JacobiND

Notations

Traditional name

Jacobi elliptic function nd

Traditional notation

$\operatorname{nd}(z \mid m)$

Mathematica StandardForm notation

```
JacobiND[z, m]
```

Primary definition

09.32.02.0001.01

$$\operatorname{nd}(z \mid m) = \frac{1}{\operatorname{dn}(z \mid m)}$$

Specific values

Specialized values

For fixed *z*

Case $m = 0$

09.32.03.0001.01

$$\operatorname{nd}(z \mid 0) = 1$$

Case $m = 1$

09.32.03.0002.01

$$\operatorname{nd}(z \mid 1) = \cosh(z)$$

09.32.03.0003.01

$$\operatorname{nd}\left(z + \frac{\pi i}{2} \,\middle|\, 1\right) = i \sinh(z)$$

09.32.03.0028.01

$$\operatorname{nd}\left(z + \frac{i \pi k}{2} \,\middle|\, 1\right) = \cosh\left(z + \frac{i \pi k}{2}\right) /; k \in \mathbb{Z}$$

For fixed *m*

Values at quarter-period points in the fundamental period parallelogram

09.32.03.0004.01

$$\mathrm{nd}(0 \mid m) = 1$$

09.32.03.0005.01

$$\mathrm{nd}(K(m) \mid m) = \frac{1}{\sqrt{1-m}}$$

09.32.03.0006.01

$$\mathrm{nd}(2\,K(m) \mid m) = 1$$

09.32.03.0007.01

$$\mathrm{nd}(3\,K(m) \mid m) = \frac{1}{\sqrt{1-m}}$$

09.32.03.0008.01

$$\mathrm{nd}(4\,K(m) \mid m) = 1$$

09.32.03.0009.01

$$\mathrm{nd}(i\,K(1-m) \mid m) = 0$$

09.32.03.0010.01

$$\mathrm{nd}(2\,i\,K(1-m) \mid m) = -1$$

09.32.03.0011.01

$$\mathrm{nd}(3\,i\,K(1-m) \mid m) = 0$$

09.32.03.0012.01

$$\mathrm{nd}(4\,i\,K(1-m) \mid m) = 1$$

09.32.03.0013.01

$$\mathrm{nd}(K(m) + i\,K(1-m) \mid m) = \tilde{\infty}$$

09.32.03.0014.01

$$\mathrm{nd}(2\,K(m) + i\,K(1-m) \mid m) = 0$$

09.32.03.0015.01

$$\mathrm{nd}(3\,K(m) + i\,K(1-m) \mid m) = \tilde{\infty}$$

09.32.03.0016.01

$$\mathrm{nd}(4\,K(m) + i\,K(1-m) \mid m) = 0$$

09.32.03.0017.01

$$\mathrm{nd}(K(m) + 2\,i\,K(1-m) \mid m) = -\frac{1}{\sqrt{1-m}}$$

09.32.03.0018.01

$$\mathrm{nd}(2\,K(m) + 2\,i\,K(1-m) \mid m) = -1$$

09.32.03.0019.01

$$\mathrm{nd}(3\,K(m) + 2\,i\,K(1-m) \mid m) = -\frac{1}{\sqrt{1-m}}$$

09.32.03.0020.01

$$\mathrm{nd}(4\,K(m) + 2\,i\,K(1-m) \mid m) = -1$$

09.32.03.0021.01

$$\operatorname{nd}(K(m)+3\,i\,K(1-m)\mid m)=\tilde{\infty}$$

09.32.03.0022.01

$$\operatorname{nd}((2\,r+1)\,K(m)+i\,(2\,s+1)\,K(1-m)\mid m)=\tilde{\infty}\;/;\;\{r,s\}\in\mathbb{Z}$$

09.32.03.0023.01

$$\operatorname{nd}(K(m)+4\,i\,K(1-m)\mid m)=\frac{1}{\sqrt{1-m}}$$

09.32.03.0024.01

$$\operatorname{nd}(2\,K(m)+4\,i\,K(1-m)\mid m)=1$$

Values at half-quarter-period points

09.32.03.0025.01

$$\operatorname{nd}\left(\frac{K(m)}{2}\,\middle|\,m\right)=\frac{1}{\sqrt[4]{1-m}}$$

09.32.03.0026.01

$$\operatorname{nd}\left(\frac{i\,K(1-m)}{2}\,\middle|\,m\right)=\frac{1}{\sqrt{1+\sqrt{m}}}$$

09.32.03.0027.01

$$\operatorname{nd}\left(\frac{K(m)}{2}+\frac{i\,K(1-m)}{2}\,\middle|\,m\right)=\frac{\sqrt{2}}{\sqrt[4]{1-m}}\left(\sqrt{1+\sqrt{1-m}}-i\sqrt{1-\sqrt{1-m}}\right)^{-1}$$

General characteristics

Domain and analyticity

$\operatorname{nd}(z\mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.32.04.0001.01

$$(z*m)\longrightarrow\operatorname{nd}(z\mid m)::(\mathbb{C}\otimes\mathbb{C})\longrightarrow\mathbb{C}$$

Symmetries and periodicities

Parity

$\operatorname{nd}(z\mid m)$ is an even function with respect to z.

09.32.04.0002.01

$$\operatorname{nd}(-z\mid m)=\operatorname{nd}(z\mid m)$$

Mirror symmetry

09.32.04.0003.01

$$\operatorname{nd}(\bar{z}\mid\bar{m})=\overline{\operatorname{nd}(z\mid m)}$$

Periodicity

$\operatorname{nd}(z\mid m)$ is a doubly periodic function with respect to z with periods $4\,i\,K(1-m)$ and $2\,K(m)$.

09.32.04.0004.01

$\mathrm{nd}(z+2K(m)\mid m)=\mathrm{nd}(z\mid m)$

09.32.04.0005.01

$\mathrm{nd}(z+2iK(1-m)\mid m)=-\mathrm{nd}(z\mid m)$

09.32.04.0006.01

$\mathrm{nd}(z+4iK(1-m)\mid m)=\mathrm{nd}(z\mid m)$

09.32.04.0007.01

$\mathrm{nd}(z+2K(m)+2iK(1-m)\mid m)=-\mathrm{nd}(z\mid m)$

09.32.04.0008.01

$\mathrm{nd}(z+2isK(1-m)+2rK(m)\mid m)=(-1)^s\,\mathrm{nd}(z\mid m)\;/;\;\{r,s\}\in\mathbb{Z}$

Poles and essential singularities

With respect to *z*

For fixed m, the function $\mathrm{nd}(z\mid m)$ has an infinite set of singular points:

a) $z=(2r+1)K(m)+i(2s+1)K(1-m)$, $\{r,s\}\in\mathbb{Z}$, are the simple poles with residues $\frac{(-1)^{s-1}i}{\sqrt{1-m}}$;

b) $z=\tilde{\infty}$ is an essential singular point.

09.32.04.0009.01

$\mathcal{S}ing_z(\mathrm{nd}(z\mid m))=\{\{\{(2s+1)iK(1-m)+(2r+1)K(m),1\}\;/;\;\{r,s\}\in\mathbb{Z}\},\{\tilde{\infty},\infty\}\}$

09.32.04.0010.01

$$\mathrm{res}_z(\mathrm{nd}(z\mid m))((2s+1)iK(1-m)+(2r+1)K(m))=\frac{(-1)^{s-1}i}{\sqrt{1-m}}\;/;\;\{r,s\}\in\mathbb{Z}$$

Branch points

With respect to *m*

For fixed z, the function $\mathrm{nd}(z\mid m)$ is a meromorphic function in m that has no branch points.

09.32.04.0013.01

$\mathcal{BP}_m(\mathrm{nd}(z\mid m))=\{\}$

P. Walker

With respect to *z*

For fixed m, the function $\mathrm{nd}(z\mid m)$ does not have branch points.

09.32.04.0011.01

$\mathcal{BP}_z(\mathrm{nd}(z\mid m))=\{\}$

Branch cuts

With respect to *m*

For fixed z, the function $\mathrm{nd}(z\mid m)$ is a meromorphic function in m that has no branch cuts.

09.32.04.0014.01

$\mathcal{BC}_m(\text{nd}(z \mid m)) = \{\}$

P. Walker

With respect to *z*

For fixed m, the function $\text{nd}(z \mid m)$ does not have branch cuts.

09.32.04.0012.01

$\mathcal{BC}_z(\text{nd}(z \mid m)) = \{\}$

Series representations

Generalized power series

Expansions at *z* == 0

09.32.06.0005.01

$$\text{nd}(z \mid m) \propto 1 + \frac{m z^2}{2} + \frac{1}{24}\left(-4 m + 5 m^2\right) z^4 + \ldots \;/;\; (z \to 0)$$

09.32.06.0001.02

$$\text{nd}(z \mid m) \propto 1 + \frac{m z^2}{2} + \frac{1}{24}\left(-4 m + 5 m^2\right) z^4 + \frac{1}{720}\left(16 m - 76 m^2 + 61 m^3\right) z^6 + \frac{\left(-64 m + 1104 m^2 - 2424 m^3 + 1385 m^4\right) z^8}{40320} + \frac{\left(256 m - 16832 m^2 + 79728 m^3 - 113672 m^4 + 50521 m^5\right) z^{10}}{3628800} + O\left(z^{12}\right)$$

09.32.06.0006.01

$$\text{nd}(z \mid m) = \sum_{k=0}^{\infty} (k+1) \sum_{r=0}^{k} \frac{(-1)^r}{r+1} \binom{k}{r} p_{r,k}\, z^{2k} \;/;\; p_{j,0} = 1 \bigwedge p_{j,k} = \frac{1}{k} \sum_{i=1}^{k} \frac{(j\, i + i - k)\, (-1)^i\, \text{dn}_i(m)\, p_{j,k-i}}{(2 i)!} \bigwedge$$
$$k \in \mathbb{N}^+ \bigwedge \text{sn}_0(m) = 1 \bigwedge \text{sn}_n(m) = \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{2 n}{2 j} \text{cn}_j(m)\, \text{dn}_k(m)\, \delta_{j+k-n} \bigwedge \text{cn}_0(m) = 1 \bigwedge$$
$$\text{cn}_n(m) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2 n - 1}{2 j + 1} \text{sn}_j(m)\, \text{dn}_k(m)\, \delta_{j+k-n+1} \bigwedge \text{dn}_0(m) = 1 \bigwedge \text{dn}_n(m) = m \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2 n - 1}{2 j + 1} \text{sn}_j(m)\, \text{cn}_k(m)\, \delta_{j+k-n+1}$$

09.32.06.0007.01

$$\text{nd}(z \mid m) \propto 1 + O\left(z^2\right)$$

Expansions at *z* == (2 *r* + 1) *K*(*m*) + (2 *s* + 1) *i* *K*(1 − *m*)

09.32.06.0008.01

$$\text{nd}(z \mid m) \propto \frac{i\, (-1)^{s-1}}{\sqrt{1-m}} \left(\frac{1}{z - z_0} + \frac{1}{6} (m-2)\, (z - z_0) + \frac{1}{360}\left(7 m^2 + 8 m - 8\right) (z - z_0)^3 + \ldots\right) /;$$
$$(z \to z_0) \bigwedge z_0 = (2 r + 1)\, K(m) + (2 s + 1)\, i\, K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

09.32.06.0009.01

$$\operatorname{nd}(z\mid m)=\frac{i\,(-1)^{s-1}}{\sqrt{1-m}}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(j+1)\,(-1)^{k-j}\operatorname{cn}_{k-j}(m)}{(2k-2j)!}\sum_{r=0}^{j}\frac{(-1)^r}{r+1}\binom{j}{r}q_{r,j}\,(z-z_0)^{2k-1}\;/;$$

$$z_0=(2r+1)K(m)+(2s+1)\,i\,K(1-m)\bigwedge r\in\mathbb{Z}\bigwedge s\in\mathbb{Z}\bigwedge q_{j,0}=1\bigwedge q_{j,k}=\frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i+i-k)\,(-1)^i\operatorname{sn}_i(m)\,q_{j,k-i}}{(2i+1)!}\bigwedge$$

$$k\in\mathbb{N}^+\bigwedge\operatorname{sn}_0(m)=1\bigwedge\operatorname{sn}_n(m)=\sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2n}{2j}\operatorname{cn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n}\bigwedge\operatorname{cn}_0(m)=1\bigwedge$$

$$\operatorname{cn}_n(m)=\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n+1}\bigwedge\operatorname{dn}_0(m)=1\bigwedge\operatorname{dn}_n(m)=m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{cn}_k(m)\,\delta_{j+k-n+1}$$

09.32.06.0010.01

$$\operatorname{nd}(z\mid m)\propto\frac{i\,(-1)^{s-1}}{\sqrt{1-m}\;(z-z_0)}\left(1+O\!\left((z-z_0)^2\right)\right)\;/;\;z_0=(2r+1)K(m)+(2s+1)\,i\,K(1-m)\land r\in\mathbb{Z}\land s\in\mathbb{Z}$$

Expansions at $m == 0$

09.32.06.0011.01

$$\operatorname{nd}(z\mid m)\propto 1+\frac{1}{2}\sin^2(z)\,m-\frac{1}{32}\sin(z)\,(8z\cos(z)-11\sin(z)+\sin(3z))\,m^2+\ldots\;/;\;(m\to 0)$$

09.32.06.0012.01

$$\begin{aligned}
\operatorname{nd}(z\mid m)\propto{}&1+\frac{1}{2}\sin^2(z)\,m-\frac{1}{32}\sin(z)\,(8z\cos(z)-11\sin(z)+\sin(3z))\,m^2-\\
&\frac{1}{1024}\left(-\left(32z^2-159\right)\cos(2z)-20\cos(4z)+\cos(6z)+136z\sin(2z)-16z\sin(4z)-140\right)m^3+\\
&\frac{1}{49\,152}\left(12\left(176z^2-553\right)\cos(2z)-12\left(32z^2-85\right)\cos(4z)-84\cos(6z)+\right.\\
&\quad\left.3\cos(8z)+8z\left(32z^2-789\right)\sin(2z)+1200z\sin(4z)-72z\sin(6z)+5697\right)m^4-\\
&\frac{1}{786\,432}\left(\left(512z^4-37\,632z^2+95\,013\right)\cos(2z)+720\left(16z^2-23\right)\cos(4z)-24\left(36z^2-73\right)\cos(6z)-108\cos(8z)+\right.\\
&\quad\left.3\cos(10z)-96z\left(72z^2-1001\right)\sin(2z)+64z\left(32z^2-363\right)\sin(4z)+2376z\sin(6z)-96z\sin(8z)-80\,100\right)m^5+\\
&\frac{1}{62\,914\,560}\left(-20\left(4096z^4-156\,864z^2+346\,599\right)\cos(2z)+5\left(8192z^4-250\,752z^2+264\,093\right)\cos(4z)+480\left(342z^2-341\right)\right.\\
&\quad\cos(6z)-60\left(128z^2-223\right)\cos(8z)-660\cos(10z)+15\cos(12z)-8z\left(512z^4-87\,360z^2+913\,065\right)\sin(2z)-\\
&\quad\left.80z\left(4480z^2-25\,707\right)\sin(4z)+34\,560z\left(z^2-8\right)\sin(6z)+19\,680z\sin(8z)-600z\sin(10z)+5\,762\,460\right)m^6+\\
&\frac{1}{3\,019\,898\,880}\left(\left(16\,384z^6-5\,568\,000z^4+152\,570\,880z^2-307\,546\,425\right)\cos(2z)+\right.\\
&\quad 60\left(81\,920z^4-1\,205\,568z^2+1\,044\,537\right)\cos(4z)-90\left(6912z^4-140\,688z^2+96\,553\right)\cos(6z)-\\
&\quad 720\left(1472z^2-1183\right)\cos(8z)+720\left(50z^2-79\right)\cos(10z)+2340\cos(12z)-45\cos(14z)-\\
&\quad 48z\left(9472z^4-794\,640z^2+6\,956\,025\right)\sin(2z)+48z\left(8192z^4-542\,080z^2+2\,182\,305\right)\sin(4z)+\\
&\quad\left.1080z\left(4128z^2-15\,533\right)\sin(6z)-1920z\left(128z^2-837\right)\sin(8z)-88\,200z\sin(10z)+2160z\sin(12z)+252\,766\,800\right)\\
&\;m^7+\frac{1}{338\,228\,674\,560}\left(84\left(57\,344z^6-9\,164\,800z^4+203\,281\,440z^2-382\,829\,805\right)\cos(2z)-\right.\\
&\quad 28\left(262\,144z^6-32\,747\,520z^4+326\,691\,360z^2-246\,944\,475\right)\cos(4z)-
\end{aligned}$$

$$2520\left(82\,944\,z^4-766\,584\,z^2+414\,119\right)\cos(6\,z)+420\left(32\,768\,z^4-524\,160\,z^2+277\,311\right)\cos(8\,z)+$$
$$100\,800\left(135\,z^2-94\right)\cos(10\,z)-1260\left(288\,z^2-425\right)\cos(12\,z)-18\,900\cos(14\,z)+315\cos(16\,z)+$$
$$8\,z\left(16\,384\,z^6-9\,988\,608\,z^4+576\,959\,040\,z^2-4\,465\,358\,415\right)\sin(2\,z)+1680\,z\left(73\,728\,z^4-2\,254\,336\,z^2+7\,243\,269\right)$$
$$\sin(4\,z)-3024\,z\left(6912\,z^4-290\,640\,z^2+730\,025\right)\sin(6\,z)-10\,080\,z\left(8704\,z^2-25\,441\right)\sin(8\,z)+$$
$$33\,600\,z\left(100\,z^2-573\right)\sin(10\,z)+861\,840\,z\sin(12\,z)-17\,640\,z\sin(14\,z)+26\,179\,325\,865\Big)\,m^8-$$
$$\frac{1}{5\,411\,658\,792\,960}\Big(2\left(65\,536\,z^8-66\,404\,352\,z^6+7\,085\,084\,160\,z^4-135\,511\,382\,160\,z^2+242\,044\,097\,715\right)\cos(2\,z)+$$
$$560\left(655\,360\,z^6-36\,642\,816\,z^4+284\,593\,104\,z^2-194\,260\,275\right)\cos(4\,z)-$$
$$63\left(1\,327\,104\,z^6-101\,606\,400\,z^4+611\,222\,400\,z^2-278\,728\,555\right)\cos(6\,z)-$$
$$1680\left(458\,752\,z^4-3\,196\,224\,z^2+1\,287\,903\right)\cos(8\,z)+210\left(160\,000\,z^4-2\,180\,400\,z^2+966\,489\right)\cos(10\,z)+$$
$$15\,120\left(1488\,z^2-935\right)\cos(12\,z)-2520\left(196\,z^2-275\right)\cos(14\,z)-21\,420\cos(16\,z)+$$
$$315\cos(18\,z)-8\,z\left(770\,048\,z^6-213\,126\,144\,z^4+9\,695\,105\,280\,z^2-68\,466\,371\,715\right)\sin(2\,z)+$$
$$64\,z\left(262\,144\,z^6-56\,340\,480\,z^4+1\,135\,602\,720\,z^2-3\,112\,652\,025\right)\sin(4\,z)+$$
$$3024\,z\left(366\,336\,z^4-6\,796\,800\,z^2+13\,139\,615\right)\sin(6\,z)-2688\,z\left(32\,768\,z^4-1\,048\,960\,z^2+1\,972\,635\right)\sin(8\,z)-$$
$$21\,000\,z\left(9440\,z^2-23\,283\right)\sin(10\,z)+181\,440\,z\left(32\,z^2-167\right)\sin(12\,z)+$$
$$1\,146\,600\,z\sin(14\,z)-20\,160\,z\sin(16\,z)-390\,888\,160\,740\Big)\,m^9+\frac{1}{779\,278\,866\,186\,240}$$
$$\Big(-72\left(851\,968\,z^8-380\,821\,504\,z^6+31\,281\,384\,960\,z^4-535\,531\,809\,960\,z^2+916\,547\,119\,425\right)\cos(2\,z)+$$
$$9\left(33\,554\,432\,z^8-11\,589\,910\,528\,z^6+417\,380\,597\,760\,z^4-2\,713\,786\,145\,280\,z^2+1\,710\,278\,838\,765\right)\cos(4\,z)+$$
$$2268\left(19\,243\,008\,z^6-634\,798\,080\,z^4+2\,883\,765\,600\,z^2-1\,157\,463\,185\right)\cos(6\,z)-$$
$$1008\left(4\,194\,304\,z^6-238\,141\,440\,z^4+1\,047\,009\,600\,z^2-347\,193\,495\right)\cos(8\,z)-$$
$$7560\left(2\,560\,000\,z^4-14\,721\,000\,z^2+4\,843\,803\right)\cos(10\,z)+22\,680\left(27\,648\,z^4-335\,952\,z^2+130\,913\right)\cos(12\,z)+$$
$$3\,175\,200\left(98\,z^2-57\right)\cos(14\,z)-11\,340\left(512\,z^2-691\right)\cos(16\,z)-215\,460\cos(18\,z)+2835\cos(20\,z)-$$
$$16\,z\left(65\,536\,z^8-104\,103\,936\,z^6+18\,753\,928\,704\,z^4-720\,595\,355\,760\,z^2+4\,737\,962\,919\,375\right)\sin(2\,z)-$$
$$2880\,z\left(2\,883\,584\,z^6-270\,219\,264\,z^4+4\,153\,099\,776\,z^2-10\,103\,019\,129\right)\sin(4\,z)+$$
$$1944\,z\left(1\,327\,104\,z^6-169\,731\,072\,z^4+2\,006\,780\,160\,z^2-3\,218\,991\,965\right)\sin(6\,z)+$$
$$24\,192\,z\left(1\,998\,848\,z^4-27\,271\,680\,z^2+38\,385\,765\right)\sin(8\,z)-75\,600\,z\left(32\,000\,z^4-852\,400\,z^2+1\,314\,741\right)\sin(10\,z)-$$
$$136\,080\,z\left(25\,728\,z^2-56\,137\right)\sin(12\,z)+846\,720\,z\left(98\,z^2-477\right)\sin(14\,z)+$$
$$13\,245\,120\,z\sin(16\,z)-204\,120\,z\sin(18\,z)+52\,907\,861\,917\,260\Big)\,m^{10}+O\left(m^{11}\right)$$

09.32.06.0013.01

$$\operatorname{nd}(z\mid m)\propto 1+O(m)$$

Expansions at *m* == 1

09.32.06.0014.01

$$\operatorname{nd}(z\mid m)\propto\cosh(z)+\frac{1}{4}\sinh(z)\,(z+\cosh(z)\sinh(z))\,(m-1)+$$
$$\frac{1}{256}\left(\left(8\,z^2+7\right)\cosh(z)-8\cosh(3\,z)+\cosh(5\,z)-24\,z\sinh(z)+12\,z\sinh(3\,z)\right)(m-1)^2+\ldots\;/;\;(m\to 1)$$

09.32.06.0015.01

$$\operatorname{nd}(z\mid m)\propto\cosh(z)+\frac{1}{4}\sinh(z)\,(z+\cosh(z)\sinh(z))\,(m-1)+$$
$$\frac{1}{256}\left(\left(8\,z^2+7\right)\cosh(z)-8\cosh(3\,z)+\cosh(5\,z)-24\,z\sinh(z)+12\,z\sinh(3\,z)\right)(m-1)^2+$$

$$\frac{1}{12\,288}\left(-3\left(88z^2+67\right)\cosh(z)+6\left(36z^2+41\right)\cosh(3z)-48\cosh(5z)+\right.$$
$$\left.3\cosh(7z)+32z\left(z^2+21\right)\sinh(z)-468z\sinh(3z)+60z\sinh(5z)\right)(m-1)^3+$$
$$\frac{1}{196\,608}\left(2\left(16z^4+1560z^2+1107\right)\cosh(z)-81\left(48z^2+35\right)\cosh(3z)+30\left(20z^2+23\right)\cosh(5z)-72\cosh(7z)+\right.$$
$$\left.3\cosh(9z)-4z\left(128z^2+1845\right)\sinh(z)+72z\left(12z^2+83\right)\sinh(3z)-1260z\sinh(5z)+84z\sinh(7z)\right)(m-1)^4+$$
$$\frac{1}{15\,728\,640}\left(-120\left(28z^4+1628z^2+1101\right)\cosh(z)+45\left(288z^4+6600z^2+3887\right)\cosh(3z)-240\left(325z^2+204\right)\cosh(5z)+\right.$$
$$30\left(196z^2+221\right)\cosh(7z)-480\cosh(9z)+15\cosh(11z)+8z\left(16z^4+4580z^2+55\,245\right)\sinh(z)-$$
$$\left.1440z\left(69z^2+272\right)\sinh(3z)+200z\left(100z^2+531\right)\sinh(5z)-12\,180z\sinh(7z)+540z\sinh(9z)\right)(m-1)^5+$$
$$\frac{1}{754\,974\,720}\left(8\left(32z^6+21\,180z^4+955\,575z^2+622\,845\right)\cosh(z)-270\left(4032z^4+48\,564z^2+25\,081\right)\cosh(3z)+\right.$$
$$15\left(20\,000z^4+301\,800z^2+139\,557\right)\cosh(5z)-720\left(833z^2+465\right)\cosh(7z)+90\left(324z^2+359\right)\cosh(9z)-$$
$$1800\cosh(11z)+45\cosh(13z)-12z\left(832z^4+129\,240z^2+1\,397\,055\right)\sinh(z)+648z\left(144z^4+8180z^2+24\,385\right)$$
$$\left.\sinh(3z)-300z\left(6200z^2+16\,761\right)\sinh(5z)+840z\left(196z^2+909\right)\sinh(7z)-59\,940z\sinh(9z)+1980z\sinh(11z)\right)$$
$$(m-1)^6+\frac{1}{84\,557\,168\,640}\left(-14\left(3968z^6+1\,331\,760z^4+51\,350\,220z^2+32\,480\,415\right)\cosh(z)+\right.$$
$$63\left(20\,736z^6+2\,453\,760z^4+21\,261\,240z^2+10\,022\,255\right)\cosh(3z)-$$
$$315\left(240\,000z^4+1\,750\,000z^2+668\,241\right)\cosh(5z)+735\left(10\,976z^4+133\,560z^2+51\,573\right)\cosh(7z)-$$
$$5040\left(1701z^2+874\right)\cosh(9z)+630\left(484z^2+529\right)\cosh(11z)-15\,120\cosh(13z)+315\cosh(15z)+$$
$$4z\left(256z^6+337\,344z^4+38\,446\,800z^2+384\,987\,645\right)\sinh(z)-756z\left(28\,512z^4+801\,960z^2+2\,009\,245\right)\sinh(3z)+$$
$$2100z\left(4000z^4+135\,400z^2+257\,937\right)\sinh(5z)-8820z\left(5096z^2+11\,175\right)\sinh(7z)+$$
$$\left.22\,680z\left(108z^2+461\right)\sinh(9z)-623\,700z\sinh(11z)+16\,380z\sinh(13z)\right)(m-1)^7+$$
$$\frac{1}{1\,352\,914\,698\,240}\left(\left(512z^8+1\,211\,392z^6+285\,670\,560z^4+9\,875\,050\,920z^2+6\,089\,965\,245\right)\cosh(z)-\right.$$
$$63\left(787\,968z^6+44\,102\,880z^4+310\,448\,520z^2+136\,673\,455\right)\cosh(3z)+$$
$$70\left(400\,000z^6+26\,280\,000z^4+130\,369\,500z^2+43\,550\,109\right)\cosh(5z)-$$
$$5880\left(60\,368z^4+334\,950z^2+102\,195\right)\cosh(7z)+315\left(69\,984z^4+745\,848z^2+253\,307\right)\cosh(9z)-$$
$$55\,440\left(275z^2+133\right)\cosh(11z)+630\left(676z^2+731\right)\cosh(13z)-17\,640\cosh(15z)+$$
$$315\cosh(17z)-12z\left(3072z^6+1\,950\,368z^4+183\,014\,720z^2+1\,730\,146\,845\right)\sinh(z)+$$
$$648z\left(3456z^6+758\,016z^4+14\,749\,280z^2+32\,740\,365\right)\sinh(3z)-21\,000z\left(16\,400z^4+257\,560z^2+392\,841\right)$$
$$\sinh(5z)+588z\left(76\,832z^4+1\,977\,640z^2+2\,906\,355\right)\sinh(7z)-11\,340z\left(10\,152z^2+19\,451\right)\sinh(9z)+$$
$$\left.9240z\left(484z^2+1953\right)\sinh(11z)-868\,140z\sinh(13z)+18\,900z\sinh(15z)\right)(m-1)^8+$$
$$\frac{1}{194\,819\,716\,546\,560}\left(-18\left(10\,496z^8+11\,495\,680z^6+2\,165\,834\,160z^4+69\,005\,718\,180z^2+41\,633\,079\,075\right)\cosh(z)+\right.$$
$$162\left(186\,624z^8+69\,745\,536z^6+2\,616\,757\,920z^4+15\,922\,895\,940z^2+6\,643\,081\,375\right)\cosh(3z)-$$
$$630\left(18\,400\,000z^6+542\,880\,000z^4+2\,102\,352\,300z^2+635\,484\,717\right)\cosh(5z)+$$
$$126\left(15\,059\,072z^6+719\,147\,520z^4+2\,605\,542\,660z^2+673\,078\,005\right)\cosh(7z)-$$
$$45\,360\left(227\,448z^4+1\,059\,399z^2+274\,924\right)\cosh(9z)+945\left(468\,512z^4+4\,562\,184z^2+1\,409\,247\right)\cosh(11z)-$$
$$45\,360\left(4901z^2+2264\right)\cosh(13z)+28\,350\left(180z^2+193\right)\cosh(15z)-181\,440\cosh(17z)+$$
$$2835\cosh(19z)+8z\left(256z^8+1\,008\,000z^6+434\,436\,912z^4+35\,584\,337\,880z^2+321\,447\,804\,615\right)\sinh(z)-$$
$$3888z\left(222\,912z^6+22\,302\,000z^4+342\,903\,750z^2+695\,415\,875\right)\sinh(3z)+$$
$$900z\left(800\,000z^6+92\,400\,000z^4+959\,477\,400z^2+1\,251\,408\,501\right)\sinh(5z)-$$

$$5292\,z\left(3\,764\,768\,z^4+42\,904\,400\,z^2+48\,678\,885\right)\sinh(7\,z)+20\,412\,z\left(69\,984\,z^4+1\,517\,400\,z^2+1\,876\,715\right)\sinh(9\,z)-41\,580\,z\left(53\,240\,z^2+92\,781\right)\sinh(11\,z)+98\,280\,z\left(676\,z^2+2619\right)\sinh(13\,z)-10\,376\,100\,z\sinh(15\,z)+192\,780\,z\sinh(17\,z)\Big)(m-1)^9+\frac{1}{15\,585\,577\,323\,724\,800}$$

$$\Big(\left(4096\,z^{10}+25\,320\,960\,z^8+18\,304\,957\,440\,z^6+2\,944\,262\,714\,400\,z^4+88\,003\,507\,762\,800\,z^2+52\,079\,055\,504\,525\right)\cosh(z)-1215\left(5\,971\,968\,z^8+988\,533\,504\,z^6+28\,639\,981\,440\,z^4+156\,265\,593\,960\,z^2+62\,399\,823\,955\right)\cosh(3\,z)+225\left(40\,000\,000\,z^8+7\,554\,400\,000\,z^6+143\,925\,600\,000\,z^4+467\,583\,076\,800\,z^2+130\,698\,250\,767\right)\cosh(5\,z)-945\left(542\,126\,592\,z^6+11\,186\,739\,200\,z^4+30\,686\,626\,320\,z^2+6\,994\,157\,055\right)\cosh(7\,z)+17\,010\left(2\,519\,424\,z^6+98\,210\,880\,z^4+289\,458\,900\,z^2+61\,983\,655\right)\cosh(9\,z)-113\,400\left(1\,171\,280\,z^4+4\,826\,206\,z^2+1\,109\,661\right)\cosh(11\,z)+4725\left(913\,952\,z^4+8\,335\,080\,z^2+2\,394\,237\right)\cosh(13\,z)-680\,400\left(2475\,z^2+1103\right)\cosh(15\,z)+28\,350\left(1156\,z^2+1231\right)\cosh(17\,z)-1\,020\,600\cosh(19\,z)+14\,175\cosh(21\,z)-160\,z\left(2944\,z^8+5\,196\,384\,z^6+1\,745\,402\,148\,z^4+129\,145\,913\,190\,z^2+1\,124\,016\,300\,855\right)\sinh(z)+131\,220\,z\left(1536\,z^8+917\,760\,z^6+59\,976\,224\,z^4+780\,845\,800\,z^2+1\,475\,445\,055\right)\sinh(3\,z)-540\,000\,z\left(340\,000\,z^6+17\,170\,300\,z^4+136\,270\,295\,z^2+158\,031\,489\right)\sinh(5\,z)+17\,640\,z\left(2\,151\,296\,z^6+174\,254\,976\,z^4+1\,268\,918\,700\,z^2+1\,197\,550\,395\right)\sinh(7\,z)-918\,540\,z\left(443\,232\,z^4+4\,107\,600\,z^2+3\,812\,645\right)\sinh(9\,z)+41\,580\,z\left(468\,512\,z^4+9\,026\,600\,z^2+9\,864\,255\right)\sinh(11\,z)-737\,100\,z\left(28\,392\,z^2+46\,115\right)\sinh(13\,z)+1\,701\,000\,z\left(300\,z^2+1127\right)\sinh(15\,z)-66\,509\,100\,z\sinh(17\,z)+1\,077\,300\,z\sinh(19\,z)\Big)(m-1)^{10}+O\left((m-1)^{11}\right)$$

09.32.06.0016.01

$$\operatorname{nd}(z\mid m)\propto\cosh(z)\,(1+O(m-1))$$

q-series

09.32.06.0002.01

$$\operatorname{nd}(z\mid m)=\frac{\pi}{2\sqrt{1-m}\;K(m)}+\frac{2\pi}{\sqrt{1-m}\;K(m)}\sum_{k=1}^{\infty}\frac{(-1)^k q(m)^k}{q(m)^{2k}+1}\cos\left(\frac{k\pi z}{K(m)}\right)$$

Other series representations

09.32.06.0003.01

$$\operatorname{nd}(z\mid m)=\frac{\pi}{2\sqrt{1-m}\;K(1-m)}\sum_{k=-\infty}^{\infty}\operatorname{sech}\left(\pi\frac{K(m)}{K(1-m)}\left(k+\frac{1}{2}+\frac{z}{2K(m)}\right)\right)$$

09.32.06.0004.01

$$\operatorname{nd}(z\mid m)\propto\frac{(-1)^{s-1}\,i}{\sqrt{1-m}\;(z-i(2s+1)K(1-m)-(2r+1)K(m))}+O(1)\;/;\;(z\to(2s+1)\,i\,K(1-m)+(2r+1)K(m))\bigwedge\{r,s\}\in\mathbb{Z}$$

Product representations

09.32.08.0001.01

$$\operatorname{nd}(z\mid m)=\frac{1}{\sqrt[4]{1-m}}\prod_{n=1}^{\infty}\frac{1-2q(m)^{2n-1}\cos\left(\frac{\pi z}{K(m)}\right)+q(m)^{4n-2}}{1+2q(m)^{2n-1}\cos\left(\frac{\pi z}{K(m)}\right)+q(m)^{4n-2}}$$

Differential equations

Ordinary nonlinear differential equations

09.32.13.0001.01

$$w''(z) + w(z)\left(2\,(1-m)\,w(z)^2 + m - 2\right) = 0 \;/;\; w(z) = \mathrm{nd}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.32.16.0001.01

$$\mathrm{nd}(i\,z \mid m) = \frac{\mathrm{nd}(z \mid 1-m)}{\mathrm{nc}(z \mid 1-m)}$$

09.32.16.0002.01

$$\mathrm{nd}(z \mid 1-m) = \frac{\mathrm{nd}(i\,z \mid m)}{\mathrm{nc}(i\,z \mid m)}$$

09.32.16.0003.01

$$\mathrm{nd}(i\,z \mid 1-m) = \frac{\mathrm{nd}(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.32.16.0007.01

$$\mathrm{nd}(x + i\,y \mid m) = \frac{\mathrm{cn}(y \mid 1-m)^2 + m\,\mathrm{sn}(x \mid m)^2\,\mathrm{sn}(y \mid 1-m)^2}{\mathrm{dn}(x \mid m)\,\mathrm{cn}(y \mid 1-m)\,\mathrm{dn}(y \mid 1-m) - i\,m\,\mathrm{sn}(x \mid m)\,\mathrm{cn}(x \mid m)\,\mathrm{sn}(y \mid 1-m)} \;/;\; \{x, y\} \in \mathbb{R}$$

09.32.16.0008.01

$$\mathrm{nd}\left(\sqrt{1-m}\;z \,\middle|\, \frac{m}{m-1}\right) = \mathrm{dn}(z \mid m)$$

09.32.16.0009.01

$$\mathrm{nd}\left(\sqrt{m}\;z \,\middle|\, \frac{1}{m}\right) = \mathrm{nc}(z \mid m)$$

09.32.16.0010.01

$$\mathrm{nd}\left(i\,\sqrt{1-m}\;z \,\middle|\, \frac{1}{1-m}\right) = \mathrm{cn}(z \mid m)$$

09.32.16.0011.01

$$\mathrm{nd}\left(i\,\sqrt{m}\;z \,\middle|\, \frac{m-1}{m}\right) = \frac{\mathrm{nc}(z \mid m)}{\mathrm{nd}(z \mid m)}$$

Landen's transformation:

09.32.16.0012.01

$$\mathrm{nd}\left(\left(1+\sqrt{1-m}\right)z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{\mathrm{dn}(z \mid m)}{1-\left(1-\sqrt{1-m}\right)\mathrm{sn}(z \mid m)^2}$$

Gauss' transformation:

09.32.16.0013.01

$$\operatorname{nd}\left(\left(1+\sqrt{m}\right)z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{1+\sqrt{m}\,\operatorname{sn}(z\mid m)^2}{1-\sqrt{m}\,\operatorname{sn}(z\mid m)^2}$$

n th degree transformations:

09.32.16.0014.01

$$\operatorname{nd}\left(\frac{z}{M} \,\middle|\, l\right) = \operatorname{nd}(z\mid m)\prod_{r=1}^{\frac{n-1}{2}} \frac{1-m\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2}{1-m\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2} \;/;$$

$$\frac{n+1}{2}\in\mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n-1}{2}} \operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n-1}{2}} \frac{\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2}$$

09.32.16.0015.01

$$\operatorname{nd}\left(\frac{z}{M}+\frac{K(m)}{nM} \,\middle|\, l\right) = \frac{\operatorname{dn}(z\mid m)}{\sqrt{1-l}}\prod_{r=1}^{\frac{n}{2}} \frac{1-m\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2}{1-m\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2 \operatorname{sn}(z\mid m)^2} \;/;$$

$$\frac{n}{2}\in\mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n}{2}} \operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n}{2}} \frac{\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2}$$

Argument involving half-periods

09.32.16.0004.01

$$\operatorname{nd}(z+K(m)\mid m) = \frac{1}{\sqrt{1-m}}\operatorname{dn}(z\mid m)$$

09.32.16.0023.01

$$\operatorname{nd}(z-K(m)\mid m) = \frac{\operatorname{dn}(z\mid m)}{\sqrt{1-m}}$$

09.32.16.0024.01

$$\operatorname{nd}(z+3K(m)\mid m) = \frac{\operatorname{dn}(z\mid m)}{\sqrt{1-m}}$$

09.32.16.0025.01

$$\operatorname{nd}(z+(2r+1)K(m)\mid m) = \frac{1}{\sqrt{1-m}}\operatorname{dn}(z\mid m) \;/;\; r\in\mathbb{Z}$$

09.32.16.0005.01

$$\operatorname{nd}(z+iK(1-m)\mid m) = i\operatorname{sc}(z\mid m)$$

09.32.16.0026.01

$$\operatorname{nd}(z-iK(1-m)\mid m) = -i\operatorname{sc}(z\mid m)$$

09.32.16.0027.01

$$\operatorname{nd}(z+3iK(1-m)\mid m) = -i\operatorname{sc}(z\mid m) \;/;\; s\in\mathbb{Z}$$

09.32.16.0028.01

$$\operatorname{nd}(z + (2s+1)\,i\,K(1-m) \mid m) = (-1)^s\,i\,\operatorname{sc}(z \mid m) \;/;\; s \in \mathbb{Z}$$

09.32.16.0006.01

$$\operatorname{nd}(z + i\,K(1-m) + K(m) \mid m) = -\frac{i\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.32.16.0029.01

$$\operatorname{nd}(z - i\,K(1-m) + K(m) \mid m) = \frac{i\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.32.16.0030.01

$$\operatorname{nd}(z + i\,K(1-m) - K(m) \mid m) = -\frac{i\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.32.16.0031.01

$$\operatorname{nd}(z - i\,K(1-m) - K(m) \mid m) = \frac{i\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.32.16.0032.01

$$\operatorname{nd}(z - i\,K(1-m) + 3\,K(m) \mid m) = \frac{i\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.32.16.0033.01

$$\operatorname{nd}(z + (4s+1)\,i\,K(1-m) + (2r+1)\,K(m) \mid m) = -\frac{i\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.32.16.0034.01

$$\operatorname{nd}(z + (4s-1)\,i\,K(1-m) + (2r+1)\,K(m) \mid m) = \frac{i\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.32.16.0035.01

$$\operatorname{nd}(z + (2s+1)\,i\,K(1-m) + (2r+1)\,K(m) \mid m) = \frac{(-1)^{s-1}\,i\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

Argument involving inverse Jacobi functions

09.32.16.0036.01

$$\operatorname{nd}\left(\operatorname{cd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m z^2 - 1}{m-1}$$

09.32.16.0037.01

$$\operatorname{nd}\left(\operatorname{cn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{1}{m z^2 - m + 1}$$

09.32.16.0038.01

$$\operatorname{nd}\left(\operatorname{cs}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2 + 1}{z^2 - m + 1}$$

09.32.16.0039.01

$$\operatorname{nd}\left(\operatorname{dc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m - z^2}{(m-1)\,z^2}$$

09.32.16.0040.01

$$\operatorname{nd}\left(\operatorname{dn}^{-1}(z \mid m) \mid m\right) = \frac{1}{z}$$

09.32.16.0041.01

$$\operatorname{nd}\left(\operatorname{ds}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2 + m}{z^2}$$

09.32.16.0042.01

$$\operatorname{nd}\left(\operatorname{nc}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2}{(1-m)\, z^2 + m}$$

09.32.16.0043.01

$$\operatorname{nd}\left(\operatorname{ns}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2}{z^2 - m}$$

09.32.16.0044.01

$$\operatorname{nd}\left(\operatorname{sc}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2 + 1}{(1-m)\, z^2 + 1}$$

09.32.16.0045.01

$$\operatorname{nd}\left(\operatorname{sd}^{-1}(z \mid m) \mid m\right)^2 = m\, z^2 + 1$$

09.32.16.0046.01

$$\operatorname{nd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)^2 = \frac{1}{1 - m\, z^2}$$

Addition formulas

09.32.16.0016.01

$$\operatorname{nd}(u + v \mid m) = \frac{1 - m \operatorname{sn}(u \mid m)^2 \operatorname{sn}(v \mid m)^2}{\operatorname{dn}(u \mid m) \operatorname{dn}(v \mid m) - m \operatorname{sn}(u \mid m) \operatorname{cn}(u \mid m) \operatorname{sn}(v \mid m) \operatorname{cn}(v \mid m)}$$

09.32.16.0017.01

$$\operatorname{nd}(u + v \mid m) \operatorname{nd}(u - v \mid m) = \frac{1 - m \operatorname{sn}(u \mid m)^2 \operatorname{sn}(v \mid m)^2}{\operatorname{dn}(v \mid m)^2 - m \operatorname{cn}(v \mid m)^2 \operatorname{sn}(u \mid m)^2}$$

Half-angle formulas

09.32.16.0018.01

$$\operatorname{nd}\left(\frac{z}{2} \,\middle|\, m\right)^2 = \frac{1 + \operatorname{dn}(z \mid m)}{1 - m + \operatorname{dn}(z \mid m) + m \operatorname{cn}(z \mid m)}$$

Multiple arguments

Double angle formulas

09.32.16.0019.01

$$\operatorname{nd}(2\, z \mid m) = \frac{1 - m \operatorname{sn}(z \mid m)^4}{\operatorname{dn}(z \mid m)^2 - m \operatorname{sn}(z \mid m)^2 \operatorname{cn}(z \mid m)^2}$$

09.32.16.0020.01

$$\operatorname{nd}(2\,z\mid m)=\frac{\operatorname{dn}(z\mid m)^2-\operatorname{cn}(z\mid m)^2\left(\operatorname{dn}(z\mid m)^2-1\right)}{\left(\operatorname{dn}(z\mid m)^2-1\right)\operatorname{cn}(z\mid m)^2+\operatorname{dn}(z\mid m)^2}$$

Multiple angle formulas

09.32.16.0021.01

$$\operatorname{nd}(n\,z\mid m)=(1-m)^{\frac{n^2-1}{4}}\prod_{\mu,\nu=0}^{n-1}\operatorname{nd}\left(z+\frac{4\,K(m)\,(\mu+\nu\,\tau)}{n}\,\middle|\,m\right)/;\ \frac{n+1}{2}\in\mathbb{Z}^+$$

09.32.16.0022.01

$$\operatorname{nd}\left(\frac{2\,n}{\pi}\,K\left(\lambda\left(\frac{n\log(q(m))}{\pi\,i}\right)\right)x\,\middle|\,\lambda\left(\frac{n}{\pi\,i}\log(q(m))\right)\right)=\frac{(1-m)^{n/4}}{\sqrt[4]{1-\lambda\left(\frac{n\log(q(m))}{\pi\,i}\right)}}\prod_{r=0}^{n-1}\operatorname{nd}\left(\frac{2\,K(m)}{\pi}\left(x+\frac{\pi\,r}{n}\right)\,\middle|\,m\right)/;\ \frac{n+1}{2}\in\mathbb{Z}^+$$

Identities

Functional identities

09.32.17.0001.01

$$(m-1)\,w(z)^4+2\,w(z)^2+\left((m-1)\,w(z)^4+2\,(1-m)\,w(z)^2-1\right)w(2\,z)-1=0\ /;\ w(z)=\operatorname{nd}(z\mid m)$$

Complex characteristics

Real part

09.32.19.0001.01

$$\operatorname{Re}(\operatorname{nd}(x+i\,y\mid m))=\frac{\operatorname{cn}(y\mid 1-m)\operatorname{dn}(x\mid m)\operatorname{dn}(y\mid 1-m)\left(\operatorname{cn}(y\mid 1-m)^2+m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)}{\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2+m^2\operatorname{cn}(x\mid m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}\ /;\ \{x,y,m\}\in\mathbb{R}$$

Imaginary part

09.32.19.0002.01

$$\operatorname{Im}(\operatorname{nd}(x+i\,y\mid m))=\frac{m\operatorname{cn}(x\mid m)\operatorname{sn}(x\mid m)\operatorname{sn}(y\mid 1-m)\left(\operatorname{cn}(y\mid 1-m)^2+m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)}{\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2+m^2\operatorname{cn}(x\mid m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}\ /;\ \{x,y,m\}\in\mathbb{R}$$

Absolute value

09.32.19.0003.01

$$|\operatorname{nd}(x+i\,y\mid m)|=\left(\sqrt{\left(\left(\operatorname{cn}(y\mid 1-m)^2+m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)^2\left(\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2+m^2\operatorname{cn}(x\mid m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)\right)}\right)\Big/\left(\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2+m^2\operatorname{cn}(x\mid m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)\ /;\ \{x,y,m\}\in\mathbb{R}$$

Argument

09.32.19.0004.01

$$\arg(\mathrm{nd}(x+i\,y\mid m)) = \tan^{-1}\Big(\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)\left(\mathrm{cn}(y\mid 1-m)^2+m\,\mathrm{sn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2\right), \\ m\,\mathrm{cn}(x\mid m)\,\mathrm{sn}(x\mid m)\,\mathrm{sn}(y\mid 1-m)\left(\mathrm{cn}(y\mid 1-m)^2+m\,\mathrm{sn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2\right)\Big)\;/;\;\{x,y,m\}\in\mathbb{R}$$

Conjugate value

09.32.19.0005.01

$$\overline{\mathrm{nd}(x+i\,y\mid m)} = \frac{\mathrm{cn}(y\mid 1-m)^2+m\,\mathrm{sn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2}{\mathrm{dn}(x\mid m)\,\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(y\mid 1-m)+i\,m\,\mathrm{sn}(x\mid m)\,\mathrm{cn}(x\mid m)\,\mathrm{sn}(y\mid 1-m)}\;/;\;\{x,y,m\}\in\mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *z*

09.32.20.0001.01

$$\frac{\partial\,\mathrm{nd}(z\mid m)}{\partial z} = m\,\mathrm{cd}(z\mid m)\,\mathrm{sd}(z\mid m)$$

09.32.20.0002.01

$$\frac{\partial^2\,\mathrm{nd}(z\mid m)}{\partial z^2} = m\,\mathrm{nd}(z\mid m)\left(\mathrm{cd}(z\mid m)^2+(m-1)\,\mathrm{sd}(z\mid m)^2\right)$$

With respect to *m*

09.32.20.0003.01

$$\frac{\partial\,\mathrm{nd}(z\mid m)}{\partial m} = \frac{\mathrm{sd}(z\mid m)\,\mathrm{cd}(z\mid m)\,((1-m)\,z-E(\mathrm{am}(z\mid m)\mid m)+\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m))}{2\,(1-m)}$$

09.32.20.0004.01

$$\frac{\partial^2\,\mathrm{nd}(z\mid m)}{\partial m^2} = \frac{1}{4\,(m-1)^2}\Bigg(-2\,((m-1)\,z+E(\mathrm{am}(z\mid m)\mid m)-\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m))\,\mathrm{sd}(z\mid m)\,\mathrm{cd}(z\mid m)+ \\ (1-m)\,\mathrm{sd}(z\mid m)\Bigg(-2\,z+\frac{F(\mathrm{am}(z\mid m)\mid m)-E(\mathrm{am}(z\mid m)\mid m)}{m}+ \\ \frac{1}{m-1}\,(\mathrm{cn}(z\mid m)\,\mathrm{sc}(z\mid m)\,(-m\,z+z-E(\mathrm{am}(z\mid m)\mid m)+\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m))\,\mathrm{sn}(z\mid m))+ \\ \frac{1}{(m-1)\,m}\,(\mathrm{dc}(z\mid m)\,\mathrm{dn}(z\mid m)\,\mathrm{nc}(z\mid m)\,((m-1)\,z+E(\mathrm{am}(z\mid m)\mid m)-m\,\mathrm{cd}(z\mid m)\,\mathrm{sn}(z\mid m)))+ \\ \frac{1}{(m-1)\,m}\left(\left(m\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(z\mid m)-((m-1)\,z+E(\mathrm{am}(z\mid m)\mid m))\,\mathrm{dn}(z\mid m)\right)\sqrt{1-m\,\mathrm{sn}(z\mid m)^2}\right)\Bigg)\,\mathrm{cd}(z\mid m)+ \\ \frac{1}{m}\Big(\mathrm{cd}(z\mid m)^2\,\mathrm{nd}(z\mid m)\,((m-1)\,z+E(\mathrm{am}(z\mid m)\mid m)-\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m)) \\ ((m-1)\,z+E(\mathrm{am}(z\mid m)\mid m)-m\,\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m))\Big)+ \\ \frac{1}{m}\left((m-1)\,((m-1)\,z+E(\mathrm{am}(z\mid m)\mid m))\,\mathrm{nd}(z\mid m)\,((m-1)\,z+E(\mathrm{am}(z\mid m)\mid m)-\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m))\,\mathrm{sd}(z\mid m)^2\right)\Bigg)$$

Symbolic differentiation

With respect to z

09.32.20.0007.01

$$\frac{\partial^n \operatorname{nd}(z \mid m)}{\partial z^n} = \operatorname{nd}(z \mid m)\, \delta_n + m \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\partial^j \operatorname{sd}(z \mid m)}{\partial z^j} \frac{\partial^{-j+n-1} \operatorname{cd}(z \mid m)}{\partial z^{-j+n-1}} \;/;\; n \in \mathbb{N}$$

09.32.20.0005.01

$$\frac{\partial^n \operatorname{nd}(z \mid m)}{\partial z^n} = \frac{2\, \pi^{n+1}}{\sqrt{1-m}\; K(m)^{n+1}} \sum_{k=1}^{\infty} \frac{(-1)^k\, k^n\, q(m)^k}{q(m)^{2k}+1} \cos\left(\frac{\pi\, n}{2} + \frac{k\, \pi\, z}{K(m)}\right) /;\; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to z

09.32.20.0006.01

$$\frac{\partial^\alpha \operatorname{nd}(z \mid m)}{\partial z^\alpha} = \frac{\pi\, z^{-\alpha}}{2\sqrt{1-m}\; K(m)\, \Gamma(1-\alpha)} + \frac{2^{\alpha+1}\, \pi^{3/2}\, z^{-\alpha}}{\sqrt{1-m}\; K(m)} \sum_{k=1}^{\infty} \frac{(-1)^k\, q(m)^k}{q(m)^{2k}+1}\, {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{k^2\, \pi^2\, z^2}{4\, K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.32.21.0001.01

$$\int \operatorname{nd}(z \mid m)\, dz = \frac{\sqrt{1-\operatorname{cd}(z \mid m)^2}\; \cos^{-1}(\operatorname{cd}(z \mid m))}{(1-m)\, \operatorname{sd}(z \mid m)}$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

09.32.21.0002.01

$$\int \operatorname{nd}(z \mid m)^2\, dz = \frac{E(\operatorname{am}(z \mid m) \mid m)\, \operatorname{dn}(z \mid m)}{(1-m)\sqrt{1-m\, \operatorname{sn}(z \mid m)^2}} - \frac{m\, (\operatorname{cn}(z \mid m)\, \operatorname{sn}(z \mid m))}{(1-m)\, \operatorname{dn}(z \mid m)}$$

Representations through equivalent functions

With inverse function

09.32.27.0001.01

$$\operatorname{nd}\left(\operatorname{nd}^{-1}(z \mid m) \,\middle|\, m\right) = z$$

With related functions

Involving one other Jacobi elliptic function

Involving cd

09.32.27.0004.01

$$\mathrm{nd}(z \mid m) = \mathrm{cd}(i\, z \mid 1 - m)$$

09.32.27.0005.01

$$\mathrm{nd}(z \mid m)^2 = \frac{m\, \mathrm{cd}(z \mid m)^2 - 1}{m - 1}$$

Involving cn

09.32.27.0007.01

$$\mathrm{nd}(z \mid m)^2 = \frac{1}{m\, \mathrm{cn}(z \mid m)^2 - m + 1}$$

Involving cs

09.32.27.0008.01

$$\mathrm{nd}(z \mid m)^2 = \frac{\mathrm{cs}(z \mid m)^2 + 1}{\mathrm{cs}(z \mid m)^2 - m + 1}$$

Involving dc

09.32.27.0010.01

$$\mathrm{nd}(z \mid m)^2 = \frac{m - \mathrm{dc}(z \mid m)^2}{(m - 1)\, \mathrm{dc}(z \mid m)^2}$$

Involving dn

09.32.27.0011.01

$$\mathrm{nd}(z \mid m) = \frac{1}{\mathrm{dn}(z \mid m)}$$

Involving ds

09.32.27.0014.01

$$\mathrm{nd}(z \mid m)^2 = \frac{\mathrm{ds}(z \mid m)^2 + m}{\mathrm{ds}(z \mid m)^2}$$

Involving nc

09.32.27.0015.01

$$\operatorname{nd}(z \mid m)^2 = \frac{\operatorname{nc}(z \mid m)^2}{(1-m)\operatorname{nc}(z \mid m)^2 + m}$$

Involving ns

09.32.27.0017.01

$$\operatorname{nd}(z \mid m)^2 = \frac{\operatorname{ns}(z \mid m)^2}{\operatorname{ns}(z \mid m)^2 - m}$$

Involving sc

09.32.27.0018.01

$$\operatorname{nd}(z \mid m)^2 = \frac{\operatorname{sc}(z \mid m)^2 + 1}{(1-m)\operatorname{sc}(z \mid m)^2 + 1}$$

Involving sd

09.32.27.0020.01

$$\operatorname{nd}(z \mid m)^2 = m \operatorname{sd}(z \mid m)^2 + 1$$

Involving sn

09.32.27.0022.01

$$\operatorname{nd}(z \mid m) = \frac{1}{\sqrt{1-m}\ \operatorname{sn}(K(1-m) - i K(m) - i z \mid 1-m)}$$

09.32.27.0021.01

$$\operatorname{nd}(z \mid m)^2 = \frac{1}{1 - m \operatorname{sn}(z \mid m)^2}$$

Involving two other Jacobi elliptic functions

Involving cd and cn

09.32.27.0002.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)}{\operatorname{cn}(z \mid m)}$$

Involving cd and nc

09.32.27.0003.01

$$\operatorname{nd}(z \mid m) = \operatorname{nc}(z \mid m)\operatorname{cd}(z \mid m)$$

Involving cn and dc

09.32.27.0006.01

$$\operatorname{nd}(z \mid m) = \frac{1}{\operatorname{cn}(z \mid m)\operatorname{dc}(z \mid m)}$$

09.32.27.0028.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(m - \operatorname{dc}(z \mid m)^2\right)}{(m-1)\operatorname{dc}(z \mid m)}$$

Involving cs and dn

09.32.27.0029.01

$$\operatorname{nd}(z \mid m) = -\frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{dn}(z \mid m)}{-\operatorname{cs}(z \mid m)^2 + m - 1}$$

Involving dc and dn

09.32.27.0030.01

$$\operatorname{nd}(z \mid m) = \frac{\left(m - \operatorname{dc}(z \mid m)^2\right)\operatorname{dn}(z \mid m)}{(m-1)\operatorname{dc}(z \mid m)^2}$$

Involving dc and nc

09.32.27.0009.01

$$\operatorname{nd}(z \mid m) == \frac{\operatorname{nc}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

Involving dn and nc

09.32.27.0031.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)^2}{m\operatorname{nc}(z \mid m)^2 - \operatorname{nc}(z \mid m)^2 - m}$$

Involving dn and ns

09.32.27.0032.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)^2}{\operatorname{ns}(z \mid m)^2 - m}$$

Involving dn and sc

09.32.27.0033.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1}$$

Involving ds and ns

09.32.27.0012.01

$$\operatorname{nd}(z \mid m) == \frac{\operatorname{ns}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

Involving ds and sn

09.32.27.0013.01

$$\mathrm{nd}(z \mid m) = \frac{1}{\mathrm{sn}(z \mid m)\, \mathrm{ds}(z \mid m)}$$

Involving ns and sd

09.32.27.0016.01

$$\mathrm{nd}(z \mid m) = \mathrm{ns}(z \mid m)\, \mathrm{sd}(z \mid m)$$

Involving sd and sn

09.32.27.0019.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{sd}(z \mid m)}{\mathrm{sn}(z \mid m)}$$

Involving three other Jacobi elliptic functions

09.32.27.0034.01

$$\mathrm{nd}(z \mid m) = -\frac{\mathrm{cn}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 + 1\right)}{\mathrm{cd}(z \mid m)\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right)}$$

09.32.27.0035.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 + 1\right)}{\mathrm{cs}(z \mid m)^2\, \mathrm{dc}(z \mid m)}$$

09.32.27.0036.01

$$\mathrm{nd}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right) \mathrm{dn}(z \mid m)}{\mathrm{cs}(z \mid m)^2\, \mathrm{dc}(z \mid m)^2}$$

09.32.27.0037.01

$$\mathrm{nd}(z \mid m) = \frac{m\, \mathrm{cn}(z \mid m) - \mathrm{dc}(z \mid m)\, \mathrm{dn}(z \mid m)}{(m-1)\, \mathrm{dc}(z \mid m)}$$

09.32.27.0038.01

$$\mathrm{nd}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right) \mathrm{dn}(z \mid m)}{\mathrm{ds}(z \mid m)^2}$$

09.32.27.0039.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 + 1\right)}{\mathrm{cs}(z \mid m)\, \mathrm{ds}(z \mid m)}$$

09.32.27.0040.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\left(\mathrm{dc}(z \mid m)^2 + \mathrm{ds}(z \mid m)^2\right)}{\mathrm{dc}(z \mid m)\, \mathrm{ds}(z \mid m)^2}$$

09.32.27.0041.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{dc}(z \mid m)^2 + \mathrm{ds}(z \mid m)^2\right)}{\mathrm{dc}(z \mid m)^2\, \mathrm{ds}(z \mid m)^2}$$

09.32.27.0042.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m)^2 \operatorname{nc}(z \mid m)}$$

09.32.27.0043.01

$$\operatorname{nd}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m)}{(m - 1) \operatorname{nc}(z \mid m)}$$

09.32.27.0044.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dc}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{ds}(z \mid m)^2 (\operatorname{nc}(z \mid m) - 1) (\operatorname{nc}(z \mid m) + 1)}$$

09.32.27.0045.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)^2}{\operatorname{ds}(z \mid m)^2 (\operatorname{nc}(z \mid m) - 1) (\operatorname{nc}(z \mid m) + 1)}$$

09.32.27.0046.01

$$\operatorname{nd}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) - \operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{(m - 1) \operatorname{nc}(z \mid m)}$$

09.32.27.0047.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.32.27.0048.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{dc}(z \mid m) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

09.32.27.0049.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{dc}(z \mid m)^2 (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

09.32.27.0050.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{nc}(z \mid m) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}$$

09.32.27.0051.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{cd}(z \mid m) \left(\operatorname{ns}(z \mid m)^2 - m\right)}$$

09.32.27.0052.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ns}(z \mid m)}{(\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1) \operatorname{sc}(z \mid m)}$$

09.32.27.0053.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) (\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m) - m \operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.32.27.0054.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{dc}(z \mid m)}$$

09.32.27.0055.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{dc}(z \mid m)^2}$$

09.32.27.0056.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{nc}(z \mid m)}$$

09.32.27.0057.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{ds}(z \mid m)^2\, \mathrm{sc}(z \mid m)^2}$$

09.32.27.0058.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{ds}(z \mid m)\, \mathrm{sc}(z \mid m)}$$

09.32.27.0059.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{ns}(z \mid m)\, \mathrm{sc}(z \mid m)}$$

09.32.27.0060.01

$$\mathrm{nd}(z \mid m) = -\frac{\mathrm{cn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{cd}(z \mid m)\left(m\, \mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}$$

09.32.27.0061.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 + 1\right)}{\mathrm{cs}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 - m + 1\right) \mathrm{sd}(z \mid m)}$$

09.32.27.0062.01

$$\mathrm{nd}(z \mid m) = -\frac{\mathrm{ns}(z \mid m)}{\left(-\mathrm{cs}(z \mid m)^2 + m - 1\right) \mathrm{sd}(z \mid m)}$$

09.32.27.0063.01

$$\mathrm{nd}(z \mid m) = -\frac{\mathrm{cn}(z \mid m)\, \mathrm{sc}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\left(m\, \mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right) \mathrm{sd}(z \mid m)}$$

09.32.27.0064.01

$$\mathrm{nd}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right) \mathrm{sd}(z \mid m)}{\mathrm{cs}(z \mid m)\, \mathrm{nc}(z \mid m)}$$

09.32.27.0065.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\, \mathrm{sd}(z \mid m)}{\mathrm{cs}(z \mid m)\, (\mathrm{nc}(z \mid m) - 1)\, (\mathrm{nc}(z \mid m) + 1)}$$

09.32.27.0066.01

$$\mathrm{nd}(z \mid m) = -\frac{\mathrm{sd}(z \mid m)}{(\mathrm{cn}(z \mid m) - 1)\, (\mathrm{cn}(z \mid m) + 1)\, \mathrm{ns}(z \mid m)}$$

09.32.27.0067.01

$$\mathrm{nd}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right) \mathrm{sd}(z \mid m)}{\mathrm{ns}(z \mid m)}$$

09.32.27.0068.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)^2 \operatorname{sd}(z \mid m)}{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)\operatorname{ns}(z \mid m)}$$

09.32.27.0069.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)}$$

09.32.27.0070.01

$$\operatorname{nd}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sd}(z \mid m)}{\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m)^2}$$

09.32.27.0071.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sd}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.32.27.0072.01

$$\operatorname{nd}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sd}(z \mid m)}{\operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.32.27.0073.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{nc}(z \mid m)\operatorname{sd}(z \mid m)^2}{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)}$$

09.32.27.0074.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m) + m\operatorname{sd}(z \mid m)}{\operatorname{ns}(z \mid m)}$$

09.32.27.0075.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)}$$

09.32.27.0076.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)\left(\operatorname{cs}(z \mid m)^2 - m + 1\right)}$$

09.32.27.0077.01

$$\operatorname{nd}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)\operatorname{dc}(z \mid m)}$$

09.32.27.0078.01

$$\operatorname{nd}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

09.32.27.0079.01

$$\operatorname{nd}(z \mid m) = \frac{\left(\operatorname{dc}(z \mid m)^2 + \operatorname{ds}(z \mid m)^2\right)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)^2\operatorname{ds}(z \mid m)}$$

09.32.27.0080.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)^2\operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m)(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)}$$

09.32.27.0081.01

$$\operatorname{nd}(z \mid m) = \frac{\left(m - \operatorname{dc}(z \mid m)^2\right) \operatorname{sn}(z \mid m)}{(m-1) \operatorname{dc}(z \mid m) \operatorname{sc}(z \mid m)}$$

09.32.27.0082.01

$$\operatorname{nd}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{sc}(z \mid m)^2}$$

09.32.27.0083.01

$$\operatorname{nd}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m) \operatorname{sc}(z \mid m)}$$

09.32.27.0084.01

$$\operatorname{nd}(z \mid m) = -\frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m) \operatorname{sc}(z \mid m) \left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.32.27.0085.01

$$\operatorname{nd}(z \mid m) = -\frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right) \operatorname{sd}(z \mid m)}$$

09.32.27.0086.01

$$\operatorname{nd}(z \mid m) = \frac{\left(m - \operatorname{dc}(z \mid m)^2\right) \operatorname{sn}(z \mid m)}{(m-1) \operatorname{dc}(z \mid m)^2 \operatorname{sd}(z \mid m)}$$

09.32.27.0087.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)^2 \operatorname{sn}(z \mid m)}{\left(m \operatorname{nc}(z \mid m)^2 - \operatorname{nc}(z \mid m)^2 - m\right) \operatorname{sd}(z \mid m)}$$

09.32.27.0088.01

$$\operatorname{nd}(z \mid m) = -\frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}{\left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right) \operatorname{sd}(z \mid m)}$$

09.32.27.0089.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)}{\operatorname{dc}(z \mid m) (\operatorname{sn}(z \mid m) - 1) (\operatorname{sn}(z \mid m) + 1)}$$

09.32.27.0090.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)}{\operatorname{nc}(z \mid m) (\operatorname{sn}(z \mid m) - 1) (\operatorname{sn}(z \mid m) + 1)}$$

09.32.27.0091.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) \operatorname{cs}(z \mid m) \operatorname{sn}(z \mid m)}{(\operatorname{sn}(z \mid m) - 1) (\operatorname{sn}(z \mid m) + 1)}$$

09.32.27.0092.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)}{\operatorname{cd}(z \mid m) \left(m \operatorname{sn}(z \mid m)^2 - 1\right)}$$

Involving four other Jacobi elliptic functions

09.32.27.0093.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,(\operatorname{dc}(z \mid m) + \operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m))}{\operatorname{ds}(z \mid m)^2}$$

09.32.27.0094.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{dc}(z \mid m) + \operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m))}{\operatorname{dc}(z \mid m)\operatorname{ds}(z \mid m)^2}$$

09.32.27.0095.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{ds}(z \mid m)^2}$$

09.32.27.0096.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)^2\,(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}$$

09.32.27.0097.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dc}(z \mid m) + \operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{ds}(z \mid m)^2\operatorname{nc}(z \mid m)}$$

09.32.27.0098.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)^2\operatorname{nc}(z \mid m)}$$

09.32.27.0099.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)}{\operatorname{ds}(z \mid m)^2\operatorname{nc}(z \mid m)}$$

09.32.27.0100.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)^2 + \operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)}{\operatorname{ds}(z \mid m)^2\operatorname{nc}(z \mid m)}$$

09.32.27.0101.01

$$\operatorname{nd}(z \mid m) = \frac{m\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{(m-1)\operatorname{ns}(z \mid m)}$$

09.32.27.0102.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sc}(z \mid m)}$$

09.32.27.0103.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m)\operatorname{dc}(z \mid m)}$$

09.32.27.0104.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m)\operatorname{dc}(z \mid m)^2}$$

09.32.27.0105.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ds}(z \mid m)}$$

09.32.27.0106.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m)\operatorname{nc}(z \mid m)}$$

09.32.27.0107.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ns}(z \mid m)}$$

09.32.27.0108.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ds}(z \mid m)^2 \operatorname{sc}(z \mid m)}$$

09.32.27.0109.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cd}(z \mid m)\,(\operatorname{cs}(z \mid m) - m \operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.32.27.0110.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m) + m \operatorname{cd}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{ns}(z \mid m)}$$

09.32.27.0111.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{ds}(z \mid m)^2 \operatorname{sc}(z \mid m)}$$

09.32.27.0112.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{ns}(z \mid m) + \operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m))}{\operatorname{ns}(z \mid m) - m \operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m) + \operatorname{nc}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.32.27.0113.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right)\operatorname{sd}(z \mid m)}$$

09.32.27.0114.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)\operatorname{sd}(z \mid m)}{(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\operatorname{ns}(z \mid m)}$$

09.32.27.0115.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.32.27.0116.01

$$\operatorname{nd}(z \mid m) = \frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sd}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.32.27.0117.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{cn}(z \mid m)}$$

09.32.27.0118.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2}{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.32.27.0119.01

$$\operatorname{nd}(z \mid m) = \frac{m \operatorname{sd}(z \mid m) - \operatorname{dc}(z \mid m) \operatorname{sc}(z \mid m)}{(m-1) \operatorname{nc}(z \mid m) \operatorname{sc}(z \mid m)}$$

09.32.27.0120.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m) + m \operatorname{cd}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{cd}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.32.27.0121.01

$$\operatorname{nd}(z \mid m) = \frac{m \operatorname{cs}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{(m-1) \operatorname{nc}(z \mid m)}$$

09.32.27.0122.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) (\operatorname{cs}(z \mid m) + \operatorname{dc}(z \mid m) \operatorname{sd}(z \mid m))}{\operatorname{cs}(z \mid m) \operatorname{dc}(z \mid m)}$$

09.32.27.0123.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) (\operatorname{cs}(z \mid m) + \operatorname{dc}(z \mid m) \operatorname{sd}(z \mid m))}{\operatorname{cs}(z \mid m) \operatorname{dc}(z \mid m)^2}$$

09.32.27.0124.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m) + \operatorname{dc}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{dc}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.32.27.0125.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{sd}(z \mid m) \operatorname{dc}(z \mid m)^2 + \operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)^2 \operatorname{ns}(z \mid m)}$$

09.32.27.0126.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ns}(z \mid m) + \operatorname{nc}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{nc}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.32.27.0127.01

$$\operatorname{nd}(z \mid m) = \operatorname{cn}(z \mid m) (\operatorname{cd}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m))$$

09.32.27.0128.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.32.27.0129.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{ns}(z \mid m) \operatorname{sc}(z \mid m)}$$

09.32.27.0130.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) (\operatorname{cd}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m))}{\operatorname{cd}(z \mid m) - m \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m)}$$

09.32.27.0131.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m) + m \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.32.27.0132.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) \left(\operatorname{sc}(z \mid m)^2 + 1\right)}{-\operatorname{cd}(z \mid m) + m \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m)}$$

09.32.27.0133.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{sd}(z \mid m)\,(\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m))}{\operatorname{sc}(z \mid m)}$$

09.32.27.0134.01

$$\operatorname{nd}(z \mid m) = \frac{1}{\operatorname{cn}(z \mid m)}\left(\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{dn}(z \mid m)^2 + \operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m) + m\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)\right)$$

09.32.27.0135.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cd}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.32.27.0136.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)^2 + \operatorname{cn}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.32.27.0137.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)}{\operatorname{cs}(z \mid m)\operatorname{nc}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.32.27.0138.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)}{\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.32.27.0139.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)}{\operatorname{dc}(z \mid m)\,(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m))}$$

09.32.27.0140.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)\,(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m))}$$

09.32.27.0141.01

$$\operatorname{nd}(z \mid m) = \frac{(\operatorname{dc}(z \mid m) + \operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m))\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{ds}(z \mid m)}$$

09.32.27.0142.01

$$\operatorname{nd}(z \mid m) = \frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.32.27.0143.01

$$\operatorname{nd}(z \mid m) = \frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.32.27.0144.01

$$\operatorname{nd}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)\,(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.32.27.0145.01

$$\operatorname{nd}(z \mid m) = -\frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.32.27.0146.01

$$\operatorname{nd}(z \mid m) = \frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sn}(z \mid m)}{(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sd}(z \mid m)}$$

09.32.27.0147.01

$$\mathrm{nd}(z \mid m) = \frac{(\mathrm{cs}(z \mid m) + \mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m))\,\mathrm{sn}(z \mid m)}{\mathrm{dc}(z \mid m)}$$

09.32.27.0148.01

$$\mathrm{nd}(z \mid m) = \frac{(\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m))\,\mathrm{sn}(z \mid m)}{\mathrm{sc}(z \mid m)}$$

09.32.27.0149.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,(\mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m)\,\mathrm{dn}(z \mid m) + \mathrm{sn}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.32.27.0150.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\,\mathrm{ds}(z \mid m)\,\mathrm{cd}(z \mid m)^2 + \mathrm{sn}(z \mid m)}{\mathrm{ds}(z \mid m)}$$

09.32.27.0151.01

$$\mathrm{nd}(z \mid m) = \frac{m\,\mathrm{sn}(z \mid m) - \mathrm{dc}(z \mid m)\,\mathrm{dn}(z \mid m)\,\mathrm{sc}(z \mid m)}{(m-1)\,\mathrm{dc}(z \mid m)\,\mathrm{sc}(z \mid m)}$$

09.32.27.0152.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dn}(z \mid m) + \mathrm{dc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)^2}$$

09.32.27.0153.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{sn}(z \mid m)\,\mathrm{dc}(z \mid m)^2 + \mathrm{dn}(z \mid m)\,\mathrm{ds}(z \mid m)}{\mathrm{dc}(z \mid m)^2\,\mathrm{ds}(z \mid m)}$$

09.32.27.0154.01

$$\mathrm{nd}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m)\,\mathrm{dn}(z \mid m)^2 + \mathrm{sn}(z \mid m)\,\mathrm{dn}(z \mid m) - \mathrm{cd}(z \mid m)\,\mathrm{cs}(z \mid m)}{(m-1)\,\mathrm{sn}(z \mid m)}$$

09.32.27.0155.01

$$\mathrm{nd}(z \mid m) = \frac{m\,\mathrm{cd}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{nc}(z \mid m)\,\mathrm{sn}(z \mid m)}{(m-1)\,\mathrm{nc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.32.27.0156.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cs}(z \mid m) + \mathrm{nc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\mathrm{ds}(z \mid m)\,\mathrm{nc}(z \mid m)}$$

09.32.27.0157.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,\mathrm{ds}(z \mid m) + \mathrm{nc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\mathrm{ds}(z \mid m)\,\mathrm{nc}(z \mid m)}$$

09.32.27.0158.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cn}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\mathrm{dc}(z \mid m)}$$

09.32.27.0159.01

$$\mathrm{nd}(z \mid m) = \frac{\mathrm{cn}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\mathrm{ds}(z \mid m)\,\mathrm{sc}(z \mid m)}$$

09.32.27.0160.01

$$\operatorname{nd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.32.27.0161.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{sd}(z \mid m)(\operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m))}{\operatorname{sc}(z \mid m)}$$

09.32.27.0162.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)(\operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m))}{\operatorname{cn}(z \mid m) - m\operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.32.27.0163.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + m\operatorname{cd}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)}$$

09.32.27.0164.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

Involving five other Jacobi elliptic functions

09.32.27.0165.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.32.27.0166.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.32.27.0167.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{cd}(z \mid m) - m\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.32.27.0168.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)\operatorname{dc}(z \mid m)}$$

09.32.27.0169.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

09.32.27.0170.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m)(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.32.27.0171.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{dc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{ds}(z \mid m)}$$

09.32.27.0172.01

$$\operatorname{nd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m) - m\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m) + \operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}$$

Involving Weierstrass functions

09.32.27.0023.01

$$\mathrm{nd}(z \mid m) = \frac{\sigma_3\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right)}{\sigma_2\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right)} /;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

09.32.27.0024.01

$$\mathrm{nd}(z \mid m)^2 = \frac{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_3}{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_2} /;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

Involving theta functions

09.32.27.0025.01

$$\mathrm{nd}(z \mid m) = (1-m)^{-1/4} \frac{\vartheta_4\left(\frac{\pi z}{2 K(m)}, q(m)\right)}{\vartheta_3\left(\frac{\pi z}{2 K(m)}, q(m)\right)}$$

09.32.27.0026.01

$$\mathrm{nd}(z \mid m) = \frac{\vartheta_3(0, q(m))}{\vartheta_4(0, q(m))} \frac{\vartheta_4\left(\frac{z}{\vartheta_3(0,q(m))^2}, q(m)\right)}{\vartheta_3\left(\frac{z}{\vartheta_3(0,q(m))^2}, q(m)\right)}$$

09.32.27.0027.01

$$\mathrm{nd}(z \mid m) = \frac{\vartheta_n(z \mid m)}{\vartheta_d(z \mid m)}$$

Zeros

09.32.30.0001.01

$$\mathrm{nd}(2 r K(m) + i (2 s + 1) K(1-m) \mid m) = 0 /; \{r, s\} \in \mathbb{Z}$$

History

- C. G. J. Jacobi (1827)
- N.H. Abel (1827)
- J. Glaisher (1882) introduced the notation nd

Copyright