KelvinKer2

Notations

Traditional name

Kelvin function of the second kind

Traditional notation

$\operatorname{ker}_\nu(z)$

Mathematica StandardForm notation

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KelvinKer[ν, z]
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Primary definition

03.20.02.0001.01

$$\operatorname{ker}_\nu(z) = \frac{1}{4} e^{\frac{1}{4}(-3) i \pi \nu} \pi z^{-\nu} \left(\sqrt[4]{-1}\, z\right)^{-\nu} \csc(\pi \nu) \left(\left(\sqrt[4]{-1}\, z\right)^{2\nu} \left(I_{-\nu}\left(\sqrt[4]{-1}\, z\right) + e^{\frac{3 i \pi \nu}{2}} J_{-\nu}\left(\sqrt[4]{-1}\, z\right)\right) - e^{\frac{i \pi \nu}{2}} z^{2\nu} \left(I_\nu\left(\sqrt[4]{-1}\, z\right) + e^{\frac{i \pi \nu}{2}} J_\nu\left(\sqrt[4]{-1}\, z\right)\right)\right) /; \nu \notin \mathbb{Z}$$

03.20.02.0002.01

$$\operatorname{ker}_\nu(z) = \lim_{\mu \to \nu} \operatorname{ker}_\mu(z) /; \nu \in \mathbb{Z}$$

Specific values

Specialized values

For fixed ν

03.20.03.0001.01

$$\operatorname{ker}_\nu(0) = ¿$$

For fixed z

Explicit rational ν

03.20.03.0002.01

$$\operatorname{ker}_0(z) = \operatorname{ker}(z)$$

03.20.03.0003.01

$$\operatorname{ker}_{-\frac{14}{3}}(z) = -\frac{(-1)^{3/4}\,\pi}{243\, 2^{5/6}\, \sqrt[6]{3}\, z^{8/3}\, ((1+i)\,z)^{5/3}} \Biggl(144\, \sqrt[3]{3}\, i\, (9\, z^2 + 110\, i) \Bigl(2^{2/3}\, ((1+i)\,z)^{2/3} - \bigl(-i + \sqrt{3}\bigr) z^{2/3}\Bigr) \operatorname{Ai}\Bigl(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\,z)^{2/3}\Bigr) \sqrt[3]{z} + 48\, \sqrt[3]{3} \left(110\, \bigl(3\, i + \sqrt{3}\bigr)\, i\, z^{2/3} + 18\, \sqrt[3]{-1}\, \sqrt{3}\, z^{8/3} - 110\, 2^{2/3}\, \sqrt{3}\, ((1+i)\,z)^{2/3} + \frac{9\, \sqrt{3}\, ((1+i)\,z)^{8/3}}{\sqrt[3]{2}}\right) \operatorname{Bi}\Bigl(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\,z)^{2/3}\Bigr) \sqrt[3]{z} + \frac{144\, \sqrt[3]{3}\, z\, (9\, i\, z^2 + 110) \Bigl(2^{2/3}\, (1+i)\, \sqrt[3]{z} + \bigl(i + \sqrt{3}\bigr) \sqrt[3]{(1+i)\,z}\Bigr) \operatorname{Ai}\bigl(\frac{1}{2}\, 3^{2/3}\, ((1+i)\,z)^{2/3}\bigr)}{\sqrt[3]{(1+i)\,z}} + \frac{1}{((1+i)\,z)^{4/3}} \Bigl(3 \Bigl(-14\,080\, 2^{2/3}\, ((1+i)\,z)^{2/3}\, \sqrt[3]{z} - 4320\, i\, 2^{2/3}\, ((1+i)\,z)^{2/3}\, z^{7/3} + 81\, 2^{2/3}\, ((1+i)\,z)^{2/3}\, z^{13/3} + 162\, \sqrt[6]{-1}\, z^5 - 8640\, (-1)^{2/3}\, z^3 - 28\,160\, \sqrt[6]{-1}\, z\Bigr) \operatorname{Ai}'\Bigl(\frac{1}{2}\, 3^{2/3}\, ((1+i)\,z)^{2/3}\Bigr)\Bigr) + \frac{1}{((1+i)\,z)^{4/3}} \Bigl(3\, i \Bigl(-14\,080\, i\, 2^{2/3}\, ((1+i)\,z)^{2/3}\, \sqrt[3]{z} - 4320\, 2^{2/3}\, ((1+i)\,z)^{2/3}\, z^{7/3} + 81\, 2^{2/3}\, i\, ((1+i)\,z)^{2/3}\, z^{13/3} - 162\, \sqrt[3]{-1}\, z^5 - 8640\, (-1)^{5/6}\, z^3 + 28\,160\, \sqrt[3]{-1}\, z\Bigr) \operatorname{Ai}'\Bigl(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\,z)^{2/3}\Bigr)\Bigr) + \frac{48\, \sqrt[3]{3}\, z\, (9\, i\, z^2 + 110) \Bigl(2^{2/3}\, \sqrt{3}\, (1+i)\, \sqrt[3]{z} + \bigl(-3 - i\, \sqrt{3}\bigr) \sqrt[3]{(1+i)\,z}\Bigr) \operatorname{Bi}\bigl(\frac{1}{2}\, 3^{2/3}\, ((1+i)\,z)^{2/3}\bigr)}{\sqrt[3]{(1+i)\,z}} - \frac{1}{((1+i)\,z)^{4/3}} \Bigl(\sqrt{3}\, \sqrt[3]{z} \Bigl(-28\,160\, \sqrt[6]{-1}\, z^{2/3} - 8640\, (-1)^{2/3}\, z^{8/3} + 162\, \sqrt[6]{-1}\, z^{14/3} - 81\, 2^{2/3}\, ((1+i)\,z)^{2/3}\, z^4 + 4320\, 2^{2/3}\, i\, ((1+i)\,z)^{2/3}\, z^2 + 14\,080\, 2^{2/3}\, ((1+i)\,z)^{2/3}\Bigr) \operatorname{Bi}'\Bigl(\frac{1}{2}\, 3^{2/3}\, ((1+i)\,z)^{2/3}\Bigr)\Bigr) - \frac{1}{((1+i)\,z)^{4/3}} \Bigl(\sqrt{3}\, \sqrt[3]{z} \Bigl(28\,160\, (-1)^{5/6}\, z^{2/3} + 8640\, \sqrt[3]{-1}\, z^{8/3} - 162\, (-1)^{5/6}\, z^{14/3} + 81\, 2^{2/3}\, ((1+i)\,z)^{2/3}\, z^4 + 4320\, 2^{2/3}\, i\, ((1+i)\,z)^{2/3}\, z^2 - 14\,080\, 2^{2/3}\, ((1+i)\,z)^{2/3}\Bigr) \operatorname{Bi}'\Bigl(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\,z)^{2/3}\Bigr)\Bigr)\Biggr)$$

03.20.03.0004.01

$$\operatorname{ker}_{-\frac{9}{2}}(z) = \frac{(-1)^{7/8}}{2\, z^{9/2}}\, e^{-\sqrt[4]{-1}\, z} \sqrt{\frac{\pi}{2}} \Bigl(z \Bigl(105\, i - z \Bigl(z \bigl(\sqrt[4]{-1}\, z + 10\bigr) - 45\, (-1)^{3/4}\Bigr)\Bigr) + 105\, \sqrt[4]{-1} + e^{i\, \sqrt{2}\, z} \Bigl(105 - z \Bigl(z \bigl(z^2 + \sqrt{2}\, (5 + 5\, i)\, z + 45\, i\bigr) + 105\, (-1)^{3/4}\Bigr)\Bigr)\Bigr)$$

03.20.03.0005.01

$$\operatorname{ker}_{-\frac{13}{3}}(z) = -\frac{(-1)^{3/4}\,\pi}{324\,\sqrt[3]{6}\,z^{13/3}\,((1+i)\,z)^{4/3}}
\Bigg(2\,\sqrt{3}\,\Big(4480\,z^{2/3} - 3024\,i\,z^{8/3} - 81\,z^{14/3} + 81\,\sqrt[6]{-1}\,\big(\sqrt[4]{-1}\,z\big)^{2/3}\,z^4 - 4480\,\sqrt[6]{-1}\,\big(\sqrt[4]{-1}\,z\big)^{2/3} + 3024\,\sqrt[6]{-1}\,\big(\sqrt[4]{-1}\,z\big)^{8/3}\Big)$$

$$\operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} -$$

$$42\,\sqrt[6]{3}\,\big(9\,z^2 - 80\,i\big)\Big(4\,z^{2/3} + 2^{2/3}\,\big(-i+\sqrt{3}\,\big)\,((1+i)\,z)^{2/3}\Big)\operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3} +$$

$$\frac{i\,z^2\,\big(81\,z^4 - 3024\,i\,z^2 - 4480\big)\Big(4\,\sqrt{3}\,i\,z^{2/3} + 2^{2/3}\,\big(3\,i+\sqrt{3}\,\big)\,((1+i)\,z)^{2/3}\Big)\operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{((1+i)\,z)^{4/3}} -$$

$$84\,\sqrt[6]{3}\,z^{5/3}\,\big(9\,z^2 + 80\,i\big)\Big(2^{2/3}\,\big(i+\sqrt{3}\,\big)\,\sqrt[3]{z} - (2-2\,i)\,\sqrt[3]{(1+i)\,z}\Big)\operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) -$$

$$\frac{1}{((1+i)\,z)^{2/3}}\,z^{5/3}\,\big(81\,z^4 - 3024\,i\,z^2 - 4480\big)\Big(2^{2/3}\,\big(-i+\sqrt{3}\,\big)\,\sqrt[3]{z} - (2-2\,i)\,\sqrt[3]{(1+i)\,z}\Big)\operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) +$$

$$\frac{i\,z^2\,\big(81\,z^4 + 3024\,i\,z^2 - 4480\big)\Big(4\,z^{2/3} + 2^{2/3}\,\big(i+\sqrt{3}\,\big)\,((1+i)\,z)^{2/3}\Big)\operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{((1+i)\,z)^{4/3}} +$$

$$\frac{28\,\sqrt[6]{3}\,z^2\,\big(9\,i\,z^2 + 80\big)\Big(4\,\sqrt{3}\,z^{2/3} + 2^{2/3}\,\big(3\,i+\sqrt{3}\,\big)\,i\,((1+i)\,z)^{2/3}\Big)\operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{((1+i)\,z)^{2/3}} -$$

$$\frac{28\,\sqrt[6]{3}\,z^2\,\big(9\,z^2 + 80\,i\big)\Big(4\,\sqrt{3}\,z^{2/3} + 2^{2/3}\,\big(3+i\,\sqrt{3}\,\big)\,((1+i)\,z)^{2/3}\Big)\operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{((1+i)\,z)^{2/3}}\Bigg)$$

03.20.03.0006.01

$$\operatorname{ker}_{-\frac{11}{3}}(z) = \frac{i\,\pi}{324\; 2^{5/6}\,\sqrt[6]{3}\; z^{14/3}} \Bigg(\sqrt{3}\,(20+20\,i)\left(64\,\sqrt[6]{-1}\;z^{2/3} + 18\,(-1)^{2/3}\,z^{8/3} - 9\,i\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2 - 32\;2^{2/3}\,((1+i)\,z)^{2/3}\right)\operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z} +$$
$$\sqrt{3}\,(20+20\,i)\left(64\,\sqrt[3]{-1}\;z^{2/3} - 18\,(-1)^{5/6}\,z^{8/3} + 9\;2^{2/3}\,((1+i)\,z)^{2/3}\,z^2 + 32\;2^{2/3}\,i\,((1+i)\,z)^{2/3}\right)\operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)$$
$$\sqrt[3]{z} - 9\,i\,\sqrt[3]{3}\,\left(9\,z^2 - 160\,i\right)\left(2^{2/3}\,(1-i)\,\sqrt[3]{z} + \left(1 - i\,\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2 +$$
$$9\,\sqrt[3]{3}\,\left(9\,z^2 + 160\,i\right)\left(2^{2/3}\,(1-i)\,\sqrt[3]{z} + \left(1 + i\,\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2 +$$
$$3\,\sqrt[3]{3}\;i\left(9\,i\,z^2 + 160\right)\left(2^{2/3}\,\sqrt{3}\,(1+i)\,\sqrt[3]{z} + \left(-3 - i\,\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2 -$$
$$3\,\sqrt[3]{3}\,\left(9\,z^2 + 160\,i\right)\left(2^{2/3}\,\sqrt{3}\,(-1+i)\,\sqrt[3]{z} + \left(3\,i + \sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2 -$$
$$(60+60\,i)\left(32\;2^{2/3}\,((1+i)\,z)^{2/3}\,\sqrt[3]{z} + 9\;2^{2/3}\,i\,((1+i)\,z)^{2/3}\,z^{7/3} + 18\,(-1)^{2/3}\,z^3 + 64\,\sqrt[6]{-1}\;z\right)\operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) +$$
$$(60+60\,i)\left(32\;2^{2/3}\,i\,((1+i)\,z)^{2/3}\,\sqrt[3]{z} + 9\;2^{2/3}\,((1+i)\,z)^{2/3}\,z^{7/3} + 18\,(-1)^{5/6}\,z^3 - 64\,\sqrt[3]{-1}\;z\right)\operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Bigg)$$

03.20.03.0007.01

$$\operatorname{ker}_{-\frac{7}{2}}(z) = \frac{(-1)^{3/8}}{2\,z^{7/2}}\,e^{-\sqrt[4]{-1}\,z}\,\sqrt{\frac{\pi}{2}}\,\left(15\,i - z\left(z\left(\sqrt[4]{-1}\;z + 6\right) - 15\,(-1)^{3/4}\right) + e^{i\,\sqrt{2}\,z}\left(z\left(z^2 + 6\,\sqrt[4]{-1}\;z + 15\,i\right) + 15\,(-1)^{3/4}\right)\right)$$

03.20.03.0008.01

$$
\begin{aligned}
\operatorname{ker}_{-\frac{10}{3}}(z) = -\frac{i\,\pi\,z^{10/3}}{54\,2^{2/3}\,\sqrt[3]{3}} \Bigg(& 16\sqrt{3}\,\left(-9\,i\,z^2-14\right)\left(\frac{1}{\left(\sqrt[4]{-1}\,z\right)^{20/3}}+\frac{\sqrt[3]{-1}}{z^{20/3}}\right)\operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\frac{1}{z^{20/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}} \\
& 16\sqrt{3}\left(14\,i\,z^{2/3}+9\,z^{8/3}-14\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}+9\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\frac{1}{z^{20/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}} \\
& 3\,\sqrt[6]{3}\,((1+i)\,z)^{2/3}\left(112\,i\,z^{2/3}-9\,z^{8/3}+112\,\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}+9\,\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+ \\
& \frac{1}{z^{20/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}}\,3\,\sqrt[6]{3}\,((1+i)\,z)^{2/3}\left(112\,i\,z^{2/3}+9\,z^{8/3}-112\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}+9\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right) \\
& \operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+16\left(-9\,i\,z^2-14\right)\left(\frac{\sqrt[3]{-1}}{z^{20/3}}-\frac{1}{\left(\sqrt[4]{-1}\,z\right)^{20/3}}\right)\operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+ \\
& \frac{16\,i\left(-14\,z^{2/3}+9\,i\,z^{8/3}-14\,\sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}+9\,\sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{z^{20/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}}+ \\
& \frac{1}{z^{20/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}}\,3^{2/3}\,((1+i)\,z)^{2/3}\left(-112\,i\,z^{2/3}+9\,z^{8/3}+112\,\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}+9\,\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right) \\
& \operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)-\frac{1}{z^{20/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}} \\
& 3^{2/3}\,((1+i)\,z)^{2/3}\left(112\,i\,z^{2/3}+9\,z^{8/3}+9\,\sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}z^2+112\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Bigg)
\end{aligned}
$$

03.20.03.0009.01

$$\operatorname{ker}_{-\frac{8}{3}}(z)=\frac{(1-i)(-1)^{3/4}\pi}{216\sqrt[6]{6}\,z^{11/3}}\Bigg(6\,i\left(9z^2+40\,i\right)\left(\sqrt[3]{-2}\;i\,z^{2/3}+((1+i)\,z)^{2/3}\right)\operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z}\,+$$
$$2\sqrt{3}\left(40\sqrt[6]{-1}\;\sqrt[3]{2}\;z^{2/3}+9\,(-1)^{2/3}\sqrt[3]{2}\;z^{8/3}-9\,i\,((1+i)\,z)^{2/3}z^2-40\,((1+i)\,z)^{2/3}\right)\operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z}\,+$$
$$2\sqrt{3}\left(9z^2+40\,i\right)\left(\sqrt[3]{-2}\;z^{2/3}+i\,((1+i)\,z)^{2/3}\right)\operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{z}\,+$$
$$\sqrt[3]{3}\,(45+45\,i)\left((2+2\,i)\sqrt[3]{z}+\sqrt[3]{2}\left(i+\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2\,+$$
$$\sqrt[3]{3}\,(-45-45\,i)\left((-2-2\,i)\sqrt[3]{z}+\sqrt[3]{2}\left(-i+\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2\,-$$
$$(15+15\,i)\sqrt[3]{3}\left(\sqrt{3}\,(-2-2\,i)\sqrt[3]{z}+\sqrt[3]{2}\left(3+i\sqrt{3}\right)\sqrt[3]{(1+i)\,z}\right)\operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2\,+$$
$$3^{5/6}\,(30-30\,i)\left((-1+i)\sqrt[3]{z}+\sqrt[3]{-2}\;\sqrt[3]{(1+i)\,z}\right)\operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^2\,-$$
$$6\left(40\,((1+i)\,z)^{2/3}\sqrt[3]{z}+9\,i\,((1+i)\,z)^{2/3}z^{7/3}+9\,(-1)^{2/3}\sqrt[3]{2}\;z^3+40\sqrt[6]{-1}\;\sqrt[3]{2}\;z\right)\operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Bigg)$$

03.20.03.0010.01

$$\operatorname{ker}_{-\frac{5}{2}}(z)=\frac{(-1)^{7/8}}{2\,z^{5/2}}\,e^{-\sqrt[4]{-1}\,z}\sqrt{\frac{\pi}{2}}\left(z\left(\sqrt[4]{-1}\;z+3\right)-3\,(-1)^{3/4}+e^{i\sqrt{2}\,z}\left(z^2+3\sqrt[4]{-1}\;z+3\,i\right)\right)$$

03.20.03.0011.01

$$\operatorname{ker}_{-\frac{7}{3}}(z) = \frac{\pi z^{4/3}}{6\, 2^{2/3} \sqrt[3]{3}} \left(\frac{3 \sqrt[6]{3}\, i \left(z^{2/3} - \sqrt[6]{-1} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3}}{z^{8/3} \left(\sqrt[4]{-1}\, z\right)^{2/3}} + \right.$$

$$\frac{3^{2/3} i \left(z^{2/3} + (-1)^{5/6} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3}}{z^{8/3} \left(\sqrt[4]{-1}\, z\right)^{2/3}} -$$

$$\frac{3 i \sqrt[6]{3} \left(z^{2/3} - (-1)^{5/6} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3}}{z^{8/3} \left(\sqrt[4]{-1}\, z\right)^{2/3}} -$$

$$\frac{i\, 3^{2/3} \left(z^{2/3} + \sqrt[6]{-1} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3}}{z^{8/3} \left(\sqrt[4]{-1}\, z\right)^{2/3}} -$$

$$2 \sqrt{3} \left(\frac{1}{\left(\sqrt[4]{-1}\, z\right)^{8/3}} - \frac{\sqrt[3]{-1}}{z^{8/3}} \right) \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 2 (-1)^{2/3} \sqrt{3} \left(\frac{\sqrt[3]{-1}}{\left(\sqrt[4]{-1}\, z\right)^{8/3}} - \frac{1}{z^{8/3}} \right) \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) -$$

$$\left. 2 \left(-\frac{1}{\left(\sqrt[4]{-1}\, z\right)^{8/3}} - \frac{\sqrt[3]{-1}}{z^{8/3}} \right) \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 2 (-1)^{2/3} \left(\frac{\sqrt[3]{-1}}{\left(\sqrt[4]{-1}\, z\right)^{8/3}} + \frac{1}{z^{8/3}} \right) \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) \right)$$

03.20.03.0012.01

$$\operatorname{ker}_{-\frac{5}{3}}(z) = \frac{\sqrt[4]{-1}\,\pi\, z^{5/3}}{36\sqrt[3]{2}\,\sqrt[6]{3}} \left(-9\sqrt[3]{3}\left(\frac{1}{\left(\sqrt[4]{-1}\,z\right)^{10/3}} - \frac{\sqrt[6]{-1}}{z^{10/3}}\right)\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3} + 3\,2^{2/3}\,3^{5/6}\left(3+i\sqrt{3}\right)i\left(\frac{\left(-\frac{1}{2}\right)^{2/3}}{2\,z^{10/3}} + \frac{(-1)^{5/6}}{((1+i)\,z)^{10/3}}\right)\right.$$

$$\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3} + \frac{3\,3^{5/6}\left(((1+i)\,z)^{2/3} - \sqrt[6]{-1}\,\sqrt[3]{2}\,z^{2/3}\right)\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3}}{\sqrt[3]{2}\,z^4} +$$

$$\frac{3\,3^{5/6}\,i\left(((1+i)\,z)^{2/3} - (-1)^{5/6}\sqrt[3]{2}\,z^{2/3}\right)\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3}}{\sqrt[3]{2}\,z^4} +$$

$$\frac{24\left(\sqrt[3]{-1}\,z^{2/3} - i\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{z^4} + \frac{8\sqrt{3}\left(\sqrt[6]{-1}\,z^{2/3} - \left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{z^4} -$$

$$\left.\frac{24\left(\sqrt[6]{-1}\,z^{2/3} + \left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{z^4} - \frac{8\sqrt{3}\left(\sqrt[3]{-1}\,z^{2/3} + i\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{z^4}\right)$$

03.20.03.0013.01

$$\operatorname{ker}_{-\frac{3}{2}}(z) = \frac{(-1)^{3/8}}{2\,z^{3/2}}\,e^{-\sqrt[4]{-1}\,z}\sqrt{\frac{\pi}{2}}\left(\sqrt[4]{-1}\,z - e^{i\sqrt{2}\,z}\left(z+\sqrt[4]{-1}\right)+1\right)$$

03.20.03.0014.01

$$\operatorname{ker}_{-\frac{3}{2}}(z) = \frac{\sqrt{\pi}}{4\,z^{3/2}}\,e^{-\frac{z}{\sqrt{2}}}$$

$$\left(-z\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi\right)\right) + \cos\left(\frac{1}{8}\left(\pi-4\sqrt{2}\,z\right)\right) + \left(z+\sqrt{2}\right)\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi\right)\right) - \left(\sqrt{2}\,z+1\right)\sin\left(\frac{1}{8}\left(\pi-4\sqrt{2}\,z\right)\right)\right)$$

03.20.03.0015.01

$$\operatorname{ker}_{-\frac{4}{3}}(z) = \frac{\pi z^{4/3}}{6\, 2^{2/3} \sqrt[3]{3}} \left(\frac{3 \sqrt[6]{3}\, i \left(z^{2/3} - \sqrt[6]{-1} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3}}{z^{8/3} \left(\sqrt[4]{-1}\, z\right)^{2/3}} + \right.$$

$$\frac{3^{2/3} i \left(z^{2/3} + (-1)^{5/6} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3}}{z^{8/3} \left(\sqrt[4]{-1}\, z\right)^{2/3}} -$$

$$\frac{3 i \sqrt[6]{3} \left(z^{2/3} - (-1)^{5/6} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3}}{z^{8/3} \left(\sqrt[4]{-1}\, z\right)^{2/3}} -$$

$$\frac{i\, 3^{2/3} \left(z^{2/3} + \sqrt[6]{-1} \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3}}{z^{8/3} \left(\sqrt[4]{-1}\, z\right)^{2/3}} -$$

$$2 \sqrt{3} \left(\frac{1}{\left(\sqrt[4]{-1}\, z\right)^{8/3}} - \frac{\sqrt[3]{-1}}{z^{8/3}} \right) \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 2 (-1)^{2/3} \sqrt{3} \left(\frac{\sqrt[3]{-1}}{\left(\sqrt[4]{-1}\, z\right)^{8/3}} - \frac{1}{z^{8/3}} \right) \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) -$$

$$\left. 2 \left(-\frac{1}{\left(\sqrt[4]{-1}\, z\right)^{8/3}} - \frac{\sqrt[3]{-1}}{z^{8/3}} \right) \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 2 (-1)^{2/3} \left(\frac{\sqrt[3]{-1}}{\left(\sqrt[4]{-1}\, z\right)^{8/3}} + \frac{1}{z^{8/3}} \right) \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) \right)$$

03.20.03.0016.01

$$\operatorname{ker}_{-\frac{2}{3}}(z) = \frac{(i-1) \sqrt[4]{-1}\, \pi}{6\, 2^{5/6} \sqrt[6]{3}\, z^{4/3}}$$

$$\left(3 \left(\sqrt[6]{-1}\, z^{2/3} + \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 3 \left((-1)^{5/6} z^{2/3} + \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \right.$$

$$\left. \sqrt{3} \left(\left(\left(\sqrt[4]{-1}\, z\right)^{2/3} - \sqrt[6]{-1}\, z^{2/3} \right) \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \left((-1)^{5/6} z^{2/3} - \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) \right) \right)$$

03.20.03.0017.01

$$\operatorname{ker}_{-\frac{1}{2}}(z) = -\frac{(-1)^{7/8}}{2 \sqrt{z}} e^{-\sqrt[4]{-1}\, z} \sqrt{\frac{\pi}{2}} \left(\sqrt[4]{-1} + e^{i \sqrt{2}\, z}\right)$$

03.20.03.0018.01

$$\operatorname{ker}_{-\frac{1}{2}}(z) = \frac{1}{\sqrt{z}} \sqrt{\frac{\pi}{2}}\, e^{-\frac{z}{\sqrt{2}}} \cos\left(\frac{\pi}{8} - \frac{z}{\sqrt{2}}\right)$$

03.20.03.0019.01

$$\operatorname{ker}_{-\frac{1}{3}}(z) = -\frac{(-1)^{3/4}\pi}{2\sqrt[3]{6}\sqrt[3]{z}\,((1+i)z)^{2/3}} \left(\sqrt{3}\left(i z^{2/3} + \sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt{3}\left(\sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3} - z^{2/3}\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3} - i z^{2/3}\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(z^{2/3} + \sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.20.03.0020.01

$$\operatorname{ker}_{\frac{1}{3}}(z) = -\frac{(-1)^{3/4}\pi}{2\sqrt[3]{6}\sqrt[3]{z}\,((1+i)z)^{2/3}} \left(\sqrt{3}\left(\sqrt[6]{-1}\,z^{2/3} + \left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt[3]{-1}\sqrt{3}\left(\sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3} - z^{2/3}\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(\left(\sqrt[4]{-1}\,z\right)^{2/3} - \sqrt[6]{-1}\,z^{2/3}\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt[3]{-1}\left(z^{2/3} + \sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.20.03.0021.01

$$\operatorname{ker}_{\frac{1}{2}}(z) = \frac{(-1)^{3/8}}{2\sqrt{z}}\, e^{-\sqrt[4]{-1}\,z}\left(-\sqrt[4]{-1} + e^{i\sqrt{2}\,z}\right)\sqrt{\frac{\pi}{2}}$$

03.20.03.0022.01

$$\operatorname{ker}_{\frac{1}{2}}(z) = \frac{1}{\sqrt{z}}\sqrt{\frac{\pi}{2}}\, e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{\pi}{8} - \frac{z}{\sqrt{2}}\right)$$

03.20.03.0023.01

$$\operatorname{ker}_{\frac{2}{3}}(z) = \frac{i\pi}{6\sqrt[6]{6}\,z^{2/3}\sqrt[3]{(1+i)z}} \left(3\left(\sqrt[12]{-1}\sqrt[3]{z} + \sqrt[3]{\sqrt[4]{-1}\,z}\right)\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - 3\left(\sqrt[3]{\sqrt[4]{-1}\,z} - (-1)^{5/12}\sqrt[3]{z}\right)\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt{3}\left(\left(\sqrt[12]{-1}\sqrt[3]{z} - \sqrt[3]{\sqrt[4]{-1}\,z}\right)\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left((-1)^{5/12}\sqrt[3]{z} + \sqrt[3]{\sqrt[4]{-1}\,z}\right)\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)\right)$$

03.20.03.0024.01

$$\operatorname{ker}_{\frac{4}{3}}(z) = -\frac{\pi}{3\sqrt[3]{6}\,z^{4/3}((1+i)z)^{8/3}} \left(-3i\sqrt[6]{3}\,z^2\left(\sqrt[6]{-1}\,z^{2/3} + \left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 3\sqrt[6]{3}\,i z^2\left((-1)^{5/6}z^{2/3} + \left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 3^{2/3}z^2\left((-1)^{2/3}z^{2/3} - i\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 3^{2/3}z^2\left(\sqrt[3]{-1}\,z^{2/3} + i\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 2\sqrt{3}\left((-1)^{2/3}z^{8/3} + \left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - 2\sqrt[3]{-1}\sqrt{3}\left(z^{8/3} + (-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - 2\left((-1)^{2/3}z^{8/3} - \left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 2\sqrt[3]{-1}\left(\sqrt[6]{-1}\,z^2\left(\sqrt[4]{-1}\,z\right)^{2/3} + z^{8/3}\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.20.03.0025.01

$$\operatorname{ker}_{\frac{3}{2}}(z) = \frac{(-1)^{3/8}}{2\, z^{3/2}}\, e^{-\sqrt[4]{-1}\, z} \sqrt{\frac{\pi}{2}} \left(i + e^{i \sqrt{2}\, z} \left(i\, z + (-1)^{3/4}\right) + (-1)^{3/4}\, z\right)$$

03.20.03.0026.01

$$\operatorname{ker}_{\frac{3}{2}}(z) = -\frac{\sqrt{\pi}}{4\, z^{3/2}}\, e^{-\frac{z}{\sqrt{2}}} \left(\left(z + \sqrt{2}\right) \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) + \left(\sqrt{2}\, z + 1\right) \cos\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) + z \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) + \sin\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right)\right)$$

03.20.03.0027.01

$$\operatorname{ker}_{\frac{5}{3}}(z) = \frac{\pi}{288\, \sqrt[3]{2}\, \sqrt[6]{3}\, z^{2/3} \left(\sqrt[4]{-1}\, z\right)^{13/3}} \Bigg(-72\, \sqrt[3]{3} \left((-1)^{5/6}\, z^{10/3} + \left(\sqrt[4]{-1}\, z\right)^{10/3}\right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} +$$
$$6\, \sqrt[3]{-2}\, 3^{5/6} \left(-3\, i + \sqrt{3}\right) z^3 \left((1+i)\, \sqrt[3]{(1+i)\, z} - (-2)^{2/3}\, \sqrt[3]{z}\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} -$$
$$24\, 3^{5/6} \left((-1)^{5/6}\, z^{10/3} - \left(\sqrt[4]{-1}\, z\right)^{10/3}\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} +$$
$$6\, \sqrt[3]{-2}\, 3^{5/6} \left(\sqrt[6]{-1}\, ((1+i)\, z)^{10/3} - 2\, \sqrt[3]{-1}\, 2^{2/3}\, z^{10/3}\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} +$$
$$192 \left((-1)^{5/6}\, z^{10/3} + \left(\sqrt[4]{-1}\, z\right)^{10/3}\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$
$$192\, (-1)^{2/3}\, z^3 \left(\sqrt[3]{z} + (-1)^{7/12}\, \sqrt[3]{\sqrt[4]{-1}\, z}\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 64\, \sqrt{3} \left((-1)^{5/6}\, z^{10/3} - \left(\sqrt[4]{-1}\, z\right)^{10/3}\right)$$
$$\operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - 64\, (-1)^{2/3}\, \sqrt{3}\, z \left(\sqrt[3]{-1}\, z \left(\sqrt[4]{-1}\, z\right)^{4/3} - z^{7/3}\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\Bigg)$$

03.20.03.0028.01

$$\operatorname{ker}_{\frac{7}{3}}(z) = \frac{\sqrt[4]{-1}\, 2^{2/3}\, \pi}{9\, \sqrt[3]{3}\, z^{7/3}\, ((1+i)\, z)^{14/3}}$$
$$\Bigg(24\, \sqrt[6]{3}\, z^4 \left(\sqrt[6]{-1}\, z^{2/3} + \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + 24\, \sqrt[6]{3}\, i\, z^4 \left((-1)^{5/6}\, z^{2/3} + \left(\sqrt[4]{-1}\, z\right)^{2/3}\right)$$
$$\operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + 8\, 3^{2/3}\, z^4 \left(\left(\sqrt[4]{-1}\, z\right)^{2/3} - \sqrt[6]{-1}\, z^{2/3}\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} +$$
$$8\, 3^{2/3}\, z^4 \left(\sqrt[3]{-1}\, z^{2/3} + i \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} +$$
$$\sqrt{3} \left(16 \left(\left(\sqrt[4]{-1}\, z\right)^{14/3} - \sqrt[6]{-1}\, z^{14/3}\right) - 9\, i\, z^6 \left(\sqrt[6]{-1}\, z^{2/3} + \left(\sqrt[4]{-1}\, z\right)^{2/3}\right)\right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$
$$\sqrt{3}\, z^4 \left(-16\, \sqrt[3]{-1}\, z^{2/3} + 9\, (-1)^{5/6}\, z^{8/3} + 9 \left(\sqrt[4]{-1}\, z\right)^{2/3} z^2 + 16\, i \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$
$$z^4 \left(16\, \sqrt[6]{-1}\, z^{2/3} + 9\, (-1)^{2/3}\, z^{8/3} - 9\, i \left(\sqrt[4]{-1}\, z\right)^{2/3} z^2 - 16 \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) +$$
$$z^4 \left(16\, \sqrt[3]{-1}\, z^{2/3} - 9\, (-1)^{5/6}\, z^{8/3} + 9 \left(\sqrt[4]{-1}\, z\right)^{2/3} z^2 + 16\, i \left(\sqrt[4]{-1}\, z\right)^{2/3}\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\Bigg)$$

03.20.03.0029.01

$$\operatorname{ker}_{\frac{5}{2}}(z) = \frac{(-1)^{3/8}}{2\,z^{5/2}}\,e^{-\sqrt[4]{-1}\,z}\sqrt{\frac{\pi}{2}}\left(z\left(\sqrt[4]{-1}\,z+3\right)-3\,(-1)^{3/4}-e^{i\sqrt{2}\,z}\left(z^2+3\sqrt[4]{-1}\,z+3\,i\right)\right)$$

03.20.03.0030.01

$$\operatorname{ker}_{\frac{8}{3}}(z) = -\frac{\pi\,z^{7/3}}{108\sqrt[3]{2}\,\sqrt[6]{3}\left(\sqrt[4]{-1}\,z\right)^{16/3}}\left(\frac{60\sqrt[3]{2}\,3^{5/6}\left(z^{2/3}+(-1)^{5/6}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^{7/3}}{((1+i)\,z)^{4/3}}+\right.$$
$$\frac{60\sqrt[3]{2}\,3^{5/6}\left(z^{2/3}+\sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)z^{7/3}}{((1+i)\,z)^{4/3}}+$$
$$90\left(\sqrt[3]{-3}\,((1+i)\,z)^{4/3}\sqrt[3]{z}+\sqrt[3]{6}\,i\,((1+i)\,z)^{2/3}\,z\right)\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$45\sqrt[3]{3}\,((1+i)\,z)^{4/3}\left(\left(1-i\sqrt{3}\right)\sqrt[3]{z}+\sqrt[3]{2}\,(1+i)\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\frac{3\sqrt[3]{z}\left(9\,z^2-40\,i\right)\left(4\,z^{2/3}+2^{2/3}\left(-i+\sqrt{3}\right)((1+i)\,z)^{2/3}\right)\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{2^{2/3}\,((1+i)\,z)^{2/3}}+$$
$$\frac{3\left(9\,z^2+40\,i\right)\left(2^{2/3}\left(i+\sqrt{3}\right)\sqrt[3]{z}-(2-2\,i)\sqrt[3]{(1+i)\,z}\right)\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)}{2^{2/3}}-\frac{1}{((1+i)\,z)^{2/3}}\,2\sqrt[3]{2}\,\sqrt{3}\,\sqrt[3]{z}$$
$$\left(-40\,i\,z^{2/3}+9\,z^{8/3}+40\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}+9\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{8/3}\right)\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\frac{1}{((1+i)\,z)^{2/3}}$$
$$\left.2\sqrt[3]{2}\,\sqrt{3}\,\sqrt[3]{z}\left(40\,i\,z^{2/3}+9\,z^{8/3}+9\sqrt[6]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}z^2+40\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}\right)\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.20.03.0031.01

$$\operatorname{ker}_{\frac{10}{3}}(z) = \frac{\pi}{108\sqrt[3]{6}\, z^{10/3}\,((1+i)z)^{2/3}} \Bigg(-3\sqrt[6]{3}\,(9z^2-112i)\left(2\sqrt[6]{-1}\, z^{2/3}+2^{2/3}((1+i)z)^{2/3}\right)\operatorname{Ai}'\!\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} - 3^{2/3}\left(224(-1)^{2/3}z^{2/3}-18\sqrt[6]{-1}\, z^{8/3}+9\,2^{2/3}((1+i)z)^{2/3}z^2-112i\,2^{2/3}((1+i)z)^{2/3}\right)\operatorname{Bi}'\!\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) ((1+i)z)^{2/3} + 3^{2/3}\left(224\sqrt[3]{-1}\, z^{2/3}-18(-1)^{5/6}z^{8/3}+9\,2^{2/3}((1+i)z)^{2/3}z^2+112\,2^{2/3}i((1+i)z)^{2/3}\right) \operatorname{Bi}'\!\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 16\sqrt{3}\left(-28(-1)^{2/3}z^{2/3}+18\sqrt[6]{-1}\, z^{8/3}+9\,2^{2/3}((1+i)z)^{2/3}z^2-14i\,2^{2/3}((1+i)z)^{2/3}\right)\operatorname{Ai}\!\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 32\sqrt{3}\left(-14\sqrt[3]{-1}\, z^{2/3}+9(-1)^{5/6}z^{8/3}+9\left(\sqrt[4]{-1}\, z\right)^{2/3}z^2+14i\left(\sqrt[4]{-1}\, z\right)^{2/3}\right)\operatorname{Ai}\!\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \frac{3\sqrt[6]{3}\,((1+i)z)^{4/3}(9z^2+112i)\left(2^{2/3}\sqrt[3]{z}+\sqrt[3]{-1}\,(1+i)\sqrt[3]{(1+i)z}\right)\operatorname{Ai}'\!\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)}{\sqrt[3]{z}} - 16i\left(-28\sqrt[6]{-1}\, z^{2/3}-18(-1)^{2/3}z^{8/3}+14\,2^{2/3}((1+i)z)^{2/3}+\frac{9((1+i)z)^{8/3}}{\sqrt[3]{2}}\right)\operatorname{Bi}\!\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 16\left(28\sqrt[3]{-1}\, z^{2/3}-18(-1)^{5/6}z^{8/3}+9\,2^{2/3}((1+i)z)^{2/3}z^2+14\,2^{2/3}i((1+i)z)^{2/3}\right)\operatorname{Bi}\!\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\Bigg)$$

03.20.03.0032.01

$$\operatorname{ker}_{\frac{7}{2}}(z) = -\frac{(-1)^{7/8}}{2z^{7/2}}e^{-\sqrt[4]{-1}\, z}\sqrt{\frac{\pi}{2}}\left(-15i+z\left(z\left(\sqrt[4]{-1}\, z+6\right)-15(-1)^{3/4}\right)+e^{i\sqrt{2}\, z}\left(z\left(z^2+6\sqrt[4]{-1}\, z+15i\right)+15(-1)^{3/4}\right)\right)$$

03.20.03.0033.01

$$\operatorname{ker}_{\frac{11}{3}}(z) = \frac{(i-1)\pi}{648\, 2^{5/6} \sqrt[6]{3}\, z^{13/3}} \Bigg(-9 \sqrt[3]{3} \left(9 z^2 + 160 i\right) \left(2^{2/3} \left(i + \sqrt{3}\right) \sqrt[3]{z} - (2 - 2 i) \sqrt[3]{(1+i) z}\right) \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) z^{5/3} +$$

$$3 \sqrt[3]{3} \left(9 z^2 - 160 i\right) \left((2+2 i) \sqrt{3} \sqrt[3]{(1+i) z} - 2^{2/3} \left(3 i + \sqrt{3}\right) \sqrt[3]{z}\right) \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) z^{5/3} +$$

$$3 \sqrt[3]{3}\, i \left(9 z^2 + 160 i\right) \left((2+2 i) \sqrt{3} \sqrt[3]{(1+i) z} - 2^{2/3} \left(-3 i + \sqrt{3}\right) \sqrt[3]{z}\right) \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) z^{5/3} +$$

$$\frac{18 \sqrt[3]{3}\, z^4 \left(9 z^2 - 160 i\right) \left(4 z^{2/3} + 2^{2/3} \left(-i + \sqrt{3}\right) ((1+i) z)^{2/3}\right) \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)}{((1+i) z)^{8/3}} +$$

$$60 \left(9 z^2 - 32 i\right) \left(4 z^{2/3} + 2^{2/3} \left(-i + \sqrt{3}\right) ((1+i) z)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) +$$

$$\frac{120 i z^{5/3} \left(9 z^2 + 32 i\right) \left(2^{2/3} \left(1 - i \sqrt{3}\right) \sqrt[3]{z} + (2+2 i) \sqrt[3]{(1+i) z}\right) \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)}{((1+i) z)^{4/3}} -$$

$$20 i \left(9 z^2 - 32 i\right) \left(2^{2/3} \left(3 i + \sqrt{3}\right) ((1+i) z)^{2/3} - 4 i \sqrt{3}\, z^{2/3}\right) \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) -$$

$$20 i \left(9 z^2 + 32 i\right) \left(4 \sqrt{3}\, z^{2/3} + 2^{2/3} \left(3 + i \sqrt{3}\right) ((1+i) z)^{2/3}\right) \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) \Bigg)$$

03.20.03.0034.01

$$\operatorname{ker}_{\frac{13}{3}}(z)=\frac{2\,(-1)^{3/4}\,2^{2/3}\,\pi\,z^{11/3}}{81\,\sqrt[3]{3}\,((1+i)\,z)^{26/3}}\Bigg(-\sqrt{3}\,\Big(-8960\,\sqrt[6]{-1}\,z^{2/3}-6048\,(-1)^{2/3}\,z^{8/3}+162\,\sqrt[6]{-1}\,z^{14/3}+81\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^4-3024\,i\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2-4480\,2^{2/3}\,((1+i)\,z)^{2/3}\Big)\,\operatorname{Ai}\Big(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)+\sqrt{3}\,\Big(-8960\,\sqrt[3]{-1}\,z^{2/3}+6048\,(-1)^{5/6}\,z^{8/3}+162\,\sqrt[3]{-1}\,z^{14/3}-81\,i\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^4+3024\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2+4480\,2^{2/3}\,i\,((1+i)\,z)^{2/3}\Big)\,\operatorname{Ai}\Big(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)+\frac{1}{((1+i)\,z)^{5/3}}\Bigg(2\,z^{5/3}\Bigg(\sqrt[6]{3}\,(-84+84\,i)\,z\Big(80\,2^{2/3}\,i\,\sqrt[3]{z}+9\,2^{2/3}\,z^{7/3}+9\,\sqrt[3]{-1}\,((1+i)\,z)^{4/3}\,z+(-1)^{5/6}\,(80+80\,i)\,\sqrt[3]{(1+i)\,z}\Big)\,\operatorname{Ai}'\Big(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)+\frac{1}{\sqrt[3]{(1+i)\,z}}\Big(84\,\sqrt[6]{3}\,\big(9\,z^2-80\,i\big)\Big(2^{2/3}\,((1+i)\,z)^{2/3}\,\sqrt[3]{z}+2\,\sqrt[6]{-1}\,z\Big)\,\operatorname{Ai}'\Big(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\,((1+i)\,z)^{2/3}+\Big(-4480\,2^{2/3}\,((1+i)\,z)^{2/3}\,\sqrt[3]{z}+3024\,2^{2/3}\,i\,((1+i)\,z)^{2/3}\,z^{7/3}+81\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^{13/3}-162\,(-1)^{5/6}\,z^5+6048\,\sqrt[3]{-1}\,z^3+8960\,(-1)^{5/6}\,z\Big)\,\operatorname{Bi}\Big(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)+i\,\Big(28\,3^{2/3}\,\Big(\Big(160\,\sqrt[6]{-1}\,z^{2/3}+18\,(-1)^{2/3}\,z^{8/3}-9\,i\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2-80\,2^{2/3}\,((1+i)\,z)^{2/3}\Big)\,\operatorname{Bi}'\Big(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\,\sqrt[3]{z}+\Big(80\,2^{2/3}\,i\,((1+i)\,z)^{2/3}\,\sqrt[3]{z}+9\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^{7/3}-18\,(-1)^{5/6}\,z^3+160\,\sqrt[3]{-1}\,z\Big)\,\operatorname{Bi}'\Big(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\Big)\,((1+i)\,z)^{2/3}+\Big(4480\,2^{2/3}\,((1+i)\,z)^{2/3}\,\sqrt[3]{z}+3024\,2^{2/3}\,i\,((1+i)\,z)^{2/3}\,z^{7/3}-81\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^{13/3}+162\,\sqrt[6]{-1}\,z^5-6048\,(-1)^{2/3}\,z^3-8960\,\sqrt[6]{-1}\,z\Big)\,\operatorname{Bi}\Big(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\Big)\Big)\Big)\Bigg)\Bigg)\Bigg)$$

03.20.03.0035.01

$$\operatorname{ker}_{\frac{9}{2}}(z)=\frac{(-1)^{3/8}}{2\,z^{9/2}}\,e^{-\sqrt[4]{-1}\,z}\,\sqrt{\frac{\pi}{2}}\,\Big(z\Big(105\,i-z\Big(z\Big(\sqrt[4]{-1}\,z+10\Big)-45\,(-1)^{3/4}\Big)\Big)+105\,\sqrt[4]{-1}+e^{i\sqrt{2}\,z}\Big(z\Big(z\Big(z^2+\sqrt{2}\,(5+5\,i)\,z+45\,i\Big)+105\,(-1)^{3/4}\Big)-105\Big)\Big)$$

03.20.03.0036.01

$$\begin{aligned}
\operatorname{ker}_{\frac{14}{3}}(z) = & -\frac{i\,\pi}{972\,\sqrt[3]{2}\,\sqrt[6]{3}\,z^{14/3}\left(\sqrt[4]{-1}\,z\right)^{28/3}} \\
& \Big(-48\,\sqrt[6]{-1}\,\sqrt[3]{2}\,3^{5/6}\,z^{28/3}\big(\sqrt[12]{-1}\,\sqrt{6}\,(110+110\,i)\,z^{2/3}+9\,(3\,i+\sqrt{3})\,i\,z^{8/3}+9\,(-2)^{2/3}\sqrt{3}\,((1+i)\,z)^{2/3}\,z^2+ \\
& \quad 110\,\sqrt[6]{-1}\,2^{2/3}\sqrt{3}\,((1+i)\,z)^{2/3}\big)\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}-24\,\sqrt[3]{-2}\,3^{5/6}\,z^{28/3}\,(9\,z^2+110\,i) \\
& \big(2^{2/3}\,(3-i\,\sqrt{3})\,((1+i)\,z)^{2/3}-2\,(-3\,i+\sqrt{3})\,z^{2/3}\big)\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}-48\,\sqrt[4]{-1}\,\sqrt[3]{2}\,3^{5/6} \\
& z^{28/3}\big(\sqrt{2}\,(-110-110\,i)\,z^{2/3}+9\,\sqrt[12]{-1}\,(1-i\,\sqrt{3})\,z^{8/3}+9\,(-1)^{7/12}\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2+110\,\sqrt[12]{-1}\,2^{2/3}\,((1+i)\,z)^{2/3}\big) \\
& \operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}-48\,\sqrt[6]{-1}\,\sqrt[3]{2}\,3^{5/6}\,z^{28/3} \\
& \big(220\,\sqrt[3]{-1}\,z^{2/3}-18\,(-1)^{5/6}\,z^{8/3}+9\,2^{2/3}\,((1+i)\,z)^{2/3}\,z^2+110\,2^{2/3}\,i\,((1+i)\,z)^{2/3}\big)\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3}+ \\
& \Big(25\,920\,(-1)^{3/4}\left(\sqrt[12]{-1}\,\sqrt[3]{z}+\sqrt[3]{\sqrt[4]{-1}\,z}\right)z^{11}-486\left(\sqrt[3]{-1}\,z^{28/3}+\left(\sqrt[4]{-1}\,z\right)^{28/3}\right)z^4+ \\
& \quad 84\,480\left(\sqrt[3]{-1}\,z^{28/3}+\left(\sqrt[4]{-1}\,z\right)^{28/3}\right)\Big)\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)- \\
& \frac{1}{\left(\sqrt[4]{-1}\,z\right)^{2/3}}\Big(6\,z^{28/3}\Big(14\,080\,i\,z^{2/3}+4320\,z^{8/3}-81\,i\,z^{14/3}+81\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}z^4- \\
& \quad 14\,080\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}+4320\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{8/3}\Big)\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Big)+ \\
& \frac{1}{\left(\sqrt[4]{-1}\,z\right)^{2/3}}\Big(2\,\sqrt{3}\,z^{28/3}\Big(-14\,080\,i\,z^{2/3}+4320\,z^{8/3}+81\,i\,z^{14/3}-81\,\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}z^4+ \\
& \quad 14\,080\,\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{2/3}+4320\,\sqrt[3]{-1}\left(\sqrt[4]{-1}\,z\right)^{8/3}\Big)\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Big)- \\
& \frac{1}{\left(\sqrt[4]{-1}\,z\right)^{2/3}}\Big(2\,\sqrt{3}\,z^{28/3}\Big(-14\,080\,i\,z^{2/3}-4320\,z^{8/3}+81\,i\,z^{14/3}+81\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}z^4- \\
& \quad 14\,080\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{2/3}+4320\,(-1)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{8/3}\Big)\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\Big)\Big)
\end{aligned}$$

Symbolic rational ν

03.20.03.0037.01

$$\operatorname{ker}_\nu(z) = -\frac{(-1)^{5/8}}{2\sqrt{z}}\, e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}} \sqrt{\frac{\pi}{2}} \left(\sum_{k=0}^{\left\lfloor \frac{1}{4}(2|\nu|-3)\right\rfloor} \frac{\left(2k+|\nu|+\frac{1}{2}\right)!\, i^{-k} z^{-2k-1}}{2^{2k+1}(2k+1)!\left(-2k+|\nu|-\frac{3}{2}\right)!}\left(1-\frac{1}{\sqrt[4]{-1}}\, e^{i\left(\pi k+\sqrt{2}\, z+\pi\nu\right)}\right)+\right.$$

$$\left.\sum_{k=0}^{\left\lfloor \frac{1}{4}(2|\nu|-1)\right\rfloor} \frac{\left(2k+|\nu|-\frac{1}{2}\right)!\, i^{-k} z^{-2k}}{2^{2k}(2k)!\left(-2k+|\nu|-\frac{1}{2}\right)!}\left(\sqrt[4]{-1}-e^{i\left(\sqrt{2}\, z+\pi\left(k+\nu-\frac{1}{2}\right)\right)}\right)\right) /;\ \nu-\frac{1}{2}\in\mathbb{Z}$$

03.20.03.0038.01

$$\operatorname{ker}_\nu(z) =$$

$$\frac{2^{\frac{1}{2}(\nu+3|\nu|-6)}\sqrt[6]{3}\, e^{\frac{1}{4}(-3)i\pi\nu}\,\pi z^{-\nu}((1+i)z)^{-\nu-|\nu|}\csc(\pi\nu)}{\Gamma(1-|\nu|)}\,\Gamma\!\left(\frac{2}{3}\right)\left(\frac{1}{2}\sqrt[6]{3}\,((1+i)z)^{2/3}\sum_{k=0}^{|\nu|-\frac{4}{3}}\frac{4^{-k}\left(i z^2\right)^k\left(-k+|\nu|-\frac{4}{3}\right)!}{k!\left(-2k+|\nu|-\frac{4}{3}\right)!\left(\frac{4}{3}\right)_k(1-|\nu|)_k}\right.$$

$$\left(3\, e^{i\pi\nu}\left(i^{\left(|\nu|-\frac{1}{3}\right)(1-\operatorname{sgn}(\nu))}\, e^{\frac{i\pi\nu}{2}}\left(\sqrt[4]{-1}\, z\right)^{2\nu}-i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)}\, z^{2\nu}\right)\operatorname{Ai}'\!\left(-\frac{1}{2}\,3^{2/3}((1+i)z)^{2/3}\right)\operatorname{sgn}(\nu)^2+\right.$$

$$\left(\sqrt{3}\, e^{i\pi\nu}\left(i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)}\, z^{2\nu}+e^{\frac{i\pi\nu}{2}}\, i^{\left(|\nu|-\frac{1}{3}\right)(1-\operatorname{sgn}(\nu))}\left(\sqrt[4]{-1}\, z\right)^{2\nu}\right)\operatorname{Bi}'\!\left(-\frac{1}{2}\,3^{2/3}((1+i)z)^{2/3}\right)-\right.$$

$$\left.3(-1)^k\left(e^{\frac{i\pi\nu}{2}}\, z^{2\nu}+\left(\sqrt[4]{-1}\, z\right)^{2\nu}\right)\operatorname{Ai}'\!\left(\frac{1}{2}\,3^{2/3}((1+i)z)^{2/3}\right)\right)\operatorname{sgn}(\nu)+$$

$$\left.(-1)^k\sqrt{3}\left(e^{\frac{i\pi\nu}{2}}\, z^{2\nu}-\left(\sqrt[4]{-1}\, z\right)^{2\nu}\right)\operatorname{Bi}'\!\left(\frac{1}{2}\,3^{2/3}((1+i)z)^{2/3}\right)\right)+$$

$$\sum_{k=0}^{|\nu|-\frac{1}{3}}\frac{4^{-k}\left(i z^2\right)^k\left(-k+|\nu|-\frac{1}{3}\right)!}{k!\left(-2k+|\nu|-\frac{1}{3}\right)!\left(\frac{1}{3}\right)_k(1-|\nu|)_k}\left(\sqrt{3}\, e^{i\pi\nu}\left(i^{\left(|\nu|-\frac{1}{3}\right)(1-\operatorname{sgn}(\nu))}\, e^{\frac{i\pi\nu}{2}}\left(\sqrt[4]{-1}\, z\right)^{2\nu}-i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)}\, z^{2\nu}\right)\right.$$

$$\operatorname{Ai}\!\left(-\frac{1}{2}\,3^{2/3}((1+i)z)^{2/3}\right)\operatorname{sgn}(\nu)^2+\left((-1)^k\sqrt{3}\left(e^{\frac{i\pi\nu}{2}}\, z^{2\nu}+\left(\sqrt[4]{-1}\, z\right)^{2\nu}\right)\operatorname{Ai}\!\left(\frac{1}{2}\,3^{2/3}((1+i)z)^{2/3}\right)+\right.$$

$$\left.e^{i\pi\nu}\left(i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)}\, z^{2\nu}+e^{\frac{i\pi\nu}{2}}\, i^{\left(|\nu|-\frac{1}{3}\right)(1-\operatorname{sgn}(\nu))}\left(\sqrt[4]{-1}\, z\right)^{2\nu}\right)\operatorname{Bi}\!\left(-\frac{1}{2}\,3^{2/3}((1+i)z)^{2/3}\right)\right)\operatorname{sgn}(\nu)-$$

$$\left.\left.(-1)^k\left(e^{\frac{i\pi\nu}{2}}\, z^{2\nu}-\left(\sqrt[4]{-1}\, z\right)^{2\nu}\right)\operatorname{Bi}\!\left(\frac{1}{2}\,3^{2/3}((1+i)z)^{2/3}\right)\right)\right) /;\ |\nu|-\frac{1}{3}\in\mathbb{Z}$$

03.20.03.0039.01

$$\ker_\nu(z) = \frac{2^{|\nu|-3} e^{\frac{1}{4}(-3) i \pi \nu} \pi z^{-\nu} \left(\sqrt[4]{-1}\, z\right)^{-\nu-|\nu|} \csc(\pi \nu) \Gamma\left(\frac{1}{3}\right) \operatorname{sgn}(\nu)}{3^{2/3} \Gamma(1-|\nu|)}$$

$$\left(\frac{3^{5/6}\, 3}{8} ((1+i) z)^{4/3} \sum_{k=0}^{|\nu|-\frac{5}{3}} \frac{4^{-k} \left(i z^2\right)^k \left(-k+|\nu|-\frac{5}{3}\right)!}{k! \left(-2k+|\nu|-\frac{5}{3}\right)! \left(\frac{5}{3}\right)_k (1-|\nu|)_k} \left((-1)^k \sqrt{3} \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} + \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \right.\right.$$

$$\sqrt{3}\, e^{i\pi\nu} \left(i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)} z^{2\nu} + e^{\frac{i\pi\nu}{2}} i^{\left(|\nu|-\frac{2}{3}\right)(1-\operatorname{sgn}(\nu))} \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) +$$

$$(-1)^k \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} - \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) \operatorname{sgn}(\nu) +$$

$$\left. e^{i\pi\nu} \left(i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)} z^{2\nu} - i^{\left(|\nu|-\frac{2}{3}\right)(1-\operatorname{sgn}(\nu))} e^{\frac{i\pi\nu}{2}} \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) \operatorname{sgn}(\nu)\right) +$$

$$\sum_{k=0}^{|\nu|-\frac{2}{3}} \frac{4^{-k} \left(i z^2\right)^k \left(-k+|\nu|-\frac{2}{3}\right)!}{k! \left(-2k+|\nu|-\frac{2}{3}\right)! \left(\frac{2}{3}\right)_k (1-|\nu|)_k} \left(-3 (-1)^k \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} + \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - \right.$$

$$3 e^{i\pi\nu} \left(i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)} z^{2\nu} + e^{\frac{i\pi\nu}{2}} i^{\left(|\nu|-\frac{2}{3}\right)(1-\operatorname{sgn}(\nu))} \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) -$$

$$(-1)^k \sqrt{3} \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} - \left(\sqrt[4]{-1}\, z\right)^{2\nu}\right) \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) \operatorname{sgn}(\nu) +$$

$$\left.\left. \sqrt{3}\, e^{i\pi\nu} \left(i^{\left(|\nu|-\frac{2}{3}\right)(1-\operatorname{sgn}(\nu))} e^{\frac{i\pi\nu}{2}} \left(\sqrt[4]{-1}\, z\right)^{2\nu} - i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)} z^{2\nu}\right) \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) \operatorname{sgn}(\nu)\right)\right) /;\ |\nu| - \frac{2}{3} \in \mathbb{Z}$$

Values at fixed points

03.20.03.0040.01

$$\ker_0(0) = ¿$$

Values at infinities

03.20.03.0041.01

$$\lim_{x\to\infty} \ker_\nu(x) = 0$$

03.20.03.0042.01

$$\lim_{x\to-\infty} \ker_\nu(x) = \tilde{\infty}$$

General characteristics

Domain and analyticity

$\ker_\nu(z)$ is an analytical function of ν and z, which is defined in $\mathbb{C}^2$.

03.20.04.0001.01

$$(\nu * z) \longrightarrow \ker_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

03.20.04.0002.01

$\operatorname{ker}_{-n}(z) = (-1)^n \operatorname{ker}_n(z) /; n \in \mathbb{Z}$

Mirror symmetry

03.20.04.0003.01

$\operatorname{ker}_{\bar{\nu}}(\bar{z}) = \overline{\operatorname{ker}_\nu(z)} /; z \notin (-\infty, 0)$

Periodicity

No periodicity

Poles and essential singularities

With respect to *z*

For fixed ν, the function $\operatorname{ker}_\nu(z)$ has an essential singularity at $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is a branch point for generic ν.

03.20.04.0004.01

$\mathcal{S}ing_z(\operatorname{ker}_\nu(z)) = \{\{\tilde{\infty}, \infty\}\}$

With respect to *ν*

For fixed z, the function $\operatorname{ker}_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

03.20.04.0005.01

$\mathcal{S}ing_\nu(\operatorname{ker}_\nu(z)) = \{\{\tilde{\infty}, \infty\}\}$

Branch points

With respect to *z*

For fixed ν, the function $\operatorname{ker}_\nu(z)$ has two branch points: $z = 0$, $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is an essential singularity.

03.20.04.0006.01

$\mathcal{BP}_z(\operatorname{ker}_\nu(z)) = \{0, \tilde{\infty}\}$

03.20.04.0007.01

$\mathcal{R}_z(\operatorname{ker}_\nu(z), 0) = \log /; \nu \notin \mathbb{Q}$

03.20.04.0008.01

$\mathcal{R}_z\left(\operatorname{ker}_{\frac{p}{q}}(z), 0\right) = q /; p \in \mathbb{Z} \land q - 1 \in \mathbb{N}^+ \land \gcd(p, q) = 1$

03.20.04.0009.01

$\mathcal{R}_z(\operatorname{ker}_\nu(z), \tilde{\infty}) = \log /; \nu \notin \mathbb{Q}$

03.20.04.0010.01

$\mathcal{R}_z\left(\operatorname{ker}_{\frac{p}{q}}(z), \tilde{\infty}\right) = q /; p \in \mathbb{Z} \land q - 1 \in \mathbb{N}^+ \land \gcd(p, q) = 1$

With respect to ν

For fixed z, the function $\mathrm{ker}_\nu(z)$ does not have branch points.

03.20.04.0011.01

$$\mathcal{BP}_\nu(\mathrm{ker}_\nu(z)) = \{\}$$

Branch cuts

With respect to z

For fixed ν, the function $\mathrm{ker}_\nu(z)$ has one infinitely long branch cut. For fixed ν, the function $\mathrm{ker}_\nu(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

03.20.04.0012.01

$$\mathcal{BC}_z(\mathrm{ker}_\nu(z)) = \{\{(-\infty, 0), -i\}\} /; \nu \notin \mathbb{Z}$$

03.20.04.0013.01

$$\lim_{\epsilon \to +0} \mathrm{ker}_\nu(x + i\,\epsilon) = \mathrm{ker}_\nu(x) /; x \in \mathbb{R} \land x < 0$$

03.20.04.0014.01

$$\lim_{\epsilon \to +0} \mathrm{ker}_\nu(x - i\,\epsilon) = \frac{1}{2} e^{-2 i \pi \nu} \pi \left(-\mathrm{bei}_\nu(x) + e^{4 i \pi \nu} \csc(\pi \nu)\, \mathrm{ber}_{-\nu}(x) - \cot(\pi \nu)\, \mathrm{ber}_\nu(x)\right) /; \nu \notin \mathbb{Z} \bigwedge x \in \mathbb{R} \bigwedge x < 0$$

03.20.04.0015.01

$$\lim_{\epsilon \to +0} \mathrm{ker}_\nu(x - i\,\epsilon) = 2\, i\, \pi \cos(\pi \nu)\, \mathrm{ber}_{-\nu}(x) + e^{-2 i \pi \nu} \mathrm{ker}_\nu(x) /; x \in \mathbb{R} \land x < 0$$

03.20.04.0016.01

$$\lim_{\epsilon \to +0} \mathrm{ker}_\nu(x - i\,\epsilon) = 2\, i\, \pi\, \mathrm{ber}_\nu(x) + \mathrm{ker}_\nu(x) /; \nu \in \mathbb{Z} \land x \in \mathbb{R} \land x < 0$$

With respect to ν

For fixed z, the function $\mathrm{ker}_\nu(z)$ is an entire function of ν and does not have branch cuts.

03.20.04.0017.01

$$\mathcal{BC}_\nu(\mathrm{ker}_\nu(z)) = \{\}$$

Series representations

Generalized power series

Expansions at $\nu == \pm n$

03.20.06.0001.01

$$\mathrm{ker}_\nu(z) \propto \mathrm{ker}_n(z) + \left(\frac{\pi}{2} \mathrm{kei}_n(z) - \pi\, 2^{n-2}\, n!\, z^{-n} \sum_{k=0}^{n-1} \frac{(-1)^{n-k}\, 2^{-k}\, z^k}{(n-k)\, k!} \left(\cos\left(\frac{1}{4}(k-n)\pi\right) \mathrm{bei}_k(z) - \sin\left(\frac{1}{4}(k-n)\pi\right) \mathrm{ber}_k(z)\right) + \frac{(-1)^n}{4} \mathrm{ber}_{-n}^{(2,0)}(z) - \frac{1}{4} \mathrm{ber}_n^{(2,0)}(z)\right)(\nu - n) + \ldots /; (\nu \to n) \land n \in \mathbb{N}$$

03.20.06.0002.01

$$\operatorname{ker}_\nu(z) \propto (-1)^n \operatorname{ker}_n(z) + \left(\frac{(-1)^n \pi}{2} \operatorname{kei}_n(z) + (-1)^n 2^{n-2} \pi\, n!\, z^{-n} \sum_{k=0}^{n-1} \frac{(-1)^{n-k} 2^{-k} z^k}{(n-k)\, k!} \left(\cos\left(\frac{1}{4}(k-n)\pi\right) \operatorname{bei}_k(z) - \sin\left(\frac{1}{4}(k-n)\pi\right) \operatorname{ber}_k(z) \right) - \frac{1}{4} \operatorname{ber}_{-n}^{(2,0)}(z) + \frac{(-1)^n}{4} \operatorname{ber}_n^{(2,0)}(z) \right)(n+\nu) + \ldots \,/;\, (\nu \to -n) \land n \in \mathbb{N}$$

Expansions at generic point $z == z_0$

03.20.06.0003.01

$$\operatorname{ker}_\nu(z) \propto \left(\operatorname{ker}_\nu(z_0) \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} - 2\, i\, \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{ber}_{-\nu}(z_0) \right) + \frac{1}{2\sqrt{2}} \left(2\, i\, \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor (\operatorname{bei}_{-\nu-1}(z_0) - \operatorname{bei}_{1-\nu}(z_0) + \operatorname{ber}_{-\nu-1}(z_0) - \operatorname{ber}_{1-\nu}(z_0)) - \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} (\operatorname{kei}_{\nu-1}(z_0) - \operatorname{kei}_{\nu+1}(z_0) + \operatorname{ker}_{\nu-1}(z_0) - \operatorname{ker}_{\nu+1}(z_0)) \right)(z-z_0) - \frac{1}{8} \left(2\, i\, \pi \cos(\pi \nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor (\operatorname{bei}_{-\nu-2}(z_0) + \operatorname{bei}_{2-\nu}(z_0) - 2 \operatorname{bei}_{-\nu}(z_0)) - \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} (\operatorname{kei}_{\nu-2}(z_0) - 2 \operatorname{kei}_\nu(z_0) + \operatorname{kei}_{\nu+2}(z_0)) \right)(z-z_0)^2 + \ldots \,/;\, (z \to z_0)$$

03.20.06.0004.01

$$\operatorname{ker}_\nu(z) = \sum_{k=0}^{\infty} \frac{\operatorname{ker}_\nu^{(0,k)}(z_0)\,(z-z_0)^k}{k!} \,/;\, |\arg(z_0)| < \pi$$

03.20.06.0005.01

$$\operatorname{ker}_\nu(z) = \frac{1}{4} \sum_{k=0}^{\infty} \frac{1}{k!} G_{5,9}^{4,4}\left(\frac{z_0}{4}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{k}{4}, \frac{1-k}{4}, \frac{2-k}{4}, \frac{3-k}{4}, \frac{2-k+2\nu}{4} \\ \frac{2-k+\nu}{4}, \frac{\nu-k}{4}, \frac{2-k-\nu}{4}, -\frac{k+\nu}{4}, \frac{2-k+2\nu}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix} \right) (z-z_0)^k \,/;\, |\arg(z_0)| < \pi$$

03.20.06.0006.01

$$\operatorname{ker}_\nu(z) = \frac{\pi^{3/2}}{4} \sum_{k=0}^{\infty} \frac{2^k z_0^{-k}}{k!} \left(2^{2\nu} z_0^{-\nu} \csc(\pi\nu)\,\Gamma(1-\nu) \left(\frac{1}{z_0}\right)^{-\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{-\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(e^{-\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1-k-\nu}{2}, \frac{2-k-\nu}{2}, 1-\nu; \frac{i z_0^2}{4}\right) + e^{\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1-k-\nu}{2}, \frac{2-k-\nu}{2}, 1-\nu; -\frac{i z_0^2}{4}\right)\right) - 2^{-2\nu} z_0^{\nu} (i + \cot(\pi\nu))\,\Gamma(\nu+1) \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(e^{-\frac{5 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-k+\nu}{2}, \frac{2-k+\nu}{2}, \nu+1; \frac{i z_0^2}{4}\right) + e^{-\frac{3 i \pi \nu}{4}} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-k+\nu}{2}, \frac{2-k+\nu}{2}, \nu+1; -\frac{i z_0^2}{4}\right)\right)\right) (z-z_0)^k \;/;\; \nu \notin \mathbb{Z}$$

03.20.06.0007.01

$$\operatorname{ker}_\nu(z) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{2^{-\frac{3k}{2}} (i-1)^k}{k!} \left(\sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \binom{k}{2j} \left(i\left(1-i^k\right) \left(\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{kei}_{4j-k+\nu}(z_0) - 2 i \pi (-1)^k \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{bei}_{-4j+k-\nu}(z_0)\right) + \left(1+i^k\right) \left(\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{ker}_{4j-k+\nu}(z_0) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{ber}_{-4j+k-\nu}(z_0)\right)\right) - \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{2j+1} \left(i\left(1-i^k\right) \left(\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{kei}_{4j-k+\nu+2}(z_0) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{bei}_{-4j+k-\nu-2}(z_0)\right) + \left(1+i^k\right) \left(\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{ker}_{4j-k+\nu+2}(z_0) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{ber}_{-4j+k-\nu-2}(z_0)\right)\right)\right) (z-z_0)^k$$

03.20.06.0008.01

$$\operatorname{ker}_\nu(z) = \sum_{k=0}^{\infty} \frac{z_0^{-k}}{k!} \sum_{m=0}^{k} (-1)^{k+m} \binom{k}{m} (-\nu)_{k-m} \sum_{i=0}^{m} \frac{(-1)^i\, 2^{2i-m}\, (-m)_{2(m-i)}\, (\nu)_i}{(m-i)!} \left(\frac{1}{4} z^2 \operatorname{kei}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{i-1}{2} \right\rfloor} \frac{\left((-1)^j\, (i-2j-1)!\right) \left(\frac{z}{2}\right)^{4j}}{(2j+1)!\, (i-4j-2)!\, (-i-\nu+1)_{2j+1}\, (\nu)_{2j+1}} + \frac{z\, (\operatorname{kei}_{\nu-1}(z) + \operatorname{ker}_{\nu-1}(z))}{2\sqrt{2}} \sum_{j=0}^{\left\lfloor \frac{i-1}{2} \right\rfloor} \frac{\left((-1)^j\, (i-2j-1)!\right) \left(\frac{z}{2}\right)^{4j}}{(2j)!\, (i-4j-1)!\, (-i-\nu+1)_{2j}\, (\nu)_{2j+1}} + \frac{z^3\, (\operatorname{kei}_{\nu-1}(z) - \operatorname{ker}_{\nu-1}(z))}{8\sqrt{2}} \sum_{j=0}^{\left\lfloor \frac{i-2}{2} \right\rfloor} \frac{\left((-1)^j\, (i-2j-2)!\right) \left(\frac{z}{2}\right)^{4j}}{(2j+1)!\, (i-4j-3)!\, (-i-\nu+1)_{2j+1}\, (\nu)_{2j+2}} + \operatorname{ker}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{i}{2} \right\rfloor} \frac{\left((-1)^j\, (i-2j)!\right) \left(\frac{z}{2}\right)^{4j}}{(2j)!\, (i-4j)!\, (-i-\nu+1)_{2j}\, (\nu)_{2j}} \right) (z-z_0)^k \;/;\; |\arg(z_0)| < \pi$$

03.20.06.0009.01

$$\operatorname{ker}_\nu(z) \propto \left(\operatorname{ker}_\nu(z_0) \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} - 2\, i\, \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0)+\pi}{2\pi} \right\rfloor \operatorname{ber}_{-\nu}(z_0) \right) (1 + O(z-z_0))$$

Expansions on branch cuts

03.20.06.0010.01

$$\operatorname{ker}_\nu(z) \propto \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{ker}_\nu(x) - 2\, i\, \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{ber}_{-\nu}(x) \right) + \frac{1}{2\sqrt{2}} \left(2\, i\, \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor (\operatorname{bei}_{-\nu-1}(x) - \operatorname{bei}_{1-\nu}(x) + \operatorname{ber}_{-\nu-1}(x) - \operatorname{ber}_{1-\nu}(x)) - e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (\operatorname{kei}_{\nu-1}(x) - \operatorname{kei}_{\nu+1}(x) + \operatorname{ker}_{\nu-1}(x) - \operatorname{ker}_{\nu+1}(x)) \right) (z-x) - \frac{1}{8} \left(2\, i\, \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor (\operatorname{bei}_{-\nu-2}(x) + \operatorname{bei}_{2-\nu}(x) - 2 \operatorname{bei}_{-\nu}(x)) - e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (\operatorname{kei}_{\nu-2}(x) - 2 \operatorname{kei}_\nu(x) + \operatorname{kei}_{\nu+2}(x)) \right) (z-x)^2 + \ldots \;/;\; (z \to x) \land x \in \mathbb{R} \land x < 0$$

03.20.06.0011.01

$$\operatorname{ker}_\nu(z) = \frac{\pi^{3/2}}{4} \sum_{k=0}^{\infty} \frac{2^k x^{-k}}{k!} \Bigg(2^{2\nu} x^{-\nu} \csc(\pi\nu)\,\Gamma(1-\nu)\, e^{-2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \Bigg(e^{-\frac{3 i \pi \nu}{4}}\, {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1}{2}(-k-\nu+1), \frac{1}{2}(-k-\nu+2), 1-\nu; \frac{i x^2}{4}\right) + e^{\frac{3 i \pi \nu}{4}}\, {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; \frac{1}{2}(-k-\nu+1), \frac{1}{2}(-k-\nu+2), 1-\nu; -\frac{i x^2}{4}\right)\Bigg) - 2^{-2\nu} x^{\nu} (i + \cot(\pi\nu))\,\Gamma(\nu+1)\, e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \Bigg(e^{-\frac{5 i \pi \nu}{4}}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-k+\nu+1), \frac{1}{2}(-k+\nu+2), \nu+1; \frac{i x^2}{4}\right) + e^{-\frac{3 i \pi \nu}{4}}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-k+\nu+1), \frac{1}{2}(-k+\nu+2), \nu+1; -\frac{i x^2}{4}\right)\Bigg)\Bigg)(z-x)^k \;/;\; \nu \notin \mathbb{Z} \bigwedge x \in \mathbb{R} \bigwedge x < 0$$

03.20.06.0012.01

$$\operatorname{ker}_\nu(z) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{2^{-\frac{3k}{2}} (i-1)^k}{k!} \Bigg(\sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \binom{k}{2j} \Bigg(i\left(1-i^k\right) \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{kei}_{4j-k+\nu}(x) - 2 i \pi (-1)^k \cos(\pi\nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{bei}_{-4j+k-\nu}(x)\right) + \left(1+i^k\right) \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{ker}_{4j-k+\nu}(x) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{ber}_{-4j+k-\nu}(x)\right)\Bigg) - \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{2j+1} \Bigg(i\left(1-i^k\right) \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{kei}_{4j-k+\nu+2}(x) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{bei}_{-4j+k-\nu-2}(x)\right) + \left(1+i^k\right) \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{ker}_{4j-k+\nu+2}(x) - (-1)^k 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{ber}_{-4j+k-\nu-2}(x)\right)\Bigg)\Bigg)(z-x)^k \;/;\; x \in \mathbb{R} \land x < 0$$

03.20.06.0013.01

$$\operatorname{ker}_\nu(z) \propto \left(e^{2 i \pi \nu \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{ker}_\nu(x) - 2 i \pi \cos(\pi\nu) \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor \operatorname{ber}_{-\nu}(x)\right)(1 + O(z-x)) \;/;\; x \in \mathbb{R} \land x < 0$$

Expansions at $z == 0$

For the function itself

General case

03.20.06.0014.01

$$\operatorname{ker}_\nu(z) \propto -2^{\nu-3}\,\Gamma(\nu-1)\sin\left(\frac{3\pi\nu}{4}\right)z^{2-\nu}\left(1-\frac{z^4}{96(\nu-3)(\nu-2)}+\frac{z^8}{30\,720(\nu-5)(\nu-4)(\nu-3)(\nu-2)}+\ldots\right)+$$
$$2^{\nu-1}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu)\,z^{-\nu}\left(1-\frac{z^4}{32(\nu-2)(\nu-1)}+\frac{z^8}{6144(\nu-4)(\nu-3)(\nu-2)(\nu-1)}+\ldots\right)+$$
$$2^{-\nu-1}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\,z^{\nu}\left(1-\frac{z^4}{32(\nu+1)(\nu+2)}+\frac{z^8}{6144(\nu+1)(\nu+2)(\nu+3)(\nu+4)}+\ldots\right)-$$
$$2^{-\nu-3}\,\Gamma(-\nu-1)\sin\left(\frac{\pi\nu}{4}\right)z^{\nu+2}\left(1-\frac{z^4}{96(\nu+2)(\nu+3)}+\frac{z^8}{30\,720(\nu+2)(\nu+3)(\nu+4)(\nu+5)}+\ldots\right)\;/;\;(z\to 0)\bigwedge \nu\notin\mathbb{Z}$$

03.20.06.0015.01

$$\operatorname{ker}_\nu(z) = z^{\nu}\,\frac{\Gamma(-\nu)}{2^{\nu+1}}\sum_{k=0}^{\infty}\frac{1}{(\nu+1)_k\,k!}\cos\left(\frac{\pi}{4}(\nu-2k)\right)\left(\frac{z}{2}\right)^{2k}+z^{-\nu}\,\frac{\Gamma(\nu)}{2^{-\nu+1}}\sum_{k=0}^{\infty}\frac{1}{(1-\nu)_k\,k!}\cos\left(\frac{\pi}{4}(3\nu-2k)\right)\left(\frac{z}{2}\right)^{2k}\;/;\;\nu\notin\mathbb{Z}$$

03.20.06.0016.01

$$\operatorname{ker}_\nu(z) = -2^{\nu-3}\,\Gamma(\nu-1)\sin\left(\frac{3\pi\nu}{4}\right)z^{2-\nu}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(1-\frac{\nu}{2}\right)_k\left(\frac{3-\nu}{2}\right)_k\left(\frac{3}{2}\right)_k\,k!}+2^{\nu-1}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu)\,z^{-\nu}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(\frac{1-\nu}{2}\right)_k\left(1-\frac{\nu}{2}\right)_k\left(\frac{1}{2}\right)_k\,k!}+$$
$$2^{-\nu-1}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\;z^{\nu}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu+1}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(\frac{1}{2}\right)_k\,k!}-2^{-\nu-3}\,\Gamma(-\nu-1)\sin\left(\frac{\pi\nu}{4}\right)z^{\nu+2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu}{2}+1\right)_k\left(\frac{\nu+3}{2}\right)_k\left(\frac{3}{2}\right)_k\,k!}\;/;\;\nu\notin\mathbb{Z}$$

03.20.06.0017.01

$$\operatorname{ker}_\nu(z) = -2^{\nu-3}\,\Gamma(\nu-1)\sin\left(\frac{3\pi\nu}{4}\right)z^{2-\nu}\,{}_0F_3\left(;\frac{3}{2},1-\frac{\nu}{2},\frac{3-\nu}{2};-\frac{z^4}{256}\right)+$$
$$2^{\nu-1}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu)\,z^{-\nu}\,{}_0F_3\left(;\frac{1}{2},\frac{1-\nu}{2},1-\frac{\nu}{2};-\frac{z^4}{256}\right)+2^{-\nu-1}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\,z^{\nu}\,{}_0F_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)-$$
$$2^{-\nu-3}\,\Gamma(-\nu-1)\sin\left(\frac{\pi\nu}{4}\right)z^{\nu+2}\,{}_0F_3\left(;\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu+3}{2};-\frac{z^4}{256}\right)\;/;\;\nu\notin\mathbb{Z}$$

03.20.06.0018.01

$$\operatorname{ker}_\nu(z) = 2^{2\nu-5}\,\pi^2\csc(\pi\nu)\sin\left(\frac{3\pi\nu}{4}\right)z^{2-\nu}\,{}_0\tilde{F}_3\left(;\frac{3}{2},1-\frac{\nu}{2},\frac{3-\nu}{2};-\frac{z^4}{256}\right)+$$
$$2^{2\nu-1}\,\pi^2\cos\left(\frac{3\pi\nu}{4}\right)\csc(\pi\nu)\,z^{-\nu}\,{}_0\tilde{F}_3\left(;\frac{1}{2},\frac{1-\nu}{2},1-\frac{\nu}{2};-\frac{z^4}{256}\right)-2^{-2\nu-1}\,\pi^2\cos\left(\frac{\pi\nu}{4}\right)\csc(\pi\nu)\,z^{\nu}$$
$${}_0\tilde{F}_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)-2^{-2\nu-5}\,\pi^2\csc(\pi\nu)\sin\left(\frac{\pi\nu}{4}\right)z^{\nu+2}\,{}_0\tilde{F}_3\left(;\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu+3}{2};-\frac{z^4}{256}\right)\;/;\;\nu\notin\mathbb{Z}$$

03.20.06.0019.01

$$\operatorname{ker}_\nu(z) \propto 2^{\nu-1}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu)\,z^{-\nu}\left(1+O\left(z^2\right)\right)+2^{-\nu-1}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\,z^{\nu}\left(1+O\left(z^2\right)\right)\;/;\;\nu\notin\mathbb{Z}$$

03.20.06.0020.01

$$\operatorname{ker}_\nu(z) \propto \begin{cases} -\log(z) & \nu = 0 \\ (-1)^{\frac{|\nu|}{4}} 2^{|\nu|-1} z^{-|\nu|} (|\nu|-1)! & \frac{\nu}{4} \in \mathbb{Z} \\ (-1)^{\theta\left(\frac{\nu-1}{4}\right)} (-1)^{\frac{\nu-1}{4}} 2^{|\nu|-\frac{3}{2}} z^{-|\nu|} (|\nu|-1)! & \frac{\nu-1}{4} \in \mathbb{Z} \\ (-1)^{\theta\left(\frac{\nu+2}{4}\right)} (-1)^{\frac{\nu+2}{4}} 2^{|\nu|-3} z^{2-|\nu|} (|\nu|-2)! & \frac{\nu-2}{4} \in \mathbb{Z} \\ -(-1)^{\theta\left(\frac{\nu-3}{4}\right)} (-1)^{\frac{\nu-3}{4}} 2^{|\nu|-\frac{3}{2}} z^{-|\nu|} (|\nu|-1)! & \frac{\nu-3}{4} \in \mathbb{Z} \\ 2^{\nu-1} \cos\left(\frac{3\pi\nu}{4}\right) \Gamma(\nu) z^{-\nu} + 2^{-\nu-1} \cos\left(\frac{\pi\nu}{4}\right) \Gamma(-\nu) z^{\nu} & \text{True} \end{cases} \; /; (z \to 0)$$

03.20.06.0021.01

$$\operatorname{ker}_\nu(z) = F_\infty(z, \nu) \; /;$$
$$\Bigg(\Bigg(F_n(z,\nu) = \frac{z^\nu \Gamma(-\nu)}{2^{\nu+1}} \sum_{k=0}^{n} \frac{\cos\left(\frac{1}{4}\pi(\nu-2k)\right)}{(\nu+1)_k\, k!} \left(\frac{z}{2}\right)^{2k} + \frac{z^{-\nu}\Gamma(\nu)}{2^{1-\nu}} \sum_{k=0}^{n} \frac{\cos\left(\frac{1}{4}\pi(3\nu-2k)\right)}{(1-\nu)_k\, k!} \left(\frac{z}{2}\right)^{2k} = \operatorname{ker}_\nu(z) - i(-i)^n 2^{-2n-\nu-4}$$
$$e^{-\frac{3i\pi\nu}{4}} \pi z^{2n-\nu+2} \csc(\pi\nu) \left(e^{\frac{i\pi\nu}{2}} z^{2\nu} \left(e^{\frac{i\pi\nu}{2}} {}_1\tilde{F}_2\left(1; n+2, n+\nu+2; -\frac{i z^2}{4}\right) - (-1)^n {}_1\tilde{F}_2\left(1; n+2, n+\nu+2; \frac{i z^2}{4}\right)\right) + \right.$$
$$\left.(-1)^n 4^\nu {}_1\tilde{F}_2\left(1; n+2, n-\nu+2; \frac{i z^2}{4}\right) - 4^\nu e^{\frac{3i\pi\nu}{2}} {}_1\tilde{F}_2\left(1; n+2, n-\nu+2; -\frac{i z^2}{4}\right)\right)\Bigg) \bigwedge n \in \mathbb{N}\Bigg)$$

Summed form of the truncated series expansion.

Logarithmic cases

03.20.06.0022.01

$$\operatorname{ker}_0(z) = \frac{\pi z^2}{16}\left(1 - \frac{z^4}{576} + \frac{z^8}{3\,686\,400} + \ldots\right) +$$
$$\frac{1}{4}\left(-4\left(\log\left(\frac{z}{2}\right) + \gamma\right) + \frac{1}{32}\left(2\log\left(\frac{z}{2}\right) + 2\gamma - 3\right) z^4 - \frac{\left(2\log\left(\frac{z}{2}\right) - \frac{25}{6} + 2\gamma\right)}{73\,728} z^8 + \ldots\right) /; (z \to 0)$$

03.20.06.0023.01

$$\operatorname{ker}_1(z) \propto -\frac{1}{\sqrt{2}\, z} + \frac{\pi z}{8\sqrt{2}}\left(1 - \frac{z^4}{192} + \frac{z^8}{737\,280} + \ldots\right) - \frac{\pi z^3}{64\sqrt{2}}\left(1 - \frac{z^4}{1152} + \frac{z^8}{11\,059\,200} + \ldots\right) +$$
$$\frac{z}{8}\left(\sqrt{2}\left(2\log\left(\frac{z}{2}\right) + 2\gamma - 1\right) + \frac{2\log\left(\frac{z}{2}\right) - \frac{5}{2} + 2\gamma}{4\sqrt{2}} z^2 - \frac{2\log\left(\frac{z}{2}\right) - \frac{10}{3} + 2\gamma}{96\sqrt{2}} z^4 + \ldots\right) /; (z \to 0)$$

03.20.06.0024.01

$$\operatorname{ker}_2(z) \propto \frac{1}{2} - \frac{\pi z^2}{32}\left(1 - \frac{z^4}{384} + \frac{z^8}{2\,211\,840} + \ldots\right) -$$
$$\frac{z^4}{16}\left(\frac{1}{12}\left(2\log\left(\frac{z}{2}\right) - \frac{17}{6} + 2\gamma\right) - \frac{2\log\left(\frac{z}{2}\right) - \frac{247}{60} + 2\gamma}{23\,040} z^4 + \frac{2\log\left(\frac{z}{2}\right) - \frac{512}{105} + 2\gamma}{309\,657\,600} z^8 + \ldots\right) /; (z \to 0)$$

03.20.06.0025.01

$$\operatorname{ker}_n(z) \propto \frac{1}{4}\left(\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{\left(e^{\frac{3i\pi n}{4}}+(-1)^k e^{-\frac{3i\pi n}{4}}\right)(n-k-1)!}{k!}\left(\frac{i z^2}{4}\right)^k - 2^{-n-2}(-1)^n z^n\left(\frac{2\cos\left(\frac{n\pi}{4}\right)}{n!}\left(2\log\left(\frac{z}{2}\right)-\psi(n+1)+\gamma\right)+\frac{\sin\left(\frac{n\pi}{4}\right)}{2(n+1)!}\left(2\log\left(\frac{z}{2}\right)-\psi(n+2)+\gamma-1\right)z^2-\frac{\cos\left(\frac{n\pi}{4}\right)}{32(n+2)!}\left(2\log\left(\frac{z}{2}\right)-\psi(n+3)-\frac{3}{2}+\gamma\right)z^4+\ldots\right)+\frac{2^{-n-2}\pi z^n\sin\left(\frac{3n\pi}{4}\right)}{n!}\left(1-\frac{z^4}{32(n+1)(n+2)}+\frac{z^8}{6144(n+1)(n+2)(n+3)(n+4)}+\ldots\right)+\frac{2^{-n-4}\pi z^{n+2}\cos\left(\frac{3n\pi}{4}\right)}{(n+1)!}\left(1-\frac{z^4}{96(n+2)(n+3)}+\frac{z^8}{30\,720(n+2)(n+3)(n+4)(n+5)}+\ldots\right)\;/;\;(z\to 0)\land n\in\mathbb{N}$$

03.20.06.0026.01

$$\operatorname{ker}_n(z) = \frac{1}{4}\left(\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{\left(e^{\frac{3i\pi n}{4}}+(-1)^k e^{-\frac{3i\pi n}{4}}\right)(n-k-1)!}{k!}\left(\frac{i z^2}{4}\right)^k+\frac{\pi}{4}\sum_{k=0}^{\infty}\frac{\sin\left(\frac{1}{4}\pi(2k+3n)\right)}{k!(k+n)!}\left(\frac{z}{2}\right)^{2k+n} - 2^{-n-2}(-1)^n z^n\sum_{k=0}^{\infty}\frac{\left(e^{-\frac{i\pi n}{4}}+(-1)^k e^{\frac{i\pi n}{4}}\right)\left(2\log\left(\frac{z}{2}\right)-\psi(k+1)-\psi(k+n+1)\right)}{k!(k+n)!}\left(\frac{i z^2}{4}\right)^k$$

03.20.06.0027.01

$$\operatorname{ker}_\nu(z) = \frac{1}{4}\left(\frac{z}{2}\right)^{-|\nu|}\sum_{k=0}^{|\nu|-1}\frac{\left(e^{\frac{1}{4}i\pi(2\nu+|\nu|)}+(-1)^k e^{-\frac{1}{4}(i\pi(2\nu+|\nu|))}\right)(|\nu|-k-1)!}{k!}\left(\frac{i z^2}{4}\right)^k+\frac{\pi}{4}\sum_{k=0}^{\infty}\frac{\sin\left(\frac{1}{4}\pi(2(k+\nu)+|\nu|)\right)}{k!(k+|\nu|)!}\left(\frac{z}{2}\right)^{2k+|\nu|} - \frac{1}{4}\left(\frac{i z}{2}\right)^{|\nu|} e^{\frac{i\pi\nu}{2}}\sum_{k=0}^{\infty}\frac{\left(e^{-\frac{1}{4}(i\pi|\nu|)}+(-1)^k e^{\frac{1}{4}i\pi|\nu|}\right)\left(2\log\left(\frac{z}{2}\right)-\psi(k+1)-\psi(k+|\nu|+1)\right)}{k!(k+|\nu|)!}\left(\frac{i z^2}{4}\right)^k\;/;\;\nu\in\mathbb{Z}$$

03.20.06.0028.01

$$\operatorname{ker}_n(z) = \frac{1}{8}\Bigg(-i^{n+1}\pi I_n\left(\sqrt[4]{-1}\,z\right)+(-1)^n\pi i J_n\left(\sqrt[4]{-1}\,z\right)+4(-i)^n K_n\left(\sqrt[4]{-1}\,z\right) - 2(-1)^n\pi Y_n\left(\sqrt[4]{-1}\,z\right)-4\left(i^n I_n\left(\sqrt[4]{-1}\,z\right)+(-1)^n J_n\left(\sqrt[4]{-1}\,z\right)\right)\left(\log(z)-\log\left(\sqrt[4]{-1}\,z\right)\right)+e^{\frac{3in\pi}{4}} n!\sum_{k=0}^{n-1}\frac{(-1)^{k/4}2^{-k+n+1}z^{k-n}}{(k-n)k!}\left((-1)^{k+\frac{n}{2}} I_k\left(\sqrt[4]{-1}\,z\right)+J_k\left(\sqrt[4]{-1}\,z\right)\right) - \frac{2^{1-n}(-1)^{n/4}z^n}{n!}\sum_{j=1}^{n}\frac{1}{j}\left((-1)^n\,{}_1F_2\left(j;\,j+1,\,n+1;\,-\frac{i z^2}{4}\right)+i^n\,{}_1F_2\left(j;\,j+1,\,n+1;\,\frac{i z^2}{4}\right)\right)+(-1)^{-\frac{n}{4}}\sum_{k=0}^{n-1}\frac{\left(2^{-2k+n+1}i^k z^{2k-n}(n-k-1)!\right)\left(i^n(-1)^{n-k}+(-1)^n\right)}{k!}\Bigg)\;/;\;n\in\mathbb{N}$$

03.20.06.0029.01

$$\operatorname{ker}_\nu(z) = \frac{2^{-|\nu|-4}\,\pi\,z^{|\nu|+2}}{\Gamma(|\nu|+2)}\cos\left(\frac{1}{4}\pi(2\nu+|\nu|)\right){}_0F_3\left(;\frac{3}{2},\frac{|\nu|}{2}+1,\frac{|\nu|}{2}+\frac{3}{2};-\frac{z^4}{256}\right)+\frac{2^{-|\nu|-2}\,\pi\,z^{|\nu|}}{\Gamma(|\nu|+1)}\sin\left(\frac{1}{4}\pi(2\nu+|\nu|)\right)$$
$${}_0F_3\left(;\frac{1}{2},\frac{|\nu|}{2}+\frac{1}{2},\frac{|\nu|}{2}+1;-\frac{z^4}{256}\right)+\frac{1}{4}\left(\frac{z}{2}\right)^{-|\nu|}\sum_{k=0}^{|\nu|-1}\frac{\left(\left(e^{\frac{1}{4}i\pi(2\nu+|\nu|)}+(-1)^k e^{-\frac{1}{4}(i\pi(2\nu+|\nu|))}\right)(|\nu|-k-1)!\right.}{k!}\left(\frac{i z^2}{4}\right)^k -$$
$$\frac{1}{4}\left(\frac{i z}{2}\right)^{|\nu|} e^{\frac{i\pi\nu}{2}}\sum_{k=0}^{\infty}\frac{\left(e^{-\frac{1}{4}(i\pi|\nu|)}+(-1)^k e^{\frac{1}{4}i\pi|\nu|}\right)\left(2\log\left(\frac{z}{2}\right)-\psi(k+1)-\psi(k+|\nu|+1)\right)}{k!\,(k+|\nu|)!}\left(\frac{i z^2}{4}\right)^k \;/;\; \nu\in\mathbb{Z}$$

03.20.06.0030.01

$$\operatorname{ker}_\nu(z) = 2^{-2(|\nu|+3)}\pi^2 z^{|\nu|+2}\cos\left(\frac{1}{4}\pi(2\nu+|\nu|)\right){}_0\tilde{F}_3\left(;\frac{3}{2},\frac{|\nu|}{2}+1,\frac{|\nu|}{2}+\frac{3}{2};-\frac{z^4}{256}\right)+2^{-2(|\nu|+1)}\pi^2 z^{|\nu|}\sin\left(\frac{1}{4}\pi(2\nu+|\nu|)\right)$$
$${}_0\tilde{F}_3\left(;\frac{1}{2},\frac{|\nu|}{2}+\frac{1}{2},\frac{|\nu|}{2}+1;-\frac{z^4}{256}\right)+\frac{1}{4}\left(\frac{z}{2}\right)^{-|\nu|}\sum_{k=0}^{|\nu|-1}\frac{\left(\left(e^{\frac{1}{4}i\pi(2\nu+|\nu|)}+(-1)^k e^{-\frac{1}{4}(i\pi(2\nu+|\nu|))}\right)(|\nu|-k-1)!\right.}{k!}\left(\frac{i z^2}{4}\right)^k -$$
$$\frac{1}{4}\left(\frac{i z}{2}\right)^{|\nu|} e^{\frac{i\pi\nu}{2}}\sum_{k=0}^{\infty}\frac{\left(e^{-\frac{1}{4}(i\pi|\nu|)}+(-1)^k e^{\frac{1}{4}i\pi|\nu|}\right)\left(2\log\left(\frac{z}{2}\right)-\psi(k+1)-\psi(k+|\nu|+1)\right)}{k!\,(k+|\nu|)!}\left(\frac{i z^2}{4}\right)^k \;/;\; \nu\in\mathbb{Z}$$

03.20.06.0031.01

$$\operatorname{ker}_n(z) = \frac{1}{4}\left(\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{\left(e^{\frac{3i\pi n}{4}}+(-1)^k e^{-\frac{3i\pi n}{4}}\right)(n-k-1)!}{k!}\left(\frac{i z^2}{4}\right)^k+\frac{\pi}{4}\sum_{k=0}^{\infty}\frac{\sin\left(\frac{1}{4}\pi(2k+3n)\right)}{k!\,(k+n)!}\left(\frac{z}{2}\right)^{2k+n}-$$
$$2^{-n-2}(-1)^n z^n\sum_{k=0}^{\infty}\frac{\left(e^{-\frac{i\pi n}{4}}+(-1)^k e^{\frac{i\pi n}{4}}\right)\left(2\log\left(\frac{z}{2}\right)-\psi(k+1)-\psi(k+n+1)\right)}{k!\,(k+n)!}\left(\frac{i z^2}{4}\right)^k$$

03.20.06.0032.01

$$\operatorname{ker}_\nu(z) = \frac{i\pi}{8}(-1)^{-\frac{|\nu|}{4}}\left(e^{-\frac{1}{4}i\pi(2\nu+|\nu|)}J_{|\nu|}\left(\sqrt[4]{-1}\,z\right)-e^{\frac{1}{4}i\pi(2\nu+|\nu|)}I_{|\nu|}\left(\sqrt[4]{-1}\,z\right)\right)+$$
$$\frac{1}{4}\left(\frac{z}{2}\right)^{-|\nu|}\sum_{k=0}^{|\nu|-1}\frac{\left(e^{\frac{1}{4}i\pi(2\nu+|\nu|)}+(-1)^k e^{-\frac{1}{4}(i\pi(2\nu+|\nu|))}\right)(|\nu|-k-1)!}{k!}\left(\frac{i z^2}{4}\right)^k -$$
$$\frac{1}{4}\left(\frac{i z}{2}\right)^{|\nu|} e^{\frac{i\pi\nu}{2}}\sum_{k=0}^{\infty}\frac{\left(e^{-\frac{1}{4}(i\pi|\nu|)}+(-1)^k e^{\frac{1}{4}i\pi|\nu|}\right)\left(2\log\left(\frac{z}{2}\right)-\psi(k+1)-\psi(k+|\nu|+1)\right)}{k!\,(k+|\nu|)!}\left(\frac{i z^2}{4}\right)^k \;/;\; \nu\in\mathbb{Z}$$

03.20.06.0033.01

$$\operatorname{ker}_0(z) \propto -\left(\log\left(\frac{z}{2}\right)+\gamma\right)\left(1+O\left(z^4\right)\right)+\frac{\pi z^2}{16}\left(1+O\left(z^4\right)\right)$$

03.20.06.0034.01

$$\operatorname{ker}_1(z) \propto -\frac{1}{\sqrt{2}\,z}\left(1+O\left(z^2\right)\right)+\frac{z\left(2\log\left(\frac{z}{2}\right)+2\gamma-1\right)}{4\sqrt{2}}\left(1+O\left(z^2\right)\right)$$

03.20.06.0035.01

$$\operatorname{ker}_2(z) \propto \frac{1}{2}\left(1+O\left(z^2\right)\right)-\frac{z^4 \log(z)}{96}\left(1+O\left(z^4\right)\right)$$

Asymptotic series expansions

Expansions inside Stokes sectors

Expansions containing $z \to \infty$

In trigonometric form ||| In trigonometric form

03.20.06.0036.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}\, e^{-\frac{z}{\sqrt{2}}}}{\sqrt{2z}}\left(\cos\left(\frac{1}{8}\left(4\sqrt{2}\, z+\pi(4\nu+1)\right)\right)-\frac{1-4\nu^2}{8z}\sin\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\, z\right)\right)+\frac{16\nu^4-40\nu^2+9}{128z^2}\sin\left(\frac{1}{8}\left(-4\sqrt{2}\, z-\pi(4\nu+1)\right)\right)+\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\cos\left(\frac{1}{8}\left(4\sqrt{2}\, z-\pi(1-4\nu)\right)\right)+\ldots\right) /; (|z|\to\infty)$$

03.20.06.0037.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}\, e^{-\frac{z}{\sqrt{2}}}}{\sqrt{2z}}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{1}{4z^2}\right)^k\cos\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\, z+\pi(4\nu+1)\right)\right)-\frac{1}{2z}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{1}{4z^2}\right)^k\sin\left(\frac{\pi k}{2}+\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\, z\right)\right)+\ldots\right) /; (|z|\to\infty)\land n\in\mathbb{N}$$

03.20.06.0038.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}\, e^{-\frac{z}{\sqrt{2}}}}{\sqrt{2z}}\left(\cos\left(\frac{1}{8}\left(4\sqrt{2}\, z+\pi(4\nu+1)\right)\right){}_8F_3\left(\frac{1}{8}(1-2\nu),\frac{1}{8}(3-2\nu),\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(2\nu+1),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7);\frac{1}{4},\frac{1}{2},\frac{3}{4};-\frac{16}{z^4}\right)-\frac{1-4\nu^2}{8z}\sin\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\, z\right)\right){}_8F_3\left(\frac{1}{8}(3-2\nu),\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9);\frac{1}{2},\frac{3}{4},\frac{5}{4};-\frac{16}{z^4}\right)-\frac{16\nu^4-40\nu^2+9}{128z^2}\sin\left(\frac{1}{8}\left(4\sqrt{2}\, z+\pi(4\nu+1)\right)\right){}_8F_3\left(\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11);\frac{3}{4},\frac{5}{4},\frac{3}{2};-\frac{16}{z^4}\right)+\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\cos\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\, z\right)\right){}_8F_3\left(\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(13-2\nu),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11),\frac{1}{8}(2\nu+13);\frac{5}{4},\frac{3}{2},\frac{7}{4};-\frac{16}{z^4}\right)\right) /; (|z|\to\infty)$$

03.20.06.0039.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}\, e^{-\frac{z}{\sqrt{2}}}}{\sqrt{2z}} \left(\cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right)\left(1 + O\left(\frac{1}{z^4}\right)\right) - \frac{1-4\nu^2}{8z}\sin\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right)\left(1 + O\left(\frac{1}{z^4}\right)\right) - \frac{16\nu^4 - 40\nu^2 + 9}{128 z^2}\sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right)\left(1 + O\left(\frac{1}{z^4}\right)\right) + \frac{-64\nu^6 + 560\nu^4 - 1036\nu^2 + 225}{3072 z^3}\cos\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right)\left(1 + O\left(\frac{1}{z^4}\right)\right)\right) /;\ (|z| \to \infty)$$

Expansions containing $z \to -\infty$

In trigonometric form ||| In trigonometric form

03.20.06.0040.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}}{\sqrt{2}\sqrt{-z}} \left(2 e^{\frac{z}{\sqrt{2}}}\cos(\pi\nu)\cos\left(\frac{1}{8}\left(-4\sqrt{2}\, z + 4\pi\nu + \pi\right)\right) + e^{-\frac{z}{\sqrt{2}}}\, i \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu-3)\right)\right) + \frac{1-4\nu^2}{8z}\left(2 e^{\frac{z}{\sqrt{2}}}\cos(\pi\nu)\cos\left(\frac{1}{8}\left(\pi(4\nu+3) - 4\sqrt{2}\, z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu-1)\right)\right)\right) + \frac{16\nu^4 - 40\nu^2 + 9}{128 z^2}\left(e^{-\frac{z}{\sqrt{2}}}\, i\cos\left(\frac{1}{8}\left(-4\sqrt{2}\, z - \pi(4\nu-3)\right)\right) + 2 e^{\frac{z}{\sqrt{2}}}\cos(\pi\nu)\sin\left(\frac{1}{8}\left(4\sqrt{2}\, z - 4\pi\nu - \pi\right)\right)\right) + \frac{-64\nu^6 + 560\nu^4 - 1036\nu^2 + 225}{3072 z^3}\left(2 e^{\frac{z}{\sqrt{2}}}\cos(\pi\nu)\sin\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi(4\nu+3)\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(-4\sqrt{2}\, z - \pi(4\nu-1)\right)\right)\right) + \ldots\right) /;\ (z \to -\infty)$$

03.20.06.0041.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}}{\sqrt{2}\sqrt{-z}} \left(\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}\left(\frac{1}{4z^2}\right)^k}{(2k)!} \left(2 e^{\frac{z}{\sqrt{2}}}\cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(-4\sqrt{2}\, z + 4\pi\nu + \pi\right)\right)\cos(\pi\nu) + e^{-\frac{z}{\sqrt{2}}}\, i\sin\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu-3)\right)\right)\right) + \frac{1}{2z}\sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}\left(\frac{1}{4z^2}\right)^k}{(2k+1)!} \left(2\cos(\pi\nu)\, e^{\frac{z}{\sqrt{2}}}\cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(\pi(4\nu+3) - 4\sqrt{2}\, z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu-1)\right)\right)\right) + \ldots\right) /;\ (z \to -\infty) \land n \in \mathbb{N}$$

03.20.06.0042.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}}{\sqrt{2}\,\sqrt{-z}} \Bigg(\Bigg(2\,e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(-4\sqrt{2}\,z+4\pi\nu+\pi\big)\Big)\cos(\pi\nu)+e^{-\frac{z}{\sqrt{2}}}\,i\sin\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu-3)\big)\Big)\Bigg)\,{}_8F_3\Big(\frac{1}{8}(1-2\nu),\frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(2\nu+1),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7);\frac{1}{4},\frac{1}{2},\frac{3}{4};-\frac{16}{z^4}\Big)+ \frac{1-4\nu^2}{8z}\Bigg(2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(\pi(4\nu+3)-4\sqrt{2}\,z\big)\Big)-i\,e^{-\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu-1)\big)\Big)\Bigg)\,{}_8F_3\Big(\frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9);\frac{1}{2},\frac{3}{4},\frac{5}{4};-\frac{16}{z^4}\Big)+ \frac{16\nu^4-40\nu^2+9}{128z^2}\Bigg(i\,e^{-\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu-3)\big)\Big)-2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\big(-4\sqrt{2}\,z+4\pi\nu+\pi\big)\Big)\Bigg) {}_8F_3\Big(\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11);\frac{3}{4},\frac{5}{4},\frac{3}{2};-\frac{16}{z^4}\Big)+\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3} \Bigg(e^{-\frac{z}{\sqrt{2}}}(-i)\cos\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu-1)\big)\Big)-2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\big(\pi(4\nu+3)-4\sqrt{2}\,z\big)\Big)\Bigg) {}_8F_3\Big(\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(13-2\nu),\frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11),\frac{1}{8}(2\nu+13);\frac{5}{4},\frac{3}{2},\frac{7}{4};-\frac{16}{z^4}\Big)+\ldots\Bigg)\;/;\;(z\to-\infty)$$

03.20.06.0043.01

$$\operatorname{ker}_\nu(z) \propto \frac{\sqrt{\pi}}{\sqrt{2}\,\sqrt{-z}} \Bigg(\Bigg(2\,e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(-4\sqrt{2}\,z+4\pi\nu+\pi\big)\Big)\cos(\pi\nu)+e^{-\frac{z}{\sqrt{2}}}\,i\sin\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu-3)\big)\Big)\Bigg)\Big(1+O\Big(\frac{1}{z^4}\Big)\Big)+ \frac{1-4\nu^2}{8z}\Bigg(2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(\pi(4\nu+3)-4\sqrt{2}\,z\big)\Big)-i\,e^{-\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu-1)\big)\Big)\Bigg)\Big(1+O\Big(\frac{1}{z^4}\Big)\Big)+ \frac{16\nu^4-40\nu^2+9}{128z^2}\Bigg(i\,e^{-\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu-3)\big)\Big)-2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\big(-4\sqrt{2}\,z+4\pi\nu+\pi\big)\Big)\Bigg) \Big(1+O\Big(\frac{1}{z^4}\Big)\Big)+\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\Bigg(e^{-\frac{z}{\sqrt{2}}}(-i)\cos\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu-1)\big)\Big)- 2\cos(\pi\nu)\,e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\big(\pi(4\nu+3)-4\sqrt{2}\,z\big)\Big)\Bigg)\Big(1+O\Big(\frac{1}{z^4}\Big)\Big)+\ldots\Bigg)\;/;\;(z\to-\infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-free arguments

General case

03.20.06.0044.01

$$\operatorname{ker}_{\nu}(z) \propto$$

$$-\frac{\sqrt{\pi}\,\csc(\pi\nu)}{4\sqrt{2}}\Bigg(e^{\frac{z}{\sqrt{2}}}\Big(z^{\nu}\Big(e^{\frac{3i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\Big(\frac{\sqrt{-iz^2}\,\cos(\pi\nu)}{z}-\sqrt[4]{-1}\,\sin(\pi\nu)\Big)\big(-\sqrt[4]{-1}\,z\big)^{-\nu-\frac{1}{2}}+\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\big((-1)^{3/4}z\big)^{-\nu-\frac{1}{2}}\Big)-$$

$$z^{-\nu}\Big(\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\big((-1)^{3/4}z\big)^{\nu-\frac{1}{2}}-e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\big(-\sqrt[4]{-1}\,z\big)^{\nu-\frac{1}{2}}\Big(-\frac{\sqrt{-iz^2}\,\cos(\pi\nu)}{z}-\sqrt[4]{-1}\,\sin(\pi\nu)\Big)\Big)\Big)+$$

$$e^{-\frac{z}{\sqrt{2}}}\Big(z^{\nu}\Big(\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{4}}\big(-\sqrt[4]{-1}\,z\big)^{-\nu-\frac{1}{2}}+e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\big((-1)^{3/4}z\big)^{-\nu-\frac{1}{2}}\Big(\frac{i\sqrt{iz^2}\,\cos(\pi\nu)}{z}-\sqrt[4]{-1}\,\sin(\pi\nu)\Big)\Big)-$$

$$z^{-\nu}\Big(\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\big(-\sqrt[4]{-1}\,z\big)^{\nu-\frac{1}{2}}+e^{-\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\big((-1)^{3/4}z\big)^{\nu-\frac{1}{2}}\Big(\frac{i\sqrt{iz^2}\,\cos(\pi\nu)}{z}+\sqrt[4]{-1}\,\sin(\pi\nu)\Big)\Big)\Big)\Bigg)+$$

$$\frac{1-4\nu^2}{8z}\Bigg(e^{\frac{z}{\sqrt{2}}}\Big(z^{-\nu}\Big(i\,e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\big(-\sqrt[4]{-1}\,z\big)^{\nu-\frac{1}{2}}\Big(\frac{(-1)^{3/4}\sqrt{-iz^2}\,\cos(\pi\nu)}{z}-\sin(\pi\nu)\Big)-e^{\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\big((-1)^{3/4}z\big)^{\nu-\frac{1}{2}}\Big)+$$

$$z^{\nu}\Big(e^{\frac{3i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\big(-\sqrt[4]{-1}\,z\big)^{-\nu-\frac{1}{2}}\Big(\frac{\sqrt[4]{-1}\,\big(\sqrt{-iz^2}\,\cos(\pi\nu)\big)}{z}-i\sin(\pi\nu)\Big)+e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\big((-1)^{3/4}z\big)^{-\nu-\frac{1}{2}}\Big)\Big)+$$

$$e^{-\frac{z}{\sqrt{2}}}\Big(z^{-\nu}\Big(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\,i\big(-\sqrt[4]{-1}\,z\big)^{\nu-\frac{1}{2}}+e^{-\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\big((-1)^{3/4}z\big)^{\nu-\frac{1}{2}}\Big(\frac{\sqrt[4]{-1}\,\sqrt{iz^2}\,\cos(\pi\nu)}{z}+\sin(\pi\nu)\Big)\Big)+$$

$$z^{\nu}\Big(-i\,e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{4}}\big(-\sqrt[4]{-1}\,z\big)^{-\nu-\frac{1}{2}}-e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\big((-1)^{3/4}z\big)^{-\nu-\frac{1}{2}}\Big(\frac{\sqrt[4]{-1}\,\sqrt{iz^2}\,\cos(\pi\nu)}{z}-\sin(\pi\nu)\Big)\Big)\Big)\Bigg)+$$

$$\frac{16\nu^4-40\nu^2+9}{128z^2}\Bigg(e^{\frac{z}{\sqrt{2}}}\Big(z^{-\nu}\Big((-1)^{3/4}e^{\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\big((-1)^{3/4}z\big)^{\nu-\frac{1}{2}}-e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\big(-\sqrt[4]{-1}\,z\big)^{\nu-\frac{1}{2}}$$

$$\Big(\frac{i\sqrt{-iz^2}\,\cos(\pi\nu)}{z}+(-1)^{3/4}\sin(\pi\nu)\Big)\Big)+$$

$$z^{\nu}\Big(e^{\frac{3i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\big(-\sqrt[4]{-1}\,z\big)^{-\nu-\frac{1}{2}}\Big(\frac{i\sqrt{-iz^2}\,\cos(\pi\nu)}{z}-(-1)^{3/4}\sin(\pi\nu)\Big)-(-1)^{3/4}e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\big((-1)^{3/4}z\big)^{-\nu-\frac{1}{2}}\Big)\Big)+$$

$$e^{-\frac{z}{\sqrt{2}}}\left(z^{\nu}\left((-1)^{3/4}\,e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{4}}\left(-\sqrt[4]{-1}\;z\right)^{-\nu-\frac{1}{2}}+e^{\frac{i \pi \nu}{4}-\frac{i z}{\sqrt{2}}}\left((-1)^{3/4}\,z\right)^{-\nu-\frac{1}{2}}\left(\frac{\sqrt{i z^{2}}\;\cos(\pi \nu)}{z}+(-1)^{3/4}\sin(\pi \nu)\right)\right)-\right.$$

$$\left.\left. z^{-\nu}\left((-1)^{3/4}\,e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{4}}\left(-\sqrt[4]{-1}\;z\right)^{\nu-\frac{1}{2}}+e^{-\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\left((-1)^{3/4}\,z\right)^{\nu-\frac{1}{2}}\left(\frac{\sqrt{i z^{2}}\;\cos(\pi \nu)}{z}-(-1)^{3/4}\sin(\pi \nu)\right)\right)\right)\right)-$$

$$\frac{64\,\nu^{6}-560\,\nu^{4}+1036\,\nu^{2}-225}{3072\,z^{3}}\left(e^{\frac{z}{\sqrt{2}}}\left(z^{-\nu}\left(i\,e^{\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\left((-1)^{3/4}\,z\right)^{\nu-\frac{1}{2}}-e^{\frac{i \pi \nu}{4}-\frac{i z}{\sqrt{2}}}\left(-\sqrt[4]{-1}\;z\right)^{\nu-\frac{1}{2}}\right.\right.\right.$$

$$\left.\left.\left(\frac{(-1)^{3/4}\sqrt{-i z^{2}}\;\cos(\pi \nu)}{z}-\sin(\pi \nu)\right)\right)+\right.$$

$$\left. z^{\nu}\left(e^{\frac{3 i \pi \nu}{4}-\frac{i z}{\sqrt{2}}}\left(-\sqrt[4]{-1}\;z\right)^{-\nu-\frac{1}{2}}\left(\frac{(-1)^{3/4}\sqrt{-i z^{2}}\;\cos(\pi \nu)}{z}+\sin(\pi \nu)\right)-i\,e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{4}}\left((-1)^{3/4}\,z\right)^{-\nu-\frac{1}{2}}\right)\right)+$$

$$e^{-\frac{z}{\sqrt{2}}}\left(z^{-\nu}\left(-e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{4}}\left(-\sqrt[4]{-1}\;z\right)^{\nu-\frac{1}{2}}-e^{-\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\left((-1)^{3/4}\,z\right)^{\nu-\frac{1}{2}}\left(\frac{(-1)^{3/4}\sqrt{i z^{2}}\;\cos(\pi \nu)}{z}+i\sin(\pi \nu)\right)\right)+\right.$$

$$z^{\nu}\left(e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{4}}\left(-\sqrt[4]{-1}\;z\right)^{-\nu-\frac{1}{2}}+e^{\frac{i \pi \nu}{4}-\frac{i z}{\sqrt{2}}}\left((-1)^{3/4}\,z\right)^{-\nu-\frac{1}{2}}\right.$$

$$\left.\left.\left.\left(\frac{(-1)^{3/4}\sqrt{i z^{2}}\;\cos(\pi \nu)}{z}-i\sin(\pi \nu)\right)\right)\right)\right)+\ldots\Bigg)\,/;\;(|z|\to\infty)\bigwedge \nu\notin\mathbb{Z}$$

03.20.06.0045.01

$\operatorname{ker}_{\nu}(z)\propto$

$$-\frac{\sqrt{\pi}\;\csc(\pi \nu)}{4\sqrt{2}}\left(z^{\nu}\left(e^{-\frac{z}{\sqrt{2}}}\left((-1)^{-3/4}\,e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{4}}\left(-\sqrt[4]{-1}\;z\right)^{\nu-\frac{1}{2}}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^{2}}\right)^{k}+O\!\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\right)+e^{-\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\right.\right.\right.$$

$$\left.\left.\left((-1)^{3/4}\,z\right)^{\nu-\frac{1}{2}}\left((-1)^{-3/4}\sin(\pi \nu)-\frac{i\sqrt{i z^{2}}\;\cos(\pi \nu)}{z}\right)\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^{2}}\right)^{k}+O\!\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\right)\right)\right)+$$

$$e^{\frac{z}{\sqrt{2}}}\left((-1)^{-3/4}\,e^{\frac{i z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}}\left((-1)^{3/4}\,z\right)^{\nu-\frac{1}{2}}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^{2}}\right)^{k}+O\!\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\right)-e^{\frac{i \pi \nu}{4}-\frac{i z}{\sqrt{2}}}\right.$$

$$\left.\left(-\sqrt[4]{-1}\;z\right)^{\nu-\frac{1}{2}}\left(\frac{\sqrt{-i z^{2}}\;\cos(\pi \nu)}{z}-(-1)^{-3/4}\sin(\pi \nu)\right)\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^{2}}\right)^{k}+O\!\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\right)\right)+$$

$$\frac{1}{2z}\Bigg(e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\, i\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}\Bigg(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\Bigg)+$$

$$e^{-\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\,\sqrt{i z^2}\,\cos(\pi\nu)}{z}+\sin(\pi\nu)\right)\Bigg(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}$$

$$\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\Bigg)\Bigg)-e^{\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(\frac{\sqrt[4]{-1}\,\sqrt{-i z^2}\,\cos(\pi\nu)}{z}+i\sin(\pi\nu)\right)$$

$$\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}\Bigg(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\Bigg)+$$

$$e^{\frac{iz}{\sqrt{2}}-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}\Bigg(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$z^\nu\Bigg(e^{-\frac{z}{\sqrt{2}}}\Bigg(\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\Bigg(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\Bigg)+e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}$$

$$\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\left(\frac{i\sqrt{i z^2}\,\cos(\pi\nu)}{z}-\sqrt[4]{-1}\,\sin(\pi\nu)\right)\Bigg(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\Bigg)\Bigg)+$$

$$e^{\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{3i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(\frac{\sqrt{-i z^2}\,\cos(\pi\nu)}{z}-\sqrt[4]{-1}\,\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\Bigg(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^2}\right)^k+$$

$$O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\Bigg)+\sqrt[4]{-1}\,e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\Bigg(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\Bigg)\Bigg)\Bigg)+$$

$$\frac{1}{2z}\Bigg(e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\left(\sin(\pi\nu)-\frac{\sqrt[4]{-1}\,\sqrt{i z^2}\,\cos(\pi\nu)}{z}\right)$$

$$\Bigg(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\Bigg)-$$

$$i\,e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\Bigg(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\Bigg)\Bigg)+$$

$$e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{-\nu-\frac{1}{2}}\left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right)-\right.$$

$$i\,e^{\frac{3i\pi\nu}{4}-\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\left(\frac{(-1)^{3/4}\sqrt{-iz^2}\cos(\pi\nu)}{z}+\sin(\pi\nu)\right)$$

$$\left.\left.\left.\left.\left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right)\right)\right)\right)\right)\ /;\ (|z|\to\infty)\bigwedge\nu\notin\mathbb{Z}\bigwedge n\in\mathbb{N}$$

03.20.06.0046.01

$$\mathrm{ker}_\nu(z)\propto-\frac{\sqrt{\pi}\csc(\pi\nu)}{4\sqrt{2}}\left(z^{-\nu}\left(e^{\frac{i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}\left((-1)^{-3/4}e^{(-1)^{3/4}z}-e^{-(-1)^{3/4}z}\left(\frac{\sqrt{-iz^2}\cos(\pi\nu)}{z}-(-1)^{-3/4}\sin(\pi\nu)\right)\right)\right.\right.$$

$$\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\right)+$$

$$e^{-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}\left(e^{-\sqrt[4]{-1}\,z}\left((-1)^{-3/4}\sin(\pi\nu)-\frac{i\sqrt{iz^2}\cos(\pi\nu)}{z}\right)+(-1)^{-3/4}e^{\sqrt[4]{-1}\,z}\right)$$

$$\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n}{2}\right\rfloor+2}}\right)\right)+$$

$$\frac{1}{2z}\left(e^{\frac{i\pi\nu}{4}}\left(-\sqrt[4]{-1}\,z\right)^{\nu-\frac{1}{2}}\left(i\,e^{(-1)^{3/4}z}-e^{-(-1)^{3/4}z}\left(\frac{\sqrt[4]{-1}\sqrt{-iz^2}\cos(\pi\nu)}{z}+i\sin(\pi\nu)\right)\right)\right.$$

$$\left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right)+$$

$$e^{-\frac{5i\pi\nu}{4}}\left((-1)^{3/4}z\right)^{\nu-\frac{1}{2}}\left(e^{-\sqrt[4]{-1}\,z}\left(\frac{\sqrt[4]{-1}\sqrt{iz^2}\cos(\pi\nu)}{z}+\sin(\pi\nu)\right)-e^{\sqrt[4]{-1}\,z}\right)$$

$$\left.\left.\left(\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\left\lfloor\frac{n-1}{2}\right\rfloor+2}}\right)\right)\right)\right)+$$

$$z^\nu\left(-e^{\frac{3i\pi\nu}{4}-(-1)^{3/4}z}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\left(\sqrt[4]{-1}\left(\sin(\pi\nu)-e^{2(-1)^{3/4}z}\right)-\frac{\sqrt{-iz^2}\cos(\pi\nu)}{z}\right)\right.$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k} \left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right)\right) + e^{\frac{i\pi\nu}{4}-\sqrt[4]{-1}\, z} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}$$

$$\left(\frac{i\sqrt{i z^2}\, \cos(\pi\nu)}{z} + \sqrt[4]{-1} \left(e^{2\sqrt[4]{-1}\, z} - \sin(\pi\nu)\right)\right)\left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k} \left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right)\right) -$$

$$\frac{1}{2z}\left(i\, e^{\frac{3i\pi\nu}{4}-(-1)^{3/4} z} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(\frac{(-1)^{3/4}\sqrt{-i z^2}\, \cos(\pi\nu)}{z} + e^{2(-1)^{3/4} z} + \sin(\pi\nu)\right)\right.$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{2^{-2k} \left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right)\right) +$$

$$e^{\frac{i\pi\nu}{4}-\sqrt[4]{-1}\, z} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi\nu)}{z} - e^{2\sqrt[4]{-1}\, z} - \sin(\pi\nu)\right)$$

$$\left.\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{2^{-2k} \left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right)\right)\right)\Bigg) \,/;\, (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z} \bigwedge n \in \mathbb{N}$$

03.20.06.0047.01

$$\operatorname{ker}_{\nu}(z) \propto -\frac{1}{64} e^{\frac{i \pi \nu}{4}-\frac{(1+i) z}{\sqrt{2}}} \sqrt{\frac{\pi}{2}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} z^{-\nu-2}$$
$$\left(e^{\sqrt{2}\, z+\frac{5 i \pi \nu}{2}} \left((-1)^{3/4} \sqrt{-i z^2}-i z\right) z^{2 \nu}+e^{\sqrt{2}\, z+\frac{i \pi \nu}{2}} \left(i z+(-1)^{3/4} \sqrt{-i z^2}\right) z^{2 \nu}+2 e^{\sqrt{2}\, i z+\frac{3 i \pi \nu}{2}} z^{2 \nu+1}-2 e^{i\left(\sqrt{2}\, z+\pi \nu\right)}\right.$$
$$\left.\left(-\sqrt[4]{-1}\, z\right)^{2 \nu} z-e^{\sqrt{2}\, z+2 i \pi \nu} \left(-\sqrt[4]{-1}\, z\right)^{2 \nu} \left(i z+(-1)^{3/4} \sqrt{-i z^2}\right)+e^{\sqrt{2}\, z} \left(-\sqrt[4]{-1}\, z\right)^{2 \nu} \left(i z-(-1)^{3/4} \sqrt{-i z^2}\right)\right)$$
$$\left(4 \nu^2-1\right) (i \cot (\pi \nu)+1)\, {}_4F_1\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}; \frac{i}{z^2}\right)+\frac{1}{64} \sqrt{\frac{\pi}{2}}\, e^{-\frac{(1+i) z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}$$
$$z^{-\nu-2} \left(e^{\frac{7 i \pi \nu}{2}} \left((-1)^{3/4} \sqrt{i z^2}-z\right) z^{2 \nu}+e^{\frac{3 i \pi \nu}{2}} \left(z+(-1)^{3/4} \sqrt{i z^2}\right) z^{2 \nu}-2 e^{\sqrt{2}\, (1+i) z+\frac{5 i \pi \nu}{2}} i z^{2 \nu+1}+\right.$$
$$\left.2 e^{\sqrt{2}\, (1+i) z+i \pi \nu} i \left((-1)^{3/4} z\right)^{2 \nu} z-e^{2 i \pi \nu} \left((-1)^{3/4} z\right)^{2 \nu} \left(z+(-1)^{3/4} \sqrt{i z^2}\right)+\left((-1)^{3/4} z\right)^{2 \nu} \left(z-(-1)^{3/4} \sqrt{i z^2}\right)\right)$$
$$\left(4 \nu^2-1\right) (i \cot (\pi \nu)+1)\, {}_4F_1\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}; -\frac{i}{z^2}\right)-$$
$$\frac{1}{4} e^{-\frac{(1+i) z}{\sqrt{2}}-\frac{i \pi \nu}{4}} \sqrt{\frac{\pi}{2}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} z^{-\nu-1} \left(e^{\sqrt{2}\, z} \cos (\pi \nu) \left(i \left((-1)^{3/4} z+\sqrt{-i z^2}\right)+\sqrt{-i z^2} \cot (\pi \nu)\right) z^{2 \nu}+\right.$$
$$(-1)^{3/4} \left(e^{i \sqrt{2}\, z} z^{2 \nu}-e^{\sqrt{2}\, z} \sin (\pi \nu) z^{2 \nu}+e^{\sqrt{2}\, z+\frac{i \pi \nu}{2}} i \left(-\sqrt[4]{-1}\, z\right)^{2 \nu}+e^{\frac{1}{2} i \left(2 \sqrt{2}\, z+\pi \nu\right)} i \left(-\sqrt[4]{-1}\, z\right)^{2 \nu} \csc (\pi \nu)\right) z+$$
$$\left.\left(\sqrt[4]{-1}\, e^{i \sqrt{2}\, z} z^{2 \nu+1}-e^{\sqrt{2}\, z+\frac{i \pi \nu}{2}} \left(-\sqrt[4]{-1}\, z\right)^{2 \nu} \sqrt{-i z^2}\right) \cot (\pi \nu)\right) {}_4F_1\left(\frac{1}{4}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}; \frac{i}{z^2}\right)-$$
$$\frac{1}{4} e^{-\frac{(1+i) z}{\sqrt{2}}-\frac{5 i \pi \nu}{4}} \sqrt{\frac{\pi}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{3}{2}} z^{-\nu} \left(e^{\frac{5 i \pi \nu}{2}} \cos (\pi \nu) \left(z-\sqrt[4]{-1} \sqrt{i z^2} \cot (\pi \nu)+(-1)^{3/4} \sqrt{i z^2}\right) z^{2 \nu}+\right.$$
$$\left(e^{\sqrt{2}\, (1+i) z+\frac{5 i \pi \nu}{2}} i z^{2 \nu}-i e^{\frac{5 i \pi \nu}{2}} \sin (\pi \nu) z^{2 \nu}+\left((-1)^{3/4} z\right)^{2 \nu}+e^{(1+i) \sqrt{2}\, z} \left((-1)^{3/4} z\right)^{2 \nu} \csc (\pi \nu)\right) z+$$
$$\left.\left(\sqrt[4]{-1} \left((-1)^{3/4} z\right)^{2 \nu} \sqrt{i z^2}-e^{\sqrt{2}\, (1+i) z+\frac{5 i \pi \nu}{2}} z^{2 \nu+1}\right) \cot (\pi \nu)\right)$$
$${}_4F_1\left(\frac{1}{4}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}; -\frac{i}{z^2}\right) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.20.06.0048.01

$$\operatorname{ker}_{\nu}(z) \propto \frac{1}{4}\sqrt{\frac{\pi}{2}} \Bigg(e^{\frac{z}{\sqrt{2}}} \Bigg(\sqrt[4]{-1}\, e^{\frac{i\pi\nu}{4}} \csc(\pi\nu)\, z^{-\nu} \Bigg(e^{\frac{iz}{\sqrt{2}}-\frac{3i\pi\nu}{2}} \left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}} \left(1+O\left(\frac{1}{z^2}\right)\right) - e^{-\frac{iz}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}} \left(\frac{(-1)^{3/4}\sqrt{-i z^2}\, \cos(\pi\nu)}{z} - \sin(\pi\nu)\right) \left(1+O\left(\frac{1}{z^2}\right)\right) \Bigg) + \sqrt[4]{-1}\, e^{-\frac{i\pi\nu}{4}} z^{\nu} \Bigg(e^{-\frac{iz}{\sqrt{2}}} (i+\cot(\pi\nu)) \left(\frac{(-1)^{3/4}\sqrt{-i z^2}\, \cos(\pi\nu)}{z} + \sin(\pi\nu)\right) \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(1+O\left(\frac{1}{z^2}\right)\right) + e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} (i-\cot(\pi\nu)) \left(1+O\left(\frac{1}{z^2}\right)\right) \Bigg) \Bigg) + e^{-\frac{z}{\sqrt{2}}} \Bigg(\sqrt[4]{-1}\, e^{\frac{i\pi\nu}{4}} \csc(\pi\nu)\, z^{-\nu} \Bigg(e^{\frac{iz}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{\nu-\frac{1}{2}} \left(1+O\left(\frac{1}{z^2}\right)\right) + e^{-\frac{iz}{\sqrt{2}}-\frac{3i\pi\nu}{2}} \left((-1)^{3/4} z\right)^{\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\sqrt{i z^2}\, \cos(\pi\nu)}{z} + \sin(\pi\nu)\right) \left(1+O\left(\frac{1}{z^2}\right)\right) \Bigg) + \sqrt[4]{-1}\, e^{-\frac{i\pi\nu}{4}} z^{\nu} \Bigg(e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} (i-\cot(\pi\nu)) \left(\frac{\sqrt[4]{-1}\sqrt{i z^2}\, \cos(\pi\nu)}{z} - \sin(\pi\nu)\right) \left(1+O\left(\frac{1}{z^2}\right)\right) - e^{\frac{iz}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} (i+\cot(\pi\nu)) \left(1+O\left(\frac{1}{z^2}\right)\right) \Bigg) \Bigg) \Bigg) /; (|z| \to \infty) \bigwedge \nu \notin \mathbb{Z}$$

03.20.06.0049.01

$$\operatorname{ker}_{\nu}(z) \propto \begin{cases} \frac{\sqrt[8]{-1}\sqrt{\pi}}{2\sqrt{2z}} \left(-(-1)^{3/4} e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}} + e^{(-1)^{3/4} z+\frac{i\pi\nu}{2}}\right) & \arg(z) \leq \frac{\pi}{4} \\ \frac{\sqrt[8]{-1}\sqrt{\pi}}{2\sqrt{2z}} \left(-(-1)^{3/4} e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}} + \sqrt[4]{-1}\, e^{\sqrt[4]{-1}\, z+\frac{i\pi\nu}{2}} + e^{(-1)^{3/4} z+\frac{i\pi\nu}{2}} + \sqrt[4]{-1}\, e^{\sqrt[4]{-1}\, z-\frac{3i\pi\nu}{2}}\right) & \frac{\pi}{4} < \arg(z) \leq \\ \frac{\sqrt[8]{-1}\sqrt{\pi}}{2\sqrt{2z}} \left(-(-1)^{3/4} e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}} + i\, e^{-(-1)^{3/4} z-\frac{i\pi\nu}{2}} + \sqrt[4]{-1}\, e^{\sqrt[4]{-1}\, z+\frac{i\pi\nu}{2}} + e^{(-1)^{3/4} z+\frac{i\pi\nu}{2}} + \sqrt[4]{-1}\, e^{\sqrt[4]{-1}\, z-\frac{3i\pi\nu}{2}} + i\, e^{\frac{3i\pi\nu}{2}-(-1)^{3/4} z}\right) & \text{True} \end{cases}$$

Logarithmic cases

03.20.06.0050.01

$$\operatorname{ker}_{\nu}(z) \propto -\frac{e^{\frac{i\pi\nu}{2}}}{8\sqrt{2\pi}} \Bigg(e^{-\frac{z}{\sqrt{2}}} \Bigg(\frac{e^{i\pi\nu-\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \bigg(-\frac{4 i \sqrt{i z^2} \left(\log\left((-1)^{3/4} z\right) - \log(z)\right)}{z} - \frac{3\pi\sqrt{i z^2}}{z} - 4\sqrt[4]{-1}\,\pi \bigg) -$$

$$\frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \left(4\sqrt[4]{-1} \left(\log\left(-\sqrt[4]{-1}\, z\right) - \log(z)\right) - (-1)^{3/4}\pi \right) \Bigg) +$$

$$e^{\frac{z}{\sqrt{2}}} \Bigg(\frac{e^{i\pi\nu-\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \bigg(-4\sqrt[4]{-1}\,\pi + \frac{\pi i \sqrt{-i z^2}}{z} + \frac{4\sqrt{-i z^2} \left(\log(z) - \log\left(-\sqrt[4]{-1}\, z\right)\right)}{z} \bigg) + \frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}}$$

$$\left(3(-1)^{3/4}\pi - 4\sqrt[4]{-1}\left(\log\left((-1)^{3/4} z\right) - \log(z)\right)\right) \Bigg) \Bigg) \left(1 + O\left(\frac{1}{z^4}\right)\right) - \frac{e^{\frac{i\pi\nu}{2} - \frac{(1+i)z}{\sqrt{2}}}}{8\sqrt{2\pi}\sqrt{-\sqrt[4]{-1}\, z}\left((-1)^{3/4} z\right)^{3/2}}$$

$$\Bigg(-\frac{(-1)^{3/4}\left(1-4\nu^2\right)}{8z} \Bigg(\sqrt{(-1)^{3/4} z} \bigg(\frac{1}{2}\left((1+i)(-1)^{\nu} e^{\sqrt{2}\, z}\pi\left((4+4i)z - i\sqrt{2}\sqrt{-i z^2}\right) - 2 e^{i\sqrt{2}\, z}\pi z\right) +$$

$$4\left((-1)^{\nu+\frac{1}{4}} e^{\sqrt{2}\, z}\sqrt{-i z^2} - i e^{i\sqrt{2}\, z} z\right)\left(\log\left(-\sqrt[4]{-1}\, z\right) - \log(z)\right) \bigg) +$$

$$\sqrt{-\sqrt[4]{-1}\, z} \bigg(\pi\left(-3 i e^{(1+i)\sqrt{2}\, z} z - 4(-1)^{\nu} z + 3(-1)^{\nu+\frac{3}{4}}\sqrt{i z^2}\right) +$$

$$4\left(e^{(1+i)\sqrt{2}\, z} z - (-1)^{\nu+\frac{1}{4}}\sqrt{i z^2}\right)\left(\log\left((-1)^{3/4} z\right) - \log(z)\right) \bigg) \Bigg) \left(1 + O\left(\frac{1}{z^4}\right)\right) +$$

$$\frac{i\left(16\nu^4 - 40\nu^2 + 9\right)}{128 z^2} \Bigg(\sqrt{(-1)^{3/4} z} \bigg(4\left(e^{i\sqrt{2}\, z} z - (-1)^{\nu+\frac{3}{4}} e^{\sqrt{2}\, z}\sqrt{-i z^2}\right)\left(\log\left(-\sqrt[4]{-1}\, z\right) - \log(z)\right) -$$

$$\frac{\pi}{\sqrt{2}}\left(\sqrt{2}\, e^{i\sqrt{2}\, z} i z + (-1)^{\nu} e^{\sqrt{2}\, z}(1+i)\left(\sqrt{2}(-2+2i)z + \sqrt{-i z^2}\right)\right) \bigg) -$$

$$\sqrt{-\sqrt[4]{-1}\, z} \bigg(\pi\left(\frac{(-1)^{\nu}\sqrt{i z^2}\,(3-3i)}{\sqrt{2}} - 3 i e^{(1+i)\sqrt{2}\, z} z + 4(-1)^{\nu} z\right) + 4\left(e^{(1+i)\sqrt{2}\, z} z + (-1)^{\nu+\frac{1}{4}}\sqrt{i z^2}\right)$$

$$\left(\log\left((-1)^{3/4} z\right) - \log(z)\right) \bigg) \Bigg) \left(1 + O\left(\frac{1}{z^4}\right)\right) - \frac{\sqrt[4]{-1}\left(64\nu^6 - 560\nu^4 + 1036\nu^2 - 225\right)}{3072 z^3}$$

$$\Bigg(\sqrt{(-1)^{3/4} z} \bigg(\frac{1}{2}\left((1+i)(-1)^{\nu} e^{\sqrt{2}\, z}\pi\left((4+4i)z - i\sqrt{2}\sqrt{-i z^2}\right) - 2 e^{i\sqrt{2}\, z}\pi z\right) +$$

$$4\left((-1)^{\nu+\frac{1}{4}} e^{\sqrt{2}\, z}\sqrt{-i z^2} - i e^{i\sqrt{2}\, z} z\right)\left(\log\left(-\sqrt[4]{-1}\, z\right) - \log(z)\right) \bigg) -$$

$$\sqrt{-\sqrt[4]{-1}\, z} \bigg(\pi\left(-3 i e^{(1+i)\sqrt{2}\, z} z - 4(-1)^{\nu} z + 3(-1)^{\nu+\frac{3}{4}}\sqrt{i z^2}\right) +$$

03.20.06.0051.01

$$\mathrm{ker}_{\nu}(z) \propto -\frac{e^{\frac{\pi i \nu}{2}-\frac{(1+i) z}{\sqrt{2}}}}{8 \sqrt{2 \pi} \sqrt{-\sqrt[4]{-1}\, z}\, \left((-1)^{3/4} z\right)^{3/2}}$$

$$\Bigg(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{4 z^2}\right)^k \Bigg(\frac{\pi}{\sqrt{2}} \Big((-1)^{k+\frac{3}{4}} \sqrt{2} \left(4 (-1)^{\nu}-3 i e^{(1+i) \sqrt{2} z}\right) \left(-\sqrt[4]{-1}\, z\right)^{3/2}+3 (-1)^{k+\nu} (1-i) \sqrt{i z^2}$$

$$\sqrt{-\sqrt[4]{-1}\, z}+\sqrt{(-1)^{3/4} z} \left(\sqrt{2} e^{i \sqrt{2} z} (-i) z-(1+i) (-1)^{\nu} e^{\sqrt{2} z} \left(2 \sqrt{2} (-1+i) z+\sqrt{-i z^2}\right)\right)\Big)+$$

$$4 \sqrt{(-1)^{3/4} z} \left(e^{i \sqrt{2} z} z-(-1)^{\nu+\frac{3}{4}} e^{\sqrt{2} z} \sqrt{-i z^2}\right) \left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z)\right)+4 (-1)^k \sqrt{-\sqrt[4]{-1}\, z}$$

$$\left(e^{(1+i) \sqrt{2} z} z+(-1)^{\nu+\frac{1}{4}} \sqrt{i z^2}\right) \left(\log\left((-1)^{3/4} z\right)-\log(z)\right)\Bigg)-\frac{(-1)^{3/4}}{2 z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(\frac{i}{4 z^2}\right)^k$$

$$\Bigg(\frac{(1+i) \pi}{2} \Big((-1)^{k+\frac{3}{4}} \left(4 (-1)^{\nu}+3 i e^{(1+i) \sqrt{2} z}\right) (-1+i) \left(-\sqrt[4]{-1}\, z\right)^{3/2}+3 (-1)^{k+\nu+\frac{3}{4}} (1-i) \sqrt{i z^2} \sqrt{-\sqrt[4]{-1}\, z}+$$

$$\sqrt{(-1)^{3/4} z} \left(e^{i \sqrt{2} z} (-1+i) z+(-1)^{\nu} e^{\sqrt{2} z} \left(4 (1+i) z-i \sqrt{2} \sqrt{-i z^2}\right)\right)\Big)+$$

$$4 \sqrt{(-1)^{3/4} z} \left((-1)^{\nu+\frac{1}{4}} e^{\sqrt{2} z} \sqrt{-i z^2}-i e^{i \sqrt{2} z} z\right) \left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z)\right)+4 (-1)^k$$

$$\sqrt{-\sqrt[4]{-1}\, z} \left(e^{(1+i) \sqrt{2} z} z-(-1)^{\nu+\frac{1}{4}} \sqrt{i z^2}\right) \left(\log\left((-1)^{3/4} z\right)-\log(z)\right)\Bigg)+\ldots\Bigg) \;/;\; (|z| \to \infty) \land \nu \in \mathbb{Z} \land n \in \mathbb{N}$$

03.20.06.0052.01

$$\mathrm{ker}_{\nu}(z) \propto$$

$$\frac{e^{\frac{i \pi \nu}{2}-\frac{(1+i) z}{\sqrt{2}}}}{8 \sqrt{2 \pi} \sqrt{-\sqrt[4]{-1}\, z}\, \left((-1)^{3/4} z\right)^{3/2}} \Bigg(\Bigg(\sqrt{(-1)^{3/4} z} \Big(4 \left(e^{i \sqrt{2} z} z-(-1)^{\nu+\frac{3}{4}} e^{\sqrt{2} z} \sqrt{-i z^2}\right) \left(\log\left(-\sqrt[4]{-1}\, z\right)-\log(z)\right)-\frac{\pi}{\sqrt{2}}$$

$$\left(\sqrt{2} e^{i \sqrt{2} z} i z+(-1)^{\nu} e^{\sqrt{2} z} (1+i) \left(\sqrt{2} (-2+2 i) z+\sqrt{-i z^2}\right)\right)\Big)+$$

$$\sqrt{-\sqrt[4]{-1}\, z} \Bigg(\pi \left(\frac{(-1)^{\nu} \sqrt{i z^2} (3-3 i)}{\sqrt{2}}-3 i e^{(1+i) \sqrt{2} z} z+4 (-1)^{\nu} z\right)+$$

$$4 \left(e^{(1+i) \sqrt{2} z} z+(-1)^{\nu+\frac{1}{4}} \sqrt{i z^2}\right) \left(\log\left((-1)^{3/4} z\right)-\log(z)\right)\Bigg)\Bigg)\, {}_8F_3\Bigg(\frac{1}{8} (1-2 \nu), \frac{1}{8} (3-2 \nu),$$

$$\frac{1}{8} (5-2 \nu), \frac{1}{8} (7-2 \nu), \frac{1}{8} (2 \nu+1), \frac{1}{8} (2 \nu+3), \frac{1}{8} (2 \nu+5), \frac{1}{8} (2 \nu+7); \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{16}{z^4}\Bigg)-$$

$$\frac{(-1)^{3/4} \left(1-4 \nu^2\right)}{8 z} \Bigg(\sqrt{(-1)^{3/4} z} \left(\frac{1}{2} \left((1+i) (-1)^{\nu} e^{\sqrt{2} z} \pi \left((4+4 i) z-i \sqrt{2} \sqrt{-i z^2}\right)-2 e^{i \sqrt{2} z} \pi z\right)+\right.$$

$$4\left((-1)^{\nu+\frac{1}{4}} e^{\sqrt{2}\, z} \sqrt{-i z^2} - i\, e^{i \sqrt{2}\, z} z\right)\left(\log\left(-\sqrt[4]{-1}\, z\right) - \log(z)\right)\Big) +$$

$$\sqrt{-\sqrt[4]{-1}\, z}\, \Big(\pi\left(-3\, i\, e^{(1+i)\sqrt{2}\, z} z - 4\,(-1)^{\nu} z + 3\,(-1)^{\nu+\frac{3}{4}} \sqrt{i z^2}\right) +$$

$$4\left(e^{(1+i)\sqrt{2}\, z} z - (-1)^{\nu+\frac{1}{4}} \sqrt{i z^2}\right)\left(\log\left((-1)^{3/4} z\right) - \log(z)\right)\Big)\Big)\, {}_8F_3\Big(\frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu),$$

$$\frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(2\nu+3), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9); \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; -\frac{16}{z^4}\Big) +$$

$$\frac{i\left(16\nu^4 - 40\nu^2 + 9\right)}{128 z^2}\Bigg(\sqrt{(-1)^{3/4} z}\, \Big(4\left(e^{i\sqrt{2}\, z} z - (-1)^{\nu+\frac{3}{4}} e^{\sqrt{2}\, z} \sqrt{-i z^2}\right)\left(\log\left(-\sqrt[4]{-1}\, z\right) - \log(z)\right) -$$

$$\frac{\pi}{\sqrt{2}}\left(\sqrt{2}\, e^{i\sqrt{2}\, z}\, i z + (-1)^{\nu} e^{\sqrt{2}\, z} (1+i)\left(\sqrt{2}\,(-2+2i) z + \sqrt{-i z^2}\right)\right)\Big) -$$

$$\sqrt{-\sqrt[4]{-1}\, z}\, \Big(\pi\left(\frac{(-1)^{\nu} \sqrt{i z^2}\,(3-3i)}{\sqrt{2}} - 3\, i\, e^{(1+i)\sqrt{2}\, z} z + 4\,(-1)^{\nu} z\right) +$$

$$4\left(e^{(1+i)\sqrt{2}\, z} z + (-1)^{\nu+\frac{1}{4}} \sqrt{i z^2}\right)\left(\log\left((-1)^{3/4} z\right) - \log(z)\right)\Big)\Bigg)$$

$${}_8F_3\Big(\frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(11-2\nu), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9),$$

$$\frac{1}{8}(2\nu+11); \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -\frac{16}{z^4}\Big) - \frac{\sqrt[4]{-1}\left(64\nu^6 - 560\nu^4 + 1036\nu^2 - 225\right)}{3072 z^3}$$

$$\Bigg(\sqrt{(-1)^{3/4} z}\, \Big(\frac{1}{2}\left((1+i)(-1)^{\nu} e^{\sqrt{2}\, z} \pi\left((4+4i) z - i\sqrt{2}\, \sqrt{-i z^2}\right) - 2\, e^{i\sqrt{2}\, z} \pi z\right) +$$

$$4\left((-1)^{\nu+\frac{1}{4}} e^{\sqrt{2}\, z} \sqrt{-i z^2} - i\, e^{i\sqrt{2}\, z} z\right)\left(\log\left(-\sqrt[4]{-1}\, z\right) - \log(z)\right)\Big) -$$

$$\sqrt{-\sqrt[4]{-1}\, z}\, \Big(\pi\left(-3\, i\, e^{(1+i)\sqrt{2}\, z} z - 4\,(-1)^{\nu} z + 3\,(-1)^{\nu+\frac{3}{4}} \sqrt{i z^2}\right) +$$

$$4\left(e^{(1+i)\sqrt{2}\, z} z - (-1)^{\nu+\frac{1}{4}} \sqrt{i z^2}\right)\left(\log\left((-1)^{3/4} z\right) - \log(z)\right)\Big)\Bigg)$$

$${}_8F_3\Big(\frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(11-2\nu), \frac{1}{8}(13-2\nu), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9), \frac{1}{8}(2\nu+11),$$

$$\frac{1}{8}(2\nu+13); \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{16}{z^4}\Big)\Bigg) \;/;\; (|z| \to \infty) \land \nu \in \mathbb{Z}$$

03.20.06.0053.01

$$\operatorname{ker}_\nu(z) \propto -\frac{e^{\frac{i\pi\nu}{2}}}{8\sqrt{2\pi}} \left(e^{-\frac{z}{\sqrt{2}}} \left(\frac{e^{i\pi\nu - \frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \left(-\frac{4 i \sqrt{i z^2} \left(\log\left((-1)^{3/4} z\right) - \log(z)\right)}{z} - \frac{3\pi \sqrt{i z^2}}{z} - 4\sqrt[4]{-1}\,\pi \right) - \frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \left(4\sqrt[4]{-1} \left(\log\left(-\sqrt[4]{-1}\, z\right) - \log(z)\right) - (-1)^{3/4}\pi\right) \right) + e^{\frac{z}{\sqrt{2}}} \left(\frac{e^{i\pi\nu - \frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \left(-4\sqrt[4]{-1}\,\pi + \frac{\pi i \sqrt{-i z^2}}{z} + \frac{4\sqrt{-i z^2}\left(\log(z) - \log\left(-\sqrt[4]{-1}\, z\right)\right)}{z} \right) + \frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \left(3(-1)^{3/4}\pi - 4\sqrt[4]{-1}\left(\log\left((-1)^{3/4} z\right) - \log(z)\right)\right) \right) \right) \left(1 + O\left(\frac{1}{z}\right)\right) /; (|z| \to \infty) \land \nu \in \mathbb{Z}$$

03.20.06.0054.01

$$\operatorname{ker}_\nu(z) \propto \begin{cases} \frac{\sqrt{\pi}\,\sqrt[8]{-1}}{2\sqrt{2z}} \left(e^{(-1)^{3/4} z + \frac{i\pi\nu}{2}} - (-1)^{3/4} e^{\frac{3i\pi\nu}{2} - \sqrt[4]{-1}\, z} \right) & \arg(z) \le \frac{\pi}{4} \\ \frac{\sqrt{\pi}\,\sqrt[8]{-1}}{2\sqrt{2z}} \left(2\sqrt[4]{-1}\, e^{\sqrt[4]{-1}\, z + \frac{i\pi\nu}{2}} - (-1)^{3/4} e^{\frac{3i\pi\nu}{2} - \sqrt[4]{-1}\, z} + e^{(-1)^{3/4} z + \frac{i\pi\nu}{2}} \right) & \frac{\pi}{4} < \arg(z) \le \frac{3\pi}{4} \\ \frac{\sqrt{\pi}\,\sqrt[8]{-1}}{2\sqrt{2z}} \left(2\sqrt[4]{-1}\, e^{\sqrt[4]{-1}\, z + \frac{i\pi\nu}{2}} + e^{(-1)^{3/4} z + \frac{i\pi\nu}{2}} - (-1)^{3/4} e^{\frac{3i\pi\nu}{2} - \sqrt[4]{-1}\, z} + 2 i e^{\frac{3i\pi\nu}{2} - (-1)^{3/4} z} \right) & \text{True} \end{cases} /; (|z| \to \infty) \land \nu \in \mathbb{Z}$$

Residue representations

03.20.06.0055.01

$$\operatorname{ker}_\nu(z) = \frac{1}{4} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(\frac{z}{4}\right)^{-4s} \Gamma\left(s + \frac{\nu}{4}\right) \Gamma\left(s - \frac{\nu}{4}\right) \Gamma\left(s + \frac{2-\nu}{4}\right)}{\Gamma\left(s + \frac{\nu+1}{2}\right) \Gamma\left(\frac{1-\nu}{2} - s\right)} \Gamma\left(\frac{\nu+2}{4} + s\right) \right) \left(-j - \frac{\nu+2}{4}\right) + \frac{1}{4} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(\frac{z}{4}\right)^{-4s} \Gamma\left(s + \frac{\nu}{4}\right) \Gamma\left(s - \frac{\nu}{4}\right) \Gamma\left(s + \frac{\nu+2}{4}\right)}{\Gamma\left(s + \frac{\nu+1}{2}\right) \Gamma\left(\frac{1-\nu}{2} - s\right)} \Gamma\left(s + \frac{2-\nu}{4}\right) \right) \left(-j - \frac{2-\nu}{4}\right) + \frac{1}{4} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(\frac{z}{4}\right)^{-4s} \Gamma\left(s - \frac{\nu}{4}\right) \Gamma\left(\frac{\nu+2}{4} + s\right) \Gamma\left(s + \frac{2-\nu}{4}\right)}{\Gamma\left(s + \frac{\nu+1}{2}\right) \Gamma\left(\frac{1-\nu}{2} - s\right)} \Gamma\left(s + \frac{\nu}{4}\right) \right) \left(-j - \frac{\nu}{4}\right) + \frac{1}{4} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\left(\frac{z}{4}\right)^{-4s} \Gamma\left(s + \frac{\nu}{4}\right) \Gamma\left(\frac{\nu+2}{4} + s\right) \Gamma\left(s + \frac{2-\nu}{4}\right)}{\Gamma\left(s + \frac{\nu+1}{2}\right) \Gamma\left(\frac{1-\nu}{2} - s\right)} \Gamma\left(s - \frac{\nu}{4}\right) \right) \left(-j + \frac{\nu}{4}\right) /; \nu \notin \mathbb{Z}$$

Integral representations

On the real axis

Contour integral representations

03.20.07.0001.01

$$\operatorname{ker}_\nu(z) = \frac{1}{8\pi i}\int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{4}\right)\Gamma\left(s-\frac{\nu}{4}\right)\Gamma\left(\frac{\nu+2}{4}+s\right)\Gamma\left(s+\frac{2-\nu}{4}\right)}{\Gamma\left(s+\frac{\nu+1}{2}\right)\Gamma\left(\frac{1-\nu}{2}-s\right)}\left(\frac{z}{4}\right)^{-4s} ds$$

Limit representations

Generating functions

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.20.13.0001.01

$$w^{(4)}(z)\, z^4 + 2\, w^{(3)}(z)\, z^3 - \left(2\nu^2+1\right) w''(z)\, z^2 + \left(2\nu^2+1\right) w'(z)\, z + \left(z^4+\nu^4-4\nu^2\right) w(z) = 0 \;/;\; w(z) = \operatorname{ber}_\nu(z)\, c_1 + \operatorname{bei}_\nu(z)\, c_2 + \operatorname{ker}_\nu(z)\, c_3 + \operatorname{kei}_\nu(z)\, c_4$$

03.20.13.0002.01

$$W_z(\operatorname{ber}_\nu(z), \operatorname{bei}_\nu(z), \operatorname{ker}_\nu(z), \operatorname{kei}_\nu(z)) = -\frac{1}{z^2}$$

03.20.13.0003.01

$$\begin{aligned} & g(z)^4\, g'(z)^3\, w^{(4)}(z) + 2\, g(z)^3 \left(g'(z)^2 - 3\, g(z)\, g''(z)\right) g'(z)^2\, w^{(3)}(z) - \\ & g(z)^2 \left(\left(2\nu^2+1\right) g'(z)^4 + 6\, g(z)\, g''(z)\, g'(z)^2 + 4\, g(z)^2\, g^{(3)}(z)\, g'(z) - 15\, g(z)^2\, g''(z)^2\right) g'(z)\, w''(z) + \\ & g(z) \left(\left(2\nu^2+1\right) g'(z)^6 + \left(2\nu^2+1\right) g(z)\, g''(z)\, g'(z)^4 - 2\, g(z)^2\, g^{(3)}(z)\, g'(z)^3 + \right. \\ & \left. g(z)^2 \left(6\, g''(z)^2 - g(z)\, g^{(4)}(z)\right) g'(z)^2 + 10\, g(z)^3\, g''(z)\, g^{(3)}(z)\, g'(z) - 15\, g(z)^3\, g''(z)^3\right) w'(z) + \\ & \left(\nu^4 - 4\nu^2 + g(z)^4\right) g'(z)^7\, w(z) = 0 \;/;\; w(z) = c_1 \operatorname{ber}_\nu(g(z)) + c_2 \operatorname{bei}_\nu(g(z)) + c_3 \operatorname{ker}_\nu(g(z)) + c_4 \operatorname{kei}_\nu(g(z)) \end{aligned}$$

03.20.13.0004.01

$$W_z(\operatorname{ber}_\nu(g(z)), \operatorname{bei}_\nu(g(z)), \operatorname{ker}_\nu(g(z)), \operatorname{kei}_\nu(g(z))) = -\frac{g'(z)^6}{g(z)^2}$$

03.20.13.0005.01

$$g(z)^4\, g'(z)^3\; h(z)^4\, w^{(4)}(z) + 2\, g(z)^3\, g'(z)^2 \left(h(z) \left(g'(z)^2 - 3\, g(z)\, g''(z)\right) - 2\, g(z)\, g'(z)\, h'(z)\right) h(z)^3\, w^{(3)}(z) + g(z)^2\, g'(z) \left(-\left(\left(2\, \nu^2 + 1\right) g'(z)^4 + 6\, g(z)\, g''(z)\, g'(z)^2 + 4\, g(z)^2\, g^{(3)}(z)\, g'(z) - 15\, g(z)^2\, g''(z)^2\right) h(z)^2 - 6\, g(z)\, g'(z) \left(h'(z)\, g'(z)^2 + g(z)\, h''(z)\, g'(z) - 3\, g(z)\, h'(z)\, g''(z)\right) h(z) + 12\, g(z)^2\, g'(z)^2\, h'(z)^2\right) h(z)^2\, w''(z) + g(z) \left(\left(\left(\left(2\, \nu^2 + 1\right) g'(z)^6 + \left(2\, \nu^2 + 1\right) g(z)\, g''(z)\, g'(z)^4 - 2\, g(z)^2\, g^{(3)}(z)\, g'(z)^3 + g(z)^2 \left(6\, g''(z)^2 - g(z)\, g^{(4)}(z)\right) g'(z)^2 + 10\, g(z)^3\, g''(z)\, g^{(3)}(z)\, g'(z) - 15\, g(z)^3\, g''(z)^3\right) h(z)^3 + 2\, g(z)\, g'(z) \left(\left(2\, \nu^2 + 1\right) h'(z)\, g'(z)^4 - 3\, g(z)\, h''(z)\, g'(z)^3 - 2\, g(z) \left(g(z)\, h^{(3)}(z) - 3\, h'(z)\, g''(z)\right) g'(z)^2 + g(z)^2 \left(9\, g''(z)\, h''(z) + 4\, h'(z)\, g^{(3)}(z)\right) g'(z) - 15\, g(z)^2\, h'(z)\, g''(z)^2\right) h(z)^2 + 12\, g(z)^2\, g'(z)^2\, h'(z) \left(h'(z)\, g'(z)^2 + 2\, g(z)\, h''(z)\, g'(z) - 3\, g(z)\, h'(z)\, g''(z)\right) h(z) - 24\, g(z)^3\, g'(z)^3\, h'(z)^3\right) h(z)\, w'(z) + \left(\left(\nu^4 - 4\, \nu^2 + g(z)^4\right) h(z)^4\, g'(z)^7 + g(z)^4 \left(24\, h'(z)^4 - 36\, h(z)\, h''(z)\, h'(z)^2 + 8\, h(z)^2\, h^{(3)}(z)\, h'(z) + h(z)^2 \left(6\, h''(z)^2 - h(z)\, h^{(4)}(z)\right)\right) g'(z)^3 - 2\, g(z)^3\, h(z) \left(g'(z)^2 - 3\, g(z)\, g''(z)\right) \left(6\, h'(z)^3 - 6\, h(z)\, h''(z)\, h'(z) + h(z)^2\, h^{(3)}(z)\right) g'(z)^2 + g(z)^2\, h(z)^2 \left(h(z)\, h''(z) - 2\, h'(z)^2\right) \left(\left(2\, \nu^2 + 1\right) g'(z)^4 + 6\, g(z)\, g''(z)\, g'(z)^2 + 4\, g(z)^2\, g^{(3)}(z)\, g'(z) - 15\, g(z)^2\, g''(z)^2\right) g'(z) - g(z)\, h(z)^3\, h'(z) \left(\left(2\, \nu^2 + 1\right) g'(z)^6 + \left(2\, \nu^2 + 1\right) g(z)\, g''(z)\, g'(z)^4 - 2\, g(z)^2\, g^{(3)}(z)\, g'(z)^3 + g(z)^2 \left(6\, g''(z)^2 - g(z)\, g^{(4)}(z)\right) g'(z)^2 + 10\, g(z)^3\, g''(z)\, g^{(3)}(z)\, g'(z) - 15\, g(z)^3\, g''(z)^3\right)\right) w(z) = 0 \;/;\; w(z) = c_1\, h(z)\, \mathrm{ber}_\nu(g(z)) + c_2\, h(z)\, \mathrm{bei}_\nu(g(z)) + c_3\, h(z)\, \mathrm{ker}_\nu(g(z)) + c_4\, h(z)\, \mathrm{kei}_\nu(g(z))$$

03.20.13.0006.01

$$W_z(h(z)\, \mathrm{ber}_\nu(g(z)),\, h(z)\, \mathrm{bei}_\nu(g(z)),\, h(z)\, \mathrm{ker}_\nu(g(z)),\, h(z)\, \mathrm{kei}_\nu(g(z))) = -\frac{h(z)^4\, g'(z)^6}{g(z)^2}$$

03.20.13.0007.01

$$z^4\, w^{(4)}(z) + (6 - 4\, r - 4\, s)\, z^3\, w^{(3)}(z) + \left(7 - 2\left(\nu^2 - 2\right) r^2 + 12\, (s - 1)\, r + 6\, (s - 2)\, s\right) z^2\, w''(z) + (2\, r + 2\, s - 1) \left(2\, r^2\, \nu^2 - 2\, (s - 1)\, s + r\, (2 - 4\, s) - 1\right) z\, w'(z) + \left(\left(a^4\, z^{4 r} + \nu^4 - 4\, \nu^2\right) r^4 - 4\, s\, \nu^2\, r^3 - 2\, s^2 \left(\nu^2 - 2\right) r^2 + 4\, s^3\, r + s^4\right) w(z) = 0 \;/;\; w(z) = c_1\, z^s\, \mathrm{ber}_\nu(a\, z^r) + c_2\, z^s\, \mathrm{bei}_\nu(a\, z^r) + c_3\, z^s\, \mathrm{ker}_\nu(a\, z^r) + c_4\, z^s\, \mathrm{kei}_\nu(a\, z^r)$$

03.20.13.0008.01

$$W_z(z^s\, \mathrm{ber}_\nu(a\, z^r),\, z^s\, \mathrm{bei}_\nu(a\, z^r),\, z^s\, \mathrm{ker}_\nu(a\, z^r),\, z^s\, \mathrm{kei}_\nu(a\, z^r)) = -a^4\, r^6\, z^{4 r + 4 s - 6}$$

03.20.13.0009.01

$$w^{(4)}(z) - 4\, (\log(r) + \log(s))\, w^{(3)}(z) + 2\left(-\left(\nu^2 - 2\right) \log^2(r) + 6\, \log(s)\, \log(r) + 3\, \log^2(s)\right) w''(z) + 4\, (\log(r) + \log(s)) \left(\nu^2\, \log^2(r) - 2\, \log(s)\, \log(r) - \log^2(s)\right) w'(z) + \left(\left(a^4\, r^{4 z} + \nu^4 - 4\, \nu^2\right) \log^4(r) - 4\, \nu^2\, \log(s)\, \log^3(r) - 2\left(\nu^2 - 2\right) \log^2(s)\, \log^2(r) + 4\, \log^3(s)\, \log(r) + \log^4(s)\right) w(z) = 0 \;/;\; w(z) = c_1\, s^z\, \mathrm{ber}_\nu(a\, r^z) + c_2\, s^z\, \mathrm{bei}_\nu(a\, r^z) + c_3\, s^z\, \mathrm{ker}_\nu(a\, r^z) + c_4\, s^z\, \mathrm{kei}_\nu(a\, r^z)$$

03.20.13.0010.01

$$W_z(s^z\, \mathrm{ber}_\nu(a\, r^z),\, s^z\, \mathrm{bei}_\nu(a\, r^z),\, s^z\, \mathrm{ker}_\nu(a\, r^z),\, s^z\, \mathrm{kei}_\nu(a\, r^z)) = -a^4\, r^{4 z}\, s^{4 z}\, \log^6(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.20.16.0001.01

$$\mathrm{ker}_\nu(-z) = (-z)^\nu\, \mathrm{ker}_\nu(z)\, z^{-\nu} + \frac{1}{2}\, \pi\, ((-z)^{-\nu}\, z^\nu - (-z)^\nu\, z^{-\nu})\, \csc(\pi\, \nu)\, \mathrm{ber}_{-\nu}(z) \;/;\; \nu \notin \mathbb{Z}$$

03.20.16.0002.01

$$\mathrm{ker}_\nu(-z) = (-1)^\nu\, \mathrm{ker}_\nu(z) + (-1)^\nu\, \mathrm{ber}_\nu(z)\, (\log(z) - \log(-z)) \;/;\; \nu \in \mathbb{Z}$$

03.20.16.0003.01

$$\operatorname{ker}_\nu(i z) = \frac{1}{2} \pi \csc(\pi \nu) \left((i z)^{-\nu} z^\nu \left(\cos\left(\frac{3 \pi \nu}{2}\right) \operatorname{ber}_{-\nu}(z) - \operatorname{bei}_{-\nu}(z) \sin\left(\frac{3 \pi \nu}{2}\right) \right) - (i z)^\nu z^{-\nu} \left(\cos\left(\frac{\pi \nu}{2}\right) \operatorname{ber}_\nu(z) + \operatorname{bei}_\nu(z) \sin\left(\frac{\pi \nu}{2}\right) \right) \right) /; \nu \notin \mathbb{Z}$$

03.20.16.0004.01

$$\operatorname{ker}_\nu(i z) = \frac{1}{2} (1 + (-1)^\nu) \operatorname{ker}_\nu(z) + \frac{1}{2} ((-1)^\nu - 1) i \operatorname{kei}_\nu(z) - \frac{1}{4} (2 (1 + (-1)^\nu) (\log(i z) - \log(z)) + \pi i (1 - (-1)^\nu)) \operatorname{ber}_\nu(z) - \frac{1}{4} (\pi (1 + (-1)^\nu) - 2 i (1 - (-1)^\nu) (\log(i z) - \log(z))) \operatorname{bei}_\nu(z) /; \nu \in \mathbb{Z}$$

03.20.16.0005.01

$$\operatorname{ker}_\nu(-i z) = \frac{1}{2} \pi \csc(\pi \nu) \left((-i z)^{-\nu} z^\nu \left(\cos\left(\frac{3 \pi \nu}{2}\right) \operatorname{ber}_{-\nu}(z) - \operatorname{bei}_{-\nu}(z) \sin\left(\frac{3 \pi \nu}{2}\right) \right) - (-i z)^\nu z^{-\nu} \left(\cos\left(\frac{\pi \nu}{2}\right) \operatorname{ber}_\nu(z) + \operatorname{bei}_\nu(z) \sin\left(\frac{\pi \nu}{2}\right) \right) \right) /; \nu \notin \mathbb{Z}$$

03.20.16.0006.01

$$\operatorname{ker}_\nu(-i z) = \frac{1}{2} (1 + (-1)^\nu) \operatorname{ker}_\nu(z) + \frac{1}{2} (1 - (-1)^\nu) i \operatorname{kei}_\nu(z) - \frac{1}{4} (\pi (1 + (-1)^\nu) + 2 (-1 + (-1)^\nu) i \log(z) - 2 i (-1 + (-1)^\nu) \log(-i z)) \operatorname{bei}_\nu(z) - \frac{1}{4} (i (-1 + (-1)^\nu) \pi + 2 (1 + (-1)^\nu) \log(-i z) - 2 (1 + (-1)^\nu) \log(z)) \operatorname{ber}_\nu(z) /; \nu \in \mathbb{Z}$$

03.20.16.0007.01

$$\operatorname{ker}_\nu\left(\frac{1}{\sqrt[4]{-1}} z\right) = \frac{1}{2} \pi \csc(\pi \nu) \left(\left(\frac{1}{\sqrt[4]{-1}} z\right)^{-\nu} \left(\sqrt[4]{-1}\, z\right)^\nu \left(\cos\left(\frac{3 \pi \nu}{2}\right) \operatorname{ber}_{-\nu}\left(\sqrt[4]{-1}\, z\right) - \sin\left(\frac{3 \pi \nu}{2}\right) \operatorname{bei}_{-\nu}\left(\sqrt[4]{-1}\, z\right) \right) - \left(\frac{1}{\sqrt[4]{-1}} z\right)^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} \left(\cos\left(\frac{\pi \nu}{2}\right) \operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right) + \operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) \sin\left(\frac{\pi \nu}{2}\right) \right) \right) /; \nu \notin \mathbb{Z}$$

03.20.16.0008.01

$$\operatorname{ker}_\nu\left(\frac{1}{\sqrt[4]{-1}} z\right) = \frac{1}{2} (1 + (-1)^\nu) \operatorname{ker}_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{1}{2} (1 - (-1)^\nu) i \operatorname{kei}_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{1}{4} \operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right) \left(i (-1 + (-1)^\nu) \pi - 2 (1 + (-1)^\nu) \log\left(\sqrt[4]{-1}\, z\right) + 2 (1 + (-1)^\nu) \log\left(-(-1)^{3/4} z\right) \right) - \frac{1}{4} \operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) \left(\pi (1 + (-1)^\nu) + 2 (-1 + (-1)^\nu) i \log\left(\sqrt[4]{-1}\, z\right) - 2 i (-1 + (-1)^\nu) \log\left(-(-1)^{3/4} z\right) \right) /; \nu \in \mathbb{Z}$$

03.20.16.0009.01

$$\operatorname{ker}_\nu\left((-1)^{-3/4} z\right) = \left((-1)^{-3/4} z\right)^\nu \operatorname{ker}_\nu\left(\sqrt[4]{-1}\, z\right) \left(\sqrt[4]{-1}\, z\right)^{-\nu} + \frac{1}{2} \pi \left(\left((-1)^{-3/4} z\right)^{-\nu} \left(\sqrt[4]{-1}\, z\right)^\nu - \left((-1)^{-3/4} z\right)^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} \right) \csc(\pi \nu) \operatorname{ber}_{-\nu}\left(\sqrt[4]{-1}\, z\right) /; \nu \notin \mathbb{Z}$$

03.20.16.0010.01

$$\operatorname{ker}_\nu\left((-1)^{-3/4} z\right) = (-1)^\nu \operatorname{ker}_\nu\left(\sqrt[4]{-1}\, z\right) + (-1)^\nu \left(\log\left(\sqrt[4]{-1}\, z\right) - \log\left(-\sqrt[4]{-1}\, z\right) \right) \operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right) /; \nu \in \mathbb{Z} /; \nu \in \mathbb{Z}$$

03.20.16.0011.01

$$\operatorname{ker}_\nu\left((-1)^{3/4} z\right) = \frac{1}{2} \pi \csc(\pi \nu) \left(\left((-1)^{3/4} z\right)^{-\nu} \left(\sqrt[4]{-1}\, z\right)^\nu \left(\cos\left(\frac{3 \pi \nu}{2}\right) \operatorname{ber}_{-\nu}\left(\sqrt[4]{-1}\, z\right) - \sin\left(\frac{3 \pi \nu}{2}\right) \operatorname{bei}_{-\nu}\left(\sqrt[4]{-1}\, z\right)\right) - \left((-1)^{3/4} z\right)^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} \left(\cos\left(\frac{\pi \nu}{2}\right) \operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right) + \operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) \sin\left(\frac{\pi \nu}{2}\right)\right)\right) /;\ \nu \notin \mathbb{Z}$$

03.20.16.0012.01

$$\operatorname{ker}_\nu\left((-1)^{3/4} z\right) = \frac{1}{2} (1 + (-1)^\nu) \operatorname{ker}_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{1}{2} (-1 + (-1)^\nu)\, i \operatorname{kei}_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{1}{4} \operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right) \left(i (1 - (-1)^\nu) \pi + 2 (1 + (-1)^\nu) \left(\log\left((-1)^{3/4} z\right) - \log\left(\sqrt[4]{-1}\, z\right)\right)\right) - \frac{1}{4} \operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) \left((1 + (-1)^\nu) \pi - 2 i (1 - (-1)^\nu) \left(\log\left((-1)^{3/4} z\right) - \log\left(\sqrt[4]{-1}\, z\right)\right)\right) /;\ \nu \in \mathbb{Z}$$

03.20.16.0013.01

$$\operatorname{ker}_\nu\left(\sqrt[4]{z^4}\right) = \frac{1}{2} z^{\nu-2} \left(z^4\right)^{-\frac{\nu}{4}} \left(z^2 + \left(z^2 - \sqrt{z^4}\right) \cos\left(\frac{3 \pi \nu}{2}\right) + \sqrt{z^4}\right) \operatorname{ker}_\nu(z) + \frac{1}{2} \sin\left(\frac{3 \pi \nu}{2}\right) z^{\nu-2} \left(z^4\right)^{-\frac{\nu}{4}} \left(\sqrt{z^4} - z^2\right) \operatorname{kei}_\nu(z) + \frac{1}{8} \pi z^{-\nu-2} \left(z^4\right)^{-\frac{\nu}{4}} \left(z^{2\nu} - \left(z^4\right)^{\nu/2}\right) \left(2 \left(z^2 + \sqrt{z^4}\right) \cot(\pi \nu) + \left(z^2 - \sqrt{z^4}\right) \csc\left(\frac{\pi \nu}{2}\right)\right) \operatorname{ber}_\nu(z) + \frac{1}{8} \pi z^{-\nu-2} \left(z^4\right)^{-\frac{\nu}{4}} \left(2 \left(z^2 + \sqrt{z^4}\right) \left(z^{2\nu} - \left(z^4\right)^{\nu/2}\right) + \left(\sqrt{z^4} - z^2\right) \left(\left(z^4\right)^{\nu/2} + z^{2\nu}\right) \sec\left(\frac{\pi \nu}{2}\right)\right) \operatorname{bei}_\nu(z) /;\ \nu \notin \mathbb{Z}$$

03.20.16.0014.01

$$\operatorname{ker}_\nu\left(\sqrt[4]{z^4}\right) = \frac{1}{8} z^{-\nu-2} \left(z^4\right)^{-\frac{\nu}{4}} \left(4 \left(z^2 + \left(z^2 - \sqrt{z^4}\right) \cos\left(\frac{3 \nu \pi}{2}\right) + \sqrt{z^4}\right) z^{2\nu} + \left(\left(-2 + i^\nu + e^{\frac{3 i \nu \pi}{2}}\right) \sqrt{z^4} - \left(2 + i^\nu + e^{\frac{3 i \nu \pi}{2}}\right) z^2\right) \left(z^{2\nu} - \left(z^4\right)^{\nu/2}\right)\right) \operatorname{ker}_\nu(z) + \frac{1}{32} z^{-\nu-2} \left(z^4\right)^{-\frac{\nu}{4}} \left(-4 i i^\nu (-1 + (-1)^\nu) \pi z^{2\nu} \left(\sqrt{z^4} - z^2\right) - \left(\left(-2 + i^\nu + e^{\frac{3 i \nu \pi}{2}}\right) \sqrt{z^4} - \left(2 + i^\nu + e^{\frac{3 i \nu \pi}{2}}\right) z^2\right) \left(\left(z^4\right)^{\nu/2} + z^{2\nu}\right) \left(4 \log(z) - \log\left(z^4\right)\right)\right) \operatorname{ber}_\nu(z) + \frac{1}{32} z^{-\nu-2} \left(z^4\right)^{-\frac{\nu}{4}} \left(4 \pi \left(-z^2 \left(z^4\right)^{\nu/2} - \left(z^4\right)^{\frac{\nu+1}{2}} + \left(1 + e^{\frac{i \nu \pi}{2}} + e^{\frac{3 i \nu \pi}{2}}\right) z^{2\nu} \sqrt{z^4} - \left(-1 + e^{\frac{i \nu \pi}{2}} + e^{\frac{3 i \nu \pi}{2}}\right) z^{2\nu+2}\right) - 4 i i^\nu (-1 + (-1)^\nu) \left(\sqrt{z^4} - z^2\right) \left(z^{2\nu} - \left(z^4\right)^{\nu/2}\right) \log(z) - 2 e^{i \nu \pi} \left(\sqrt{z^4} - z^2\right) \left(z^{2\nu} - \left(z^4\right)^{\nu/2}\right) \log\left(z^4\right) \sin\left(\frac{\nu \pi}{2}\right)\right) \operatorname{bei}_\nu(z) + \frac{1}{8} z^{-\nu-2} \left(z^4\right)^{-\frac{\nu}{4}} \left(\sqrt{z^4} - z^2\right) \left(i i^\nu (-1 + (-1)^\nu) \left(\left(z^4\right)^{\nu/2} + z^{2\nu}\right) + 4 z^{2\nu} \sin\left(\frac{3 \nu \pi}{2}\right)\right) \operatorname{kei}_\nu(z) /;\ \nu \in \mathbb{Z}$$

03.20.16.0015.01

$$\operatorname{ker}_{-\nu}(z) = \cos(\pi \nu) \operatorname{ker}_\nu(z) - \sin(\pi \nu) \operatorname{kei}_\nu(z)$$

Addition formulas

03.20.16.0016.01

$$\operatorname{ker}_\nu(z_1 - z_2) = \sum_{k=-\infty}^{\infty} \left(\operatorname{ber}_k(z_2) \operatorname{ker}_{k+\nu}(z_1) - \operatorname{bei}_k(z_2) \operatorname{kei}_{k+\nu}(z_1)\right) /;\ \left|\frac{z_2}{z_1}\right| < 1$$

03.20.16.0017.01

$$\operatorname{ker}_\nu(z_1 + z_2) = \sum_{k=-\infty}^{\infty} \left(\operatorname{ber}_k(z_2) \operatorname{ker}_{\nu-k}(z_1) - \operatorname{bei}_k(z_2) \operatorname{kei}_{\nu-k}(z_1)\right) /;\ \left|\frac{z_2}{z_1}\right| < 1$$

Multiple arguments

03.20.16.0018.01

$$\operatorname{ker}_\nu(z_1 z_2) = z_1^\nu \sum_{k=0}^{\infty} \frac{(1-z_1^2)^k \left(\frac{z_2}{2}\right)^k}{k!} \left(\cos\left(\frac{3 k \pi}{4}\right) \operatorname{ker}_{k+\nu}(z_2) - \sin\left(\frac{3 k \pi}{4}\right) \operatorname{kei}_{k+\nu}(z_2)\right) /; \left|z_1^2 - 1\right| < 1$$

Related transformations

Involving $\operatorname{kei}_\nu(z)$

03.20.16.0019.01

$$\operatorname{ker}_\nu(z) + i \operatorname{kei}_\nu(z) = \frac{\pi \csc(\pi \nu)}{2} \left(\frac{e^{-\frac{3}{4} i \pi \nu} \left(\sqrt[4]{-1}\, z\right)^\nu}{z^\nu} I_{-\nu}\left(\sqrt[4]{-1}\, z\right) - \frac{z^\nu}{e^{\frac{i \pi \nu}{4}} \left(\sqrt[4]{-1}\, z\right)^\nu} I_\nu\left(\sqrt[4]{-1}\, z\right)\right) /; \nu \notin \mathbb{Z}$$

03.20.16.0020.01

$$\operatorname{ker}_\nu(z) + i \operatorname{kei}_\nu(z) = K_\nu\left(\sqrt[4]{-1}\, z\right) (-i)^\nu + \frac{1}{4} i^\nu I_\nu\left(\sqrt[4]{-1}\, z\right) (-i \pi - \log(4) - 4 \log(z) + 4 \log((1+i) z)) /; \nu \in \mathbb{Z}$$

03.20.16.0021.01

$$\operatorname{ker}_\nu(z) - i \operatorname{kei}_\nu(z) = \frac{\pi \csc(\pi \nu)}{2} \left(\frac{e^{\frac{3 i \pi \nu}{4}} \left((-1)^{3/4} z\right)^\nu}{z^\nu} I_{-\nu}\left((-1)^{3/4} z\right) - \frac{e^{\frac{i \pi \nu}{4}} z^\nu}{\left((-1)^{3/4} z\right)^\nu} I_\nu\left((-1)^{3/4} z\right)\right) /; \nu \notin \mathbb{Z}$$

03.20.16.0022.01

$$\operatorname{ker}_\nu(z) - i \operatorname{kei}_\nu(z) = \frac{(-1)^{\nu-1}}{4} \left(2 \pi Y_\nu\left(\sqrt[4]{-1}\, z\right) + J_\nu\left(\sqrt[4]{-1}\, z\right) \left(-i \pi + 4 \log(z) - 4 \log\left(\sqrt[4]{-1}\, z\right)\right)\right) /; \nu \in \mathbb{Z}$$

Identities

Recurrence identities

Consecutive neighbors

03.20.17.0001.01

$$\operatorname{ker}_\nu(z) = -\frac{\sqrt{2}\, (\nu+1)}{z} (\operatorname{ker}_{\nu+1}(z) - \operatorname{kei}_{\nu+1}(z)) - \operatorname{ker}_{\nu+2}(z)$$

03.20.17.0002.01

$$\operatorname{ker}_\nu(z) = \frac{\sqrt{2}\, (\nu-1)}{z} (\operatorname{kei}_{\nu-1}(z) - \operatorname{ker}_{\nu-1}(z)) - \operatorname{ker}_{\nu-2}(z)$$

Distant neighbors

Increasing

03.20.17.0003.01

$$\mathrm{ker}_{\nu}(z) = (\nu+1)_{n-1}\left((n+\nu)\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{(-1)^k (n-k)!\, 2^{n-2k} z^{2k-n}}{k!\,(n-2k)!\,(-n-\nu)_k (\nu+1)_k}\left(\cos\left(\frac{1}{4}(2k-3n)\pi\right)\mathrm{ker}_{n+\nu}(z) - \sin\left(\frac{1}{4}(2k-3n)\pi\right)\mathrm{kei}_{n+\nu}(z)\right) + \sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \frac{(-1)^k (-k+n-1)!\, 2^{-2k+n-1} z^{2k-n+1}}{k!\,(-2k+n-1)!\,(-n-\nu+1)_k (\nu+1)_k} \left(\cos\left(\frac{1}{4}(2k-3n-1)\pi\right)\mathrm{ker}_{n+\nu+1}(z) - \sin\left(\frac{1}{4}(2k-3n-1)\pi\right)\mathrm{kei}_{n+\nu+1}(z)\right)\right) /;\, n \in \mathbb{N}$$

03.20.17.0004.01

$$\mathrm{ker}_{\nu}(z) = -\frac{4(\nu+1)(\nu+2)}{z^2}\mathrm{kei}_{\nu+2}(z) - \mathrm{ker}_{\nu+2}(z) + \frac{\sqrt{2}\,(\nu+1)}{z}\mathrm{ker}_{\nu+3}(z) - \frac{\sqrt{2}\,(\nu+1)}{z}\mathrm{kei}_{\nu+3}(z)$$

03.20.17.0005.01

$$\mathrm{ker}_{\nu}(z) = \frac{2\sqrt{2}\,(\nu+2)\left(-z^2+2\nu^2+8\nu+6\right)}{z^3}\mathrm{kei}_{\nu+3}(z) + \frac{4(\nu+1)(\nu+2)}{z^2}\mathrm{kei}_{\nu+4}(z) + \frac{2\sqrt{2}\,(\nu+2)\left(z^2+2\nu^2+8\nu+6\right)}{z^3}\mathrm{ker}_{\nu+3}(z) + \mathrm{ker}_{\nu+4}(z)$$

03.20.17.0006.01

$$\mathrm{ker}_{\nu}(z) = \frac{12(\nu+2)(\nu+3)}{z^2}\mathrm{kei}_{\nu+4}(z) + \frac{2\sqrt{2}\,(\nu+2)\left(z^2-2\left(\nu^2+4\nu+3\right)\right)}{z^3}\mathrm{kei}_{\nu+5}(z) + \frac{\left(z^4-16\left(\nu^4+10\nu^3+35\nu^2+50\nu+24\right)\right)}{z^4}\mathrm{ker}_{\nu+4}(z) - \frac{2\sqrt{2}\,(\nu+2)\left(z^2+2\nu^2+8\nu+6\right)}{z^3}\mathrm{ker}_{\nu+5}(z)$$

03.20.17.0007.01

$$\mathrm{ker}_{\nu}(z) = -\frac{\sqrt{2}\,(\nu+3)\left(-3z^4+16\left(\nu^2+6\nu+8\right)z^2+16\left(\nu^4+12\nu^3+49\nu^2+78\nu+40\right)\right)}{z^5}\mathrm{kei}_{\nu+5}(z) + \frac{\sqrt{2}\,(\nu+3)\left(-3z^4-16\left(\nu^2+6\nu+8\right)z^2+16\left(\nu^4+12\nu^3+49\nu^2+78\nu+40\right)\right)}{z^5}\mathrm{ker}_{\nu+5}(z) - \frac{12(\nu+2)(\nu+3)}{z^2}\mathrm{kei}_{\nu+6}(z) - \frac{\left(z^4-16\left(\nu^4+10\nu^3+35\nu^2+50\nu+24\right)\right)}{z^4}\mathrm{ker}_{\nu+6}(z)$$

Decreasing

03.20.17.0008.01

$$\operatorname{ker}_\nu(z) = (1-\nu)_{n-1} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(-1)^k (-k+n-1)!\, 2^{-2k+n-1} z^{2k-n+1}}{k!\,(-2k+n-1)!\,(1-\nu)_k (-n+\nu+1)_k} \left(\sin\left(\frac{1}{4}(2k+n-1)\pi\right) \operatorname{kei}_{-n+\nu-1}(z) - \cos\left(\frac{1}{4}(2k+n-1)\pi\right) \operatorname{ker}_{-n+\nu-1}(z) \right) + (n-\nu) \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k (n-k)!\, 2^{n-2k} z^{2k-n}}{k!\,(n-2k)!\,(1-\nu)_k (\nu-n)_k} \left(\cos\left(\frac{1}{4}(2k+n)\pi\right) \operatorname{ker}_{\nu-n}(z) - \sin\left(\frac{1}{4}(2k+n)\pi\right) \operatorname{kei}_{\nu-n}(z) \right) \right) /;\, n \in \mathbb{N}^+$$

03.20.17.0009.01

$$\operatorname{ker}_\nu(z) = -\frac{\sqrt{2}\,(\nu-1)}{z} \operatorname{kei}_{\nu-3}(z) + \frac{\sqrt{2}\,(\nu-1)}{z} \operatorname{ker}_{\nu-3}(z) - \operatorname{ker}_{\nu-2}(z) - \frac{4(\nu-2)(\nu-1)}{z^2} \operatorname{kei}_{\nu-2}(z)$$

03.20.17.0010.01

$$\operatorname{ker}_\nu(z) = \frac{4(\nu-2)(\nu-1)}{z^2} \operatorname{kei}_{\nu-4}(z) + \operatorname{ker}_{\nu-4}(z) + \frac{2\sqrt{2}\,(\nu-2)\left(z^2+2\nu^2-8\nu+6\right)}{z^3} \operatorname{ker}_{\nu-3}(z) - \frac{2\sqrt{2}\,(\nu-2)\left(z^2-2\nu^2+8\nu-6\right)}{z^3} \operatorname{kei}_{\nu-3}(z)$$

03.20.17.0011.01

$$\operatorname{ker}_\nu(z) = \frac{2\sqrt{2}\,(\nu-2)\left(z^2-2\nu^2+8\nu-6\right)}{z^3} \operatorname{kei}_{\nu-5}(z) + \frac{12(\nu-3)(\nu-2)}{z^2} \operatorname{kei}_{\nu-4}(z) + \frac{\left(z^4-16\left(\nu^4-10\nu^3+35\nu^2-50\nu+24\right)\right)}{z^4} \operatorname{ker}_{\nu-4}(z) - \frac{2\sqrt{2}\,(\nu-2)\left(z^2+2\nu^2-8\nu+6\right)}{z^3} \operatorname{ker}_{\nu-5}(z)$$

03.20.17.0012.01

$$\operatorname{ker}_\nu(z) = -\frac{12(\nu-3)(\nu-2)}{z^2} \operatorname{kei}_{\nu-6}(z) + \frac{\sqrt{2}\,(\nu-3)\left(3z^4-16\left(\nu^2-6\nu+8\right)z^2-16\left(\nu^4-12\nu^3+49\nu^2-78\nu+40\right)\right)}{z^5} \operatorname{kei}_{\nu-5}(z) + \frac{\sqrt{2}\,(\nu-3)\left(-3z^4-16\left(\nu^2-6\nu+8\right)z^2+16\left(\nu^4-12\nu^3+49\nu^2-78\nu+40\right)\right)}{z^5} \operatorname{ker}_{\nu-5}(z) - \frac{\left(z^4-16\left(\nu^4-10\nu^3+35\nu^2-50\nu+24\right)\right)}{z^4} \operatorname{ker}_{\nu-6}(z)$$

Functional identities

Relations between contiguous functions

03.20.17.0013.01

$$\operatorname{ker}_\nu(z) = -\frac{z}{2\sqrt{2}\,\nu} \left(\operatorname{kei}_{\nu-1}(z) + \operatorname{kei}_{\nu+1}(z) + \operatorname{ker}_{\nu-1}(z) + \operatorname{ker}_{\nu+1}(z)\right)$$

Differentiation

Low-order differentiation

With respect to ν

03.20.20.0001.01

$$\operatorname{ker}_{\nu}^{(1,0)}(z) = 2^{\nu-1} \pi \csc(\pi \nu) z^{-\nu} \sum_{k=0}^{\infty} \frac{\cos\left(\frac{1}{4} \pi (2k - 3\nu)\right) \psi(k-\nu+1)}{k! \, \Gamma(k-\nu+1)} \left(\frac{z}{2}\right)^{2k} +$$
$$2^{-\nu-1} \pi \csc(\pi \nu) z^{\nu} \sum_{k=0}^{\infty} \frac{\cos\left(\frac{1}{4} \pi (2k-\nu)\right) \psi(k+\nu+1)}{k! \, \Gamma(k+\nu+1)} \left(\frac{z}{2}\right)^{2k} + \frac{1}{4}$$
$$\left(3 \pi \operatorname{kei}_{\nu}(z) - \pi \left(4 \cot(\pi \nu) \log\left(\frac{z}{2}\right) + \pi\right) \operatorname{ber}_{\nu}(z) - 4 \left(\pi \cot(\pi \nu) + \log\left(\frac{z}{2}\right)\right) \operatorname{ker}_{\nu}(z) + \pi \left(\pi \cot(\pi \nu) - 4 \log\left(\frac{z}{2}\right)\right) \operatorname{bei}_{\nu}(z)\right) /; \nu \notin \mathbb{Z}$$

03.20.20.0002.01

$$\operatorname{ker}_{n}^{(1,0)}(z) = -\pi \, 2^{n-2} \, n! \, z^{-n} \sum_{k=0}^{n-1} \frac{(-1)^{n-k} 2^{-k} z^{k}}{(n-k)! \, k!} \left(\cos\left(\frac{1}{4} (k-n) \pi\right) \operatorname{bei}_{k}(z) - \sin\left(\frac{1}{4} (k-n) \pi\right) \operatorname{ber}_{k}(z)\right) +$$
$$\frac{1}{2} \pi \operatorname{kei}_{n}(z) + \frac{1}{4} (-1)^{n} \operatorname{ber}_{-n}^{(2,0)}(z) - \frac{1}{4} \operatorname{ber}_{n}^{(2,0)}(z) /; n \in \mathbb{N}$$

03.20.20.0003.01

$$\operatorname{ker}_{-n}^{(1,0)}(z) = (-1)^{n} 2^{n-2} \pi \, n! \, z^{-n} \sum_{k=0}^{n-1} \frac{(-1)^{n-k} 2^{-k} z^{k}}{(n-k)! \, k!} \left(\cos\left(\frac{1}{4} (k-n) \pi\right) \operatorname{bei}_{k}(z) - \sin\left(\frac{1}{4} (k-n) \pi\right) \operatorname{ber}_{k}(z)\right) +$$
$$\frac{1}{2} ((-1)^{n} \pi) \operatorname{kei}_{n}(z) - \frac{1}{4} \operatorname{ber}_{-n}^{(2,0)}(z) + \frac{1}{4} (-1)^{n} \operatorname{ber}_{n}^{(2,0)}(z) /; n \in \mathbb{N}$$

03.20.20.0004.01

$$\operatorname{ker}_{n+\frac{1}{2}}^{(1,0)}(z) = \frac{1}{8} \pi \left(3 (-1)^{n} \pi \operatorname{bei}_{-n-\frac{1}{2}}(z) - 4 \left(\log(z) - \log\left(\sqrt[4]{-1} \, z\right)\right)\right) \left(\operatorname{bei}_{n+\frac{1}{2}}(z) + e^{i n \pi} \operatorname{ber}_{-n-\frac{1}{2}}(z)\right) + \pi \operatorname{ber}_{n+\frac{1}{2}}(z)\right) -$$
$$\frac{(-1)^{7/8} 2^{-n-\frac{5}{2}} e^{\frac{i n \pi}{4}} \sqrt{\pi} \, z^{-n-\frac{1}{2}}}{n!} e^{-\sqrt[4]{-1} \, z} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{2k} \binom{n}{2k} (2n-2k)!$$
$$\left((-1)^{n} \sqrt{2} (-1+i) \left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right) + e^{(1+i)\sqrt{2} \, z} \left(\operatorname{Chi}\left((1+i)\sqrt{2} \, z\right) - \operatorname{Shi}\left((1+i)\sqrt{2} \, z\right)\right)\right) +\right.$$
$$\left. 2 (-1)^{k} e^{\frac{i \pi n}{2} + \sqrt{2} \, z} \left(-i \operatorname{Ci}\left((1+i)\sqrt{2} \, z\right) - i \, e^{2(-1)^{3/4} z} \left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) + \operatorname{Si}\left((1+i)\sqrt{2} \, z\right)\right)\right) i^{k} z^{2k} +$$
$$\frac{\sqrt[8]{-1} \, 2^{-n-\frac{1}{2}} e^{\frac{i n \pi}{4}} \sqrt{\pi} \, z^{\frac{1}{2}-n}}{n!} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} 2^{2k} \binom{n}{2k+1} (-2k+2n-1)!$$
$$\left((-1)^{3/4} (-1)^{n} e^{i (-1)^{3/4} z} \left(-e^{(1+i)\sqrt{2} \, z} \operatorname{Chi}\left((1+i)\sqrt{2} \, z\right) + \psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right) + e^{2\sqrt[4]{-1} \, z} \operatorname{Shi}\left(2\sqrt[4]{-1} \, z\right)\right) -\right.$$
$$i (-1)^{k} e^{\frac{i n \pi}{2}} \sin\left(\sqrt[4]{-1} \, z\right) \left(\operatorname{Ci}\left((1+i)\sqrt{2} \, z\right) + \psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right) + i \operatorname{Si}\left((1+i)\sqrt{2} \, z\right)\right) +$$
$$\left.(-1)^{k} e^{\frac{i n \pi}{2}} \cos\left(\sqrt[4]{-1} \, z\right) \left(\operatorname{Ci}\left((1+i)\sqrt{2} \, z\right) - \psi\left(k+\frac{3}{2}\right) + \psi\left(k-n+\frac{1}{2}\right) + i \operatorname{Si}\left((1+i)\sqrt{2} \, z\right)\right)\right) i^{k} z^{2k} /; n \in \mathbb{N}$$

03.20.20.0005.01

$$\ker_{-n-\frac{1}{2}}^{(1,0)}(z)=\frac{1}{8}\pi\left(\pi\,\mathrm{ber}_{-n-\frac{1}{2}}(z)-4\left(\log(z)-\log\left(\sqrt[4]{-1}\,z\right)\right)\left(\mathrm{bei}_{-n-\frac{1}{2}}(z)-(-1)^n\,\mathrm{ber}_{n+\frac{1}{2}}(z)\right)-3\,(-1)^n\,\pi\,\mathrm{bei}_{n+\frac{1}{2}}(z)\right)+$$
$$\frac{(-1)^{3/8}\,2^{-n-\frac{3}{2}}\,e^{\frac{i n\pi}{4}}\sqrt{\pi}\,z^{-n-\frac{1}{2}}}{n!}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}2^{2k}\binom{n}{2k}(2n-2k)!\,i^k$$
$$\left(\frac{1}{\sqrt[4]{-1}}\left(\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right)-\psi\left(k+\frac{1}{2}\right)+\psi\left(k-n+\frac{1}{2}\right)\right)\sinh\left(\sqrt[4]{-1}\,z\right)-\cosh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right)+\right.$$
$$\frac{1}{\sqrt[4]{-1}}\left(\cosh\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right)+\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)-\sinh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right)-$$
$$i\,(-1)^k\,e^{\frac{3 i n\pi}{2}}\left(\cos\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right)+\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)+\sin\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right)-$$
$$\left.(-1)^k\,e^{\frac{3 i n\pi}{2}}\left(\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right)-\psi\left(k+\frac{1}{2}\right)+\psi\left(k-n+\frac{1}{2}\right)\right)\sin\left(\sqrt[4]{-1}\,z\right)-\cos\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right)z^{2k}+$$
$$\frac{(-1)^{5/8}\,2^{-n-\frac{1}{2}}\,e^{\frac{i n\pi}{4}}\sqrt{\pi}\,z^{\frac{1}{2}-n}}{n!}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}2^{2k}\binom{n}{2k+1}(-2k+2n-1)!\,i^k$$
$$\left(-\frac{1}{\sqrt[4]{-1}}\left(\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right)+\psi\left(k+\frac{3}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)\sinh\left(\sqrt[4]{-1}\,z\right)-\cosh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right)-\right.$$
$$\frac{1}{\sqrt[4]{-1}}\left(\cosh\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right)-\psi\left(k+\frac{3}{2}\right)+\psi\left(k-n+\frac{1}{2}\right)\right)-\sinh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right)+$$
$$(-1)^k\,e^{\frac{3 i n\pi}{2}}\left(\cos\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right)-\psi\left(k+\frac{3}{2}\right)+\psi\left(k-n+\frac{1}{2}\right)\right)+\sin\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right)-$$
$$\left.i\,(-1)^k\,e^{\frac{3 i n\pi}{2}}\left(\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right)+\psi\left(k+\frac{3}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)\sin\left(\sqrt[4]{-1}\,z\right)-\cos\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right)z^{2k}\;/;\;n\in\mathbb{N}$$

With respect to *z*

03.20.20.0006.01

$$\frac{\partial\ker_\nu(z)}{\partial z}=-\frac{1}{\sqrt{2}\,z}\left(z\,\mathrm{kei}_{\nu-1}(z)+z\,\ker_{\nu-1}(z)+\sqrt{2}\,\nu\ker_\nu(z)\right)$$

03.20.20.0007.01

$$\frac{\partial\ker_\nu(z)}{\partial z}=\frac{1}{2\sqrt{2}}\left(-\mathrm{kei}_{\nu-1}(z)+\mathrm{kei}_{\nu+1}(z)-\ker_{\nu-1}(z)+\ker_{\nu+1}(z)\right)$$

03.20.20.0008.01

$$\frac{\partial(z^\nu\ker_\nu(z))}{\partial z}=-\frac{z^\nu}{\sqrt{2}}\left(\mathrm{kei}_{\nu-1}(z)+\ker_{\nu-1}(z)\right)$$

03.20.20.0009.01

$$\frac{\partial(z^{-\nu}\ker_\nu(z))}{\partial z}=\frac{z^{-\nu}}{\sqrt{2}}\left(\mathrm{kei}_{\nu+1}(z)+\ker_{\nu+1}(z)\right)$$

03.20.20.0010.01

$$\frac{\partial^2 \operatorname{ker}_\nu(z)}{\partial z^2} = \frac{1}{4}\left(\operatorname{kei}_{\nu-2}(z) - 2\operatorname{kei}_\nu(z) + \operatorname{kei}_{\nu+2}(z)\right)$$

03.20.20.0011.01

$$\frac{\partial^2 \operatorname{ker}_\nu(z)}{\partial z^2} = \frac{\operatorname{kei}_{\nu-1}(z)}{\sqrt{2}\, z} - \operatorname{kei}_\nu(z) + \frac{\operatorname{ker}_{\nu-1}(z)}{\sqrt{2}\, z} + \frac{(\nu(\nu+1))\operatorname{ker}_\nu(z)}{z^2}$$

Symbolic differentiation

With respect to ν

03.20.20.0012.01

$$\operatorname{ker}_\nu^{(m,0)}(z) = -\frac{\pi}{2}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{z}{2}\right)^{2k}\frac{\partial^m \frac{\left(\frac{z}{2}\right)^\nu \sin\left(\frac{1}{4}\pi(2k+3\nu)\right)}{\Gamma(k+\nu+1)}}{\partial \nu^m} - \right.$$
$$\sum_{j=0}^{m}\binom{m}{j}\left(\pi^{m-j}(-i)^{-j+m+1}\sum_{i=0}^{m-j}\frac{(-1)^i\, i!\, \mathcal{S}_{m-j}^{(i)}}{2^i}\left(\left(i\cot\left(\frac{\pi\nu}{2}\right)+1\right)^i\left(i\cot\left(\frac{\pi\nu}{2}\right)-1\right) - 2^{m-j}(i\cot(\pi\nu)+1)^i(i\cot(\pi\nu)-1)\right)\right)$$
$$\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{z}{2}\right)^{2k}\frac{\partial^j \frac{\left(\frac{z}{2}\right)^{-\nu}\cos\left(\frac{1}{4}\pi(2k-3\nu)\right)}{\Gamma(k-\nu+1)}}{\partial \nu^j} +$$
$$\sum_{j=0}^{m}\binom{m}{j}\pi^{m-j}\left((-i)^{-j+m+1}2^{m-j}(i\cot(\pi\nu)-1)\sum_{i=0}^{m-j}\frac{(-1)^i\, i!\, \mathcal{S}_{m-j}^{(i)}(i\cot(\pi\nu)+1)^i}{2^i} - \delta_{m-j}\, i\right)$$
$$\left.\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{z}{2}\right)^{2k}\frac{\partial^j \frac{\left(\frac{z}{2}\right)^{\nu}\cos\left(\frac{1}{4}\pi(2k+3\nu)\right)}{\Gamma(k+\nu+1)}}{\partial \nu^j}\right) /;\ \nu \notin \mathbb{Z}$$

With respect to z

03.20.20.0013.01

$$\frac{\partial^n \mathrm{ker}_\nu(z)}{\partial z^n} = z^{-n} \sum_{m=0}^{n} (-1)^{m+n} \binom{n}{m} (-\nu)_{n-m} \sum_{k=0}^{m} \frac{(-1)^k\, 2^{2k-m} (-m)_{2(m-k)} (\nu)_k}{(m-k)!} \left(\mathrm{ker}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \frac{(-1)^j (k-2j)!}{(2j)!\,(k-4j)!\,(-k-\nu+1)_{2j} (\nu)_{2j}} \left(\frac{z}{2}\right)^{4j} + \right.$$

$$\frac{z}{2\sqrt{2}} (\mathrm{kei}_{\nu-1}(z) + \mathrm{ker}_{\nu-1}(z)) \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{(-1)^j (-2j+k-1)!}{(2j)!\,(-4j+k-1)!\,(-k-\nu+1)_{2j} (\nu)_{2j+1}} \left(\frac{z}{2}\right)^{4j} +$$

$$\frac{z^2}{4} \mathrm{kei}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{(-1)^j (-2j+k-1)!}{(2j+1)!\,(-4j+k-2)!\,(-k-\nu+1)_{2j+1} (\nu)_{2j+1}} \left(\frac{z}{2}\right)^{4j} +$$

$$\left. \frac{z^3}{8\sqrt{2}} (\mathrm{kei}_{\nu-1}(z) - \mathrm{ker}_{\nu-1}(z)) \sum_{j=0}^{\left\lfloor \frac{k-2}{2} \right\rfloor} \frac{(-1)^j (-2j+k-2)!}{(2j+1)!\,(-4j+k-3)!\,(-k-\nu+1)_{2j+1} (\nu)_{2j+2}} \left(\frac{z}{2}\right)^{4j} \right) /;\, n \in \mathbb{N}$$

03.20.20.0014.01

$$\frac{\partial^n \mathrm{ker}_\nu(z)}{\partial z^n} = 2^{n+2\nu-2} e^{\frac{1}{4}(-3) i \pi \nu} \pi^{3/2} \csc(\pi\nu)\, \Gamma(1-\nu)\, z^{-n-\nu}\, {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2};\, \frac{1}{2}(-n-\nu+1), \frac{1}{2}(-n-\nu+2), 1-\nu;\, \frac{i z^2}{4} \right) +$$

$$2^{n+2\nu-2} e^{\frac{3 i \pi \nu}{4}} \pi^{3/2} \csc(\pi\nu)\, \Gamma(1-\nu)\, z^{-n-\nu}\, {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2};\, \frac{1}{2}(-n-\nu+1), \frac{1}{2}(-n-\nu+2), 1-\nu;\, -\frac{1}{4}\left(i z^2\right) \right) -$$

$$2^{n-2\nu-2} e^{\frac{3 i \pi \nu}{4}} \pi^{3/2} (-i + \cot(\pi\nu))\, \Gamma(\nu+1)\, z^{\nu-n}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2};\, \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+1;\, \frac{i z^2}{4} \right) -$$

$$2^{n-2\nu-2} e^{\frac{1}{4}(-3) i \pi \nu} \pi^{3/2} (i + \cot(\pi\nu))\, \Gamma(\nu+1)\, z^{\nu-n}$$

$${}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2};\, \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+1;\, -\frac{1}{4}\left(i z^2\right) \right) /;\, \nu \notin \mathbb{Z} \bigwedge n \in \mathbb{N}$$

03.20.20.0015.01

$$\frac{\partial^n \mathrm{ker}_\nu(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1} (i-1)^n \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{2k} \left(i (1-i^n)\, \mathrm{kei}_{4k-n+\nu}(z) + (1+i^n)\, \mathrm{ker}_{4k-n+\nu}(z)\right) + \right.$$

$$\left. \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n}{2k+1} \left(-i (1-i^n)\, \mathrm{kei}_{4k-n+\nu+2}(z) - (1+i^n)\, \mathrm{ker}_{4k-n+\nu+2}(z)\right) \right) /;\, n \in \mathbb{N}$$

03.20.20.0016.01

$$\frac{\partial^n \mathrm{ker}_\nu(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1} (i-1)^n \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \left(\frac{n+1}{2k+1} \binom{n}{2k} \left(\left(i - i^{n+1}\right) \mathrm{kei}_{4k-n+\nu}(z) + (1+i^n)\, \mathrm{ker}_{4k-n+\nu}(z) \right) - \right.$$

$$\left. \frac{(1+i)\sqrt{2}\,(4k-n+\nu+1)}{z} \binom{n}{2k+1} \left((-i+i^n)\, \mathrm{kei}_{4k-n+\nu+1}(z) + \left(-1+i^{n+1}\right) \mathrm{ker}_{4k-n+\nu+1}(z) \right) \right) /;\, n \in \mathbb{N}$$

03.20.20.0017.01

$$\frac{\partial^n \operatorname{ker}_\nu(z)}{\partial z^n} = \frac{1}{4} G_{5,9}^{4,4}\left(\frac{z}{4}, \frac{1}{4} \middle| \begin{array}{c} -\frac{n}{4}, \frac{1-n}{4}, \frac{2-n}{4}, \frac{3-n}{4}, \frac{1}{4}(-n+2\nu+2) \\ \frac{1}{4}(-n+\nu+2), \frac{\nu-n}{4}, \frac{1}{4}(-n-\nu+2), \frac{1}{4}(-n-\nu), \frac{1}{4}(-n+2\nu+2), 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array}\right) /; n \in \mathbb{Z} \land n \geq 2$$

Fractional integro-differentiation

With respect to z

03.20.20.0018.01

$$\frac{\partial^\alpha \operatorname{ker}_\nu(z)}{\partial z^\alpha} = \frac{2^{\nu-2} e^{-\frac{3 i \pi \nu}{4}} \pi z^{-\alpha-\nu} \csc(\pi \nu)}{\Gamma(1-\alpha-\nu)} \left({}_2F_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; 1-\nu, \frac{1-\alpha-\nu}{2}, 1-\frac{\alpha+\nu}{2}; \frac{i z^2}{4}\right) + e^{\frac{3 i \pi \nu}{2}} {}_2F_3\left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}; 1-\nu, \frac{1-\alpha-\nu}{2}, 1-\frac{\alpha+\nu}{2}; -\frac{i z^2}{4}\right)\right) - \frac{2^{-\nu-2} e^{-\frac{i \pi \nu}{4}} \pi z^{\nu-\alpha} \csc(\pi \nu)}{\Gamma(1-\alpha+\nu)} \left({}_2F_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1; \nu+1, \frac{1-\alpha+\nu}{2}, 1-\frac{\alpha-\nu}{2}; \frac{i z^2}{4}\right) + e^{\frac{i \pi \nu}{2}} {}_2F_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1; \nu+1, \frac{1-\alpha+\nu}{2}, 1-\frac{\alpha-\nu}{2}; -\frac{i z^2}{4}\right)\right) /; \nu \notin \mathbb{Z}$$

03.20.20.0019.01

$$\frac{\partial^\alpha \operatorname{ker}_\nu(z)}{\partial z^\alpha} = 2^{|\nu|-2} z^{-\alpha-|\nu|} \sum_{k=\left\lfloor \frac{|\nu|-1}{2} \right\rfloor+1}^{|\nu|-1} \frac{\left(e^{\frac{1}{4} i \pi (2\nu+|\nu|)} + (-1)^k e^{-\frac{1}{4}(i \pi (2\nu+|\nu|))}\right) (|\nu|-k-1)! \, \Gamma(2k-|\nu|+1)}{k! \, \Gamma(2k-\alpha-|\nu|+1)} \left(\frac{i z^2}{4}\right)^k + (-1)^{|\nu|-1} 2^{|\nu|-2} z^{-\alpha-|\nu|} \sum_{k=0}^{\left\lfloor \frac{|\nu|-1}{2} \right\rfloor} \frac{\left(e^{\frac{1}{4} i \pi (2\nu+|\nu|)} + (-1)^k e^{-\frac{1}{4}(i \pi (2\nu+|\nu|))}\right) (|\nu|-k-1)! \, (\log(z) - \psi(2k-\alpha-|\nu|+1) + \psi(|\nu|-2k))}{k! \, (|\nu|-2k-1)! \, \Gamma(2k-\alpha-|\nu|+1)} \left(\frac{i z^2}{4}\right)^k + 2^{-|\nu|-2} \pi z^{|\nu|-\alpha} \sum_{k=0}^{\infty} \frac{\sin\left(\frac{1}{4} \pi (2(k+\nu)+|\nu|)\right) \Gamma(2k+|\nu|+1)}{k! \, (k+|\nu|)! \, \Gamma(2k-\alpha+|\nu|+1)} \left(\frac{z}{2}\right)^{2k} - i^{\nu+|\nu|} 2^{-|\nu|-1} z^{|\nu|-\alpha} \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{1}{4}(i \pi |\nu|)} + (-1)^k e^{\frac{1}{4} i \pi |\nu|}\right) \mathcal{FC}_{\log}^{(\alpha)}(z, 2k+|\nu|)}{k! \, (k+|\nu|)!} \left(\frac{i z^2}{4}\right)^k + 2^{-|\nu|-2} i^{\nu+|\nu|} z^{|\nu|-\alpha} \sum_{k=0}^{\infty} \frac{\left(e^{-\frac{1}{4}(i \pi |\nu|)} + (-1)^k e^{\frac{1}{4} i \pi |\nu|}\right) \Gamma(2k+|\nu|+1) (2\log(2) + \psi(k+1) + \psi(k+|\nu|+1))}{k! \, (k+|\nu|)! \, \Gamma(2k-\alpha+|\nu|+1)} \left(\frac{i z^2}{4}\right)^k /; \nu \in \mathbb{Z}$$

Integration

Indefinite integration

03.20.21.0001.01

$$\int \operatorname{ker}_\nu(a z) \, dz = \frac{1}{16} z \, G_{2,6}^{4,1}\left(\frac{a z}{4}, \frac{1}{4} \middle| \begin{array}{c} \frac{3}{4}, \frac{\nu+1}{2} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{1}{4}, \frac{\nu+1}{2} \end{array}\right)$$

Definite integration

03.20.21.0002.01

$$\int_0^\infty t^{\alpha-1} e^{-p t} \operatorname{ker}_\nu(t)\, dt = 2^{-\nu-3} p^{-\alpha-\nu} \left(4^\nu \Gamma(\alpha-\nu)\Gamma(\nu-1) p^{2\nu} \left(4(\nu-1)\cos\left(\frac{3\pi\nu}{4}\right) {}_4F_3\left(\frac{\alpha}{4}-\frac{\nu}{4}, \frac{\alpha}{4}-\frac{\nu}{4}+\frac{1}{4}, \frac{\alpha}{4}+\frac{1}{2}-\frac{\nu}{4}, \frac{\alpha}{4}-\frac{\nu}{4}+\frac{3}{4}; \frac{1}{2}, \frac{1}{2}-\frac{\nu}{2}, 1-\frac{\nu}{2}; -\frac{1}{p^4}\right) - \frac{(\alpha-\nu)(\alpha-\nu+1)\sin\left(\frac{3\pi\nu}{4}\right)}{p^2} {}_4F_3\left(\frac{\alpha}{4}+\frac{1}{2}-\frac{\nu}{4}, \frac{\alpha}{4}-\frac{\nu}{4}+\frac{3}{4}, \frac{\alpha}{4}-\frac{\nu}{4}+1, \frac{\alpha}{4}-\frac{\nu}{4}+\frac{5}{4}; \frac{3}{2}, 1-\frac{\nu}{2}, \frac{3}{2}-\frac{\nu}{2}; -\frac{1}{p^4}\right)\right) + \Gamma(-\nu-1)\Gamma(\alpha+\nu)\left(-4(\nu+1)\cos\left(\frac{\pi\nu}{4}\right) {}_4F_3\left(\frac{\alpha}{4}+\frac{\nu}{4}, \frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{4}, \frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{2}, \frac{\alpha}{4}+\frac{\nu}{4}+\frac{3}{4}; \frac{1}{2}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1; -\frac{1}{p^4}\right) - \frac{(\alpha+\nu)(\alpha+\nu+1)\sin\left(\frac{\pi\nu}{4}\right)}{p^2} {}_4F_3\left(\frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{2}, \frac{\alpha}{4}+\frac{\nu}{4}+\frac{3}{4}, \frac{\alpha}{4}+\frac{\nu}{4}+1, \frac{\alpha}{4}+\frac{\nu}{4}+\frac{5}{4}; \frac{3}{2}, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}; -\frac{1}{p^4}\right)\right)\right) /;$$

$$\operatorname{Re}(\alpha+\nu) > 0 \bigwedge \operatorname{Re}(\alpha-\nu) > 0 \bigwedge \operatorname{Re}(p) > -\frac{1}{\sqrt{2}} \bigwedge \nu \notin \mathbb{Z}$$

Integral transforms

Laplace transforms

03.20.22.0001.01

$$\mathcal{L}_t[\operatorname{ker}_\nu(t)](z) = 2^{-\nu-3}\pi z^{-\nu-3}\left(4^{\nu+1}\cos\left(\frac{3\pi\nu}{4}\right)\csc(\pi\nu) z^{2\nu+2} {}_4F_3\left(\frac{1}{4}-\frac{\nu}{4}, \frac{1}{2}-\frac{\nu}{4}, \frac{3}{4}-\frac{\nu}{4}, 1-\frac{\nu}{4}; \frac{1}{2}, \frac{1}{2}-\frac{\nu}{2}, 1-\frac{\nu}{2}; -\frac{1}{z^4}\right) - \csc\left(\frac{\pi\nu}{4}\right)\sec\left(\frac{\pi\nu}{2}\right) z^2 {}_4F_3\left(\frac{\nu}{4}+\frac{1}{4}, \frac{\nu}{4}+\frac{1}{2}, \frac{\nu}{4}+\frac{3}{4}, \frac{\nu}{4}+1; \frac{1}{2}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1; -\frac{1}{z^4}\right) - \csc(\pi\nu)\left((\nu+2)\sin\left(\frac{\pi\nu}{4}\right) {}_4F_3\left(\frac{\nu}{4}+\frac{3}{4}, \frac{\nu}{4}+1, \frac{\nu}{4}+\frac{5}{4}, \frac{\nu}{4}+\frac{3}{2}; \frac{3}{2}, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}; -\frac{1}{z^4}\right) - z^{2\nu}\left(2^{2\nu+1}-4^\nu \nu\right) \sin\left(\frac{3\pi\nu}{4}\right) {}_4F_3\left(\frac{3}{4}-\frac{\nu}{4}, 1-\frac{\nu}{4}, \frac{5}{4}-\frac{\nu}{4}, \frac{3}{2}-\frac{\nu}{4}; \frac{3}{2}, 1-\frac{\nu}{2}, \frac{3}{2}-\frac{\nu}{2}; -\frac{1}{z^4}\right)\right)\right) /; |\operatorname{Re}(\nu)| < 1 \bigwedge \operatorname{Re}(z) > -\frac{1}{\sqrt{2}}$$

Mellin transforms

03.20.22.0002.01

$$\mathcal{M}_t[\operatorname{ker}_\nu(t)](z) = 2^{z-2}\cos\left(\frac{1}{4}\pi(z+2\nu)\right)\Gamma\left(\frac{z-\nu}{2}\right)\Gamma\left(\frac{z+\nu}{2}\right) /; \operatorname{Re}(z+\nu) > 0 \land \operatorname{Re}(z-\nu) > 0$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

03.20.26.0001.01

$$\operatorname{ker}_\nu(z) = 2^{-\nu-3}\,\pi^2\csc(\pi\nu)\left(2^{3\nu-2}\sin\left(\frac{3\pi\nu}{4}\right)z^{2-\nu}\,{}_0\tilde{F}_3\left(;\frac{3}{2},\frac{3}{2}-\frac{\nu}{2},1-\frac{\nu}{2};-\frac{z^4}{256}\right)+2^{3\nu+2}\cos\left(\frac{3\pi\nu}{4}\right)z^{-\nu}\,{}_0\tilde{F}_3\left(;\frac{1}{2},\frac{1}{2}-\frac{\nu}{2},1-\frac{\nu}{2};-\frac{z^4}{256}\right)-2^{2-\nu}\cos\left(\frac{\pi\nu}{4}\right)z^{\nu}\,{}_0\tilde{F}_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu+2}{2};-\frac{z^4}{256}\right)-2^{-\nu-2}\sin\left(\frac{\pi\nu}{4}\right)z^{\nu+2}\,{}_0\tilde{F}_3\left(;\frac{3}{2},\frac{\nu+3}{2},\frac{\nu+2}{2};-\frac{z^4}{256}\right)\right)/;\neg\,\nu\in\mathbb{Z}$$

Involving ${}_pF_q$

03.20.26.0002.01

$$\operatorname{ker}_\nu(z) = 2^{-\nu-3}z^{-\nu}\left(z^{2\nu}\left(4\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)\,{}_0F_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu+2}{2};-\frac{z^4}{256}\right)-z^2\,\Gamma(-\nu-1)\sin\left(\frac{\pi\nu}{4}\right){}_0F_3\left(;\frac{3}{2},\frac{\nu+3}{2},\frac{\nu+2}{2};-\frac{z^4}{256}\right)\right)+4^{\nu}\left(4\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu)\,{}_0F_3\left(;\frac{1}{2},\frac{1}{2}-\frac{\nu}{2},1-\frac{\nu}{2};-\frac{z^4}{256}\right)-z^2\,\Gamma(\nu-1)\sin\left(\frac{3\pi\nu}{4}\right){}_0F_3\left(;\frac{3}{2},\frac{3}{2}-\frac{\nu}{2},1-\frac{\nu}{2};-\frac{z^4}{256}\right)\right)\right)/;\nu\notin\mathbb{Z}$$

Involving hypergeometric U

03.20.26.0003.01

$$\operatorname{ker}_\nu(z) = -2^{-\nu-2}z^{-\nu}e^{-\frac{3i\pi\nu}{4}}\pi\csc(\pi\nu)\left(e^{\frac{i\pi\nu}{2}}z^{2\nu}-\left(\sqrt[4]{-1}\,z\right)^{2\nu}\right){}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)+2^{-\nu-2}z^{-\nu}e^{\frac{i\pi\nu}{4}}\pi\csc(\pi\nu)\left(e^{\frac{i\pi\nu}{2}}\left((-1)^{3/4}z\right)^{2\nu}-z^{2\nu}\right){}_0\tilde{F}_1\left(;\nu+1;-\frac{i z^2}{4}\right)+2^{\nu-1}e^{-\sqrt[4]{-1}\,z-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^{2\nu}U\left(\nu+\frac{1}{2},2\nu+1,2\sqrt[4]{-1}\,z\right)+2^{\nu-1}e^{\frac{3i\pi\nu}{4}-(-1)^{3/4}z}\sqrt{\pi}\,z^{-\nu}\left((-1)^{3/4}z\right)^{2\nu}U\left(\nu+\frac{1}{2},2\nu+1,2(-1)^{3/4}z\right)/;\nu\notin\mathbb{Z}$$

03.20.26.0004.01

$$\operatorname{ker}_\nu(z) = 2^{\nu-1}e^{-\sqrt[4]{-1}\,z-\frac{i\pi\nu}{4}}\sqrt{\pi}\,z^{\nu}U\left(\nu+\frac{1}{2},2\nu+1,2\sqrt[4]{-1}\,z\right)+(-1)^{\nu/4}2^{\nu-1}e^{-(-1)^{3/4}z}\sqrt{\pi}\,z^{\nu}U\left(\nu+\frac{1}{2},2\nu+1,2(-1)^{3/4}z\right)+2^{-\nu-3}e^{\frac{3i\pi\nu}{4}}\left(-i\pi-4\log(z)+4\log\left(\sqrt[4]{-1}\,z\right)\right)z^{\nu}\,{}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)+(-1)^{\frac{5\nu}{4}}2^{-\nu-3}\left(i\pi-4\log(z)+4\log\left((-1)^{3/4}z\right)\right)z^{\nu}\,{}_0\tilde{F}_1\left(;\nu+1;-\frac{i z^2}{4}\right)/;\nu\in\mathbb{Z}$$

Through Meijer G

Classical cases for the direct function itself

03.20.26.0005.01

$$\operatorname{ker}_\nu(z) = \frac{1}{4}G_{1,5}^{4,0}\left(\frac{z^4}{256}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{\nu+1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.20.26.0006.01

$$\operatorname{ker}_{-\nu}(z)+\operatorname{ker}_\nu(z) = \frac{1}{2}\cos\left(\frac{\pi\nu}{2}\right)G_{1,5}^{4,0}\left(\frac{z^4}{256}\,\middle|\,\begin{matrix}\frac{1}{2}\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.20.26.0007.01

$$\operatorname{ker}_\nu(z)-\operatorname{ker}_{-\nu}(z)=-\frac{1}{2}\sin\left(\frac{\pi\nu}{2}\right)G_{1,5}^{4,0}\left(\frac{z^4}{256}\,\middle|\,\begin{matrix}0\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},0\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Classical cases for powers of ker

03.20.26.0008.01

$$\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)^2=\frac{1}{16\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)+\frac{\sqrt{\pi}}{2^{7/2}}G_{3,7}^{6,0}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0009.01

$$\operatorname{ker}_\nu(z)^2=\frac{1}{16\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)+\frac{\sqrt{\pi}}{2^{7/2}}G_{3,7}^{6,0}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases for products of ker

03.20.26.0010.01

$$\operatorname{ker}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=\frac{\cos(\pi\nu)}{16\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}G_{2,6}^{5,0}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{matrix}\right)$$

03.20.26.0011.01

$$\operatorname{ker}_{-\nu}(z)\operatorname{ker}_\nu(z)=\frac{\cos(\pi\nu)}{16\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}G_{2,6}^{5,0}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Classical cases involving bei

03.20.26.0012.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},0,-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0013.01

$$\operatorname{bei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right)-\frac{1}{8\sqrt{2\pi}}G_{3,7}^{4,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu,\frac{\nu+1}{2}\end{matrix}\right)$$

03.20.26.0014.01

$$\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},0,-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.20.26.0015.01

$$\operatorname{bei}_{-\nu}(z)\operatorname{ker}_\nu(z)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right)-\frac{1}{8\sqrt{2\pi}}G_{3,7}^{4,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu,\frac{\nu+1}{2}\end{matrix}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Classical cases involving ber

03.20.26.0016.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)+\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0017.01

$$\operatorname{ber}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2}\end{matrix}\right)+\frac{1}{8\sqrt{2\pi}}G_{3,7}^{4,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu+\frac{1}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

03.20.26.0018.01

$$\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)+\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.20.26.0019.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{ker}_\nu(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2}\end{matrix}\right)+\frac{1}{8\sqrt{2\pi}}G_{3,7}^{4,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu+\frac{1}{2},\frac{\nu+1}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Classical cases involving powers of kei

03.20.26.0020.01

$$\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)^2+\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)^2=\frac{1}{8\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0021.01

$$\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)^2-\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)^2=-\frac{\sqrt{\pi}}{2^{5/2}}G_{3,7}^{6,0}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0022.01

$$\operatorname{kei}_\nu(z)^2+\operatorname{ker}_\nu(z)^2=\frac{1}{8\sqrt{\pi}}G_{0,4}^{4,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.20.26.0023.01

$$\operatorname{kei}_\nu(z)^2-\operatorname{ker}_\nu(z)^2=-\frac{\sqrt{\pi}}{2^{5/2}}G_{3,7}^{6,0}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving kei

03.20.26.0024.01

$$\operatorname{kei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_{\nu}\left(\sqrt[4]{z}\right)=-\frac{1}{8}\sqrt{\frac{\pi}{2}}\;G_{3,7}^{6,0}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},\nu\\0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\nu\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0025.01

$$\operatorname{kei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_{\nu}\left(\sqrt[4]{z}\right)=\frac{\sin(\pi\nu)}{16\sqrt{\pi}}\;G_{0,4}^{4,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\;G_{2,6}^{5,0}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},0\end{array}\right)$$

03.20.26.0026.01

$$\operatorname{kei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_{\nu}\left(\sqrt[4]{z}\right)+\operatorname{kei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_{-\nu}\left(\sqrt[4]{z}\right)=-2^{-\frac{5}{2}}\sqrt{\pi}\,\cos(\pi\nu)\,G_{2,6}^{5,0}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},0\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0027.01

$$\operatorname{kei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_{\nu}\left(\sqrt[4]{z}\right)-\operatorname{kei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ker}_{-\nu}\left(\sqrt[4]{z}\right)=-2^{-\frac{5}{2}}\sqrt{\pi}\,\sin(\pi\nu)\,G_{2,6}^{5,0}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0028.01

$$\operatorname{kei}_{\nu}(z)\operatorname{ker}_{\nu}(z)=-\frac{1}{8}\sqrt{\frac{\pi}{2}}\;G_{3,7}^{6,0}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},\nu\\0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\nu\end{array}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.20.26.0029.01

$$\operatorname{kei}_{-\nu}(z)\operatorname{ker}_{\nu}(z)=\frac{\sin(\pi\nu)}{16\sqrt{\pi}}\;G_{0,4}^{4,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\;G_{2,6}^{5,0}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},0\end{array}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

03.20.26.0030.01

$$\operatorname{kei}_{\nu}(z)\operatorname{ker}_{\nu}(z)+\operatorname{kei}_{-\nu}(z)\operatorname{ker}_{-\nu}(z)=-2^{-\frac{5}{2}}\sqrt{\pi}\,\cos(\pi\nu)\,G_{2,6}^{5,0}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},0\end{array}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.20.26.0031.01

$$\operatorname{kei}_{\nu}(z)\operatorname{ker}_{\nu}(z)-\operatorname{kei}_{-\nu}(z)\operatorname{ker}_{-\nu}(z)=-2^{-\frac{5}{2}}\sqrt{\pi}\,\sin(\pi\nu)\,G_{2,6}^{5,0}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{array}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving ber, bei and kei

03.20.26.0032.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)+\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{array}{c}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0033.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)-\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0034.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)+\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0035.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)-\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{array}{c}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0036.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{array}\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.20.26.0037.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.20.26.0038.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{array}\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.20.26.0039.01

$$\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{array}\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving Bessel *J*

03.20.26.0040.01

$$J_\nu\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}_\nu(z)=\frac{1}{8}e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\left(G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)-i\,G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)+\frac{1}{\pi\sqrt{2}}\left(G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+i\,G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},0\end{matrix}\right)\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.20.26.0041.01

$$J_{-\nu}\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}_\nu(z)=\frac{1}{8}e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{\nu}\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(e^{-i\pi\nu}\left(G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)-i\,G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)\right)+\frac{e^{i\pi\nu}}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+i\,G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Classical cases involving Bessel *I*

03.20.26.0042.01

$$I_\nu\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}_\nu(z)=\frac{1}{8}e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\left(G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)+i\,G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)+\frac{1}{\pi\sqrt{2}}\left(G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)-i\,G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},0\end{matrix}\right)\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.20.26.0043.01

$$I_{-\nu}\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}_\nu(z)=\frac{1}{8}e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{\nu}\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(e^{i\pi\nu}\left(G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(1-3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{matrix}\right)+i\,G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}-\frac{1}{2}(3\nu)\\0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{matrix}\right)\right)+\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)-i\,G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Classical cases involving Bessel *K*

03.20.26.0044.01

$$K_\nu\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}_\nu(z) = -\frac{1}{16}\,e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\left(\sqrt[4]{-1}\,z\right)^{-\nu}(i+\cot(\pi\nu))\,z^\nu\,G_{0,4}^{3,0}\!\left(-\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right) + \frac{1}{16}\left(e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\csc(\pi\nu)\right)G_{0,4}^{3,0}\!\left(-\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right) - \frac{e^{-\frac{3i\pi\nu}{4}}\pi^{5/2}z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\csc(\pi\nu)\csc\left(\pi\left(\nu+\frac{1}{4}\right)\right)}{4\sqrt{2}}\,G_{3,5}^{2,1}\!\left(i z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4},-\nu-\frac{1}{4}\\0,-\nu,\nu,\frac{1}{4},-\nu-\frac{1}{4}\end{matrix}\right) + \frac{e^{\frac{3i\pi\nu}{4}}\pi^{5/2}z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}(-i+\cot(\pi\nu))\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)}{4\sqrt{2}}\,G_{3,5}^{2,1}\!\left(i z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4},\nu-\frac{1}{4}\\0,\nu,\frac{1}{4},-\nu,\nu-\frac{1}{4}\end{matrix}\right)\;/;\;\nu\notin\mathbb{Z}\bigwedge-\frac{\pi}{2}<\arg(z)\le 0$$

Classical cases involving ${}_0F_1$

03.20.26.0045.01

$${}_0F_1\!\left(;\nu+1;\frac{i\sqrt{z}}{4}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right) = \frac{1}{8}\,e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,\Gamma(\nu+1)\left(2^{-\frac{\nu}{2}}\left(i\,G_{1,5}^{3,0}\!\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{5\nu}{4}\\\frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},-\frac{1}{4}(3\nu),\frac{5\nu}{4}\end{matrix}\right)+G_{1,5}^{3,0}\!\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{4}(5\nu+2)\\\frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},-\frac{1}{4}(3\nu),\frac{1}{4}(5\nu+2)\end{matrix}\right)\right)+\frac{1}{\sqrt{2}\,\pi}\,G_{2,6}^{3,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1-\nu}{4},\frac{3-\nu}{4}\\-\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-3\nu),\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\end{matrix}\right)-i\,G_{2,6}^{3,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1-\nu}{4},\frac{3-\nu}{4}\\\frac{2-\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu),-\frac{\nu}{4}\end{matrix}\right)\right)$$

03.20.26.0046.01

$${}_0F_1\!\left(;1-\nu;\frac{i\sqrt{z}}{4}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right) = \frac{1}{8}\,e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,\Gamma(1-\nu)\left(2^{\nu/2}e^{i\pi\nu}\left(G_{1,5}^{3,0}\!\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{4}(2-5\nu)\\-\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-5\nu),\frac{3\nu}{4}\end{matrix}\right)+i\,G_{1,5}^{3,0}\!\left(\frac{z}{64}\,\middle|\,\begin{matrix}-\frac{1}{4}(5\nu)\\-\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},-\frac{1}{4}(5\nu),\frac{3\nu}{4}\end{matrix}\right)\right)+\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\\frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)-i\,G_{2,6}^{3,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\\frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)\right)\right)$$

03.20.26.0047.01

$${}_0F_1\!\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{ker}_\nu(z) = \frac{1}{8}\,e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,\Gamma(\nu+1)\left(2^{-\frac{\nu}{2}}\left(i\,G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{5\nu}{4}\\\frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},-\frac{1}{4}(3\nu),\frac{5\nu}{4}\end{matrix}\right)+G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{4}(5\nu+2)\\\frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},-\frac{1}{4}(3\nu),\frac{1}{4}(5\nu+2)\end{matrix}\right)\right)+\frac{1}{\sqrt{2}\,\pi}\,G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1-\nu}{4},\frac{3-\nu}{4}\\-\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-3\nu),\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\end{matrix}\right)-i\,G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1-\nu}{4},\frac{3-\nu}{4}\\\frac{2-\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu),-\frac{\nu}{4}\end{matrix}\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.20.26.0048.01

$$ {}_0F_1\left(;1-\nu;\frac{i z^2}{4}\right)\mathrm{ker}_\nu(z) = \frac{1}{8} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, \Gamma(1-\nu)\left(2^{\nu/2} e^{i \pi \nu}\left(G_{1,5}^{3,0}\left(\frac{z^4}{64}\middle| \begin{matrix} \frac{1}{4}(2-5\nu) \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(2-5\nu), \frac{3\nu}{4}\end{matrix}\right) + i\, G_{1,5}^{3,0}\left(\frac{z^4}{64}\middle| \begin{matrix} -\frac{1}{4}(5\nu) \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{1}{4}(5\nu), \frac{3\nu}{4}\end{matrix}\right)\right) + \frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi} \left(G_{2,6}^{3,2}\left(\frac{z^4}{16}\middle| \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, \frac{3\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(3\nu+2)\end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\middle| \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu+2}{4}, \frac{\nu}{4}, \frac{3\nu}{4}, \frac{1}{4}(3\nu+2)\end{matrix}\right)\right)\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4} $$

Classical cases involving ${}_0\tilde{F}_1$

03.20.26.0049.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i\sqrt{z}}{4}\right)\mathrm{ker}_\nu\left(\sqrt[4]{z}\right) = \frac{1}{8} e^{-\frac{3 i \pi \nu}{4}} \sqrt{\pi}\left(2^{-\frac{\nu}{2}}\left(i\, G_{1,5}^{3,0}\left(\frac{z}{64}\middle| \begin{matrix} \frac{5\nu}{4} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, -\frac{1}{4}(3\nu), \frac{5\nu}{4}\end{matrix}\right) + G_{1,5}^{3,0}\left(\frac{z}{64}\middle| \begin{matrix} \frac{1}{4}(5\nu+2) \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, -\frac{1}{4}(3\nu), \frac{1}{4}(5\nu+2)\end{matrix}\right)\right) + \frac{1}{\sqrt{2}\,\pi}\right) G_{2,6}^{3,2}\left(\frac{z}{16}\middle| \begin{matrix} \frac{1-\nu}{4}, \frac{3-\nu}{4} \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(2-3\nu), \frac{2-\nu}{4}, -\frac{1}{4}(3\nu)\end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z}{16}\middle| \begin{matrix} \frac{1-\nu}{4}, \frac{3-\nu}{4} \\ \frac{2-\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(2-3\nu), -\frac{1}{4}(3\nu), -\frac{\nu}{4}\end{matrix}\right) $$

03.20.26.0050.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i\sqrt{z}}{4}\right)\mathrm{ker}_\nu\left(\sqrt[4]{z}\right) = \frac{1}{8} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\left(2^{\nu/2} e^{i\pi\nu}\left(G_{1,5}^{3,0}\left(\frac{z}{64}\middle| \begin{matrix} \frac{1}{4}(2-5\nu) \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(2-5\nu), \frac{3\nu}{4}\end{matrix}\right) + i\, G_{1,5}^{3,0}\left(\frac{z}{64}\middle| \begin{matrix} -\frac{1}{4}(5\nu) \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{1}{4}(5\nu), \frac{3\nu}{4}\end{matrix}\right)\right) + \frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z}{16}\middle| \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, \frac{3\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(3\nu+2)\end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z}{16}\middle| \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu+2}{4}, \frac{\nu}{4}, \frac{3\nu}{4}, \frac{1}{4}(3\nu+2)\end{matrix}\right)\right)\right) $$

03.20.26.0051.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\mathrm{ker}_\nu(z) = \frac{1}{8} e^{-\frac{3 i \pi \nu}{4}} \sqrt{\pi}\left(2^{-\frac{\nu}{2}}\left(i\, G_{1,5}^{3,0}\left(\frac{z^4}{64}\middle| \begin{matrix} \frac{5\nu}{4} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, -\frac{1}{4}(3\nu), \frac{5\nu}{4}\end{matrix}\right) + G_{1,5}^{3,0}\left(\frac{z^4}{64}\middle| \begin{matrix} \frac{1}{4}(5\nu+2) \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, -\frac{1}{4}(3\nu), \frac{1}{4}(5\nu+2)\end{matrix}\right)\right) + \frac{1}{\sqrt{2}\,\pi} G_{2,6}^{3,2}\left(\frac{z^4}{16}\middle| \begin{matrix} \frac{1-\nu}{4}, \frac{3-\nu}{4} \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(2-3\nu), \frac{2-\nu}{4}, -\frac{1}{4}(3\nu)\end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\middle| \begin{matrix} \frac{1-\nu}{4}, \frac{3-\nu}{4} \\ \frac{2-\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(2-3\nu), -\frac{1}{4}(3\nu), -\frac{\nu}{4}\end{matrix}\right)\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4} $$

03.20.26.0052.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ker}_\nu(z) = \frac{1}{8} e^{\frac{3 i\pi\nu}{4}}\sqrt{\pi}\left(2^{\nu/2} e^{i\pi\nu}\left(G_{1,5}^{3,0}\left(\frac{z^4}{64}\middle| \begin{matrix}\frac{1}{4}(2-5\nu)\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-5\nu),\frac{3\nu}{4}\end{matrix}\right) + i\, G_{1,5}^{3,0}\left(\frac{z^4}{64}\middle|\begin{matrix}-\frac{1}{4}(5\nu)\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},-\frac{1}{4}(5\nu),\frac{3\nu}{4}\end{matrix}\right)\right)+\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z^4}{16}\middle|\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z^4}{16}\middle|\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)\right)\right) /; -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4} $$

Generalized cases for the direct function itself

03.20.26.0053.01

$$ \operatorname{ker}_\nu(z) = \frac{1}{4} G_{1,5}^{4,0}\left(\frac{z}{4},\frac{1}{4}\middle|\begin{matrix}\frac{\nu+1}{2}\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{\nu+2}{4},\frac{\nu+1}{2}\end{matrix}\right) $$

03.20.26.0054.01

$$ \operatorname{ker}_{-\nu}(z)+\operatorname{ker}_\nu(z) = \frac{1}{2}\cos\left(\frac{\pi\nu}{2}\right) G_{1,5}^{4,0}\left(\frac{z}{4},\frac{1}{4}\middle|\begin{matrix}\frac{1}{2}\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{\nu+2}{4},\frac{1}{2}\end{matrix}\right) $$

03.20.26.0055.01

$$ \operatorname{ker}_\nu(z)-\operatorname{ker}_{-\nu}(z) = -\frac{1}{2}\sin\left(\frac{\pi\nu}{2}\right) G_{1,5}^{4,0}\left(\frac{z}{4},\frac{1}{4}\middle|\begin{matrix}0\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{\nu+2}{4},0\end{matrix}\right) $$

Generalized cases for powers of ker

03.20.26.0056.01

$$ \operatorname{ker}_\nu(z)^2 = \frac{1}{16\sqrt{\pi}} G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\middle| 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right) + \frac{\sqrt{\pi}}{2^{7/2}} G_{3,7}^{6,0}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{matrix}\frac{1}{4},\frac{3}{4},\nu+\frac{1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},\nu+\frac{1}{2}\end{matrix}\right) $$

Brychkov Yu.A. (2006)

Generalized cases for products of ker

03.20.26.0057.01

$$ \operatorname{ker}_{-\nu}(z)\operatorname{ker}_\nu(z) = \frac{\cos(\pi\nu)}{16\sqrt{\pi}} G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\middle| 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right) + \frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2},\frac{1}{2}\end{matrix}\right) $$

Generalized cases involving bei

03.20.26.0058.01

$$ \operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right) - \frac{1}{2^{7/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},0,-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right) $$

Brychkov Yu.A. (2006)

03.20.26.0059.01

$$ \operatorname{bei}_{-\nu}(z)\operatorname{ker}_\nu(z) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\middle| 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right) - \frac{1}{8\sqrt{2\pi}} G_{3,7}^{4,2}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{matrix}\frac{1}{4},\frac{3}{4},\nu\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2},\nu,\frac{\nu+1}{2}\end{matrix}\right) $$

Generalized cases involving ber

03.20.26.0060.01

$$\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{matrix}\right) + \frac{1}{2^{7/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0061.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{ker}_\nu(z) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu}{2} \end{matrix}\right) + \frac{1}{8\sqrt{2\pi}} G_{3,7}^{4,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \nu+\frac{1}{2} \\ 0, \frac{1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \nu+\frac{1}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

Generalized cases involving powers of kei

03.20.26.0062.01

$$\operatorname{kei}_\nu(z)^2 + \operatorname{ker}_\nu(z)^2 = \frac{1}{8\sqrt{\pi}} G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0063.01

$$\operatorname{kei}_\nu(z)^2 - \operatorname{ker}_\nu(z)^2 = -\frac{\sqrt{\pi}}{2^{5/2}} G_{3,7}^{6,0}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \nu+\frac{1}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \nu+\frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving kei

03.20.26.0064.01

$$\operatorname{kei}_\nu(z)\operatorname{ker}_\nu(z) = -\frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{3,7}^{6,0}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \nu \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, \nu \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0065.01

$$\operatorname{kei}_{-\nu}(z)\operatorname{ker}_\nu(z) = \frac{\sin(\pi\nu)}{16\sqrt{\pi}} G_{0,4}^{4,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}\right) - \frac{1}{8}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, 0 \end{matrix}\right)$$

03.20.26.0066.01

$$\operatorname{kei}_\nu(z)\operatorname{ker}_\nu(z) + \operatorname{kei}_{-\nu}(z)\operatorname{ker}_{-\nu}(z) = -2^{-\frac{5}{2}}\sqrt{\pi}\,\cos(\pi\nu)\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, 0 \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0067.01

$$\operatorname{kei}_\nu(z)\operatorname{ker}_\nu(z) - \operatorname{kei}_{-\nu}(z)\operatorname{ker}_{-\nu}(z) = -2^{-\frac{5}{2}}\sqrt{\pi}\,\sin(\pi\nu)\, G_{2,6}^{5,0}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving ber, bei and kei

03.20.26.0068.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0069.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{3,7}^{4,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},\frac{1}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0070.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{3,7}^{4,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},0\\ 0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{array}\right)$$

Brychkov Yu.A. (2006)

03.20.26.0071.01

$$\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving Bessel *J*

03.20.26.0072.01

$$J_\nu\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}_\nu(z)=$$

$$\frac{1}{8}e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^{-\nu}\left(\sqrt[4]{-1}\,z\right)^\nu\left(G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{array}\right)-i\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{array}\right)+\right.$$

$$\left.\frac{1}{\pi\sqrt{2}}\left(G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)+i\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},0\end{array}\right)\right)\right)$$

03.20.26.0073.01

$$J_{-\nu}\left(\sqrt[4]{-1}\,z\right)\operatorname{ker}_\nu(z)=\frac{1}{8}e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}$$

$$\left(e^{-i\pi\nu}\left(G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{2}(1-3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{1}{2}(1-3\nu),\frac{\nu}{2}\end{array}\right)-i\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}-\frac{1}{2}(3\nu)\\ 0,\frac{1}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{\nu}{2}\end{array}\right)\right)+\right.$$

$$\left.\frac{e^{i\pi\nu}}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)+i\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},0,\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)\right)\right)$$

Generalized cases involving Bessel *I*

03.20.26.0074.01

$$I_\nu\left(\sqrt[4]{-1}\, z\right) \operatorname{ker}_\nu(z) = \frac{1}{8} e^{\frac{1}{4}(-3) i \pi \nu} \sqrt{\pi}\, z^{-\nu} \left(\sqrt[4]{-1}\, z\right)^\nu \left(G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{matrix}\right) + i\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{3\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{3\nu}{2} \end{matrix}\right) + \frac{1}{\pi\sqrt{2}} \left(G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{1}{2}-\frac{\nu}{2}, 0 \end{matrix}\right)\right)\right)$$

03.20.26.0075.01

$$I_{-\nu}\left(\sqrt[4]{-1}\, z\right) \operatorname{ker}_\nu(z) = \frac{1}{8} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^{\nu} \left(\sqrt[4]{-1}\, z\right)^{-\nu} \left(e^{i\pi\nu} \left(i\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{1}{2}(3\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, -\frac{1}{2}(3\nu), \frac{\nu}{2} \end{matrix}\right) + G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{2}(1-3\nu) \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1}{2}(1-3\nu), \frac{\nu}{2} \end{matrix}\right)\right) + \frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi} \left(G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, 0, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)\right)\right)$$

Generalized cases involving Bessel K

03.20.26.0076.01

$$K_\nu\left(\sqrt[4]{-1}\, z\right) \operatorname{ker}_\nu(z) = -\frac{1}{16} e^{-\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^{\nu} \left(\sqrt[4]{-1}\, z\right)^{-\nu} (i + \cot(\pi\nu))\, G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}\right) + \frac{1}{16} e^{\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, z^{-\nu} \left(\sqrt[4]{-1}\, z\right)^{\nu} \csc(\pi\nu)\, G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}\right) - \frac{e^{-\frac{3 i \pi \nu}{4}} \pi^{5/2} z^{-\nu} \left(\sqrt[4]{-1}\, z\right)^{\nu} \csc(\pi\nu) \csc\left(\pi\left(\nu + \frac{1}{4}\right)\right)}{4\sqrt{2}} G_{3,5}^{2,1}\left(\sqrt[4]{-1}\, z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{4}, -\nu - \frac{1}{4} \\ 0, -\nu, \nu, \frac{1}{4}, -\nu - \frac{1}{4} \end{matrix}\right) + \frac{e^{\frac{3 i \pi \nu}{4}} \pi^{5/2} z^{\nu} \left(\sqrt[4]{-1}\, z\right)^{-\nu} (-i + \cot(\pi\nu)) \csc\left(\pi\left(\nu + \frac{3}{4}\right)\right)}{4\sqrt{2}} G_{3,5}^{2,1}\left(\sqrt[4]{-1}\, z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{4}, \nu - \frac{1}{4} \\ 0, \nu, \frac{1}{4}, -\nu, \nu - \frac{1}{4} \end{matrix}\right) /;\ \nu \notin \mathbb{Z}$$

Generalized cases involving ${}_0F_1$

03.20.26.0077.01

$${}_0F_1\left(; \nu+1; \frac{i z^2}{4}\right) \operatorname{ker}_\nu(z) = \frac{1}{8} e^{-\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, \Gamma(\nu+1) \left(2^{-\frac{\nu}{2}} \left(i\, G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{5\nu}{4} \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, -\frac{1}{4}(3\nu), \frac{5\nu}{4} \end{matrix}\right) + G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{4}(5\nu+2) \\ \frac{2-\nu}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, -\frac{1}{4}(3\nu), \frac{1}{4}(5\nu+2) \end{matrix}\right)\right) + \frac{1}{\sqrt{2}\,\pi} \left(G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{4}, \frac{3-\nu}{4} \\ -\frac{\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(2-3\nu), \frac{2-\nu}{4}, -\frac{1}{4}(3\nu) \end{matrix}\right) - i\, G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{4}, \frac{3-\nu}{4} \\ \frac{2-\nu}{4}, \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(2-3\nu), -\frac{1}{4}(3\nu), -\frac{\nu}{4} \end{matrix}\right)\right)\right)$$

03.20.26.0078.01

$$ {}_0F_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ker}_\nu(z)=\frac{1}{8}e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\,\Gamma(1-\nu)\left(2^{\nu/2}e^{i\pi\nu}\left(i\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}-\frac{1}{4}(5\nu)\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},-\frac{1}{4}(5\nu),\frac{3\nu}{4}\end{array}\right)+G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4}(2-5\nu)\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-5\nu),\frac{3\nu}{4}\end{array}\right)\right)+\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2)\end{array}\right)-i\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},\frac{1}{4}(3\nu+2)\end{array}\right)\right)\right) $$

Generalized cases involving ${}_0\tilde{F}_1$

03.20.26.0079.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{ker}_\nu(z)=\frac{1}{8}e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\left(2^{-\frac{\nu}{2}}\left(i\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{5\nu}{4}\\ \frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},-\frac{1}{4}(3\nu),\frac{5\nu}{4}\end{array}\right)+G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4}(5\nu+2)\\ \frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},-\frac{1}{4}(3\nu),\frac{1}{4}(5\nu+2)\end{array}\right)\right)+\frac{1}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1-\nu}{4},\frac{3-\nu}{4}\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-3\nu),\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\end{array}\right)-i\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1-\nu}{4},\frac{3-\nu}{4}\\ \frac{2-\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu),-\frac{\nu}{4}\end{array}\right)\right)\right) $$

03.20.26.0080.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ker}_\nu(z)=\frac{1}{8}e^{\frac{3i\pi\nu}{4}}\sqrt{\pi}\left(2^{\nu/2}e^{i\pi\nu}\left(i\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}-\frac{1}{4}(5\nu)\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},-\frac{1}{4}(5\nu),\frac{3\nu}{4}\end{array}\right)+G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4}(2-5\nu)\\ -\frac{\nu}{4},\frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(2-5\nu),\frac{3\nu}{4}\end{array}\right)\right)+\frac{e^{-i\pi\nu}}{\sqrt{2}\,\pi}\left(G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2)\end{array}\right)-i\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{2-\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},\frac{1}{4}(3\nu+2)\end{array}\right)\right)\right) $$

Representations through equivalent functions

With related functions

03.20.27.0001.01

$$ \operatorname{ker}_\nu(z)=-\frac{1}{2}\pi\left(\operatorname{bei}_\nu(z)-\csc(\pi\nu)\operatorname{ber}_{-\nu}(z)+\cot(\pi\nu)\operatorname{ber}_\nu(z)\right)\;/;\;\nu\notin\mathbb{Z} $$

03.20.27.0002.01

$$ \operatorname{ker}_\nu(z)=\csc(\pi\nu)\operatorname{kei}_{-\nu}(z)-\cot(\pi\nu)\operatorname{kei}_\nu(z)\;/;\;\nu\notin\mathbb{Z} $$

03.20.27.0003.01

$$ \operatorname{ker}_\nu(z)=\frac{1}{4}\left(2K_\nu\left(\sqrt[4]{-1}\,z\right)(-i)^\nu-\pi(-1)^\nu Y_\nu\left(\sqrt[4]{-1}\,z\right)+\pi\operatorname{bei}_\nu(z)-(\log(4)+4\log(z)-4\log((1+i)z))\operatorname{ber}_\nu(z)\right)\;/;\;\nu\in\mathbb{Z} $$

03.20.27.0004.01

$$\operatorname{ker}_\nu(z) == \frac{1}{4} \pi z^{-\nu} \left(-z^4\right)^{\frac{1}{4}(-\nu-2)} \csc(\pi \nu) \left(\left(I_{-\nu}\left(\sqrt[4]{-z^4}\right)\left(\sin\left(\frac{3\pi\nu}{4}\right) z^2 + \sqrt{-z^4} \cos\left(\frac{3\pi\nu}{4}\right)\right) + J_{-\nu}\left(\sqrt[4]{-z^4}\right)\left(\sqrt{-z^4} \cos\left(\frac{3\pi\nu}{4}\right) - z^2 \sin\left(\frac{3\pi\nu}{4}\right)\right)\right)\left(-z^4\right)^{\nu/2} - z^{2\nu} I_\nu\left(\sqrt[4]{-z^4}\right)\left(\sin\left(\frac{\pi\nu}{4}\right) z^2 + \sqrt{-z^4} \cos\left(\frac{\pi\nu}{4}\right)\right) - z^{2\nu} J_\nu\left(\sqrt[4]{-z^4}\right)\left(\sqrt{-z^4} \cos\left(\frac{\pi\nu}{4}\right) - z^2 \sin\left(\frac{\pi\nu}{4}\right)\right)\right) /; \nu \notin \mathbb{Z}$$

03.20.27.0005.01

$$\operatorname{ker}_\nu(z) = \frac{1}{2} e^{-\frac{3 i \pi \nu}{4}} z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu K_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{1}{4} \pi e^{\frac{3 i \pi \nu}{4}} z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu Y_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{\pi}{4}\left(e^{-\frac{3 i \pi \nu}{4}} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu}(i + \cot(\pi\nu)) - e^{\frac{3 i \pi \nu}{4}} z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu \cot(\pi\nu)\right) J_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{\pi}{4}\left(e^{-\frac{3 i \pi \nu}{4}} z^{-\nu}\left(\sqrt[4]{-1}\, z\right)^\nu \csc(\pi\nu) - e^{\frac{3 i \pi \nu}{4}} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu}(-i + \cot(\pi\nu))\right) I_\nu\left(\sqrt[4]{-1}\, z\right) /; \nu \notin \mathbb{Z}$$

03.20.27.0006.01

$$\operatorname{ker}_\nu(z) = -\frac{1}{8}(-1)^\nu \left(-i\pi + 4\log(z) - 4\log\left(\sqrt[4]{-1}\, z\right)\right) J_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{1}{2}(-i)^\nu K_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{1}{4}(-1)^\nu \pi Y_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{1}{8} i^\nu I_\nu\left(\sqrt[4]{-1}\, z\right)\left(-i\pi - 4\log(z) + 4\log\left(\sqrt[4]{-1}\, z\right)\right) /; \nu \in \mathbb{Z}$$

03.20.27.0007.01

$$\operatorname{ker}_\nu(z) = \begin{cases} -\frac{\pi}{4}\left(e^{-i\pi\nu} Y_\nu\left(\sqrt[4]{-1}\, z\right) + (3 i \cos(\pi\nu) - \sin(\pi\nu)) J_\nu\left(\sqrt[4]{-1}\, z\right)\right) - i e^{-\frac{i\pi\nu}{2}} \pi \cos(\pi\nu) I_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{1}{2} e^{-\frac{5 i\pi\nu}{2}} K_\nu\left(\sqrt[4]{-1}\, z\right) & \frac{3\pi}{4} < a \\ \frac{1}{2} e^{-\frac{i\pi\nu}{2}} K_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{\pi}{4} e^{i\pi\nu}\left(Y_\nu\left(\sqrt[4]{-1}\, z\right) - i J_\nu\left(\sqrt[4]{-1}\, z\right)\right) & \mathrm{T} \end{cases} /; \nu \in \mathbb{Z}$$

03.20.27.0008.01

$$\operatorname{ker}_\nu(z) == \frac{\pi}{4}\csc(\pi\nu) z^{-\nu}\left(e^{\frac{i\pi\nu}{4}} Y_{-\nu}\left(\sqrt[4]{-1}\, z\right)\left(e^{\frac{i\pi\nu}{2}}\left(\sqrt[4]{-1}\, z\right)^{2\nu}\cot(\pi\nu) - z^{2\nu}\csc(\pi\nu)\right)\left(\sqrt[4]{-1}\, z\right)^{-\nu} + e^{\frac{i\pi\nu}{4}} Y_\nu\left(\sqrt[4]{-1}\, z\right)\left(z^{2\nu}\cot(\pi\nu) - e^{\frac{i\pi\nu}{2}}\left(\sqrt[4]{-1}\, z\right)^{2\nu}\csc(\pi\nu)\right)\left(\sqrt[4]{-1}\, z\right)^{-\nu} + e^{\frac{1}{4}(-3) i\pi\nu}\left((-1)^{3/4} z\right)^{-\nu} Y_{-\nu}\left((-1)^{3/4} z\right)\left(\left((-1)^{3/4} z\right)^{2\nu}\cot(\pi\nu) - e^{\frac{i\pi\nu}{2}} z^{2\nu}\csc(\pi\nu)\right) + e^{\frac{1}{4}(-3) i\pi\nu}\left((-1)^{3/4} z\right)^{-\nu} Y_\nu\left((-1)^{3/4} z\right)\left(e^{\frac{i\pi\nu}{2}} z^{2\nu}\cot(\pi\nu) - \left((-1)^{3/4} z\right)^{2\nu}\csc(\pi\nu)\right)\right) /; \nu \notin \mathbb{Z}$$

03.20.27.0009.01

$$\operatorname{ker}_\nu(z) + i \operatorname{kei}_\nu(z) = e^{-\frac{3 i\pi\nu}{4}}\left(\sqrt[4]{-1}\, z\right)^\nu K_\nu\left(\sqrt[4]{-1}\, z\right) z^{-\nu} + \frac{\pi}{2} I_\nu\left(\sqrt[4]{-1}\, z\right)\left(e^{-\frac{3 i\pi\nu}{4}}\csc(\pi\nu)\left(\sqrt[4]{-1}\, z\right)^\nu z^{-\nu} + e^{\frac{3 i\pi\nu}{4}}\left(\sqrt[4]{-1}\, z\right)^{-\nu}(i - \cot(\pi\nu)) z^\nu\right) /; \nu \notin \mathbb{Z}$$

03.20.27.0010.01

$$\operatorname{ker}_\nu(z) + i \operatorname{kei}_\nu(z) = \begin{cases} e^{-\frac{5 i\pi\nu}{2}} K_\nu\left(\sqrt[4]{-1}\, z\right) - 2\pi i e^{-\frac{i\pi\nu}{2}}\cos(\pi\nu) I_\nu\left(\sqrt[4]{-1}\, z\right) & \frac{3\pi}{4} < \arg(z) \leq \pi \\ e^{-\frac{i\pi\nu}{2}} K_\nu\left(\sqrt[4]{-1}\, z\right) & \text{True} \end{cases} /; \nu \in \mathbb{Z}$$

03.20.27.0011.01

$$\mathrm{ker}_\nu(z) - i\,\mathrm{kei}_\nu(z) = -\frac{\pi}{2}$$
$$\left(e^{\frac{3 i \pi \nu}{4}} z^{-\nu} \left(\sqrt[4]{-1}\; z\right)^{\nu} Y_\nu\left(\sqrt[4]{-1}\; z\right) + \left(e^{-\frac{3 i \pi \nu}{4}} z^{\nu} \left(\sqrt[4]{-1}\; z\right)^{-\nu} (i + \cot(\pi \nu)) - e^{\frac{3 i \pi \nu}{4}} z^{-\nu} \left(\sqrt[4]{-1}\; z\right)^{\nu} \cot(\pi \nu)\right) J_\nu\left(\sqrt[4]{-1}\; z\right)\right) /;\ \nu \notin \mathbb{Z}$$

03.20.27.0012.01

$$\mathrm{ker}_\nu(z) - i\,\mathrm{kei}_\nu(z) = \begin{cases} -\frac{\pi}{2}\left(e^{-i \pi \nu} Y_\nu\left(\sqrt[4]{-1}\; z\right) + (3\, i \cos(\pi \nu) - \sin(\pi \nu))\, J_\nu\left(\sqrt[4]{-1}\; z\right)\right) & \frac{3 \pi}{4} < \arg(z) \leq \pi \\ -\frac{\pi}{2} e^{i \pi \nu}\left(Y_\nu\left(\sqrt[4]{-1}\; z\right) - i\, J_\nu\left(\sqrt[4]{-1}\; z\right)\right) & \text{True} \end{cases} /;\ \nu \in \mathbb{Z}$$

Theorems

History

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