JacobiSC

View the online version at
functions.wolfram.com

Download the
PDF File

Notations

Traditional name

Jacobi elliptic function sc

Traditional notation

$\operatorname{sc}(z \mid m)$

Mathematica StandardForm notation

`JacobiSC[z, m]`

Primary definition

09.34.02.0001.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{sn}(z \mid m)}{\operatorname{cn}(z \mid m)}$$

Specific values

Specialized values

For fixed *z*

Case *m* = 0

09.34.03.0001.01

$$\operatorname{sc}(z \mid 0) = \tan(z)$$

09.34.03.0002.01

$$\operatorname{sc}\left(z + \frac{\pi}{2} \,\middle|\, 0\right) = -\cot(z)$$

09.34.03.0030.01

$$\operatorname{sc}\left(z + \frac{\pi k}{2} \,\middle|\, 0\right) = \tan\left(z + \frac{\pi k}{2}\right) /; k \in \mathbb{Z}$$

Case *m* = 1

09.34.03.0003.01

$$\operatorname{sc}(z \mid 1) = \sinh(z)$$

09.34.03.0004.01

$$\mathrm{sc}\left(z + \frac{\pi i}{2} \middle| 1\right) = i \cosh(z)$$

09.34.03.0031.01

$$\mathrm{sc}\left(z + \frac{i \pi k}{2} \middle| 1\right) = \sinh\left(z + \frac{i \pi k}{2}\right) /; k \in \mathbb{Z}$$

For fixed *m*

Values at quarter-period points in the fundamental period parallelogram

09.34.03.0005.01

$$\mathrm{sc}(0 \mid m) = 0$$

09.34.03.0006.01

$$\mathrm{sc}(K(m) \mid m) = \tilde{\infty}$$

09.34.03.0007.01

$$\mathrm{sc}(2 K(m) \mid m) = 0$$

09.34.03.0008.01

$$\mathrm{sc}(3 K(m) \mid m) = \tilde{\infty}$$

09.34.03.0009.01

$$\mathrm{sc}(4 K(m) \mid m) = 0$$

09.34.03.0010.01

$$\mathrm{sc}(i K(1-m) \mid m) = i$$

09.34.03.0011.01

$$\mathrm{sc}(2 i K(1-m) \mid m) = 0$$

09.34.03.0012.01

$$\mathrm{sc}(3 i K(1-m) \mid m) = -i$$

09.34.03.0013.01

$$\mathrm{sc}(4 i K(1-m) \mid m) = 0$$

09.34.03.0014.01

$$\mathrm{sc}(K(m) + i K(1-m) \mid m) = \frac{i}{\sqrt{1-m}}$$

09.34.03.0015.01

$$\mathrm{sc}(2 K(m) + i K(1-m) \mid m) = i$$

09.34.03.0016.01

$$\mathrm{sc}(3 K(m) + i K(1-m) \mid m) = \frac{i}{\sqrt{1-m}}$$

09.34.03.0017.01

$$\mathrm{sc}(4 K(m) + i K(1-m) \mid m) = i$$

09.34.03.0018.01

$$\mathrm{sc}(K(m) + 2 i K(1-m) \mid m) = \tilde{\infty}$$

09.34.03.0019.01

$$\mathrm{sc}(2 K(m) + 2 i K(1-m) \mid m) = 0$$

09.34.03.0020.01
$$\operatorname{sc}(3\,K(m) + 2\,i\,K(1-m) \mid m) == \tilde{\infty}$$

09.34.03.0021.01
$$\operatorname{sc}(4\,K(m) + 2\,i\,K(1-m) \mid m) == 0$$

09.34.03.0022.01
$$\operatorname{sc}(K(m) + 3\,i\,K(1-m) \mid m) == -\frac{i}{\sqrt{1-m}}$$

09.34.03.0023.01
$$\operatorname{sc}(2\,K(m) + 3\,i\,K(1-m) \mid m) == -i$$

09.34.03.0024.01
$$\operatorname{sc}(K(m) + 4\,i\,K(1-m) \mid m) == \tilde{\infty}$$

09.34.03.0025.01
$$\operatorname{sc}((2\,r+1)\,K(m) + 2\,s\,i\,K(1-m) \mid m) == \tilde{\infty} \;/;\; \{r,\, s\} \in \mathbb{Z}$$

09.34.03.0026.01
$$\operatorname{sc}(2\,K(m) + 4\,i\,K(1-m) \mid m) == 0$$

Values at half-quarter-period points

09.34.03.0027.01
$$\operatorname{sc}\left(\frac{K(m)}{2} \,\middle|\, m\right) == \frac{1}{\sqrt[4]{1-m}}$$

09.34.03.0028.01
$$\operatorname{sc}\left(\frac{i\,K(1-m)}{2} \,\middle|\, m\right) == \frac{i}{\sqrt{1+\sqrt{m}}}$$

09.34.03.0029.01
$$\operatorname{sc}\left(\frac{K(m)}{2} + \frac{i\,K(1-m)}{2} \,\middle|\, m\right) == \frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(i\,\sqrt{1-\sqrt{m}} + \sqrt{1+\sqrt{m}}\right)}{(1-m)^{1/4}}$$

General characteristics

Domain and analyticity

$\operatorname{sc}(z \mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.34.04.0001.01
$$(z * m) \longrightarrow \operatorname{sc}(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\operatorname{sc}(z \mid m)$ is an odd function with respect to z.

09.34.04.0002.01
$$\operatorname{sc}(-z \mid m) == -\operatorname{sc}(z \mid m)$$

Mirror symmetry

09.34.04.0003.01

$\mathrm{sc}(\bar{z} \mid \bar{m}) = \overline{\mathrm{sc}(z \mid m)}$

Periodicity

$\mathrm{sc}(z \mid m)$ is a doubly periodic function with respect to z with periods $4\, i\, K(1-m)$ and $2\, K(m)$.

09.34.04.0004.01

$\mathrm{sc}(z + 2\, K(m) \mid m) = \mathrm{sc}(z \mid m)$

09.34.04.0005.01

$\mathrm{sc}(z + 2\, i\, K(1-m) \mid m) = -\mathrm{sc}(z \mid m)$

09.34.04.0006.01

$\mathrm{sc}(z + 4\, i\, K(1-m) \mid m) = \mathrm{sc}(z \mid m)$

09.34.04.0007.01

$\mathrm{sc}(z + 2\, K(m) + 2\, i\, K(1-m) \mid m) = -\mathrm{sc}(z \mid m)$

09.34.04.0008.01

$\mathrm{sc}(z + 2\, i\, s\, K(1-m) + 2\, r\, K(m) \mid m) = (-1)^s\, \mathrm{sc}(z \mid m) \;/;\; \{r, s\} \in \mathbb{Z}$

Poles and essential singularities

With respect to *z*

For fixed m, the function $\mathrm{sc}(z \mid m)$ has an infinite set of singular points:

a) $z = (2\, r + 1)\, K(m) + 2\, s\, i\, K(1-m)$, $\{r, s\} \in \mathbb{Z}$, are the simple poles with residues $\frac{(-1)^{s-1}}{\sqrt{1-m}}$;

b) $z = \tilde{\infty}$ is an essential singular point.

09.34.04.0009.01

$\mathcal{S}ing_z(\mathrm{sc}(z \mid m)) = \{\{\{2\, s\, i\, K(1-m) + (2\, r + 1)\, K(m), 1\} \;/;\; \{r, s\} \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}\}$

09.34.04.0010.01

$$\mathrm{res}_z(\mathrm{sc}(z \mid m))\, (2\, s\, i\, K(1-m) + (2\, r + 1)\, K(m)) = \frac{(-1)^{s-1}}{\sqrt{1-m}} \;/;\; \{r, s\} \in \mathbb{Z}$$

Branch points

With respect to *m*

For fixed z, the function $\mathrm{sc}(z \mid m)$ is a meromorphic function in m that has no branch points.

09.34.04.0013.01

$\mathcal{BP}_m(\mathrm{sc}(z \mid m)) = \{\}$

P. Walker

With respect to *z*

For fixed m, the function $\mathrm{sc}(z \mid m)$ does not have branch points.

09.34.04.0011.01

$\mathcal{BP}_z(\mathrm{sc}(z \mid m)) == \{\}$

Branch cuts

With respect to *m*

For fixed z, the function $\mathrm{sc}(z \mid m)$ is a meromorphic function in m that has no branch cuts.

09.34.04.0014.01

$\mathcal{BC}_m(\mathrm{sc}(z \mid m)) == \{\}$

P. Walker

With respect to *z*

For fixed m, the function $\mathrm{sc}(z \mid m)$ does not have branch cuts.

09.34.04.0012.01

$\mathcal{BC}_z(\mathrm{sc}(z \mid m)) == \{\}$

Series representations

Generalized power series

Expansions at *z* == 0

09.34.06.0005.01

$$\mathrm{sc}(z \mid m) \propto z + \frac{1}{6}(2-m)z^3 + \frac{1}{120}\left(16 - 16m + m^2\right)z^5 + \ldots \;/;\; (z \to 0)$$

09.34.06.0001.02

$$\mathrm{sc}(z \mid m) \propto z + \frac{1}{6}(2-m)z^3 + \frac{1}{120}\left(16 - 16m + m^2\right)z^5 + \frac{\left(272 - 408m + 138m^2 - m^3\right)z^7}{5040} + \frac{\left(7936 - 15872m + 9168m^2 - 1232m^3 + m^4\right)z^9}{362880} + O\left(z^{11}\right)$$

09.34.06.0006.01

$$\mathrm{sc}(z \mid m) = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(j+1)(-1)^{k-j}\,\mathrm{sn}_{k-j}(m)}{(2k-2j+1)!}\sum_{r=0}^{j}\frac{(-1)^r}{r+1}\binom{j}{r}q_{r,j}\,z^{2k+1} \;/;\; q_{j,0} = 1 \bigwedge q_{j,k} = \frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i + i - k)(-1)^i\,\mathrm{cn}_i(m)\,q_{j,k-i}}{(2i)!} \bigwedge$$
$$k \in \mathbb{N}^+ \bigwedge \mathrm{sn}_0(m) = 1 \bigwedge \mathrm{sn}_n(m) = \sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2n}{2j}\mathrm{cn}_j(m)\,\mathrm{dn}_k(m)\,\delta_{j+k-n} \bigwedge \mathrm{cn}_0(m) = 1 \bigwedge$$
$$\mathrm{cn}_n(m) = \sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\mathrm{sn}_j(m)\,\mathrm{dn}_k(m)\,\delta_{j+k-n+1} \bigwedge \mathrm{dn}_0(m) = 1 \bigwedge \mathrm{dn}_n(m) = m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\mathrm{sn}_j(m)\,\mathrm{cn}_k(m)\,\delta_{j+k-n+1}$$

09.34.06.0007.01

$$\mathrm{sc}(z \mid m) \propto z + O\left(z^3\right)$$

Expansions at *z* == (2 *r* + 1) *K*(*m*) + 2 *i s K*(1 − *m*)

09.34.06.0008.01

$$\operatorname{sc}(z \mid m) \propto \frac{(-1)^{s-1}}{\sqrt{1-m}} \left(\frac{1}{z-z_0} + \frac{1}{6}(m-2)(z-z_0) + \frac{1}{360}\left(7m^2+8m-8\right)(z-z_0)^3 + \ldots \right) /;$$
$$(z \to z_0) \bigwedge z_0 = (2r+1)K(m) + 2is K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

09.34.06.0009.01

$$\operatorname{sc}(z \mid m) = \frac{(-1)^{s-1}}{\sqrt{1-m}} \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{(j+1)(-1)^{k-j} \operatorname{cn}_{k-j}(m)}{(2k-2j)!} \sum_{r=0}^{j} \frac{(-1)^r}{r+1} \binom{j}{r} q_{r,j} (z-z_0)^{2k-1} /;$$

$$z_0 = (2r+1)K(m) + 2isK(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z} \bigwedge q_{j,0} = 1 \bigwedge q_{j,k} = \frac{1}{k} \sum_{i=1}^{k} \frac{(j\,i+i-k)(-1)^i \operatorname{sn}_i(m)\, q_{j,k-i}}{(2i+1)!} \bigwedge$$

$$k \in \mathbb{N}^+ \bigwedge \operatorname{sn}_0(m) = 1 \bigwedge \operatorname{sn}_n(m) = \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{2n}{2j} \operatorname{cn}_j(m) \operatorname{dn}_k(m)\, \delta_{j+k-n} \bigwedge \operatorname{cn}_0(m) = 1 \bigwedge$$

$$\operatorname{cn}_n(m) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \operatorname{sn}_j(m) \operatorname{dn}_k(m)\, \delta_{j+k-n+1} \bigwedge \operatorname{dn}_0(m) = 1 \bigwedge \operatorname{dn}_n(m) = m \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \operatorname{sn}_j(m) \operatorname{cn}_k(m)\, \delta_{j+k-n+1}$$

09.34.06.0010.01

$$\operatorname{sc}(z \mid m) \propto \frac{(-1)^{s-1}}{\sqrt{1-m}\,(z-z_0)} \left(1 + O\left((z-z_0)^2\right)\right) /; z_0 = (2r+1)K(m) + 2isK(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

Expansions at *m* == 0

09.34.06.0011.01

$$\operatorname{sc}(z \mid m) \propto \tan(z) + \frac{1}{4}\left(\tan(z) - z\sec^2(z)\right) m - \frac{1}{512}\left(72z\cos(z) + 2\left(-16z^2 - 18\cos(2z) + \cos(4z) - 19\right)\sin(z)\right) \sec^3(z)\, m^2 + \ldots /;$$
$$(m \to 0)$$

09.34.06.0012.01

$$\operatorname{sc}(z \mid m) \propto \tan(z) + \frac{1}{4}\left(\tan(z) - z\sec^2(z)\right) m -$$
$$\frac{1}{512}\left(72z\cos(z) + 2\left(-16z^2 - 18\cos(2z) + \cos(4z) - 19\right)\sin(z)\right)\sec^3(z)\, m^2 + \frac{1}{12\,288}\left(-128z^3 + 2\left(32z^2 - 303\right)\cos(2z)\, z + \right.$$
$$\left. 24\cos(4z)\, z + 6\cos(6z)\, z - 624z + 3\left(112z^2 + 115\right)\sin(2z) + 150\sin(4z) - 15\sin(6z)\right)\sec^4(z)\, m^3 +$$
$$\frac{1}{1\,572\,864}\left(-96z\left(152z^2 + 975\right)\cos(z) + 16z\left(304z^2 - 1725\right)\cos(3z) + 3000z\cos(5z) + 600z\cos(7z) + \right.$$
$$\left(5632z^4 + 19\,392z^2 + 17\,715\right)\sin(z) - \left(512z^4 - 19\,776z^2 - 25\,059\right)\sin(3z) +$$
$$\left. 3\left(160z^2 + 2113\right)\sin(5z) + 6\left(16z^2 - 167\right)\sin(7z) + 3\sin(9z)\right)\sec^5(z)\, m^4 +$$
$$\frac{1}{62\,914\,560}\left(-33\,792z^5 - 303\,360z^3 + 4\left(6656z^4 - 51\,840z^2 - 497\,775\right)\cos(2z)\, z - \right.$$
$$8\left(128z^4 - 11\,800z^2 + 45\,135\right)\cos(4z)\, z - 120\left(16z^2 - 717\right)\cos(6z)\, z - 10\left(32z^2 - 1413\right)\cos(8z)\, z -$$
$$60\cos(10z)\, z - 1\,558\,170z + 30\left(5120z^4 + 24\,320z^2 + 23\,841\right)\sin(2z) - 30\left(512z^4 - 13\,000z^2 - 17\,193\right)\sin(4z) +$$
$$\left. 45\left(480z^2 + 1753\right)\sin(6z) + 45\left(80z^2 - 439\right)\sin(8z) + 135\sin(10z)\right)\sec^6(z)\, m^5 +$$
$$\frac{1}{24\,159\,191\,040}\left(-15\,360z\left(928z^4 + 10\,290z^2 + 54\,723\right)\cos(z) + 480z\left(18\,560z^4 - 50\,960z^2 - 787\,377\right)\cos(3z) - \right.$$
$$48z\left(7424z^4 - 330\,400z^2 + 721\,995\right)\cos(5z) - 1680z\left(560z^2 - 12\,423\right)\cos(7z) - 240z\left(560z^2 - 11\,931\right)\cos(9z) -$$
$$29\,520z\cos(11z) + 4\left(1\,236\,992z^6 + 8\,613\,120z^4 + 33\,029\,280z^2 + 29\,634\,165\right)\sin(z) -$$

$$3\left(311\,296\,z^6-10\,296\,320\,z^4-68\,224\,320\,z^2-67\,579\,665\right)\sin(3\,z)+$$
$$\left(16\,384\,z^6-3\,609\,600\,z^4+78\,102\,720\,z^2+95\,850\,225\right)\sin(5\,z)-30\left(1792\,z^4-215\,040\,z^2-265\,557\right)\sin(7\,z)-$$
$$30\left(256\,z^4-29\,952\,z^2+121\,377\right)\sin(9\,z)-45\left(128\,z^2-885\right)\sin(11\,z)-45\sin(13\,z)\Big)\sec^7(z)\,m^6+\frac{1}{1\,352\,914\,698\,240}$$
$$\Big(-39\,583\,744\,z^7-526\,934\,016\,z^5-4\,415\,416\,320\,z^3+6\left(6\,504\,448\,z^6-32\,657\,408\,z^4-860\,762\,560\,z^2-4\,958\,239\,335\right)$$
$$\cos(2\,z)\,z-3840\left(1024\,z^6-82\,880\,z^4+93\,758\,z^2+2\,528\,085\right)\cos(4\,z)\,z+$$
$$2\left(16\,384\,z^6-6\,289\,920\,z^4+175\,230\,720\,z^2-6\,228\,495\right)\cos(6\,z)\,z+672\left(256\,z^4-65\,440\,z^2+1\,024\,365\right)\cos(8\,z)\,z+$$
$$42\left(512\,z^4-125\,120\,z^2+1\,899\,015\right)\cos(10\,z)\,z+6720\left(8\,z^2-207\right)\cos(12\,z)\,z+1890\cos(14\,z)\,z-$$
$$20\,661\,268\,320\,z+35\left(6\,823\,936\,z^6+55\,971\,840\,z^4+257\,347\,584\,z^2+226\,081\,791\right)\sin(2\,z)-$$
$$28\left(1\,949\,696\,z^6-28\,876\,800\,z^4-273\,056\,400\,z^2-260\,531\,505\right)\sin(4\,z)+$$
$$7\left(139\,264\,z^6-16\,972\,800\,z^4+338\,054\,400\,z^2+352\,864\,395\right)\sin(6\,z)-840\left(5120\,z^4-284\,832\,z^2-72\,189\right)\sin(8\,z)-$$
$$105\left(5120\,z^4-264\,960\,z^2+917\,943\right)\sin(10\,z)-1260\left(368\,z^2-1157\right)\sin(12\,z)-4095\sin(14\,z)\Big)\sec^8(z)\,m^7+$$
$$\frac{1}{173\,173\,081\,374\,720}\Big(-16\,800\,z\left(372\,736\,z^6+5\,562\,240\,z^4+53\,071\,720\,z^2+241\,268\,799\right)\cos(z)+$$
$$6048\,z\left(905\,216\,z^6+1\,580\,160\,z^4-59\,648\,680\,z^2-367\,609\,745\right)\cos(3\,z)-$$
$$168\,z\left(3\,620\,864\,z^6-137\,978\,880\,z^4+64\,018\,880\,z^2+3\,076\,030\,125\right)\cos(5\,z)+$$
$$24\,z\left(212\,992\,z^6-35\,374\,080\,z^4+633\,644\,480\,z^2+2\,522\,909\,025\right)\cos(7\,z)+$$
$$30\,240\,z\left(1152\,z^4-127\,432\,z^2+1\,640\,373\right)\cos(9\,z)+1680\,z\left(2304\,z^4-222\,224\,z^2+2\,905\,971\right)\cos(11\,z)+$$
$$2520\,z\left(4352\,z^2-49\,343\right)\cos(13\,z)+430\,920\,z\cos(15\,z)+$$
$$2\left(1\,023\,606\,784\,z^8+11\,283\,980\,288\,z^6+64\,624\,062\,720\,z^4+276\,612\,940\,800\,z^2+226\,518\,668\,775\right)\sin(z)-$$
$$9\left(62\,521\,344\,z^8-1\,935\,130\,624\,z^6-20\,219\,404\,800\,z^4-114\,095\,358\,720\,z^2-96\,412\,306\,235\right)\sin(3\,z)+$$
$$2\left(16\,187\,392\,z^8-2\,528\,296\,960\,z^6+22\,150\,947\,840\,z^4+312\,004\,672\,560\,z^2+275\,867\,986\,905\right)\sin(5\,z)-$$
$$\left(131\,072\,z^8-96\,108\,544\,z^6+8\,858\,572\,800\,z^4-167\,025\,852\,000\,z^2-138\,260\,046\,645\right)\sin(7\,z)+$$
$$63\left(16\,384\,z^6-7\,687\,680\,z^4+293\,400\,000\,z^2-74\,361\,395\right)\sin(9\,z)+$$
$$224\left(512\,z^6-221\,040\,z^4+7\,951\,050\,z^2-25\,759\,215\right)\sin(11\,z)+$$
$$105\left(8192\,z^4-516\,384\,z^2+1\,063\,449\right)\sin(13\,z)+315\left(288\,z^2-1691\right)\sin(15\,z)+315\sin(17\,z)\Big)\sec^9(z)\,m^8+$$
$$\frac{1}{12\,468\,461\,858\,979\,840}\Big(-20\,472\,135\,680\,z^9-367\,470\,673\,920\,z^7-3\,702\,333\,035\,520\,z^5-31\,824\,706\,032\,000\,z^3+$$
$$4\left(5\,782\,503\,424\,z^8-11\,574\,558\,720\,z^6-826\,602\,803\,712\,z^4-11\,241\,612\,285\,120\,z^2-50\,828\,437\,643\,445\right)\cos(2\,z)\,z-$$
$$2\left(1\,914\,699\,776\,z^8-142\,726\,938\,624\,z^6-663\,875\,218\,944\,z^4+6\,913\,553\,623\,200\,z^2+43\,597\,401\,570\,765\right)\cos(4\,z)\,z+$$
$$8\left(16\,449\,536\,z^8-4\,428\,693\,504\,z^6+112\,644\,725\,760\,z^4-33\,210\,581\,040\,z^2-1\,635\,148\,968\,705\right)\cos(6\,z)\,z-$$
$$4\left(65\,536\,z^8-71\,589\,888\,z^6+7\,153\,429\,248\,z^4-66\,253\,556\,880\,z^2-1\,097\,919\,700\,605\right)\cos(8\,z)\,z-$$
$$2880\left(1024\,z^6-782\,880\,z^4+58\,308\,600\,z^2-676\,097\,415\right)\cos(10\,z)\,z-$$
$$18\left(16\,384\,z^6-10\,633\,728\,z^4+693\,621\,600\,z^2-9\,057\,217\,365\right)\cos(12\,z)\,z-756\left(8192\,z^4-1\,027\,360\,z^2+7\,440\,975\right)$$
$$\cos(14\,z)\,z-22\,680\left(72\,z^2-1493\right)\cos(16\,z)\,z-22\,680\cos(18\,z)\,z-131\,813\,346\,602\,700\,z+$$
$$63\left(2\,332\,819\,456\,z^8+29\,091\,840\,000\,z^6+179\,510\,599\,680\,z^4+871\,387\,205\,760\,z^2+682\,578\,963\,495\right)\sin(2\,z)-$$
$$63\left(833\,355\,776\,z^8-9\,007\,718\,400\,z^6-129\,806\,760\,960\,z^4-914\,279\,256\,720\,z^2-730\,307\,555\,235\right)\sin(4\,z)+$$
$$9\left(354\,680\,832\,z^8-25\,156\,812\,800\,z^6+136\,480\,081\,920\,z^4+3\,095\,763\,602\,400\,z^2+2\,456\,267\,583\,735\right)\sin(6\,z)-$$
$$72\left(180\,224\,z^8-66\,483\,200\,z^6+5\,102\,603\,520\,z^4-92\,641\,079\,790\,z^2-56\,820\,978\,585\right)\sin(8\,z)+$$
$$315\left(409\,600\,z^6-75\,909\,120\,z^4+2\,395\,379\,520\,z^2-1\,350\,388\,863\right)\sin(10\,z)+$$
$$315\left(40\,960\,z^6-5\,698\,560\,z^4+188\,126\,064\,z^2-611\,766\,981\right)\sin(12\,z)+$$
$$7560\left(14\,336\,z^4-382\,788\,z^2+602\,157\right)\sin(14\,z)+5670\left(2232\,z^2-5525\right)\sin(16\,z)+48\,195\sin(18\,z)\Big)\sec^{10}(z)\,m^9+$$

$$\frac{1}{7\,979\,815\,589\,747\,097\,600}\Big(-240\,z\,\big(72\,741\,552\,128\,z^8+1\,430\,099\,066\,880\,z^6+15\,175\,126\,987\,776\,z^4+143\,545\,963\,216\,320\,z^2+567\,588\,084\,861\,735\big)\cos(z)+480\,z\,\big(39\,405\,420\,544\,z^8+218\,540\,298\,240\,z^6-1\,371\,786\,038\,784\,z^4-39\,228\,923\,137\,320\,z^2-175\,326\,548\,057\,325\big)\cos(3\,z)-26\,880\,z\,\big(134\,815\,744\,z^8-4\,081\,820\,160\,z^6-28\,787\,057\,856\,z^4+175\,581\,265\,545\,z^2+1\,053\,553\,433\,850\big)\cos(5\,z)+480\,z\,\big(268\,140\,544\,z^8-32\,222\,699\,520\,z^6+631\,247\,395\,968\,z^4-298\,989\,472\,320\,z^2-3\,869\,987\,716\,665\big)\cos(7\,z)-1120\,z\,\big(229\,376\,z^8-101\,376\,000\,z^6+7\,125\,757\,056\,z^4+5\,253\,474\,240\,z^2-1\,884\,308\,786\,235\big)\cos(9\,z)-31\,680\,z\,\big(112\,640\,z^6-29\,022\,336\,z^4+1\,873\,361\,700\,z^2-20\,927\,561\,445\big)\cos(11\,z)-5760\,z\,\big(56\,320\,z^6-6\,640\,704\,z^4+522\,347\,490\,z^2-8\,177\,342\,355\big)\cos(13\,z)-7560\,z\,\big(999\,424\,z^4-51\,359\,520\,z^2+279\,537\,015\big)\cos(15\,z)-113\,400\,z\,\big(19\,296\,z^2-165\,199\big)\cos(17\,z)-33\,112\,800\,z\cos(19\,z)+2\,\big(2\,748\,011\,511\,808\,z^{10}+41\,952\,187\,514\,880\,z^8+345\,480\,654\,274\,560\,z^6+1\,807\,745\,271\,897\,600\,z^4+8\,497\,718\,945\,704\,800\,z^2+6\,288\,200\,451\,412\,425\big)\sin(z)-2\,\big(954\,606\,813\,184\,z^{10}-26\,964\,933\,672\,960\,z^8-452\,624\,165\,683\,200\,z^6-3\,108\,972\,461\,011\,200\,z^4-17\,443\,458\,335\,167\,200\,z^2-12\,988\,772\,629\,429\,275\big)\sin(3\,z)+10\,\big(20\,065\,550\,336\,z^{10}-2\,815\,274\,188\,800\,z^8+12\,926\,866\,046\,976\,z^6+297\,957\,576\,349\,440\,z^4+2\,660\,842\,809\,545\,760\,z^2+1\,977\,158\,789\,043\,165\big)\sin(5\,z)-4\,\big(1\,062\,207\,488\,z^{10}-453\,524\,520\,960\,z^8+20\,777\,484\,349\,440\,z^6-63\,238\,275\,072\,000\,z^4-2\,708\,553\,310\,722\,000\,z^2-1\,874\,819\,891\,427\,525\big)\sin(7\,z)+4\,\big(1\,048\,576\,z^{10}-1\,931\,673\,600\,z^8+493\,750\,333\,440\,z^6-33\,127\,034\,572\,800\,z^4+590\,265\,992\,458\,800\,z^2+245\,614\,921\,985\,475\big)\sin(9\,z)-45\,\big(1\,441\,792\,z^8-1\,587\,052\,544\,z^6+194\,838\,336\,000\,z^4-5\,805\,285\,837\,120\,z^2+4\,550\,752\,841\,985\big)\sin(11\,z)-45\,\big(131\,072\,z^8-100\,237\,312\,z^6+7\,884\,817\,920\,z^4-360\,300\,104\,640\,z^2+1\,262\,320\,276\,455\big)\sin(13\,z)-315\,\big(1\,048\,576\,z^6-231\,298\,560\,z^4+3\,856\,707\,360\,z^2-5\,043\,389\,535\big)\sin(15\,z)-14\,175\,\big(13\,824\,z^4-666\,272\,z^2+1\,016\,955\big)\sin(17\,z)-14\,175\,\big(512\,z^2-2753\big)\sin(19\,z)-14\,175\sin(21\,z)\Big)\sec^{11}(z)\,m^{10}+O\big(m^{11}\big)$$

09.34.06.0013.01

$$\operatorname{sc}(z\mid m)\propto\tan(z)\,(1+O(m))$$

Expansions at *m* == 1

09.34.06.0014.01

$$\operatorname{sc}(z\mid m)\propto\sinh(z)-\frac{1}{8}\cosh(z)\,(\sinh(2\,z)-2\,z)\,(m-1)+\frac{1}{256}\big(-24\,z\cosh(z)-12\,z\cosh(3\,z)+\big(8\,z^2+7\big)\sinh(z)+8\sinh(3\,z)+\sinh(5\,z)\big)\,(m-1)^2+\ldots\;/;\;(m\to1)$$

09.34.06.0015.01

$$\operatorname{sc}(z\mid m)\propto\sinh(z)-\frac{1}{8}\cosh(z)\,(\sinh(2\,z)-2\,z)\,(m-1)+\frac{1}{256}\big(-24\,z\cosh(z)-12\,z\cosh(3\,z)+\big(8\,z^2+7\big)\sinh(z)+8\sinh(3\,z)+\sinh(5\,z)\big)\,(m-1)^2+\frac{1}{12\,288}\big(32\,z\,\big(z^2+21\big)\cosh(z)+468\,z\cosh(3\,z)+60\,z\cosh(5\,z)-3\,\big(88\,z^2+67\big)\sinh(z)-6\,\big(36\,z^2+41\big)\sinh(3\,z)-48\sinh(5\,z)-3\sinh(7\,z)\big)\,(m-1)^3+\frac{1}{196\,608}\big(-4\,z\,\big(128\,z^2+1845\big)\cosh(z)-72\,z\,\big(12\,z^2+83\big)\cosh(3\,z)-1260\,z\cosh(5\,z)-84\,z\cosh(7\,z)+2\,\big(16\,z^4+1560\,z^2+1107\big)\sinh(z)+81\,\big(48\,z^2+35\big)\sinh(3\,z)+30\,\big(20\,z^2+23\big)\sinh(5\,z)+72\sinh(7\,z)+3\sinh(9\,z)\big)\,(m-1)^4+$$

$$\frac{1}{15\,728\,640}\Big(8\,z\,(16\,z^4+4580\,z^2+55\,245)\cosh(z)+1440\,z\,(69\,z^2+272)\cosh(3\,z)+200\,z\,(100\,z^2+531)\cosh(5\,z)+12\,180\,z\cosh(7\,z)+540\,z\cosh(9\,z)-120\,(28\,z^4+1628\,z^2+1101)\sinh(z)-45\,(288\,z^4+6600\,z^2+3887)\sinh(3\,z)-240\,(325\,z^2+204)\sinh(5\,z)-30\,(196\,z^2+221)\sinh(7\,z)-480\sinh(9\,z)-15\sinh(11\,z)\Big)(m-1)^5+$$

$$\frac{1}{754\,974\,720}\Big(-12\,z\,(832\,z^4+129\,240\,z^2+1\,397\,055)\cosh(z)-648\,z\,(144\,z^4+8180\,z^2+24\,385)\cosh(3\,z)-300\,z\,(6200\,z^2+16\,761)\cosh(5\,z)-840\,z\,(196\,z^2+909)\cosh(7\,z)-59\,940\,z\cosh(9\,z)-1980\,z\cosh(11\,z)+8\,(32\,z^6+21\,180\,z^4+955\,575\,z^2+622\,845)\sinh(z)+270\,(4032\,z^4+48\,564\,z^2+25\,081)\sinh(3\,z)+15\,(20\,000\,z^4+301\,800\,z^2+139\,557)\sinh(5\,z)+720\,(833\,z^2+465)\sinh(7\,z)+90\,(324\,z^2+359)\sinh(9\,z)+1800\sinh(11\,z)+45\sinh(13\,z)\Big)(m-1)^6+$$

$$\frac{1}{84\,557\,168\,640}\Big(4\,z\,(256\,z^6+337\,344\,z^4+38\,446\,800\,z^2+384\,987\,645)\cosh(z)+756\,z\,(28\,512\,z^4+801\,960\,z^2+2\,009\,245)\cosh(3\,z)+2100\,z\,(4000\,z^4+135\,400\,z^2+257\,937)\cosh(5\,z)+8820\,z\,(5096\,z^2+11\,175)\cosh(7\,z)+22\,680\,z\,(108\,z^2+461)\cosh(9\,z)+623\,700\,z\cosh(11\,z)+16\,380\,z\cosh(13\,z)-14\,(3968\,z^6+1\,331\,760\,z^4+51\,350\,220\,z^2+32\,480\,415)\sinh(z)-63\,(20\,736\,z^6+2\,453\,760\,z^4+21\,261\,240\,z^2+10\,022\,255)\sinh(3\,z)-315\,(240\,000\,z^4+1\,750\,000\,z^2+668\,241)\sinh(5\,z)-735\,(10\,976\,z^4+133\,560\,z^2+51\,573)\sinh(7\,z)-5040\,(1701\,z^2+874)\sinh(9\,z)-630\,(484\,z^2+529)\sinh(11\,z)-15\,120\sinh(13\,z)-315\sinh(15\,z)\Big)(m-1)^7+$$

$$\frac{1}{1\,352\,914\,698\,240}\Big(-12\,z\,(3072\,z^6+1\,950\,368\,z^4+183\,014\,720\,z^2+1\,730\,146\,845)\cosh(z)-648\,z\,(3456\,z^6+758\,016\,z^4+14\,749\,280\,z^2+32\,740\,365)\cosh(3\,z)-21\,000\,z\,(16\,400\,z^4+257\,560\,z^2+392\,841)\cosh(5\,z)-588\,z\,(76\,832\,z^4+1\,977\,640\,z^2+2\,906\,355)\cosh(7\,z)-11\,340\,z\,(10\,152\,z^2+19\,451)\cosh(9\,z)-9240\,z\,(484\,z^2+1953)\cosh(11\,z)-868\,140\,z\cosh(13\,z)-18\,900\,z\cosh(15\,z)+(512\,z^8+1\,211\,392\,z^6+285\,670\,560\,z^4+9\,875\,050\,920\,z^2+6\,089\,965\,245)\sinh(z)+63\,(787\,968\,z^6+44\,102\,880\,z^4+310\,448\,520\,z^2+136\,673\,455)\sinh(3\,z)+70\,(400\,000\,z^6+26\,280\,000\,z^4+130\,369\,500\,z^2+43\,550\,109)\sinh(5\,z)+5880\,(60\,368\,z^4+334\,950\,z^2+102\,195)\sinh(7\,z)+315\,(69\,984\,z^4+745\,848\,z^2+253\,307)\sinh(9\,z)+55\,440\,(275\,z^2+133)\sinh(11\,z)+630\,(676\,z^2+731)\sinh(13\,z)+17\,640\sinh(15\,z)+315\sinh(17\,z)\Big)(m-1)^8+$$

$$\frac{1}{194\,819\,716\,546\,560}\Big(8\,z\,(256\,z^8+1\,008\,000\,z^6+434\,436\,912\,z^4+35\,584\,337\,880\,z^2+321\,447\,804\,615)\cosh(z)+3888\,z\,(222\,912\,z^6+22\,302\,000\,z^4+342\,903\,750\,z^2+695\,415\,875)\cosh(3\,z)+900\,z\,(800\,000\,z^6+92\,400\,000\,z^4+959\,477\,400\,z^2+1\,251\,408\,501)\cosh(5\,z)+5292\,z\,(3\,764\,768\,z^4+42\,904\,400\,z^2+48\,678\,885)\cosh(7\,z)+20\,412\,z\,(69\,984\,z^4+1\,517\,400\,z^2+1\,876\,715)\cosh(9\,z)+41\,580\,z\,(53\,240\,z^2+92\,781)\cosh(11\,z)+98\,280\,z\,(676\,z^2+2619)\cosh(13\,z)+10\,376\,100\,z\cosh(15\,z)+192\,780\,z\cosh(17\,z)-18\,(10\,496\,z^8+11\,495\,680\,z^6+2\,165\,834\,160\,z^4+69\,005\,718\,180\,z^2+41\,633\,079\,075)\sinh(z)-162\,(186\,624\,z^8+69\,745\,536\,z^6+2\,616\,757\,920\,z^4+15\,922\,895\,940\,z^2+6\,643\,081\,375)\sinh(3\,z)-630\,(18\,400\,000\,z^6+542\,880\,000\,z^4+2\,102\,352\,300\,z^2+635\,484\,717)\sinh(5\,z)-126\,(15\,059\,072\,z^6+719\,147\,520\,z^4+2\,605\,542\,660\,z^2+673\,078\,005)\sinh(7\,z)-45\,360\,(227\,448\,z^4+1\,059\,399\,z^2+274\,924)\sinh(9\,z)-945\,(468\,512\,z^4+4\,562\,184\,z^2+1\,409\,247)\sinh(11\,z)-45\,360\,(4901\,z^2+2264)\sinh(13\,z)-28\,350\,(180\,z^2+193)\sinh(15\,z)-181\,440\sinh(17\,z)-2835\sinh(19\,z)\Big)(m-1)^9+$$

$$\frac{1}{15\,585\,577\,323\,724\,800}\Big(-160\,z\,(2944\,z^8+5\,196\,384\,z^6+1\,745\,402\,148\,z^4+129\,145\,913\,190\,z^2+1\,124\,016\,300\,855)\cosh(z)-131\,220\,z\,(1536\,z^8+917\,760\,z^6+59\,976\,224\,z^4+780\,845\,800\,z^2+1\,475\,445\,055)\cosh(3\,z)-$$

$$540\,000\, z \left(340\,000\, z^6 + 17\,170\,300\, z^4 + 136\,270\,295\, z^2 + 158\,031\,489\right) \cosh(5\, z) -$$
$$17\,640\, z \left(2\,151\,296\, z^6 + 174\,254\,976\, z^4 + 1\,268\,918\,700\, z^2 + 1\,197\,550\,395\right) \cosh(7\, z) -$$
$$918\,540\, z \left(443\,232\, z^4 + 4\,107\,600\, z^2 + 3\,812\,645\right) \cosh(9\, z) -$$
$$41\,580\, z \left(468\,512\, z^4 + 9\,026\,600\, z^2 + 9\,864\,255\right) \cosh(11\, z) - 737\,100\, z \left(28\,392\, z^2 + 46\,115\right) \cosh(13\, z) -$$
$$1\,701\,000\, z \left(300\, z^2 + 1127\right) \cosh(15\, z) - 66\,509\,100\, z \cosh(17\, z) - 1\,077\,300\, z \cosh(19\, z) +$$
$$\left(4096\, z^{10} + 25\,320\,960\, z^8 + 18\,304\,957\,440\, z^6 + 2\,944\,262\,714\,400\, z^4 + 88\,003\,507\,762\,800\, z^2 + 52\,079\,055\,504\,525\right)$$
$$\sinh(z) + 1215 \left(5\,971\,968\, z^8 + 988\,533\,504\, z^6 + 28\,639\,981\,440\, z^4 + 156\,265\,593\,960\, z^2 + 62\,399\,823\,955\right) \sinh(3\, z) +$$
$$225 \left(40\,000\,000\, z^8 + 7\,554\,400\,000\, z^6 + 143\,925\,600\,000\, z^4 + 467\,583\,076\,800\, z^2 + 130\,698\,250\,767\right) \sinh(5\, z) +$$
$$945 \left(542\,126\,592\, z^6 + 11\,186\,739\,200\, z^4 + 30\,686\,626\,320\, z^2 + 6\,994\,157\,055\right) \sinh(7\, z) +$$
$$17\,010 \left(2\,519\,424\, z^6 + 98\,210\,880\, z^4 + 289\,458\,900\, z^2 + 61\,983\,655\right) \sinh(9\, z) +$$
$$113\,400 \left(1\,171\,280\, z^4 + 4\,826\,206\, z^2 + 1\,109\,661\right) \sinh(11\, z) +$$
$$4725 \left(913\,952\, z^4 + 8\,335\,080\, z^2 + 2\,394\,237\right) \sinh(13\, z) + 680\,400 \left(2475\, z^2 + 1103\right) \sinh(15\, z) +$$
$$28\,350 \left(1156\, z^2 + 1231\right) \sinh(17\, z) + 1\,020\,600 \sinh(19\, z) + 14\,175 \sinh(21\, z)\Big) (m-1)^{10} + O\left((m-1)^{11}\right)$$

09.34.06.0016.01

$$\operatorname{sc}(z \mid m) \propto \sinh(z)\,(1 + O(m-1))$$

q-series

09.34.06.0002.01

$$\operatorname{sc}(z \mid m) = \frac{\pi}{2\sqrt{1-m}\; K(m)} \tan\left(\frac{\pi z}{2 K(m)}\right) + \frac{2\pi}{\sqrt{1-m}\; K(m)} \sum_{k=1}^{\infty} \frac{(-1)^k q(m)^{2k}}{q(m)^{2k} + 1} \sin\left(\frac{k \pi z}{K(m)}\right)$$

Other series representations

09.34.06.0003.01

$$\operatorname{sc}(z \mid m) = -\frac{\pi}{2\sqrt{1-m}\; K(1-m)} \sum_{k=-\infty}^{\infty} \operatorname{csch}\left(\pi \frac{K(m)}{K(1-m)} \left(k + \frac{1}{2} + \frac{z}{2 K(m)}\right)\right)$$

09.34.06.0004.01

$$\operatorname{sc}(z \mid m) \propto \frac{(-1)^{s-1}}{\sqrt{1-m}\; (z - 2 s i K(1-m) - (2r+1) K(m))} + O(1) \;/;\; (z \to 2 s i K(1-m) + (2r+1) K(m)) \bigwedge \{r, s\} \in \mathbb{Z}$$

Product representations

09.34.08.0001.01

$$\operatorname{sc}(z \mid m) = \frac{1}{\sqrt[4]{1-m}} \tan\left(\frac{\pi z}{2 K(m)}\right) \prod_{k=1}^{\infty} \frac{1 - 2 q(m)^{2k} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k}}{1 + 2 q(m)^{2k} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k}}$$

Differential equations

Ordinary nonlinear differential equations

09.34.13.0001.01

$$w''(z) - w(z) \left(2 (1-m) w(z)^2 - m + 2\right) = 0 \;/;\; w(z) = \operatorname{sc}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.34.16.0001.01

$$\operatorname{sc}(i\, z,\, m) = i\, \operatorname{sn}(z,\, 1-m)$$

09.34.16.0002.01

$$\operatorname{sc}(z \mid 1-m) = -i\, \operatorname{sn}(i\, z \mid m)$$

09.34.16.0003.01

$$\operatorname{sc}(i\, z \mid 1-m) = i\, \operatorname{sn}(z \mid m)$$

09.34.16.0007.01

$$\operatorname{sc}(x + i\, y \mid m) = \frac{\operatorname{dn}(y \mid 1-m)\operatorname{sn}(x \mid m) + i\operatorname{cn}(x \mid m)\operatorname{cn}(y \mid 1-m)\operatorname{dn}(x \mid m)\operatorname{sn}(y \mid 1-m)}{\operatorname{cn}(x \mid m)\operatorname{cn}(y \mid 1-m) - i\operatorname{sn}(x \mid m)\operatorname{dn}(x \mid m)\operatorname{sn}(y \mid 1-m)\operatorname{dn}(y \mid 1-m)} \;/;\; \{x,\, y\} \in \mathbb{R}$$

09.34.16.0008.01

$$\operatorname{sc}\left(\sqrt{1-m}\, z \,\middle|\, \frac{m}{m-1}\right) = \sqrt{1-m}\, \operatorname{sc}(z \mid m)$$

09.34.16.0009.01

$$\operatorname{sc}\left(\sqrt{m}\, z \,\middle|\, \frac{1}{m}\right) = \sqrt{m}\, \operatorname{sd}(z \mid m)$$

09.34.16.0010.01

$$\operatorname{sc}\left(i\sqrt{1-m}\, z \,\middle|\, \frac{1}{1-m}\right) = i\sqrt{1-m}\, \operatorname{sd}(z \mid m)$$

09.34.16.0011.01

$$\operatorname{sc}\left(i\sqrt{m}\, z \,\middle|\, \frac{m-1}{m}\right) = i\sqrt{m}\, \operatorname{sn}(z \mid m)$$

Landen's transformation:

09.34.16.0012.01

$$\operatorname{sc}\left(\left(1+\sqrt{1-m}\right) z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{\left(1+\sqrt{1-m}\right)\operatorname{sn}(z \mid m)\operatorname{cn}(z \mid m)}{1-\left(1+\sqrt{1-m}\right)\operatorname{sn}(z \mid m)^2}$$

Gauss' transformation:

09.34.16.0013.01

$$\operatorname{sc}\left(\left(1+\sqrt{m}\right) z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{\left(1+\sqrt{m}\right)\operatorname{sn}(z \mid m)}{\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}$$

n th degree transformations:

09.34.16.0014.01

$$\operatorname{sc}\left(\frac{z}{M} \,\middle|\, l\right) == \frac{1}{M} \operatorname{sc}(z \mid m) \prod_{r=1}^{\frac{n-1}{2}} \frac{1 - \operatorname{sn}(z \mid m)^2 \operatorname{ns}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^2}{1 - \operatorname{sn}(z \mid m)^2 \operatorname{ns}\left(\frac{(2 r-1) K(m)}{n} \,\middle|\, m\right)^2} \;/;$$

$$\frac{n+1}{2} \in \mathbb{Z}^+ \bigwedge l == m^n \prod_{r=1}^{\frac{n-1}{2}} \operatorname{sn}\left(\frac{(2 r - 1) K(m)}{n} \,\middle|\, m\right)^8 \bigwedge M == \prod_{r=1}^{\frac{n-1}{2}} \frac{\operatorname{sn}\left(\frac{(2 r-1) K(m)}{n} \,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^2}$$

09.34.16.0015.01

$$\operatorname{sc}\left(\frac{z}{M} + \frac{K(m)}{n M} \,\middle|\, l\right) == -\frac{M}{\sqrt{1-l}} \operatorname{cs}(z \mid m) \prod_{r=1}^{\frac{n}{2}} \frac{1 - \operatorname{sn}(z \mid m)^2 \operatorname{ns}\left(\frac{(2 r-1) K(m)}{n} \,\middle|\, m\right)^2}{1 - \operatorname{sn}(z \mid m)^2 \operatorname{ns}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^2} \;/;$$

$$\frac{n}{2} \in \mathbb{Z}^+ \bigwedge l == m^n \prod_{r=1}^{\frac{n}{2}} \operatorname{sn}\left(\frac{(2 r - 1) K(m)}{n} \,\middle|\, m\right)^8 \bigwedge M == \prod_{r=1}^{\frac{n}{2}} \frac{\operatorname{sn}\left(\frac{(2 r-1) K(m)}{n} \,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^2}$$

Argument involving half-periods

09.34.16.0004.01

$$\operatorname{sc}(z + K(m) \mid m) == -\frac{1}{\sqrt{1-m}} \operatorname{cs}(z \mid m)$$

09.34.16.0020.01

$$\operatorname{sc}(z - K(m) \mid m) = -\frac{\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.34.16.0021.01

$$\operatorname{sc}(z + 3 K(m) \mid m) = -\frac{\operatorname{cs}(z \mid m)}{\sqrt{1-m}}$$

09.34.16.0022.01

$$\operatorname{sc}(z + (2 r + 1) K(m) \mid m) = -\frac{1}{\sqrt{1-m}} \operatorname{cs}(z \mid m) \;/;\; r \in \mathbb{Z}$$

09.34.16.0005.01

$$\operatorname{sc}(z + i K(1 - m) \mid m) == i \operatorname{nd}(z \mid m)$$

09.34.16.0023.01

$$\operatorname{sc}(z - i K(1 - m) \mid m) = -i \operatorname{nd}(z \mid m)$$

09.34.16.0024.01

$$\operatorname{sc}(z + 3 i K(1 - m) \mid m) = -i \operatorname{nd}(z \mid m)$$

09.34.16.0025.01

$$\operatorname{sc}(z + (2 s + 1) i K(1 - m) \mid m) = (-1)^s i \operatorname{nd}(z \mid m) \;/;\; s \in \mathbb{Z}$$

09.34.16.0006.01

$$\operatorname{sc}(z + i K(1 - m) + K(m) \mid m) == \frac{i}{\sqrt{1-m}} \operatorname{dn}(z \mid m)$$

09.34.16.0026.01

$$\operatorname{sc}(z - i\,K(1-m) + K(m) \mid m) = -\frac{i\,\operatorname{dn}(z \mid m)}{\sqrt{1-m}}$$

09.34.16.0027.01

$$\operatorname{sc}(z + i\,K(1-m) - K(m) \mid m) = \frac{i\,\operatorname{dn}(z \mid m)}{\sqrt{1-m}}$$

09.34.16.0028.01

$$\operatorname{sc}(z - i\,K(1-m) - K(m) \mid m) = -\frac{i\,\operatorname{dn}(z \mid m)}{\sqrt{1-m}}$$

09.34.16.0029.01

$$\operatorname{sc}(z + i\,K(1-m) + 3\,K(m) \mid m) = \frac{i\,\operatorname{dn}(z \mid m)}{\sqrt{1-m}}$$

09.34.16.0030.01

$$\operatorname{sc}(z + (4\,s+1)\,i\,K(1-m) + (2\,r+1)\,K(m) \mid m) = \frac{i\,\operatorname{dn}(z \mid m)}{\sqrt{1-m}} \;/;\; \{r, s\} \in \mathbb{Z}$$

09.34.16.0031.01

$$\operatorname{sc}(z + (4\,s-1)\,i\,K(1-m) + (2\,r+1)\,K(m) \mid m) = -\frac{i\,\operatorname{dn}(z \mid m)}{\sqrt{1-m}} \;/;\; \{r, s\} \in \mathbb{Z}$$

09.34.16.0032.01

$$\operatorname{sc}(z + (2\,s+1)\,i\,K(1-m) + (2\,r+1)\,K(m) \mid m) = \frac{(-1)^s\,i\,\operatorname{dn}(z \mid m)}{\sqrt{1-m}} \;/;\; \{r, s\} \in \mathbb{Z}$$

Argument involving inverse Jacobi functions

09.34.16.0033.01

$$\operatorname{sc}\left(\operatorname{cd}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2-1}{(m-1)\,z^2}$$

09.34.16.0034.01

$$\operatorname{sc}\left(\operatorname{cn}^{-1}(z \mid m) \mid m\right)^2 = \frac{1-z^2}{z^2}$$

09.34.16.0035.01

$$\operatorname{sc}\left(\operatorname{cs}^{-1}(z \mid m) \mid m\right) = \frac{1}{z}$$

09.34.16.0036.01

$$\operatorname{sc}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right)^2 = \frac{1-z^2}{m-1}$$

09.34.16.0037.01

$$\operatorname{sc}\left(\operatorname{dn}^{-1}(z \mid m) \mid m\right)^2 = \frac{1-z^2}{z^2+m-1}$$

09.34.16.0038.01

$$\operatorname{sc}\left(\operatorname{ds}^{-1}(z \mid m) \mid m\right)^2 = \frac{1}{z^2+m-1}$$

09.34.16.0039.01

$$\operatorname{sc}\left(\operatorname{nc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = z^2 - 1$$

09.34.16.0040.01

$$\operatorname{sc}\left(\operatorname{nd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2 - 1}{(m-1)\, z^2 + 1}$$

09.34.16.0041.01

$$\operatorname{sc}\left(\operatorname{ns}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{1}{z^2 - 1}$$

09.34.16.0042.01

$$\operatorname{sc}\left(\operatorname{sd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2}{(m-1)\, z^2 + 1}$$

09.34.16.0043.01

$$\operatorname{sc}\left(\operatorname{sn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2}{1 - z^2}$$

Addition formulas

09.34.16.0016.01

$$\operatorname{sc}(u + v \mid m) = \frac{\operatorname{cn}(v \mid m)\operatorname{dn}(v \mid m)\operatorname{sn}(u \mid m) + \operatorname{cn}(u \mid m)\operatorname{dn}(u \mid m)\operatorname{sn}(v \mid m)}{\operatorname{cn}(u \mid m)\operatorname{cn}(v \mid m) - \operatorname{sn}(u \mid m)\operatorname{dn}(u \mid m)\operatorname{sn}(v \mid m)\operatorname{dn}(v \mid m)}$$

09.34.16.0017.01

$$\operatorname{sc}(u + v \mid m)\operatorname{sc}(u - v \mid m) = \frac{\operatorname{sn}(u \mid m)^2 - \operatorname{sn}(v \mid m)^2}{\operatorname{cn}(v \mid m)^2 - \operatorname{dn}(v \mid m)^2 \operatorname{sn}(u \mid m)^2}$$

Half-angle formulas

09.34.16.0018.01

$$\operatorname{sc}\left(\frac{z}{2} \,\middle|\, m\right)^2 = \frac{1 - \operatorname{cn}(z \mid m)}{\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)}$$

Multiple arguments

Double angle formulas

09.34.16.0019.01

$$\operatorname{sc}(2 z \mid m) = \frac{2 \operatorname{sn}(z \mid m)\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{cn}(z \mid m)^2 - \operatorname{sn}(z \mid m)^2 \operatorname{dn}(z \mid m)^2}$$

Identities

Functional identities

09.34.17.0001.01

$$\left((m-1)\, w(z)^4 + 1\right)^2 w(2 z)^2 - 4 \left(w(z)^2 + 1\right)\left((1-m)\, w(z)^2 + 1\right) w(z)^2 = 0 \;/;\; w(z) = \operatorname{sc}(z \mid m)$$

Complex characteristics

Real part

09.34.19.0001.01

$$\operatorname{Re}(\operatorname{sc}(x + i\, y \mid m)) = \frac{\operatorname{cn}(x \mid m)\, \operatorname{cn}(y \mid 1 - m)\, \operatorname{dn}(y \mid 1 - m)\, \operatorname{sn}(x \mid m) \left(1 - \operatorname{dn}(x \mid m)^2\, \operatorname{sn}(y \mid 1 - m)^2\right)}{\operatorname{cn}(x \mid m)^2\, \operatorname{cn}(y \mid 1 - m)^2 + \operatorname{dn}(x \mid m)^2\, \operatorname{dn}(y \mid 1 - m)^2\, \operatorname{sn}(x \mid m)^2\, \operatorname{sn}(y \mid 1 - m)^2} \;/;\; \{x, y, m\} \in \mathbb{R}$$

Imaginary part

09.34.19.0002.01

$$\operatorname{Im}(\operatorname{sc}(x + i\, y \mid m)) = \frac{\operatorname{dn}(x \mid m) \left(\operatorname{cn}(x \mid m)^2\, \operatorname{cn}(y \mid 1 - m)^2 + \operatorname{dn}(y \mid 1 - m)^2\, \operatorname{sn}(x \mid m)^2\right) \operatorname{sn}(y \mid 1 - m)}{\operatorname{cn}(x \mid m)^2\, \operatorname{cn}(y \mid 1 - m)^2 + \operatorname{dn}(x \mid m)^2\, \operatorname{dn}(y \mid 1 - m)^2\, \operatorname{sn}(x \mid m)^2\, \operatorname{sn}(y \mid 1 - m)^2} \;/;\; \{x, y, m\} \in \mathbb{R}$$

Absolute value

09.34.19.0003.01

$$|\operatorname{sc}(x + i\, y \mid m)| = \sqrt{\frac{\operatorname{dn}(y \mid 1 - m)^2\, \operatorname{sn}(x \mid m)^2 + \operatorname{cn}(x \mid m)^2\, \operatorname{cn}(y \mid 1 - m)^2\, \operatorname{dn}(x \mid m)^2\, \operatorname{sn}(y \mid 1 - m)^2}{\operatorname{cn}(x \mid m)^2\, \operatorname{cn}(y \mid 1 - m)^2 + \operatorname{dn}(x \mid m)^2\, \operatorname{dn}(y \mid 1 - m)^2\, \operatorname{sn}(x \mid m)^2\, \operatorname{sn}(y \mid 1 - m)^2}} \;/;\; \{x, y, m\} \in \mathbb{R}$$

Argument

09.34.19.0004.01

$$\arg(\operatorname{sc}(x + i\, y \mid m)) = \tan^{-1}\!\Big(\operatorname{cn}(x \mid m)\, \operatorname{cn}(y \mid 1 - m)\, \operatorname{dn}(y \mid 1 - m)\, \operatorname{sn}(x \mid m) \left(1 - \operatorname{dn}(x \mid m)^2\, \operatorname{sn}(y \mid 1 - m)^2\right),$$
$$\operatorname{dn}(x \mid m) \left(\operatorname{cn}(x \mid m)^2\, \operatorname{cn}(y \mid 1 - m)^2 + \operatorname{dn}(y \mid 1 - m)^2\, \operatorname{sn}(x \mid m)^2\right) \operatorname{sn}(y \mid 1 - m)\Big) \;/;\; \{x, y, m\} \in \mathbb{R}$$

Conjugate value

09.34.19.0005.01

$$\overline{\operatorname{sc}(x + i\, y \mid m)} = \frac{\operatorname{dn}(y \mid 1 - m)\, \operatorname{sn}(x \mid m) - i\, \operatorname{cn}(x \mid m)\, \operatorname{cn}(y \mid 1 - m)\, \operatorname{dn}(x \mid m)\, \operatorname{sn}(y \mid 1 - m)}{\operatorname{cn}(x \mid m)\, \operatorname{cn}(y \mid 1 - m) + i\, \operatorname{sn}(x \mid m)\, \operatorname{dn}(x \mid m)\, \operatorname{sn}(y \mid 1 - m)\, \operatorname{dn}(y \mid 1 - m)} \;/;\; \{x, y, m\} \in \mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *z*

09.34.20.0001.01

$$\frac{\partial \operatorname{sc}(z \mid m)}{\partial z} = \operatorname{dc}(z \mid m)\, \operatorname{nc}(z \mid m)$$

09.34.20.0002.01

$$\frac{\partial^2 \operatorname{sc}(z \mid m)}{\partial z^2} = \left(\operatorname{dc}(z \mid m)^2 - (m - 1)\, \operatorname{nc}(z \mid m)^2\right) \operatorname{sc}(z \mid m)$$

With respect to *m*

09.34.20.0003.01

$$\frac{\partial \operatorname{sc}(z \mid m)}{\partial m} = \frac{\operatorname{nc}(z \mid m) \operatorname{dc}(z \mid m) ((1-m) z - E(\operatorname{am}(z \mid m) \mid m) + m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m))}{2 m (1-m)}$$

09.34.20.0004.01

$$\frac{\partial^2 \operatorname{sc}(z \mid m)}{\partial m^2} = \frac{1}{4 (m-1)^2 m^2} \Big(-(m-1) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{sc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)) \operatorname{nc}(z \mid m)^2 - 2 (m-1) \operatorname{dc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)) \operatorname{nc}(z \mid m) - 2 m \operatorname{dc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)) \operatorname{nc}(z \mid m) + (1-m) m \operatorname{dc}(z \mid m) \Big(-2 z + \frac{F(\operatorname{am}(z \mid m) \mid m) - E(\operatorname{am}(z \mid m) \mid m)}{m} + 2 \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m) + ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m) - \frac{1}{m-1} (\operatorname{cd}(z \mid m) \operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m) (-m z + z - E(\operatorname{am}(z \mid m) \mid m) + m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m))) + \frac{1}{(m-1) m} \Big((m \operatorname{cn}(z \mid m) \operatorname{sn}(z \mid m) - ((m-1) z + E(\operatorname{am}(z \mid m) \mid m)) \operatorname{dn}(z \mid m)) \sqrt{1 - m \operatorname{sn}(z \mid m)^2} \Big) \Big) \operatorname{nc}(z \mid m) + \operatorname{dc}(z \mid m)^2 \operatorname{sc}(z \mid m) ((m-1) z + E(\operatorname{am}(z \mid m) \mid m) - m \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m))^2 \Big)$$

Symbolic differentiation

With respect to z

09.34.20.0007.01

$$\frac{\partial^n \operatorname{sc}(z \mid m)}{\partial z^n} = \operatorname{sc}(z \mid m) \delta_n + \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\partial^j \operatorname{dc}(z \mid m)}{\partial z^j} \frac{\partial^{-j+n-1} \operatorname{nc}(z \mid m)}{\partial z^{-j+n-1}} /; n \in \mathbb{N}$$

09.34.20.0005.01

$$\frac{\partial^n \operatorname{sc}(z \mid m)}{\partial z^n} = \frac{z^{-n-1}}{2 \sqrt{1-m}} \sum_{k=1}^{\infty} \frac{(-1)^{k-1} \left(2^{2k} - 1\right) B_{2k}}{k (2k - n - 1)!} \left(\frac{\pi z}{K(m)}\right)^{2k} + \frac{2 \pi^{n+1}}{\sqrt{1-m}\, K(m)^{n+1}} \sum_{k=1}^{\infty} \frac{(-1)^k k^n q(m)^{2k}}{q(m)^{2k} + 1} \sin\left(\frac{\pi n}{2} + \frac{k \pi z}{K(m)}\right) /; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to z

09.34.20.0006.01

$$\frac{\partial^\alpha \operatorname{sc}(z \mid m)}{\partial z^\alpha} = \frac{1}{2 \sqrt{1-m}} \sum_{k=1}^{\infty} \frac{(-1)^{k-1} \left(2^{2k} - 1\right) \pi^{2k} z^{2k-\alpha-1} B_{2k} K(m)^{-2k}}{k \Gamma(2k - \alpha)} + \frac{2^\alpha \pi^{5/2} z^{1-\alpha}}{\sqrt{1-m}\, K(m)^2} \sum_{k=1}^{\infty} \frac{(-1)^k k\, q(m)^{2k}}{q(m)^{2k} + 1} \, {}_1\tilde{F}_2\left(1; 1 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{k^2 \pi^2 z^2}{4 K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.34.21.0001.01

$$\int \mathrm{sc}(z \mid m)\, dz = \frac{\log\left(\mathrm{dc}(z \mid m) + \sqrt{1-m}\ \mathrm{nc}(z \mid m)\right)}{\sqrt{1-m}}$$

Representations through equivalent functions

With inverse function

09.34.27.0001.01

$$\mathrm{sc}\left(\mathrm{sc}^{-1}(z \mid m) \,\middle|\, m\right) = z$$

With related functions

Involving am

09.34.27.0028.01

$$\mathrm{sc}(z \mid m) = \tan(\mathrm{am}(z \mid m))$$

Involving one other Jacobi elliptic function

Involving cd

09.34.27.0004.01

$$\mathrm{sc}(z \mid m)^2 = \frac{\mathrm{cd}(z \mid m)^2 - 1}{(m-1)\,\mathrm{cd}(z \mid m)^2}$$

Involving cn

09.34.27.0007.01

$$\mathrm{sc}(z \mid m)^2 = \frac{1 - \mathrm{cn}(z \mid m)^2}{\mathrm{cn}(z \mid m)^2}$$

Involving cs

09.34.27.0008.01

$$\mathrm{sc}(z \mid m) = \frac{1}{\mathrm{cs}(z \mid m)}$$

Involving dc

09.34.27.0011.01

$$\mathrm{sc}(z \mid m)^2 = \frac{1 - \mathrm{dc}(z \mid m)^2}{m-1}$$

Involving dn

09.34.27.0012.01

$$\operatorname{sc}(z \mid m)^2 = \frac{1 - \operatorname{dn}(z \mid m)^2}{\operatorname{dn}(z \mid m)^2 + m - 1}$$

Involving ds

09.34.27.0013.01

$$\operatorname{sc}(z \mid m)^2 = \frac{1}{\operatorname{ds}(z \mid m)^2 + m - 1}$$

Involving nc

09.34.27.0016.01

$$\operatorname{sc}(z \mid m)^2 = \operatorname{nc}(z \mid m)^2 - 1$$

Involving nd

09.34.27.0017.01

$$\operatorname{sc}(z \mid m)^2 = \frac{\operatorname{nd}(z \mid m)^2 - 1}{(m-1)\operatorname{nd}(z \mid m)^2 + 1}$$

Involving ns

09.34.27.0018.01

$$\operatorname{sc}(z \mid m) = -\frac{i}{\operatorname{ns}(i z \mid 1 - m)}$$

09.34.27.0019.01

$$\operatorname{sc}(z \mid m)^2 = \frac{1}{\operatorname{ns}(z \mid m)^2 - 1}$$

Involving sd

09.34.27.0020.01

$$\operatorname{sc}(z \mid m)^2 = \frac{\operatorname{sd}(z \mid m)^2}{(m-1)\operatorname{sd}(z \mid m)^2 + 1}$$

Involving sn

09.34.27.0021.01

$$\operatorname{sc}(z \mid m) = -i \operatorname{sn}(i z \mid 1 - m)$$

09.34.27.0022.01

$$\operatorname{sc}(z \mid m)^2 = \frac{\operatorname{sn}(z \mid m)^2}{1 - \operatorname{sn}(z \mid m)^2}$$

Involving two other Jacobi elliptic functions

Involving cd and ds

09.34.27.0002.01

$$\mathrm{sc}(z \mid m) = \frac{1}{\mathrm{ds}(z \mid m)\,\mathrm{cd}(z \mid m)}$$

Involving cd and sd

09.34.27.0003.01

$$\mathrm{sc}(z \mid m) = \frac{\mathrm{sd}(z \mid m)}{\mathrm{cd}(z \mid m)}$$

Involving cn and cs

09.34.27.0029.01

$$\mathrm{sc}(z \mid m) = -\frac{(\mathrm{cn}(z \mid m) - 1)\,(\mathrm{cn}(z \mid m) + 1)\,\mathrm{cs}(z \mid m)}{\mathrm{cn}(z \mid m)^2}$$

Involving cn and ns

09.34.27.0005.01

$$\mathrm{sc}(z \mid m) = \frac{1}{\mathrm{ns}(z \mid m)\,\mathrm{cn}(z \mid m)}$$

09.34.27.0030.01

$$\mathrm{sc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{ns}(z \mid m)}{(\mathrm{ns}(z \mid m) - 1)\,(\mathrm{ns}(z \mid m) + 1)}$$

Involving cn and sn

09.34.27.0006.01

$$\mathrm{sc}(z \mid m) = \frac{\mathrm{sn}(z \mid m)}{\mathrm{cn}(z \mid m)}$$

09.34.27.0031.01

$$\mathrm{sc}(z \mid m) = -\frac{\mathrm{cn}(z \mid m)\,\mathrm{sn}(z \mid m)}{(\mathrm{sn}(z \mid m) - 1)\,(\mathrm{sn}(z \mid m) + 1)}$$

Involving cs and dn

09.34.27.0032.01

$$\mathrm{sc}(z \mid m) = -\frac{\mathrm{cs}(z \mid m)\,(\mathrm{dn}(z \mid m) - 1)\,(\mathrm{dn}(z \mid m) + 1)}{\mathrm{dn}(z \mid m)^2 + m - 1}$$

Involving cs and sn

09.34.27.0033.01

$$\mathrm{sc}(z \mid m) = -\frac{\mathrm{cs}(z \mid m)\,\mathrm{sn}(z \mid m)^2}{(\mathrm{sn}(z \mid m) - 1)\,(\mathrm{sn}(z \mid m) + 1)}$$

Involving dc and ds

09.34.27.0009.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

Involving dc and sd

09.34.27.0010.01

$$\operatorname{sc}(z \mid m) = \operatorname{sd}(z \mid m)\operatorname{dc}(z \mid m)$$

Involving nc and ns

09.34.27.0014.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{nc}(z \mid m)}{\operatorname{ns}(z \mid m)}$$

Involving nc and sn

09.34.27.0015.01

$$\operatorname{sc}(z \mid m) = \operatorname{sn}(z \mid m)\operatorname{nc}(z \mid m)$$

Involving three other Jacobi elliptic functions

09.34.27.0034.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)(\operatorname{dc}(z \mid m) - \operatorname{dn}(z \mid m))(\operatorname{dc}(z \mid m) + \operatorname{dn}(z \mid m))}{\operatorname{dn}(z \mid m)^2}$$

09.34.27.0035.01

$$\operatorname{sc}(z \mid m) = \frac{m\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{ds}(z \mid m)}$$

09.34.27.0036.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)\operatorname{ds}(z \mid m)}{\operatorname{cn}(z \mid m)^2}$$

09.34.27.0037.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)^2}{(\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m))(\operatorname{dn}(z \mid m) + \operatorname{ds}(z \mid m))}$$

09.34.27.0038.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m)^2\operatorname{ds}(z \mid m)}{(\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m))(\operatorname{dn}(z \mid m) + \operatorname{ds}(z \mid m))}$$

09.34.27.0039.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{dn}(z \mid m)\left(\operatorname{ds}(z \mid m)^2 + m - 1\right)}$$

09.34.27.0040.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{cn}(z \mid m)}$$

09.34.27.0041.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{ds}(z \mid m)^2 + m\right)}{\operatorname{ds}(z \mid m)\left(\operatorname{ds}(z \mid m)^2 + m - 1\right)\operatorname{nc}(z \mid m)}$$

09.34.27.0042.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\,(\operatorname{cd}(z \mid m) - \operatorname{nd}(z \mid m))\,(\operatorname{cd}(z \mid m) + \operatorname{nd}(z \mid m))}{\operatorname{cd}(z \mid m)^2}$$

09.34.27.0043.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\,(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))}{\operatorname{dn}(z \mid m) + m\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.34.27.0044.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)}{(\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - 1)\,(\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) + 1)}$$

09.34.27.0045.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)}{(\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - 1)\,(\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) + 1)}$$

09.34.27.0046.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{ns}(z \mid m)}{\operatorname{cn}(z \mid m)}$$

09.34.27.0047.01

$$\operatorname{sc}(z \mid m) = \frac{m\operatorname{cn}(z \mid m)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{ns}(z \mid m)}$$

09.34.27.0048.01

$$\operatorname{sc}(z \mid m) = -(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\operatorname{ns}(z \mid m)$$

09.34.27.0049.01

$$\operatorname{sc}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - 1)\,(\operatorname{dn}(z \mid m) + 1)\operatorname{ns}(z \mid m)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{nc}(z \mid m)}$$

09.34.27.0050.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)}{(\operatorname{ns}(z \mid m) - 1)\,(\operatorname{ns}(z \mid m) + 1)}$$

09.34.27.0051.01

$$\operatorname{sc}(z \mid m) = \frac{m\operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m)}{(\operatorname{nd}(z \mid m) - 1)\,(\operatorname{nd}(z \mid m) + 1)\,(\operatorname{ns}(z \mid m) - 1)\operatorname{ns}(z \mid m)\,(\operatorname{ns}(z \mid m) + 1)}$$

09.34.27.0052.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)\,(\operatorname{ns}(z \mid m) - 1)\,(\operatorname{ns}(z \mid m) + 1)}$$

09.34.27.0053.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nd}(z \mid m)\,(\operatorname{ns}(z \mid m) - 1)\,(\operatorname{ns}(z \mid m) + 1)}$$

09.34.27.0054.01
$$\operatorname{sc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{cd}(z \mid m) \left(\operatorname{ns}(z \mid m)^2 - m\right)}$$

09.34.27.0055.01
$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) (\operatorname{cn}(z \mid m) - 1) (\operatorname{cn}(z \mid m) + 1)}{\operatorname{cn}(z \mid m)^2 \operatorname{sd}(z \mid m)}$$

09.34.27.0056.01
$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m)}{(m - 1) \operatorname{sd}(z \mid m)}$$

09.34.27.0057.01
$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) (\operatorname{dn}(z \mid m) - 1) (\operatorname{dn}(z \mid m) + 1)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right) \operatorname{sd}(z \mid m)}$$

09.34.27.0058.01
$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) (\operatorname{dn}(z \mid m) - 1) (\operatorname{dn}(z \mid m) + 1)}{\operatorname{dn}(z \mid m) \left(\operatorname{dn}(z \mid m)^2 + m - 1\right) \operatorname{sd}(z \mid m)}$$

09.34.27.0059.01
$$\operatorname{sc}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - 1) (\operatorname{dn}(z \mid m) + 1) \operatorname{nc}(z \mid m)}{m \operatorname{dn}(z \mid m) \operatorname{sd}(z \mid m)}$$

09.34.27.0060.01
$$\operatorname{sc}(z \mid m) = -\frac{(\operatorname{cn}(z \mid m) - 1) (\operatorname{cn}(z \mid m) + 1) \operatorname{nd}(z \mid m)}{\operatorname{cn}(z \mid m) \operatorname{sd}(z \mid m)}$$

09.34.27.0061.01
$$\operatorname{sc}(z \mid m) = \frac{\operatorname{dc}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}{m \operatorname{sd}(z \mid m)}$$

09.34.27.0062.01
$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}{\left(m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right) \operatorname{sd}(z \mid m)}$$

09.34.27.0063.01
$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)}{(\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1) \operatorname{sd}(z \mid m)}$$

09.34.27.0064.01
$$\operatorname{sc}(z \mid m) = \frac{m \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{nc}(z \mid m) \left(m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right)}$$

09.34.27.0065.01
$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{sd}(z \mid m)}{(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m)) (\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}$$

09.34.27.0066.01
$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m)}{(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m)) (\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}$$

09.34.27.0067.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{sd}(z \mid m)^2}{(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}$$

09.34.27.0068.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{dn}(z \mid m)\left(m \operatorname{sd}(z \mid m)^2 - \operatorname{sd}(z \mid m)^2 + 1\right)}$$

09.34.27.0069.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.34.27.0070.01

$$\operatorname{sc}(z \mid m) = \frac{(\operatorname{cd}(z \mid m) - 1)(\operatorname{cd}(z \mid m) + 1)\operatorname{cn}(z \mid m)}{(m-1)\operatorname{cd}(z \mid m)^2 \operatorname{sn}(z \mid m)}$$

09.34.27.0071.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)(\operatorname{dc}(z \mid m) - 1)(\operatorname{dc}(z \mid m) + 1)}{(m-1)\operatorname{sn}(z \mid m)}$$

09.34.27.0072.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)(\operatorname{dn}(z \mid m) - 1)(\operatorname{dn}(z \mid m) + 1)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{sn}(z \mid m)}$$

09.34.27.0073.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\left(\operatorname{ds}(z \mid m)^2 + m - 1\right)\operatorname{sn}(z \mid m)}$$

09.34.27.0074.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{sn}(z \mid m)}$$

09.34.27.0075.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)(\operatorname{nd}(z \mid m) - 1)(\operatorname{nd}(z \mid m) + 1)}{\left(m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}$$

09.34.27.0076.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{sd}(z \mid m)^2}{\left(m \operatorname{sd}(z \mid m)^2 - \operatorname{sd}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}$$

09.34.27.0077.01

$$\operatorname{sc}(z \mid m) = \frac{m \operatorname{dn}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)}$$

09.34.27.0078.01

$$\operatorname{sc}(z \mid m) = \frac{\left(m \operatorname{cd}(z \mid m)^2 - 1\right)\operatorname{sn}(z \mid m)}{(m-1)\operatorname{cd}(z \mid m)^2 \operatorname{nc}(z \mid m)}$$

09.34.27.0079.01

$$\operatorname{sc}(z \mid m) = \frac{\left(m - \operatorname{dc}(z \mid m)^2\right)\operatorname{sn}(z \mid m)}{(m-1)\operatorname{nc}(z \mid m)}$$

09.34.27.0080.01

$$\operatorname{sc}(z \mid m) = \frac{m \operatorname{sn}(z \mid m)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right) \operatorname{nc}(z \mid m)}$$

09.34.27.0081.01

$$\operatorname{sc}(z \mid m) = \frac{\left(\operatorname{ds}(z \mid m)^2 + m\right) \operatorname{sn}(z \mid m)}{\left(\operatorname{ds}(z \mid m)^2 + m - 1\right) \operatorname{nc}(z \mid m)}$$

09.34.27.0082.01

$$\operatorname{sc}(z \mid m) = \frac{\left(m - \operatorname{dc}(z \mid m)^2\right) \operatorname{sn}(z \mid m)}{(m - 1) \operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m)}$$

09.34.27.0083.01

$$\operatorname{sc}(z \mid m) = \frac{m \operatorname{nd}(z \mid m)^2 \operatorname{sn}(z \mid m)}{\operatorname{nc}(z \mid m) \left(m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right)}$$

09.34.27.0084.01

$$\operatorname{sc}(z \mid m) = \frac{\left(m \operatorname{sd}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}{\operatorname{nc}(z \mid m) \left(m \operatorname{sd}(z \mid m)^2 - \operatorname{sd}(z \mid m)^2 + 1\right)}$$

09.34.27.0085.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.34.27.0086.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m) \operatorname{sn}(z \mid m)^2}{(\operatorname{sn}(z \mid m) - 1) (\operatorname{sn}(z \mid m) + 1)}$$

09.34.27.0087.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)^2}{\operatorname{sd}(z \mid m) (\operatorname{sn}(z \mid m) - 1) (\operatorname{sn}(z \mid m) + 1)}$$

09.34.27.0088.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{sn}(z \mid m)}$$

Involving four other Jacobi elliptic functions

09.34.27.0089.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m) (\operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m) - \operatorname{dc}(z \mid m))}{\operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m)}$$

09.34.27.0090.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) \operatorname{cs}(z \mid m) \operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m)}{\operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m)}$$

09.34.27.0091.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m) (\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{cn}(z \mid m)}$$

09.34.27.0092.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{ds}(z \mid m)\,(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{dn}(z \mid m)}$$

09.34.27.0093.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{ds}(z \mid m)\,(\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{dn}(z \mid m)}$$

09.34.27.0094.01

$$\operatorname{sc}(z \mid m) = \frac{m\operatorname{cn}(z \mid m) - m\operatorname{nc}(z \mid m) + \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m)}$$

09.34.27.0095.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\,(\operatorname{dn}(z \mid m) - \operatorname{cd}(z \mid m)\operatorname{nc}(z \mid m))}{\operatorname{dn}(z \mid m) + m\operatorname{cd}(z \mid m)\operatorname{nc}(z \mid m) - \operatorname{cd}(z \mid m)\operatorname{nc}(z \mid m)}$$

09.34.27.0096.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\,(\operatorname{dn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{nc}(z \mid m))}{\operatorname{dn}(z \mid m)}$$

09.34.27.0097.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m)\operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

09.34.27.0098.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\,(\operatorname{cn}(z \mid m) - \operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m))}{\operatorname{cn}(z \mid m) + m\operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.34.27.0099.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\,(\operatorname{cn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m))}{\operatorname{cn}(z \mid m)}$$

09.34.27.0100.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\left(\operatorname{dn}(z \mid m) - \operatorname{dc}(z \mid m)^2\operatorname{nd}(z \mid m)\right)}{\operatorname{dn}(z \mid m)}$$

09.34.27.0101.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{cn}(z \mid m)}$$

09.34.27.0102.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m)^2\operatorname{nd}(z \mid m)}$$

09.34.27.0103.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{ds}(z \mid m)^2\operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.34.27.0104.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\,(\operatorname{cd}(z \mid m) - \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m))}{\operatorname{cd}(z \mid m)}$$

09.34.27.0105.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\left(\operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)^2\right)}{\operatorname{cd}(z \mid m)}$$

09.34.27.0106.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2}{\operatorname{cd}(z \mid m)}$$

09.34.27.0107.01

$$\operatorname{sc}(z \mid m) = \frac{(\operatorname{cd}(z \mid m)\operatorname{nc}(z \mid m) - \operatorname{dn}(z \mid m))\operatorname{ns}(z \mid m)}{m\operatorname{cd}(z \mid m)}$$

09.34.27.0108.01

$$\operatorname{sc}(z \mid m) = \frac{(\operatorname{dc}(z \mid m)\operatorname{nc}(z \mid m) - \operatorname{dn}(z \mid m))\operatorname{ns}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.34.27.0109.01

$$\operatorname{sc}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{ns}(z \mid m)}{m\operatorname{cd}(z \mid m)}$$

09.34.27.0110.01

$$\operatorname{sc}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)(\operatorname{dn}(z \mid m) + m\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))}$$

09.34.27.0111.01

$$\operatorname{sc}(z \mid m) = \frac{(\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cd}(z \mid m))\operatorname{ns}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

09.34.27.0112.01

$$\operatorname{sc}(z \mid m) = \frac{-\operatorname{dn}(z \mid m)\operatorname{cs}(z \mid m)^2 + \operatorname{cd}(z \mid m)\operatorname{ns}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{dn}(z \mid m)}{(m-1)\operatorname{cd}(z \mid m)\operatorname{ns}(z \mid m)}$$

09.34.27.0113.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

09.34.27.0114.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{(m-1)\operatorname{cd}(z \mid m)}$$

09.34.27.0115.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m)\operatorname{ns}(z \mid m)}$$

09.34.27.0116.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.34.27.0117.01

$$\operatorname{sc}(z \mid m) = \operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)$$

09.34.27.0118.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{cd}(z \mid m)}$$

09.34.27.0119.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2 - \operatorname{sd}(z \mid m)}$$

09.34.27.0120.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2 - \operatorname{sd}(z \mid m)}$$

09.34.27.0121.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sd}(z \mid m)}$$

09.34.27.0122.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sd}(z \mid m)}$$

09.34.27.0123.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{cn}(z \mid m)}{(m-1)\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.34.27.0124.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m)^2 - \operatorname{dc}(z \mid m)}{\operatorname{dn}(z \mid m)^2\operatorname{sd}(z \mid m)}$$

09.34.27.0125.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{cn}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.34.27.0126.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.34.27.0127.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.34.27.0128.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) - \operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)}{(m-1)\operatorname{sd}(z \mid m)}$$

09.34.27.0129.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{cn}(z \mid m) - \operatorname{nd}(z \mid m)}{\operatorname{cn}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.34.27.0130.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{sd}(z \mid m)}$$

09.34.27.0131.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))}{m\operatorname{sd}(z \mid m)}$$

09.34.27.0132.01

$$\operatorname{sc}(z \mid m) = -\frac{(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)) \operatorname{nd}(z \mid m)}{\operatorname{sd}(z \mid m)}$$

09.34.27.0133.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) (\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))}{(\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)) \operatorname{sd}(z \mid m)}$$

09.34.27.0134.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}{(\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)) \operatorname{sd}(z \mid m)}$$

09.34.27.0135.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{nd}(z \mid m) (\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m) - \operatorname{cn}(z \mid m))}{\operatorname{sd}(z \mid m)}$$

09.34.27.0136.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) - \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)}{\operatorname{sd}(z \mid m)}$$

09.34.27.0137.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m)^2}{\operatorname{sd}(z \mid m)}$$

09.34.27.0138.01

$$\operatorname{sc}(z \mid m) = \frac{m \operatorname{sd}(z \mid m)}{\operatorname{dc}(z \mid m) + m \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m) - \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)}$$

09.34.27.0139.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m)^2 - \operatorname{sd}(z \mid m)}$$

09.34.27.0140.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m) (\operatorname{ds}(z \mid m) + m \operatorname{sd}(z \mid m) - \operatorname{sd}(z \mid m))}$$

09.34.27.0141.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{(m - 1) \operatorname{sd}(z \mid m)}$$

09.34.27.0142.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)}{\operatorname{sd}(z \mid m)}$$

09.34.27.0143.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m)^2}{\operatorname{sd}(z \mid m)}$$

09.34.27.0144.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{ns}(z \mid m) - \operatorname{dn}(z \mid m) \operatorname{sd}(z \mid m)}$$

09.34.27.0145.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m)^2}{\operatorname{dn}(z \mid m)^2 \operatorname{sd}(z \mid m) - \operatorname{ds}(z \mid m)}$$

09.34.27.0146.01

$$\operatorname{sc}(z \mid m) = -\frac{1}{\operatorname{dn}(z \mid m)} \left(-\operatorname{nc}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{cs}(z \mid m)^2 + \operatorname{dn}(z \mid m)\operatorname{cs}(z \mid m) + m \operatorname{nc}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{nc}(z \mid m)\operatorname{sd}(z \mid m)\right)$$

09.34.27.0147.01

$$\operatorname{sc}(z \mid m) = -\frac{1}{\operatorname{cn}(z \mid m)} \left(-\operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{cs}(z \mid m)^2 + \operatorname{cn}(z \mid m)\operatorname{cs}(z \mid m) + m \operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m)\right)$$

09.34.27.0148.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{nc}(z \mid m)(\operatorname{ns}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{dn}(z \mid m))}{m \operatorname{sd}(z \mid m)}$$

09.34.27.0149.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)(\operatorname{dn}(z \mid m) - \operatorname{ns}(z \mid m)\operatorname{sd}(z \mid m))}{\operatorname{dn}(z \mid m) + m \operatorname{ns}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{ns}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.34.27.0150.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{cd}(z \mid m)}{\operatorname{sd}(z \mid m)}$$

09.34.27.0151.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)^2 - \operatorname{nd}(z \mid m)}$$

09.34.27.0152.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.34.27.0153.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)(\operatorname{cd}(z \mid m) - \operatorname{dc}(z \mid m))}{(m-1)\operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.34.27.0154.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m)}{(m-1)\operatorname{sn}(z \mid m)}$$

09.34.27.0155.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))}{(\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{sn}(z \mid m)}$$

09.34.27.0156.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{sn}(z \mid m)}$$

09.34.27.0157.01

$$\operatorname{sc}(z \mid m) = \frac{m \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m) + m \operatorname{nc}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.34.27.0158.01

$$\operatorname{sc}(z \mid m) = \frac{m \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)(\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))}$$

09.34.27.0159.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.34.27.0160.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}{m \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) + \operatorname{sn}(z \mid m)}$$

09.34.27.0161.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cs}(z \mid m) (\operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{sn}(z \mid m))}{m \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) + \operatorname{sn}(z \mid m)}$$

09.34.27.0162.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ds}(z \mid m) + m \operatorname{sn}(z \mid m)}{\left(\operatorname{ds}(z \mid m)^2 + m - 1\right) \operatorname{nc}(z \mid m)}$$

09.34.27.0163.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m) \operatorname{ds}(z \mid m) + m \operatorname{sn}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.34.27.0164.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{nc}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m)}$$

09.34.27.0165.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m)}{\operatorname{sn}(z \mid m)}$$

09.34.27.0166.01

$$\operatorname{sc}(z \mid m) = \frac{-\operatorname{sn}(z \mid m) \operatorname{cs}(z \mid m)^2 + \operatorname{cd}(z \mid m) \operatorname{nd}(z \mid m) \operatorname{cs}(z \mid m) - \operatorname{sn}(z \mid m)}{(m - 1) \operatorname{cd}(z \mid m) \operatorname{nd}(z \mid m)}$$

09.34.27.0167.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{dc}(z \mid m) \operatorname{sn}(z \mid m)}{(m - 1) \operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m)}$$

09.34.27.0168.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m) \operatorname{dn}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{dn}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{ds}(z \mid m)}$$

09.34.27.0169.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m) \operatorname{dn}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{dc}(z \mid m)}{\operatorname{dn}(z \mid m) \operatorname{sn}(z \mid m)}$$

09.34.27.0170.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{sn}(z \mid m)}$$

09.34.27.0171.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) \operatorname{sd}(z \mid m)}{\operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.34.27.0172.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{nd}(z \mid m)}$$

Involving five other Jacobi elliptic functions

09.34.27.0173.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cn}(z \mid m)}{(\operatorname{dn}(z \mid m) + m\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{sd}(z \mid m)}$$

09.34.27.0174.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.34.27.0175.01

$$\operatorname{sc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.34.27.0176.01

$$\operatorname{sc}(z \mid m) = \frac{-\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}{(m-1)\operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.34.27.0177.01

$$\operatorname{sc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{cd}(z \mid m)\operatorname{nd}(z \mid m)}{m\operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m) + \operatorname{sn}(z \mid m)}$$

Involving Weierstrass functions

09.34.27.0023.01

$$\operatorname{sc}(z \mid m) = \sqrt{e_1 - e_3}\, \frac{\sigma\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right)}{\sigma_1\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right)} \;/;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

09.34.27.0024.01

$$\operatorname{sc}\left(z\sqrt{e_1 - e_3} \,\middle|\, m\right)^2 = \frac{e_1 - e_3}{\wp(z; g_2, g_3) - e_1} \;/;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

Involving theta functions

09.34.27.0025.02

$$\operatorname{sc}(z \mid m) = (1-m)^{-1/4} \frac{\vartheta_1\left(\frac{\pi z}{2K(m)}, q(m)\right)}{\vartheta_2\left(\frac{\pi z}{2K(m)}, q(m)\right)}$$

09.34.27.0026.01

$$\operatorname{sc}(z \mid m) = \frac{\vartheta_3(0, q(m))\,\vartheta_1\left(\frac{z}{\vartheta_3(0,q(m))^2}, q(m)\right)}{\vartheta_4(0, q(m))\,\vartheta_2\left(\frac{z}{\vartheta_3(0,q(m))^2}, q(m)\right)}$$

09.34.27.0027.01

$$\operatorname{sc}(z \mid m) = \frac{\vartheta_s(z \mid m)}{\vartheta_c(z \mid m)}$$

Zeros

09.34.30.0001.01

$$\operatorname{sc}(2\, r\, K(m) + 2\, s\, i\, K(1 - m) \mid m) = 0 \;/;\; \{r, s\} \in \mathbb{Z}$$

Theorems

Solution of one functional equation

The function pair $f(z) = \operatorname{sc}(z \mid m)$ and $g(z) = \operatorname{sc}(z \mid m) \Big/ \sqrt{\left(\operatorname{sc}(z \mid m)^2 + 1\right)\left((1 - m) \operatorname{sc}(z \mid m)^2 + 1\right)}$ is a solution of the functional equation

$$\frac{f\,(x + y)}{f\,(x - y)} = \frac{g\,(x) + g\,(y)}{g\,(x) - g\,(y)}.$$

History

– C. G. J. Jacobi (1827)

– N. H. Abel (1827)

– J. Glaisher (1882) introduced the notation sc

Copyright