JacobiDN

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Notations

Traditional name

Jacobi elliptic function dn

Traditional notation

$\mathrm{dn}(z \mid m)$

Mathematica StandardForm notation

`JacobiDN[z, m]`

Primary definition

09.29.02.0001.01

$$\mathrm{dn}(z \mid m) == \frac{\partial \mathrm{am}(z \mid m)}{\partial z}$$

Specific values

Specialized values

For fixed *z*

Case *m* = 0

09.29.03.0001.01

$$\mathrm{dn}(z \mid 0) == 1$$

Case *m* = 1

09.29.03.0002.01

$$\mathrm{dn}(z \mid 1) == \mathrm{sech}(z)$$

09.29.03.0003.01

$$\mathrm{dn}\left(z + \frac{\pi i}{2} \middle| 1\right) == -i\,\mathrm{csch}(z)$$

09.29.03.0029.01

$$\mathrm{dn}\left(z + \frac{i\pi k}{2} \middle| 1\right) == \mathrm{sech}\left(z + \frac{i\pi k}{2}\right) /; k \in \mathbb{Z}$$

For fixed m

Values at quarter-period points in the fundamental period parallelogram

09.29.03.0004.01

$$\mathrm{dn}(0 \mid m) == 1$$

09.29.03.0005.01

$$\mathrm{dn}(K(m) \mid m) == \sqrt{1-m}$$

09.29.03.0006.01

$$\mathrm{dn}(2\,K(m) \mid m) == 1$$

09.29.03.0007.01

$$\mathrm{dn}(3\,K(m) \mid m) == \sqrt{1-m}$$

09.29.03.0008.01

$$\mathrm{dn}(4\,K(m) \mid m) == 1$$

09.29.03.0009.01

$$\mathrm{dn}(i\,K(1-m) \mid m) == \tilde{\infty}$$

09.29.03.0010.01

$$\mathrm{dn}(2\,i\,K(1-m) \mid m) == -1$$

09.29.03.0011.01

$$\mathrm{dn}(3\,i\,K(1-m) \mid m) == \tilde{\infty}$$

09.29.03.0012.01

$$\mathrm{dn}(4\,i\,K(1-m) \mid m) == 1$$

09.29.03.0013.01

$$\mathrm{dn}(K(m) + i\,K(1-m) \mid m) == 0$$

09.29.03.0014.01

$$\mathrm{dn}(2\,K(m) + i\,K(1-m) \mid m) == \tilde{\infty}$$

09.29.03.0015.01

$$\mathrm{dn}(3\,K(m) + i\,K(1-m) \mid m) == 0$$

09.29.03.0016.01

$$\mathrm{dn}(4\,K(m) + i\,K(1-m) \mid m) == \tilde{\infty}$$

09.29.03.0017.01

$$\mathrm{dn}(K(m) + 2\,i\,K(1-m) \mid m) == -\sqrt{1-m}$$

09.29.03.0018.01

$$\mathrm{dn}(2\,K(m) + 2\,i\,K(1-m) \mid m) == -1$$

09.29.03.0019.01

$$\mathrm{dn}(3\,K(m) + 2\,i\,K(1-m) \mid m) == -\sqrt{1-m}$$

09.29.03.0020.01

$$\mathrm{dn}(4\,K(m) + 2\,i\,K(1-m) \mid m) == -1$$

09.29.03.0021.01

$$\mathrm{dn}(K(m) + 3\,i\,K(1-m) \mid m) == 0$$

09.29.03.0022.01

$$\mathrm{dn}(2\,K(m) + 3\,i\,K(1-m) \mid m) == \tilde{\infty}$$

09.29.03.0023.01

$$\mathrm{dn}(2\,r\,K(m)+i\,(2\,s+1)\,K(1-m)\mid m) == \tilde{\infty}\ /;\ \{r,\,s\}\in\mathbb{Z}$$

09.29.03.0024.01

$$\mathrm{dn}(K(m)+4\,i\,K(1-m)\mid m) == \sqrt{1-m}$$

09.29.03.0025.01

$$\mathrm{dn}(2\,K(m)+4\,i\,K(1-m)\mid m) == 1$$

Values at half-quarter-period points

09.29.03.0026.01

$$\mathrm{dn}\left(\frac{K(m)}{2}\,\middle|\,m\right) == \sqrt[4]{1-m}$$

09.29.03.0027.01

$$\mathrm{dn}\left(\frac{i\,K(1-m)}{2}\,\middle|\,m\right) == \sqrt{1+\sqrt{m}}$$

09.29.03.0028.01

$$\mathrm{dn}\left(\frac{K(m)}{2}+\frac{i\,K(1-m)}{2}\,\middle|\,m\right) == \frac{\sqrt[4]{1-m}}{\sqrt{2}}\left(\sqrt{\sqrt{1-m}+1}-i\sqrt{1-\sqrt{1-m}}\right)$$

General characteristics

Domain and analyticity

dn$(z\mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.29.04.0001.01

$$(z * m)\longrightarrow \mathrm{dn}(z\mid m)\,::\,(\mathbb{C}\otimes\mathbb{C})\longrightarrow\mathbb{C}$$

Symmetries and periodicities

Parity

dn$(z\mid m)$ is an even function with respect to z.

09.29.04.0002.01

$$\mathrm{dn}(-z\mid m) == \mathrm{dn}(z\mid m)$$

Mirror symmetry

09.29.04.0003.01

$$\mathrm{dn}(\bar{z}\mid\bar{m}) == \overline{\mathrm{dn}(z\mid m)}$$

Periodicity

dn$(z\mid m)$ is a doubly periodic function with respect to z with periods $4\,i\,K(1-m)$ and $2\,K(m)$.

09.29.04.0004.01

$$\mathrm{dn}(z+2\,K(m)\mid m) == \mathrm{dn}(z\mid m)$$

09.29.04.0005.01

$$\mathrm{dn}(z+2\,i\,K(1-m)\mid m) == -\mathrm{dn}(z\mid m)$$

09.29.04.0006.01
$\mathrm{dn}(z + 4\,i\,K(1-m) \mid m) == \mathrm{dn}(z \mid m)$

09.29.04.0007.01
$\mathrm{dn}(z + 2\,K(m) + 2\,i\,K(1-m) \mid m) == -\mathrm{dn}(z \mid m)$

09.29.04.0008.01
$\mathrm{dn}(z + 2\,i\,s\,K(1-m) + 2\,r\,K(m) \mid m) == (-1)^s\,\mathrm{dn}(z \mid m)\ /;\ \{r, s\} \in \mathbb{Z}$

Poles and essential singularities

With respect to *z*

For fixed *m*, the function dn(*z* | *m*) has an infinite set of singular points:

a) $z = 2\,r\,K(m) + i\,(2\,s+1)\,K(1-m)$, $\{r, s\} \in \mathbb{Z}$, are the simple poles with residues $(-1)^{s-1}\,i$;

b) $z == \tilde{\infty}$ is an essential singular point.

09.29.04.0009.01
$\mathcal{S}ing_z(\mathrm{dn}(z \mid m)) == \{\{(2\,s+1)\,i\,K(1-m) + 2\,r\,K(m),\ 1\}\ /;\ \{r, s\} \in \mathbb{Z}\},\ \{\tilde{\infty},\ \infty\}\}$

09.29.04.0010.01
$\mathrm{res}_z(\mathrm{dn}(z \mid m))\,((2\,s+1)\,i\,K(1-m) + 2\,r\,K(m)) == (-1)^{s-1}\,i\ /;\ \{r, s\} \in \mathbb{Z}$

Branch points

With respect to *m*

For fixed *z*, the function cdn(*z* | *m*) is a meromorphic function in *m* that has no branch points.

09.29.04.0013.01
$\mathcal{BP}_m(\mathrm{dn}(z \mid m)) == \{\}$

P. Walker

With respect to *z*

For fixed *m*, the function dn(*z* | *m*) does not have branch points.

09.29.04.0011.01
$\mathcal{BP}_z(\mathrm{dn}(z \mid m)) == \{\}$

Branch cuts

With respect to *m*

For fixed *z*, the function dn(*z* | *m*) is a meromorphic function in *m* that has no branch cuts.

09.29.04.0014.01
$\mathcal{BC}_m(\mathrm{dn}(z \mid m)) == \{\}$

P. Walker

With respect to *z*

For fixed *m*, the function dn(*z* | *m*) does not have branch cuts.

09.29.04.0012.01

$\mathcal{BC}_z(\mathrm{dn}(z \mid m)) == \{\}$

Series representations

Generalized power series

Expansions at $z == 0$

09.29.06.0006.01

$$\mathrm{dn}(z \mid m) \propto 1 - \frac{m z^2}{2} + \frac{m (m+4)}{24} z^4 + \ldots /; (z \to 0)$$

09.29.06.0001.02

$$\mathrm{dn}(z \mid m) \propto 1 - \frac{m z^2}{2} + \frac{m (m+4)}{24} z^4 + \frac{1}{720}\left(-16 m - 44 m^2 - m^3\right) z^6 + \frac{\left(64 m + 912 m^2 + 408 m^3 + m^4\right) z^8}{40\,320} + \frac{\left(-256 m - 15\,808 m^2 - 30\,768 m^3 - 3688 m^4 - m^5\right) z^{10}}{3\,628\,800} + O\left(z^{12}\right)$$

09.29.06.0007.01

$$\mathrm{dn}(z \mid m) = \sum_{k=0}^{\infty} \frac{(-1)^k \, \mathrm{dn}_k(m) \, z^{2k}}{(2k)!} /; \mathrm{sn}_0(m) = 1 \bigwedge \mathrm{sn}_n(m) = \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{2n}{2j} \mathrm{cn}_j(m) \, \mathrm{dn}_k(m) \, \delta_{j+k-n} \bigwedge \mathrm{cn}_0(m) = 1 \bigwedge$$

$$\mathrm{cn}_n(m) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \mathrm{sn}_j(m) \, \mathrm{dn}_k(m) \, \delta_{j+k-n+1} \bigwedge \mathrm{dn}_0(m) = 1 \bigwedge \mathrm{dn}_n(m) = m \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \mathrm{sn}_j(m) \, \mathrm{cn}_k(m) \, \delta_{j+k-n+1}$$

09.29.06.0008.01

$$\mathrm{dn}(z \mid m) \propto 1 + O\left(z^2\right)$$

Expansions at $z == 2 r K(m) + (2 s + 1) i K(1 - m)$

09.29.06.0009.01

$$\mathrm{dn}(z \mid m) \propto i \, (-1)^{s-1} \left(\frac{1}{z - z_0} + \frac{1}{6}(m-2)(z - z_0) + \frac{1}{360}\left(7 m^2 + 8 m - 8\right)(z - z_0)^3 + \ldots\right) /;$$

$$(z \to z_0) \bigwedge z_0 = 2 r K(m) + (2 s + 1) i K(1 - m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

09.29.06.0010.01

$$\mathrm{dn}(z \mid m) = i \, (-1)^{s-1} \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{(j+1)(-1)^{k-j} \, \mathrm{cn}_{k-j}(m)}{(2k-2j)!} \sum_{r=0}^{j} \frac{(-1)^r}{r+1} \binom{j}{r} q_{r,j} \, (z - z_0)^{2k-1} /;$$

$$z_0 = 2 r K(m) + (2 s + 1) i K(1 - m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z} \bigwedge q_{j,0} = 1 \bigwedge q_{j,k} = \frac{1}{k} \sum_{i=1}^{k} \frac{(j i + i - k)(-1)^i \, \mathrm{sn}_i(m) \, q_{j,k-i}}{(2i+1)!} \bigwedge$$

$$k \in \mathbb{N}^+ \bigwedge \mathrm{sn}_0(m) = 1 \bigwedge \mathrm{sn}_n(m) = \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{2n}{2j} \mathrm{cn}_j(m) \, \mathrm{dn}_k(m) \, \delta_{j+k-n} \bigwedge \mathrm{cn}_0(m) = 1 \bigwedge$$

$$\mathrm{cn}_n(m) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \mathrm{sn}_j(m) \, \mathrm{dn}_k(m) \, \delta_{j+k-n+1} \bigwedge \mathrm{dn}_0(m) = 1 \bigwedge \mathrm{dn}_n(m) = m \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \mathrm{sn}_j(m) \, \mathrm{cn}_k(m) \, \delta_{j+k-n+1}$$

09.29.06.0011.01

$$\mathrm{dn}(z \mid m) \propto \frac{i\,(-1)^{s-1}}{z - z_0}\left(1 + O\left((z - z_0)^2\right)\right) /;\ z_0 = 2\,r\,K(m) + (2\,s+1)\,i\,K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

Expansions at $m == 0$

09.29.06.0012.01

$$\mathrm{dn}(z \mid m) \propto 1 - \frac{1}{2}\sin^2(z)\,m - \frac{1}{32}\sin(z)\,(-8\,z\cos(z) + 5\sin(z) + \sin(3\,z))\,m^2 + \ldots /;\ (m \to 0)$$

09.29.06.0013.01

$$\begin{aligned}\mathrm{dn}(z \mid m) \propto\ & 1 - \frac{1}{2}\sin^2(z)\,m - \frac{1}{32}\sin(z)\,(-8\,z\cos(z) + 5\sin(z) + \sin(3\,z))\,m^2 + \\ & \frac{1}{1024}\left(\left(31 - 32\,z^2\right)\cos(2\,z) + 12\cos(4\,z) + \cos(6\,z) + 72\,z\sin(2\,z) + 16\,z\sin(4\,z) - 44\right)m^3 + \\ & \frac{1}{49\,152}\left(-12\left(112\,z^2 - 75\right)\cos(2\,z) - 12\left(32\,z^2 - 37\right)\cos(4\,z) + 60\cos(6\,z) + \right. \\ & \left. 3\cos(8\,z) - 8\,z\left(32\,z^2 - 285\right)\sin(2\,z) + 816\,z\sin(4\,z) + 72\,z\sin(6\,z) - 1407\right)m^4 + \\ & \frac{1}{786\,432}\left(\left(512\,z^4 - 17\,664\,z^2 + 9765\right)\cos(2\,z) - 48\left(176\,z^2 - 119\right)\cos(4\,z) - 24\left(36\,z^2 - 41\right)\cos(6\,z) + 84\cos(8\,z) + \right. \\ & \left. 3\cos(10\,z) - 32\,z\left(152\,z^2 - 825\right)\sin(2\,z) - 64\,z\left(32\,z^2 - 189\right)\sin(4\,z) + 1800\,z\sin(6\,z) + 96\,z\sin(8\,z) - 16\,548\right)m^5 + \\ & \frac{1}{62\,914\,560}\left(60\left(1024\,z^4 - 19\,520\,z^2 + 9471\right)\cos(2\,z) + 5\left(8192\,z^4 - 146\,304\,z^2 + 75\,933\right)\cos(4\,z) - 1440\left(90\,z^2 - 53\right)\right. \\ & \cos(6\,z) - 60\left(128\,z^2 - 143\right)\cos(8\,z) + 540\cos(10\,z) + 15\cos(12\,z) + 8\,z\left(512\,z^4 - 47\,680\,z^2 + 202\,665\right)\sin(2\,z) - \\ & \left. 240\,z\left(1152\,z^2 - 3641\right)\sin(4\,z) - 2880\,z\left(12\,z^2 - 59\right)\sin(6\,z) + 15\,840\,z\sin(8\,z) + 600\,z\sin(10\,z) - 1\,033\,380\right)m^6 + \\ & \frac{1}{3\,019\,898\,880}\left(-\left(16\,384\,z^6 - 3\,356\,160\,z^4 + 47\,416\,320\,z^2 - 20\,807\,865\right)\cos(2\,z) + \right. \\ & 60\left(65\,536\,z^4 - 587\,328\,z^2 + 258\,687\right)\cos(4\,z) + 270\left(2304\,z^4 - 30\,768\,z^2 + 12\,899\right)\cos(6\,z) - \\ & 720\left(1216\,z^2 - 649\right)\cos(8\,z) - 3600\left(10\,z^2 - 11\right)\cos(10\,z) + 1980\cos(12\,z) + 45\cos(14\,z) + \\ & 48\,z\left(7424\,z^4 - 355\,600\,z^2 + 1\,296\,405\right)\sin(2\,z) + 48\,z\left(8192\,z^4 - 342\,400\,z^2 + 787\,305\right)\sin(4\,z) - \\ & \left. 1080\,z\left(3360\,z^2 - 8077\right)\sin(6\,z) - 1920\,z\left(128\,z^2 - 567\right)\sin(8\,z) + 73\,800\,z\sin(10\,z) + 2160\,z\sin(12\,z) - 40\,320\,720\right) \\ & m^7 + \frac{1}{338\,228\,674\,560}\left(-28\left(139\,264\,z^6 - 13\,900\,800\,z^4 + 162\,684\,000\,z^2 - 65\,714\,805\right)\cos(2\,z) - \right. \\ & 28\left(262\,144\,z^6 - 21\,934\,080\,z^4 + 137\,187\,360\,z^2 - 53\,897\,355\right)\cos(4\,z) + 22\,680\left(7680\,z^4 - 48\,024\,z^2 + 16\,237\right)\cos(6\,z) + \\ & 420\left(32\,768\,z^4 - 370\,560\,z^2 + 133\,503\right)\cos(8\,z) - 20\,160\left(575\,z^2 - 286\right)\cos(10\,z) - 1260\left(288\,z^2 - 313\right)\cos(12\,z) + \\ & 16\,380\cos(14\,z) + 315\cos(16\,z) - 8\,z\left(16\,384\,z^6 - 6\,461\,952\,z^4 + 219\,428\,160\,z^2 - 717\,669\,855\right)\sin(2\,z) + \\ & 336\,z\left(303\,104\,z^4 - 6\,054\,400\,z^2 + 11\,396\,145\right)\sin(4\,z) + \\ & 9072\,z\left(2304\,z^4 - 66\,800\,z^2 + 110\,135\right)\sin(6\,z) - 3360\,z\left(22\,016\,z^2 - 44\,835\right)\sin(8\,z) - \\ & \left. 67\,200\,z\left(50\,z^2 - 207\right)\sin(10\,z) + 740\,880\,z\sin(12\,z) + 17\,640\,z\sin(14\,z) - 3\,779\,643\,735\right)m^8 + \\ & \frac{1}{5\,411\,658\,792\,960}\left(2\left(65\,536\,z^8 - 45\,301\,760\,z^6 + 3\,093\,081\,600\,z^4 - 31\,716\,896\,400\,z^2 + 11\,928\,745\,395\right)\cos(2\,z) - \right. \\ & 336\left(917\,504\,z^6 - 35\,307\,520\,z^4 + 175\,588\,080\,z^2 - 63\,335\,115\right)\cos(4\,z) - \\ & 63\left(1\,327\,104\,z^6 - 72\,852\,480\,z^4 + 303\,200\,640\,z^2 - 88\,136\,555\right)\cos(6\,z) + \\ & 5040\left(131\,072\,z^4 - 660\,672\,z^2 + 184\,747\right)\cos(8\,z) + 210\left(160\,000\,z^4 - 1\,623\,600\,z^2 + 524\,121\right)\cos(10\,z) - \\ & 5040\left(3888\,z^2 - 1835\right)\cos(12\,z) - 2520\left(196\,z^2 - 211\right)\cos(14\,z) + 18\,900\cos(16\,z) + \\ & \left. 315\cos(18\,z) - 24\,z\left(212\,992\,z^6 - 39\,244\,800\,z^4 + 1\,071\,069\,440\,z^2 - 3\,225\,618\,585\right)\sin(2\,z) - \right.\end{aligned}$$

$$64\,z\left(262\,144\,z^6-39\,481\,344\,z^4+532\,415\,520\,z^2-871\,069\,815\right)\sin(4\,z)+15\,120\,z\left(62\,208\,z^4-816\,000\,z^2+1\,056\,475\right)$$
$$\sin(6\,z)+2688\,z\left(32\,768\,z^4-767\,360\,z^2+1\,026\,525\right)\sin(8\,z)-12\,600\,z\left(13\,600\,z^2-24\,757\right)\sin(10\,z)-$$
$$60\,480\,z\left(96\,z^2-379\right)\sin(12\,z)+1\,005\,480\,z\sin(14\,z)+20\,160\,z\sin(16\,z)-51\,741\,682\,020\Big)\,m^9+$$
$$\frac{1}{779\,278\,866\,186\,240}\Big(72\left(720\,896\,z^8-225\,075\,200\,z^6+12\,041\,433\,600\,z^4-111\,798\,981\,000\,z^2+39\,463\,527\,915\right)\cos(2\,z)+$$
$$9\left(33\,554\,432\,z^8-8\,419\,016\,704\,z^6+212\,868\,956\,160\,z^4-900\,514\,460\,160\,z^2+303\,767\,329\,005\right)\cos(4\,z)-$$
$$11\,340\left(3\,317\,760\,z^6-80\,123\,904\,z^4+256\,126\,176\,z^2-66\,479\,231\right)\cos(6\,z)-$$
$$1008\left(4\,194\,304\,z^6-179\,159\,040\,z^4+579\,919\,680\,z^2-135\,446\,535\right)\cos(8\,z)+$$
$$7560\left(2\,240\,000\,z^4-9\,765\,000\,z^2+2\,375\,529\right)\cos(10\,z)+22\,680\left(27\,648\,z^4-259\,920\,z^2+77\,249\right)\cos(12\,z)-$$
$$90\,720\left(3038\,z^2-1377\right)\cos(14\,z)-11\,340\left(512\,z^2-547\right)\cos(16\,z)+192\,780\cos(18\,z)+2835\cos(20\,z)+$$
$$16\,z\left(65\,536\,z^8-74\,022\,912\,z^6+9\,060\,871\,680\,z^4-212\,000\,166\,000\,z^2+597\,775\,583\,475\right)\sin(2\,z)-$$
$$576\,z\left(12\,320\,768\,z^6-827\,430\,912\,z^4+8\,661\,085\,440\,z^2-12\,758\,172\,525\right)\sin(4\,z)-$$
$$5832\,z\left(442\,368\,z^6-41\,868\,288\,z^4+356\,581\,120\,z^2-389\,052\,335\right)\sin(6\,z)+$$
$$24\,192\,z\left(1\,736\,704\,z^4-17\,671\,680\,z^2+17\,979\,285\right)\sin(8\,z)+75\,600\,z\left(32\,000\,z^4-650\,800\,z^2+753\,081\right)\sin(10\,z)-$$
$$408\,240\,z\left(7552\,z^2-12\,675\right)\sin(12\,z)-423\,360\,z\left(196\,z^2-747\right)\sin(14\,z)+$$
$$11\,793\,600\,z\sin(16\,z)+204\,120\,z\sin(18\,z)-6\,485\,526\,884\,340\Big)\,m^{10}+O\left(m^{11}\right)$$

09.29.06.0014.01

$$\mathrm{dn}(z\mid m)\propto 1+O\left(m^2\right)$$

Expansions at $m == 1$

09.29.06.0015.01

$$\mathrm{dn}(z\mid m)\propto \mathrm{sech}(z)-\frac{1}{4}\tanh(z)\,(z+\cosh(z)\sinh(z))\,\mathrm{sech}(z)\,(m-1)+$$
$$\frac{1}{512}\left(8\cosh(2\,z)\,z^2-24\,z^2-4\sinh(2\,z)\,z-4\sinh(4\,z)\,z+5\cosh(4\,z)-5\right)\mathrm{sech}^3(z)\,(m-1)^2+\ldots\,/;\,(m\to 1)$$

09.29.06.0016.01

$$\mathrm{dn}(z\mid m)\propto \mathrm{sech}(z)-\frac{1}{4}\,\mathrm{sech}(z)\tanh(z)\,(z+\cosh(z)\sinh(z))\,(m-1)+$$
$$\frac{1}{512}\,\mathrm{sech}^3(z)\left(8\cosh(2\,z)\,z^2-24\,z^2-4\sinh(2\,z)\,z-4\sinh(4\,z)\,z+5\cosh(4\,z)-5\right)(m-1)^2\;+$$
$$\frac{1}{49\,152}\,\mathrm{sech}^4(z)\left(3\left(160\,z^2+83\right)\cosh(z)-3\left(88\,z^2+43\right)\cosh(3\,z)-3\left(8\,z^2+41\right)\cosh(5\,z)+\right.$$
$$\left.3\cosh(7\,z)+32\,z\left(23\,z^2+6\right)\sinh(z)-8\,z\left(4\,z^2-39\right)\sinh(3\,z)+120\,z\sinh(5\,z)\right)(m-1)^3+$$
$$\frac{1}{1\,572\,864}\,\mathrm{sech}^5(z)\left(3680\,z^4-2760\,z^2-4\left(1976\,z^2+1527\right)\sinh(2\,z)\,z+4\left(56\,z^2-1227\right)\sinh(4\,z)\,z-\right.$$
$$4\left(8\,z^2+297\right)\sinh(6\,z)\,z+36\sinh(8\,z)\,z-\left(2432\,z^4-312\,z^2+1167\right)\cosh(2\,z)+$$
$$\left.2\left(16\,z^4+1716\,z^2+1287\right)\cosh(4\,z)+3\left(120\,z^2+389\right)\cosh(6\,z)-60\cosh(8\,z)-2514\right)\;(m-1)^4-$$
$$\frac{1}{251\,658\,240}\,\mathrm{sech}^6(z)\left(840\left(476\,z^4-87\,z^2-411\right)\cosh(z)-30\left(6288\,z^4-10\,116\,z^2-2755\right)\cosh(3\,z)+\right.$$
$$5\left(736\,z^4+44\,904\,z^2+41\,811\right)\cosh(5\,z)+10\left(16\,z^4+1848\,z^2+5835\right)\cosh(7\,z)-30\left(36\,z^2+161\right)\cosh(9\,z)+$$
$$15\cosh(11\,z)+16\,z\left(13\,456\,z^4-28\,940\,z^2-23\,745\right)\sinh(z)-24\,z\left(1264\,z^4+18\,980\,z^2+27\,975\right)\sinh(3\,z)+$$
$$\left.8\,z\left(16\,z^4+540\,z^2-44\,145\right)\sinh(5\,z)-20\,z\left(160\,z^2+2859\right)\sinh(7\,z)+4500\,z\sinh(9\,z)\right)(m-1)^5\;+$$
$$\frac{1}{24\,159\,191\,040}\,\mathrm{sech}^7(z)\left(-3\,014\,144\,z^6+17\,125\,920\,z^4+2\,772\,000\,z^2+\right.$$

$$24\left(485\,296\,z^4-1\,407\,720\,z^2-1\,672\,185\right)\sinh(2\,z)\,z-60\left(31\,808\,z^4+276\,960\,z^2+638\,391\right)\sinh(4\,z)\,z+$$
$$180\left(32\,z^4+408\,z^2-80\,953\right)\sinh(6\,z)\,z-24\left(16\,z^4+2760\,z^2+62\,415\right)\sinh(8\,z)\,z+180\left(72\,z^2+1381\right)\sinh(10\,z)\,z-$$
$$900\sinh(12\,z)\,z+2\left(1\,349\,504\,z^6+4\,620\,960\,z^4+7\,884\,360\,z^2-4\,624\,785\right)\cosh(2\,z)-$$
$$4\left(46\,208\,z^6+1\,922\,880\,z^4-5\,262\,840\,z^2-2\,644\,965\right)\cosh(4\,z)+$$
$$\left(256\,z^6+204\,480\,z^4+8\,508\,960\,z^2+9\,467\,235\right)\cosh(6\,z)+30\left(400\,z^4+11\,904\,z^2+58\,617\right)\cosh(8\,z)-$$
$$45\left(2160\,z^2+4837\right)\cosh(10\,z)+1620\cosh(12\,z)-12\,339\,990\Big)(m-1)^6+$$
$$\frac{1}{5\,411\,658\,792\,960}\operatorname{sech}^8(z)\left(63\left(9\,614\,336\,z^6-67\,529\,280\,z^4-48\,535\,200\,z^2+46\,854\,555\right)\cosh(z)-\right.$$
$$140\left(3\,269\,504\,z^6+1\,174\,224\,z^4+27\,202\,662\,z^2+992\,079\right)\cosh(3\,z)+$$
$$70\left(478\,976\,z^6+10\,396\,272\,z^4-37\,828\,116\,z^2-25\,073\,235\right)\cosh(5\,z)-$$
$$35\left(1792\,z^6+490\,272\,z^4+20\,050\,128\,z^2+27\,450\,513\right)\cosh(7\,z)-7\left(256\,z^6-84\,960\,z^4-1\,830\,960\,z^2+16\,994\,475\right)$$
$$\cosh(9\,z)+630\left(432\,z^4+22\,276\,z^2+35\,213\right)\cosh(11\,z)-2520\left(25\,z^2+109\right)\cosh(13\,z)+$$
$$315\cosh(15\,z)+8\,z\left(33\,244\,544\,z^6-169\,790\,880\,z^4+373\,682\,400\,z^2+460\,109\,475\right)\sinh(z)-$$
$$12\,z\left(5\,176\,064\,z^6+94\,597\,440\,z^4-370\,032\,320\,z^2-594\,245\,715\right)\sinh(3\,z)+$$
$$16\,z\left(139\,456\,z^6+13\,918\,800\,z^4+88\,750\,830\,z^2+293\,798\,925\right)\sinh(5\,z)-$$
$$4\,z\left(256\,z^6+94\,080\,z^4+9\,042\,600\,z^2-336\,907\,305\right)\sinh(7\,z)+420\,z\left(192\,z^4-20\,072\,z^2+162\,363\right)\sinh(9\,z)-$$
$$\left.2520\,z\left(1260\,z^2+11\,297\right)\sinh(11\,z)+258\,300\,z\sinh(13\,z)\right)(m-1)^7+$$
$$\frac{1}{173\,173\,081\,374\,720}\operatorname{sech}^9(z)\left(1\,196\,803\,584\,z^8-11\,621\,272\,320\,z^6+59\,265\,712\,800\,z^4+51\,670\,006\,920\,z^2-\right.$$
$$28\left(225\,685\,504\,z^6-1\,385\,289\,216\,z^4+3\,419\,854\,200\,z^2+5\,152\,437\,585\right)\sinh(2\,z)\,z+$$
$$4\left(464\,069\,120\,z^6+3\,558\,439\,584\,z^4-18\,625\,005\,840\,z^2-39\,050\,472\,195\right)\sinh(4\,z)\,z-$$
$$144\left(480\,320\,z^6+23\,703\,624\,z^4+113\,082\,060\,z^2+540\,952\,755\right)\sinh(6\,z)\,z+$$
$$4\left(5888\,z^6+6\,093\,024\,z^4+375\,391\,800\,z^2-4\,243\,004\,955\right)\sinh(8\,z)\,z-$$
$$32\left(32\,z^6-129\,948\,z^4-12\,802\,230\,z^2-2\,131\,605\right)\sinh(10\,z)\,z+$$
$$756\left(864\,z^4+95\,200\,z^2+578\,675\right)\sinh(12\,z)\,z-420\left(1000\,z^2+16\,371\right)\sinh(14\,z)\,z+8820\sinh(16\,z)\,z-$$
$$2\left(636\,233\,728\,z^8+1\,416\,993\,536\,z^6-30\,708\,972\,000\,z^4-53\,216\,291\,520\,z^2+17\,945\,797\,275\right)\cosh(2\,z)+$$
$$4\left(42\,446\,336\,z^8+2\,036\,204\,800\,z^6-2\,051\,632\,800\,z^4+22\,241\,422\,350\,z^2+6\,187\,089\,195\right)\cosh(4\,z)-$$
$$63\left(53\,248\,z^8+10\,174\,720\,z^6+163\,077\,600\,z^4-668\,248\,200\,z^2-553\,255\,345\right)\cosh(6\,z)+$$
$$2\left(256\,z^8+760\,704\,z^6+12\,390\,000\,z^4+3\,620\,465\,100\,z^2+6\,758\,870\,265\right)\cosh(8\,z)+35$$
$$\left(1792\,z^6-2\,028\,000\,z^4-25\,849\,368\,z^2+29\,453\,985\right)\cosh(10\,z)-1260\left(8640\,z^4+195\,426\,z^2+251\,863\right)\cosh(12\,z)+$$
$$\left.1260\left(2300\,z^2+4459\right)\cosh(14\,z)-16\,380\cosh(16\,z)-37\,948\,733\,550\right)(m-1)^8-\frac{1}{49\,873\,847\,435\,919\,360}$$
$$\left(\operatorname{sech}^{10}(z)\left(63\left(5\,765\,895\,168\,z^8-64\,101\,931\,520\,z^6+358\,754\,198\,880\,z^4+461\,814\,328\,800\,z^2-210\,208\,477\,185\right)\cosh(z)-\right.\right.$$
$$216\left(1\,654\,016\,128\,z^8-3\,795\,293\,152\,z^6-30\,335\,368\,280\,z^4-117\,788\,351\,835\,z^2+5\,787\,713\,820\right)\cosh(3\,z)+$$
$$720\left(74\,894\,880\,z^8+1\,610\,434\,336\,z^6-3\,479\,640\,570\,z^4+22\,032\,817\,380\,z^2+9\,879\,190\,965\right)\cosh(5\,z)-$$
$$63\left(17\,245\,696\,z^8+1\,590\,476\,032\,z^6+22\,333\,139\,520\,z^4-86\,714\,291\,880\,z^2-90\,423\,595\,035\right)\cosh(7\,z)+$$
$$9\left(24\,064\,z^8-26\,967\,808\,z^6-5\,753\,905\,920\,z^4+53\,348\,533\,560\,z^2+184\,980\,199\,635\right)\cosh(9\,z)+$$
$$36\left(128\,z^8-2\,892\,288\,z^6-583\,461\,480\,z^4-5\,821\,526\,340\,z^2+1\,616\,170\,815\right)\cosh(11\,z)-$$
$$189\left(62\,208\,z^6+12\,659\,840\,z^4+187\,133\,400\,z^2+213\,575\,595\right)\cosh(13\,z)+945\left(20\,000\,z^4+800\,040\,z^2+997\,737\right)$$
$$\cosh(15\,z)-5670\left(196\,z^2+839\right)\cosh(17\,z)+2835\cosh(19\,z)+8\,z\left(17\,769\,803\,264\,z^8-\right.$$
$$154\,337\,008\,896\,z^6+662\,044\,192\,992\,z^4-1\,413\,285\,787\,620\,z^2-2\,207\,834\,702\,955\Big)\sinh(z)-$$
$$4\,z\left(11\,114\,481\,664\,z^8+218\,002\,922\,496\,z^6-1\,813\,216\,327\,776\,z^4+4\,986\,817\,337\,880\,z^2+9\,150\,568\,206\,255\right)$$
$$\sinh(3\,z)+20\,z\left(179\,849\,216\,z^8+17\,458\,283\,520\,z^6+75\,235\,589\,856\,z^4-516\,684\,231\,000\,z^2-1\,408\,419\,633\,915\right)$$

$$\sinh(5\,z) - 64\,z\left(629\,536\,z^8 + 211\,179\,024\,z^6 + 6\,915\,622\,266\,z^4 + 21\,606\,985\,575\,z^2 + 173\,396\,382\,075\right)\sinh(7\,z) +$$
$$128\,z\left(16\,z^8 + 22\,536\,z^6 + 88\,216\,317\,z^4 + 3\,070\,844\,595\,z^2 - 14\,032\,657\,485\right)\sinh(9\,z) -$$
$$36\,z\left(10\,240\,z^6 - 65\,446\,752\,z^4 - 2\,648\,877\,000\,z^2 - 3\,662\,620\,605\right)\sinh(11\,z) + 11\,340\,z$$
$$\left(23\,328\,z^4 + 1\,054\,072\,z^2 + 5\,123\,503\right)\sinh(13\,z) - 56\,700\,z\left(3400\,z^2 + 23\,959\right)\sinh(15\,z) + 4\,524\,660\,z\sinh(17\,z)\Big)\Big)$$

$$(m-1)^9 - \frac{1}{7\,979\,815\,589\,747\,097\,600}\operatorname{sech}^{11}(z)\left(3\,127\,485\,374\,464\,z^{10} - 43\,994\,001\,300\,480\,z^8 +\right.$$
$$330\,560\,432\,294\,400\,z^6 - 1\,585\,067\,158\,382\,400\,z^4 - 2\,197\,145\,541\,729\,600\,z^2 - 1080$$
$$\left(18\,545\,000\,960\,z^8 - 183\,880\,478\,592\,z^6 + 874\,548\,867\,248\,z^4 - 1\,915\,061\,306\,760\,z^2 - 3\,460\,899\,395\,415\right)\sinh(2\,z)\,z +$$
$$120\left(69\,804\,123\,904\,z^8 + 410\,456\,968\,704\,z^6 - 5\,520\,309\,884\,208\,z^4 + 16\,449\,365\,164\,680\,z^2 + 36\,940\,206\,271\,815\right)$$
$$\sinh(4\,z)\,z - 160$$
$$\left(4\,557\,007\,168\,z^8 + 197\,971\,975\,584\,z^6 + 530\,033\,745\,528\,z^4 - 4\,517\,746\,166\,430\,z^2 - 16\,561\,877\,387\,265\right)\sinh(6\,z)\,z +$$
$$40\left(206\,488\,576\,z^8 + 31\,450\,871\,808\,z^6 + 739\,283\,503\,392\,z^4 + 653\,416\,076\,880\,z^2 + 21\,042\,320\,833\,485\right)\sinh(8\,z)\,z -$$
$$320\left(992\,z^8 + 17\,632\,080\,z^6 + 6\,037\,860\,528\,z^4 + 148\,398\,278\,175\,z^2 - 293\,396\,228\,055\right)\sinh(10\,z)\,z +$$
$$20\left(512\,z^8 - 53\,001\,216\,z^6 - 18\,679\,375\,008\,z^4 - 500\,341\,983\,120\,z^2 - 880\,724\,436\,795\right)\sinh(12\,z)\,z -$$
$$540\left(186\,624\,z^6 + 62\,952\,736\,z^4 + 1\,786\,255\,800\,z^2 + 7\,697\,400\,375\right)\sinh(14\,z)\,z +$$
$$18\,900\left(20\,000\,z^4 + 1\,596\,816\,z^2 + 7\,072\,143\right)\sinh(16\,z)\,z - 18\,900\left(2744\,z^2 + 41\,535\right)\sinh(18\,z)\,z +$$
$$510\,300\sinh(20\,z)\,z - \left(3\,714\,757\,763\,072\,z^{10} + 725\,097\,369\,600\,z^8 - 264\,064\,801\,674\,240\,z^6 +\right.$$
$$\left.2\,031\,769\,019\,901\,600\,z^4 + 4\,031\,463\,722\,718\,600\,z^2 - 969\,296\,803\,112\,925\right)\cosh(2\,z) +$$
$$2\left(365\,160\,251\,392\,z^{10} + 18\,368\,995\,703\,040\,z^8 - 80\,431\,555\,034\,880\,z^6 - 126\,842\,760\,399\,600\,z^4 -\right.$$
$$\left.1\,454\,744\,457\,558\,000\,z^2 - 200\,711\,093\,812\,425\right)\cosh(4\,z) -$$
$$\left(37\,339\,713\,536\,z^{10} + 6\,399\,354\,193\,920\,z^8 + 85\,954\,169\,468\,160\,z^6 - 302\,198\,583\,304\,800\,z^4 +\right.$$
$$\left.1\,412\,940\,934\,326\,600\,z^2 + 861\,620\,257\,661\,625\right)\cosh(6\,z) + 32\left(7\,556\,864\,z^{10} + 4\,110\,213\,600\,z^8 +\right.$$
$$\left.266\,226\,090\,480\,z^6 + 3\,641\,863\,424\,850\,z^4 - 10\,988\,271\,798\,525\,z^2 - 15\,273\,874\,335\,825\right)\cosh(8\,z) -$$
$$\left(4096\,z^{10} + 32\,532\,480\,z^8 - 133\,951\,507\,200\,z^6 - 9\,643\,178\,109\,600\,z^4 - 4\,911\,089\,891\,400\,z^2 + 110\,501\,743\,056\,075\right)$$
$$\cosh(10\,z) - 90\left(11\,520\,z^8 - 344\,065\,792\,z^6 - 27\,964\,693\,680\,z^4 - 238\,785\,870\,960\,z^2 - 6\,743\,301\,075\right)\cosh(12\,z) +$$
$$4725\left(622\,080\,z^6 + 47\,775\,520\,z^4 + 557\,948\,568\,z^2 + 597\,795\,471\right)\cosh(14\,z) -$$
$$4725\left(1\,120\,000\,z^4 + 18\,727\,968\,z^2 + 17\,561\,343\right)\cosh(16\,z) + 2\,778\,300\left(124\,z^2 + 221\right)\cosh(18\,z) -$$
$$\left.963\,900\cosh(20\,z) + 889\,662\,247\,584\,075\right)(m-1)^{10} + O\left((m-1)^{11}\right)$$

09.29.06.0017.01

$$\operatorname{dn}(z \mid m) \propto \operatorname{sech}(z) + O(m-1)$$

q-series

09.29.06.0002.01

$$\operatorname{dn}(z \mid m) == \frac{\pi}{2\,K(m)} + \frac{2\,\pi}{K(m)}\sum_{n=1}^{\infty}\frac{q(m)^n}{1+q(m)^{2\,n}}\cos\left(\frac{n\,\pi\,z}{K(m)}\right)$$

09.29.06.0003.01

$$\log(\operatorname{dn}(2\,K(m)\,z \mid m)) == -8\sum_{r=1}^{\infty}\frac{q(m)^{2\,r-1}}{(2\,r-1)\left(1-q(m)^{4\,r-2}\right)}\sin^2((2\,r-1)\,\pi\,z)$$

Other series representations

09.29.06.0004.01

$$\mathrm{dn}(z \mid m) == \frac{\pi}{2\,K(1-m)} \sum_{k=-\infty}^{\infty} \operatorname{sech}\left(\pi \frac{K(m)}{K(1-m)}\left(k + \frac{z}{2\,K(m)}\right)\right)$$

09.29.06.0005.01

$$\mathrm{dn}(z \mid m) \propto \frac{(-1)^{s-1}\, i}{z - i\,(2\,s+1)\,K(1-m) - 2\,r\,K(m)} + O(1) \,/;\, (z \to (2\,s+1)\,i\,K(1-m) + 2\,r\,K(m)) \bigwedge \{r, s\} \in \mathbb{Z}$$

Product representations

09.29.08.0001.01

$$\mathrm{dn}(z \mid m) == \sqrt[4]{1-m} \prod_{n=1}^{\infty} \frac{1 + 2\,q(m)^{2\,n-1} \cos\left(\frac{\pi\,z}{K(m)}\right) + q\,(m)^{4\,n-2}}{1 - 2\,q(m)^{2\,n-1} \cos\left(\frac{\pi\,z}{K(m)}\right) + q\,(m)^{4\,n-2}}$$

Differential equations

Ordinary nonlinear differential equations

With respect to *m*

09.29.13.0003.01

$$\begin{aligned}
&z^2\,w(m)^{12} + \left(3\,m\,z^2 - 6\,z^2 + 1\right) w(m)^{10} + \left(3\,m^2\,z^2 - 15\,m\,z^2 + 15\,z^2 - 4\right) w(m)^8 + \left(z^2\,m^3 - 12\,z^2\,m^2 + 30\,z^2\,m - 20\,z^2 + 6\right) w(m)^6 + \\
&\quad \left(-3\,z^2\,m^3 + 18\,z^2\,m^2 - 30\,z^2\,m + 15\,z^2 - 4\right) w(m)^4 + \left(3\,z^2\,m^3 - 12\,z^2\,m^2 + 15\,z^2\,m - 6\,z^2 + 1\right) w(m)^2 + \\
&\quad \left(64\,(m-1)^2\,m^2\,w(m)^6 + 64\,(m-2)\,(m-1)^2\,m^2\,w(m)^4 + 16\,(m-2)^2\,(m-1)^2\,m^2\,w(m)^2\right) w'(m)^4 + \\
&\quad \left(-64\,(m-1)\,m\,(2\,m-1)\,w(m)^7 - 32\,(m-1)\,m\left(4\,m^2 - 13\,m + 6\right) w(m)^5 - \right. \\
&\qquad \left. 32\,(m-1)\,m\left(m^3 - 8\,m^2 + 14\,m - 6\right) w(m)^3 + 32\,(m-2)\,(m-1)^3\,m\,w(m)\right) w'(m)^3 - (m-1)^3\,z^2 + \\
&\quad \left(16\left(5\,m^2 - 5\,m + 1\right) w(m)^8 + 8\left(9\,m^3 - 43\,m^2 + 38\,m - 8\right) w(m)^6 + 16\,(m-2)\left(m^3 - 11\,m^2 + 12\,m - 3\right) w(m)^4 - \right. \\
&\qquad \left. 8\,(m-1)\left(4\,m^3 - 21\,m^2 + 26\,m - 8\right) w(m)^2 + 16\,(m-1)^4\right) w'(m)^2 + \\
&\quad \left(16\,(m-1)^2\,m^2\,w(m)^8 + 32\,(m-2)\,(m-1)^2\,m^2\,w(m)^6 + 16\,(m-1)^2\,m^2\left(m^2 - 6\,m + 6\right) w(m)^4 - \right. \\
&\qquad \left. 32\,(m-2)\,(m-1)^3\,m^2\,w(m)^2 + 16\,(m-1)^4\,m^2\right) w''(m)^2 + \\
&\quad \left(-8\,(2\,m-1)\,w(m)^9 - 8\left(m^2 - 8\,m + 4\right) w(m)^7 + 24\left(m^2 - 4\,m + 2\right) w(m)^5 - 8\,(m-2)\,(3\,m-2)\,w(m)^3 + 8\,(m-1)^2\,w(m)\right) \\
&\quad w'(m) + \left(-8\,(m-1)\,m\,w(m)^9 - 8\,(m-4)\,(m-1)\,m\,w(m)^7 + 24\,(m-2)\,(m-1)\,m\,w(m)^5 - \right. \\
&\qquad \left. 8\,(m-1)\,m\,(3\,m-4)\,w(m)^3 + 8\,(m-1)^2\,m\,w(m) + \left(-64\,(m-1)^2\,m^2\,w(m)^7 - 96\,(m-2)\,(m-1)^2\,m^2\,w(m)^5 - \right. \right. \\
&\qquad\quad \left. \left. 32\,(m-1)^2\,m^2\left(m^2 - 6\,m + 6\right) w(m)^3 + 32\,(m-2)\,(m-1)^3\,m^2\,w(m)\right) w'(m)^2 + \right. \\
&\qquad \left(32\,(m-1)\,m\,(2\,m-1)\,w(m)^8 + 32\,(m-1)\,m\left(3\,m^2 - 9\,m + 4\right) w(m)^6 + 32\,(m-1)\,m\left(m^3 - 9\,m^2 + 15\,m - 6\right) w(m)^4 - \right. \\
&\qquad\quad \left. 32\,(m-1)^2\,m\left(2\,m^2 - 7\,m + 4\right) w(m)^2 + 32\,(m-1)^4\,m\right) w'(m)\Big) w''(m) \,/;\, w(m) == \mathrm{dn}(z \mid m)
\end{aligned}$$

With respect to *z*

09.29.13.0001.01

$$w''(z) + \left(2\,w(z)^2 + m - 2\right) w(z) == 0 \,/;\, w(z) == \mathrm{dn}(z \mid m)$$

09.29.13.0002.01

$$w'(z)^2 == \left(1 - w(z)^2\right)\left(m - 1 + w(z)^2\right) \,/;\, w(z) == \mathrm{dn}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.29.16.0001.01

$$\mathrm{dn}(i\,z \mid m) == \frac{\mathrm{dn}(z \mid 1-m)}{\mathrm{cn}(z \mid 1-m)}$$

09.29.16.0002.01

$$\mathrm{dn}(z \mid 1-m) == \frac{\mathrm{dn}(i\,z \mid m)}{\mathrm{cn}(i\,z \mid m)}$$

09.29.16.0003.01

$$\mathrm{dn}(i\,z \mid 1-m) == \frac{\mathrm{dn}(z \mid m)}{\mathrm{cn}(z \mid m)}$$

09.29.16.0007.01

$$\mathrm{dn}(x + i\,y \mid m) == (\mathrm{dn}(x \mid m)\,\mathrm{cn}(y \mid 1-m)\,\mathrm{dn}(y \mid 1-m) - i\,m\,\mathrm{sn}(x \mid m)\,\mathrm{cn}(x \mid m)\,\mathrm{sn}(y \mid 1-m))\Big/ \left(\mathrm{cn}(y \mid 1-m)^2 + m\,\mathrm{sn}(x \mid m)^2\,\mathrm{sn}(y \mid 1-m)^2\right) /;\ \{x, y\} \in \mathbb{R}$$

09.29.16.0008.01

$$\mathrm{dn}\left(\sqrt{1-m}\ z \,\middle|\, \frac{m}{m-1}\right) == \frac{1}{\mathrm{dn}(z \mid m)}$$

09.29.16.0009.01

$$\mathrm{dn}\left(\sqrt{m}\ z \,\middle|\, \frac{1}{m}\right) == \mathrm{cn}(z \mid m)$$

09.29.16.0010.01

$$\mathrm{dn}\left(i\,\sqrt{1-m}\ z \,\middle|\, \frac{1}{1-m}\right) == \frac{1}{\mathrm{cn}(z \mid m)}$$

09.29.16.0011.01

$$\mathrm{dn}\left(i\,\sqrt{m}\ z \,\middle|\, \frac{m-1}{m}\right) == \frac{\mathrm{cn}(z \mid m)}{\mathrm{dn}(z \mid m)}$$

Landen's transformation:

09.29.16.0012.01

$$\mathrm{dn}\left(\left(1+\sqrt{1-m}\right)z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) == \frac{1-\left(1-\sqrt{1-m}\right)\mathrm{sn}(z \mid m)^2}{\mathrm{dn}(z \mid m)}$$

Gauss' transformation:

09.29.16.0013.01

$$\mathrm{dn}\left(\left(1+\sqrt{m}\right)z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) == \frac{1-\sqrt{m}\ \mathrm{sn}(z \mid m)^2}{1+\sqrt{m}\ \mathrm{sn}(z \mid m)^2}$$

n th degree transformations:

09.29.16.0014.01

$$\mathrm{dn}\left(\frac{z}{M}\,\middle|\, l\right) == \mathrm{dn}(z \mid m) \prod_{r=1}^{\frac{n-1}{2}} \frac{1 - m\,\mathrm{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2 \mathrm{sn}(z \mid m)^2}{1 - m\,\mathrm{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2 \mathrm{sn}(z \mid m)^2} /;$$

$$\frac{n+1}{2} \in \mathbb{Z}^+ \bigwedge l == m^n \prod_{r=1}^{\frac{n-1}{2}} \mathrm{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^8 \bigwedge M == \prod_{r=1}^{\frac{n-1}{2}} \frac{\mathrm{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2}{\mathrm{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2}$$

09.29.16.0015.01

$$\mathrm{dn}\left(\frac{z}{M} + \frac{K(m)}{n\,M}\,\middle|\, l\right) == \frac{\sqrt{1-l}}{\mathrm{dn}(z \mid m)} \prod_{r=1}^{\frac{n}{2}} \frac{1 - m\,\mathrm{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2 \mathrm{sn}(z \mid m)^2}{1 - m\,\mathrm{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2 \mathrm{sn}(z \mid m)^2} /;$$

$$\frac{n}{2} \in \mathbb{Z}^+ \bigwedge l == m^n \prod_{r=1}^{\frac{n}{2}} \mathrm{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^8 \bigwedge M == \prod_{r=1}^{\frac{n}{2}} \frac{\mathrm{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2}{\mathrm{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2}$$

Argument involving half-periods

09.29.16.0004.01

$$\mathrm{dn}(z + K(m) \mid m) == \sqrt{1-m}\ \mathrm{nd}(z \mid m)$$

09.29.16.0169.01

$$\mathrm{dn}(z - K(m) \mid m) = \sqrt{1-m}\ \mathrm{nd}(z \mid m)$$

09.29.16.0170.01

$$\mathrm{dn}(z + 3\,K(m) \mid m) = \sqrt{1-m}\ \mathrm{nd}(z \mid m)$$

09.29.16.0171.01

$$\mathrm{dn}(z + (2r+1)\,K(m) \mid m) = \sqrt{1-m}\ \mathrm{nd}(z \mid m)\ /;\ r \in \mathbb{Z}$$

09.29.16.0005.01

$$\mathrm{dn}(z + i\,K(1-m) \mid m) == -i\,\mathrm{cs}(z \mid m)$$

09.29.16.0172.01

$$\mathrm{dn}(z - i\,K(1-m) \mid m) = i\,\mathrm{cs}(z \mid m)$$

09.29.16.0173.01

$$\mathrm{dn}(z + 3\,i\,K(1-m) \mid m) = i\,\mathrm{cs}(z \mid m)\ /;\ s \in \mathbb{Z}$$

09.29.16.0174.01

$$\mathrm{dn}(z + (2s+1)\,i\,K(1-m) \mid m) = (-1)^{s-1}\,i\,\mathrm{cs}(z \mid m)\ /;\ s \in \mathbb{Z}$$

09.29.16.0006.01

$$\mathrm{dn}(z + K(m) + i\,K(1-m) \mid m) == i\,\sqrt{1-m}\ \mathrm{sc}(z \mid m)$$

09.29.16.0175.01

$$\mathrm{dn}(z - i\,K(1-m) + K(m) \mid m) = -i\,\sqrt{1-m}\ \mathrm{sc}(z \mid m)$$

09.29.16.0176.01

$$\mathrm{dn}(z + i\,K(1-m) - K(m) \mid m) = i\,\sqrt{1-m}\ \mathrm{sc}(z \mid m)$$

09.29.16.0177.01

$$\mathrm{dn}(z - i\,K(1-m) - K(m) \mid m) = -i\,\sqrt{1-m}\ \mathrm{sc}(z \mid m)$$

09.29.16.0178.01

$$\mathrm{dn}(z + i\,K(1-m) + 3\,K(m) \mid m) = i\,\sqrt{1-m}\ \mathrm{sc}(z \mid m)$$

09.29.16.0179.01

$$\mathrm{dn}(z + (4\,s+1)\,i\,K(1-m) + (2\,r+1)\,K(m) \mid m) = i\,\sqrt{1-m}\ \mathrm{sc}(z \mid m)$$

09.29.16.0180.01

$$\mathrm{dn}(z + (4\,s-1)\,i\,K(1-m) + (2\,r+1)\,K(m) \mid m) = -i\,\sqrt{1-m}\ \mathrm{sc}(z \mid m)$$

09.29.16.0181.01

$$\mathrm{dn}(z + (2\,s+1)\,i\,K(1-m) + (2\,r+1)\,K(m) \mid m) = i\,(-1)^s\,\sqrt{1-m}\ \mathrm{sc}(z \mid m)$$

Argument involving inverse Jacobi functions

09.29.16.0182.01

$$\mathrm{dn}\left(\mathrm{cd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m-1}{m\,z^2-1}$$

09.29.16.0183.01

$$\mathrm{dn}\left(\mathrm{cn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = m\,z^2 - m + 1$$

09.29.16.0184.01

$$\mathrm{dn}\left(\mathrm{cs}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2-m+1}{z^2+1}$$

09.29.16.0185.01

$$\mathrm{dn}\left(\mathrm{dc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{(m-1)\,z^2}{m-z^2}$$

09.29.16.0186.01

$$\mathrm{dn}\left(\mathrm{ds}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2}{z^2+m}$$

09.29.16.0187.01

$$\mathrm{dn}\left(\mathrm{nc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m}{z^2} - m + 1$$

09.29.16.0188.01

$$\mathrm{dn}\left(\mathrm{nd}^{-1}(z \mid m) \,\middle|\, m\right) = \frac{1}{z}$$

09.29.16.0189.01

$$\mathrm{dn}\left(\mathrm{ns}^{-1}(z \mid m) \,\middle|\, m\right)^2 = 1 - \frac{m}{z^2}$$

09.29.16.0190.01

$$\mathrm{dn}\left(\mathrm{sc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{(1-m)\,z^2+1}{z^2+1}$$

09.29.16.0191.01

$$\mathrm{dn}\left(\mathrm{sd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{1}{m\,z^2+1}$$

09.29.16.0192.01

$$\mathrm{dn}\left(\mathrm{sn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = 1 - m\,z^2$$

Addition formulas

09.29.16.0016.01

$$\mathrm{dn}(a+v\mid m)==\frac{\mathrm{dn}(u\mid m)\,\mathrm{dn}(v\mid m)-m\,\mathrm{sn}(u\mid m)\,\mathrm{cn}(u\mid m)\,\mathrm{sn}(v\mid m)\,\mathrm{cn}(v\mid m)}{1-m\,\mathrm{sn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2}$$

09.29.16.0017.01

$$\mathrm{dn}(u+v\mid m)+\mathrm{dn}(u-v\mid m)==\frac{2\,\mathrm{dn}(u\mid m)\,\mathrm{dn}(v\mid m)}{1-m\,\mathrm{sn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2}$$

09.29.16.0018.01

$$\mathrm{dn}(u+v\mid m)-\mathrm{dn}(u-v\mid m)==-\frac{2\,m\,\mathrm{sn}(u\mid m)\,\mathrm{cn}(u\mid m)\,\mathrm{sn}(v\mid m)\,\mathrm{cn}(v\mid m)}{1-m\,\mathrm{sn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2}$$

09.29.16.0019.01

$$\mathrm{dn}(u+v\mid m)\,\mathrm{dn}(u-v\mid m)==\frac{\mathrm{dn}(v\mid m)^2-m\,\mathrm{cn}(v\mid m)^2\,\mathrm{sn}(u\mid m)^2}{1-m\,\mathrm{sn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2}$$

09.29.16.0020.01

$$\mathrm{dn}(u+v\mid m)\,\mathrm{dn}(u-v\mid m)==\frac{\mathrm{dn}(u\mid m)^2+\mathrm{dn}(v\mid m)^2}{1-m\,\mathrm{sn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2}-1$$

09.29.16.0021.01

$$\mathrm{dn}(u+v\mid m)\,\mathrm{dn}(u-v\mid m)==1-\frac{m\left(\mathrm{cn}(v\mid m)^2\,\mathrm{sn}(u\mid m)^2+\mathrm{cn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2\right)}{1-m\,\mathrm{sn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2}$$

09.29.16.0022.01

$$(1+\mathrm{dn}(u+v\mid m))\,(1+\mathrm{dn}(u-v\mid m))==\frac{(\mathrm{dn}(u\mid m)+\mathrm{dn}(v\mid m))^2}{1-m\,\mathrm{sn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2}$$

09.29.16.0023.01

$$(1+\mathrm{dn}(u+v\mid m))\,(1-\mathrm{dn}(u-v\mid m))==\frac{m\,(\mathrm{sn}(u\mid m)\,\mathrm{cn}(v\mid m)-\mathrm{sn}(v\mid m)\,\mathrm{cn}(u\mid m))^2}{1-m\,\mathrm{sn}(u\mid m)^2\,\mathrm{sn}(v\mid m)^2}$$

09.29.16.0024.01

$$\mathrm{sn}(v\mid m)\,\mathrm{cn}(u\mid m)\,\mathrm{dn}(u+v\mid m)==\mathrm{dn}(v\mid m)\,\mathrm{sn}(u+v\mid m)-\mathrm{cn}(v\mid m)\,\mathrm{sn}(u\mid m)$$

09.29.16.0025.01

$$\mathrm{cn}(v\mid m)\,\mathrm{cn}(u\mid m)\,\mathrm{dn}(u+v\mid m)==\mathrm{cn}(u+v\mid m)\,\mathrm{dn}(v\mid m)\,\mathrm{dn}(u\mid m)+(1-m)\,\mathrm{sn}(v\mid m)\,\mathrm{sn}(u\mid m)$$

09.29.16.0026.01

$$\mathrm{dn}(v\mid m)\,\mathrm{cn}(u\mid m)\,\mathrm{sn}(u+v\mid m)==\mathrm{dn}(u+v\mid m)\,\mathrm{sn}(v\mid m)+\mathrm{cn}(u+v\mid m)\,\mathrm{sn}(u\mid m)$$

09.29.16.0027.01

$$\frac{1}{m}(\mathrm{cn}(v\mid m)\,\mathrm{cn}(u\mid m)\,\mathrm{cn}(u+v\mid m)==\mathrm{dn}(v\mid m)\,\mathrm{dn}(u\mid m)\,\mathrm{dn}(u+v\mid m)-1)+1$$

09.29.16.0028.01

$$\frac{1}{m}(\mathrm{cn}(v\mid m)\,\mathrm{cn}(u\mid m)\,\mathrm{cn}(u+v\mid m)==\mathrm{dn}(v\mid m)\,\mathrm{dn}(u\mid m)\,\mathrm{dn}(u+v\mid m)-1)+1$$

09.29.16.0029.01

$$\mathrm{sn}(v\mid m)\,\mathrm{sn}(u\mid m)\,\mathrm{sn}(u+v\mid m)==-\frac{\mathrm{sn}(u+v\mid m)}{\mathrm{dn}(u+v\mid m)}(\mathrm{cn}(u+v\mid m)-\mathrm{cn}(v\mid m)\,\mathrm{cn}(u\mid m))$$

09.29.16.0030.01

$$\text{sn}(v \mid m)\,\text{sn}(u \mid m)\,\text{sn}(u+v \mid m) == -\frac{\text{sn}(u+v \mid m)}{m\,\text{cn}(u+v \mid m)}\left(\text{dn}(u+v \mid m) - \text{dn}(v \mid m)\,\text{dn}(u \mid m)\right)$$

09.29.16.0031.01

$$\text{sn}(u+v \mid m)\,\text{cn}(v \mid m)\,\text{dn}(u \mid m) == \text{cn}(u+v \mid m)\,\text{sn}(v \mid m) + \text{dn}(u+v \mid m)\,\text{sn}(u \mid m)$$

09.29.16.0032.01

$$\text{cn}(u+v \mid m)\,\text{cn}(v \mid m)\,\text{dn}(u \mid m) == \text{dn}(u+v \mid m)\,\text{cn}(u \mid m)\,\text{dn}(v \mid m) - (1-m)\,\text{sn}(u+v \mid m)\,\text{sn}(v \mid m)$$

09.29.16.0033.01

$$\text{dn}(u+v \mid m)\,\text{cn}(v \mid m)\,\text{sn}(u \mid m) == \text{sn}(u+v \mid m)\,\text{dn}(u \mid m) - \text{sn}(v \mid m)\,\text{cn}(u \mid m)$$

Half-angle formulas

09.29.16.0034.01

$$\text{dn}\left(\frac{z}{2}\,\middle|\,m\right)^2 == \frac{1-m+\text{dn}(z \mid m)+m\,\text{cn}(z \mid m)}{1+\text{dn}(z \mid m)}$$

Multiple arguments

Double angle formulas

09.29.16.0035.01

$$\text{dn}(2\,z \mid m) == \frac{\text{dn}(z \mid m)^2 + \text{cn}(z \mid m)^2\left(\text{dn}(z \mid m)^2 - 1\right)}{\text{dn}(z \mid m)^2 - \text{cn}(z \mid m)^2\left(\text{dn}(z \mid m)^2 - 1\right)}$$

09.29.16.0036.01

$$\text{dn}(2\,z \mid m) == \frac{\text{dn}(z \mid m)^2 - m\,\text{sn}(z \mid m)^2\,\text{cn}(z \mid m)^2}{1-m\,\text{sn}(z \mid m)^4}$$

09.29.16.0037.01

$$\frac{1-\text{dn}(2\,z \mid m)}{1+\text{dn}(2\,z \mid m)} == \frac{m\,\text{sn}(z \mid m)^2\,\text{cn}(z \mid m)^2}{\text{dn}(z \mid m)^2}$$

Multiple angle formulas

09.29.16.0038.01

$$\text{dn}(n\,z \mid m) == \left(\frac{1}{1-m}\right)^{\frac{n^2-1}{4}} \prod_{\mu,\nu=0}^{n-1} \text{dn}\left(z + \frac{4\,K(m)\,(\mu+\nu\,\tau)}{n}\,\middle|\,m\right) /;\ \frac{n+1}{2} \in \mathbb{Z}^+$$

09.29.16.0039.01

$$n\,\text{dn}(n\,z \mid m) == (-1)^{\frac{1-n}{2}} \sum_{r,s=0}^{n-1} \text{dn}\left(z + \frac{4\,(K(m)\,r + K(m)\,s\,\tau)}{n}\,\middle|\,m\right) /;\ \frac{n+1}{2} \in \mathbb{Z}^+$$

09.29.16.0040.01

$$\text{dn}\left(\frac{2\,n}{\pi}\,K\left(\lambda\left(\frac{n\log(q(m))}{\pi\,i}\right)\right)x\,\middle|\,\lambda\left(\frac{n}{\pi\,i}\log(q(m))\right)\right) == \frac{\sqrt[4]{1-\lambda\left(\frac{n\log(q(m))}{\pi\,i}\right)}}{(1-m)^{n/4}} \prod_{r=0}^{n-1} \text{dn}\left(\frac{2\,K(m)}{\pi}\left(\frac{\pi\,r}{n}+x\right)\,\middle|\,m\right) /;\ \frac{n+1}{2} \in \mathbb{Z}^+$$

Products of a single Jacobi function

09.29.16.0072.01

$$m^{\frac{p-1}{2}} \prod_{k=0}^{p-1} \mathrm{sn}\left(z + \frac{4\,k\,K(m)}{p} \middle| m\right) == (-1)^{\frac{p-1}{2}} \left(\prod_{k=1}^{\frac{p-1}{2}} \mathrm{ns}\left(\frac{4\,k\,K(m)}{p} \middle| m\right)^2\right) \sum_{k=0}^{p-1} \mathrm{sn}\left(z + \frac{4\,k\,K(m)}{p} \middle| m\right) /;\ \frac{p-1}{2} \in \mathbb{N}$$

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09.29.16.0073.01

$$m^{\frac{p-1}{2}} \prod_{k=0}^{p-1} \mathrm{cn}\left(z + \frac{4\,k\,K(m)}{p} \middle| m\right) == \left(\prod_{k=1}^{\frac{p-1}{2}} \mathrm{ds}\left(\frac{4\,k\,K(m)}{p} \middle| m\right)^2\right) \sum_{k=0}^{p-1} \mathrm{cn}\left(z + \frac{4\,k\,K(m)}{p} \middle| m\right) /;\ \frac{p-1}{2} \in \mathbb{N} \bigwedge r \in \mathbb{N}^+ \bigwedge r < p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0074.01

$$m^{p/2} \prod_{k=0}^{p-1} \mathrm{sn}\left(z + \frac{2\,k\,K(m)}{p} \middle| m\right) == \left(\prod_{k=1}^{p/2-1} \mathrm{ns}\left(\frac{2\,k\,K(m)}{p} \middle| m\right)^2\right) \sum_{k=0}^{p-1} (-1)^k\, \mathrm{Z}\left(\mathrm{am}\left(z + \frac{2\,k\,K(m)}{p} \middle| m\right) \middle| m\right) /;$$
$$\frac{p}{2} \in \mathbb{N}^+ \bigwedge r \in \mathbb{N}^+ \bigwedge r < p \bigwedge \gcd(p, r) == 1 \bigwedge 1 - m > 0$$

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09.29.16.0075.01

$$m^{p/2} \prod_{k=0}^{p-1} \mathrm{cn}\left(z + \frac{2\,k\,K(m)}{p} \middle| m\right) == \sqrt{1-m}\ (-1)^{p/2} \left(\prod_{k=1}^{p/2-1} \mathrm{ds}\left(\frac{2\,k\,K(m)}{p} \middle| m\right)^2\right) \sum_{k=0}^{p-1} (-1)^k\, \mathrm{Z}\left(\mathrm{am}\left(z + \frac{2\,k\,K(m)}{p} \middle| m\right) \middle| m\right) /;$$
$$\frac{p}{2} \in \mathbb{N}^+ \bigwedge r \in \mathbb{N}^+ \bigwedge r < p \bigwedge \gcd(p, r) == 1 \bigwedge 1 - m > 0$$

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Sums over products of two Jacobi functions

09.29.16.0041.01

$$\mathrm{dn}(z \mid m)\, \mathrm{dn}(z + K(m) \mid m) == \sqrt{1-m}$$

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09.29.16.0042.01

$$\mathrm{dn}(z \mid m)\, \mathrm{dn}\left(z + \frac{2\,K(m)}{3} \middle| m\right) + \mathrm{dn}\left(z + \frac{2\,K(m)}{3} \middle| m\right) \mathrm{dn}\left(z + \frac{4\,K(m)}{3} \middle| m\right) + \mathrm{dn}\left(z + \frac{4\,K(m)}{3} \middle| m\right) \mathrm{dn}(z \mid m) ==$$
$$\mathrm{dn}\left(\frac{2\,K(m)}{3} \middle| m\right)\left(\mathrm{dn}\left(\frac{2\,K(m)}{3} \middle| m\right) + 2\right)$$

Khare/Sukhatme_2002

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09.29.16.0043.01

$$\mathrm{dn}(z \mid m)\,\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z+K(m) \mid m)+$$
$$\mathrm{dn}(z+K(m) \mid m)\,\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z \mid m)==2\sqrt[4]{1-m}\,\left(1+\sqrt{1-m}\right)$$

Khare/Sukhatme_2002

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09.29.16.0044.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,(k+1)\,K(m)}{p}\,\middle|\,m\right)==\sum_{k=0}^{p-1}\mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(\frac{2\,(k+1)\,K(m)}{p}\,\middle|\,m\right)/;\ p-2\in\mathbb{N}$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0045.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,(k+n)\,K(m)}{p}\,\middle|\,m\right)==\sum_{k=0}^{p-1}\mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(\frac{2\,(k+n)\,K(m)}{p}\,\middle|\,m\right)/;$$
$$p-2\in\mathbb{N}\bigwedge n\in\mathbb{Z}\bigwedge 1\le n\le\frac{p+p\bmod 2}{2}$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0076.01

$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\left(\mathrm{cn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\right)==0\ /;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0077.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)==p\,\mathrm{dn}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\left(1-\frac{\left|\mathrm{cn}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right|\mathrm{Z}\left(\sin^{-1}\left(\mathrm{sn}\left(\frac{2\,K(m)}{p}\,\middle|\,m\right)\right)\,\middle|\,m\right)}{\left|\mathrm{sn}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right|\mathrm{dn}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)}\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1\bigwedge m\in\mathbb{R}\bigwedge m<1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0078.01

$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)==\frac{p\,\mathrm{cn}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{Z}\left(\sin^{-1}\left(\mathrm{sn}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)\,\middle|\,m\right)}{m\left|\mathrm{sn}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right|\left|\mathrm{cn}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right|}/;$$
$$p-2\in\mathbb{N}\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1\bigwedge m\in\mathbb{R}\bigwedge m<1$$

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Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0079.01

$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\mathrm{cn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right) == p\,\mathrm{cn}\left(\frac{2rK(m)}{p}\middle|m\right)\left(1-\frac{\mathrm{dn}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{Z}\left(\sin^{-1}\left(\mathrm{sn}\left(\frac{2rK(m)}{p}\middle|m\right)\right)\middle|m\right)}{m\left|\mathrm{sn}\left(\frac{2rK(m)}{p}\middle|m\right)\right|\left|\mathrm{cn}\left(\frac{2rK(m)}{p}\middle|m\right)\right|}\right)/;$$

$$p-2\in\mathbb{N}\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1\bigwedge m\in\mathbb{R}\bigwedge m<1$$

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Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0080.01

$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{4kK(m)}{p}\middle|m\right)\left(\mathrm{dn}\left(z+\frac{4(k-r)K(m)}{p}\middle|m\right)+\mathrm{dn}\left(z+\frac{4(k+r)K(m)}{p}\middle|m\right)\right) == 0\ /;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0081.01

$$\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{4kK(m)}{p}\middle|m\right)\left(\mathrm{sn}\left(z+\frac{4(k-r)K(m)}{p}\middle|m\right)+\mathrm{sn}\left(z+\frac{4(k+r)K(m)}{p}\middle|m\right)\right) == 0\ /;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0082.01

$$\sum_{k=0}^{p-1} (-1)^k\,\mathrm{sn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\left(\mathrm{cn}\left(z+\frac{2(k-r)K(m)}{p}\middle|m\right)+\mathrm{cn}\left(z+\frac{2(k+r)K(m)}{p}\middle|m\right)\right) == 0\ /;$$

$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0083.01

$$\sum_{k=0}^{p-1} (-1)^k\,\mathrm{dn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2(k+r)K(m)}{p}\middle|m\right) == -2\,\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)\sum_{k=0}^{p-1}(-1)^k\,\mathrm{Z}\left(\mathrm{am}\left(z+\frac{2kK(m)}{p}\middle|m\right)\middle|m\right)/;$$

$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge \gcd(p,r)==1\bigwedge 1-m>0$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0084.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)==\frac{2}{m}\operatorname{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}(-1)^k\,\mathrm{Z}\left(\operatorname{am}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\middle|\,m\right)/;$$

$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge \gcd(p,\,r)==1\bigwedge 1-m>0$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0085.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)==-\frac{2}{m}\operatorname{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}(-1)^k\,\mathrm{Z}\left(\operatorname{am}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\middle|\,m\right)/;$$

$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge \gcd(p,\,r)==1\bigwedge 1-m>0$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

Sums over products of three Jacobi functions

09.29.16.0046.01

$$\operatorname{dn}(z\mid m)^2\operatorname{dn}(z+K(m)\mid m)+\operatorname{dn}(z\mid m)\operatorname{dn}(z+K(m)\mid m)^2==\sqrt{1-m}\;(\operatorname{dn}(z\mid m)+\operatorname{dn}(z+K(m)\mid m))$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0047.01

$$\operatorname{dn}(z\mid m)^2\operatorname{dn}(z+K(m)\mid m)-\operatorname{dn}(z\mid m)\operatorname{dn}(z+K(m)\mid m)^2==\sqrt{1-m}\;(\operatorname{dn}(z\mid m)-\operatorname{dn}(z+K(m)\mid m))$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0048.01

$$\operatorname{dn}(z\mid m)\operatorname{dn}\left(z+\frac{2\,K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,K(m)}{3}\,\middle|\,m\right)==$$

$$\frac{\operatorname{dn}\left(\frac{2\,K(m)}{3}\,\middle|\,m\right)^2+m-1}{1-\operatorname{dn}\left(\frac{2\,K(m)}{3}\,\middle|\,m\right)^2}\left(\operatorname{dn}(z\mid m)+\operatorname{dn}\left(z+\frac{2\,K(m)}{3}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{4\,K(m)}{3}\,\middle|\,m\right)\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0049.01

$$\mathrm{dn}(z\mid m)^2\left(\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)^2\left(\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}(z\mid m)\right)+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)^2\left(\mathrm{dn}(z\mid m)+\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\right)==$$
$$2\left(\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)-m+1\right)\Big/\left(1+\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\right)\left(\mathrm{dn}(z\mid m)+\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0050.01

$$\mathrm{dn}(z\mid m)\,\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z+K(m)\mid m)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z+K(m)\mid m)\,\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)+$$
$$\mathrm{dn}(z+K(m)\mid m)\,\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z\mid m)+\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z\mid m)\,\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)==$$
$$\sqrt{1-m}\left(\mathrm{dn}(z\mid m)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}(z+K(m)\mid m)+\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0051.01

$$\mathrm{dn}(z\mid m)\,\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z+K(m)\mid m)-\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z+K(m)\mid m)\,\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)+$$
$$\mathrm{dn}(z+K(m)\mid m)\,\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z\mid m)-\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\mathrm{dn}(z\mid m)\,\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)==$$
$$\sqrt{1-m}\left(-\mathrm{dn}(z\mid m)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)-\mathrm{dn}(z+K(m)\mid m)+\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0052.01

$$\mathrm{dn}(z\mid m)^2\left(\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\right)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)^2\left(\mathrm{dn}(z+K(m)\mid m)+\mathrm{dn}(z\mid m)\right)+$$
$$\mathrm{dn}(z+K(m)\mid m)^2\left(\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\right)+\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)^2\left(\mathrm{dn}(z\mid m)+\mathrm{dn}(z+K(m)\mid m)\right)==$$
$$2\sqrt[4]{1-m}\left(1-\sqrt[4]{1-m}+\sqrt{1-m}\right)\left(\mathrm{dn}(z\mid m)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}(z+K(m)\mid m)+\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0053.01

$$\mathrm{dn}(z\mid m)^2\left(\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)\right)-\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)^2(\mathrm{dn}(z+K(m)\mid m)+\mathrm{dn}(z\mid m))+$$
$$\mathrm{dn}(z+K(m)\mid m)^2\left(\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\right)-\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)^2(\mathrm{dn}(z\mid m)+\mathrm{dn}(z+K(m)\mid m))==$$
$$2\sqrt[4]{1-m}\left(1+\sqrt[4]{1-m}+\sqrt{1-m}\right)\left(\mathrm{dn}(z\mid m)-\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}(z+K(m)\mid m)-\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0054.01

$$\mathrm{dn}(z\mid m)^2\,\mathrm{dn}(z+K(m)\mid m)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)^2\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}(z+K(m)\mid m)^2\,\mathrm{dn}(z\mid m)+$$
$$\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)^2\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)==\sqrt{1-m}\left(\mathrm{dn}(z\mid m)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}(z+K(m)\mid m)+\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0055.01

$$\mathrm{dn}(z\mid m)^2\,\mathrm{dn}(z+K(m)\mid m)-\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)^2\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}(z+K(m)\mid m)^2\,\mathrm{dn}(z\mid m)-$$
$$\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)^2\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)==\sqrt{1-m}\left(\mathrm{dn}(z\mid m)-\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}(z+K(m)\mid m)-\mathrm{dn}\left(z+\frac{3\,K(m)}{2}\,\middle|\,m\right)\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0056.01

$$\frac{\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2\left(\mathrm{dn}\left(z+\frac{2\,(k-1)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{2\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\right)}{\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)}==$$
$$\frac{\sum_{k=0}^{p-1}\mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2\left(\mathrm{dn}\left(\frac{2\,(k-1)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(\frac{2\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\right)}{\sum_{k=0}^{p-1}\mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)}\;/;\;p-2\in\mathbb{N}$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0057.01

$$\frac{\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 \left(\mathrm{dn}\left(z+\frac{2\,(k-n)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{2\,(k+n)\,K(m)}{p}\,\middle|\,m\right)\right)}{\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1} \mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 \left(\mathrm{dn}\left(\frac{2\,(k-n)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(\frac{2\,(k+n)\,K(m)}{p}\,\middle|\,m\right)\right)}{\sum_{k=0}^{p-1} \mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)} /;\ p-2\in\mathbb{N}\bigwedge n\in\mathbb{Z}\bigwedge 1\le n\le\frac{p+p \bmod 2}{2}$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0058.01

$$\frac{\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \mathrm{dn}\left(z+\frac{2\,(k+1)\,K(m)}{p}\,\middle|\,m\right) \mathrm{dn}\left(z+\frac{2\,(k+2)\,K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1} \mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \mathrm{dn}\left(\frac{2\,(k+1)\,K(m)}{p}\,\middle|\,m\right) \mathrm{dn}\left(\frac{2\,(k+2)\,K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1} \mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)} /;\ p-2\in\mathbb{N}$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0059.01

$$\frac{\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \mathrm{dn}\left(z+\frac{2\,K(m)\,(k+n_1)}{p}\,\middle|\,m\right) \mathrm{dn}\left(z+\frac{2\,K(m)\,(k+n_2)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1} \mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \mathrm{dn}\left(\frac{2\,(k+n_1)\,K(m)}{p}\,\middle|\,m\right) \mathrm{dn}\left(\frac{2\,(k+n_2)\,K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1} \mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)} /;\ p-2\in\mathbb{N}\bigwedge n_1\in\mathbb{Z}\bigwedge n_2\in\mathbb{Z}\bigwedge 1\le n_1<n_2<p$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0060.01

$$\frac{\sum_{k=0}^{p-1} (-1)^k \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 \left(\mathrm{dn}\left(z+\frac{2\,(k-1)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{2\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\right)}{\sum_{k=0}^{p-1} (-1)^k \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1} (-1)^k \mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 \left(\mathrm{dn}\left(\frac{2\,(k-1)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(\frac{2\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\right)}{\sum_{k=0}^{p-1} (-1)^k \mathrm{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)} /;\ \frac{p}{2}\in\mathbb{N}^+$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0061.01

$$\frac{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)^2\left(\operatorname{dn}\left(z+\frac{2(k-n)K(m)}{p}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{2(k+n)K(m)}{p}\,\middle|\,m\right)\right)}{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(\frac{2kK(m)}{p}\,\middle|\,m\right)^2\left(\operatorname{dn}\left(\frac{2(k-n)K(m)}{p}\,\middle|\,m\right)+\operatorname{dn}\left(\frac{2(k+n)K(m)}{p}\,\middle|\,m\right)\right)}{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(\frac{2kK(m)}{p}\,\middle|\,m\right)} /;\ \frac{p}{2}\in\mathbb{N}^+ \bigwedge n\in\mathbb{Z} \bigwedge 1\le n\le\frac{p}{2}$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0062.01

$$\sum_{k=0}^{p-1}\operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)^2\operatorname{dn}\left(z+\frac{(2k+p)K(m)}{p}\,\middle|\,m\right)==\sqrt{1-m}\sum_{k=0}^{p-1}\operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right) /;\ \frac{p}{2}\in\mathbb{N}^+$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0063.01

$$\sum_{k=0}^{p-1}(-1)^k\operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)^2\operatorname{dn}\left(z+\frac{(2k+p)K(m)}{p}\,\middle|\,m\right)==\sqrt{1-m}\sum_{k=0}^{p-1}(-1)^k\operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right) /;\ \frac{p}{2}\in\mathbb{N}^+$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0064.01

$$\frac{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{2(k+1)K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{2(k+2)K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(\frac{2kK(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{2(k+1)K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{2(k+2)K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(\frac{2kK(m)}{p}\,\middle|\,m\right)} /;\ \frac{p}{2}\in\mathbb{N}^+$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0065.01

$$\frac{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{2K(m)(k+n_1)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{2K(m)(k+n_2)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(\frac{2kK(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{2(k+n_1)K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{2(k+n_2)K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(\frac{2kK(m)}{p}\,\middle|\,m\right)} /;\ \frac{p}{2}\in\mathbb{N}^+ \bigwedge n_1\in\mathbb{Z} \bigwedge n_2\in\mathbb{Z} \bigwedge 1\le n_1<n_2<p$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0086.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^2\left(\mathrm{dn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)+\mathrm{dn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\right)==$$

$$2\left(\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)-\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0087.01

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2K(m)k}{p}\middle|m\right)$$

$$\left(\mathrm{cn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)+\mathrm{cn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\right)==$$

$$-\frac{2}{m}\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)\left(\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)-\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0088.01

$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2kK(m)}{p}\middle|m\right)$$

$$\left(\mathrm{dn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)+\mathrm{dn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\right)==$$

$$-\frac{2}{m}\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)\left(\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)-\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0089.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)$$

$$\left(\mathrm{dn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2K(m)(k-s)}{p}\middle|m\right)+\mathrm{dn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2K(m)(k+s)}{p}\middle|m\right)\right)==$$

$$-2\left(\mathrm{cs}\left(\frac{2(r-s)K(m)}{p}\middle|m\right)\left(\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)-\mathrm{cs}\left(\frac{2sK(m)}{p}\middle|m\right)\right)+\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{cs}\left(\frac{2sK(m)}{p}\middle|m\right)\right)$$

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0090.01

$$\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\left(\mathrm{cn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)\mathrm{cn}\left(z+\frac{2K(m)(k-s)}{p}\middle|m\right)+\mathrm{cn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\mathrm{cn}\left(z+\frac{2K(m)(k+s)}{p}\middle|m\right)\right)==-\frac{2}{m}\left(\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2sK(m)}{p}\middle|m\right)+\mathrm{ds}\left(\frac{2(r-s)K(m)}{p}\middle|m\right)\left(\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)-\mathrm{cs}\left(\frac{2sK(m)}{p}\middle|m\right)\right)\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\wedge r\in\mathbb{N}^+\wedge r<p\wedge s\in\mathbb{N}^+\wedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0091.01

$$\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\left(\mathrm{sn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2K(m)(k-s)}{p}\middle|m\right)+\mathrm{sn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2K(m)(k+s)}{p}\middle|m\right)\right)==\frac{2}{m}\left(\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ns}\left(\frac{2sK(m)}{p}\middle|m\right)+\mathrm{ns}\left(\frac{2(r-s)K(m)}{p}\middle|m\right)\left(\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)-\mathrm{cs}\left(\frac{2sK(m)}{p}\middle|m\right)\right)\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\wedge r\in\mathbb{N}^+\wedge r<p\wedge s\in\mathbb{N}^+\wedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0092.01

$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\left(\mathrm{cn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2K(m)(k-s)}{p}\middle|m\right)+\mathrm{cn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2K(m)(k+s)}{p}\middle|m\right)\right)==-\frac{2}{m}\left(\left(\mathrm{cs}\left(\frac{2sK(m)}{p}\middle|m\right)+\mathrm{cs}\left(\frac{2(r-s)K(m)}{p}\middle|m\right)\right)\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)-\mathrm{ds}\left(\frac{2(r-s)K(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2sK(m)}{p}\middle|m\right)\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\wedge r\in\mathbb{N}^+\wedge r<p\wedge s\in\mathbb{N}^+\wedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0093.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)$$
$$\left(\text{dn}\left(z+\frac{2\,K(m)\,(k-s)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{2\,K(m)\,(k+s)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\right)==$$
$$\frac{2}{m}\left(\left(\text{cs}\left(\frac{2\,s\,K(m)}{p}\,\middle|\,m\right)+\text{cs}\left(\frac{2\,(r-s)\,K(m)}{p}\,\middle|\,m\right)\right)\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)-\text{ns}\left(\frac{2\,(r-s)\,K(m)}{p}\,\middle|\,m\right)\text{ns}\left(\frac{2\,s\,K(m)}{p}\,\middle|\,m\right)\right)$$
$$\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0094.01

$$\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\left(\text{sn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)+\text{sn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\right)==0\ /;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0095.01

$$\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\left(\text{cn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\right)==0\ /;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0096.01

$$\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\left(\text{dn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\right)==0\ /;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0097.01

$$\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)$$
$$\left(\text{dn}\left(z+\frac{2\,K(m)\,(k-s)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{2\,K(m)\,(k+s)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\right)==$$
$$0\ /;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1\bigwedge s\in\mathbb{N}^+\bigwedge s<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0098.01

$$\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^2\left(\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$\frac{2}{m}\left(\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2-\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0099.01

$$\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$\frac{2}{m}\,\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\left(\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)-\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0100.01

$$\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$2\,\text{ns}\left(\frac{8\,r\,K(m)}{p}\,\middle|\,m\right)\left(\text{ds}\left(\frac{8\,r\,K(m)}{p}\,\middle|\,m\right)-\text{cs}\left(\frac{8\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0101.01

$$\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{cn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{cn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$-\frac{2}{m}\left(\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)+\text{ds}\left(\frac{4\,(r-s)\,K(m)}{p}\,\middle|\,m\right)\left(\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)-\text{ns}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\right)\right)$$
$$\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0102.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$

$$\left(\text{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$-2\left(\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{cs}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)+\text{cs}\left(\frac{4\,(r-s)\,K(m)}{p}\,\middle|\,m\right)\left(\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)-\text{ns}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\right)\right)$$

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0103.01

$$\sum_{k=0}^{p-1} \text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$

$$\left(\text{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$-2\,\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\left(\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)-\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0104.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$

$$\left(\text{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$-\frac{2}{m}\,\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\left(\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)-\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0105.01

$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\mathrm{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$\frac{2}{m}\left(\left(\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)-\mathrm{ds}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\right)\mathrm{ns}\left(\frac{4\,(r-s)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\right)$$
$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\,/;\,p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0106.01

$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\mathrm{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$-2\left(\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{cs}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)+\mathrm{cs}\left(\frac{4\,(r-s)\,K(m)}{p}\,\middle|\,m\right)\left(\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)-\mathrm{ds}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\right)\right)$$
$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\,/;\,p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0107.01

$$\sum_{k=0}^{p-1}(-1)^k\,\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,(k+2\,r)\,K(m)}{p}\,\middle|\,m\right)==$$
$$-\left(\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2+2\,\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}(-1)^k\,\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,\,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0108.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2\left(\operatorname{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$2\left(\operatorname{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2+\operatorname{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\operatorname{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;$$

$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,\,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0109.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)$$

$$\left(\operatorname{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\operatorname{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$-\frac{2}{m}\operatorname{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\left(\operatorname{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)+\operatorname{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;$$

$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,\,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0110.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)$$

$$\left(\operatorname{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$\frac{2}{m}\operatorname{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\left(\operatorname{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)+\operatorname{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;$$

$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,\,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

Sums over products of four Jacobi functions

09.29.16.0066.01

$$\operatorname{dn}(z\mid m)^3\operatorname{dn}(z+K(m)\mid m)+\operatorname{dn}(z\mid m)\operatorname{dn}(z+K(m)\mid m)^3==\sqrt{1-m}\left(\operatorname{dn}(z\mid m)^2+\operatorname{dn}(z+K(m)\mid m)^2\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0067.01

$$\mathrm{dn}(z \mid m)^3\,\mathrm{dn}(z+K(m) \mid m) - \mathrm{dn}(z \mid m)\,\mathrm{dn}(z+K(m) \mid m)^3 == \sqrt{1-m}\,\left(\mathrm{dn}(z \mid m)^2 - \mathrm{dn}(z+K(m) \mid m)^2\right)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0068.01

$$\mathrm{dn}(z \mid m)^2\,\mathrm{dn}(z+K(m) \mid m)^2 == 1-m$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0069.01

$$\mathrm{dn}(z \mid m)^3\left(\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)^3\left(\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}(z \mid m)\right)+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)^3\left(\mathrm{dn}(z \mid m)+\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\right) ==$$
$$\frac{2m\,\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)}{1-\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^2}\left(\mathrm{dn}(z \mid m)^2+\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)^2+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)^2\right)-2(1-m)$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0111.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)\,k}{p}\,\middle|\,m\right)^2$$
$$\left(\mathrm{cn}\left(z+\frac{2K(m)\,(k-r)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2K(m)\,(k-r)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{2K(m)\,(k+r)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2K(m)\,(k+r)}{p}\,\middle|\,m\right)\right) ==$$
$$-2\left(\mathrm{cs}\left(\frac{2rK(m)}{p}\,\middle|\,m\right)^2+\mathrm{ds}\left(\frac{2rK(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{2rK(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2K(m)\,k}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2K(m)\,k}{p}\,\middle|\,m\right) /;\ p\in\mathbb{N}^+\bigwedge$$
$$r\in\mathbb{N}^+\bigwedge r<p$$

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09.29.16.0112.01

$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2K(m)\,k}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2K(m)\,k}{p}\,\middle|\,m\right)$$
$$\left(\mathrm{cn}\left(z+\frac{2K(m)\,(k-r)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2K(m)\,(k-r)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{2K(m)\,(k+r)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2K(m)\,(k+r)}{p}\,\middle|\,m\right)\right) ==$$
$$-2\,\mathrm{cs}\left(\frac{2rK(m)}{p}\,\middle|\,m\right)\left(\mathrm{ds}\left(\frac{2rK(m)}{p}\,\middle|\,m\right)+\mathrm{ns}\left(\frac{2rK(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2K(m)\,k}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2K(m)\,k}{p}\,\middle|\,m\right) /;\ p\in\mathbb{N}^+\bigwedge$$
$$r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0113.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right) \text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)$$
$$\left(\text{dn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{2\,K(m)\,(k-s)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{2\,K(m)\,(k+s)}{p}\,\middle|\,m\right)\right)==$$
$$-2\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{cs}\left(\frac{2\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0114.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right) \text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)$$
$$\left(\text{cn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)\text{cn}\left(z+\frac{2\,K(m)\,(k-s)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\text{cn}\left(z+\frac{2\,K(m)\,(k+s)}{p}\,\middle|\,m\right)\right)==$$
$$-\frac{2}{m}\,\text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{2\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0115.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right) \text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)$$
$$\left(\text{sn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,(k-s)}{p}\,\middle|\,m\right)+\text{sn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,(k+s)}{p}\,\middle|\,m\right)\right)==$$
$$\frac{2}{m}\,\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{ns}\left(\frac{2\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0116.01

$$\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right) \mathrm{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right)$$
$$\left(\mathrm{dn}\left(z+\frac{2\,K(m)\,(k-s)}{p}\,\middle|\,m\right) \mathrm{sn}\left(z+\frac{2\,K(m)\,(k-r)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{2\,K(m)\,(k+s)}{p}\,\middle|\,m\right) \mathrm{sn}\left(z+\frac{2\,K(m)\,(k+r)}{p}\,\middle|\,m\right)\right) ==$$
$$-2\,\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \mathrm{cs}\left(\frac{2\,s\,K(m)}{p}\,\middle|\,m\right) \sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right) \mathrm{sn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right) /;$$
$$p \in \mathbb{N}^+ \bigwedge r \in \mathbb{N}^+ \bigwedge r < p \bigwedge s \in \mathbb{N}^+ \bigwedge s < r$$

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09.29.16.0117.01

$$\sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 \mathrm{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^2 ==$$
$$\frac{p}{2\,K(m)}\left(\int_0^{2\,K(m)} \mathrm{dn}(t\mid m)^2\,\mathrm{dn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2 dt + 4\,E(m)\,\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\right) -$$
$$2\,\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2 \sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 /;\; p \in \mathbb{N}^+ \bigwedge r \in \mathbb{N}^+ \bigwedge r < p-1$$

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09.29.16.0118.01

$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\mathrm{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right) \mathrm{sn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right) \mathrm{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$
$$\frac{4}{m^2}\,\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \sum_{k=0}^{p-1} \mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 + \frac{p}{2\,K(m)}$$
$$\left(\int_0^{2\,K(m)} \mathrm{cn}(t\mid m)\,\mathrm{sn}(t\mid m)\left(\mathrm{cn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \mathrm{sn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \mathrm{sn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right) dt -$$
$$\frac{8}{m^2}\,\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) E(m)\Bigg) /;\; p \in \mathbb{N}^+ \bigwedge r \in \mathbb{N}^+ \bigwedge r < p-1$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0119.01

$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$-\frac{4}{m}\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1} \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2+\frac{p}{2\,K(m)}$$
$$\left(\int_0^{2\,K(m)} \text{cn}(t\mid m)\,\text{dn}(t\mid m)\left(\text{cn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)dt+$$
$$\frac{8}{m}\,E(m)\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\Bigg)\,/;\,p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0120.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$\frac{4}{m}\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1} \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2+\frac{p}{2\,K(m)}$$
$$\left(\int_0^{2\,K(m)} \text{sn}(t\mid m)\,\text{dn}(t\mid m)\left(\text{dn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)dt-$$
$$\frac{8}{m}\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)E(m)\Bigg)\,/;\,p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0121.01

$$\sum_{k=0}^{p-1} \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^3\left(\text{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$2\,\text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1} \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2+$$
$$\frac{p}{2\,K(m)}\left(\int_0^{2\,K(m)} \text{dn}(t\mid m)^3\left(\text{dn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)dt-4\,\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right) \text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)E(m)\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0122.01

$$\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^3\left(\text{sn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$\frac{2}{m^2}\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2+$$

$$\frac{p}{2\,K(m)}\left(\int_0^{2\,K(m)}\text{sn}(t\mid m)^3\left(\text{sn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)+\text{sn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)dt-\right.$$

$$\left.\frac{4}{m^2}\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)E(m)\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0123.01

$$\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^3\left(\text{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$\frac{p}{2\,K(m)}\left(\int_0^{2\,K(m)}\text{cn}(t\mid m)^3\left(\text{cn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)dt-\frac{4}{m^2}\,\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)E(m)\right)+$$

$$\frac{2}{m^2}\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2\ /;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0124.01

$$\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\left(\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^2+\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^2\right)==$$

$$\frac{2}{m}\left(\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)$$

$$\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0125.01

$$\sum_{k=0}^{p-1} \operatorname{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\operatorname{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\operatorname{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$\frac{2}{m}\operatorname{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\left(\operatorname{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)+\operatorname{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0126.01

$$\sum_{k=0}^{p-1} \operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\operatorname{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\operatorname{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$\frac{2}{m}\operatorname{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\operatorname{ns}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0127.01

$$\sum_{k=0}^{p-1} \operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\operatorname{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{cn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\operatorname{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\operatorname{cn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$-\frac{2}{m}\operatorname{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\operatorname{ds}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0128.01

$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$
$$-2\,\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right) \text{cs}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right) \sum_{k=0}^{p-1} \text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) /;$$
$$p \in \mathbb{N}^+ \bigwedge r \in \mathbb{N}^+ \bigwedge r < p \bigwedge s \in \mathbb{N}^+ \bigwedge s < r$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0129.01

$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$
$$\frac{2}{m}\,\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right) \text{ns}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right) \sum_{k=0}^{p-1} \text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) /;$$
$$p \in \mathbb{N}^+ \bigwedge r \in \mathbb{N}^+ \bigwedge r < p \bigwedge s \in \mathbb{N}^+ \bigwedge s < r$$

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09.29.16.0130.01

$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^2$$
$$\left(\text{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$
$$-\frac{2}{m}\left(\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right) \text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right) \sum_{k=0}^{p-1} \text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) /;\; p \in \mathbb{N}^+ \bigwedge$$
$$r \in \mathbb{N}^+ \bigwedge r < p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0131.01

$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$-\frac{2}{m}\,\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\left(\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)+\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge$$
$$r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0132.01

$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^2 \text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\left(\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$\frac{2}{m}\,\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0133.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{cn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{cn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$-\frac{2}{m}\,\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0134.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\text{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\right)\text{dn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right)==$$
$$-2\,\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{cs}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0135.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)+\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$
$$\frac{2}{m}\,\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{ns}\left(\frac{2\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0136.01

$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{cn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$
$$-\frac{2}{m}\,\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0137.01

$$\sum_{k=0}^{p-1}(-1)^k\,\text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2$$
$$\left(\text{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$
$$2\left(\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2-\text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}(-1)^k\,\text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;$$
$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,\,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0138.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\middle|m\right)\operatorname{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\middle|m\right)\operatorname{dn}\left(z+\frac{2\,(k+2\,r)\,K(m)}{p}\middle|m\right)\operatorname{dn}\left(z+\frac{2\,(k+3\,r)\,K(m)}{p}\middle|m\right)==$$

$$2\left(\operatorname{cs}\left(\frac{4\,r\,K(m)}{p}\middle|m\right)\operatorname{cs}\left(\frac{6\,r\,K(m)}{p}\middle|m\right)\operatorname{cs}\left(\frac{2\,r\,K(m)}{p}\middle|m\right)+\operatorname{cs}\left(\frac{2\,r\,K(m)}{p}\middle|m\right)^2\operatorname{cs}\left(\frac{4\,r\,K(m)}{p}\middle|m\right)\right)$$

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{Z}\left(\operatorname{am}\left(z+\frac{2\,k\,K(m)}{p}\middle|m\right)\middle|m\right)/;\ \frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge \gcd(p,\,r)==1\bigwedge 1-m>0$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0139.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\middle|m\right)^3\left(\operatorname{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\middle|m\right)+\operatorname{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\middle|m\right)\right)==$$

$$2\operatorname{ns}\left(\frac{2\,r\,K(m)}{p}\middle|m\right)\operatorname{ds}\left(\frac{2\,r\,K(m)}{p}\middle|m\right)\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\middle|m\right)^2/;\ \frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,\,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0140.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{cn}\left(z+\frac{2\,k\,K(m)}{p}\middle|m\right)^3\left(\operatorname{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\middle|m\right)+\operatorname{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\middle|m\right)\right)==$$

$$\frac{2}{m^2}\operatorname{cs}\left(\frac{2\,r\,K(m)}{p}\middle|m\right)\operatorname{ns}\left(\frac{2\,r\,K(m)}{p}\middle|m\right)\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\middle|m\right)^2/;\ \frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,\,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0141.01

$$\sum_{k=0}^{p-1}(-1)^k \operatorname{sn}\left(z+\frac{2\,k\,K(m)}{p}\middle|m\right)^3\left(\operatorname{sn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\middle|m\right)+\operatorname{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\middle|m\right)\right)==$$

$$\frac{2}{m^2}\operatorname{cs}\left(\frac{2\,r\,K(m)}{p}\middle|m\right)\operatorname{ds}\left(\frac{2\,r\,K(m)}{p}\middle|m\right)\sum_{k=0}^{p-1}(-1)^k \operatorname{dn}\left(z+\frac{2\,k\,K(m)}{p}\middle|m\right)^2/;\ \frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge \gcd(p,\,r)==1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

Sums over products of five Jacobi functions

09.29.16.0142.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^4\left(\mathrm{dn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)+\mathrm{dn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\right)==$$

$$2\,\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^3+2\,\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2$$

$$\left(\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2-\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0143.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^3\left(\mathrm{dn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)^2+\mathrm{dn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)^2\right)==$$

$$-2\,\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^3+$$

$$2\left(\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)^2\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2+\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)^2\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)^2+\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)^2\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2-\right.$$

$$\left.3\,\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0144.01

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\left(\mathrm{dn}\left(z+\frac{2(k-r)K(m)}{p}\middle|m\right)^2+\mathrm{dn}\left(z+\frac{2(k+r)K(m)}{p}\middle|m\right)^2\right)==$$

$$-2\,\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2kK(m)}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0145.01

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2kK(m)}{p}\middle|m\right)^2\mathrm{dn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\left(\mathrm{sn}\left(z+\frac{2(k-r)K(m)}{p}\middle|m\right)+\mathrm{sn}\left(z+\frac{2(k+r)K(m)}{p}\middle|m\right)\right)==$$

$$-\frac{2}{m}\,\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2kK(m)}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2kK(m)}{p}\middle|m\right)/;$$

$$p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0146.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\left(\text{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$

$$\frac{2}{m}\,\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;$$

$p\in\mathbb{N}^+ \bigwedge r\in\mathbb{N}^+ \bigwedge r<p-1$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0147.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^4 \left(\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right) ==$$

$$-\frac{2}{m}\,\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^3+\frac{2}{m^2}\,\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2$$

$$\left(\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2-\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+ \bigwedge r\in\mathbb{N}^+ \bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0148.01

$$\sum_{k=0}^{p-1} \text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^3 \left(\text{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^2+\text{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^2\right) ==$$

$$\frac{2}{m}\,\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^3+$$

$$\frac{2}{m^2}\left(\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2-\right.$$

$$\left.3\,\text{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\text{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\right)\sum_{k=0}^{p-1}\text{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+ \bigwedge r\in\mathbb{N}^+ \bigwedge r<p$$

Khare/Lakshminarayan/Sukhatme_2002

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09.29.16.0149.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^2\left(\mathrm{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$-2\,\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^3+\frac{2}{m}\,\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^3$$

$$\left(m\,\mathrm{sn}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\mathrm{cn}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2-\mathrm{cn}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\;p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

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$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^3\left(\mathrm{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^2+\mathrm{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^2\right)==$$

$$-\frac{2}{m}\,\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^3+$$

$$\frac{2}{m^2}\left(\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2-\right.$$

$$\left.3\,\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\;p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

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$$\sum_{k=0}^{p-1}(-1)^k\,\mathrm{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)$$

$$\mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\left(\mathrm{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$-\frac{4}{m^2}\,\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}(-1)^k\,\mathrm{Z}\left(\mathrm{am}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\middle|\,m\right)+$$

$$\frac{2}{m}\,\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}(-1)^k\,\mathrm{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;$$

$$\frac{p}{2}\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge\gcd(p,r)==1\bigwedge 1-m>0$$

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Sums over products of six Jacobi functions

09.29.16.0152.01

$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^4\left(\mathrm{cn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)+\mathrm{cn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\right)==$$
$$-2\,\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\mathrm{cn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^2+$$
$$2\,\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2\left(\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2+3\,\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\right)$$
$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\mathrm{cn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

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$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^4\left(\mathrm{cn}\left(z+\frac{2K(m)(k-s)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2K(m)(k-r)}{p}\middle|m\right)+\mathrm{cn}\left(z+\frac{2K(m)(k+s)}{p}\middle|m\right)\mathrm{sn}\left(z+\frac{2K(m)(k+r)}{p}\middle|m\right)\right)==$$
$$-2\,\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2sK(m)}{p}\middle|m\right)\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\mathrm{cn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\mathrm{dn}\left(z+\frac{2K(m)k}{p}\middle|m\right)^2+$$
$$2\left(\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{cs}\left(\frac{2sK(m)}{p}\middle|m\right)\mathrm{ns}\left(\frac{2sK(m)}{p}\middle|m\right)+\right.$$
$$\left.\mathrm{ns}\left(\frac{2rK(m)}{p}\middle|m\right)\mathrm{ds}\left(\frac{2sK(m)}{p}\middle|m\right)\left(\mathrm{cs}\left(\frac{2rK(m)}{p}\middle|m\right)^2+\mathrm{cs}\left(\frac{2sK(m)}{p}\middle|m\right)^2\right)\right)$$
$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2K(m)k}{p}\middle|m\right)\mathrm{cn}\left(z+\frac{2K(m)k}{p}\middle|m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p\bigwedge s\in\mathbb{N}^+\bigwedge s<r$$

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$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^3\left(\mathrm{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^3\right)==$$
$$\frac{p}{2\,K(m)}\left(24\,E(m)\,\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2+\right.$$
$$\left.\int_0^{2\,K(m)}\mathrm{dn}(t\mid m)^3\left(\mathrm{dn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{dn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^3\right)dt\right)-$$
$$12\,\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2\ /;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

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$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^3\left(\mathrm{sn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^3\right)==$$
$$\frac{12}{m^3}\,\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2+$$
$$\frac{p}{2\,K(m)}\left(\int_0^{2\,K(m)}\mathrm{sn}(t\mid m)^3\left(\mathrm{sn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{sn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^3\right)dt-\right.$$
$$\left.\frac{24}{m^3}\,\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)E(m)\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

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$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^3\left(\mathrm{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^3\right)==$$
$$-\frac{12}{m^3}\,\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2+$$
$$\frac{p}{2\,K(m)}\left(\int_0^{2\,K(m)}\mathrm{cn}(t\mid m)^3\left(\mathrm{cn}\left(t+\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{cn}\left(t-\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^3\right)dt+\right.$$
$$\left.\frac{24}{m^3}\,E(m)\,\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

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$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\left(\mathrm{sn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{sn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^3\right)==$$

$$-\frac{2}{m^2}\left(\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\right.$$

$$\left.\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+3\,\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)$$

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

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$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^4$$

$$\left(\mathrm{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)\right)==$$

$$\frac{2}{m}\,\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^2+$$

$$\frac{2}{m^2}\,\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\left(\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+3\,\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\right)$$

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

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$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^4$$

$$\left(\mathrm{cn}\left(z+\frac{4\,(k-s)\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{4\,(k+s)\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\,\middle|\,m\right)\right)==$$

$$\frac{2}{m}\,\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)^2+$$

$$\frac{2}{m^2}\left(\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{cs}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)+\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)\right.$$

$$\left.\left(\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\mathrm{ns}\left(\frac{4\,s\,K(m)}{p}\,\middle|\,m\right)^2\right)\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

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$$\sum_{k=0}^{p-1} \mathrm{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\left(\mathrm{cn}\left(z+\frac{4\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{cn}\left(z+\frac{4\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^3\right)==$$
$$-\frac{2}{m}\left(\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+\right.$$
$$\left.\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2+3\,\mathrm{cs}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ns}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{ds}\left(\frac{4\,r\,K(m)}{p}\,\middle|\,m\right)^2\right)$$
$$\sum_{k=0}^{p-1}\mathrm{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p$$

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Sums over products of seven Jacobi functions

09.29.16.0161.01

$$\sum_{k=0}^{p-1}\mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2$$
$$\left(\mathrm{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^3\right)==-4\,\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)$$
$$\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

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$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2\left(\mathrm{sn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^3+\mathrm{sn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^3\right)==$$
$$-\frac{4}{m^2}\,\mathrm{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\mathrm{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\mathrm{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)$$
$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)/;\ p\in\mathbb{N}^+\bigwedge r\in\mathbb{N}^+\bigwedge r<p-1$$

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$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)^2 \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)$$
$$\left(\text{cn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^3+\text{cn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^3\right) == -\frac{4}{m^2}\,\text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2 \text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)$$
$$\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) /;\; p\in\mathbb{N}^+ \bigwedge r\in\mathbb{N}^+ \bigwedge r<p-1$$

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09.29.16.0164.01

$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)\left(\text{dn}\left(z+\frac{2\,(k-r)\,K(m)}{p}\,\middle|\,m\right)^4+\text{dn}\left(z+\frac{2\,(k+r)\,K(m)}{p}\,\middle|\,m\right)^4\right) ==$$
$$2\left(\text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^4-\text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2 \text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2-\right.$$
$$\left.\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2 \text{cs}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2-\text{ns}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2 \text{ds}\left(\frac{2\,r\,K(m)}{p}\,\middle|\,m\right)^2\right)$$
$$\sum_{k=0}^{p-1} \text{cn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{sn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) \text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) /;\; p\in\mathbb{N}^+ \bigwedge r\in\mathbb{N}^+ \bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

Sums over products of arbitrarily many Jacobi functions

09.29.16.0165.01

$$\sum_{j=0}^{p-1}\prod_{k=0}^{l-1} \text{dn}\left(z+\frac{2\,K(m)\,(j+k\,r)}{p}\,\middle|\,m\right) ==$$
$$\left(\prod_{k=1}^{\frac{l-1}{2}} \text{cs}\left(\frac{2\,k\,r\,K(m)}{p}\,\middle|\,m\right)^2+2\,(-1)^{\frac{l-1}{2}}\sum_{k=1}^{\frac{l-1}{2}}\prod_{n=1}^{l} \text{If}\left[n==k,\,1,\,\text{cs}\left(\frac{2\,(n-k)\,r\,K(m)}{p}\,\middle|\,m\right)\right]\right)\sum_{k=0}^{p-1} \text{dn}\left(z+\frac{2\,K(m)\,k}{p}\,\middle|\,m\right) /;$$
$$p\in\mathbb{N}^+ \bigwedge r\in\mathbb{N}^+ \bigwedge r<p-1 \bigwedge \frac{l-1}{2}\in\mathbb{N} \bigwedge l\le p$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0166.01

$$\sum_{j=0}^{p-1}\prod_{k=0}^{r-1} \text{dn}\left(z+\frac{2\,K(m)\,(j+k)}{p}\,\middle|\,m\right) == \frac{p}{2\,K(m)}\int_0^{2\,K(m)}\prod_{k=0}^{r-1} \text{dn}\left(t+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right) dt\; /;\; p-3\in\mathbb{N} \bigwedge \frac{r}{2}\in\mathbb{N}^+ \bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0167.01

$$\sum_{j=0}^{p-1}\prod_{k=0}^{r-1}\text{sn}\left(z+\frac{2\,K(m)\,(j+k)}{p}\,\middle|\,m\right)==\frac{p}{2\,K(m)}\int_0^{2\,K(m)}\prod_{k=0}^{r-1}\text{sn}\left(t+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)dt\;/;\;p-3\in\mathbb{N}\bigwedge\frac{r}{2}\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0168.01

$$\sum_{j=0}^{p-1}\prod_{k=0}^{r-1}\text{cn}\left(z+\frac{2\,K(m)\,(j+k)}{p}\,\middle|\,m\right)==\frac{p}{2\,K(m)}\int_0^{2\,K(m)}\prod_{k=0}^{r-1}\text{cn}\left(t+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)dt\;/;\;p-3\in\mathbb{N}\bigwedge\frac{r}{2}\in\mathbb{N}^+\bigwedge r<p-1$$

Khare/Lakshminarayan/Sukhatme_2002

Khare/Lakshminarayan/Sukhatme_JMP_2002

09.29.16.0070.01

$$\sum_{k=0}^{p-1}\prod_{l=0}^{r-1}\text{dn}\left(z+\frac{2\,(k+n_l)\,K(m)}{p}\,\middle|\,m\right)==\sum_{k=0}^{p-1}\prod_{l=0}^{r-1}\text{dn}\left(\frac{2\,(k+n_l)\,K(m)}{p}\,\middle|\,m\right)/;$$

$$p-2\in\mathbb{N}\bigwedge\frac{r}{2}\in\mathbb{N}^+\bigwedge n_0==0\bigwedge n_l\in\mathbb{N}\bigwedge 1\le n_l<p\bigwedge n_l<n_{l+1}$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

09.29.16.0071.01

$$\frac{\sum_{k=0}^{p-1}\prod_{l=0}^{r-1}\text{dn}\left(z+\frac{2\left(k+n_l\right)K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1}\text{dn}\left(z+\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)}==\frac{\sum_{k=0}^{p-1}\prod_{l=0}^{r-1}\text{dn}\left(\frac{2\left(k+n_l\right)K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1}\text{dn}\left(\frac{2\,k\,K(m)}{p}\,\middle|\,m\right)}\;/;$$

$$p-2\in\mathbb{N}\bigwedge\frac{r-1}{2}\in\mathbb{N}^+\bigwedge n_0==0\bigwedge n_l\in\mathbb{N}\bigwedge 1\le n_l<p\bigwedge n_l<n_{l+1}$$

Khare/Sukhatme_2002

Khare/Sukhatme_JMP_2002

Identities involving the group of functions

Cyclic Identities of rank 2

$p = 3$

09.29.18.0001.01

$$\text{cn}(z\mid m)\left(\text{dn}\left(z+\frac{4\,K(m)}{3}\,\middle|\,m\right)+\text{dn}\left(z+\frac{8\,K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\text{cn}\left(z+\frac{4\,K(m)}{3}\,\middle|\,m\right)\left(\text{dn}\left(z+\frac{8\,K(m)}{3}\,\middle|\,m\right)+\text{dn}(z\mid m)\right)+\text{cn}\left(z+\frac{8\,K(m)}{3}\,\middle|\,m\right)\left(\text{dn}(z\mid m)+\text{dn}\left(z+\frac{4\,K(m)}{3}\,\middle|\,m\right)\right)==0$$

09.29.18.0002.01

$$\operatorname{sn}(z \mid m)\left(\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\operatorname{dn}(z \mid m)\right)+\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\operatorname{dn}(z \mid m)+\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==0$$

09.29.18.0003.01

$$\operatorname{cn}(z \mid m)\left(\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\operatorname{sn}(z \mid m)\right)+\operatorname{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\operatorname{sn}(z \mid m)+\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==0$$

General case: $(p-1)/2 \in \mathbb{Z}$

09.29.18.0004.01

$$\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\operatorname{dn}\left(z+\frac{4(k-1)K(m)}{p}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{4(k+1)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+$$

09.29.18.0005.01

$$\sum_{k=0}^{p-1}\operatorname{sn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\operatorname{dn}\left(z+\frac{4(k-1)K(m)}{p}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{4(k+1)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+$$

09.29.18.0006.01

$$\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\operatorname{sn}\left(z+\frac{4(k-1)K(m)}{p}\,\middle|\,m\right)+\operatorname{sn}\left(z+\frac{4(k+1)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+$$

09.29.18.0007.01

$$\sum_{k=0}^{p-1}\operatorname{sn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\operatorname{cn}\left(z+\frac{4(k-1)K(m)}{p}\,\middle|\,m\right)+\operatorname{cn}\left(z+\frac{4(k+1)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+$$

09.29.18.0008.01

$$\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\operatorname{dn}\left(z+\frac{4(k-n)K(m)}{p}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{4(k+n)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+\bigwedge n\in\mathbb{Z}\bigwedge 1\le n\le\frac{p+1}{2}$$

09.29.18.0009.01

$$\sum_{k=0}^{p-1}\operatorname{sn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\operatorname{dn}\left(z+\frac{4(k-n)K(m)}{p}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{4(k+n)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+\bigwedge n\in\mathbb{Z}\bigwedge 1\le n\le\frac{p+1}{2}$$

09.29.18.0010.01

$$\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\operatorname{sn}\left(z+\frac{4(k-n)K(m)}{p}\,\middle|\,m\right)+\operatorname{sn}\left(z+\frac{4(k+n)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+\bigwedge n\in\mathbb{Z}\bigwedge 1\le n\le\frac{p+1}{2}$$

09.29.18.0011.01

$$\sum_{k=0}^{p-1}\operatorname{sn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\operatorname{cn}\left(z+\frac{4(k-n)K(m)}{p}\,\middle|\,m\right)+\operatorname{cn}\left(z+\frac{4(k+n)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+\bigwedge n\in\mathbb{Z}\bigwedge 1\le n\le\frac{p+1}{2}$$

Cyclic Identities of rank 3

$p = 2$

09.29.18.0012.01

$$\mathrm{cn}(z \mid m)\,\mathrm{sn}(z \mid m)\,\mathrm{dn}(z + K(m) \mid m) + \mathrm{cn}(z + K(m) \mid m)\,\mathrm{sn}(z + K(m) \mid m)\,\mathrm{dn}(z \mid m) == 0$$

$p = 3$

09.29.18.0013.01

$$\mathrm{cn}(z \mid m)\left(\mathrm{sn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{dn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right) + \mathrm{sn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{dn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\right) +$$
$$\mathrm{cn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\left(\mathrm{sn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{dn}(z \mid m) + \mathrm{sn}(z \mid m)\,\mathrm{dn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\right) +$$
$$\mathrm{cn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\left(\mathrm{sn}(z \mid m)\,\mathrm{dn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) + \mathrm{sn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{dn}(z \mid m)\right) == 0$$

09.29.18.0014.01

$$\mathrm{cn}(z \mid m)\,\mathrm{dn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{dn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right) + \mathrm{cn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{dn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{dn}(z \mid m) +$$
$$\mathrm{cn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{dn}(z \mid m)\,\mathrm{dn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) == -\mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2\left(\mathrm{cn}(z \mid m) + \mathrm{cn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) + \mathrm{cn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\right)$$

09.29.18.0015.01

$$\mathrm{cn}(z \mid m)\,\mathrm{sn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{sn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right) +$$
$$\mathrm{cn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{sn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{sn}(z \mid m) + \mathrm{cn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{sn}(z \mid m)\,\mathrm{sn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) ==$$
$$\frac{-\left(1 + \mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)\right)^2}{m}\left(\mathrm{cn}(z \mid m) + \mathrm{cn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) + \mathrm{cn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\right)$$

09.29.18.0016.01

$$\mathrm{sn}(z \mid m)\,\mathrm{dn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{dn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right) +$$
$$\mathrm{sn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{dn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{dn}(z \mid m) + \mathrm{sn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{dn}(z \mid m)\,\mathrm{dn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) ==$$
$$\left(2\,\mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^3 + 3\,\mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2 - 2\,\mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right) + 3m - 3\right) \Big/ \left(1 - \mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2\right)$$
$$\left(\mathrm{sn}(z \mid m) + \mathrm{sn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) + \mathrm{sn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\right)$$

09.29.18.0017.01

$$\mathrm{sn}(z \mid m)\,\mathrm{cn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{cn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right) +$$
$$\mathrm{sn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right)\mathrm{cn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{cn}(z \mid m) + \mathrm{sn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\mathrm{cn}(z \mid m)\,\mathrm{cn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) ==$$
$$\mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)\left(\mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right) + 2\right) \Big/ \left(1 - \mathrm{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2\right)\left(\mathrm{sn}(z \mid m) + \mathrm{sn}\left(z + \frac{4K(m)}{3} \,\middle|\, m\right) + \mathrm{sn}\left(z + \frac{8K(m)}{3} \,\middle|\, m\right)\right)$$

09.29.18.0018.01

$$\mathrm{dn}(z\mid m)\,\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+$$
$$\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{cn}(z\mid m)+\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{cn}(z\mid m)\,\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)==$$
$$-\frac{\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^2}{\left(1+\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\right)^2}\left(\mathrm{dn}(z\mid m)+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)$$

09.29.18.0019.01

$$\mathrm{dn}(z\mid m)\,\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+$$
$$\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{sn}(z\mid m)+\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{sn}(z\mid m)\,\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)==$$
$$\left(-\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^3-\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^2+\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)+1-2m\right)\Big/\left(m\left(1+\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\right)\right)$$

09.29.18.0020.01

$$\mathrm{dn}(z\mid m)\left(\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\right)+$$
$$\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==$$
$$2\,\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\left(\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)+1\right)\left(\mathrm{cn}(z\mid m)+\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)$$

09.29.18.0021.01

$$\mathrm{sn}(z\mid m)\left(\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\right)+$$
$$\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)+\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==$$
$$\frac{2}{m}\,\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\left(\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)+1\right)\left(\mathrm{cn}(z\mid m)+\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)$$

09.29.18.0022.01

$$\mathrm{dn}(z\mid m)\left(\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\right)+$$
$$\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)+\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==$$
$$2\left(\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^2+2\,\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)-m+1\right)\Big/\left(1+\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\right)\left(\mathrm{sn}(z\mid m)+\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)$$

09.29.18.0023.01

$$\operatorname{cn}(z\mid m)\left(\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\operatorname{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\operatorname{cn}(z\mid m)\operatorname{sn}(z\mid m)\right)+$$
$$\operatorname{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\operatorname{cn}(z\mid m)\operatorname{sn}(z\mid m)+\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==-2\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)$$
$$\left(\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)+2\right)\Big/\left(\left(1+\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\right)\left(1-\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^2\right)\right)\left(\operatorname{sn}(z\mid m)+\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)$$

09.29.18.0024.01

$$\operatorname{cn}(z\mid m)\left(\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\operatorname{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\operatorname{cn}(z\mid m)\operatorname{dn}(z\mid m)\right)+$$
$$\operatorname{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\operatorname{cn}(z\mid m)\operatorname{dn}(z\mid m)+\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==$$
$$-2\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\Big/\left(1+\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\right)^2\left(\operatorname{dn}(z\mid m)+\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)$$

09.29.18.0025.01

$$\operatorname{sn}(z\mid m)\left(\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\operatorname{sn}(z\mid m)\operatorname{dn}(z\mid m)\right)+$$
$$\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\operatorname{sn}(z\mid m)\operatorname{dn}(z\mid m)+\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==$$
$$2\left(\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^3+\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^2-\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)-1+m\right)\Big/\left(m\left(1+\operatorname{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)\right)\right)$$
$$\left(\operatorname{dn}(z\mid m)+\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)$$

09.29.18.0026.01

$$\operatorname{cn}(z\mid m)\operatorname{sn}(z\mid m)\left(\operatorname{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)+\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\operatorname{cn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\left(\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{dn}(z\mid m)\right)+$$
$$\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\operatorname{dn}(z\mid m)+\operatorname{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\right)==0$$

09.29.18.0027.01

$$\operatorname{cn}(z\mid m)\operatorname{dn}(z\mid m)\left(\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\operatorname{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\operatorname{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\operatorname{sn}(z\mid m)\right)+$$
$$\operatorname{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\operatorname{sn}(z\mid m)+\operatorname{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==0$$

09.29.18.0028.01

$$\mathrm{dn}(z \mid m)\,\mathrm{sn}(z \mid m)\left(\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\left(\mathrm{cn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)+\mathrm{cn}(z \mid m)\right)+$$
$$\mathrm{dn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{8K(m)}{3}\,\middle|\,m\right)\left(\mathrm{cn}(z \mid m)+\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)==0$$

$p = 4$

09.29.18.0029.01

$$\mathrm{cn}(z \mid m)\,\mathrm{sn}(z \mid m)\left(\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\right)+$$
$$\mathrm{cn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)(\mathrm{dn}(z+K(m) \mid m)+\mathrm{dn}(z \mid m))+$$
$$\mathrm{cn}(z+K(m) \mid m)\,\mathrm{sn}(z+K(m) \mid m)\left(\mathrm{dn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{K(m)}{2}\,\middle|\,m\right)\right)+$$
$$\mathrm{cn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{3K(m)}{2}\,\middle|\,m\right)(\mathrm{dn}(z \mid m)+\mathrm{dn}(z+K(m) \mid m))==0$$

09.29.18.0030.01

$$\mathrm{cn}(z \mid m)\,\mathrm{sn}(z \mid m)\,\mathrm{dn}(z+K(m) \mid m)+\mathrm{cn}(z+K(m) \mid m)\,\mathrm{sn}(z+K(m) \mid m)\,\mathrm{dn}(z \mid m)==0$$

General case: $p \in \mathbb{Z}$

09.29.18.0031.01

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)\left(\mathrm{dn}\left(z+\frac{2(k-1)K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{2(k+1)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;p-2\in\mathbb{N}$$

09.29.18.0032.01

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{2kK(m)}{p}\,\middle|\,m\right)\left(\mathrm{dn}\left(z+\frac{2(k-n)K(m)}{p}\,\middle|\,m\right)+\mathrm{dn}\left(z+\frac{2(k+n)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;$$
$$p-2\in\mathbb{N}\bigwedge n\in\mathbb{Z}\bigwedge 1\le n\le\frac{p+p\bmod 2}{2}$$

General case: $(p-1)/2 \in \mathbb{Z}$

09.29.18.0033.01

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\mathrm{dn}\left(z+\frac{4(k+1)K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4(k-1)K(m)}{p}\,\middle|\,m\right)+\right.$$
$$\left.\mathrm{dn}\left(z+\frac{4(k-1)K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4(k+1)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+$$

09.29.18.0034.01

$$\sum_{k=0}^{p-1}\mathrm{cn}\left(z+\frac{4kK(m)}{p}\,\middle|\,m\right)\left(\mathrm{dn}\left(z+\frac{4(k+n)K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4(k-n)K(m)}{p}\,\middle|\,m\right)+\right.$$
$$\left.\mathrm{dn}\left(z+\frac{4(k-n)K(m)}{p}\,\middle|\,m\right)\mathrm{sn}\left(z+\frac{4(k+n)K(m)}{p}\,\middle|\,m\right)\right)==0\;/;\;\frac{p-1}{2}\in\mathbb{N}^+\bigwedge n\in\mathbb{Z}\bigwedge 1\le n<p$$

09.29.18.0035.01

$$\frac{\sum_{k=0}^{p-1} \operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,(k-1)\,K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1} \operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1} \operatorname{cn}\left(\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{4\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{4\,(k-1)\,K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1} \operatorname{cn}\left(\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)} /;\ \frac{p-1}{2}\in\mathbb{N}^+$$

09.29.18.0036.01

$$\frac{\sum_{k=0}^{p-1} \operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,(k+n)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,(k-n)\,K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1} \operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)} ==$$

$$\frac{\sum_{k=0}^{p-1} \operatorname{cn}\left(\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{4\,(k+n)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{4\,(k-n)\,K(m)}{p}\,\middle|\,m\right)}{\sum_{k=0}^{p-1} \operatorname{cn}\left(\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)} /;\ \frac{p-1}{2}\in\mathbb{N}^+ \bigwedge n\in\mathbb{Z} \bigwedge 1\le n<p$$

09.29.18.0037.01

$$\left(\sum_{k=0}^{p-1}\operatorname{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\left(\operatorname{cn}\left(z+\frac{4\,(k-1)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,(k-1)\,K(m)}{p}\,\middle|\,m\right)+\right.\right.$$
$$\left.\left.\operatorname{cn}\left(z+\frac{4\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\right)\right)\Big/\left(\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\right) ==$$

$$\left(\sum_{k=0}^{p-1}\operatorname{dn}\left(\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\left(\operatorname{cn}\left(\frac{4\,(k-1)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{4\,(k-1)\,K(m)}{p}\,\middle|\,m\right)+\operatorname{cn}\left(\frac{4\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{4\,(k+1)\,K(m)}{p}\,\middle|\,m\right)\right)\right)\Big/$$
$$\left(\sum_{k=0}^{p-1}\operatorname{cn}\left(\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\right) /;\ \frac{p-1}{2}\in\mathbb{N}^+$$

09.29.18.0038.01

$$\left(\sum_{k=0}^{p-1}\operatorname{dn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\left(\operatorname{cn}\left(z+\frac{4\,(k-n)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,(k-n)\,K(m)}{p}\,\middle|\,m\right)+\right.\right.$$
$$\left.\left.\operatorname{cn}\left(z+\frac{4\,(k+n)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(z+\frac{4\,(k+n)\,K(m)}{p}\,\middle|\,m\right)\right)\right)\Big/\left(\sum_{k=0}^{p-1}\operatorname{cn}\left(z+\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\right) ==$$

$$\left(\sum_{k=0}^{p-1}\operatorname{dn}\left(\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\left(\operatorname{cn}\left(\frac{4\,(k-n)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{4\,(k-n)\,K(m)}{p}\,\middle|\,m\right)+\operatorname{cn}\left(\frac{4\,(k+n)\,K(m)}{p}\,\middle|\,m\right)\operatorname{dn}\left(\frac{4\,(k+n)\,K(m)}{p}\,\middle|\,m\right)\right)\right)\Big/$$
$$\left(\sum_{k=0}^{p-1}\operatorname{cn}\left(\frac{4\,k\,K(m)}{p}\,\middle|\,m\right)\right) /;\ \frac{p-1}{2}\in\mathbb{N}^+ \bigwedge n\in\mathbb{Z} \bigwedge 1\le n<p$$

Cyclic Identities of rank 4 and above

$r = 4, p = 2$

09.29.18.0039.01

$$m\operatorname{cn}(z\mid m)\operatorname{cn}(z+K(m)\mid m)\operatorname{sn}(z\mid m)\operatorname{sn}(z+K(m)\mid m) == \sqrt{1-m}\,\left(1-\operatorname{sn}(z\mid m)^2-\operatorname{sn}(z+K(m)\mid m)^2\right)$$

09.29.18.0040.01

$$\operatorname{cn}(z \mid m)\operatorname{cn}(z+K(m) \mid m)\operatorname{dn}(z \mid m)\operatorname{dn}(z+K(m) \mid m) == -(1-m)\operatorname{sn}(z \mid m)\operatorname{sn}(z+K(m) \mid m)$$

09.29.18.0041.01

$$\operatorname{sn}(z \mid m)\operatorname{sn}(z+K(m) \mid m)\operatorname{dn}(z \mid m)\operatorname{dn}(z+K(m) \mid m) == -\operatorname{cn}(z \mid m)\operatorname{cn}(z+K(m) \mid m)$$

$r = 4, p = 3$

09.29.18.0042.01

$$\operatorname{sn}(z \mid m)\operatorname{cn}(z \mid m)\operatorname{dn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)+$$
$$\operatorname{sn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)\operatorname{cn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{dn}(z \mid m)+$$
$$\operatorname{sn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{cn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{dn}(z \mid m)\operatorname{dn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right) ==$$
$$\frac{\operatorname{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2+m-1}{1-\operatorname{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2}\left(\operatorname{cn}(z \mid m)\operatorname{sn}(z \mid m)+\operatorname{cn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)\operatorname{sn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)+\operatorname{cn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{sn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\right)$$

09.29.18.0043.01

$$\operatorname{sn}(z \mid m)\operatorname{dn}(z \mid m)\operatorname{cn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{cn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)+\operatorname{sn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{cn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\operatorname{cn}(z \mid m)+$$
$$\operatorname{sn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\operatorname{cn}(z \mid m)\operatorname{cn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right) ==$$
$$\frac{\operatorname{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2}{1-\operatorname{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2}\left(\operatorname{sn}(z \mid m)\operatorname{dn}(z \mid m)+\operatorname{sn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)+\operatorname{sn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\right)$$

09.29.18.0044.01

$$\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)\operatorname{sn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{sn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)+\operatorname{cn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{sn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\operatorname{sn}(z \mid m)+$$
$$\operatorname{cn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\operatorname{sn}(z \mid m)\operatorname{sn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right) ==$$
$$-\frac{1}{1-\operatorname{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2}\left(\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)+\operatorname{cn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)+\operatorname{cn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\operatorname{dn}\left(z+\frac{8K(m)}{3} \,\middle|\, m\right)\right)$$

09.29.18.0045.01

$$m^2\left(\operatorname{cn}(z \mid m)\operatorname{cn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)\operatorname{sn}(z \mid m)\operatorname{sn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)+\operatorname{cn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)\operatorname{cn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\right.$$
$$\left.\operatorname{sn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)\operatorname{sn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)+\operatorname{cn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{cn}(z \mid m)\operatorname{sn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)\operatorname{sn}(z \mid m)\right) ==$$
$$\frac{2m\operatorname{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)}{1-\operatorname{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)^2}\left(\operatorname{dn}(z \mid m)^2+\operatorname{dn}\left(z+\frac{2K(m)}{3} \,\middle|\, m\right)^2+\operatorname{dn}\left(z+\frac{4K(m)}{3} \,\middle|\, m\right)^2\right)+m-(2-m)\left(1+\operatorname{dn}\left(\frac{2K(m)}{3} \,\middle|\, m\right)\right)^2$$

$r = 5, p = 3$

09.29.18.0046.01

$$\mathrm{dn}(z \mid m)^3 \left(\mathrm{sn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)+\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)+$$
$$\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)^3\left(\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)+\mathrm{sn}(z \mid m)\,\mathrm{cn}(z \mid m)\right)+$$
$$\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)^3\left(\mathrm{sn}(z \mid m)\,\mathrm{cn}(z \mid m)+\mathrm{sn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\right)==$$
$$\frac{-2m\,\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)}{1-\mathrm{dn}\left(\frac{2K(m)}{3}\,\middle|\,m\right)^2}\left(\mathrm{sn}(z \mid m)\,\mathrm{cn}(z \mid m)\,\mathrm{dn}(z \mid m)+\mathrm{sn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{2K(m)}{3}\,\middle|\,m\right)+\right.$$
$$\left.\mathrm{sn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{cn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\mathrm{dn}\left(z+\frac{4K(m)}{3}\,\middle|\,m\right)\right)$$

Local identities of rank 2

09.29.18.0047.01

$$\mathrm{dn}(z \mid m)\,\mathrm{sn}(a+z \mid m) == \mathrm{ns}(a \mid m)\,\mathrm{cn}(z \mid m) - \mathrm{cs}(a \mid m)\,\mathrm{cn}(a+z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0048.01

$$\mathrm{dn}(z \mid m)\,\mathrm{cn}(a+z \mid m) == \mathrm{cs}(a \mid m)\,\mathrm{sn}(a+z \mid m) - \mathrm{ds}(a \mid m)\,\mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0049.01

$$m\,\mathrm{sn}(z \mid m)\,\mathrm{cn}(a+z \mid m) == \mathrm{ds}(a \mid m)\,\mathrm{dn}(z \mid m) - \mathrm{ns}(a \mid m)\,\mathrm{dn}(a+z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0050.01

$$\mathrm{dn}(z \mid m)\,\mathrm{dn}(a+z \mid m) == \mathrm{dn}(a \mid m) + \mathrm{cs}(a \mid m)\,(-\mathrm{Z}(\mathrm{am}(a \mid m) \mid m) + \mathrm{Z}(\mathrm{am}(a+z \mid m) \mid m) - \mathrm{Z}(\mathrm{am}(z \mid m) \mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0051.01

$$m\,\mathrm{sn}(z \mid m)\,\mathrm{sn}(a+z \mid m) == -\mathrm{ns}(a \mid m)\,(-\mathrm{Z}(\mathrm{am}(a \mid m) \mid m) + \mathrm{Z}(\mathrm{am}(a+z \mid m) \mid m) - \mathrm{Z}(\mathrm{am}(z \mid m) \mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0052.01

$$m\,\mathrm{cn}(z \mid m)\,\mathrm{cn}(a+z \mid m) == m\,\mathrm{cn}(a \mid m) + \mathrm{ds}(a \mid m)\,(-\mathrm{Z}(\mathrm{am}(a \mid m) \mid m) + \mathrm{Z}(\mathrm{am}(a+z \mid m) \mid m) - \mathrm{Z}(\mathrm{am}(z \mid m) \mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

Local identities of rank 3

Rank 3 identities with 3 distinct arguments

09.29.18.0053.01

$$\mathrm{dn}(z \mid m)\,\mathrm{dn}(a+z \mid m)\,\mathrm{dn}(b+z \mid m) ==$$
$$-\mathrm{cs}(a \mid m)\,\mathrm{cs}(a-b \mid m)\,\mathrm{dn}(a+z \mid m) + \mathrm{cs}(a-b \mid m)\,\mathrm{cs}(b \mid m)\,\mathrm{dn}(b+z \mid m) - \mathrm{cs}(a \mid m)\,\mathrm{cs}(b \mid m)\,\mathrm{dn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0054.01

$$m\,\mathrm{sn}(z \mid m)\,\mathrm{sn}(a+z \mid m)\,\mathrm{sn}(b+z \mid m) == \mathrm{ns}(a \mid m)\,\mathrm{ns}(a-b \mid m)\,\mathrm{sn}(a+z \mid m) - \mathrm{ns}(a-b \mid m)\,\mathrm{ns}(b \mid m)\,\mathrm{sn}(b+z \mid m) + \mathrm{ns}(a \mid m)\,\mathrm{ns}(b \mid m)\,\mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0055.01

$$m\,\mathrm{cn}(z \mid m)\,\mathrm{cn}(a+z \mid m)\,\mathrm{cn}(b+z \mid m) == -\mathrm{cn}(a+z \mid m)\,\mathrm{ds}(a \mid m)\,\mathrm{ds}(a-b \mid m) + \mathrm{cn}(b+z \mid m)\,\mathrm{ds}(b \mid m)\,\mathrm{ds}(a-b \mid m) - \mathrm{cn}(z \mid m)\,\mathrm{ds}(a \mid m)\,\mathrm{ds}(b \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0056.01

$$\mathrm{dn}(z \mid m)\,\mathrm{dn}(a+z \mid m)\,\mathrm{sn}(b+z \mid m) == -\mathrm{cs}(a \mid m)\,\mathrm{ns}(a-b \mid m)\,\mathrm{sn}(a+z \mid m) + \mathrm{cs}(a-b \mid m)\,\mathrm{cs}(b \mid m)\,\mathrm{sn}(b+z \mid m) - \mathrm{cs}(a \mid m)\,\mathrm{ns}(b \mid m)\,\mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0057.01

$$\mathrm{dn}(z \mid m)\,\mathrm{dn}(a+z \mid m)\,\mathrm{cn}(b+z \mid m) == \mathrm{cn}(b+z \mid m)\,\mathrm{cs}(a-b \mid m)\,\mathrm{cs}(b \mid m) - \mathrm{cn}(a+z \mid m)\,\mathrm{cs}(a \mid m)\,\mathrm{ds}(a-b \mid m) - \mathrm{cn}(z \mid m)\,\mathrm{cs}(a \mid m)\,\mathrm{ds}(b \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0058.01

$$m\,\mathrm{sn}(z \mid m)\,\mathrm{sn}(a+z \mid m)\,\mathrm{dn}(b+z \mid m) == \mathrm{cs}(a-b \mid m)\,\mathrm{dn}(a+z \mid m)\,\mathrm{ns}(a \mid m) + \mathrm{cs}(b \mid m)\,\mathrm{dn}(z \mid m)\,\mathrm{ns}(a \mid m) - \mathrm{dn}(b+z \mid m)\,\mathrm{ns}(a-b \mid m)\,\mathrm{ns}(b \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0059.01

$$m\,\mathrm{sn}(z \mid m)\,\mathrm{sn}(a+z \mid m)\,\mathrm{cn}(b+z \mid m) == \mathrm{cn}(a+z \mid m)\,\mathrm{ds}(a-b \mid m)\,\mathrm{ns}(a \mid m) + \mathrm{cn}(z \mid m)\,\mathrm{ds}(b \mid m)\,\mathrm{ns}(a \mid m) - \mathrm{cn}(b+z \mid m)\,\mathrm{ns}(a-b \mid m)\,\mathrm{ns}(b \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0060.01

$$m\,\mathrm{cn}(z \mid m)\,\mathrm{cn}(a+z \mid m)\,\mathrm{dn}(b+z \mid m) == -\mathrm{cs}(a-b \mid m)\,\mathrm{dn}(a+z \mid m)\,\mathrm{ds}(a \mid m) - \mathrm{cs}(b \mid m)\,\mathrm{dn}(z \mid m)\,\mathrm{ds}(a \mid m) + \mathrm{dn}(b+z \mid m)\,\mathrm{ds}(a-b \mid m)\,\mathrm{ds}(b \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0061.01

$$m\,\mathrm{cn}(z \mid m)\,\mathrm{cn}(a+z \mid m)\,\mathrm{sn}(b+z \mid m) == -\mathrm{ds}(a \mid m)\,\mathrm{ns}(a-b \mid m)\,\mathrm{sn}(a+z \mid m) + \mathrm{ds}(a-b \mid m)\,\mathrm{ds}(b \mid m)\,\mathrm{sn}(b+z \mid m) - \mathrm{ds}(a \mid m)\,\mathrm{ns}(b \mid m)\,\mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0062.01

$$m\,\mathrm{dn}(z \mid m)\,\mathrm{sn}(a+z \mid m)\,\mathrm{cn}(b+z \mid m) == \mathrm{ns}(a-b \mid m)\,(\mathrm{dn}(b \mid m) + \mathrm{cs}(b \mid m)\,(-\mathrm{Z}(\mathrm{am}(b \mid m) \mid m) + \mathrm{Z}(\mathrm{am}(b+z \mid m) \mid m) - \mathrm{Z}(\mathrm{am}(z \mid m) \mid m))) - \mathrm{ds}(a-b \mid m)\,(\mathrm{dn}(a \mid m) + \mathrm{cs}(a \mid m)\,(-\mathrm{Z}(\mathrm{am}(a \mid m) \mid m) + \mathrm{Z}(\mathrm{am}(a+z \mid m) \mid m) - \mathrm{Z}(\mathrm{am}(z \mid m) \mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

Rank 3 identities with 2 distinct arguments

09.29.18.0063.01

$$\mathrm{dn}(z \mid m)^2\,\mathrm{dn}(a+z \mid m) == -m\,\mathrm{cn}(z \mid m)\,\mathrm{sn}(z \mid m)\,\mathrm{cs}(a \mid m) - \mathrm{cs}(a \mid m)^2\,\mathrm{dn}(a+z \mid m) + \mathrm{dn}(z \mid m)\,\mathrm{ds}(a \mid m)\,\mathrm{ns}(a \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0064.01

$$m\,\mathrm{sn}(z\mid m)^2\,\mathrm{sn}(a+z\mid m)==\mathrm{sn}(a+z\mid m)\,\mathrm{ns}(a\mid m)^2-\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)-\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0065.01

$$m\,\mathrm{cn}(z\mid m)^2\,\mathrm{cn}(a+z\mid m)==-\mathrm{sn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ds}(a\mid m)+\mathrm{cn}(a+z\mid m)\left(-\mathrm{ds}(a\mid m)^2\right)+\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0066.01

$$\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(a+z\mid m)==\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)-\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(a+z\mid m)+\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0067.01

$$\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)==-\mathrm{cn}(a+z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)+\mathrm{cn}(z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{ds}(a\mid m)-\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0068.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{sn}(a+z\mid m)==\\-\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ds}(a\mid m)+\mathrm{cs}(a\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{ns}(a\mid m)+m\,\mathrm{cn}(z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0069.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(a+z\mid m)==-\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)+\mathrm{cn}(a+z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{ds}(a\mid m)+\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0070.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{cn}(a+z\mid m)==\\-\mathrm{cs}(a\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{ds}(a\mid m)-m\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{ds}(a\mid m)+\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0071.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{cn}(a+z\mid m)==\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ds}(a\mid m)-\mathrm{ns}(a\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{ds}(a\mid m)+\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0072.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(a+z\mid m)==\\\mathrm{ds}(a\mid m)\,\mathrm{dn}(z\mid m)^2-\mathrm{ds}(a\mid m)-\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0073.01

$$m\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(a+z\mid m)==\\\mathrm{ns}(a\mid m)\,\mathrm{dn}(z\mid m)^2+\mathrm{dn}(a\mid m)\,(-\mathrm{ds}(a\mid m))-\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0074.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)==\\\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)^2-\mathrm{cs}(a\mid m)-\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

Local identities of rank 4

Rank 4 identities with 4 distinct arguments

09.29.18.0075.01

$$\mathrm{dn}(z \mid m)\,\mathrm{dn}(a+z \mid m)\,\mathrm{dn}(b+z \mid m)\,\mathrm{dn}(c+z \mid m) ==$$
$$-\mathrm{cs}(a \mid m)\,\mathrm{cs}(a-b \mid m)\,(\mathrm{dn}(c-a \mid m)+\mathrm{cs}(c-a \mid m)\,(-\mathrm{Z}(\mathrm{am}(c-a \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(a+z \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m)))-$$
$$\mathrm{cs}(a-b \mid m)\,\mathrm{cs}(b \mid m)\,(\mathrm{dn}(c-b \mid m)+\mathrm{cs}(c-b \mid m)\,(-\mathrm{Z}(\mathrm{am}(c-b \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(b+z \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m)))-$$
$$\mathrm{cs}(a \mid m)\,\mathrm{cs}(b \mid m)\,(\mathrm{dn}(c \mid m)+\mathrm{cs}(c \mid m)\,(-\mathrm{Z}(\mathrm{am}(c \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(z \mid m) \mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0076.01

$$m^2\,\mathrm{sn}(z \mid m)\,\mathrm{sn}(a+z \mid m)\,\mathrm{sn}(b+z \mid m)\,\mathrm{sn}(c+z \mid m) ==$$
$$-\mathrm{ns}(a \mid m)\,\mathrm{ns}(a-b \mid m)\,\mathrm{ns}(c-a \mid m)\,(-\mathrm{Z}(\mathrm{am}(c-a \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(a+z \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m))+$$
$$\mathrm{ns}(a-b \mid m)\,\mathrm{ns}(c-b \mid m)\,\mathrm{ns}(b \mid m)\,(-\mathrm{Z}(\mathrm{am}(c-b \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(b+z \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m))-$$
$$\mathrm{ns}(a \mid m)\,\mathrm{ns}(b \mid m)\,\mathrm{ns}(c \mid m)\,(-\mathrm{Z}(\mathrm{am}(c \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(z \mid m) \mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0077.01

$$m^2\,\mathrm{cn}(z \mid m)\,\mathrm{cn}(a+z \mid m)\,\mathrm{cn}(b+z \mid m)\,\mathrm{cn}(c+z \mid m) ==$$
$$-\mathrm{ds}(a \mid m)\,\mathrm{ds}(a-b \mid m)\,(m\,\mathrm{cn}(c-a \mid m)+\mathrm{ds}(c-a \mid m)\,(-\mathrm{Z}(\mathrm{am}(c-a \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(a+z \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m)))+$$
$$\mathrm{ds}(a-b \mid m)\,\mathrm{ds}(b \mid m)\,(m\,\mathrm{cn}(c-b \mid m)+\mathrm{ds}(c-b \mid m)\,(-\mathrm{Z}(\mathrm{am}(c-b \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(b+z \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m)))-$$
$$\mathrm{ds}(a \mid m)\,\mathrm{ds}(b \mid m)\,(m\,\mathrm{cn}(c \mid m)+\mathrm{ds}(c \mid m)\,(-\mathrm{Z}(\mathrm{am}(c \mid m) \mid m)+\mathrm{Z}(\mathrm{am}(c+z \mid m) \mid m)-\mathrm{Z}(\mathrm{am}(z \mid m) \mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0078.01

$$\mathrm{dn}(z \mid m)\,\mathrm{dn}(a+z \mid m)\,\mathrm{dn}(b+z \mid m)\,\mathrm{sn}(c+z \mid m) ==$$
$$-(\mathrm{ns}(c-a \mid m)\,\mathrm{cn}(a+z \mid m)-\mathrm{cs}(c-a \mid m)\,\mathrm{cn}(c+z \mid m))\,\mathrm{cs}(a \mid m)\,\mathrm{cs}(a-b \mid m)+$$
$$(\mathrm{ns}(c-b \mid m)\,\mathrm{cn}(b+z \mid m)-\mathrm{cs}(c-b \mid m)\,\mathrm{cn}(c+z \mid m))\,\mathrm{cs}(b \mid m)\,\mathrm{cs}(a-b \mid m)-$$
$$(\mathrm{ns}(c \mid m)\,\mathrm{cn}(z \mid m)-\mathrm{cs}(c \mid m)\,\mathrm{cn}(c+z \mid m))\,\mathrm{cs}(a \mid m)\,\mathrm{cs}(b \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0079.01

$$\mathrm{dn}(z \mid m)\,\mathrm{dn}(a+z \mid m)\,\mathrm{dn}(b+z \mid m)\,\mathrm{cn}(c+z \mid m) ==$$
$$\mathrm{cs}(a \mid m)\,\mathrm{cs}(a-b \mid m)\,(\mathrm{ds}(c-a \mid m)\,\mathrm{sn}(a+z \mid m)-\mathrm{cs}(c-a \mid m)\,\mathrm{sn}(c+z \mid m))-\mathrm{cs}(a-b \mid m)\,\mathrm{cs}(b \mid m)$$
$$(\mathrm{ds}(c-b \mid m)\,\mathrm{sn}(b+z \mid m)-\mathrm{cs}(c-b \mid m)\,\mathrm{sn}(c+z \mid m))+\mathrm{cs}(a \mid m)\,\mathrm{cs}(b \mid m)\,(\mathrm{ds}(c \mid m)\,\mathrm{sn}(z \mid m)-\mathrm{cs}(c \mid m)\,\mathrm{sn}(c+z \mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0080.01

$$m\,\mathrm{sn}(z \mid m)\,\mathrm{sn}(a+z \mid m)\,\mathrm{sn}(b+z \mid m)\,\mathrm{dn}(c+z \mid m) ==$$
$$(\mathrm{cs}(c-a \mid m)\,\mathrm{cn}(a+z \mid m)-\mathrm{ns}(c-a \mid m)\,\mathrm{cn}(c+z \mid m))\,\mathrm{ns}(a \mid m)\,\mathrm{ns}(a-b \mid m)-$$
$$(\mathrm{cs}(c-b \mid m)\,\mathrm{cn}(b+z \mid m)-\mathrm{ns}(c-b \mid m)\,\mathrm{cn}(c+z \mid m))\,\mathrm{ns}(b \mid m)\,\mathrm{ns}(a-b \mid m)+$$
$$(\mathrm{cs}(c \mid m)\,\mathrm{cn}(z \mid m)-\mathrm{ns}(c \mid m)\,\mathrm{cn}(c+z \mid m))\,\mathrm{ns}(a \mid m)\,\mathrm{ns}(b \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0081.01

$$m^2\,\mathrm{sn}(z \mid m)\,\mathrm{sn}(a+z \mid m)\,\mathrm{sn}(b+z \mid m)\,\mathrm{cn}(c+z \mid m) ==$$
$$-\mathrm{ns}(b \mid m)\,(\mathrm{ds}(c-b \mid m)\,\mathrm{dn}(b+z \mid m)-\mathrm{ns}(c-b \mid m)\,\mathrm{dn}(c+z \mid m))\,\mathrm{ns}(a-b \mid m)+$$
$$(\mathrm{ds}(c-a \mid m)\,\mathrm{dn}(a+z \mid m)-\mathrm{ns}(c-a \mid m)\,\mathrm{dn}(c+z \mid m))\,\mathrm{ns}(a \mid m)\,\mathrm{ns}(a-b \mid m)+$$
$$(\mathrm{ds}(c \mid m)\,\mathrm{dn}(z \mid m)-\mathrm{ns}(c \mid m)\,\mathrm{dn}(c+z \mid m))\,\mathrm{ns}(a \mid m)\,\mathrm{ns}(b \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0082.01

$$m\,\mathrm{cn}(z\mid m)\,\mathrm{cn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m)\,\mathrm{dn}(c+z\mid m) ==$$
$$\mathrm{ds}(a\mid m)\,\mathrm{ds}(a-b\mid m)\,(\mathrm{cs}(c-a\mid m)\,\mathrm{sn}(a+z\mid m)-\mathrm{ds}(c-a\mid m)\,\mathrm{sn}(c+z\mid m))-$$
$$\mathrm{ds}(a-b\mid m)\,\mathrm{ds}(b\mid m)\,(\mathrm{cs}(c-b\mid m)\,\mathrm{sn}(b+z\mid m)-\mathrm{ds}(c-b\mid m)\,\mathrm{sn}(c+z\mid m))+$$
$$\mathrm{ds}(a\mid m)\,\mathrm{ds}(b\mid m)\,(\mathrm{cs}(c\mid m)\,\mathrm{sn}(z\mid m)-\mathrm{ds}(c\mid m)\,\mathrm{sn}(c+z\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0083.01

$$m^2\,\mathrm{cn}(z\mid m)\,\mathrm{cn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m)\,\mathrm{sn}(c+z\mid m) ==$$
$$-\mathrm{ds}(a\mid m)\,(\mathrm{ns}(c-a\mid m)\,\mathrm{dn}(a+z\mid m)-\mathrm{ds}(c-a\mid m)\,\mathrm{dn}(c+z\mid m))\,\mathrm{ds}(a-b\mid m)+$$
$$(\mathrm{ns}(c-b\mid m)\,\mathrm{dn}(b+z\mid m)-\mathrm{ds}(c-b\mid m)\,\mathrm{dn}(c+z\mid m))\,\mathrm{ds}(b\mid m)\,\mathrm{ds}(a-b\mid m)+$$
$$(\mathrm{ns}(c\mid m)\,\mathrm{dn}(z\mid m)-\mathrm{ds}(c\mid m)\,\mathrm{dn}(c+z\mid m))\,(-\mathrm{ds}(a\mid m))\,\mathrm{ds}(b\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0084.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(b+z\mid m)\,\mathrm{dn}(c+z\mid m) ==$$
$$\mathrm{ns}(a\mid m)\,\mathrm{ns}(a-b\mid m)\,(\mathrm{dn}(c-a\mid m)+\mathrm{cs}(c-a\mid m)\,(-\mathrm{Z}(\mathrm{am}(c-a\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m)))-$$
$$\mathrm{cs}(a-b\mid m)\,\mathrm{ns}(b\mid m)\,(\mathrm{dn}(c-b\mid m)+\mathrm{cs}(c-b\mid m)\,(-\mathrm{Z}(\mathrm{am}(c-b\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m)))+$$
$$\mathrm{cs}(a\mid m)\,\mathrm{ns}(b\mid m)\,(\mathrm{dn}(c\mid m)+\mathrm{cs}(c\mid m)\,(-\mathrm{Z}(\mathrm{am}(c\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0085.01

$$m\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m)\,\mathrm{dn}(c+z\mid m) ==$$
$$-\mathrm{ds}(a\mid m)\,\mathrm{ds}(a-b\mid m)\,(\mathrm{dn}(c-a\mid m)+\mathrm{cs}(c-a\mid m)\,(-\mathrm{Z}(\mathrm{am}(c-a\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m)))+$$
$$\mathrm{cs}(a-b\mid m)\,\mathrm{ds}(b\mid m)\,(\mathrm{dn}(c-b\mid m)+\mathrm{cs}(c-b\mid m)\,(-\mathrm{Z}(\mathrm{am}(c-b\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m)))-$$
$$\mathrm{cs}(a\mid m)\,\mathrm{ds}(b\mid m)\,(\mathrm{dn}(c\mid m)+\mathrm{cs}(c\mid m)\,(-\mathrm{Z}(\mathrm{am}(c\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0086.01

$$m^2\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m)\,\mathrm{sn}(c+z\mid m) ==$$
$$\mathrm{ds}(a-b\mid m)\,\mathrm{ns}(c-a\mid m)\,\mathrm{ns}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(c-a\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m))-$$
$$\mathrm{ns}(b\mid m)\,\mathrm{ds}(a-b\mid m)\,\mathrm{ns}(c-b\mid m)\,(-\mathrm{Z}(\mathrm{am}(c-b\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m))+$$
$$\mathrm{ds}(a\mid m)\,\mathrm{ds}(b\mid m)\,\mathrm{ns}(c\mid m)\,(-\mathrm{Z}(\mathrm{am}(c\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(c+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0087.01

$$m\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{dn}(b+z\mid m)\,\mathrm{sn}(c+z\mid m) == -\mathrm{cs}(a\mid m)\,\mathrm{cs}(b\mid m)\,(\mathrm{ns}(c\mid m)\,\mathrm{dn}(z\mid m)-\mathrm{ds}(c\mid m)\,\mathrm{dn}(c+z\mid m))-$$
$$\mathrm{cs}(a-b\mid m)\,(\mathrm{ns}(c-a\mid m)\,\mathrm{dn}(a+z\mid m)-\mathrm{ds}(c-a\mid m)\,\mathrm{dn}(c+z\mid m))\,\mathrm{ds}(a\mid m)+$$
$$\mathrm{cs}(a-b\mid m)\,(\mathrm{ns}(c-b\mid m)\,\mathrm{dn}(b+z\mid m)-\mathrm{ds}(c-b\mid m)\,\mathrm{dn}(c+z\mid m))\,\mathrm{ds}(b\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0088.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(b+z\mid m)\,\mathrm{cn}(c+z\mid m) ==$$
$$-\mathrm{ns}(a\mid m)\,\mathrm{ns}(a-b\mid m)\,(\mathrm{ds}(c-a\mid m)\,\mathrm{sn}(a+z\mid m)-\mathrm{cs}(c-a\mid m)\,\mathrm{sn}(c+z\mid m))+$$
$$\mathrm{cs}(a-b\mid m)\,\mathrm{ns}(b\mid m)\,(\mathrm{ds}(c-b\mid m)\,\mathrm{sn}(b+z\mid m)-\mathrm{cs}(c-b\mid m)\,\mathrm{sn}(c+z\mid m))-$$
$$\mathrm{cs}(a\mid m)\,\mathrm{ns}(b\mid m)\,(\mathrm{ds}(c\mid m)\,\mathrm{sn}(z\mid m)-\mathrm{cs}(c\mid m)\,\mathrm{sn}(c+z\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0089.01

$$m\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m)\,\mathrm{sn}(c+z\mid m) ==$$
$$-(\mathrm{ns}(c-a\mid m)\,\mathrm{cn}(a+z\mid m)-\mathrm{cs}(c-a\mid m)\,\mathrm{cn}(c+z\mid m))\,\mathrm{ds}(a\mid m)\,\mathrm{ds}(a-b\mid m)-$$
$$(\mathrm{ns}(c\mid m)\,\mathrm{cn}(z\mid m)-\mathrm{cs}(c\mid m)\,\mathrm{cn}(c+z\mid m))\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(b\mid m)+$$
$$(\mathrm{ns}(c-b\mid m)\,\mathrm{cn}(b+z\mid m)-\mathrm{cs}(c-b\mid m)\,\mathrm{cn}(c+z\mid m))\,\mathrm{cs}(a-b\mid m)\,\mathrm{ds}(b\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

Rank 4 identities with 3 distinct arguments

09.29.18.0090.01

$$\mathrm{dn}(z\mid m)^2\,\mathrm{dn}(a+z\mid m)\,\mathrm{dn}(b+z\mid m) == -\mathrm{cs}(a\mid m)\,\mathrm{cs}(b\mid m)\,\mathrm{dn}(z\mid m)^2-$$
$$\mathrm{cs}(a\mid m)\,\mathrm{cs}(a-b\mid m)\,(\mathrm{dn}(a\mid m)+\mathrm{cs}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))+$$
$$\mathrm{cs}(a-b\mid m)\,\mathrm{cs}(b\mid m)\,(\mathrm{dn}(b\mid m)+\mathrm{cs}(b\mid m)\,(-\mathrm{Z}(\mathrm{am}(b\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0091.01

$$m^2\,\mathrm{sn}(z\mid m)^2\,\mathrm{sn}(a+z\mid m)\,\mathrm{sn}(b+z\mid m) ==$$
$$m\,\mathrm{ns}(b\mid m)\,\mathrm{sn}(z\mid m)^2\,\mathrm{ns}(a\mid m)-\mathrm{ns}(a\mid m)^2\,\mathrm{ns}(a-b\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))+$$
$$\mathrm{ns}(a-b\mid m)\,\mathrm{ns}(b\mid m)^2\,(-\mathrm{Z}(\mathrm{am}(b\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0092.01

$$m^2\,\mathrm{cn}(z\mid m)^2\,\mathrm{cn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m) == -m\,\mathrm{ds}(a\mid m)\,\mathrm{ds}(b\mid m)\,\mathrm{cn}(z\mid m)^2-$$
$$\mathrm{ds}(a\mid m)\,\mathrm{ds}(a-b\mid m)\,(m\,\mathrm{cn}(a\mid m)+\mathrm{ds}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))+$$
$$\mathrm{ds}(a-b\mid m)\,\mathrm{ds}(b\mid m)\,(m\,\mathrm{cn}(b\mid m)+\mathrm{ds}(b\mid m)\,(-\mathrm{Z}(\mathrm{am}(b\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0093.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(b+z\mid m) == -m\,\mathrm{cs}(a\mid m)\,\mathrm{ns}(b\mid m)\,\mathrm{sn}(z\mid m)^2+$$
$$\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{ns}(a-b\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))-$$
$$\mathrm{cs}(b\mid m)\,\mathrm{ns}(b\mid m)\,\mathrm{cs}(a-b\mid m)\,(-\mathrm{Z}(\mathrm{am}(b\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0094.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m) == -m\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(b\mid m)\,\mathrm{cn}(z\mid m)^2-$$
$$\mathrm{cs}(a\mid m)\,\mathrm{ds}(a-b\mid m)\,(m\,\mathrm{cn}(a\mid m)+\mathrm{ds}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))+$$
$$\mathrm{cs}(a-b\mid m)\,\mathrm{cs}(b\mid m)\,(m\,\mathrm{cn}(b\mid m)+\mathrm{ds}(b\mid m)\,(-\mathrm{Z}(\mathrm{am}(b\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0095.01

$$m^2\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m) == -m\,\mathrm{ns}(a\mid m)\,\mathrm{ds}(b\mid m)\,\mathrm{sn}(z\mid m)^2+$$
$$\mathrm{ds}(a\mid m)\,\mathrm{ds}(a-b\mid m)\,\mathrm{ns}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))-$$
$$\mathrm{ds}(b\mid m)\,\mathrm{ns}(b\mid m)\,\mathrm{ns}(a-b\mid m)\,(-\mathrm{Z}(\mathrm{am}(b\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(b+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0096.01

$$\mathrm{dn}(z\mid m)^2\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(b+z\mid m) == (\mathrm{ns}(b\mid m)\,\mathrm{cn}(z\mid m)-\mathrm{cs}(b\mid m)\,\mathrm{cn}(b+z\mid m))\,\mathrm{cs}(a-b\mid m)\,\mathrm{cs}(b\mid m)-$$
$$\mathrm{cs}(a\mid m)\,\mathrm{ns}(b\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(z\mid m)+(\mathrm{ns}(a\mid m)\,\mathrm{cn}(z\mid m)-\mathrm{cs}(a\mid m)\,\mathrm{cn}(a+z\mid m))\,(-\mathrm{cs}(a\mid m))\,\mathrm{ns}(a-b\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0097.01

$$\mathrm{dn}(z \mid m)^2\, \mathrm{dn}(a+z \mid m)\, \mathrm{cn}(b+z \mid m) == -\mathrm{cs}(a \mid m)\, \mathrm{cn}(z \mid m)\, \mathrm{dn}(z \mid m)\, \mathrm{ds}(b \mid m) + \mathrm{cs}(a \mid m)\, \mathrm{ds}(a-b \mid m)\, (\mathrm{ds}(a \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{cs}(a \mid m)\, \mathrm{sn}(a+z \mid m)) - \mathrm{cs}(b \mid m)\, \mathrm{cs}(a-b \mid m)\, (\mathrm{ds}(b \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{cs}(b \mid m)\, \mathrm{sn}(b+z \mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0098.01

$$m^2\, \mathrm{cn}(z \mid m)^2\, \mathrm{sn}(a+z \mid m)\, \mathrm{cn}(b+z \mid m) == (\mathrm{ns}(a \mid m)\, \mathrm{dn}(z \mid m) - \mathrm{ds}(a \mid m)\, \mathrm{dn}(a+z \mid m))\, (-\mathrm{ds}(a \mid m))\, \mathrm{ds}(a-b \mid m) + (\mathrm{ns}(b \mid m)\, \mathrm{dn}(z \mid m) - \mathrm{ds}(b \mid m)\, \mathrm{dn}(b+z \mid m))\, \mathrm{ds}(b \mid m)\, \mathrm{ns}(a-b \mid m) - m\, \mathrm{ns}(a \mid m)\, \mathrm{ds}(b \mid m)\, \mathrm{cn}(z \mid m)\, \mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0099.01

$$m\, \mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)\, \mathrm{sn}(a+z \mid m)\, \mathrm{cn}(b+z \mid m) == \mathrm{cn}(z \mid m)\, \mathrm{dn}(z \mid m)\, \mathrm{ds}(b \mid m)\, \mathrm{ns}(a \mid m) - \mathrm{ds}(a-b \mid m)\, (\mathrm{ds}(a \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{cs}(a \mid m)\, \mathrm{sn}(a+z \mid m))\, \mathrm{ns}(a \mid m) + \mathrm{ns}(a-b \mid m)\, \mathrm{ns}(b \mid m)\, (\mathrm{ds}(b \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{cs}(b \mid m)\, \mathrm{sn}(b+z \mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0100.01

$$m\, \mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)\, \mathrm{dn}(a+z \mid m)\, \mathrm{cn}(b+z \mid m) == \mathrm{cs}(a-b \mid m)\, \mathrm{cs}(b \mid m)\, (\mathrm{ds}(b \mid m)\, \mathrm{dn}(z \mid m) - \mathrm{ns}(b \mid m)\, \mathrm{dn}(b+z \mid m)) - \mathrm{cs}(a \mid m)\, (\mathrm{ds}(a \mid m)\, \mathrm{dn}(z \mid m) - \mathrm{ns}(a \mid m)\, \mathrm{dn}(a+z \mid m))\, \mathrm{ds}(a-b \mid m) - m\, \mathrm{cn}(z \mid m)\, \mathrm{cs}(a \mid m)\, \mathrm{ds}(b \mid m)\, \mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0101.01

$$\mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)\, \mathrm{dn}(a+z \mid m)\, \mathrm{dn}(b+z \mid m) == -(\mathrm{cs}(a \mid m)\, \mathrm{cn}(z \mid m) - \mathrm{ns}(a \mid m)\, \mathrm{cn}(a+z \mid m))\, \mathrm{cs}(a \mid m)\, \mathrm{cs}(a-b \mid m) + (\mathrm{cs}(b \mid m)\, \mathrm{cn}(z \mid m) - \mathrm{ns}(b \mid m)\, \mathrm{cn}(b+z \mid m))\, \mathrm{cs}(b \mid m)\, \mathrm{cs}(a-b \mid m) - \mathrm{cs}(a \mid m)\, \mathrm{cs}(b \mid m)\, \mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0102.01

$$m\, \mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)\, \mathrm{sn}(a+z \mid m)\, \mathrm{sn}(b+z \mid m) == (\mathrm{ns}(a \mid m)\, \mathrm{cn}(z \mid m) - \mathrm{cs}(a \mid m)\, \mathrm{cn}(a+z \mid m))\, \mathrm{ns}(a \mid m)\, \mathrm{ns}(a-b \mid m) - (\mathrm{ns}(b \mid m)\, \mathrm{cn}(z \mid m) - \mathrm{cs}(b \mid m)\, \mathrm{cn}(b+z \mid m))\, \mathrm{ns}(b \mid m)\, \mathrm{ns}(a-b \mid m) + \mathrm{dn}(z \mid m)\, \mathrm{ns}(a \mid m)\, \mathrm{ns}(b \mid m)\, \mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0103.01

$$m\, \mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)\, \mathrm{cn}(a+z \mid m)\, \mathrm{cn}(b+z \mid m) == -(\mathrm{ns}(a \mid m)\, \mathrm{cn}(z \mid m) - \mathrm{cs}(a \mid m)\, \mathrm{cn}(a+z \mid m))\, \mathrm{ds}(a-b \mid m)\, \mathrm{ns}(a \mid m) + (\mathrm{ns}(b \mid m)\, \mathrm{cn}(z \mid m) - \mathrm{cs}(b \mid m)\, \mathrm{cn}(b+z \mid m))\, \mathrm{ds}(a-b \mid m)\, \mathrm{ns}(b \mid m) - \mathrm{dn}(z \mid m)\, \mathrm{ds}(a \mid m)\, \mathrm{ds}(b \mid m)\, \mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0104.01

$$m\, \mathrm{dn}(z \mid m)\, \mathrm{cn}(z \mid m)\, \mathrm{sn}(a+z \mid m)\, \mathrm{cn}(b+z \mid m) == -(\mathrm{ns}(a \mid m)\, \mathrm{cn}(z \mid m) - \mathrm{cs}(a \mid m)\, \mathrm{cn}(a+z \mid m))\, \mathrm{ds}(a \mid m)\, \mathrm{ds}(a-b \mid m) + (\mathrm{ns}(b \mid m)\, \mathrm{cn}(z \mid m) - \mathrm{cs}(b \mid m)\, \mathrm{cn}(b+z \mid m))\, \mathrm{ds}(b \mid m)\, \mathrm{ns}(a-b \mid m) - \mathrm{dn}(z \mid m)\, \mathrm{ds}(b \mid m)\, \mathrm{ns}(a \mid m)\, \mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0105.01

$$m\, \mathrm{dn}(z \mid m)\, \mathrm{cn}(z \mid m)\, \mathrm{dn}(a+z \mid m)\, \mathrm{sn}(b+z \mid m) == \mathrm{cs}(a-b \mid m)\, \mathrm{cs}(b \mid m)\, (\mathrm{ns}(b \mid m)\, \mathrm{dn}(z \mid m) - \mathrm{ds}(b \mid m)\, \mathrm{dn}(b+z \mid m)) - \mathrm{cs}(a \mid m)\, (\mathrm{ns}(a \mid m)\, \mathrm{dn}(z \mid m) - \mathrm{ds}(a \mid m)\, \mathrm{dn}(a+z \mid m))\, \mathrm{ns}(a-b \mid m) - m\, \mathrm{cn}(z \mid m)\, \mathrm{cs}(a \mid m)\, \mathrm{ns}(b \mid m)\, \mathrm{sn}(z \mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0106.01

$$\mathrm{dn}(z \mid m)\, \mathrm{cn}(z \mid m)\, \mathrm{dn}(a+z \mid m)\, \mathrm{dn}(b+z \mid m) == -\mathrm{cn}(z \mid m)\, \mathrm{cs}(a \mid m)\, \mathrm{cs}(b \mid m)\, \mathrm{dn}(z \mid m) + \mathrm{cs}(a \mid m)\, \mathrm{cs}(a-b \mid m)\, (\mathrm{cs}(a \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{ds}(a \mid m)\, \mathrm{sn}(a+z \mid m)) - \mathrm{cs}(a-b \mid m)\, \mathrm{cs}(b \mid m)\, (\mathrm{cs}(b \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{ds}(b \mid m)\, \mathrm{sn}(b+z \mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0107.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{sn}(b+z\mid m) == \mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{ns}(b\mid m) - \mathrm{ds}(a\mid m)\,\mathrm{ns}(a-b\mid m)\,(\mathrm{ds}(a\mid m)\,\mathrm{sn}(z\mid m) - \mathrm{cs}(a\mid m)\,\mathrm{sn}(a+z\mid m)) + \mathrm{ds}(b\mid m)\,\mathrm{ns}(a-b\mid m)\,(\mathrm{ds}(b\mid m)\,\mathrm{sn}(z\mid m) - \mathrm{cs}(b\mid m)\,\mathrm{sn}(b+z\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0108.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{cn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m) == -\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ds}(b\mid m) - \mathrm{ds}(a-b\mid m)\,(\mathrm{ds}(b\mid m)\,\mathrm{sn}(z\mid m) - \mathrm{cs}(b\mid m)\,\mathrm{sn}(b+z\mid m))\,\mathrm{ds}(b\mid m) + \mathrm{ds}(a\mid m)\,\mathrm{ds}(a-b\mid m)\,(\mathrm{ds}(a\mid m)\,\mathrm{sn}(z\mid m) - \mathrm{cs}(a\mid m)\,\mathrm{sn}(a+z\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0109.01

$$m^2\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{sn}(b+z\mid m) == -\mathrm{ds}(b\mid m)\,(\mathrm{ds}(b\mid m)\,\mathrm{dn}(z\mid m) - \mathrm{ns}(b\mid m)\,\mathrm{dn}(b+z\mid m))\,\mathrm{ns}(a-b\mid m) + (\mathrm{ds}(a\mid m)\,\mathrm{dn}(z\mid m) - \mathrm{ns}(a\mid m)\,\mathrm{dn}(a+z\mid m))\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a-b\mid m) + m\,\mathrm{cn}(z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{ns}(b\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0110.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{dn}(b+z\mid m) == -\mathrm{cs}(a-b\mid m)\,(\mathrm{ns}(a\mid m)\,\mathrm{dn}(z\mid m) - \mathrm{ds}(a\mid m)\,\mathrm{dn}(a+z\mid m))\,\mathrm{ns}(a\mid m) + \mathrm{cs}(a-b\mid m)\,(\mathrm{ns}(b\mid m)\,\mathrm{dn}(z\mid m) - \mathrm{ds}(b\mid m)\,\mathrm{dn}(b+z\mid m))\,\mathrm{ns}(b\mid m) - m\,\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{cs}(b\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0111.01

$$m^2\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{cn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m) == (\mathrm{ns}(a\mid m)\,\mathrm{dn}(z\mid m) - \mathrm{ds}(a\mid m)\,\mathrm{dn}(a+z\mid m))\,\mathrm{ds}(a-b\mid m)\,(-\mathrm{ns}(a\mid m)) + (\mathrm{ns}(b\mid m)\,\mathrm{dn}(z\mid m) - \mathrm{ds}(b\mid m)\,\mathrm{dn}(b+z\mid m))\,\mathrm{ds}(a-b\mid m)\,\mathrm{ns}(b\mid m) - m\,\mathrm{ds}(a\mid m)\,\mathrm{ds}(b\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0112.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(b+z\mid m) == \mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(b\mid m) + \mathrm{cs}(a-b\mid m)\,(\mathrm{cs}(b\mid m)\,\mathrm{sn}(z\mid m) - \mathrm{ds}(b\mid m)\,\mathrm{sn}(b+z\mid m))\,\mathrm{ns}(b\mid m) - \mathrm{ns}(a\mid m)\,\mathrm{ns}(a-b\mid m)\,(\mathrm{cs}(a\mid m)\,\mathrm{sn}(z\mid m) - \mathrm{ds}(a\mid m)\,\mathrm{sn}(a+z\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0113.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cn}(b+z\mid m) == -(\mathrm{cs}(a\mid m)\,\mathrm{cn}(z\mid m) - \mathrm{ns}(a\mid m)\,\mathrm{cn}(a+z\mid m))\,\mathrm{ds}(a\mid m)\,\mathrm{ds}(a-b\mid m) + (\mathrm{cs}(b\mid m)\,\mathrm{cn}(z\mid m) - \mathrm{ns}(b\mid m)\,\mathrm{cn}(b+z\mid m))\,\mathrm{cs}(a-b\mid m)\,\mathrm{ds}(b\mid m) - \mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ds}(b\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

Rank 4 identities with 2 distinct arguments

09.29.18.0114.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m) == \mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)^3 + \left(\mathrm{ds}(a\mid m)^2+1\right)(-\mathrm{cs}(a\mid m))\,\mathrm{dn}(z\mid m) + \mathrm{cs}(a\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m) + m\,\mathrm{cn}(z\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0115.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(a+z\mid m) == -m\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)^3 + \left(\mathrm{ds}(a\mid m)^2-1\right)(-\mathrm{ns}(a\mid m))\,\mathrm{sn}(z\mid m) - \mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m) + \mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(a+z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0116.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{cn}(a+z\mid m) == m\,\mathrm{ds}(a\mid m)\,\mathrm{cn}(z\mid m)^3 + \left(\mathrm{cs}(a\mid m)^2 + m\right)(-\mathrm{ds}(a\mid m))\,\mathrm{cn}(z\mid m) + \mathrm{cn}(a+z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m) + \mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0117.01

$$\mathrm{dn}(z\mid m)^3\,\mathrm{dn}(a+z\mid m) == -(-Z(\mathrm{am}(a\mid m)\mid m) + Z(\mathrm{am}(a+z\mid m)\mid m) - Z(\mathrm{am}(z\mid m)\mid m))\,\mathrm{cs}(a\mid m)^3 - \mathrm{dn}(a\mid m)\,\mathrm{cs}(a\mid m)^2 - m\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cs}(a\mid m) + \mathrm{dn}(z\mid m)^2\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0118.01

$$\mathrm{dn}(z\mid m)^3\,\mathrm{sn}(a+z\mid m) == m\,\mathrm{ns}(a\mid m)\,\mathrm{cn}(z\mid m)^3 - \mathrm{ns}(a\mid m)\left(2\,\mathrm{cs}(a\mid m)^2 - \mathrm{ds}(a\mid m)^2\right)\mathrm{cn}(z\mid m) + \mathrm{cn}(a+z\mid m)\,\mathrm{cs}(a\mid m)^3 + \mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0119.01

$$\mathrm{dn}(z\mid m)^3\,\mathrm{cn}(a+z\mid m) == \mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{cs}(a\mid m) + m\,\mathrm{ds}(a\mid m)\,\mathrm{sn}(z\mid m)^3 - \mathrm{cs}(a\mid m)^3\,\mathrm{sn}(a+z\mid m) - \mathrm{ds}(a\mid m)\left(2 - \mathrm{ns}(a\mid m)^2\right)\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0120.01

$$m\,\mathrm{sn}(z\mid m)^3\,\mathrm{dn}(a+z\mid m) == -m\,\mathrm{cs}(a\mid m)\,\mathrm{cn}(z\mid m)^3 + \mathrm{cs}(a\mid m)\left(\mathrm{ns}(a\mid m)^2 + m\right)\mathrm{cn}(z\mid m) - \mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(z\mid m) + \mathrm{cn}(a+z\mid m)\left(-\mathrm{ns}(a\mid m)^3\right)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0121.01

$$m^2\,\mathrm{sn}(z\mid m)^3\,\mathrm{sn}(a+z\mid m) == -(-Z(\mathrm{am}(a\mid m)\mid m) + Z(\mathrm{am}(a+z\mid m)\mid m) - Z(\mathrm{am}(z\mid m)\mid m))\,\mathrm{ns}(a\mid m)^3 - m\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{ns}(a\mid m) + \mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)^2\,\mathrm{ds}(a\mid m) - \mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0122.01

$$m^2\,\mathrm{sn}(z\mid m)^3\,\mathrm{cn}(a+z\mid m) == -\mathrm{ds}(a\mid m)\,\mathrm{dn}(z\mid m)^3 + \mathrm{ds}(a\mid m)\left(\mathrm{ns}(a\mid m)^2 + 1\right)\mathrm{dn}(z\mid m) - m\,\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m) - \mathrm{ns}(a\mid m)^3\,\mathrm{dn}(a+z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0123.01

$$m\,\mathrm{cn}(z\mid m)^3\,\mathrm{dn}(a+z\mid m) == \mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{ds}(a\mid m) + m\,\mathrm{cs}(a\mid m)\,\mathrm{sn}(z\mid m)^3 - \mathrm{ds}(a\mid m)^3\,\mathrm{sn}(a+z\mid m) + \mathrm{cs}(a\mid m)\left(2\,\mathrm{ds}(a\mid m)^2 - \mathrm{ns}(a\mid m)^2\right)\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0124.01

$$m^2\,\mathrm{cn}(z\mid m)^3\,\mathrm{sn}(a+z\mid m) == \mathrm{ns}(a\mid m)\,\mathrm{dn}(z\mid m)^3 - \left(2\,\mathrm{ds}(a\mid m)^2 - \mathrm{cs}(a\mid m)^2\right)\mathrm{ns}(a\mid m)\,\mathrm{dn}(z\mid m) + \mathrm{dn}(a+z\mid m)\,\mathrm{ds}(a\mid m)^3 + m\,\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0125.01

$$m^2\,\mathrm{cn}(z\mid m)^3\,\mathrm{cn}(a+z\mid m) == -(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))\,\mathrm{ds}(a\mid m)^3-$$
$$m\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{ds}(a\mid m)-\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\left(m\,\mathrm{dn}(a\mid m)^2-m+1\right)+\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)^2\,\mathrm{ns}(a\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0126.01

$$\mathrm{dn}(z\mid m)^2\,\mathrm{dn}(a+z\mid m)^2 == 2\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))\,\mathrm{cs}(a\mid m)+$$
$$\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)-\mathrm{cs}(a\mid m)^2\left(\mathrm{dn}(a+z\mid m)^2+\mathrm{dn}(z\mid m)^2\right)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0127.01

$$m\,\mathrm{dn}(z\mid m)^2\,\mathrm{sn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m) == -m\,\mathrm{sn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m)\,\mathrm{cs}(a\mid m)^2-2\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cs}(a\mid m)+$$
$$\mathrm{dn}(z\mid m)\left(\mathrm{ds}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{cs}(a\mid m)-m\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0128.01

$$m\,\mathrm{sn}(z\mid m)^2\,\mathrm{dn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m) == \mathrm{cn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{ns}(a\mid m)^2-$$
$$2\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{ns}(a\mid m)+\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{sn}(z\mid m)\,\mathrm{ns}(a\mid m)+\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ds}(a\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0129.01

$$m\,\mathrm{cn}(z\mid m)^2\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(a+z\mid m) == -\mathrm{sn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{ds}(a\mid m)^2-$$
$$2\,\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{cn}(a+z\mid m)\,\mathrm{ds}(a\mid m)+\mathrm{cn}(z\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{ds}(a\mid m)-\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0130.01

$$m^2\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m) == \mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\left(\mathrm{dn}(a+z\mid m)^2+\mathrm{dn}(z\mid m)^2\right)-$$
$$\left(\mathrm{ds}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)(\mathrm{dn}(a\mid m)+\mathrm{cs}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0131.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(a+z\mid m) ==$$
$$-\mathrm{cs}(a\mid m)\left(\mathrm{dn}(a\mid m)^2+1\right)\mathrm{ns}(a\mid m)+\mathrm{cs}(a\mid m)\left(\mathrm{dn}(a+z\mid m)^2+\mathrm{dn}(z\mid m)^2\right)\mathrm{ns}(a\mid m)-$$
$$\mathrm{ds}(a\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0132.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m) == 2\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)-\mathrm{cs}(a\mid m)\left(\mathrm{dn}(a+z\mid m)^2+\mathrm{dn}(z\mid m)^2\right)\mathrm{ds}(a\mid m)+$$
$$\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{ns}(a\mid m)\,(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m))$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0133.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(a+z\mid m) ==$$
$$-\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{dn}(a+z\mid m)\,\mathrm{ns}(a\mid m)-m\,\mathrm{cs}(a\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{ns}(a\mid m)-$$
$$m\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m)+\mathrm{dn}(z\mid m)\,\mathrm{ds}(a\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0134.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m) ==$$
$$-\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{cn}(a+z\mid m)\,\mathrm{ns}(a\mid m)-\mathrm{ds}(a\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)-$$
$$\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)+\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\left(\mathrm{ds}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0135.01

$$m\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(a+z\mid m) ==$$
$$\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{ns}(a\mid m)+\mathrm{cn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)-$$
$$\mathrm{cs}(a\mid m)\left(\mathrm{ds}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{sn}(a+z\mid m)+\mathrm{ds}(a\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

Higher order local identities

Examples of local identities of rank 5

09.29.18.0136.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(a+z\mid m) ==$$
$$m\,\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{cn}(z\mid m)^3-\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{cn}(z\mid m)+$$
$$\mathrm{cn}(a+z\mid m)\left(\mathrm{cs}(a\mid m)^2\,\mathrm{ds}(a\mid m)^2+\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{ns}(a\mid m)^2\right)+$$
$$\mathrm{cs}(a\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(a+z\mid m)+\mathrm{dn}(z\mid m)\,\mathrm{ds}(a\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0137.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m) ==$$
$$m\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{sn}(z\mid m)^3+\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{sn}(z\mid m)+$$
$$\mathrm{cn}(a+z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)+\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{ns}(a\mid m)-$$
$$\left(\mathrm{cs}(a\mid m)^2\,\mathrm{ds}(a\mid m)^2+\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{ns}(a\mid m)^2\right)\mathrm{sn}(a+z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0138.01

$$m^2\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{cn}(a+z\mid m) ==$$
$$\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{dn}(z\mid m)^3-\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{dn}(z\mid m)+$$
$$\mathrm{dn}(a+z\mid m)\left(\mathrm{cs}(a\mid m)^2\,\mathrm{ds}(a\mid m)^2+\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{ns}(a\mid m)^2\right)+$$
$$m\,\mathrm{cn}(a+z\mid m)\,\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(a+z\mid m)+m\,\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)\left(\mathrm{ds}(a\mid m)^2+\mathrm{ns}(a\mid m)^2\right)\mathrm{sn}(z\mid m)$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0139.01

$$m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{dn}(a+z\mid m)^2 ==$$
$$-m\,\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cn}(z\mid m)\,\mathrm{cs}(a\mid m)^2+\left(-\mathrm{dn}(a\mid m)^2+\mathrm{dn}(a+z\mid m)^2+2\,\mathrm{dn}(z\mid m)^2-2\right)\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{cs}(a\mid m)+$$
$$\left(\mathrm{cs}(a\mid m)^2\,\mathrm{ds}(a\mid m)^2+\left(\mathrm{cs}(a\mid m)^2+\mathrm{ds}(a\mid m)^2\right)\mathrm{ns}(a\mid m)^2\right)\left(-\mathrm{Z}(\mathrm{am}(a\mid m)\mid m)+\mathrm{Z}(\mathrm{am}(a+z\mid m)\mid m)-\mathrm{Z}(\mathrm{am}(z\mid m)\mid m)\right)$$

Khare/Lakshminarayan/Sukhatme_2003

Local identities of arbitrary rank

09.29.18.0140.01

$$\mathrm{dn}(z \mid m)^{2\,n}\,\mathrm{dn}(a+z \mid m) == (-1)^n\,\mathrm{dn}(a+z \mid m)\,\mathrm{cs}(a \mid m)^{2\,n} + (\mathrm{ds}(a \mid m)\,\mathrm{ns}(a \mid m)\,\mathrm{dn}(z \mid m) - m\,\mathrm{cs}(a \mid m)\,\mathrm{cn}(z \mid m)\,\mathrm{sn}(z \mid m)) \sum_{k=0}^{n-1} (-1)^k\,\mathrm{cs}(a \mid m)^{2\,k}\,\mathrm{dn}(z \mid m)^{2\,(-k+n-1)} \;/;\; n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0141.01

$$\mathrm{dn}(z \mid m)^{2\,n+1}\,\mathrm{dn}(a+z \mid m) == (-1)^n\,(\mathrm{dn}(a \mid m) + \mathrm{cs}(a \mid m)\,(-Z(\mathrm{am}(a \mid m) \mid m) + Z(\mathrm{am}(a+z \mid m) \mid m) - Z(\mathrm{am}(z \mid m) \mid m)))\,\mathrm{cs}(a \mid m)^{2\,n} + (\mathrm{ds}(a \mid m)\,\mathrm{ns}(a \mid m)\,\mathrm{dn}(z \mid m) - m\,\mathrm{cs}(a \mid m)\,\mathrm{cn}(z \mid m)\,\mathrm{sn}(z \mid m)) \sum_{k=0}^{n-1} (-1)^k\,\mathrm{cs}(a \mid m)^{2\,k}\,\mathrm{dn}(z \mid m)^{2\,(n-k)-1} \;/;\; n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0142.01

$$m^n\,\mathrm{sn}(z \mid m)^{2\,n}\,\mathrm{sn}(a+z \mid m) == \mathrm{ns}(a \mid m)^{2\,n}\,\mathrm{sn}(a+z \mid m) - (\mathrm{cn}(z \mid m)\,\mathrm{dn}(z \mid m)\,\mathrm{ns}(a \mid m) + \mathrm{cs}(a \mid m)\,\mathrm{ds}(a \mid m)\,\mathrm{sn}(z \mid m)) \sum_{k=0}^{n-1} m^{-k+n-1}\,\mathrm{ns}(a \mid m)^{2\,k}\,\mathrm{sn}(z \mid m)^{2\,(-k+n-1)} \;/;\; n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0143.01

$$m^{n+1}\,\mathrm{sn}(z \mid m)^{2\,n+1}\,\mathrm{sn}(a+z \mid m) == -\mathrm{ns}(a \mid m)^{2\,n+1}\,(-Z(\mathrm{am}(a \mid m) \mid m) + Z(\mathrm{am}(a+z \mid m) \mid m) - Z(\mathrm{am}(z \mid m) \mid m)) - (\mathrm{cn}(z \mid m)\,\mathrm{dn}(z \mid m)\,\mathrm{ns}(a \mid m) + \mathrm{cs}(a \mid m)\,\mathrm{ds}(a \mid m)\,\mathrm{sn}(z \mid m)) \sum_{k=0}^{n-1} m^{-k+n-1}\,\mathrm{ns}(a \mid m)^{2\,k}\,\mathrm{sn}(z \mid m)^{2\,(n-k)-1} \;/;\; n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0144.01

$$m^n\,\mathrm{cn}(z \mid m)^{2\,n}\,\mathrm{cn}(a+z \mid m) == (-1)^n\,\mathrm{cn}(a+z \mid m)\,\mathrm{ds}(a \mid m)^{2\,n} + (\mathrm{cs}(a \mid m)\,\mathrm{ns}(a \mid m)\,\mathrm{cn}(z \mid m) - \mathrm{ds}(a \mid m)\,\mathrm{sn}(z \mid m)\,\mathrm{dn}(z \mid m)) \sum_{k=0}^{n-1} (-1)^k\,m^{-k+n-1}\,\mathrm{ds}(a \mid m)^{2\,k}\,\mathrm{cn}(z \mid m)^{2\,(-k+n-1)} \;/;\; n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0145.01

$$m^{n+1}\,\mathrm{cn}(z \mid m)^{2\,n+1}\,\mathrm{cn}(a+z \mid m) == (-1)^n\,(m\,\mathrm{cn}(a \mid m) + \mathrm{ds}(a \mid m)\,(-Z(\mathrm{am}(a \mid m) \mid m) + Z(\mathrm{am}(a+z \mid m) \mid m) - Z(\mathrm{am}(z \mid m) \mid m)))\,\mathrm{ds}(a \mid m)^{2\,n} + (\mathrm{cs}(a \mid m)\,\mathrm{ns}(a \mid m)\,\mathrm{cn}(z \mid m) - \mathrm{ds}(a \mid m)\,\mathrm{sn}(z \mid m)\,\mathrm{dn}(z \mid m)) \sum_{k=0}^{n-1} (-1)^k\,m^{-k+n-1}\,\mathrm{ds}(a \mid m)^{2\,k}\,\mathrm{cn}(z \mid m)^{2\,(n-k)-1} \;/;\; n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0146.01

$$m^n\,\mathrm{cn}(z \mid m)^{2\,n}\,\mathrm{sn}(z \mid m)\,\mathrm{dn}(a+z \mid m) == m^n\,\mathrm{cs}(a \mid m)\,\mathrm{cn}(z \mid m)^{2\,n+1} + (-1)^{n+1}\,\mathrm{cn}(a+z \mid m)\,\mathrm{ds}(a \mid m)^{2\,n}\,\mathrm{ns}(a \mid m) - \mathrm{ns}(a \mid m)\,(\mathrm{cs}(a \mid m)\,\mathrm{ns}(a \mid m)\,\mathrm{cn}(z \mid m) - \mathrm{ds}(a \mid m)\,\mathrm{sn}(z \mid m)\,\mathrm{dn}(z \mid m)) \sum_{k=0}^{n-1} (-1)^k\,m^{-k+n-1}\,\mathrm{ds}(a \mid m)^{2\,k}\,\mathrm{cn}(z \mid m)^{2\,(-k+n-1)} \;/;\; n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0147.01

$$m^n \operatorname{sn}(z \mid m)^{2n} \operatorname{cn}(z \mid m) \operatorname{dn}(a+z \mid m) == -m^n \operatorname{cs}(a \mid m) \operatorname{sn}(z \mid m)^{2n+1} + \operatorname{ds}(a \mid m) \operatorname{ns}(a \mid m)^{2n} \operatorname{sn}(a+z \mid m) -$$
$$\operatorname{ds}(a \mid m) (\operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m) \operatorname{ns}(a \mid m) + \operatorname{cs}(a \mid m) \operatorname{ds}(a \mid m) \operatorname{sn}(z \mid m)) \sum_{k=0}^{n-1} m^{-k+n-1} \operatorname{ns}(a \mid m)^{2k} \operatorname{sn}(z \mid m)^{2(-k+n-1)} \text{ /; } n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0148.01

$$m \operatorname{dn}(z \mid m)^{2n} \operatorname{cn}(z \mid m) \operatorname{sn}(a+z \mid m) == \operatorname{ns}(a \mid m) \operatorname{dn}(z \mid m)^{2n+1} + (-1)^{n+1} \operatorname{cs}(a \mid m)^{2n} \operatorname{dn}(a+z \mid m) \operatorname{ds}(a \mid m) -$$
$$\operatorname{ds}(a \mid m) (\operatorname{ds}(a \mid m) \operatorname{ns}(a \mid m) \operatorname{dn}(z \mid m) - m \operatorname{cs}(a \mid m) \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m)) \sum_{k=0}^{n-1} (-1)^k \operatorname{cs}(a \mid m)^{2k} \operatorname{dn}(z \mid m)^{2(-k+n-1)} \text{ /; } n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0149.01

$$m \operatorname{dn}(z \mid m)^{2n} \operatorname{sn}(a+z \mid m) \operatorname{cn}(a+z \mid m) ==$$
$$2(-1)^n n \operatorname{dn}(a+z \mid m) \operatorname{ds}(a \mid m) \operatorname{ns}(a \mid m) \operatorname{cs}(a \mid m)^{2n-1} + (-1)^n m \operatorname{cn}(a+z \mid m) \operatorname{sn}(a+z \mid m) \operatorname{cs}(a \mid m)^{2n} +$$
$$\sum_{k=0}^{n-1} (-1)^k \operatorname{cs}(a \mid m)^{2k} \left(2k \operatorname{nc}(a \mid m) \operatorname{ns}(a \mid m) \operatorname{ds}(a \mid m)^2 + \operatorname{cs}(a \mid m) \left(2 \operatorname{ds}(a \mid m)^2 + m\right)\right) \operatorname{dn}(z \mid m)^{2(n-k)-1} -$$
$$m \operatorname{ds}(a \mid m) \operatorname{ns}(a \mid m) \sum_{k=0}^{n-1} (-1)^k (2k+1) \operatorname{cs}(a \mid m)^{2k} \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m) \operatorname{dn}(z \mid m)^{2(-k+n-1)} \text{ /; } n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0150.01

$$m \operatorname{dn}(z \mid m)^{2n+1} \operatorname{sn}(a+z \mid m) \operatorname{cn}(a+z \mid m) == (-1)^{n-1}$$
$$(-m-(2n+1) \operatorname{ds}(a \mid m) \operatorname{ns}(a \mid m) (\operatorname{dn}(a \mid m) + \operatorname{cs}(a \mid m) (-Z(\operatorname{am}(a \mid m) \mid m) + Z(\operatorname{am}(a+z \mid m) \mid m) - Z(\operatorname{am}(z \mid m) \mid m))) +$$
$$1) \operatorname{cs}(a \mid m)^{2n-1} + (-1)^{n-1} \operatorname{dn}(a+z \mid m)^2 \operatorname{cs}(a \mid m)^{2n+1} -$$
$$m \operatorname{ds}(a \mid m) \operatorname{ns}(a \mid m) \sum_{k=0}^{n-1} (-1)^k (2k+1) \operatorname{cs}(a \mid m)^{2k} \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m) \operatorname{dn}(z \mid m)^{2(n-k)-1} +$$
$$\sum_{k=0}^{n-1} (-1)^k \operatorname{cs}(a \mid m)^{2k} \left(2k \operatorname{nc}(a \mid m) \operatorname{ns}(a \mid m) \operatorname{ds}(a \mid m)^2 + \operatorname{cs}(a \mid m) \left(2 \operatorname{ds}(a \mid m)^2 + m\right)\right) \operatorname{dn}(z \mid m)^{2(n-k)} \text{ /; } n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0151.01

$$\operatorname{dn}(z \mid m)^{2n} \operatorname{sn}(a+z \mid m) \operatorname{dn}(a+z \mid m) ==$$
$$2(-1)^n n \operatorname{cn}(a+z \mid m) \operatorname{ds}(a \mid m) \operatorname{ns}(a \mid m) \operatorname{cs}(a \mid m)^{2n-1} + (-1)^n \operatorname{dn}(a+z \mid m) \operatorname{sn}(a+z \mid m) \operatorname{cs}(a \mid m)^{2n} -$$
$$\sum_{k=0}^{n-1} (-1)^k \operatorname{cs}(a \mid m)^{2k} \left(2k \operatorname{nc}(a \mid m) \operatorname{ds}(a \mid m)^2 + \operatorname{cs}(a \mid m) \operatorname{ns}(a \mid m)\right) \operatorname{sn}(z \mid m) \operatorname{dn}(z \mid m)^{2(n-k)-1} +$$
$$\operatorname{ds}(a \mid m) \sum_{k=0}^{n-1} (-1)^k \operatorname{cs}(a \mid m)^{2k} \left(\operatorname{cs}(a \mid m)^2 + (2k+1) \operatorname{ns}(a \mid m)^2\right) \operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m)^{2(-k+n-1)} \text{ /; } n \in \mathbb{N}^+$$

Khare/Lakshminarayan/Sukhatme_2003

09.29.18.0152.01

$$\mathrm{dn}(z\mid m)^{2n+1}\,\mathrm{sn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)==(-1)^{n-1}\left(\mathrm{cs}(a\mid m)^2+2\,n\,\mathrm{ds}(a\mid m)^2\right)\mathrm{ns}(a\mid m)\,\mathrm{sn}(z\mid m)\,\mathrm{cs}(a\mid m)^{2n-1}+$$
$$(-1)^{n+1}\,\mathrm{cn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cs}(a\mid m)^{2n+1}+(-1)^n\,(2\,n+1)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{cs}(a\mid m)^{2n}+$$
$$\mathrm{ds}(a\mid m)\sum_{k=0}^{n-1}(-1)^k\,\mathrm{cs}(a\mid m)^{2k}\left(\mathrm{cs}(a\mid m)^2+(2\,k+1)\,\mathrm{ns}(a\mid m)^2\right)\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)^{2(n-k)-1}-$$
$$\sum_{k=0}^{n-1}(-1)^k\,\mathrm{cs}(a\mid m)^{2k}\left(2\,k\,\mathrm{nc}(a\mid m)\,\mathrm{ds}(a\mid m)^2+\mathrm{cs}(a\mid m)\,\mathrm{ns}(a\mid m)\right)\mathrm{sn}(z\mid m)\,\mathrm{dn}(z\mid m)^{2(n-k)}\;/;\;n\in\mathbb{N}^+$$

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09.29.18.0153.01

$$\mathrm{dn}(z\mid m)^{2n}\,\mathrm{cn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)==$$
$$2\,(-1)^{n-1}\,n\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{cs}(a\mid m)^{2n-1}+(-1)^n\,\mathrm{cn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)\,\mathrm{cs}(a\mid m)^{2n}-$$
$$\sum_{k=0}^{n-1}(-1)^k\,\mathrm{cs}(a\mid m)^{2k}\left(\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)+2\,k\,\mathrm{nc}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{ds}(a\mid m)\right)\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)^{2(n-k)-1}-$$
$$\mathrm{ns}(a\mid m)\sum_{k=0}^{n-1}(-1)^k\,\mathrm{cs}(a\mid m)^{2k}\left(\mathrm{cs}(a\mid m)^2+2\,k\,\mathrm{ds}(a\mid m)^2\right)\mathrm{sn}(z\mid m)\,\mathrm{dn}(z\mid m)^{2(-k+n-1)}\;/;\;n\in\mathbb{N}^+$$

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09.29.18.0154.01

$$\mathrm{dn}(z\mid m)^{2n+1}\,\mathrm{cn}(a+z\mid m)\,\mathrm{dn}(a+z\mid m)==(-1)^{n-1}\,\mathrm{cn}(z\mid m)\,\mathrm{ds}(a\mid m)\left(\mathrm{cs}(a\mid m)^2+2\,n\,\mathrm{ns}(a\mid m)^2\right)\mathrm{cs}(a\mid m)^{2n-1}+$$
$$(-1)^n\,\mathrm{dn}(a+z\mid m)\,\mathrm{sn}(a+z\mid m)\,\mathrm{cs}(a\mid m)^{2n+1}+(-1)^n\,(2\,n+1)\,\mathrm{cn}(a+z\mid m)\,\mathrm{ds}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{cs}(a\mid m)^{2n}-$$
$$\mathrm{ns}(a\mid m)\sum_{k=0}^{n-1}(-1)^k\,\mathrm{cs}(a\mid m)^{2k}\left(\mathrm{cs}(a\mid m)^2+2\,k\,\mathrm{ds}(a\mid m)^2\right)\mathrm{sn}(z\mid m)\,\mathrm{dn}(z\mid m)^{2(n-k)-1}-$$
$$\sum_{k=0}^{n-1}(-1)^k\,\mathrm{cs}(a\mid m)^{2k}\left(\mathrm{cs}(a\mid m)\,\mathrm{ds}(a\mid m)+2\,k\,\mathrm{nc}(a\mid m)\,\mathrm{ns}(a\mid m)\,\mathrm{ds}(a\mid m)\right)\mathrm{cn}(z\mid m)\,\mathrm{dn}(z\mid m)^{2(n-k)}\;/;\;n\in\mathbb{N}^+$$

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Identities

Functional identities

09.29.17.0001.01

$$w(z)^4+2\,(m-1)\,w(z)^2-m+\left(w(z)^4-2\,w(z)^2-m+1\right)w(2\,z)+1==0\;/;\;w(z)==\mathrm{dn}(z\mid m)$$

Complex characteristics

Real part

09.29.19.0001.01

$$\mathrm{Re}(\mathrm{dn}(x+i\,y\mid m))==\frac{\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)}{\mathrm{cn}(y\mid 1-m)^2+m\,\mathrm{sn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2}\;/;\;\{x,\,y,\,m\}\in\mathbb{R}$$

Imaginary part

09.29.19.0002.01

$$\operatorname{Im}(\operatorname{dn}(x + i\, y \mid m)) == -\frac{m\, \operatorname{sn}(x \mid m)\, \operatorname{cn}(x \mid m)\, \operatorname{sn}(y \mid 1-m)}{\operatorname{cn}(y \mid 1-m)^2 + m\, \operatorname{sn}(x \mid m)^2\, \operatorname{sn}(y \mid 1-m)^2} /;\ \{x, y, m\} \in \mathbb{R}$$

Absolute value

09.29.19.0003.01

$$|\operatorname{dn}(x + i\, y \mid m)| == \frac{\sqrt{\operatorname{cn}(y \mid 1-m)^2\, \operatorname{dn}(x \mid m)^2\, \operatorname{dn}(y \mid 1-m)^2 + m^2\, \operatorname{cn}(x \mid m)^2\, \operatorname{sn}(x \mid m)^2\, \operatorname{sn}(y \mid 1-m)^2}}{\operatorname{cn}(y \mid 1-m)^2 + m\, \operatorname{sn}(x \mid m)^2\, \operatorname{sn}(y \mid 1-m)^2} /;\ \{x, y, m\} \in \mathbb{R}$$

Argument

09.29.19.0004.01

$$\arg(\operatorname{dn}(x + i\, y \mid m)) == \tan^{-1}(\operatorname{cn}(y \mid 1-m)\, \operatorname{dn}(x \mid m)\, \operatorname{dn}(y \mid 1-m),\ -m\, \operatorname{sn}(x \mid m)\, \operatorname{cn}(x \mid m)\, \operatorname{sn}(y \mid 1-m)) /;\ \{x, y, m\} \in \mathbb{R}$$

Conjugate value

09.29.19.0005.01

$$\overline{\operatorname{dn}(x + i\, y \mid m)} == \frac{\operatorname{cn}(y \mid 1-m)\, \operatorname{dn}(x \mid m)\, \operatorname{dn}(y \mid 1-m) + i\, m\, \operatorname{cn}(x \mid m)\, \operatorname{sn}(x \mid m)\, \operatorname{sn}(y \mid 1-m)}{\operatorname{cn}(y \mid 1-m)^2 + m\, \operatorname{sn}(x \mid m)^2\, \operatorname{sn}(y \mid 1-m)^2} /;\ \{x, y, m\} \in \mathbb{R}$$

Differentiation

Low-order differentiation

With respect to z

09.29.20.0001.01

$$\frac{\partial \operatorname{dn}(z \mid m)}{\partial z} == -m\, \operatorname{sn}(z \mid m)\, \operatorname{cn}(z \mid m)$$

09.29.20.0002.01

$$\frac{\partial^2 \operatorname{dn}(z \mid m)}{\partial z^2} == m\, \operatorname{dn}(z \mid m) \left(\operatorname{sn}(z \mid m)^2 - \operatorname{cn}(z \mid m)^2\right)$$

09.29.20.0003.01

$$\frac{\partial^2 \operatorname{dn}(z \mid m)}{\partial z^2} == -2\, \operatorname{dn}(z \mid m)^3 + (2 - m)\, \operatorname{dn}(z \mid m)$$

With respect to m

09.29.20.0004.01

$$\frac{\partial \operatorname{dn}(z \mid m)}{\partial m} == \frac{1}{2\,(1-m)} (\operatorname{sn}(z \mid m)\, \operatorname{cn}(z \mid m)\, ((m-1)\, z + E(\operatorname{am}(z \mid m) \mid m) - \operatorname{dn}(z \mid m)\, \operatorname{sc}(z \mid m)))$$

09.29.20.0005.01

$$\frac{\partial^2 \mathrm{dn}(z \mid m)}{\partial m^2} == \frac{1}{4 (m-1)^2} \left(2 ((m-1) z + E(\mathrm{am}(z \mid m) \mid m) - \mathrm{dn}(z \mid m) \mathrm{sc}(z \mid m)) \mathrm{sn}(z \mid m) \mathrm{cn}(z \mid m) + \right.$$
$$(1-m) \mathrm{sn}(z \mid m) \left(2 z + \frac{E(\mathrm{am}(z \mid m) \mid m) - F(\mathrm{am}(z \mid m) \mid m)}{m} + \right.$$
$$\frac{1}{m-1} (\mathrm{cn}(z \mid m) \mathrm{sc}(z \mid m) ((m-1) z + E(\mathrm{am}(z \mid m) \mid m) - \mathrm{dn}(z \mid m) \mathrm{sc}(z \mid m)) \mathrm{sn}(z \mid m)) -$$
$$\frac{1}{(m-1) m} (\mathrm{dc}(z \mid m) \mathrm{dn}(z \mid m) \mathrm{nc}(z \mid m) ((m-1) z + E(\mathrm{am}(z \mid m) \mid m) - m \mathrm{cd}(z \mid m) \mathrm{sn}(z \mid m))) +$$
$$\left. \left. \frac{1}{(m-1) m} \left(((m-1) z + E(\mathrm{am}(z \mid m) \mid m)) \mathrm{dn}(z \mid m) - m \mathrm{cn}(z \mid m) \mathrm{sn}(z \mid m)) \sqrt{1 - m \mathrm{sn}(z \mid m)^2} \right) \right) \mathrm{cn}(z \mid m) + \right.$$
$$\frac{1}{m} \left(\mathrm{cn}(z \mid m)^2 \mathrm{dn}(z \mid m) (-m z + z - E(\mathrm{am}(z \mid m) \mid m) + \mathrm{dn}(z \mid m) \mathrm{sc}(z \mid m)) \right.$$
$$((m-1) z + E(\mathrm{am}(z \mid m) \mid m) - m \mathrm{cd}(z \mid m) \mathrm{sn}(z \mid m))) + \frac{1}{m} (\mathrm{dn}(z \mid m)$$
$$\left. \left. ((m-1) z + E(\mathrm{am}(z \mid m) \mid m) - \mathrm{dn}(z \mid m) \mathrm{sc}(z \mid m)) \mathrm{sn}(z \mid m)^2 ((m-1) z + E(\mathrm{am}(z \mid m) \mid m) - m \mathrm{cd}(z \mid m) \mathrm{sn}(z \mid m))) \right) \right)$$

Symbolic differentiation

With respect to z

09.29.20.0008.01

$$\frac{\partial^n \mathrm{dn}(z \mid m)}{\partial z^n} = \mathrm{dn}(z \mid m) \delta_n - m \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\partial^j \mathrm{sn}(z \mid m)}{\partial z^j} \frac{\partial^{-j+n-1} \mathrm{cn}(z \mid m)}{\partial z^{-j+n-1}} /; n \in \mathbb{N}$$

09.29.20.0006.01

$$\frac{\partial^n \mathrm{dn}(z \mid m)}{\partial z^n} == \frac{2 \pi^{n+1}}{K(m)^{n+1}} \sum_{k=1}^{\infty} \frac{k^n q(m)^k}{q(m)^{2k} + 1} \cos\left(\frac{\pi n}{2} + \frac{k \pi z}{K(m)}\right) /; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to z

09.29.20.0007.01

$$\frac{\partial^\alpha \mathrm{dn}(z \mid m)}{\partial z^\alpha} == \frac{\pi z^{-\alpha}}{2 K(m) \Gamma(1-\alpha)} + \frac{2^{\alpha+1} \pi^{3/2} z^{-\alpha}}{K(m)} \sum_{k=1}^{\infty} \frac{q(m)^k}{1 + q(m)^{2k}} {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1 - \frac{\alpha}{2}; -\frac{k^2 \pi^2 z^2}{4 K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.29.21.0001.01

$$\int \mathrm{dn}(z \mid m) \, dz == \mathrm{am}(z \mid m)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

09.29.21.0002.01

$$\int \mathrm{dn}(z \mid m)^2 \, dz == \frac{E(\mathrm{am}(z \mid m) \mid m)\, \mathrm{dn}(z \mid m)}{\sqrt{1 - m\, \mathrm{sn}(z \mid m)^2}}$$

09.29.21.0003.01

$$\int \mathrm{dn}(z \mid m)^3 \, dz == \frac{1}{2}\left(m\, \mathrm{cn}(z \mid m)\, \mathrm{sn}(z \mid m) - (m-2)\, \mathrm{am}(z \mid m)\right)$$

09.29.21.0004.01

$$\int \frac{dz}{\mathrm{dn}(z \mid m)} == \frac{1}{i\sqrt{1-m}} \log\left(\frac{\mathrm{cn}(z \mid m) + i\sqrt{1-m}\ \mathrm{sn}(z \mid m)}{\mathrm{dn}(z \mid m)}\right)$$

09.29.21.0005.01

$$\int \frac{dz}{\mathrm{dn}(z \mid m)^2} == \frac{E(\mathrm{am}(z \mid m) \mid m)\, \mathrm{dn}(z \mid m)}{(1-m)\sqrt{1 - m\, \mathrm{sn}(z \mid m)^2}} - \frac{m\, (\mathrm{cn}(z \mid m)\, \mathrm{sn}(z \mid m))}{(1-m)\, \mathrm{dn}(z \mid m)}$$

09.29.21.0006.01

$$\int \frac{1}{\mathrm{dn}(z \mid m)^3} \, dz == -\frac{1}{2(1-m)}\left(\frac{(m-2)\cos^{-1}\left(\frac{\mathrm{cn}(z|m)}{\mathrm{dn}(z|m)}\right)\sqrt{1 - \frac{\mathrm{cn}(z|m)^2}{\mathrm{dn}(z|m)^2}}\ \mathrm{dn}(z \mid m)}{\left(\mathrm{dn}(z \mid m)^2 - m\, \mathrm{cn}(z \mid m)^2\right)\mathrm{sn}(z \mid m)} + \frac{m\, \mathrm{cn}(z \mid m)\, \mathrm{sn}(z \mid m)}{\mathrm{dn}(z \mid m)^2}\right)$$

Involving direct function and elliptic functions

Involving Jacobi functions

Involving cn

09.29.21.0007.01

$$\int \mathrm{cn}(z \mid m)\, \mathrm{dn}(z \mid m) \, dz == \mathrm{sn}(z \mid m)$$

09.29.21.0008.01

$$\int \frac{\mathrm{dn}(z \mid m)}{\mathrm{cn}(z \mid m)} \, dz == \log\left(\frac{1 + \mathrm{sn}(z \mid m)}{\mathrm{cn}(z \mid m)}\right)$$

09.29.21.0009.01

$$\int \frac{\mathrm{dn}(z \mid m)}{\mathrm{cn}(z \mid m)^2} \, du == \frac{\mathrm{sn}(z \mid m)}{\mathrm{cn}(z \mid m)}$$

09.29.21.0010.01

$$\int \frac{\mathrm{cn}(z \mid m)}{\mathrm{dn}(z \mid m)} \, dz == -\frac{1}{\sqrt{m}} \log\left(\frac{1 - \sqrt{m}\ \mathrm{sn}(z \mid m)}{\mathrm{dn}(z \mid m)}\right)$$

09.29.21.0011.01

$$\int \frac{\mathrm{cn}(z \mid m)}{\mathrm{dn}(z \mid m)^2}\, dz == \frac{\mathrm{sn}(z \mid m)}{\mathrm{dn}(z \mid m)}$$

09.29.21.0012.01

$$\int \frac{dz}{\mathrm{cn}(z \mid m)\,\mathrm{dn}(z \mid m)} == \frac{1}{1-m}\log\left(\frac{1+\mathrm{sn}(z \mid m)}{\mathrm{cn}(z \mid m)}\right) + \frac{\sqrt{m}}{1-m}\log\left(\frac{1-\sqrt{m}\ \mathrm{sn}(z \mid m)}{\mathrm{dn}(z \mid m)}\right)$$

09.29.21.0013.01

$$\int\left(\frac{\mathrm{dn}(z \mid m)}{\mathrm{cn}(z \mid m)} - \frac{m\,\mathrm{cn}(z \mid m)}{\mathrm{dn}(z \mid m)}\right) dz == \log\left(\frac{\mathrm{sn}(z \mid m)+1}{\mathrm{cn}(z \mid m)}\right) + \sqrt{m}\ \log\left(\frac{1-\sqrt{m}\ \mathrm{sn}(z \mid m)}{\mathrm{dn}(z \mid m)}\right)$$

Definite integration

Involving functions of the direct function

Involving elementary functions of the direct function

Involving products of the direct function

09.29.21.0014.01

$$\int_0^{2K(m)} \mathrm{dn}(t \mid m)^3\, \mathrm{dn}(a+t \mid m)\, dt == 2\,\mathrm{ds}(a \mid m)\,\mathrm{ns}(a \mid m)\,E(m) - 2\,K(m)\,\mathrm{cs}(a \mid m)^2\,(\mathrm{dn}(a \mid m) - \mathrm{cs}(a \mid m)\,Z(\mathrm{am}(a \mid m) \mid m))$$

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09.29.21.0015.01

$$\int_0^{2K(m)} \mathrm{dn}(t \mid m)\,\mathrm{dn}(a+t \mid m)\,\mathrm{dn}(b+t \mid m)\,\mathrm{dn}(c+t \mid m)\, dt == 2\,K(m)\,(\mathrm{dn}(a \mid m)\,\mathrm{dn}(b \mid m)\,\mathrm{dn}(c \mid m) + \mathrm{cs}(b-a \mid m)\,\mathrm{cs}(c-a \mid m)\,\mathrm{cs}(a \mid m)\,Z(\mathrm{am}(a \mid m) \mid m) - \mathrm{cs}(b \mid m)\,\mathrm{cs}(b-a \mid m)\,\mathrm{cs}(c-b \mid m)\,Z(\mathrm{am}(b \mid m) \mid m) + \mathrm{cs}(c-a \mid m)\,\mathrm{cs}(c-b \mid m)\,\mathrm{cs}(c \mid m)\,Z(\mathrm{am}(c \mid m) \mid m))$$

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Involving direct function and elliptic functions

Involving Jacobi functions

Involving cn

09.29.21.0016.01

$$\frac{1}{2\,K(m)}\int_0^{2K(m)} m\,\mathrm{cn}(t \mid m)\,\mathrm{dn}(a+t \mid m)\,\mathrm{cn}(b+t \mid m)\,\mathrm{dn}(c+t \mid m)\, dt == m\,\mathrm{cn}(b \mid m)\,\mathrm{dn}(a \mid m)\,\mathrm{dn}(c \mid m) + \mathrm{cs}(c-a \mid m)\,\mathrm{ds}(b-a \mid m)\,\mathrm{ds}(a \mid m)\,Z(\mathrm{am}(a \mid m) \mid m) - \mathrm{ds}(b \mid m)\,\mathrm{cs}(b-a \mid m)\,\mathrm{cs}(c-b \mid m)\,Z(\mathrm{am}(b \mid m) \mid m) + \mathrm{cs}(c-a \mid m)\,\mathrm{ds}(c-b \mid m)\,\mathrm{ds}(c \mid m)\,Z(\mathrm{am}(c \mid m) \mid m)$$

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Involving sn

09.29.21.0017.01

$$\int_0^{2K(m)} \mathrm{sn}(t \mid m)\,\mathrm{dn}(a+t \mid m)\,\mathrm{sn}(b+t \mid m)\,\mathrm{dn}(c+t \mid m)\,dt ==$$
$$\frac{1}{m}((2K(m))(\mathrm{cs}(c-a \mid m)\,\mathrm{ns}(b-a \mid m)(-\mathrm{ns}(a \mid m))\,Z(\mathrm{am}(a \mid m) \mid m) +$$
$$\mathrm{cs}(b-a \mid m)\,\mathrm{cs}(c-b \mid m)\,\mathrm{ns}(b \mid m)\,Z(\mathrm{am}(b \mid m) \mid m) - \mathrm{ns}(c \mid m)\,\mathrm{cs}(c-a \mid m)\,\mathrm{ns}(c-b \mid m)\,Z(\mathrm{am}(c \mid m) \mid m)))$$

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Representations through equivalent functions

With inverse function

09.29.27.0001.01

$$\mathrm{dn}\left(\mathrm{dn}^{-1}(z \mid m) \,\middle|\, m\right) == z$$

With related functions

Involving am

09.29.27.0033.01

$$\mathrm{dn}(z \mid m)^2 = 1 - m\sin^2(\mathrm{am}(z \mid m))$$

09.29.27.0002.01

$$\mathrm{dn}(z \mid m) == \sqrt{1 - m\sin^2(\mathrm{am}(z \mid m))} \;/;\; m < 1$$

Involving one other Jacobi elliptic function

Involving cd

09.29.27.0005.01

$$\mathrm{dn}(z \mid m)^2 == \frac{m-1}{m\,\mathrm{cd}(z \mid m)^2 - 1}$$

Involving cn

09.29.27.0007.01

$$\mathrm{dn}(z \mid m)^2 == 1 - m + m\,\mathrm{cn}(z \mid m)^2$$

Involving cs

09.29.27.0008.01

$$\mathrm{dn}(z \mid m)^2 == \frac{\mathrm{cs}(z \mid m)^2 - m + 1}{\mathrm{cs}(z \mid m)^2 + 1}$$

Involving dc

09.29.27.0010.01

$$\mathrm{dn}(z \mid m) == \mathrm{dc}(i\,z \mid 1-m)$$

09.29.27.0011.01

$$\mathrm{dn}(z \mid m)^2 == \frac{(m-1)\,\mathrm{dc}(z \mid m)^2}{m - \mathrm{dc}(z \mid m)^2}$$

Involving ds

09.29.27.0014.01

$$\mathrm{dn}(z \mid m)^2 == \frac{\mathrm{ds}(z \mid m)^2}{\mathrm{ds}(z \mid m)^2 + m}$$

Involving nc

09.29.27.0015.01

$$\mathrm{dn}(z \mid m)^2 == \frac{m}{\mathrm{nc}(z \mid m)^2} - m + 1$$

Involving nd

09.29.27.0016.01

$$\mathrm{dn}(z \mid m) == \frac{1}{\mathrm{nd}(z \mid m)}$$

Involving ns

09.29.27.0018.01

$$\mathrm{dn}(z \mid m)^2 == 1 - \frac{m}{\mathrm{ns}(z \mid m)^2}$$

Involving sc

09.29.27.0019.01

$$\mathrm{dn}(z \mid m)^2 == \frac{(1-m)\,\mathrm{sc}(z \mid m)^2 + 1}{\mathrm{sc}(z \mid m)^2 + 1}$$

Involving sd

09.29.27.0021.01

$$\mathrm{dn}(z \mid m)^2 == \frac{1}{m\,\mathrm{sd}(z \mid m)^2 + 1}$$

Involving sn

09.29.27.0022.01

$$\mathrm{dn}(z \mid m) == \sqrt{1-m}\ \mathrm{sn}(-i\,z + K(1-m) - i\,K(m) \mid 1-m)$$

09.29.27.0023.01

$$\mathrm{dn}(z \mid m)^2 == 1 - m\,\mathrm{sn}(z \mid m)^2$$

Involving two other Jacobi elliptic functions

Involving cd and ncnc

09.29.27.0003.01

$$\mathrm{dn}(z \mid m) == \frac{\mathrm{cn}(z \mid m)}{\mathrm{cd}(z \mid m)}$$

Involving cd and nc

09.29.27.0004.01

$$\mathrm{dn}(z \mid m) == \frac{1}{\mathrm{cd}(z \mid m)\,\mathrm{nc}(z \mid m)}$$

09.29.27.0034.01

$$\mathrm{dn}(z \mid m) = \frac{(m-1)\,\mathrm{cd}(z \mid m)\,\mathrm{nc}(z \mid m)}{m\,\mathrm{cd}(z \mid m)^2 - 1}$$

09.29.27.0035.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\left(m\,\mathrm{nc}(z \mid m)^2 - \mathrm{nc}(z \mid m)^2 - m\right)}{\mathrm{nc}(z \mid m)}$$

Involving cs and nd

09.29.27.0036.01

$$\mathrm{dn}(z \mid m) = -\frac{\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right)\mathrm{nd}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

Involving dc and cn

09.29.27.0006.01

$$\mathrm{dn}(z \mid m) == \mathrm{dc}(z \mid m)\,\mathrm{cn}(z \mid m)$$

Involving dc and nc

09.29.27.0009.01

$$\mathrm{dn}(z \mid m) == \frac{\mathrm{dc}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0037.01

$$\mathrm{dn}(z \mid m) = \frac{(m-1)\,\mathrm{dc}(z \mid m)\,\mathrm{nc}(z \mid m)}{m - \mathrm{dc}(z \mid m)^2}$$

Involving dc and nd

09.29.27.0038.01

$$\mathrm{dn}(z \mid m) = \frac{(m-1)\,\mathrm{dc}(z \mid m)^2\,\mathrm{nd}(z \mid m)}{m - \mathrm{dc}(z \mid m)^2}$$

Involving ds and ns

09.29.27.0013.01

$$\mathrm{dn}(z \mid m) == \frac{\mathrm{ds}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

Involving ds and sn

09.29.27.0012.01

$$\mathrm{dn}(z \mid m) == \mathrm{ds}(z \mid m)\,\mathrm{sn}(z \mid m)$$

Involving nc and nd

09.29.27.0039.01

$$\mathrm{dn}(z \mid m) = -\frac{\left(m\,\mathrm{nc}(z \mid m)^2 - \mathrm{nc}(z \mid m)^2 - m\right)\mathrm{nd}(z \mid m)}{\mathrm{nc}(z \mid m)^2}$$

Involving nd and ns

09.29.27.0040.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\left(\mathrm{ns}(z \mid m)^2 - m\right)}{\mathrm{ns}(z \mid m)^2}$$

Involving nd and sc

09.29.27.0041.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{nd}(z \mid m)\left(m\,\mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}{\mathrm{sc}(z \mid m)^2 + 1}$$

Involving ns and sd

09.29.27.0017.01

$$\mathrm{dn}(z \mid m) == \frac{1}{\mathrm{sd}(z \mid m)\,\mathrm{ns}(z \mid m)}$$

09.29.27.0042.01

$$\mathrm{dn}(z \mid m) = \frac{\left(\mathrm{ns}(z \mid m)^2 - m\right)\mathrm{sd}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

Involving sd and sn

09.29.27.0020.01

$$\mathrm{dn}(z \mid m) == \frac{\mathrm{sn}(z \mid m)}{\mathrm{sd}(z \mid m)}$$

Involving three other Jacobi elliptic functions

09.29.27.0043.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)^2\,\mathrm{dc}(z \mid m)}{\mathrm{cn}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 + 1\right)}$$

09.29.27.0044.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{ds}(z \mid m)}{\mathrm{cn}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 + 1\right)}$$

09.29.27.0045.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{ds}(z \mid m)^2}{\mathrm{dc}(z \mid m)\left(\mathrm{ds}(z \mid m)^2 + m - 1\right)}$$

09.29.27.0046.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right)\mathrm{nc}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0047.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)^2\,\mathrm{dc}(z \mid m)\,\mathrm{nc}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0048.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{ds}(z \mid m)\,\mathrm{nc}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0049.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\left(\mathrm{ds}(z \mid m)^2 + m - 1\right)\mathrm{nc}(z \mid m)}{\mathrm{ds}(z \mid m)^2 + m}$$

09.29.27.0050.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,\mathrm{ds}(z \mid m)^2\,\mathrm{nc}(z \mid m)}{\mathrm{cd}(z \mid m)^2\,\mathrm{ds}(z \mid m)^2 + 1}$$

09.29.27.0051.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{ds}(z \mid m)\,(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0052.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,\mathrm{ds}(z \mid m)^2\,(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0053.01

$$\mathrm{dn}(z \mid m) = \frac{m\,\mathrm{cn}(z \mid m) - m\,\mathrm{nc}(z \mid m) + \mathrm{nc}(z \mid m)}{\mathrm{dc}(z \mid m)}$$

09.29.27.0054.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)^2\,\mathrm{dc}(z \mid m)^2\,\mathrm{nd}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0055.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)^2\,\mathrm{nd}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0056.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2\,\mathrm{ds}(z \mid m)^2\,\mathrm{nd}(z \mid m)}{\mathrm{dc}(z \mid m)^2 + \mathrm{ds}(z \mid m)^2}$$

09.29.27.0057.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)^2 \, (\mathrm{nc}(z \mid m) - 1) \, (\mathrm{nc}(z \mid m) + 1) \, \mathrm{nd}(z \mid m)}{\mathrm{nc}(z \mid m)^2}$$

09.29.27.0058.01

$$\mathrm{dn}(z \mid m) = -\frac{-m \, \mathrm{cn}(z \mid m) + m \, \mathrm{dc}(z \mid m) \, \mathrm{nd}(z \mid m) - \mathrm{dc}(z \mid m) \, \mathrm{nd}(z \mid m)}{\mathrm{dc}(z \mid m)}$$

09.29.27.0059.01

$$\mathrm{dn}(z \mid m) = -\frac{-m \, \mathrm{cd}(z \mid m) + m \, \mathrm{nc}(z \mid m) \, \mathrm{nd}(z \mid m) - \mathrm{nc}(z \mid m) \, \mathrm{nd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0060.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m) \left(\mathrm{cs}(z \mid m)^2 - m + 1\right)}{\mathrm{cs}(z \mid m) \, \mathrm{ns}(z \mid m)}$$

09.29.27.0061.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m) \, \mathrm{ns}(z \mid m)}{\mathrm{cd}(z \mid m) \left(\mathrm{cs}(z \mid m)^2 + 1\right)}$$

09.29.27.0062.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m) \left(\mathrm{cs}(z \mid m)^2 - m + 1\right) \mathrm{ns}(z \mid m)}{\mathrm{cs}(z \mid m) \left(\mathrm{cs}(z \mid m)^2 + 1\right)}$$

09.29.27.0063.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m) \, \mathrm{dc}(z \mid m) \, \mathrm{ns}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0064.01

$$\mathrm{dn}(z \mid m) = -(\mathrm{cn}(z \mid m) - 1) \, (\mathrm{cn}(z \mid m) + 1) \, \mathrm{ds}(z \mid m) \, \mathrm{ns}(z \mid m)$$

09.29.27.0065.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m) \, \mathrm{ns}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0066.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2 \, \mathrm{ds}(z \mid m) \, \mathrm{ns}(z \mid m)}{\mathrm{dc}(z \mid m)^2 + \mathrm{ds}(z \mid m)^2}$$

09.29.27.0067.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m) \, (\mathrm{nc}(z \mid m) - 1) \, (\mathrm{nc}(z \mid m) + 1) \, \mathrm{ns}(z \mid m)}{\mathrm{nc}(z \mid m)^2}$$

09.29.27.0068.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m) \, (\mathrm{ns}(z \mid m) - 1) \, (\mathrm{ns}(z \mid m) + 1)}{\mathrm{cn}(z \mid m) \, \mathrm{ns}(z \mid m)^2}$$

09.29.27.0069.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m) \, \mathrm{nc}(z \mid m) \, (\mathrm{ns}(z \mid m) - 1) \, (\mathrm{ns}(z \mid m) + 1)}{\mathrm{ns}(z \mid m)^2}$$

09.29.27.0070.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2\, \mathrm{nd}(z \mid m)\, (\mathrm{ns}(z \mid m) - 1)\, (\mathrm{ns}(z \mid m) + 1)}{\mathrm{ns}(z \mid m)^2}$$

09.29.27.0071.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\, \mathrm{nc}(z \mid m)\, \left(\mathrm{ns}(z \mid m)^2 - m\right)}{\mathrm{ns}(z \mid m)^2}$$

09.29.27.0072.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\, \mathrm{ds}(z \mid m)}{\left(\mathrm{ds}(z \mid m)^2 + m - 1\right) \mathrm{sc}(z \mid m)}$$

09.29.27.0073.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\, \left(\mathrm{ds}(z \mid m)^2 + m - 1\right) \mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m)}{\mathrm{ds}(z \mid m)^2 + m}$$

09.29.27.0074.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\, (\mathrm{ns}(z \mid m) - 1)\, (\mathrm{ns}(z \mid m) + 1)\, \mathrm{sc}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.29.27.0075.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\, \left(\mathrm{ns}(z \mid m)^2 - m\right) \mathrm{sc}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.29.27.0076.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\, (\mathrm{cs}(z \mid m) - m\, \mathrm{sc}(z \mid m) + \mathrm{sc}(z \mid m))}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0077.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)}{\mathrm{cn}(z \mid m)\, \left(\mathrm{sc}(z \mid m)^2 + 1\right)}$$

09.29.27.0078.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\, \mathrm{nc}(z \mid m)}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0079.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2\, \mathrm{nd}(z \mid m)}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0080.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\, \mathrm{sc}(z \mid m)}{\mathrm{cn}(z \mid m)\, \left(\mathrm{sc}(z \mid m)^2 + 1\right)}$$

09.29.27.0081.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\, \mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m)}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0082.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\, \mathrm{ns}(z \mid m)\, \mathrm{sc}(z \mid m)}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0083.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)^2\, \mathrm{nd}(z \mid m)\, \mathrm{sc}(z \mid m)^2}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0084.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\, \mathrm{ns}(z \mid m)\, \mathrm{sc}(z \mid m)^2}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0085.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cd}(z \mid m) \left(m\, \mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0086.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\, \mathrm{nc}(z \mid m) \left(m\, \mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0087.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\, \mathrm{ns}(z \mid m)\, \mathrm{sc}(z \mid m) \left(m\, \mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0088.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m)}{\left(\mathrm{sc}(z \mid m)^2 + 1\right) \mathrm{sd}(z \mid m)}$$

09.29.27.0089.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m) \left(\mathrm{cs}(z \mid m)^2 - m + 1\right) \mathrm{nc}(z \mid m)\, \mathrm{sd}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0090.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cs}(z \mid m) \left(m\, \mathrm{nc}(z \mid m)^2 - \mathrm{nc}(z \mid m)^2 - m\right) \mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0091.01

$$\mathrm{dn}(z \mid m) = -\frac{\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right) \mathrm{sd}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.29.27.0092.01

$$\mathrm{dn}(z \mid m) = -\frac{\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right) \mathrm{ns}(z \mid m)\, \mathrm{sd}(z \mid m)}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0093.01

$$\mathrm{dn}(z \mid m) = -\frac{\left(m\, \mathrm{nc}(z \mid m)^2 - \mathrm{nc}(z \mid m)^2 - m\right) \mathrm{ns}(z \mid m)\, \mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)^2}$$

09.29.27.0094.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2\, (\mathrm{ns}(z \mid m) - 1)\, (\mathrm{ns}(z \mid m) + 1)\, \mathrm{sd}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.29.27.0095.01

$$\mathrm{dn}(z \mid m) = -\frac{\left(m\, \mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right) \mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m)}$$

09.29.27.0096.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{ns}(z \mid m)\left(m\,\mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)\mathrm{sd}(z \mid m)}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0097.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{nc}(z \mid m)\left(m\,\mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)\mathrm{sd}(z \mid m)}{\mathrm{sc}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}$$

09.29.27.0098.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,\mathrm{ns}(z \mid m) - m\,\mathrm{sd}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.29.27.0099.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,\mathrm{nc}(z \mid m)}{\mathrm{cd}(z \mid m)^2 + \mathrm{sd}(z \mid m)^2}$$

09.29.27.0100.01

$$\mathrm{dn}(z \mid m) = \frac{(m-1)\,\mathrm{nc}(z \mid m)\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{m\,\mathrm{sd}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2}$$

09.29.27.0101.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cn}(z \mid m)}{\mathrm{dc}(z \mid m)\left(m\,\mathrm{sd}(z \mid m)^2 - \mathrm{sd}(z \mid m)^2 + 1\right)}$$

09.29.27.0102.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{sc}(z \mid m)\left(m\,\mathrm{sd}(z \mid m)^2 - \mathrm{sd}(z \mid m)^2 + 1\right)}$$

09.29.27.0103.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2\,\mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{dc}(z \mid m)^2\,\mathrm{sd}(z \mid m)^2 + 1}$$

09.29.27.0104.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)}{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}$$

09.29.27.0105.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)}{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}$$

09.29.27.0106.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)}{\mathrm{nc}(z \mid m)^2\,\mathrm{sn}(z \mid m)}$$

09.29.27.0107.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,\mathrm{sc}(z \mid m)}{\left(\mathrm{sc}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}$$

09.29.27.0108.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,\mathrm{sc}(z \mid m)^2}{\left(\mathrm{sc}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}$$

09.29.27.0109.01

$$\mathrm{dn}(z \mid m) = \frac{(m-1)\,\mathrm{cd}(z \mid m)\,\mathrm{sn}(z \mid m)}{(\mathrm{cd}(z \mid m)-1)\,(\mathrm{cd}(z \mid m)+1)\,\mathrm{cs}(z \mid m)}$$

09.29.27.0110.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{dc}(z \mid m)\,(\mathrm{sn}(z \mid m)-1)\,(\mathrm{sn}(z \mid m)+1)}{\mathrm{cn}(z \mid m)}$$

09.29.27.0111.01

$$\mathrm{dn}(z \mid m) = -\mathrm{dc}(z \mid m)\,\mathrm{nc}(z \mid m)\,(\mathrm{sn}(z \mid m)-1)\,(\mathrm{sn}(z \mid m)+1)$$

09.29.27.0112.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{dc}(z \mid m)\,(\mathrm{sn}(z \mid m)-1)\,(\mathrm{sn}(z \mid m)+1)}{\mathrm{cs}(z \mid m)\,\mathrm{sn}(z \mid m)}$$

Involving four other Jacobi elliptic functions

09.29.27.0113.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{ds}(z \mid m)^2\,(\mathrm{cn}(z \mid m)-\mathrm{nc}(z \mid m))}{\mathrm{dc}(z \mid m)}$$

09.29.27.0114.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)^2\,\mathrm{nc}(z \mid m)}{\mathrm{cd}(z \mid m)\,\mathrm{ds}(z \mid m)^2+\mathrm{dc}(z \mid m)}$$

09.29.27.0115.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,\mathrm{ds}(z \mid m)^2\,\mathrm{nd}(z \mid m)}{\mathrm{dc}(z \mid m)+\mathrm{cs}(z \mid m)\,\mathrm{ds}(z \mid m)}$$

09.29.27.0116.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)^2\,(\mathrm{dc}(z \mid m)\,\mathrm{nd}(z \mid m)-\mathrm{cn}(z \mid m))}{\mathrm{dc}(z \mid m)}$$

09.29.27.0117.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)^2\,(\mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m)-\mathrm{cd}(z \mid m))}{\mathrm{nc}(z \mid m)}$$

09.29.27.0118.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,(\mathrm{ds}(z \mid m)\,\mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m)-\mathrm{cs}(z \mid m))}{\mathrm{nc}(z \mid m)}$$

09.29.27.0119.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,\mathrm{ds}(z \mid m)\,\mathrm{ns}(z \mid m)}{\mathrm{dc}(z \mid m)+\mathrm{cs}(z \mid m)\,\mathrm{ds}(z \mid m)}$$

09.29.27.0120.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{ds}(z \mid m)\,(\mathrm{cn}(z \mid m)-\mathrm{nc}(z \mid m))\,\mathrm{ns}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0121.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,(\mathrm{dc}(z \mid m)\,\mathrm{ns}(z \mid m)-\mathrm{cn}(z \mid m)\,\mathrm{ds}(z \mid m))}{\mathrm{dc}(z \mid m)}$$

09.29.27.0122.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{ds}(z \mid m)\,(\mathrm{cd}(z \mid m)\,\mathrm{ds}(z \mid m) - \mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m))}{\mathrm{nc}(z \mid m)}$$

09.29.27.0123.01

$$\mathrm{dn}(z \mid m) = -\frac{-m\,\mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m) + m\,\mathrm{nd}(z \mid m)\,\mathrm{ns}(z \mid m) - \mathrm{nd}(z \mid m)\,\mathrm{ns}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.29.27.0124.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,(\mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m) - \mathrm{sc}(z \mid m))}{\mathrm{ns}(z \mid m)}$$

09.29.27.0125.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{ds}(z \mid m)\,(\mathrm{cn}(z \mid m) - \mathrm{nc}(z \mid m))}{\mathrm{sc}(z \mid m)}$$

09.29.27.0126.01

$$\mathrm{dn}(z \mid m) = \frac{m\,\mathrm{cn}(z \mid m) - m\,\mathrm{nc}(z \mid m) + \mathrm{nc}(z \mid m)}{\mathrm{ds}(z \mid m)\,\mathrm{sc}(z \mid m)}$$

09.29.27.0127.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)}{\mathrm{cn}(z \mid m)\,(\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m))}$$

09.29.27.0128.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)}{\mathrm{cn}(z \mid m)\,(\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m))}$$

09.29.27.0129.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)\,\mathrm{nc}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0130.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,\mathrm{nc}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0131.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)^2\,\mathrm{nd}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0132.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,\mathrm{ns}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0133.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)^2\,\mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0134.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,\mathrm{ns}(z \mid m)\,\mathrm{sc}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0135.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,\mathrm{nc}(z \mid m)}{\mathrm{cd}(z \mid m)\,\mathrm{ds}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0136.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,(\mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m) - m\,\mathrm{sc}(z \mid m))}{\mathrm{ns}(z \mid m)}$$

09.29.27.0137.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,(\mathrm{cs}(z \mid m) - m\,\mathrm{sc}(z \mid m) + \mathrm{sc}(z \mid m))}{\mathrm{ns}(z \mid m)}$$

09.29.27.0138.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,\mathrm{ns}(z \mid m)\,(\mathrm{cs}(z \mid m) - m\,\mathrm{sc}(z \mid m) + \mathrm{sc}(z \mid m))}{\mathrm{cs}(z \mid m)^2 + 1}$$

09.29.27.0139.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,\mathrm{nc}(z \mid m)\,(\mathrm{cs}(z \mid m) - m\,\mathrm{sc}(z \mid m) + \mathrm{sc}(z \mid m))}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0140.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,\mathrm{ns}(z \mid m) - m\,\mathrm{cd}(z \mid m)\,\mathrm{sc}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.29.27.0141.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,(\mathrm{ns}(z \mid m) - m\,\mathrm{nc}(z \mid m)\,\mathrm{sc}(z \mid m) + \mathrm{nc}(z \mid m)\,\mathrm{sc}(z \mid m))}{\mathrm{ns}(z \mid m) + \mathrm{nc}(z \mid m)\,\mathrm{sc}(z \mid m)}$$

09.29.27.0142.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,(\mathrm{ds}(z \mid m)\,\mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m) - \mathrm{cn}(z \mid m))}{\mathrm{sc}(z \mid m)}$$

09.29.27.0143.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,(\mathrm{ns}(z \mid m)\,\mathrm{sc}(z \mid m) - \mathrm{cn}(z \mid m))}{\mathrm{sc}(z \mid m)}$$

09.29.27.0144.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\,\mathrm{ns}(z \mid m)\left(m\,\mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0145.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2\,(\mathrm{nd}(z \mid m)\,\mathrm{ns}(z \mid m) - \mathrm{sd}(z \mid m))}{\mathrm{ns}(z \mid m)}$$

09.29.27.0146.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cn}(z \mid m) - \mathrm{nc}(z \mid m)}{\mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)^2}$$

09.29.27.0147.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{cn}(z \mid m)}{\mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)^2}$$

09.29.27.0148.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cn}(z \mid m) - \mathrm{nc}(z \mid m)}{\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0149.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ns}(z \mid m)\,\mathrm{sc}(z \mid m) - \mathrm{cn}(z \mid m)}{\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0150.01

$$\mathrm{dn}(z \mid m) = -\frac{\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right)\mathrm{nc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0151.01

$$\mathrm{dn}(z \mid m) = \frac{(\mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m) - m\,\mathrm{sc}(z \mid m))\,\mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0152.01

$$\mathrm{dn}(z \mid m) = \frac{(\mathrm{cs}(z \mid m) - m\,\mathrm{sc}(z \mid m) + \mathrm{sc}(z \mid m))\,\mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0153.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ns}(z \mid m)\,(\mathrm{cs}(z \mid m) - m\,\mathrm{sc}(z \mid m) + \mathrm{sc}(z \mid m))\,\mathrm{sd}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)}$$

09.29.27.0154.01

$$\mathrm{dn}(z \mid m) = \frac{(\mathrm{ns}(z \mid m) - m\,\mathrm{nc}(z \mid m)\,\mathrm{sc}(z \mid m) + \mathrm{nc}(z \mid m)\,\mathrm{sc}(z \mid m))\,\mathrm{sd}(z \mid m)}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0155.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cn}(z \mid m)}{\mathrm{sc}(z \mid m)\,(\mathrm{ds}(z \mid m) + m\,\mathrm{sd}(z \mid m) - \mathrm{sd}(z \mid m))}$$

09.29.27.0156.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m) - \mathrm{cn}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)^2}$$

09.29.27.0157.01

$$\mathrm{dn}(z \mid m) = -\frac{m\,\mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m) - m\,\mathrm{cs}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0158.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)^2\,\mathrm{nd}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0159.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,\mathrm{ns}(z \mid m)}{\mathrm{cs}(z \mid m) + \mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0160.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{dc}(z \mid m)\,(\mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m))}{\mathrm{ns}(z \mid m)}$$

09.29.27.0161.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,\mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{cn}(z \mid m)}{\mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)^2}$$

09.29.27.0162.01

$$\mathrm{dn}(z \mid m) = -\frac{-m\,\mathrm{cd}(z \mid m) + m\,\mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0163.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nc}(z \mid m)}{\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0164.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m) - m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0165.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m) - m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.29.27.0166.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,(\mathrm{cd}(z \mid m) - m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m))}{\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0167.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{nc}(z \mid m)\,(-\mathrm{cd}(z \mid m) + m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m))}{\mathrm{sc}(z \mid m)^2 + 1}$$

09.29.27.0168.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cn}(z \mid m)}{\mathrm{dc}(z \mid m) + m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0169.01

$$\mathrm{dn}(z \mid m) = -\frac{1}{\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}\left(-\mathrm{cn}(z \mid m)\,\mathrm{nd}(z \mid m)^2 + m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)\,\mathrm{nd}(z \mid m) + \mathrm{cn}(z \mid m)\right)$$

09.29.27.0170.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nc}(z \mid m)}{\mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)^2 + \mathrm{cd}(z \mid m)}$$

09.29.27.0171.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{nc}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m)}$$

09.29.27.0172.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,(\mathrm{cs}(z \mid m)\,\mathrm{nc}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.29.27.0173.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,\mathrm{ds}(z \mid m)\,\mathrm{nc}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)^2\,\mathrm{ds}(z \mid m)}$$

09.29.27.0174.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,(\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.29.27.0175.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)^2\,\mathrm{ds}(z \mid m)}$$

09.29.27.0176.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2 \left(\mathrm{ds}(z \mid m)\, \mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m)\right)}{\mathrm{ds}(z \mid m)}$$

09.29.27.0177.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)\, \mathrm{cs}(z \mid m)}$$

09.29.27.0178.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m) \left(\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m)\right)}{\mathrm{cs}(z \mid m)}$$

09.29.27.0179.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)^2\, \mathrm{ds}(z \mid m)}$$

09.29.27.0180.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)^2 \left(\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m)\right)}{\mathrm{ds}(z \mid m)}$$

09.29.27.0181.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\, \mathrm{nc}(z \mid m) \left(\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m)\right)}{\mathrm{ns}(z \mid m)}$$

09.29.27.0182.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)}{\left(\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)\right) \mathrm{sn}(z \mid m)}$$

09.29.27.0183.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\, \mathrm{sc}(z \mid m)}{\left(\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m)\right) \mathrm{sn}(z \mid m)}$$

09.29.27.0184.01

$$\mathrm{dn}(z \mid m) = -\frac{m\, \mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m) - \mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m) - m\, \mathrm{sn}(z \mid m)}{\mathrm{dc}(z \mid m)\, \mathrm{sc}(z \mid m)}$$

09.29.27.0185.01

$$\mathrm{dn}(z \mid m) = \frac{1}{\mathrm{ds}(z \mid m)} \left(\mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m)\, \mathrm{ds}(z \mid m)^2 + m\, \mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m) - \mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m) - m\, \mathrm{sn}(z \mid m)\right)$$

09.29.27.0186.01

$$\mathrm{dn}(z \mid m) = -\frac{m\, \mathrm{dc}(z \mid m)\, \mathrm{nd}(z \mid m)\, \mathrm{sc}(z \mid m) - \mathrm{dc}(z \mid m)\, \mathrm{nd}(z \mid m)\, \mathrm{sc}(z \mid m) - m\, \mathrm{sn}(z \mid m)}{\mathrm{dc}(z \mid m)\, \mathrm{sc}(z \mid m)}$$

09.29.27.0187.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{cs}(z \mid m)\, \mathrm{nd}(z \mid m)}{\mathrm{cd}(z \mid m)^2\, \mathrm{cs}(z \mid m)}$$

09.29.27.0188.01

$$\mathrm{dn}(z \mid m) = -\frac{m\, \mathrm{dc}(z \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{ds}(z \mid m)\, \mathrm{nc}(z \mid m)}{\mathrm{dc}(z \mid m)\, \mathrm{ds}(z \mid m)}$$

09.29.27.0189.01

$$\mathrm{dn}(z \mid m) = -\frac{-\mathrm{cn}(z \mid m)\, \mathrm{ds}(z \mid m) + m\, \mathrm{dc}(z \mid m)\, \mathrm{sn}(z \mid m) - \mathrm{dc}(z \mid m)\, \mathrm{sn}(z \mid m)}{\mathrm{dc}(z \mid m)\, \mathrm{ds}(z \mid m)}$$

09.29.27.0190.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cn}(z \mid m) - m\,\mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\mathrm{dc}(z \mid m)}$$

09.29.27.0191.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,(\mathrm{cn}(z \mid m) - m\,\mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m))}{\mathrm{cn}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m)}$$

09.29.27.0192.01

$$\mathrm{dn}(z \mid m) = -\frac{-\mathrm{cn}(z \mid m) + m\,\mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m) - \mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\mathrm{ds}(z \mid m)\,\mathrm{sc}(z \mid m)}$$

Involving five other Jacobi elliptic functions

09.29.27.0193.01

$$\mathrm{dn}(z \mid m) = -\frac{\mathrm{cn}(z \mid m) - \mathrm{dc}(z \mid m)\,\mathrm{nd}(z \mid m)}{\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0194.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,\mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m) - \mathrm{cn}(z \mid m)}{\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.29.27.0195.01

$$\mathrm{dn}(z \mid m) = -\frac{1}{\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}(\mathrm{cn}(z \mid m) - \mathrm{cd}(z \mid m)\,\mathrm{nd}(z \mid m) + m\,\mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m))$$

09.29.27.0196.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m)}$$

09.29.27.0197.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m)}$$

09.29.27.0198.01

$$\mathrm{dn}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\,(\mathrm{ds}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.29.27.0199.01

$$\mathrm{dn}(z \mid m) = \frac{1}{\mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m)}(\mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m)\,\mathrm{nd}(z \mid m) - m\,\mathrm{cd}(z \mid m)\,\mathrm{sc}(z \mid m)\,\mathrm{nd}(z \mid m) + \mathrm{cd}(z \mid m)\,\mathrm{sc}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m))$$

Involving Weierstrass functions

09.29.27.0024.01

$$\mathrm{dn}(z \mid m) == \frac{\sigma_2\left(\frac{z}{\sqrt{e_1 - e_3}}; g_2, g_3\right)}{\sigma_3\left(\frac{z}{\sqrt{e_1 - e_3}}; g_2, g_3\right)} /;$$

$$\{\omega_1, \omega_2, \omega_3\} == \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m == \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n == \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

09.29.27.0025.01

$$\mathrm{dn}(z \mid m)^2 == \frac{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_2}{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_3} /;$$

$$\{\omega_1, \omega_2, \omega_3\} == \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m == \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n == \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

09.29.27.0026.01

$$\mathrm{dn}\left(z \,\middle|\, \frac{e_2 - e_3}{e_1 - e_3}\right) == \sqrt{\frac{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_2}{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_3}} /;$$

$$\{\omega_1, \omega_2, \omega_3\} == \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge e_n == \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

Involving theta functions

09.29.27.0027.01

$$\mathrm{dn}(z \mid m) == \sqrt[4]{1 - m}\, \frac{\vartheta_3\left(\frac{\pi z}{2K(m)}, q(m)\right)}{\vartheta_4\left(\frac{\pi z}{2K(m)}, q(m)\right)}$$

09.29.27.0028.01

$$\mathrm{dn}(z \mid m) == \frac{\vartheta_4(0, q(m))}{\vartheta_3(0, q(m))} \frac{\vartheta_3\left(\frac{z}{\vartheta_3(0,q(m))^2}, q(m)\right)}{\vartheta_4\left(\frac{z}{\vartheta_3(0,q(m))^2}, q(m)\right)}$$

09.29.27.0029.01

$$\mathrm{dn}(z \mid m) == \frac{\vartheta_d(z \mid m)}{\vartheta_n(z \mid m)}$$

09.29.27.0030.01

$$\mathrm{Z}(\mathrm{am}(u \mid m) \mid m) + \mathrm{Z}(\mathrm{am}(v \mid m) \mid m) - \mathrm{Z}(\mathrm{am}(u + v \mid m) \mid m) == m\, \mathrm{sn}(v \mid m)\, \mathrm{sn}(u \mid m)\, \mathrm{sn}(u + v \mid m)$$

09.29.27.0031.01

$$\mathrm{Z}(\mathrm{am}(u \mid m) \mid m) + \mathrm{Z}(\mathrm{am}(v \mid m) \mid m) - \mathrm{Z}(\mathrm{am}(u + v \mid m) \mid m) == \frac{(m\, \mathrm{sn}(v \mid m))\, (\mathrm{cn}(v \mid m) - \mathrm{cn}(u \mid m)\, \mathrm{cn}(u + v \mid m))}{\mathrm{dn}(v \mid m)}$$

09.29.27.0032.01

$$\mathrm{Z}(\mathrm{am}(u \mid m) \mid m) + \mathrm{Z}(\mathrm{am}(v \mid m) \mid m) - \mathrm{Z}(\mathrm{am}(u + v \mid m) \mid m) == \frac{\mathrm{sn}(v \mid m)\, (\mathrm{dn}(v \mid m) - \mathrm{dn}(u \mid m)\, \mathrm{dn}(u + v \mid m))}{\mathrm{cn}(v \mid m)}$$

Zeros

09.29.30.0001.01

$$\mathrm{dn}((2r + 1)\, K(m) + i\, (2s + 1)\, K(1 - m) \mid m) == 0 /; \{r, s\} \in \mathbb{Z}$$

Theorems

The zero modes of the periodic supersymmetric partner potentials

The wave functions $\psi_0^{(\pm)}(z) = \mathrm{dn}(z \mid m)^{\mp 1}$ are the zero modes of the periodic supersymmetric partner potentials $V_+(z) = 2 - m + 2\,(m-1) \Big/ \mathrm{dn}(z \mid m)^2$ and $V_-(z) = 2 - m - 2\,\mathrm{dn}(z \mid m)^2$.

A periodic solution of the nonlinear Schrödinger equation

The function $\psi(x, t) == r \exp\left(i \left(p\,x - \left(p^2 - \left(2 - k^2\right) r^2\right) t\right)\right) \mathrm{dn}\left(r\,x - 2\,p\,r\,t \mid k^2\right)$ is a periodic solution of the non-linear Schrödinger equation $i \frac{\partial \psi(x,t)}{\partial t} + \frac{\partial^2 \psi(x,t)}{\partial x^2} + 2\,\psi(x, t)^2\,\overline{\psi(x, t)} == 0$.

The time dependence of the x- and y-coordinates of a mathematical pendulum in a gravitational field

The time dependence of the x - and y-coordinates of a mathematical pendulum of length l in a gravitational field with acceleration g and maximal elongation angle α is given by

$$x(t) == 2\,l \sin\left(\frac{\alpha}{2}\right) \mathrm{sn}\left(\sqrt{\frac{g}{l}}\ t \,\middle|\, \sin^2\left(\frac{\alpha}{2}\right)\right) \mathrm{dn}\left(\sqrt{\frac{g}{l}}\ t \,\middle|\, \sin^2\left(\frac{\alpha}{2}\right)\right),\ y(t) == l\left(2\,\mathrm{dn}^2\left(\sqrt{\frac{g}{l}}\ t \,\middle|\, \sin^2\left(\frac{\alpha}{2}\right)\right) - 1\right).$$

History

– C. G. J. Jacobi (1827)
– N. H. Abel (1827)
– C. Gudermann (1838) introduced the notations dn

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