JacobiNC

Notations

Traditional name

Jacobi elliptic function nc

Traditional notation

$\mathrm{nc}(z \mid m)$

Mathematica StandardForm notation

`JacobiNC[z, m]`

Primary definition

09.31.02.0001.01

$$\mathrm{nc}(z \mid m) = \frac{1}{\mathrm{cn}(z \mid m)}$$

Specific values

Specialized values

For fixed z

Case $m = 0$

09.31.03.0001.01

$$\mathrm{nc}(z \mid 0) = \sec(z)$$

09.31.03.0002.01

$$\mathrm{nc}\left(z + \frac{\pi}{2} \middle| 0\right) = -\csc(z)$$

09.31.03.0026.01

$$\mathrm{nc}\left(z + \frac{\pi k}{2} \middle| 0\right) = \sec\left(z + \frac{\pi k}{2}\right) /; k \in \mathbb{Z}$$

Case $m = 1$

09.31.03.0003.01

$$\mathrm{nc}(z \mid 1) = \cosh(z)$$

09.31.03.0004.01

$$\mathrm{nc}\left(z+\frac{\pi i}{2}\,\middle|\,1\right) = i\sinh(z)$$

09.31.03.0027.01

$$\mathrm{nc}\left(z+\frac{i\pi k}{2}\,\middle|\,1\right) = \cosh\left(z+\frac{i\pi k}{2}\right) /; k\in\mathbb{Z}$$

For fixed *m*

Values at quarter-period points in the fundamental period parallelogram

09.31.03.0005.01

$$\mathrm{nc}(0 \mid m) = 1$$

09.31.03.0006.01

$$\mathrm{nc}(K(m) \mid m) = \tilde{\infty}$$

09.31.03.0007.01

$$\mathrm{nc}(2K(m) \mid m) = -1$$

09.31.03.0008.01

$$\mathrm{nc}(3K(m) \mid m) = \tilde{\infty}$$

09.31.03.0009.01

$$\mathrm{nc}(4K(m) \mid m) = 1$$

09.31.03.0010.01

$$\mathrm{nc}(i K(1-m) \mid m) = 0$$

09.31.03.0011.01

$$\mathrm{nc}(2 i K(1-m) \mid m) = -1$$

09.31.03.0012.01

$$\mathrm{nc}(3 i K(1-m) \mid m) = 0$$

09.31.03.0013.01

$$\mathrm{nc}(4 i K(1-m) \mid m) = 1$$

09.31.03.0014.01

$$\mathrm{nc}(K(m)+i K(1-m) \mid m) = i\frac{\sqrt{m}}{\sqrt{1-m}}$$

09.31.03.0015.01

$$\mathrm{nc}(2K(m)+i K(1-m) \mid m) = 0$$

09.31.03.0016.01

$$\mathrm{nc}(3K(m)+i K(1-m) \mid m) = -i\frac{\sqrt{m}}{\sqrt{1-m}}$$

09.31.03.0017.01

$$\mathrm{nc}(4K(m)+i K(1-m) \mid m) = 0$$

09.31.03.0018.01

$$\mathrm{nc}(K(m)+2 i K(1-m) \mid m) = \tilde{\infty}$$

09.31.03.0019.01

$\operatorname{nc}(2\,K(m) + 2\,i\,K(1-m) \mid m) == 1$

09.31.03.0020.01

$\operatorname{nc}(3\,K(m) + 2\,i\,K(1-m) \mid m) == \tilde{\infty}$

09.31.03.0021.01

$\operatorname{nc}(4\,K(m) + 2\,i\,K(1-m) \mid m) == -1$

09.31.03.0022.01

$\operatorname{nc}((2\,r+1)\,K(m) + 2\,i\,s\,K(1-m) \mid m) == \tilde{\infty} \;/;\; \{r, s\} \in \mathbb{Z}$

Values at half-quarter-period points

09.31.03.0023.01

$$\operatorname{nc}\left(\frac{K(m)}{2} \,\middle|\, m\right) == \frac{\sqrt{1+\sqrt{1-m}}}{\sqrt[4]{1-m}}$$

09.31.03.0024.01

$$\operatorname{nc}\left(\frac{i\,K(1-m)}{2} \,\middle|\, m\right) == \frac{\sqrt[4]{m}}{\sqrt{1+\sqrt{m}}}$$

09.31.03.0025.01

$$\operatorname{nc}\left(\frac{K(m)}{2} + \frac{i\,K(1-m)}{2} \,\middle|\, m\right) == \frac{\sqrt[4]{m}}{\sqrt[4]{1-m}}\,\frac{1+i}{\sqrt{2}}$$

General characteristics

Domain and analyticity

$\operatorname{nc}(z \mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.31.04.0001.01

$(z * m) \longrightarrow \operatorname{nc}(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

$\operatorname{nc}(z \mid m)$ is an even function with respect to z.

09.31.04.0002.01

$\operatorname{nc}(-z \mid m) == \operatorname{nc}(z \mid m)$

Mirror symmetry

09.31.04.0003.01

$\operatorname{nc}(\bar{z} \mid \bar{m}) == \overline{\operatorname{nc}(z \mid m)}$

Periodicity

$\operatorname{nc}(z \mid m)$ is a doubly periodic function with respect to z with periods $4\,i\,K(1-m)$ and $4\,K(m)$.

09.31.04.0004.01

$\text{nc}(z + 2\,K(m) \mid m) = -\text{nc}(z \mid m)$

09.31.04.0005.01

$\text{nc}(z + 4\,K(m) \mid m) = \text{nc}(z \mid m)$

09.31.04.0006.01

$\text{nc}(z + 2\,i\,K(1-m) \mid m) = -\text{nc}(z \mid m)$

09.31.04.0007.01

$\text{nc}(z + 4\,i\,K(1-m) \mid m) = \text{nc}(z \mid m)$

09.31.04.0008.01

$\text{nc}(z + 2\,K(m) + 2\,i\,K(1-m) \mid m) = \text{nc}(z \mid m)$

09.31.04.0009.01

$\text{nc}(z + 2\,i\,s\,K(1-m) + 2\,r\,K(m) \mid m) = (-1)^{r+s}\,\text{nc}(z \mid m) \;/;\; \{r, s\} \in \mathbb{Z}$

Poles and essential singularities

With respect to *z*

For fixed m, the function $\text{nc}(z \mid m)$ has an infinite set of singular points:

a) $z = (2\,r+1)\,K(m) + 2\,s\,i\,K(1-m)$, $\{r, s\} \in \mathbb{Z}$, are the simple poles with residues $\frac{(-1)^{r+s-1}}{\sqrt{1-m}}$;

b) $z = \tilde{\infty}$ is an essential singular point.

09.31.04.0010.01

$\mathcal{S}ing_z(\text{nc}(z \mid m)) = \{\{\{2\,s\,i\,K(1-m) + (2\,r+1)\,K(m), 1\} \;/;\; \{r, s\} \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}\}$

09.31.04.0011.01

$$\text{res}_z(\text{nc}(z \mid m))\,(2\,s\,i\,K(1-m) + (2\,r+1)\,K(m)) = \frac{(-1)^{r+s-1}}{\sqrt{1-m}} \;/;\; \{r, s\} \in \mathbb{Z}$$

Branch points

With respect to *m*

For fixed z, the function $\text{nc}(z \mid m)$ is a meromorphic function in m that has no branch points.

09.31.04.0014.01

$\mathcal{BP}_m(\text{nc}(z \mid m)) = \{\}$

P. Walker

With respect to *z*

For fixed m, the function $\text{nc}(z \mid m)$ does not have branch points.

09.31.04.0012.01

$\mathcal{BP}_z(\text{nc}(z \mid m)) = \{\}$

Branch cuts

With respect to *m*

For fixed z, the function $\text{nc}(z \mid m)$ is a meromorphic function in m that has no branch cuts.

09.31.04.0015.01

$$\mathcal{BC}_m(\text{nc}(z \mid m)) = \{\}$$

P. Walker

With respect to *z*

For fixed m, the function $\text{nc}(z \mid m)$ does not have branch cuts.

09.31.04.0013.01

$$\mathcal{BC}_z(\text{nc}(z \mid m)) = \{\}$$

Series representations

Generalized power series

Expansions at $z = 0$

09.31.06.0005.01

$$\text{nc}(z \mid m) \propto 1 + \frac{z^2}{2} + \frac{1}{24}(5 - 4m) z^4 + \ldots /; (z \to 0)$$

09.31.06.0001.02

$$\text{nc}(z \mid m) \propto 1 + \frac{z^2}{2} + \frac{1}{24}(5 - 4m) z^4 + \frac{1}{720}\left(61 - 76m + 16m^2\right) z^6 + \frac{\left(1385 - 2424m + 1104m^2 - 64m^3\right) z^8}{40320} + \frac{\left(50521 - 113672m + 79728m^2 - 16832m^3 + 256m^4\right) z^{10}}{3628800} + O\left(z^{12}\right)$$

09.31.06.0006.01

$$\text{nc}(z \mid m) = \sum_{k=0}^{\infty} (k+1) \sum_{r=0}^{k} \frac{(-1)^r}{r+1} \binom{k}{r} p_{r,k} z^{2k} /; p_{j,0} = 1 \bigwedge p_{j,k} = \frac{1}{k} \sum_{i=1}^{k} \frac{(j i + i - k)(-1)^i \text{cn}_i(m) p_{j,k-i}}{(2i)!} \bigwedge k \in \mathbb{N}^+ \bigwedge \text{sn}_0(m) = 1 \bigwedge \text{sn}_n(m) = \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{2n}{2j} \text{cn}_j(m) \text{dn}_k(m) \delta_{j+k-n} \bigwedge \text{cn}_0(m) = 1 \bigwedge \text{cn}_n(m) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \text{sn}_j(m) \text{dn}_k(m) \delta_{j+k-n+1} \bigwedge \text{dn}_0(m) = 1 \bigwedge \text{dn}_n(m) = m \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \text{sn}_j(m) \text{cn}_k(m) \delta_{j+k-n+1}$$

09.31.06.0007.01

$$\text{nc}(z \mid m) \propto 1 + O\left(z^2\right)$$

Expansions at $z = (2r+1) K(m) + 2 i s K(1-m)$

09.31.06.0008.01

$$\text{nc}(z \mid m) \propto \frac{(-1)^{r+s-1}}{\sqrt{1-m}} \left(\frac{1}{z - z_0} + \frac{1}{6}(1 - 2m)(z - z_0) + \frac{1}{360}\left(-8m^2 + 8m + 7\right)(z - z_0)^3 + \ldots \right) /; (z \to z_0) \land z_0 = (2r+1) K(m) + 2 i s K(1-m) \land r \in \mathbb{Z} \land s \in \mathbb{Z}$$

09.31.06.0009.01

$$\operatorname{nc}(z \mid m) = \frac{(-1)^{r+s-1}}{\sqrt{1-m}} \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{(j+1)\,(-1)^{k-j}\,\operatorname{dn}_{k-j}(m)}{(2k-2j)!} \sum_{r=0}^{j} \frac{(-1)^r}{r+1} \binom{j}{r} q_{r,j}\,(z-z_0)^{2k-1} \text{ /; }$$

$$z_0 = (2r+1)\,K(m) + 2\,i\,s\,K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z} \bigwedge q_{j,0} = 1 \bigwedge q_{j,k} = \frac{1}{k} \sum_{i=1}^{k} \frac{(j\,i+i-k)\,(-1)^i\,\operatorname{sn}_i(m)\,q_{j,k-i}}{(2i+1)!} \bigwedge$$

$$k \in \mathbb{N}^+ \bigwedge \operatorname{sn}_0(m) = 1 \bigwedge \operatorname{sn}_n(m) = \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{2n}{2j} \operatorname{cn}_j(m)\,\operatorname{dn}_k(m)\,\delta_{j+k-n} \bigwedge \operatorname{cn}_0(m) = 1 \bigwedge$$

$$\operatorname{cn}_n(m) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \operatorname{sn}_j(m)\,\operatorname{dn}_k(m)\,\delta_{j+k-n+1} \bigwedge \operatorname{dn}_0(m) = 1 \bigwedge \operatorname{dn}_n(m) = m \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \operatorname{sn}_j(m)\,\operatorname{cn}_k(m)\,\delta_{j+k-n+1}$$

09.31.06.0010.01

$$\operatorname{nc}(z \mid m) \propto \frac{(-1)^{r+s-1}}{\sqrt{1-m}\,(z-z_0)} \left(1 + O\!\left((z-z_0)^2\right)\right) \text{ /; } z_0 = (2r+1)\,K(m) + 2\,i\,s\,K(1-m) \land r \in \mathbb{Z} \land s \in \mathbb{Z}$$

Expansions at *m* == 0

09.31.06.0011.01

$$\operatorname{nc}(z \mid m) \propto \sec(z) + \frac{1}{4}(\sin(z) - z\sec(z))\tan(z)\,m -$$
$$\frac{1}{512}\left(8\cos(2z)\,z^2 - 24z^2 + 44\sin(2z)\,z + 4\sin(4z)\,z + 11\cos(4z) - 11\right)\sec^3(z)\,m^2 + \ldots \text{ /; } (m \to 1)$$

09.31.06.0012.01

$$\operatorname{nc}(z \mid m) \propto \sec(z) + \frac{1}{4}(\sin(z) - z\sec(z))\tan(z)\,m -$$
$$\frac{1}{512}\left(8\cos(2z)\,z^2 - 24z^2 + 44\sin(2z)\,z + 4\sin(4z)\,z + 11\cos(4z) - 11\right)\sec^3(z)\,m^2 +$$
$$\frac{1}{49\,152}\left(15\left(128z^2+55\right)\cos(z) - 3\left(72z^2+139\right)\cos(3z) + 3\left(8z^2-137\right)\cos(5z) + \right.$$
$$\left. 3\cos(7z) - 16z\left(46z^2+105\right)\sin(z) + 16z\left(2z^2-117\right)\sin(3z) - 192z\sin(5z)\right)\sec^4(z)\,m^3 +$$
$$\frac{1}{1\,572\,864}\left(3680z^4 + 28\,680z^2 - 4\left(3304z^2+12\,069\right)\sin(2z)\,z + 4\left(184z^2-7065\right)\sin(4z)\,z + \right.$$
$$4\left(8z^2-675\right)\sin(6z)\,z + 36\sin(8z)\,z - \left(2432z^4 - 26\,472z^2 - 5361\right)\cos(2z) +$$
$$\left. 2\left(16z^4 - 852z^2 - 5529\right)\cos(4z) + 3\left(168z^2-1787\right)\cos(6z) + 84\cos(8z) + 10\,974\right)\sec^5(z)\,m^4 -$$
$$\frac{1}{251\,658\,240}\left(-40\left(14\,644z^4 + 156\,513z^2 + 47\,007\right)\cos(z) + 30\left(9712z^4 - 63\,780z^2 + 13\,347\right)\cos(3z) - \right.$$
$$5\left(544z^4 - 10\,008z^2 - 226\,323\right)\cos(5z) + 10\left(16z^4 - 3720z^2 + 35\,691\right)\cos(7z) + 30\,(6z-17)\,(6z+17)\cos(9z) +$$
$$15\cos(11z) + 8z\left(26\,912z^4 + 134\,240z^2 + 423\,615\right)\sin(z) - 48z\left(632z^4 - 20\,990z^2 - 111\,945\right)\sin(3z) +$$
$$\left. 16z\left(8z^4 - 4410z^2 + 135\,615\right)\sin(5z) - 20z\left(208z^2 - 8973\right)\sin(7z) - 5940z\sin(9z)\right)\sec^6(z)\,m^5 -$$

$$\frac{1}{251\,658\,240}\left(-40\left(14\,644z^4 + 156\,513z^2 + 47\,007\right)\cos(z) + 30\left(9712z^4 - 63\,780z^2 + 13\,347\right)\cos(3z) - \right.$$
$$5\left(544z^4 - 10\,008z^2 - 226\,323\right)\cos(5z) + 10\left(16z^4 - 3720z^2 + 35\,691\right)\cos(7z) + 30\,(6z-17)\,(6z+17)\cos(9z) +$$
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$$\left. 16z\left(8z^4 - 4410z^2 + 135\,615\right)\sin(5z) - 20z\left(208z^2 - 8973\right)\sin(7z) - 5940z\sin(9z)\right)\sec^6(z)\,m^5 -$$

$$\frac{1}{24\,159\,191\,040}\Big(-3\,014\,144\,z^6-32\,743\,200\,z^4-280\,828\,800\,z^2+24\left(670\,704\,z^4+4\,085\,480\,z^2+15\,703\,545\right)\sin(2\,z)\,z-60\left(43\,712\,z^4-733\,888\,z^2-5\,397\,135\right)\sin(4\,z)\,z+60\left(224\,z^4-59\,288\,z^2+1\,666\,755\right)\sin(6\,z)\,z+24\left(16\,z^4-6480\,z^2+283\,305\right)\sin(8\,z)\,z+180\left(72\,z^2-2269\right)\sin(10\,z)\,z+900\sin(12\,z)\,z+2\left(1\,349\,504\,z^6-8\,115\,360\,z^4-183\,965\,400\,z^2-31\,171\,455\right)\cos(2\,z)-4\left(46\,208\,z^6-4\,100\,160\,z^4+21\,346\,920\,z^2-16\,308\,855\right)\cos(4\,z)+\left(256\,z^6-96\,960\,z^4+308\,160\,z^2+62\,820\,045\right)\cos(6\,z)+30\left(496\,z^4-42\,768\,z^2+479\,367\right)\cos(8\,z)+45\left(2736\,z^2-10\,603\right)\cos(10\,z)+1980\cos(12\,z)-79\,618\,410\Big)\sec^7(z)\,m^6-$$

$$\frac{1}{5\,411\,658\,792\,960}\Big(-63\left(12\,580\,864\,z^6+155\,912\,640\,z^4+1\,551\,549\,600\,z^2+353\,219\,675\right)\cos(z)+140\left(4\,273\,024\,z^6+302\,160\,z^4-340\,916\,742\,z^2+2\,529\,567\right)\cos(3\,z)-70\left(628\,480\,z^6-28\,156\,560\,z^4+127\,545\,876\,z^2-184\,163\,139\right)\cos(5\,z)+35\left(1280\,z^6-369\,120\,z^4-100\,368\,z^2+220\,288\,689\right)\cos(7\,z)-7\left(256\,z^6-18\,720\,z^4+9\,236\,880\,z^2-193\,488\,075\right)\cos(9\,z)-630\left(432\,z^4-34\,372\,z^2+91\,485\right)\cos(11\,z)-2520\left(25\,z^2-157\right)\cos(13\,z)-315\cos(15\,z)+8\,z\left(33\,244\,544\,z^6+298\,278\,624\,z^4+1\,336\,586\,160\,z^2+4\,800\,015\,045\right)\sin(z)-12\,z\left(5\,176\,064\,z^6-166\,506\,816\,z^4-1\,293\,092\,640\,z^2-5\,942\,460\,615\right)\sin(3\,z)+8\,z\left(278\,912\,z^6-48\,239\,520\,z^4+558\,161\,940\,z^2+5\,371\,069\,725\right)\sin(5\,z)-4\,z\left(256\,z^6-581\,952\,z^4+90\,658\,680\,z^2-2\,669\,422\,455\right)\sin(7\,z)+252\,z\left(384\,z^4+1560\,z^2+2\,230\,115\right)\sin(9\,z)+5040\,z\left(774\,z^2-11\,159\right)\sin(11\,z)+308\,700\,z\sin(13\,z)\Big)\sec^8(z)\,m^7+$$

$$\frac{1}{173\,173\,081\,374\,720}\Big(1\,196\,803\,584\,z^8+18\,723\,140\,352\,z^6+162\,990\,848\,160\,z^4+1\,474\,381\,190\,520\,z^2-84\left(94\,641\,152\,z^6+980\,135\,424\,z^4+4\,862\,912\,920\,z^2+19\,212\,070\,275\right)\sin(2\,z)\,z+36\left(64\,872\,960\,z^6-839\,939\,296\,z^4-8\,695\,009\,680\,z^2-46\,600\,990\,065\right)\sin(4\,z)\,z-216\left(402\,560\,z^6-33\,543\,216\,z^4+306\,998\,720\,z^2+3\,625\,985\,335\right)\sin(6\,z)\,z+4\left(12\,032\,z^6-6\,636\,000\,z^4+1\,063\,802\,040\,z^2-40\,252\,129\,785\right)\sin(8\,z)\,z+8\left(128\,z^6+437\,136\,z^4-55\,986\,000\,z^2-742\,062\,195\right)\sin(10\,z)\,z+756\left(864\,z^4-140\,320\,z^2+1\,345\,585\right)\sin(12\,z)\,z+420\left(1000\,z^2-22\,731\right)\sin(14\,z)\,z+8820\sin(16\,z)\,z-2\left(636\,233\,728\,z^8-2\,298\,564\,352\,z^6-81\,203\,428\,320\,z^4-1\,096\,786\,328\,400\,z^2-158\,774\,880\,285\right)\cos(2\,z)+4\left(42\,446\,336\,z^8-3\,271\,466\,240\,z^6-8\,264\,534\,880\,z^4+212\,940\,349\,650\,z^2-48\,463\,987\,005\right)\cos(4\,z)-63\left(53\,248\,z^8-16\,497\,920\,z^6+509\,192\,160\,z^4-2\,070\,293\,400\,z^2+4\,751\,990\,815\right)\cos(6\,z)+2\left(256\,z^8-352\,128\,z^6+227\,349\,360\,z^4-1\,054\,258\,380\,z^2-65\,327\,886\,855\right)\cos(8\,z)+7\left(10\,496\,z^6+10\,388\,640\,z^4-60\,396\,840\,z^2-2\,595\,004\,875\right)\cos(10\,z)+1260\left(10\,368\,z^4-358\,974\,z^2+760\,673\right)\cos(12\,z)+6300\left(540\,z^2-1477\right)\cos(14\,z)+18\,900\cos(16\,z)+323\,553\,254\,850\Big)\sec^9(z)\,m^8-\frac{1}{49\,873\,847\,435\,919\,360}$$

$$\Big(-693\left(640\,648\,192\,z^8+11\,114\,918\,400\,z^6+103\,544\,164\,320\,z^4+1\,011\,114\,289\,440\,z^2+183\,912\,469\,155\right)\cos(z)+216\left(2\,021\,590\,912\,z^8+7\,211\,068\,704\,z^6-88\,869\,508\,840\,z^4-1\,917\,658\,532\,895\,z^2-74\,520\,831\,420\right)\cos(3\,z)-720\left(91\,535\,840\,z^8-3\,068\,581\,152\,z^6-12\,921\,515\,670\,z^4+183\,960\,186\,480\,z^2-90\,768\,299\,517\right)\cos(5\,z)+9\left(147\,609\,088\,z^8-21\,017\,455\,872\,z^6+497\,265\,867\,840\,z^4-1\,804\,580\,711\,640\,z^2+6\,300\,177\,480\,765\right)\cos(7\,z)-27\left(5632\,z^8-22\,068\,480\,z^6+5\,277\,466\,880\,z^4-35\,863\,419\,480\,z^2-720\,548\,768\,625\right)\cos(9\,z)+36\left(128\,z^8+2\,768\,640\,z^6-772\,770\,600\,z^4+7\,816\,771\,620\,z^2+60\,456\,064\,095\right)\cos(11\,z)+189\left(62\,208\,z^6-18\,051\,200\,z^4+404\,104\,440\,z^2-740\,346\,795\right)\cos(13\,z)+945\left(20\,000\,z^4-1\,078\,440\,z^2+1\,867\,113\right)\cos(15\,z)+5670\left(196\,z^2-1095\right)\cos(17\,z)+2835\cos(19\,z)+8\,z\left(17\,769\,803\,264\,z^8+233\,220\,314\,880\,z^6+1\,637\,875\,022\,112\,z^4+7\,171\,584\,281\,820\,z^2+26\,945\,149\,506\,345\right)\sin(z)-$$

$$4\,z\left(11\,114\,481\,664\,z^8-329\,411\,764\,224\,z^6-4\,475\,056\,511\,520\,z^4-25\,403\,813\,288\,280\,z^2-109\,796\,213\,117\,115\right)\sin(3\,z)+20\,z\left(179\,849\,216\,z^8-26\,385\,661\,440\,z^6+181\,372\,280\,544\,z^4+2\,698\,100\,685\,000\,z^2+16\,371\,991\,876\,905\right)\sin(5\,z)-8\,z\left(5\,036\,288\,z^8-2\,548\,683\,648\,z^6+146\,556\,644\,976\,z^4-1\,168\,817\,466\,600\,z^2-15\,593\,642\,462\,475\right)\sin(7\,z)+8\,z\left(256\,z^8-1\,648\,512\,z^6-628\,329\,744\,z^4-30\,940\,675\,920\,z^2+2\,691\,609\,178\,545\right)\sin(9\,z)-36\,z\left(11\,776\,z^6+74\,036\,256\,z^4-3\,957\,899\,400\,z^2-12\,244\,321\,845\right)\sin(11\,z)-756\,z\left(412\,128\,z^4-27\,634\,520\,z^2+207\,626\,265\right)\sin(13\,z)-94\,500\,z\left(2360\,z^2-22\,737\right)\sin(15\,z)-5\,159\,700\,z\sin(17\,z)\Big)$$

$$\sec^{10}(z)\,m^9-\frac{1}{7\,979\,815\,589\,747\,097\,600}\Big(-3\,127\,485\,374\,464\,z^{10}-63\,366\,273\,653\,760\,z^8-724\,973\,096\,010\,240\,z^6-5\,803\,927\,864\,257\,600\,z^4-53\,237\,719\,525\,636\,800\,z^2+2160\left(11\,081\,770\,240\,z^8+161\,812\,138\,048\,z^6+1\,225\,340\,001\,032\,z^4+5\,764\,403\,332\,800\,z^2+22\,640\,041\,844\,805\right)\sin(2\,z)\,z-120\left(83\,424\,409\,856\,z^8-723\,001\,694\,976\,z^6-15\,277\,284\,541\,008\,z^4-102\,021\,535\,808\,640\,z^2-476\,425\,577\,821\,725\right)\sin(4\,z)\,z+40\left(21\,784\,779\,008\,z^8-1\,391\,637\,227\,904\,z^6+5\,072\,163\,219\,648\,z^4+126\,500\,645\,043\,360\,z^2+840\,081\,479\,030\,325\right)\sin(6\,z)\,z-40\left(246\,754\,304\,z^8-56\,320\,072\,704\,z^6+2\,612\,805\,933\,024\,z^4-19\,701\,553\,793\,040\,z^2-269\,430\,695\,986\,725\right)\sin(8\,z)\,z+40\left(15\,104\,z^8+97\,188\,480\,z^6-47\,883\,860\,640\,z^4+504\,590\,189\,040\,z^2+39\,292\,204\,182\,525\right)\sin(10\,z)\,z+20\left(512\,z^8+52\,296\,192\,z^6-25\,141\,106\,592\,z^4+908\,266\,867\,200\,z^2+225\,436\,486\,905\right)\sin(12\,z)\,z+540\left(186\,624\,z^6-87\,773\,728\,z^4+3\,699\,596\,040\,z^2-23\,994\,177\,585\right)\sin(14\,z)\,z+18\,900\left(20\,000\,z^4-2\,100\,816\,z^2+12\,561\,231\right)\sin(16\,z)\,z+18\,900\left(2744\,z^2-53\,127\right)\sin(18\,z)\,z+510\,300\sin(20\,z)\,z+\left(3\,714\,757\,763\,072\,z^{10}-1\,042\,292\,920\,320\,z^8-577\,678\,143\,006\,720\,z^6-7\,378\,698\,967\,466\,400\,z^4-84\,637\,606\,349\,986\,200\,z^2-10\,608\,569\,972\,968\,275\right)\cos(2\,z)-2\left(365\,160\,251\,392\,z^{10}-26\,458\,755\,160\,320\,z^8-177\,932\,880\,840\,960\,z^6+425\,836\,073\,804\,400\,z^4+20\,669\,808\,618\,547\,200\,z^2-1\,837\,886\,114\,913\,975\right)\cos(4\,z)+\left(37\,339\,713\,536\,z^{10}-9\,216\,831\,375\,360\,z^8+191\,012\,478\,531\,840\,z^6+1\,067\,674\,612\,687\,200\,z^4-11\,070\,187\,941\,216\,600\,z^2+9\,039\,440\,287\,308\,375\right)\cos(6\,z)-224\left(1\,079\,552\,z^{10}-846\,531\,360\,z^8+77\,688\,939\,600\,z^6-1\,459\,842\,367\,950\,z^4+4\,501\,803\,215\,475\,z^2-25\,066\,618\,232\,625\right)\cos(8\,z)+\left(4096\,z^{10}-14\,837\,760\,z^8+188\,079\,816\,960\,z^6-21\,156\,536\,469\,600\,z^4+155\,271\,976\,234\,200\,z^2+1\,580\,276\,908\,253\,925\right)\cos(10\,z)+90\left(13\,056\,z^8+394\,241\,792\,z^6-44\,166\,563\,280\,z^4+444\,642\,569\,280\,z^2+1\,590\,783\,707\,835\right)\cos(12\,z)+945\left(3\,608\,064\,z^6-405\,685\,600\,z^4+7\,052\,105\,160\,z^2-11\,795\,057\,685\right)\cos(14\,z)+4725\left(1\,280\,000\,z^4-28\,536\,480\,z^2+36\,656\,241\right)\cos(16\,z)+56\,700\left(6860\,z^2-15\,751\right)\cos(18\,z)+1\,077\,300\cos(20\,z)-9\,434\,038\,449\,457\,125\Big)\sec^{11}(z)\,m^{10}+O\left(m^{11}\right)$$

09.31.06.0013.01

$$\operatorname{nc}(z\mid m)\propto\sec(z)\,(1+O(m))$$

Expansions at $m == 1$

09.31.06.0014.01

$$\operatorname{nc}(z\mid m)\propto\cosh(z)-\frac{1}{8}\sinh(z)\,(\sinh(2\,z)-2\,z)\,(m-1)+\frac{1}{256}\left(\left(8\,z^2-9\right)\cosh(z)+8\cosh(3\,z)+\cosh(5\,z)-16\,z\sinh(z)-12\,z\sinh(3\,z)\right)(m-1)^2+\ldots\;/;\;(m\to 1)$$

09.31.06.0015.01

$$\operatorname{nc}(z\mid m)\propto\cosh(z)-\frac{1}{8}\sinh(z)\,(\sinh(2\,z)-2\,z)\,(m-1)+\frac{1}{256}\left(\left(8\,z^2-9\right)\cosh(z)+8\cosh(3\,z)+\cosh(5\,z)-16\,z\sinh(z)-12\,z\sinh(3\,z)\right)(m-1)^2+$$

$$\frac{1}{12\,288}\left(-27\left(8z^2-11\right)\cosh(z)-6\left(36z^2+41\right)\cosh(3z)-48\cosh(5z)-\right.$$
$$\left.3\cosh(7z)+8z\left(4z^2+45\right)\sinh(z)+468z\sinh(3z)+60z\sinh(5z)\right)(m-1)^3+$$
$$\frac{1}{196\,608}\left(2\left(16z^4+1128z^2-1797\right)\cosh(z)+3\left(1296z^2+943\right)\cosh(3z)+30\left(20z^2+23\right)\cosh(5z)+72\cosh(7z)+\right.$$
$$\left.3\cosh(9z)-4z\left(112z^2+843\right)\sinh(z)-72z\left(12z^2+83\right)\sinh(3z)-1260z\sinh(5z)-84z\sinh(7z)\right)(m-1)^4+$$
$$\frac{1}{15\,728\,640}\left(-40\left(76z^4+3228z^2-5751\right)\cosh(z)-15\left(864z^4+19\,800z^2+11\,597\right)\cosh(3z)-240\left(325z^2+204\right)\right.$$
$$\cosh(5z)-30\left(196z^2+221\right)\cosh(7z)-480\cosh(9z)-15\cosh(11z)+8z\left(16z^4+3660z^2+22\,185\right)\sinh(z)+$$
$$\left.360z\left(276z^2+1087\right)\sinh(3z)+200z\left(100z^2+531\right)\sinh(5z)+12\,180z\sinh(7z)+540z\sinh(9z)\right)(m-1)^5+$$
$$\frac{1}{754\,974\,720}\left(8\left(32z^6+17\,820z^4+588\,375z^2-1\,146\,195\right)\cosh(z)+90\left(12\,096z^4+145\,620z^2+74\,525\right)\cosh(3z)+\right.$$
$$15\left(20\,000z^4+301\,800z^2+139\,557\right)\cosh(5z)+720\left(833z^2+465\right)\cosh(7z)+$$
$$90\left(324z^2+359\right)\cosh(9z)+1800\cosh(11z)+45\cosh(13z)-12z\left(768z^4+96\,440z^2+502\,095\right)\sinh(z)-$$
$$432z\left(216z^4+12\,270z^2+36\,485\right)\sinh(3z)-300z\left(6200z^2+16\,761\right)\sinh(5z)-$$
$$\left.840z\left(196z^2+909\right)\sinh(7z)-59\,940z\sinh(9z)-1980z\sinh(11z)\right)(m-1)^6+$$
$$\frac{1}{84\,557\,168\,640}\left(-14\left(3712z^6+1\,058\,640z^4+29\,817\,540z^2-62\,553\,375\right)\cosh(z)-\right.$$
$$63\left(20\,736z^6+2\,453\,760z^4+21\,231\,000z^2+9\,882\,415\right)\cosh(3z)-$$
$$315\left(240\,000z^4+1\,750\,000z^2+668\,239\right)\cosh(5z)-735\left(10\,976z^4+133\,560z^2+51\,573\right)\cosh(7z)-$$
$$5040\left(1701z^2+874\right)\cosh(9z)-630\left(484z^2+529\right)\cosh(11z)-15\,120\cosh(13z)-315\cosh(15z)+$$
$$4z\left(256z^6+292\,992z^4+27\,155\,520z^2+125\,335\,035\right)\sinh(z)+6804z\left(3168z^4+89\,080z^2+222\,225\right)\sinh(3z)+$$
$$2100z\left(4000z^4+135\,400z^2+257\,937\right)\sinh(5z)+8820z\left(5096z^2+11\,175\right)\sinh(7z)+$$
$$\left.22\,680z\left(108z^2+461\right)\sinh(9z)+623\,700z\sinh(11z)+16\,380z\sinh(13z)\right)(m-1)^7+$$
$$\frac{1}{1\,352\,914\,698\,240}\left(\left(512z^8+1\,075\,200z^6+216\,777\,120z^4+5\,453\,932\,680z^2-12\,187\,866\,915\right)\cosh(z)+\right.$$
$$63\left(787\,968z^6+44\,094\,240z^4+309\,619\,800z^2+134\,140\,385\right)\cosh(3z)+350$$
$$\left(80\,000z^6+5\,256\,000z^4+26\,073\,900z^2+8\,709\,921\right)\cosh(5z)+5880\left(60\,368z^4+334\,950z^2+102\,195\right)\cosh(7z)+$$
$$315\left(69\,984z^4+745\,848z^2+253\,307\right)\cosh(9z)+55\,440\left(275z^2+133\right)\cosh(11z)+630\left(676z^2+731\right)\cosh(13z)+$$
$$17\,640\cosh(15z)+315\cosh(17z)-4z\left(8704z^6+4\,839\,072z^4+370\,434\,960z^2+1\,543\,848\,075\right)\sinh(z)-$$
$$432z\left(5184z^6+1\,137\,024z^4+22\,104\,180z^2+48\,770\,155\right)\sinh(3z)-8400z\left(41\,000z^4+643\,900z^2+982\,101\right)$$
$$\sinh(5z)-588z\left(76\,832z^4+1\,977\,640z^2+2\,906\,355\right)\sinh(7z)-11\,340z\left(10\,152z^2+19\,451\right)\sinh(9z)-$$
$$\left.9240z\left(484z^2+1953\right)\sinh(11z)-868\,140z\sinh(13z)-18\,900z\sinh(15z)\right)(m-1)^8+$$
$$\frac{1}{194\,819\,716\,546\,560}\left(-18\left(9984z^8+9\,771\,776z^6+1\,580\,310\,480z^4+36\,477\,444\,780z^2-86\,127\,832\,035\right)\cosh(z)-\right.$$
$$162\left(186\,624z^8+69\,745\,536z^6+2\,615\,185\,440z^4+15\,856\,977\,780z^2+6\,489\,123\,935\right)\cosh(3z)-$$
$$630\left(18\,400\,000z^6+542\,880\,000z^4+2\,102\,350\,500z^2+635\,467\,347\right)\cosh(5z)-$$
$$126\left(15\,059\,072z^6+719\,147\,520z^4+2\,605\,542\,660z^2+673\,078\,005\right)\cosh(7z)-$$
$$45\,360\left(227\,448z^4+1\,059\,399z^2+274\,924\right)\cosh(9z)-945\left(468\,512z^4+4\,562\,184z^2+1\,409\,247\right)\cosh(11z)-$$
$$45\,360\left(4901z^2+2264\right)\cosh(13z)-28\,350\left(180z^2+193\right)\cosh(15z)-181\,440\cosh(17z)-$$
$$2835\cosh(19z)+16z\left(128z^8+454\,464z^6+172\,469\,304z^4+11\,542\,260\,240z^2+43\,954\,585\,605\right)\sinh(z)+$$
$$1944z\left(445\,824z^6+44\,597\,952z^4+684\,627\,300z^2+1\,377\,694\,745\right)\sinh(3z)+$$
$$4500z\left(160\,000z^6+18\,480\,000z^4+191\,895\,480z^2+250\,280\,163\right)\sinh(5z)+$$

$$5292\, z \left(3\,764\,768\, z^4 + 42\,904\,400\, z^2 + 48\,678\,885\right) \sinh(7\, z) + 20\,412\, z \left(69\,984\, z^4 + 1\,517\,400\, z^2 + 1\,876\,715\right) \sinh(9\, z) + 41\,580\, z \left(53\,240\, z^2 + 92\,781\right) \sinh(11\, z) + 98\,280\, z \left(676\, z^2 + 2619\right) \sinh(13\, z) + 10\,376\,100\, z \sinh(15\, z) + 192\,780\, z \sinh(17\, z)\Big) (m-1)^9 + \frac{1}{15\,585\,577\,323\,724\,800} \Big(\left(4096\, z^{10} + 23\,109\,120\, z^8 + 15\,001\,297\,920\, z^6 + 2\,076\,748\,480\,800\, z^4 + 44\,740\,696\,971\,600\, z^2 - 110\,914\,462\,696\,275\right) \cosh(z) + 405 \left(17\,915\,904\, z^8 + 2\,965\,310\,208\, z^6 + 85\,818\,096\,000\, z^4 + 466\,095\,165\,480\, z^2 + 181\,992\,844\,775\right) \cosh(3\, z) + 225 \left(40\,000\,000\, z^8 + 7\,554\,400\,000\, z^6 + 143\,925\,600\,000\, z^4 + 467\,581\,413\,600\, z^2 + 130\,691\,579\,823\right) \cosh(5\, z) + 945 \left(542\,126\,592\, z^6 + 11\,186\,739\,200\, z^4 + 30\,686\,626\,320\, z^2 + 6\,994\,157\,025\right) \cosh(7\, z) + 17\,010 \left(2\,519\,424\, z^6 + 98\,210\,880\, z^4 + 289\,458\,900\, z^2 + 61\,983\,655\right) \cosh(9\, z) + 113\,400 \left(1\,171\,280\, z^4 + 4\,826\,206\, z^2 + 1\,109\,661\right) \cosh(11\, z) + 4725 \left(913\,952\, z^4 + 8\,335\,080\, z^2 + 2\,394\,237\right) \cosh(13\, z) + 680\,400 \left(2475\, z^2 + 1103\right) \cosh(15\, z) + 28\,350 \left(1156\, z^2 + 1231\right) \cosh(17\, z) + 1\,020\,600 \cosh(19\, z) + 14\,175 \cosh(21\, z) - 80\, z \left(5632\, z^8 + 9\,014\,976\, z^6 + 2\,676\,722\,328\, z^4 + 161\,909\,934\,480\, z^2 + 567\,983\,549\,385\right) \sinh(z) - 43\,740\, z \left(4608\, z^8 + 2\,753\,280\, z^6 + 179\,852\,064\, z^4 + 2\,336\,147\,800\, z^2 + 4\,372\,958\,135\right) \sinh(3\, z) - 54\,000\, z \left(3\,400\,000\, z^6 + 171\,703\,000\, z^4 + 1\,362\,702\,250\, z^2 + 1\,580\,291\,223\right) \sinh(5\, z) - 17\,640\, z \left(2\,151\,296\, z^6 + 174\,254\,976\, z^4 + 1\,268\,918\,700\, z^2 + 1\,197\,550\,395\right) \sinh(7\, z) - 918\,540\, z \left(443\,232\, z^4 + 4\,107\,600\, z^2 + 3\,812\,645\right) \sinh(9\, z) - 41\,580\, z \left(468\,512\, z^4 + 9\,026\,600\, z^2 + 9\,864\,255\right) \sinh(11\, z) - 737\,100\, z \left(28\,392\, z^2 + 46\,115\right) \sinh(13\, z) - 1\,701\,000\, z \left(300\, z^2 + 1127\right) \sinh(15\, z) - 66\,509\,100\, z \sinh(17\, z) - 1\,077\,300\, z \sinh(19\, z)\Big) (m-1)^{10} + O\left((m-1)^{11}\right)$$

09.31.06.0016.01

$$\mathrm{nc}(z \mid m) \propto \cosh(z)\, (1 + O(m-1))$$

q-series

09.31.06.0002.01

$$\mathrm{nc}(z \mid m) = \frac{\pi}{2\sqrt{1-m}\; K(m)} \sec\left(\frac{\pi z}{2 K(m)}\right) - \frac{2\pi}{\sqrt{1-m}\; K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k q(m)^{2k+1}}{q(m)^{2k+1}+1} \cos\left(\frac{(2k+1)\pi z}{2 K(m)}\right)$$

Other series representations

09.31.06.0003.01

$$\mathrm{nc}(z \mid m) = \frac{\pi}{2\sqrt{1-m}\; K(1-m)} \sum_{k=-\infty}^{\infty} (-1)^k \operatorname{csch}\left(\pi \frac{K(m)}{K(1-m)} \left(k + \frac{1}{2} + \frac{z}{2 K(m)}\right)\right)$$

09.31.06.0004.01

$$\mathrm{nc}(z \mid m) \propto \frac{(-1)^{r+s-1}}{\sqrt{1-m}\; (z - 2 s i K(1-m) - (2r+1) K(m))} + O(1) \;/;\; (z \to 2 s i K(1-m) + (2r+1) K(m)) \bigwedge \{r, s\} \in \mathbb{Z}$$

Product representations

09.31.08.0001.01

$$\mathrm{nc}(z \mid m) = \frac{1}{2} \frac{1}{\sqrt[4]{q(m)}} \frac{\sqrt[4]{m}}{\sqrt[4]{1-m}} \sec\left(\frac{\pi z}{2 K(m)}\right) \prod_{n=1}^{\infty} \frac{1 - 2 q(m)^{2n-1} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4n-2}}{1 + 2 q(m)^{2n} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4n}}$$

Differential equations

Ordinary nonlinear differential equations

09.31.13.0001.01

$$w''(z) - w(z)\left(2\,(1-m)\,w(z)^2 + 2\,m - 1\right) = 0\;/;\; w(z) = \mathrm{nc}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.31.16.0001.01

$$\mathrm{nc}(i\,z \mid m) = \mathrm{cn}(z \mid 1-m)$$

09.31.16.0002.01

$$\mathrm{nc}(z \mid 1-m) = \mathrm{cn}(i\,z \mid m)$$

09.31.16.0003.01

$$\mathrm{nc}(i\,z \mid 1-m) = \mathrm{cn}(z \mid m)$$

09.31.16.0007.01

$$\mathrm{nc}(x + i\,y \mid m) = \left(\mathrm{cn}(y \mid 1-m)^2 + m\,\mathrm{sn}(x \mid m)^2\,\mathrm{sn}(y \mid 1-m)^2\right) \Big/ \\ (\mathrm{cn}(x \mid m)\,\mathrm{cn}(y \mid 1-m) - i\,\mathrm{sn}(x \mid m)\,\mathrm{dn}(x \mid m)\,\mathrm{sn}(y \mid 1-m)\,\mathrm{dn}(y \mid 1-m))\;/;\;\{x, y\} \in \mathbb{R}$$

09.31.16.0008.01

$$\mathrm{nc}\left(\sqrt{1-m}\;z \,\middle|\, \frac{m}{m-1}\right) = \frac{\mathrm{nc}(z \mid m)}{\mathrm{nd}(z \mid m)}$$

09.31.16.0009.01

$$\mathrm{nc}\left(\sqrt{m}\;z \,\middle|\, \frac{1}{m}\right) = \mathrm{nd}(z \mid m)$$

09.31.16.0010.01

$$\mathrm{nc}\left(i\,\sqrt{m}\;z \,\middle|\, \frac{m-1}{m}\right) = \mathrm{dn}(z \mid m)$$

09.31.16.0011.01

$$\mathrm{nc}\left(i\,\sqrt{1-m}\;z \,\middle|\, \frac{1}{1-m}\right) = \frac{\mathrm{nd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

Landen's transformation:

09.31.16.0012.01

$$\mathrm{nc}\left(\left(1+\sqrt{1-m}\right) z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{\mathrm{dn}(z \mid m)}{1 - \left(1+\sqrt{1-m}\right)\mathrm{sn}(z \mid m)^2}$$

Gauss' transformation:

09.31.16.0013.01

$$\mathrm{nc}\left(\left(1+\sqrt{m}\right) z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{1+\sqrt{m}\;\mathrm{sn}(z \mid m)^2}{\mathrm{cn}(z \mid m)\,\mathrm{dn}(z \mid m)}$$

n th degree transformations:

09.31.16.0014.01

$$\operatorname{nc}\left(\frac{z}{M} \middle| l\right) = \operatorname{nc}(z \mid m) \prod_{r=1}^{\frac{n-1}{2}} \frac{1 - m \operatorname{sn}\left(\frac{2 r K(m)}{n} \middle| m\right)^2 \operatorname{sn}(z \mid m)^2}{1 - \frac{\operatorname{sn}(z|m)^2}{\operatorname{sn}\left(\frac{(2r-1) K(m)}{n}\middle|m\right)^2}} \;/;$$

$$\frac{n+1}{2} \in \mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n-1}{2}} \operatorname{sn}\left(\frac{(2r-1) K(m)}{n} \middle| m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n-1}{2}} \frac{\operatorname{sn}\left(\frac{(2r-1) K(m)}{n} \middle| m\right)^2}{\operatorname{sn}\left(\frac{2 r K(m)}{n} \middle| m\right)^2}$$

09.31.16.0015.01

$$\operatorname{nc}\left(\frac{z}{M} + \frac{K}{n M} \middle| l\right) = -\frac{M}{\sqrt{1-l}} \frac{\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)} \prod_{r=1}^{\frac{n}{2}} \frac{1 - m \operatorname{sn}\left(\frac{(2r-1) K(m)}{n} \middle| m\right)^2 \operatorname{sn}(z \mid m)^2}{1 - \frac{\operatorname{sn}(z|m)^2}{\operatorname{sn}\left(\frac{2 r K(m)}{n}\middle|m\right)^2}} \;/;$$

$$\frac{n}{2} \in \mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n}{2}} \operatorname{sn}\left(\frac{(2r-1) K(m)}{n} \middle| m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n}{2}} \frac{\operatorname{sn}\left(\frac{(2r-1) K(m)}{n} \middle| m\right)^2}{\operatorname{sn}\left(\frac{2 r K(m)}{n} \middle| m\right)^2}$$

Argument involving half-periods

09.31.16.0023.01

$$\operatorname{nc}(z + K(m) \mid m) = -\frac{\operatorname{ds}(z \mid m)}{\sqrt{1-m}}$$

09.31.16.0024.01

$$\operatorname{nc}(z - K(m) \mid m) = \frac{\operatorname{ds}(z \mid m)}{\sqrt{1-m}}$$

09.31.16.0025.01

$$\operatorname{nc}(z + 3 K(m) \mid m) = \frac{\operatorname{ds}(z \mid m)}{\sqrt{1-m}}$$

09.31.16.0026.01

$$\operatorname{nc}(z + (2r+1) K(m) \mid m) = \frac{(-1)^{r-1}}{\sqrt{1-m}} \operatorname{ds}(z \mid m) \;/;\; r \in \mathbb{Z}$$

09.31.16.0027.01

$$\operatorname{nc}(z + i K(1-m) \mid m) = i \sqrt{m} \operatorname{sd}(z \mid m)$$

09.31.16.0028.01

$$\operatorname{nc}(z - i K(1-m) \mid m) = -i \sqrt{m} \operatorname{sd}(z \mid m)$$

09.31.16.0029.01

$$\operatorname{nc}(z + 3 i K(1-m) \mid m) = -i \sqrt{m} \operatorname{sd}(z \mid m) \;/;\; s \in \mathbb{Z}$$

09.31.16.0030.01

$$\operatorname{nc}(z + (2s+1) i K(1-m) \mid m) = (-1)^s i \sqrt{m} \operatorname{sd}(z \mid m) \;/;\; s \in \mathbb{Z}$$

09.31.16.0031.01

$$\operatorname{nc}(z + i K(1-m) + K(m) \mid m) = \frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}}$$

09.31.16.0032.01

$$\operatorname{nc}(z - i K(1-m) + K(m) \mid m) = -\frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}}$$

09.31.16.0033.01

$$\operatorname{nc}(z + i K(1-m) - K(m) \mid m) = -\frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}}$$

09.31.16.0034.01

$$\operatorname{nc}(z - i K(1-m) - K(m) \mid m) = \frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}}$$

09.31.16.0035.01

$$\operatorname{nc}(z + i K(1-m) + 3 K(m) \mid m) = -\frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}}$$

09.31.16.0036.01

$$\operatorname{nc}(z + (4s+1) i K(1-m) + (4r+1) K(m) \mid m) = \frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}} /; \{r, s\} \in \mathbb{Z}$$

09.31.16.0037.01

$$\operatorname{nc}(z + (4s+1) i K(1-m) + (4r-1) K(m) \mid m) = -\frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}} /; \{r, s\} \in \mathbb{Z}$$

09.31.16.0038.01

$$\operatorname{nc}(z + (4s-1) i K(1-m) + (4r+1) K(m) \mid m) = -\frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}} /; \{r, s\} \in \mathbb{Z}$$

09.31.16.0039.01

$$\operatorname{nc}(z + (4s-1) i K(1-m) + (4r-1) K(m) \mid m) = \frac{i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}} /; \{r, s\} \in \mathbb{Z}$$

09.31.16.0040.01

$$\operatorname{nc}(z + (2s+1) i K(1-m) + (2r+1) K(m) \mid m) = \frac{(-1)^{r+s} i \sqrt{m} \operatorname{cn}(z \mid m)}{\sqrt{1-m}} /; \{r, s\} \in \mathbb{Z}$$

Argument involving inverse Jacobi functions

09.31.16.0041.01

$$\operatorname{nc}\left(\operatorname{cd}^{-1}(z \mid m) \middle| m\right)^2 = \frac{m z^2 - 1}{(m-1) z^2}$$

09.31.16.0042.01

$$\operatorname{nc}\left(\operatorname{cn}^{-1}(z \mid m) \middle| m\right) = \frac{1}{z}$$

09.31.16.0043.01

$$\operatorname{nc}\left(\operatorname{cs}^{-1}(z \mid m) \,\middle|\, m\right)^2 = 1 + \frac{1}{z^2}$$

09.31.16.0044.01

$$\operatorname{nc}\left(\operatorname{dc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2 - m}{1 - m}$$

09.31.16.0045.01

$$\operatorname{nc}\left(\operatorname{dn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m}{z^2 + m - 1}$$

09.31.16.0046.01

$$\operatorname{nc}\left(\operatorname{ds}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2 + m}{z^2 + m - 1}$$

09.31.16.0047.01

$$\operatorname{nc}\left(\operatorname{nd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m\, z^2}{1 - (1 - m)\, z^2}$$

09.31.16.0048.01

$$\operatorname{nc}\left(\operatorname{ns}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2}{z^2 - 1}$$

09.31.16.0049.01

$$\operatorname{nc}\left(\operatorname{sc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = z^2 + 1$$

09.31.16.0050.01

$$\operatorname{nc}\left(\operatorname{sd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m\, z^2 + 1}{(m - 1)\, z^2 + 1}$$

09.31.16.0051.01

$$\operatorname{nc}\left(\operatorname{sn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{1}{1 - z^2}$$

Addition formulas

09.31.16.0016.01

$$\operatorname{nc}(u + v \mid m) = \frac{1 - m \operatorname{sn}(u \mid m)^2 \operatorname{sn}(v \mid m)^2}{\operatorname{cn}(u \mid m) \operatorname{cn}(v \mid m) - \operatorname{sn}(u \mid m) \operatorname{dn}(u \mid m) \operatorname{sn}(v \mid m) \operatorname{dn}(v \mid m)}$$

09.31.16.0017.01

$$\operatorname{nc}(u + v \mid m) \operatorname{nc}(u - v \mid m) = \frac{1 - m \operatorname{sn}(u \mid m)^2 \operatorname{sn}(v \mid m)^2}{\operatorname{cn}(v \mid m)^2 - \operatorname{dn}(v \mid m)^2 \operatorname{sn}(u \mid m)^2}$$

Half-angle formulas

09.31.16.0018.01

$$\operatorname{nc}\left(\frac{z}{2} \,\middle|\, m\right)^2 = \frac{1 + \operatorname{dn}(z \mid m)}{\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)}$$

Multiple arguments

Double angle formulas

09.31.16.0019.01

$$\operatorname{nc}(2z\mid m) = \frac{1 - m\operatorname{sn}(z\mid m)^4}{\operatorname{cn}(z\mid m)^2 - \operatorname{sn}(z\mid m)^2\operatorname{dn}(z\mid m)^2}$$

09.31.16.0020.01

$$\operatorname{nc}(2z\mid m) = \frac{\operatorname{cn}(z\mid m)^2 + \operatorname{dn}(z\mid m)^2\operatorname{sn}(z\mid m)^2}{\operatorname{cn}(z\mid m)^2 - \operatorname{sn}(z\mid m)^2\operatorname{dn}(z\mid m)^2}$$

Multiple angle formulas

09.31.16.0021.01

$$\operatorname{nc}(n z\mid m) = \left(\frac{1-m}{m}\right)^{\frac{n^2-1}{4}} \prod_{\mu,\nu=0}^{n-1} \operatorname{nc}\left(z + \frac{4K(m)(\mu + \nu\tau)}{n} \middle| m\right) /;\ \frac{n+1}{2} \in \mathbb{Z}^+$$

09.31.16.0022.01

$$\operatorname{nc}\left(\frac{2n}{\pi} K\left(\lambda\left(\frac{n}{\pi i}\log(q(m))\right)\right) x \middle| \lambda\left(\frac{n}{\pi i}\log(q(m))\right)\right) = (-1)^{\frac{n-1}{2}} \frac{q(m)^{n/4}}{\sqrt[4]{q(m)^n}} \frac{\sqrt[4]{\lambda\left(\frac{n}{\pi i}\log(q(m))\right)}}{m^{n/4}} \frac{(1-m)^{n/4}}{\sqrt[4]{1-\lambda\left(\frac{n}{\pi i}\log(q(m))\right)}} \prod_{r=0}^{n-1} \operatorname{nc}\left(\frac{2K(m)}{\pi}\left(x + \frac{\pi r}{n}\right) \middle| m\right) /;\ \frac{n+1}{2} \in \mathbb{Z}^+$$

Identities

Functional identities

09.31.17.0001.01

$$(m-1)w(z)^4 - 2m\,w(z)^2 + m + \left((m-1)w(z)^4 + 2(1-m)w(z)^2 + m\right) w(2z) = 0 /;\ w(z) = \operatorname{nc}(z\mid m)$$

Complex characteristics

Real part

09.31.19.0001.01

$$\operatorname{Re}(\operatorname{nc}(x + i y\mid m)) = \frac{\operatorname{cn}(x\mid m)\operatorname{cn}(y\mid 1-m)\left(\operatorname{cn}(y\mid 1-m)^2 + m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)}{\operatorname{cn}(x\mid m)^2\operatorname{cn}(y\mid 1-m)^2 + \operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2} /;\ \{x, y, m\} \in \mathbb{R}$$

Imaginary part

09.31.19.0002.01

$$\operatorname{Im}(\operatorname{nc}(x + i y\mid m)) = \frac{\operatorname{dn}(x\mid m)\operatorname{dn}(y\mid 1-m)\operatorname{sn}(x\mid m)\operatorname{sn}(y\mid 1-m)\left(\operatorname{cn}(y\mid 1-m)^2 + m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)}{\operatorname{cn}(x\mid m)^2\operatorname{cn}(y\mid 1-m)^2 + \operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2} /;\ \{x, y, m\} \in \mathbb{R}$$

Absolute value

09.31.19.0003.01

$$|\text{nc}(x + i\, y \mid m)| = \left(\sqrt{\left(\left(\text{cn}(y \mid 1-m)^2 + m\,\text{sn}(x \mid m)^2\,\text{sn}(y \mid 1-m)^2\right)^2 \left(\text{cn}(x \mid m)^2\,\text{cn}(y \mid 1-m)^2 + \text{dn}(x \mid m)^2\,\text{dn}(y \mid 1-m)^2\,\text{sn}(x \mid m)^2\,\text{sn}(y \mid 1-m)^2\right)\right)}\right) \Big/ \left(\text{cn}(x \mid m)^2\,\text{cn}(y \mid 1-m)^2 + \text{dn}(x \mid m)^2\,\text{dn}(y \mid 1-m)^2\,\text{sn}(x \mid m)^2\,\text{sn}(y \mid 1-m)^2\right) /;\ \{x,\, y,\, m\} \in \mathbb{R}$$

Argument

09.31.19.0004.01

$$\arg(\text{nc}(x + i\, y \mid m)) = \tan^{-1}\left(\text{cn}(x \mid m)\,\text{cn}(y \mid 1-m)\left(\text{cn}(y \mid 1-m)^2 + m\,\text{sn}(x \mid m)^2\,\text{sn}(y \mid 1-m)^2\right),\ \text{dn}(x \mid m)\,\text{dn}(y \mid 1-m)\,\text{sn}(x \mid m)\,\text{sn}(y \mid 1-m)\left(\text{cn}(y \mid 1-m)^2 + m\,\text{sn}(x \mid m)^2\,\text{sn}(y \mid 1-m)^2\right)\right) /;\ \{x,\, y,\, m\} \in \mathbb{R}$$

Conjugate value

09.31.19.0005.01

$$\overline{\text{nc}(x + i\, y \mid m)} = \frac{\text{cn}(y \mid 1-m)^2 + m\,\text{sn}(x \mid m)^2\,\text{sn}(y \mid 1-m)^2}{\text{cn}(x \mid m)\,\text{cn}(y \mid 1-m) + i\,\text{sn}(x \mid m)\,\text{dn}(x \mid m)\,\text{sn}(y \mid 1-m)\,\text{dn}(y \mid 1-m)} /;\ \{x,\, y,\, m\} \in \mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *z*

09.31.20.0001.01

$$\frac{\partial\,\text{nc}(z \mid m)}{\partial z} = \text{dc}(z \mid m)\,\text{sc}(z \mid m)$$

09.31.20.0002.01

$$\frac{\partial^2\,\text{nc}(z \mid m)}{\partial z^2} = \text{nc}(z \mid m)\left(\text{dc}(z \mid m)^2 + (1-m)\,\text{sc}(z \mid m)^2\right)$$

With respect to *m*

09.31.20.0003.01

$$\frac{\partial\,\text{nc}(z \mid m)}{\partial m} = \frac{1}{2\,m\,(1-m)}\,(\text{sc}(z \mid m)\,\text{dc}(z \mid m)\,((1-m)\,z - E(\text{am}(z \mid m) \mid m) + m\,\text{cd}(z \mid m)\,\text{sn}(z \mid m)))$$

09.31.20.0004.01

$$\frac{\partial^2 \operatorname{nc}(z \mid m)}{\partial m^2} = \frac{1}{4 (m-1)^2 m^2} \Big(-(m-1) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{nc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)) \operatorname{sc}(z \mid m)^2 - 2 (m-1) \operatorname{dc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)) \operatorname{sc}(z \mid m) - 2 m \operatorname{dc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)) \operatorname{sc}(z \mid m) + (1-m) m \operatorname{dc}(z \mid m) \Big(-2 z + \frac{F(\operatorname{am}(z \mid m) \mid m) - E(\operatorname{am}(z \mid m) \mid m)}{m} + 2 \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m) + ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m) - \frac{1}{m-1} (\operatorname{cd}(z \mid m) \operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m) (-m z + z - E(\operatorname{am}(z \mid m) \mid m) + m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m))) + \frac{1}{(m-1) m} \Big(\Big(m \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m) - ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{dn}(z \mid m) \Big) \sqrt{1 - m \operatorname{sn}(z \mid m)^2} \Big)\Big) \operatorname{sc}(z \mid m) + \operatorname{dc}(z \mid m)^2 \operatorname{nc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m))^2\Big)$$

Symbolic differentiation

With respect to z

09.31.20.0007.01

$$\frac{\partial^n \operatorname{nc}(z \mid m)}{\partial z^n} = \operatorname{nc}(z \mid m)\, \delta_n + \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\partial^j \operatorname{sc}(z \mid m)}{\partial z^j} \, \frac{\partial^{-j+n-1} \operatorname{dc}(z \mid m)}{\partial z^{-j+n-1}} \,/;\, n \in \mathbb{N}$$

09.31.20.0005.01

$$\frac{\partial^n \operatorname{nc}(z \mid m)}{\partial z^n} = \frac{\pi}{2 \sqrt{1-m}\, z^n K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k E_{2k}}{(2k-n)!} \left(\frac{\pi z}{2 K(m)}\right)^{2k} - \frac{2^{1-n} \pi^{n+1}}{\sqrt{1-m}\, K(m)^{n+1}} \sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)^n q(m)^{2k+1}}{q(m)^{2k+1} + 1} \cos\left(\frac{\pi n}{2} + \frac{(2k+1) \pi z}{2 K(m)}\right) /;\, n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to z

09.31.20.0006.01

$$\frac{\partial^\alpha \operatorname{nc}(z \mid m)}{\partial z^\alpha} = \frac{\pi}{2 \sqrt{1-m}\, K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k K(m)^{-2k} E_{2k}}{\Gamma(2k-\alpha+1)} \left(\frac{\pi}{2}\right)^{2k} z^{2k-\alpha} - \frac{2^{\alpha+1} \pi^{3/2} z^{-\alpha}}{\sqrt{1-m}\, K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k q(m)^{2k+1}}{q(m)^{2k+1} + 1} \, {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1 - \frac{\alpha}{2}; -\frac{(2k+1)^2 \pi^2 z^2}{16 K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.31.21.0001.01

$$\int \mathrm{nc}(z \mid m)\, dz = \frac{\log\left(\mathrm{dc}(z \mid m) + \sqrt{1-m}\ \mathrm{sc}(z \mid m)\right)}{\sqrt{1-m}}$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

09.31.21.0002.01

$$\int \mathrm{nc}(z \mid m)^2\, dz = z + \frac{\mathrm{dn}(z \mid m)\,\mathrm{sn}(z \mid m)}{(1-m)\,\mathrm{cn}(z \mid m)} - \frac{E(\mathrm{am}(z \mid m) \mid m)\left(m\,\mathrm{cn}(z \mid m)^2 - m + 1\right)}{(1-m)\left(\mathrm{dn}(z \mid m)\sqrt{1 - m\,\mathrm{sn}(z \mid m)^2}\right)}$$

Representations through equivalent functions

With inverse function

09.31.27.0001.01

$$\mathrm{nc}\left(\mathrm{nc}^{-1}(z \mid m) \mid m\right) = z$$

With related functions

Involving am

09.31.27.0027.01

$$\mathrm{nc}(z \mid m) = \sec(\mathrm{am}(z \mid m))$$

Involving one other Jacobi elliptic function

Involving cd

09.31.27.0004.01

$$\mathrm{nc}(z \mid m)^2 = \frac{m\,\mathrm{cd}(z \mid m)^2 - 1}{(m-1)\,\mathrm{cd}(z \mid m)^2}$$

Involving cn

09.31.27.0005.01

$$\mathrm{nc}(z \mid m) = \frac{1}{\mathrm{cn}(z \mid m)}$$

09.31.27.0006.01

$$\mathrm{nc}(z \mid m) = \mathrm{cn}(i\,z \mid 1-m)$$

Involving cs

09.31.27.0009.01

$$\operatorname{nc}(z \mid m)^2 = 1 + \frac{1}{\operatorname{cs}(z \mid m)^2}$$

Involving dc

09.31.27.0012.01

$$\operatorname{nc}(z \mid m)^2 = \frac{\operatorname{dc}(z \mid m)^2 - m}{1 - m}$$

Involving dn

09.31.27.0013.01

$$\operatorname{nc}(z \mid m)^2 = \frac{m}{\operatorname{dn}(z \mid m)^2 + m - 1}$$

Involving ds

09.31.27.0014.01

$$\operatorname{nc}(z \mid m)^2 = \frac{\operatorname{ds}(z \mid m)^2 + m}{\operatorname{ds}(z \mid m)^2 + m - 1}$$

Involving nd

09.31.27.0015.01

$$\operatorname{nc}(z \mid m)^2 = \frac{m \operatorname{nd}(z \mid m)^2}{1 - (1 - m) \operatorname{nd}(z \mid m)^2}$$

Involving ns

09.31.27.0017.01

$$\operatorname{nc}(z \mid m)^2 = \frac{\operatorname{ns}(z \mid m)^2}{\operatorname{ns}(z \mid m)^2 - 1}$$

Involving sc

09.31.27.0019.01

$$\operatorname{nc}(z \mid m)^2 = \operatorname{sc}(z \mid m)^2 + 1$$

Involving sd

09.31.27.0020.01

$$\operatorname{nc}(z \mid m)^2 = \frac{m \operatorname{sd}(z \mid m)^2 + 1}{(m - 1) \operatorname{sd}(z \mid m)^2 + 1}$$

Involving sn

09.31.27.0021.01

$$\operatorname{nc}(z \mid m)^2 = \frac{1}{1 - \operatorname{sn}(z \mid m)^2}$$

Involving two other Jacobi elliptic functions

Involving cd and cn

09.31.27.0028.01

$$\operatorname{nc}(z \mid m) = \frac{\left(m \operatorname{cd}(z \mid m)^2 - 1\right) \operatorname{cn}(z \mid m)}{(m-1) \operatorname{cd}(z \mid m)^2}$$

Involving cd and dn

09.31.27.0002.01

$$\operatorname{nc}(z \mid m) = \frac{1}{\operatorname{dn}(z \mid m) \operatorname{cd}(z \mid m)}$$

09.31.27.0029.01

$$\operatorname{nc}(z \mid m) = \frac{\left(m \operatorname{cd}(z \mid m)^2 - 1\right) \operatorname{dn}(z \mid m)}{(m-1) \operatorname{cd}(z \mid m)}$$

Involving cd and nd

09.31.27.0003.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{nd}(z \mid m)}{\operatorname{cd}(z \mid m)}$$

09.31.27.0030.01

$$\operatorname{nc}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) \operatorname{nd}(z \mid m)}{m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1}$$

Involving cn and cs

09.31.27.0031.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m)^2}$$

Involving cn and dc

09.31.27.0032.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(m - \operatorname{dc}(z \mid m)^2\right)}{m-1}$$

Involving cn and dn

09.31.27.0033.01

$$\operatorname{nc}(z \mid m) = \frac{m \operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m)^2 + m - 1}$$

Involving cn and ds

09.31.27.0034.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{ds}(z \mid m)^2 + m\right)}{\operatorname{ds}(z \mid m)^2 + m - 1}$$

Involving cn and nd

09.31.27.0035.01

$$\operatorname{nc}(z \mid m) = \frac{m \operatorname{cn}(z \mid m) \operatorname{nd}(z \mid m)^2}{m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1}$$

Involving cn and ns

09.31.27.0036.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m)^2}{(\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

Involving cn and sc

09.31.27.0037.01

$$\operatorname{nc}(z \mid m) = \operatorname{cn}(z \mid m) \left(\operatorname{sc}(z \mid m)^2 + 1\right)$$

Involving cn and sd

09.31.27.0038.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(m \operatorname{sd}(z \mid m)^2 + 1\right)}{m \operatorname{sd}(z \mid m)^2 - \operatorname{sd}(z \mid m)^2 + 1}$$

Involving cn and sn

09.31.27.0039.01

$$\operatorname{nc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)}{(\operatorname{sn}(z \mid m) - 1) (\operatorname{sn}(z \mid m) + 1)}$$

Involving cs and ns

09.31.27.0007.01

$$\operatorname{nc}(z \mid m) == \frac{\operatorname{ns}(z \mid m)}{\operatorname{cs}(z \mid m)}$$

Involving cs and sn

09.31.27.0008.01

$$\mathrm{nc}(z \mid m) = \frac{1}{\mathrm{sn}(z \mid m)\,\mathrm{cs}(z \mid m)}$$

09.31.27.0040.01

$$\mathrm{nc}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}{\mathrm{cs}(z \mid m)}$$

Involving dc and dn

09.31.27.0010.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dc}(z \mid m)}{\mathrm{dn}(z \mid m)}$$

09.31.27.0041.01

$$\mathrm{nc}(z \mid m) = \frac{\left(m - \mathrm{dc}(z \mid m)^2\right)\mathrm{dn}(z \mid m)}{(m-1)\,\mathrm{dc}(z \mid m)}$$

Involving dc and nd

09.31.27.0011.01

$$\mathrm{nc}(z \mid m) = \mathrm{nd}(z \mid m)\,\mathrm{dc}(z \mid m)$$

Involving ns and sc

09.31.27.0016.01

$$\mathrm{nc}(z \mid m) = \mathrm{ns}(z \mid m)\,\mathrm{sc}(z \mid m)$$

Involving sc and sn

09.31.27.0018.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{sc}(z \mid m)}{\mathrm{sn}(z \mid m)}$$

09.31.27.0042.01

$$\mathrm{nc}(z \mid m) = \frac{\left(\mathrm{sc}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}{\mathrm{sc}(z \mid m)}$$

Involving three other Jacobi elliptic functions

09.31.27.0043.01

$$\mathrm{nc}(z \mid m) = -\frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{dn}(z \mid m)}{\mathrm{cd}(z \mid m)\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right)}$$

09.31.27.0044.01

$$\mathrm{nc}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{dn}(z \mid m)}{\mathrm{cs}(z \mid m)^2\,\mathrm{dc}(z \mid m)}$$

09.31.27.0045.01

$$\mathrm{nc}(z \mid m) = \frac{m\,\mathrm{cn}(z \mid m) - \mathrm{dc}(z \mid m)\,\mathrm{dn}(z \mid m)}{m-1}$$

09.31.27.0046.01

$$\operatorname{nc}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\operatorname{cs}(z \mid m) \operatorname{ds}(z \mid m)}$$

09.31.27.0047.01

$$\operatorname{nc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m)^2}{(\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m)) (\operatorname{dn}(z \mid m) + \operatorname{ds}(z \mid m))}$$

09.31.27.0048.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \left(\operatorname{ds}(z \mid m)^2 + m\right)}{\operatorname{dc}(z \mid m) \left(\operatorname{ds}(z \mid m)^2 + m - 1\right)}$$

09.31.27.0049.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{cd}(z \mid m)^2 \operatorname{ds}(z \mid m)^2 + 1\right)}{\operatorname{cd}(z \mid m)^2 \operatorname{ds}(z \mid m)^2}$$

09.31.27.0050.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \left(\operatorname{cd}(z \mid m)^2 \operatorname{ds}(z \mid m)^2 + 1\right)}{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2}$$

09.31.27.0051.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m)^2 \operatorname{nd}(z \mid m)}$$

09.31.27.0052.01

$$\operatorname{nc}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m)}{(m - 1) \operatorname{nd}(z \mid m)}$$

09.31.27.0053.01

$$\operatorname{nc}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) \operatorname{nd}(z \mid m)}{\operatorname{cs}(z \mid m)^2 (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}$$

09.31.27.0054.01

$$\operatorname{nc}(z \mid m) = \frac{m \operatorname{cd}(z \mid m)}{\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.31.27.0055.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m)}{(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) - 1) (\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) + 1)}$$

09.31.27.0056.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m)^2}{(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) - 1) (\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) + 1)}$$

09.31.27.0057.01

$$\operatorname{nc}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) \operatorname{nd}(z \mid m)}{(\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

09.31.27.0058.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m)}{(\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

09.31.27.0059.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\,\mathrm{ns}(z \mid m)^2}{\mathrm{dc}(z \mid m)\,(\mathrm{ns}(z \mid m) - 1)\,(\mathrm{ns}(z \mid m) + 1)}$$

09.31.27.0060.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,\mathrm{ns}(z \mid m)^2}{\mathrm{nd}(z \mid m)\,(\mathrm{ns}(z \mid m) - 1)\,(\mathrm{ns}(z \mid m) + 1)}$$

09.31.27.0061.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\,\mathrm{ns}(z \mid m)^2}{\mathrm{cd}(z \mid m)\left(\mathrm{ns}(z \mid m)^2 - m\right)}$$

09.31.27.0062.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{ds}(z \mid m)^2 + m\right)}{\mathrm{ds}(z \mid m)\left(\mathrm{ds}(z \mid m)^2 + m - 1\right)\mathrm{sc}(z \mid m)}$$

09.31.27.0063.01

$$\mathrm{nc}(z \mid m) = -\frac{(\mathrm{dn}(z \mid m) - 1)\,(\mathrm{dn}(z \mid m) + 1)\,\mathrm{ns}(z \mid m)}{\left(\mathrm{dn}(z \mid m)^2 + m - 1\right)\mathrm{sc}(z \mid m)}$$

09.31.27.0064.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,(\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.31.27.0065.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{ns}(z \mid m) + \mathrm{sc}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.31.27.0066.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{dc}(z \mid m)}$$

09.31.27.0067.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{nd}(z \mid m)}$$

09.31.27.0068.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{ds}(z \mid m)\,\mathrm{sc}(z \mid m)}$$

09.31.27.0069.01

$$\mathrm{nc}(z \mid m) = -\frac{\mathrm{dn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{cd}(z \mid m)\left(m\,\mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}$$

09.31.27.0070.01

$$\mathrm{nc}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\left(m\,\mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}{\mathrm{dn}(z \mid m)}$$

09.31.27.0071.01

$$\mathrm{nc}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{dn}(z \mid m)}{\mathrm{cs}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 - m + 1\right)\mathrm{sd}(z \mid m)}$$

09.31.27.0072.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ns}(z \mid m)}{(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)\operatorname{sd}(z \mid m)}$$

09.31.27.0073.01

$$\operatorname{nc}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)\operatorname{sd}(z \mid m)}$$

09.31.27.0074.01

$$\operatorname{nc}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sd}(z \mid m)}{\operatorname{cs}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.31.27.0075.01

$$\operatorname{nc}(z \mid m) = \frac{m\operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{cs}(z \mid m)(\operatorname{nd}(z \mid m) - 1)(\operatorname{nd}(z \mid m) + 1)}$$

09.31.27.0076.01

$$\operatorname{nc}(z \mid m) = \frac{m\operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m)}{\left(m\operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right)\operatorname{sc}(z \mid m)}$$

09.31.27.0077.01

$$\operatorname{nc}(z \mid m) = -\frac{m\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{(\operatorname{dn}(z \mid m) - 1)(\operatorname{dn}(z \mid m) + 1)}$$

09.31.27.0078.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sd}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.31.27.0079.01

$$\operatorname{nc}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sd}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.31.27.0080.01

$$\operatorname{nc}(z \mid m) = -\frac{\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)\operatorname{sd}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.31.27.0081.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m)}{(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}$$

09.31.27.0082.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{nd}(z \mid m)^2}{(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}$$

09.31.27.0083.01

$$\operatorname{nc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)}{(\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m) - 1)(\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m) + 1)}$$

09.31.27.0084.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(\operatorname{cd}(z \mid m)^2 + \operatorname{sd}(z \mid m)^2\right)}{\operatorname{cd}(z \mid m)^2}$$

09.31.27.0085.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{cd}(z \mid m)^2 + \mathrm{sd}(z \mid m)^2\right)}{\mathrm{cd}(z \mid m)}$$

09.31.27.0086.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(m\,\mathrm{sd}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2\right)}{(m-1)\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.31.27.0087.01

$$\mathrm{nc}(z \mid m) = -\frac{m\,\mathrm{sn}(z \mid m)}{\mathrm{cs}(z \mid m)\,(\mathrm{dn}(z \mid m) - 1)\,(\mathrm{dn}(z \mid m) + 1)}$$

09.31.27.0088.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dc}(z \mid m)\left(\mathrm{ds}(z \mid m)^2 + m\right)\mathrm{sn}(z \mid m)}{\mathrm{ds}(z \mid m)}$$

09.31.27.0089.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{ds}(z \mid m)\left(\mathrm{ds}(z \mid m)^2 + m\right)\mathrm{sn}(z \mid m)}{\mathrm{dc}(z \mid m)\left(\mathrm{ds}(z \mid m)^2 + m - 1\right)}$$

09.31.27.0090.01

$$\mathrm{nc}(z \mid m) = \frac{\left(\mathrm{cd}(z \mid m)^2\,\mathrm{ds}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}{\mathrm{cd}(z \mid m)\,\mathrm{ds}(z \mid m)}$$

09.31.27.0091.01

$$\mathrm{nc}(z \mid m) = \frac{\left(m\,\mathrm{cd}(z \mid m)^2 - 1\right)\mathrm{sn}(z \mid m)}{(m-1)\,\mathrm{cd}(z \mid m)^2\,\mathrm{sc}(z \mid m)}$$

09.31.27.0092.01

$$\mathrm{nc}(z \mid m) = \frac{\left(m - \mathrm{dc}(z \mid m)^2\right)\mathrm{sn}(z \mid m)}{(m-1)\,\mathrm{sc}(z \mid m)}$$

09.31.27.0093.01

$$\mathrm{nc}(z \mid m) = \frac{m\,\mathrm{sn}(z \mid m)}{\left(\mathrm{dn}(z \mid m)^2 + m - 1\right)\mathrm{sc}(z \mid m)}$$

09.31.27.0094.01

$$\mathrm{nc}(z \mid m) = \frac{\left(\mathrm{ds}(z \mid m)^2 + m\right)\mathrm{sn}(z \mid m)}{\left(\mathrm{ds}(z \mid m)^2 + m - 1\right)\mathrm{sc}(z \mid m)}$$

09.31.27.0095.01

$$\mathrm{nc}(z \mid m) = \frac{m\,\mathrm{nd}(z \mid m)^2\,\mathrm{sn}(z \mid m)}{\left(m\,\mathrm{nd}(z \mid m)^2 - \mathrm{nd}(z \mid m)^2 + 1\right)\mathrm{sc}(z \mid m)}$$

09.31.27.0096.01

$$\mathrm{nc}(z \mid m) = (\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m))\,\mathrm{sn}(z \mid m)$$

09.31.27.0097.01

$$\mathrm{nc}(z \mid m) = \frac{\left(m - \mathrm{dc}(z \mid m)^2\right)\mathrm{sn}(z \mid m)}{(m-1)\,\mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.31.27.0098.01

$$\operatorname{nc}(z \mid m) = \frac{\left(\operatorname{cd}(z \mid m)^2 + \operatorname{sd}(z \mid m)^2\right) \operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m) \operatorname{sd}(z \mid m)}$$

09.31.27.0099.01

$$\operatorname{nc}(z \mid m) = \frac{\left(m \operatorname{sd}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}{\operatorname{sc}(z \mid m) \left(m \operatorname{sd}(z \mid m)^2 - \operatorname{sd}(z \mid m)^2 + 1\right)}$$

09.31.27.0100.01

$$\operatorname{nc}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)}{\operatorname{dc}(z \mid m) (\operatorname{sn}(z \mid m) - 1) (\operatorname{sn}(z \mid m) + 1)}$$

09.31.27.0101.01

$$\operatorname{nc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)}{\operatorname{nd}(z \mid m) (\operatorname{sn}(z \mid m) - 1) (\operatorname{sn}(z \mid m) + 1)}$$

09.31.27.0102.01

$$\operatorname{nc}(z \mid m) = \operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m)$$

Involving four other Jacobi elliptic functions

09.31.27.0103.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)\right)}{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2}$$

09.31.27.0104.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \left(\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)\right)}{\operatorname{ds}(z \mid m)^2}$$

09.31.27.0105.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m) \operatorname{dn}(z \mid m)}{\operatorname{ds}(z \mid m)^2}$$

09.31.27.0106.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dc}(z \mid m) + \operatorname{cs}(z \mid m) \operatorname{ds}(z \mid m)}{\operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m)}$$

09.31.27.0107.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m)}$$

09.31.27.0108.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{ds}(z \mid m)}{\operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.31.27.0109.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2}{\operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.31.27.0110.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{nd}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m)}$$

09.31.27.0111.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.31.27.0112.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2}{\operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.31.27.0113.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.31.27.0114.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dc}(z \mid m) + \operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.31.27.0115.01

$$\operatorname{nc}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m)) \operatorname{ns}(z \mid m)}{(\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)) \operatorname{sc}(z \mid m)}$$

09.31.27.0116.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) (\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m) \operatorname{dc}(z \mid m)}$$

09.31.27.0117.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) (\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ds}(z \mid m)}$$

09.31.27.0118.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) (\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m) \operatorname{nd}(z \mid m)}$$

09.31.27.0119.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) (\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)}$$

09.31.27.0120.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) (\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ds}(z \mid m)}$$

09.31.27.0121.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m) + \operatorname{sc}(z \mid m)}{\operatorname{ns}(z \mid m)}$$

09.31.27.0122.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) (\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cd}(z \mid m) (\operatorname{cs}(z \mid m) - m \operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.31.27.0123.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\, \mathrm{ns}(z \mid m) + m\, \mathrm{cd}(z \mid m)\, \mathrm{sc}(z \mid m)}{\mathrm{cd}(z \mid m)\, \mathrm{ns}(z \mid m)}$$

09.31.27.0124.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\, \mathrm{ns}(z \mid m) + \mathrm{dc}(z \mid m)\, \mathrm{sc}(z \mid m)}{\mathrm{dc}(z \mid m)\, \mathrm{ns}(z \mid m)}$$

09.31.27.0125.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\, \mathrm{ds}(z \mid m) + \mathrm{dn}(z \mid m)\, \mathrm{sc}(z \mid m)}{\mathrm{ds}(z \mid m)}$$

09.31.27.0126.01

$$\mathrm{nc}(z \mid m) = \frac{m\, \mathrm{cn}(z \mid m) - \mathrm{dn}(z \mid m)\, \mathrm{ds}(z \mid m)\, \mathrm{sc}(z \mid m)}{m-1}$$

09.31.27.0127.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\, \mathrm{ns}(z \mid m) + \mathrm{nd}(z \mid m)\, \mathrm{sc}(z \mid m)}{\mathrm{nd}(z \mid m)\, \mathrm{ns}(z \mid m)}$$

09.31.27.0128.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\, \mathrm{nd}(z \mid m)}{\mathrm{ds}(z \mid m)\, \mathrm{nd}(z \mid m)^2 - \mathrm{sd}(z \mid m)}$$

09.31.27.0129.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\, \mathrm{ns}(z \mid m)}{\mathrm{nd}(z \mid m)\, \mathrm{ns}(z \mid m) - \mathrm{sd}(z \mid m)}$$

09.31.27.0130.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\, \mathrm{nd}(z \mid m)\, \mathrm{ns}(z \mid m)}{\mathrm{nd}(z \mid m)\, \mathrm{ns}(z \mid m) - \mathrm{sd}(z \mid m)}$$

09.31.27.0131.01

$$\mathrm{nc}(z \mid m) = -\frac{\mathrm{dn}(z \mid m)\, (\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m))}{\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right) \mathrm{sd}(z \mid m)}$$

09.31.27.0132.01

$$\mathrm{nc}(z \mid m) = \frac{m\, \mathrm{cs}(z \mid m)\, \mathrm{sd}(z \mid m)}{\mathrm{dn}(z \mid m) + m\, \mathrm{nd}(z \mid m) - \mathrm{nd}(z \mid m)}$$

09.31.27.0133.01

$$\mathrm{nc}(z \mid m) = -\frac{m\, \mathrm{sc}(z \mid m)\, \mathrm{sd}(z \mid m)}{\mathrm{dn}(z \mid m) - \mathrm{nd}(z \mid m)}$$

09.31.27.0134.01

$$\mathrm{nc}(z \mid m) = \frac{(\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m))\, \mathrm{sd}(z \mid m)}{\mathrm{nd}(z \mid m)}$$

09.31.27.0135.01

$$\mathrm{nc}(z \mid m) = \frac{(\mathrm{cs}(z \mid m) - m\, \mathrm{sc}(z \mid m) + \mathrm{sc}(z \mid m))\, \mathrm{sd}(z \mid m)}{\mathrm{dn}(z \mid m)}$$

09.31.27.0136.01

$$\mathrm{nc}(z \mid m) = \frac{m\, \mathrm{sd}(z \mid m) - \mathrm{dc}(z \mid m)\, \mathrm{sc}(z \mid m)}{(m-1)\, \mathrm{nd}(z \mid m)\, \mathrm{sc}(z \mid m)}$$

09.31.27.0137.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{ns}(z \mid m) + m\,\mathrm{cd}(z \mid m)\,\mathrm{sd}(z \mid m)}{(\mathrm{ns}(z \mid m) - 1)\,\mathrm{ns}(z \mid m)\,(\mathrm{ns}(z \mid m) + 1)\,\mathrm{sd}(z \mid m)^2}$$

09.31.27.0138.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{ns}(z \mid m) + \mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.31.27.0139.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{sd}(z \mid m)\,\mathrm{dc}(z \mid m)^2 + \mathrm{dn}(z \mid m)\,\mathrm{ns}(z \mid m)}{\mathrm{dc}(z \mid m)\,\mathrm{ns}(z \mid m)}$$

09.31.27.0140.01

$$\mathrm{nc}(z \mid m) = -\frac{\mathrm{dn}(z \mid m)\,\mathrm{sc}(z \mid m)^2 + \mathrm{dn}(z \mid m) - \mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m)}{(m - 1)\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.31.27.0141.01

$$\mathrm{nc}(z \mid m) = \frac{m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{dn}(z \mid m)}$$

09.31.27.0142.01

$$\mathrm{nc}(z \mid m) = \frac{m\,\mathrm{cd}(z \mid m)}{\mathrm{dn}(z \mid m) + m\,\mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.31.27.0143.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,(\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m))}{\mathrm{cd}(z \mid m)}$$

09.31.27.0144.01

$$\mathrm{nc}(z \mid m) = \mathrm{dn}(z \mid m)\,(\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m))$$

09.31.27.0145.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{nd}(z \mid m)}$$

09.31.27.0146.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.31.27.0147.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{nd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{nd}(z \mid m)}$$

09.31.27.0148.01

$$\mathrm{nc}(z \mid m) = \frac{\mathrm{cd}(z \mid m) - m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{dn}(z \mid m)}$$

09.31.27.0149.01

$$\mathrm{nc}(z \mid m) = -\frac{\mathrm{dn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{-\mathrm{cd}(z \mid m) + m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.31.27.0150.01

$$\mathrm{nc}(z \mid m) = \mathrm{cn}(z \mid m) + \mathrm{dn}(z \mid m)\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)$$

09.31.27.0151.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cd}(z \mid m)\right)}{\operatorname{cd}(z \mid m)}$$

09.31.27.0152.01

$$\operatorname{nc}(z \mid m) = \operatorname{dn}(z \mid m)\left(\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cd}(z \mid m)\right)$$

09.31.27.0153.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cd}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

09.31.27.0154.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cd}(z \mid m)}{\operatorname{ns}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.31.27.0155.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cn}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

09.31.27.0156.01

$$\operatorname{nc}(z \mid m) = \operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cn}(z \mid m)$$

09.31.27.0157.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.31.27.0158.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.31.27.0159.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.31.27.0160.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m))}$$

09.31.27.0161.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nd}(z \mid m)(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m))}$$

09.31.27.0162.01

$$\operatorname{nc}(z \mid m) = \frac{\left(\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)\right)\operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

09.31.27.0163.01

$$\operatorname{nc}(z \mid m) = (\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sn}(z \mid m)$$

09.31.27.0164.01

$$\operatorname{nc}(z \mid m) = \frac{\left(\operatorname{ds}(z \mid m)^2 + m\right)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)(\operatorname{ds}(z \mid m) + m\operatorname{sd}(z \mid m) - \operatorname{sd}(z \mid m))}$$

09.31.27.0165.01

$$\operatorname{nc}(z \mid m) = \frac{(\operatorname{cd}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m))\operatorname{sn}(z \mid m)}{\operatorname{sd}(z \mid m)}$$

09.31.27.0166.01

$$\operatorname{nc}(z \mid m) = \frac{\left(\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cd}(z \mid m)\right)\operatorname{sn}(z \mid m)}{\operatorname{sd}(z \mid m)}$$

09.31.27.0167.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)}$$

09.31.27.0168.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m)\operatorname{cd}(z \mid m)^2 + \operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)}$$

09.31.27.0169.01

$$\operatorname{nc}(z \mid m) = \frac{m\operatorname{cd}(z \mid m)\operatorname{sd}(z \mid m)}{m\operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m) + \operatorname{sn}(z \mid m)}$$

09.31.27.0170.01

$$\operatorname{nc}(z \mid m) = \frac{m\operatorname{cd}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{sd}(z \mid m)^2\operatorname{ns}(z \mid m)^3 - \operatorname{sd}(z \mid m)^2\operatorname{ns}(z \mid m) - \operatorname{ns}(z \mid m) + \operatorname{sn}(z \mid m)}$$

09.31.27.0171.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dc}(z \mid m)(\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m) + m\operatorname{sn}(z \mid m))}{\operatorname{ds}(z \mid m)}$$

09.31.27.0172.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m) + m\operatorname{sn}(z \mid m)}{\left(\operatorname{ds}(z \mid m)^2 + m - 1\right)\operatorname{sc}(z \mid m)}$$

09.31.27.0173.01

$$\operatorname{nc}(z \mid m) = \frac{m\operatorname{sn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}{(m-1)\operatorname{sc}(z \mid m)}$$

09.31.27.0174.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)\operatorname{dc}(z \mid m)}$$

09.31.27.0175.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m) + \operatorname{nd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.31.27.0176.01

$$\operatorname{nc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + m\operatorname{cd}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)^2}$$

09.31.27.0177.01

$$\operatorname{nc}(z \mid m) = \operatorname{cn}(z \mid m) + \operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m)$$

Involving five other Jacobi elliptic functions

09.31.27.0178.01

$$\operatorname{nc}(z\mid m)=\frac{\operatorname{cn}(z\mid m)\operatorname{ns}(z\mid m)+m\operatorname{cd}(z\mid m)\operatorname{sd}(z\mid m)}{\operatorname{nd}(z\mid m)\,(\operatorname{nd}(z\mid m)\operatorname{ns}(z\mid m)-\operatorname{sd}(z\mid m))}$$

09.31.27.0179.01

$$\operatorname{nc}(z\mid m)=-\frac{m\operatorname{cd}(z\mid m)\operatorname{sd}(z\mid m)}{-\operatorname{ns}(z\mid m)\operatorname{nd}(z\mid m)^2+\operatorname{sd}(z\mid m)\operatorname{nd}(z\mid m)+\operatorname{ns}(z\mid m)-\operatorname{sn}(z\mid m)}$$

09.31.27.0180.01

$$\operatorname{nc}(z\mid m)=\frac{\operatorname{cn}(z\mid m)+m\operatorname{cd}(z\mid m)\operatorname{sd}(z\mid m)\operatorname{sn}(z\mid m)}{(\operatorname{ns}(z\mid m)-1)\,(\operatorname{ns}(z\mid m)+1)\operatorname{sd}(z\mid m)^2}$$

09.31.27.0181.01

$$\operatorname{nc}(z\mid m)=\frac{\operatorname{cn}(z\mid m)+m\operatorname{cd}(z\mid m)\operatorname{sd}(z\mid m)\operatorname{sn}(z\mid m)}{(\operatorname{nd}(z\mid m)-\operatorname{sd}(z\mid m))\,(\operatorname{nd}(z\mid m)+\operatorname{sd}(z\mid m))}$$

Involving Weierstrass functions

09.31.27.0022.01

$$\operatorname{nc}(z\mid m)=\frac{\sigma_3\left(\frac{z}{\sqrt{e_1-e_3}};\,g_2,\,g_3\right)}{\sigma_1\left(\frac{z}{\sqrt{e_1-e_3}};\,g_2,\,g_3\right)}\;/;$$

$$\{\omega_1,\,\omega_2,\,\omega_3\}=\{\omega_1(g_2,\,g_3),\,-\omega_1(g_2,\,g_3)-\omega_3(g_2,\,g_3),\,\omega_3(g_2,\,g_3)\}\bigwedge m=\lambda\left(\frac{\omega_3}{\omega_1}\right)\bigwedge e_n=\wp(\omega_n;\,g_2,\,g_3)\bigwedge n\in\{1,\,2,\,3\}$$

09.31.27.0023.01

$$\operatorname{nc}(z\mid m)^2=\frac{\wp\left(\frac{z}{\sqrt{e_1-e_3}};\,g_2,\,g_3\right)-e_3}{\wp\left(\frac{z}{\sqrt{e_1-e_3}};\,g_2,\,g_3\right)-e_1}\;/;$$

$$\{\omega_1,\,\omega_2,\,\omega_3\}=\{\omega_1(g_2,\,g_3),\,-\omega_1(g_2,\,g_3)-\omega_3(g_2,\,g_3),\,\omega_3(g_2,\,g_3)\}\bigwedge m=\lambda\left(\frac{\omega_3}{\omega_1}\right)\bigwedge e_n=\wp(\omega_n;\,g_2,\,g_3)\bigwedge n\in\{1,\,2,\,3\}$$

Involving theta functions

09.31.27.0024.02

$$\operatorname{nc}(z\mid m)=\frac{\sqrt[4]{m}}{\sqrt[4]{1-m}}\,\frac{\vartheta_4\left(\frac{\pi z}{2K(m)},\,q(m)\right)}{\vartheta_2\left(\frac{\pi z}{2K(m)},\,q(m)\right)}$$

09.31.27.0025.01

$$\operatorname{nc}(z\mid m)=\frac{\vartheta_2(0,\,q(m))}{\vartheta_4(0,\,q(m))}\,\frac{\vartheta_4\left(\frac{z}{\vartheta_3(0,q(m))^2},\,q(m)\right)}{\vartheta_2\left(\frac{z}{\vartheta_3(0,q(m))^2},\,q(m)\right)}$$

09.31.27.0026.01

$$\operatorname{nc}(z\mid m)=\frac{\vartheta_n(z\mid m)}{\vartheta_c(z\mid m)}$$

Zeros

09.31.30.0001.01

$\mathrm{nc}(2\, r\, K(m) + (2\, s + 1)\, i\, K(1 - m) \mid m) == 0 \,/;\, \{r, s\} \in \mathbb{Z}$

History

– C. G. J. Jacobi (1827)

– N. H. Abel (1827)

– J. Glaisher (1882) introduced the notation nc

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