30\,801\,845\,235\right)\sinh(3\,z)+$$
$$4\left(278\,912\,z^8+67\,623\,360\,z^6+1\,340\,887\,800\,z^4+1\,630\,969\,200\,z^2+65\,465\,190\,945\right)\sinh(5\,z)-$$
$$\left(512\,z^8+655\,872\,z^6+102\,063\,360\,z^4-7\,263\,574\,920\,z^2-89\,900\,762\,805\right)\sinh(7\,z)+$$
$$21\left(3328\,z^6+274\,400\,z^4+81\,521\,280\,z^2+573\,855\,975\right)\sinh(9\,z)-$$
$$\left.2520\left(1584\,z^4+46\,153\,z^2+76\,823\right)\sinh(11\,z)+25\,200\left(25\,z^2+58\right)\sinh(13\,z)-2520\sinh(15\,z)\right)(m-1)^8-$$

$$\frac{1}{24\,936\,923\,717\,959\,680}\left(\operatorname{sech}^9(z)\left(-4\,787\,214\,336\,z^9+99\,787\,355\,136\,z^7-1\,257\,474\,347\,136\,z^5+4\,018\,881\,938\,400\,z^3+\right.\right.$$
$$32\left(159\,058\,432\,z^8+761\,193\,216\,z^6-40\,569\,297\,552\,z^4+130\,346\,098\,155\,z^2-11\,253\,659\,686\,605\right)\cosh(2\,z)\,z-$$
$$4\left(169\,785\,344\,z^8+17\,477\,176\,320\,z^6-45\,079\,100\,640\,z^4+342\,233\,262\,000\,z^2+49\,191\,077\,513\,655\right)\cosh(4\,z)\,z+$$
$$72\left(186\,368\,z^8+76\,515\,840\,z^6+3\,015\,563\,040\,z^4-29\,481\,335\,100\,z^2-942\,604\,664\,715\right)\cosh(6\,z)\,z-$$
$$8\left(256\,z^8+1\,466\,496\,z^6+444\,742\,704\,z^4+76\,644\,470\,700\,z^2+1\,626\,311\,735\,055\right)\cosh(8\,z)\,z-72$$
$$\left(5632\,z^6-6\,626\,592\,z^4+82\,926\,900\,z^2+12\,803\,092\,155\right)\cosh(10\,z)\,z+6804\left(14\,112\,z^4+825\,040\,z^2+5\,052\,795\right)$$
$$\cosh(12\,z)\,z-1\,890\,000\left(22\,z^2+183\right)\cosh(14\,z)\,z+691\,740\cosh(16\,z)\,z-219\,166\,390\,309\,860\,z+$$
$$63\left(582\,413\,312\,z^8-8\,089\,396\,736\,z^6+60\,126\,039\,840\,z^4-10\,868\,996\,160\,z^2+1\,133\,054\,482\,725\right)\sinh(2\,z)-$$
$$9\left(1\,197\,604\,864\,z^8+20\,758\,863\,104\,z^6-330\,406\,228\,320\,z^4-104\,637\,866\,760\,z^2-9\,019\,729\,066\,005\right)\sinh(4\,z)+$$
$$81\left(4\,957\,696\,z^8+556\,510\,976\,z^6+8\,539\,715\,520\,z^4+25\,553\,168\,760\,z^2+557\,722\,363\,275\right)\sinh(6\,z)-$$
$$18\left(8192\,z^8+11\,273\,472\,z^6+1\,306\,045\,440\,z^4-64\,966\,849\,920\,z^2-725\,821\,080\,075\right)\sinh(8\,z)+$$
$$9\left(512\,z^8-2\,109\,184\,z^6-350\,232\,960\,z^4+20\,508\,301\,800\,z^2+167\,613\,554\,115\right)\sinh(10\,z)-$$
$$189\left(20\,736\,z^6+5\,220\,000\,z^4+99\,037\,080\,z^2+141\,823\,235\right)\sinh(12\,z)+$$
$$\left.\left.1890\left(2000\,z^4+93\,624\,z^2+138\,363\right)\sinh(14\,z)-2835\left(56\,z^2+275\right)\sinh(16\,z)+315\sinh(18\,z)\right)\right)(m-1)^9$$

$$\frac{1}{3\,989\,907\,794\,873\,548\,800}\left(\operatorname{sech}^{10}(z)\left(60\,z\left(16\,817\,174\,528\,z^8-378\,611\,515\,392\,z^6+5\,047\,067\,174\,688\,z^4-\right.\right.\right.$$
$$\left.12\,541\,133\,626\,560\,z^2+960\,755\,170\,934\,475\right)\cosh(z)-60\,z\left(16\,540\,175\,360\,z^8-76\,926\,124\,800\,z^6-\right.$$
$$\left.1\,472\,601\,799\,872\,z^4+2\,012\,536\,486\,800\,z^2-670\,184\,901\,344\,385\right)\cosh(3\,z)+300\,z$$
$$\left(499\,297\,792\,z^8+21\,757\,585\,152\,z^6-107\,571\,721\,824\,z^4+936\,659\,223\,360\,z^2+63\,704\,433\,212\,343\right)\cosh(5\,z)-$$
$$20\,z\left(150\,906\,880\,z^8+28\,037\,781\,504\,z^6+876\,500\,345\,952\,z^4-9\,984\,229\,229\,520\,z^2-291\,923\,536\,963\,905\right)$$
$$\cosh(7\,z)+20\,z\left(28\,672\,z^8-4\,995\,072\,z^6-684\,639\,648\,z^4+2\,103\,990\,139\,440\,z^2+50\,122\,724\,308\,335\right)$$
$$\cosh(9\,z)+20\,z\left(512\,z^8-13\,918\,464\,z^6-5\,450\,947\,488\,z^4-50\,866\,492\,320\,z^2+3\,115\,889\,546\,175\right)\cosh(11\,z)-$$
$$1080\,z\left(31\,104\,z^6+13\,035\,008\,z^4+483\,566\,440\,z^2+2\,523\,588\,165\right)\cosh(13\,z)+$$
$$18\,900\,z\left(4000\,z^4+369\,480\,z^2+1\,918\,833\right)\cosh(15\,z)-37\,800\,z\left(196\,z^2+3369\right)\cosh(17\,z)+56\,700\,z\cosh(19\,z)+$$

$$\left(284316852224 z^{10}-4825025049600 z^8+44507024432640 z^6-281744300980800 z^4+5478046459200 z^2-5159604364407675\right) \sinh(z)-2\left(44457926656 z^{10}+1703842398720 z^8-30435995600640 z^6+250898785426800 z^4+52403872468500 z^2+5512444108203375\right) \sinh(3 z)+20\left(359698432 z^{10}+68225961600 z^8+624136423680 z^6-13488971580000 z^4-13746266210250 z^2-456326879156127\right) \sinh(5 z)-\left(80580608 z^{10}+52805813760 z^8+3841539206400 z^6+46922997492000 z^4+248800859420400 z^2+4202594639779725\right) \sinh(7 z)+\left(4096 z^{10}+14077440 z^8+48452947200 z^6+3341533456800 z^4-95964233576400 z^2-1049340690712125\right) \sinh(9 z)-2520\left(448 z^8-3496832 z^6-256959000 z^4+4048385535 z^2+42037315020\right) \sinh(11 z)+5670\left(186624 z^6+19053520 z^4+278440340 z^2+360007785\right) \sinh(13 z)-2835\left(400000 z^4+7818800 z^2+8605849\right) \sinh(15 z)+1587600\left(33 z^2+67\right) \sinh(17 z)-113400 \sinh(19 z)\big)(m-1)^{10}\Big)+O\left((m-1)^{11}\right)$$

09.24.06.0017.01

$$\operatorname{am}(z \mid m) \propto 2 \tan^{-1}(e^z)-\frac{\pi}{2}+O(m-1)$$

q-series

09.24.06.0002.01

$$\operatorname{am}(z \mid m)=\frac{\pi z}{2 K(m)}+2 \sum_{k=1}^{\infty} \frac{q(m)^k}{k\left(q(m)^{2k}+1\right)} \sin\left(\frac{k \pi z}{K(m)}\right)$$

Other series representations

09.24.06.0003.01

$$\operatorname{am}(z \mid m)=2 \sum_{k=-\infty}^{\infty} \tan^{-1}\left(\tanh\left(\frac{\pi K(m)}{2 K(1-m)}\left(k+\frac{z}{2 K(m)}\right)\right)\right)$$

09.24.06.0004.01

$$\operatorname{am}(z \mid m)=2 \sum_{k=-\infty}^{\infty} \tan^{-1}\left(\tanh\left(\frac{\pi z}{4 K(1-m)}+\frac{k \pi K(m)}{2 K(1-m)}\right)\right)$$

Integral representations

On the real axis

09.24.07.0001.01

$$\operatorname{am}(z \mid m)=\int_0^z \operatorname{dn}(t \mid m)\, dt$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

09.24.13.0001.01

$$\frac{\partial w(z)}{\partial z} - \mathrm{dn}(z \mid m) = 0 \;/;\; w(z) = \mathrm{am}(z \mid m)$$

Ordinary nonlinear differential equations

09.24.13.0002.01

$$w''(z)^2 + \left(w'(z)^2 + m - 1\right)\left(w'(z)^2 - 1\right) = 0 \;/;\; w(z) = \mathrm{am}(z \mid m)$$

09.24.13.0003.01

$$-w'(z)^5 + 2\, w'(z)^3 + w''(z)^2\, w'(z) - w'(z) + \left(1 - w'(z)^2\right) w^{(3)}(z) = 0 \;/;\; w(z) = \mathrm{am}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.24.16.0001.01

$$\mathrm{am}\left(\sqrt{1-m}\; z \;\middle|\; \frac{m}{m-1}\right) = \frac{\pi}{2} - \mathrm{am}(K(m) - z \mid m) \;/;\; m \notin (1, \infty)$$

09.24.16.0002.01

$$\mathrm{am}\left(\sqrt{1-m}\; z \;\middle|\; \frac{m}{m-1}\right) = -\frac{\pi}{2} + \mathrm{am}(K(m) - z \mid m) \;/;\; m > 1$$

Argument involving inverse Jacobi functions

Argument involving numeric multiples of inverse trigonometric and hyperbolic functions

09.24.16.0006.01

$$\mathrm{am}\left(\mathrm{sn}^{-1}(z \mid m) \,\middle|\, m\right) = \sin^{-1}(z)$$

Related transformations

09.24.16.0003.01

$$\cos(\mathrm{am}(z \mid m)) = \mathrm{cn}(z \mid m)$$

09.24.16.0004.01

$$\sin(\mathrm{am}(z \mid m)) = \mathrm{sn}(z \mid m)$$

09.24.16.0005.01

$$\cot^2\left(\mathrm{am}\left(-\frac{\sqrt{2}\; z}{\sqrt{\frac{a-\sqrt{a^2-4b}}{b}}} \;\middle|\; \frac{2\sqrt{a^2-4b}}{a+\sqrt{a^2-4b}}\right)\right) = \frac{2x}{a+\sqrt{a^2-4b}} \;/;\; \{x, y\} = \mathrm{eexp}(z; a, b)$$

Differentiation

Low-order differentiation

With respect to *z*

09.24.20.0001.01

$$\frac{\partial \operatorname{am}(z \mid m)}{\partial z} = \operatorname{dn}(z \mid m)$$

09.24.20.0002.01

$$\frac{\partial^2 \operatorname{am}(z \mid m)}{\partial z^2} = -m \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m)$$

With respect to *m*

09.24.20.0003.01

$$\frac{\partial \operatorname{am}(z \mid m)}{\partial m} = \frac{((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{dn}(z \mid m) - m \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m)}{2 (m-1) m}$$

09.24.20.0007.01

$$\frac{\partial \operatorname{am}(z \mid m)}{\partial m} = \frac{(z - \Pi(m; \operatorname{am}(z \mid m) \mid m)) \operatorname{dn}(z \mid m)}{2 m}$$

09.24.20.0008.01

$$\frac{\partial \operatorname{am}(z \mid m)}{\partial m} = \frac{\operatorname{dn}(z \mid m)}{2 (m-1) m} \left(E(\operatorname{am}(z \mid m) \mid m) + (m-1) F(\operatorname{am}(z \mid m) \mid m) - \frac{m \operatorname{cn}(z \mid m)}{\operatorname{ds}(z \mid m)}\right)$$

09.24.20.0004.01

$$\frac{\partial^2 \operatorname{am}(z \mid m)}{\partial m^2} = \frac{1}{4 (m-1)^2 m^2} \Big(((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \sqrt{1 - m \operatorname{sn}(z \mid m)^2} \operatorname{dn}(z \mid m)^2 + \Big(-m^2 \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)^3 + m ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{sn}(z \mid m)^2 + m \operatorname{cn}(z \mid m) \left(((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{sc}(z \mid m) - \sqrt{1 - m \operatorname{sn}(z \mid m)^2} \right) \operatorname{sn}(z \mid m) - 2 m^2 z + 4 m z - 2 z - 3 m E(\operatorname{am}(z \mid m) \mid m) + E(\operatorname{am}(z \mid m) \mid m) - m F(\operatorname{am}(z \mid m) \mid m) + F(\operatorname{am}(z \mid m) \mid m) + m \operatorname{cn}(z \mid m)^2 (-m z + z - E(\operatorname{am}(z \mid m) \mid m) + m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)) \Big) \operatorname{dn}(z \mid m) - m \left(m^2 z^2 + z^2 + 2 (m-1) E(\operatorname{am}(z \mid m) \mid m) z + E(\operatorname{am}(z \mid m) \mid m)^2 - 2 m \left(z^2 + 1\right)\right) \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m) \Big)$$

09.24.20.0009.01

$$\frac{\partial^2 \operatorname{am}(z \mid m)}{\partial m^2} = -\frac{1}{4 (m-1) m^2} \Big(m (z - 2 \Pi(m; \operatorname{am}(z \mid m) \mid m)) \operatorname{dn}(z \mid m) + 2 (m-1) (z - \Pi(m; \operatorname{am}(z \mid m) \mid m)) \operatorname{nd}(z \mid m) + m \operatorname{cn}(z \mid m) \left((m-1) (z - \Pi(m; \operatorname{am}(z \mid m) \mid m))^2 + \operatorname{nd}(z \mid m)^2\right) \operatorname{sn}(z \mid m)\Big)$$

Symbolic differentiation

With respect to *z*

09.24.20.0010.01

$$\frac{\partial^n \operatorname{am}(z \mid m)}{\partial z^n} = \operatorname{dn}(z \mid m) \delta_{n-1} + \operatorname{am}(z \mid m) \delta_n - m \sum_{j=0}^{n-2} \binom{n-2}{j} \frac{\partial^j \operatorname{sn}(z \mid m)}{\partial z^j} \frac{\partial^{-j+n-2} \operatorname{cn}(z \mid m)}{\partial z^{-j+n-2}} \;/;\; n \in \mathbb{N}$$

09.24.20.0005.02

$$\frac{\partial^n \operatorname{am}(z \mid m)}{\partial z^n} = \frac{\pi (2-n)_n z^{1-n}}{2 K(m)} + \frac{2 \pi^n}{K(m)^n} \sum_{k=1}^{\infty} \frac{k^{n-1} q(m)^k}{q(m)^{2k}+1} \sin\left(\frac{\pi n}{2} + \frac{k \pi z}{K(m)}\right) /; n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to z

09.24.20.0006.01

$$\frac{\partial^\alpha \operatorname{am}(z \mid m)}{\partial z^\alpha} = \frac{\pi z^{1-\alpha}}{2 K(m) \Gamma(2-\alpha)} - \frac{2^\alpha \pi^{3/2} z^{1-\alpha}}{K(m)} \sum_{k=1}^{\infty} \frac{q(m)^k}{q(m)^{2k}+1} {}_1\tilde{F}_2\left(1; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{k^2 \pi^2 z^2}{4 K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.24.21.0001.01

$$\int \operatorname{am}(z \mid m) \, dz = \frac{\pi z^2}{4 K(m)} - \frac{2 K(m)}{\pi} \sum_{k=1}^{\infty} \frac{q(m)^k}{k \left(q(m)^{2k}+1\right)} \left(\cos\left(\frac{k \pi z}{K(m)}\right) - 1\right)$$

Representations through equivalent functions

With inverse function

09.24.27.0001.02

$$\operatorname{am}(F(z \mid m) \mid m) = z$$

09.24.27.0002.02

$$F(\operatorname{am}(z \mid m) \mid m) = z - 2 \left\lfloor \frac{i \operatorname{Im}(z)}{2 K(1-m)} \right\rceil K(1-m) /; 0 < m < 1$$

With related functions

Involving one other Jacobi elliptic function

Involving sn

09.24.27.0003.01

$$\operatorname{am}(z \mid m) = (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil + \left\lfloor \frac{\operatorname{Re}\left(\sin^{-1}(\operatorname{sn}(z|m))\right)}{\pi} \right\rceil} \sin^{-1}(\operatorname{sn}(z \mid m)) + \pi \left(\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rceil + \left\lfloor \frac{\operatorname{Re}\left(\sin^{-1}(\operatorname{sn}(z \mid m))\right)}{\pi} \right\rceil\right)$$

09.24.27.0004.01

$$\operatorname{am}(z \mid m) = (-1)^{\left\lfloor \frac{\operatorname{Re}(\operatorname{am}(z|m))}{\pi} \right\rceil} \sin^{-1}(\operatorname{sn}(z \mid m)) + \pi \left\lfloor \frac{\operatorname{Re}(\operatorname{am}(z \mid m))}{\pi} \right\rceil$$

Zeros

09.24.30.0001.01

$\operatorname{am}(2\,s\,i\,K(1-m)\mid m) == 0\ /;\ s\in\mathbb{Z}$

History

- C. G. J. Jacobi (1827)
– V. A. Puiseux (1850) showed am is multivalued

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