Log

Notations

Traditional name

Natural logarithm

Traditional notation

$\log(z)$

Mathematica StandardForm notation

Log[z]

Primary definition

01.04.02.0001.01

$$\log(z) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1} (z-1)^k}{k} \;/;\; |z-1| < 1$$

Specific values

Specialized values

01.04.03.0001.01

$$\log(e^n) = n \;/;\; n \in \mathbb{Z}$$

Values at fixed points

01.04.03.0002.01

$$\log(0) = -\infty$$

01.04.03.0003.01

$$\lim_{z \to +0} \log(z) = -\infty$$

01.04.03.0004.01

$$\log(1) = 0$$

01.04.03.0005.01

$$\log(-1) = i\pi$$

01.04.03.0006.01

$$\log(e) = 1$$

01.04.03.0007.01

$$\log(-e) = 1 + i\pi$$

01.04.03.0008.01

$$\log(i) = \frac{\pi i}{2}$$

01.04.03.0009.01

$$\log(-i) = -\frac{\pi i}{2}$$

01.04.03.0010.01

$$\log(\varphi) = \operatorname{csch}^{-1}(2)$$

Values at infinities

01.04.03.0011.01

$$\log(\infty) = \infty$$

01.04.03.0012.01

$$\log(-\infty) = \infty$$

01.04.03.0013.01

$$\log(i\infty) = \infty$$

01.04.03.0014.01

$$\log(-i\infty) = \infty$$

01.04.03.0015.01

$$\log(\tilde{\infty}) = \infty$$

General characteristics

Domain and analyticity

$\log(z)$ is an analytical function of z which is defined over the whole complex z-plane.

01.04.04.0001.01

$$z \longrightarrow \log(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

01.04.04.0002.01

$$\log(\bar{z}) = \overline{\log(z)} \;/;\; z \notin (-\infty, 0)$$

Quasi-homogeneity

01.04.04.0003.01

$$\log(\lambda z) = \log(z) + \log(\lambda) \;/;\; \lambda > 0 \lor z + \lambda \geq 0$$

Scale symmetry

01.04.04.0004.01

$$\log(z^a) = a\log(z) \;/;\; z > 0 \land a \in \mathbb{R} \lor -\pi < \operatorname{Im}(a\log(z)) \leq \pi$$

Periodicity

No periodicity

Poles and essential singularities

The function $\log(z)$ does not have poles and essential singularities.

01.04.04.0005.01

$$\mathcal{S}ing_z(\log(z)) == \{\}$$

Branch points

The function $\log(z)$ has two branch points: $z == 0$, $z = \tilde{\infty}$.

01.04.04.0006.01

$$\mathcal{BP}_z(\log(z)) == \{0, \tilde{\infty}\}$$

01.04.04.0007.01

$$\mathcal{R}_z(\log(z), 0) == \log$$

01.04.04.0008.01

$$\mathcal{R}_z(\log(z), \tilde{\infty}) == \log$$

Branch cuts

The function $\log(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$ where it is continuous from above.

01.04.04.0009.01

$$\mathcal{BC}_z(\log(z)) == \{\{(-\infty, 0), -i\}\}$$

01.04.04.0010.01

$$\lim_{\epsilon \to +0} \log(x + i\,\epsilon) == \log(x) \;/;\; x < 0$$

01.04.04.0013.01

$$\lim_{\epsilon \to +0} \log(x + i\,\epsilon) = \log(|x|) + \pi\, i \;/;\; x < 0$$

01.04.04.0014.01

$$\lim_{\epsilon \to +0} \log(x + i\,\epsilon) = \log(-x) + \pi\, i \;/;\; x < 0$$

01.04.04.0015.01

$$\lim_{\epsilon \to +0} \log(x - i\,\epsilon) = \log(|x|) - \pi\, i \;/;\; x < 0$$

01.04.04.0011.01

$$\lim_{\epsilon \to +0} \log(x - i\,\epsilon) == \log(-x) - i\,\pi \;/;\; x < 0$$

01.04.04.0012.01

$$\lim_{\epsilon \to +0} \log(x - i\,\epsilon) == \log(x) - 2\, i\,\pi \;/;\; x < 0$$

Analytic continuations

$\log(z)$ is chosen to be the principal branch of the general logarithmic function that has infinitely many sheets, each differing from $\log(z)$ by $2\pi i k,\ k \in \mathbb{Z}$.

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

01.04.06.0013.01

$$\log(z) \propto \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \log(z_0) + \frac{z-z_0}{z_0} - \frac{(z-z_0)^2}{2 z_0^2} + \ldots \text{ /; } (z \to z_0)$$

01.04.06.0014.01

$$\log(z) \propto \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \log(z_0) + \frac{z-z_0}{z_0} - \frac{(z-z_0)^2}{2 z_0^2} + O\left((z-z_0)^3\right)$$

01.04.06.0015.01

$$\log(z) = \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^{k-1} z_0^{-k}}{k} (z-z_0)^k$$

01.04.06.0016.01

$$\log(z) == 2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{z}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^{k-1} z_0^{-k}}{k} (z-z_0)^k$$

01.04.06.0017.01

$$\log(z) = \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \log(z_0) + \frac{z-z_0}{z_0}\, {}_2F_1\left(1, 1; 2; -\frac{z-z_0}{z_0}\right)$$

01.04.06.0018.01

$$\log(z) \propto \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \log(z_0)\,(1 + O(z-z_0))$$

Expansions of $\log(f(z))$ at $z == z_0$

01.04.06.0019.01

$$\log(f(z)) \propto 2 i \pi \left\lfloor \frac{\pi - \arg(f(z_0)) - \arg\left(\frac{f(z)}{f(z_0)}\right)}{2\pi} \right\rfloor + \log(f(z_0)) + \frac{f'(z_0)\,(z-z_0)}{f(z_0)} +$$
$$\frac{1}{2}\left(\frac{f''(z_0)}{f(z_0)} - \frac{f'(z_0)^2}{f(z_0)^2}\right)(z-z_0)^2 + \frac{1}{6}\left(\frac{2 f'(z_0)^3}{f(z_0)^3} - \frac{3 f''(z_0)\, f'(z_0)}{f(z_0)^2} + \frac{f^{(3)}(z_0)}{f(z_0)}\right)(z-z_0)^3 + \ldots \text{ /; } (z \to z_0) \land f(z_0) \neq 0$$

01.04.06.0020.01

$$\log(f(z)) = 2\,i\,\pi \left\lfloor \frac{\pi - \arg(f(z_0)) - \arg\left(\frac{f(z)}{f(z_0)}\right)}{2\,\pi} \right\rfloor + \log(f(z_0)) + \sum_{s=0}^{\infty} \sum_{k=0}^{s} \frac{(-1)^k}{k+1} \left(\frac{f'(z_0)}{f(z_0)}\right)^{k+1} p_{k+1,s-k}\,(z-z_0)^{s+1} \,/;$$

$$f(z_0) \neq 0 \bigwedge f'(z_0) \neq 0 \bigwedge p_{j,0} = 1 \bigwedge p_{j,k} = \frac{1}{f'(z_0)\,k} \sum_{m=1}^{k} \frac{j\,m + m - k}{(m+1)!} f^{(m+1)}(z_0)\, p_{j,k-m} \bigwedge k \in \mathbb{N}^+$$

01.04.06.0021.01

$$\log(f(z)) = 2\,i\,\pi \left\lfloor \frac{\pi - \arg(f(z_0)) - \arg\left(\frac{f(z)}{f(z_0)}\right)}{2\,\pi} \right\rfloor + \log(f(z_0)) + \sum_{s=0}^{\infty} \sum_{k=0}^{\frac{s}{u}} \frac{(-1)^k}{k+1} \left(\frac{f^{(u)}(z_0)}{f(z_0)\,u!}\right)^{k+1} p_{k+1,s-u\,k}\,(z-z_0)^{s+u} \,/;$$

$$f(z_0) \neq 0 \bigwedge \left(f^{(k)}(z_0) = 0 \,/;\, 1 \leq k \leq u-1\right) \bigwedge f^{(u)}(z_0) \neq 0 \bigwedge$$
$$p_{j,0} = 1 \bigwedge p_{j,k} = \frac{u!}{f^{(u)}(z_0)\,k} \sum_{m=1}^{k} \frac{j\,m + m - k}{(m+u)!\,(m+1)!} f^{(m+u)}(z_0)\, p_{j,k-m} \bigwedge k \in \mathbb{N}^+$$

01.04.06.0022.01

$$\log(f(z)) \propto \left(2\,i\,\pi \left\lfloor \frac{\pi - \arg(f(z_0)) - \arg\left(\frac{f(z)}{f(z_0)}\right)}{2\,\pi} \right\rfloor + \log(f(z_0))\right)(1 + O(z - z_0)) \,/;\, (z \to z_0) \land f(z_0) \neq 0$$

Expansions on branch cuts

For the function itself

01.04.06.0023.01

$$\log(z) \propto 2\,i\,\pi \left\lfloor \frac{\arg(z-x)}{2\,\pi} \right\rfloor + \log(x) + \frac{z-x}{x} - \frac{(z-x)^2}{2\,x^2} + \ldots \,/;\, (z \to x) \land x < 0$$

01.04.06.0024.01

$$\log(z) \propto 2\,i\,\pi \left\lfloor \frac{\arg(z-x)}{2\,\pi} \right\rfloor + \log(x) + \frac{z-x}{x} - \frac{(z-x)^2}{2\,x^2} + O\left((z-x)^3\right) \,/;\, x < 0$$

01.04.06.0025.01

$$\log(z) = 2\,i\,\pi \left\lfloor \frac{\arg(z-x)}{2\,\pi} \right\rfloor + \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^{k-1}\,x^{-k}}{k} (z-x)^k \,/;\, x < 0$$

01.04.06.0026.01

$$\log(z) = 2\,i\,\pi \left\lfloor \frac{\arg(z-x)}{2\,\pi} \right\rfloor + \frac{z-x}{x}\, {}_2F_1\left(1, 1; 2; -\frac{z-x}{x}\right) + \log(x) \,/;\, x < 0$$

01.04.06.0027.01

$$\log(z) \propto 2\,i\,\pi \left\lfloor \frac{\arg(z-x)}{2\,\pi} \right\rfloor + \log(x)\,(1 + O(z-x)) \,/;\, x < 0$$

Expansions at $z = 1$

For the function itself

01.04.06.0005.02

$$\log(z) \propto z - 1 - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} - \ldots \;/;\; (z \to 1)$$

01.04.06.0028.01

$$\log(z) \propto z - 1 - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} - O\big((z-1)^4\big)$$

01.04.06.0006.01

$$\log(z) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1} (z-1)^k}{k} \;/;\; |z-1| < 1$$

01.04.06.0007.01

$$\log(z) = (z-1)\, {}_2F_1(1, 1; 2; 1-z)$$

01.04.06.0008.02

$$\log(z) \propto z - 1 + O\big((z-1)^2\big)$$

01.04.06.0029.01

$$\log(z) = F_\infty(z) \;/;\; \left(\left(F_n(z) = (z-1) \sum_{k=0}^{n} \frac{(-1)^k (z-1)^k}{k+1} = \log(z) + \mathrm{B}_{1-z}(n+2, 0)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions of log(1 + *z*) at *z* == 0

For the function itself

01.04.06.0001.02

$$\log(1+z) \propto z - \frac{z^2}{2} + \frac{z^3}{3} - \ldots \;/;\; (z \to 0)$$

01.04.06.0030.01

$$\log(1+z) \propto z - \frac{z^2}{2} + \frac{z^3}{3} - O\big(z^4\big)$$

01.04.06.0002.01

$$\log(1+z) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1} z^k}{k} \;/;\; |z| < 1$$

01.04.06.0003.01

$$\log(1+z) = z\, {}_2F_1(1, 1; 2; -z)$$

01.04.06.0004.02

$$\log(1+z) \propto z + O\big(z^2\big)$$

01.04.06.0031.01

$$\log(1+z) = F_\infty(z) \;/;\; \left(\left(F_n(z) = z \sum_{k=0}^{n} \frac{(-1)^k z^k}{k+1} = \log(1+z) + \mathrm{B}_{-z}(n+2, 0)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions of $\log(f(z))$ at $z == 0$

01.04.06.0032.01

$$\log(f(z)) \propto 2\,i\,\pi \left\lfloor \frac{\pi - \arg\left(\frac{f(z)}{c_0}\right) - \arg(c_0)}{2\,\pi} \right\rfloor + \log(c_0) + \frac{c_1}{c_0}\,z + \left(\frac{c_2}{c_0} - \frac{c_1^2}{2\,c_0^2}\right) z^2 + \left(\frac{c_1^3}{3\,c_0^3} - \frac{c_2\,c_1}{c_0^2} + \frac{c_3}{c_0}\right) z^3 + \ldots \;/;$$

$$(z \to 0) \bigwedge f(z) == \sum_{k=0}^{\infty} c_k\,z^k \bigwedge c_0 \neq 0$$

01.04.06.0033.01

$$\log(f(z)) == z \sum_{s=0}^{\infty} \sum_{k=0}^{s} \frac{(-1)^k}{k+1} \left(\frac{c_1}{c_0}\right)^{k+1} p_{k+1,s-k}\,z^s + 2\,i\,\pi \left\lfloor \frac{\pi - \arg\left(\frac{f(z)}{c_0}\right) - \arg(c_0)}{2\,\pi} \right\rfloor + \log(c_0) \;/;$$

$$f(z) == \sum_{k=0}^{\infty} c_k\,z^k \bigwedge c_0 \neq 0 \bigwedge c_1 \neq 0 \bigwedge p_{j,0} == 1 \bigwedge p_{j,k} == \frac{1}{c_1\,k} \sum_{m=1}^{k} (j\,m + m - k)\,c_{m+1}\,p_{j,k-m} \bigwedge k \in \mathbb{N}^+$$

01.04.06.0034.01

$$\log(f(z)) == z^u \sum_{s=0}^{\infty} \sum_{k=0}^{\frac{s}{u}} \frac{(-1)^k}{k+1} \left(\frac{c_u}{c_0}\right)^{k+1} p_{k+1,s-u\,k}\,z^s + 2\,i\,\pi \left\lfloor \frac{\pi - \arg\left(\frac{f(z)}{c_0}\right) - \arg(c_0)}{2\,\pi} \right\rfloor + \log(c_0) \;/;\; f(z) == \sum_{k=0}^{\infty} c_k\,z^k \bigwedge$$

$$c_0 \neq 0 \bigwedge (c_k == 0 \;/;\; 1 \leq k \leq u-1) \bigwedge c_u \neq 0 \bigwedge p_{j,0} == 1 \bigwedge p_{j,k} == \frac{1}{c_u\,k} \sum_{m=1}^{k} (j\,m + m - k)\,c_{m+u}\,p_{j,k-m} \bigwedge k \in \mathbb{N}^+$$

01.04.06.0035.01

$$\log(f(z)) \propto \left(2\,i\,\pi \left\lfloor \frac{\pi - \arg\left(\frac{f(z)}{c_0}\right) - \arg(c_0)}{2\,\pi} \right\rfloor + \log(c_0)\right)(1 + O(z)) \;/;\; (z \to 0) \bigwedge f(z) == \sum_{k=0}^{\infty} c_k\,z^k \bigwedge c_0 \neq 0$$

Expansions of $(c \log(1+z))^{\alpha}$ at $z == 0$

01.04.06.0036.01

$$(c \log(1+z))^{\alpha} = e^{2\,i\,\alpha\,\pi \left\lfloor -\frac{\arg(c)}{2\,\pi} - \frac{1}{2\,\pi} \arg\left(\frac{\log(z+1)}{e}\right) + \frac{1}{2} - \frac{\arg(z)}{2\,\pi} \right\rfloor} c^{\alpha}\,z^{\alpha}\,\alpha \sum_{k=0}^{\infty} \binom{k-\alpha}{k} \sum_{j=0}^{k} \frac{(-1)^j}{\alpha - j} \binom{k}{j} p_{j,k}\,z^k \;/;$$

$$c \neq 0 \bigwedge c_k = \frac{c\,(-1)^k}{k+1} \bigwedge p_{j,0} = 1 \bigwedge p_{j,k} = \frac{1}{c\,k} \sum_{m=1}^{k} (j\,m - k + m)\,c_m\,p_{j,k-m} \bigwedge k \in \mathbb{N}^+$$

Expansions of $\log(1 + z)$ at $z == \infty$

For the function itself

01.04.06.0037.01

$$\log(1+z) \propto \log(z) + \frac{1}{z} - \frac{1}{2\,z^2} + \frac{1}{3\,z^3} - \ldots \;/;\; (|z| \to \infty)$$

01.04.06.0038.01

$$\log(1+z) \propto \log(z) + \frac{1}{z} - \frac{1}{2z^2} + \frac{1}{3z^3} + O\left(\frac{1}{z^4}\right)$$

01.04.06.0039.01

$$\log(1+z) = \log(z) + \sum_{k=1}^{\infty} \frac{(-1)^{k-1} z^{-k}}{k} \;/;\; |z| > 1$$

01.04.06.0040.01

$$\log(1+z) = 2\,i\,\pi \left\lfloor \frac{\pi - \arg(z) - \arg\left(1 + \frac{1}{z}\right)}{2\pi} \right\rfloor + \log\left(1 + \frac{1}{z}\right) + \log(z)$$

01.04.06.0041.01

$$\log(1+z) = \log(z) + \log\left(1 + \frac{1}{z}\right) \;/;\; |z| > 1$$

01.04.06.0042.01

$$\log(1+z) = \log(z) + \frac{1}{z}\, {}_2F_1\left(1, 1; 2; -\frac{1}{z}\right) \;/;\; |z| > 1$$

01.04.06.0043.01

$$\log(1+z) \propto \log(z) + O\left(\frac{1}{z}\right)$$

Asymptotic series expansions

01.04.06.0009.01

$$\log(z) \propto \log(z) \;/;\; (z \to 0)$$

01.04.06.0010.01

$$\log(z) \propto \log(z) \;/;\; (|z| \to \infty)$$

Residue representations

01.04.06.0011.01

$$\log(1+z) = \sum_{j=1}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(-s)^2 z^{-s}}{\Gamma(1-s)}\,\Gamma(s+1)\right)(-j) \;/;\; |z| < 1$$

01.04.06.0012.02

$$\log(1+z) = \operatorname{res}_s\left(\Gamma(s+1)\, z^{-s}\, \frac{\Gamma(-s)}{s}\right)(0) + \sum_{j=1}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s+1)\, z^{-s}}{s}\,\Gamma(-s)\right)(j) \;/;\; |z| > 1$$

Integral representations

On the real axis

Of the direct function

01.04.07.0001.01

$$\log(z) = \int_1^z \frac{1}{t}\, dt$$

Contour integral representations

01.04.07.0002.01

$$\log(z+1) = \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s+1)\,\Gamma(-s)^2\,z^{-s}}{\Gamma(1-s)}\,ds \;/;\; |\arg(z)| < \pi$$

01.04.07.0003.01

$$\log(z+1) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(s+1)\,\Gamma(-s)^2\,z^{-s}}{\Gamma(1-s)}\,ds \;/;\; -1 < \gamma < 0 \land |\arg(z)| < \pi$$

Product representations

01.04.08.0001.01

$$\log(z) = \frac{z-1}{e-1}\prod_{k=1}^{\infty}\frac{e^{2^{-k}}+1}{z^{2^{-k}}+1}$$

Ruffa Anthony

Limit representations

01.04.09.0001.01

$$\log(z) = \lim_{\epsilon\to 0}\frac{z^{\epsilon}-1}{\epsilon}$$

01.04.09.0002.01

$$\log(z) = \lim_{\omega\to\infty}\omega\left(z^{1/\omega}-1\right)$$

Continued fraction representations

01.04.10.0001.01

$$\log(1+z) = \cfrac{z}{1+\cfrac{z}{2+\cfrac{z}{3+\cfrac{4z}{4+\cfrac{4z}{5+\cfrac{9z}{6+\cfrac{9z}{7+\ldots}}}}}}} \;/;\; z \notin (-\infty, -1)$$

01.04.10.0002.01

$$\log(1+z) = \frac{z}{1+\mathrm{K}_k\left(\left\lfloor\frac{k+1}{2}\right\rfloor^2 z,\, k+1\right)_1^{\infty}} \;/;\; z \notin (-\infty, -1)$$

01.04.10.0003.01

$$\log(1+z) = \cfrac{z}{1+\cfrac{z}{2+\cfrac{z}{3+\cfrac{2z}{2+\cfrac{2z}{5+\cfrac{3z}{2+\cfrac{3z}{7+\ldots}}}}}}} \;/;\; z \notin (-\infty, -1)$$

01.04.10.0004.01

$$\log(1+z) = \frac{z}{1+\mathrm{K}_k\left(\left\lfloor \frac{k+1}{2} \right\rfloor z, \frac{1}{2}\left((-1)^k (k-1)+k+3\right)\right)_1^\infty} \;/;\; z \notin (-\infty, -1)$$

01.04.10.0005.01

$$\log\left(\frac{1+z}{1-z}\right) = \cfrac{2z}{1-\cfrac{z^2}{3-\cfrac{4z^2}{5-\cfrac{9z^2}{7-\cfrac{16z^2}{9-\cfrac{25z^2}{11-\cfrac{36z^2}{13-\ldots}}}}}}} \;/;\; z \notin (-\infty, -1) \land z \notin (1, \infty)$$

01.04.10.0006.01

$$\log\left(\frac{1+z}{1-z}\right) = \frac{2z}{1+\mathrm{K}_k\left(-k^2 z^2, 2k+1\right)_1^\infty} \;/;\; z \notin (-\infty, -1) \land z \notin (1, \infty)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

01.04.13.0001.01

$$z\, w'(z) - 1 = 0 \;/;\; w(z) = \log(z) \land w(1) = 0$$

01.04.13.0002.01

$$z\, w''(z) + w'(z) = 0 \;/;\; w(z) = c_1 + c_2 \log(z)$$

01.04.13.0003.01

$$W_z(1, \log(z)) = \frac{1}{z}$$

01.04.13.0004.01

$$w''(z) + \left(\frac{g'(z)}{g(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) = 0 \;/;\; w(z) = c_1 \log(g(z)) + c_2$$

01.04.13.0005.01

$$W_z(\log(g(z)),\, 1) = -\frac{g'(z)}{g(z)}$$

01.04.13.0006.01

$$w''(z) + \left(\frac{g'(z)}{g(z)} - \frac{2\,h'(z)}{h(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) + \left(\frac{2\,h'(z)^2}{h(z)^2} + \frac{g''(z)\,h'(z)}{h(z)\,g'(z)} - \frac{g'(z)\,h'(z)}{g(z)\,h(z)} - \frac{h''(z)}{h(z)}\right) w(z) = 0\;/;$$
$$w(z) = c_1\,h(z)\log(g(z)) + c_2\,h(z)$$

01.04.13.0007.01

$$W_z(h(z)\log(g(z)),\, h(z)) = -\frac{h(z)^2\,g'(z)}{g(z)}$$

01.04.13.0008.01

$$z^2\,w''(z) + (1 - 2\,s)\,z\,w'(z) + w(z)\,s^2 = 0\;/;\; w(z) = c_1\,z^s\log(a\,z^r) + c_2\,z^s$$

01.04.13.0009.01

$$W_z(z^s\log(a\,z^r),\, z^s) = -r\,z^{2s-1}$$

01.04.13.0010.01

$$w''(z) - 2\log(s)\,w'(z) + \log^2(s)\,w(z) = 0\;/;\; w(z) = c_1\,s^z\log(a\,r^z) + c_2\,s^z$$

01.04.13.0011.01

$$W_z(s^z\log(a\,r^z),\, s^z) = -s^{2z}\log(r)$$

Transformations

Transformations and argument simplifications

Argument involving complex characteristics

01.04.16.0003.01

$$\log(|z|) = \log(z) - i\arg(z)$$

01.04.16.0002.01

$$\log\left(\frac{z}{|z|}\right) = i\arg(z)$$

01.04.16.0001.01

$$\log(\operatorname{sgn}(z)) = i\arg(z)$$

01.04.16.0046.01

$$\log(\bar{z}) = \log(z) - 2\,i\arg(z)\;/;\; \arg(z) \neq \pi$$

01.04.16.0047.01

$$\log(\bar{z}) = \log(z) - 2\,i\left(\arg(z) - \pi\left\lfloor\frac{\arg(z)+\pi}{2\pi}\right\rfloor\right)$$

Argument involving basic arithmetic operations

01.04.16.0007.01

$$\log(-z) = \log(z) - \pi\,i\;/;\; \operatorname{Im}(z) > 0 \lor \operatorname{Im}(z) = 0 \land z < 0$$

01.04.16.0008.01

$$\log(-z) = \log(z) + \pi\,i\;/;\; \operatorname{Im}(z) < 0 \lor \operatorname{Im}(z) = 0 \land z > 0$$

01.04.16.0048.01

$$\log(-z) = \log(z) - \pi i \;/;\; \arg(z) > 0$$

01.04.16.0049.01

$$\log(-z) = \log(z) + \pi i \;/;\; \arg(z) \leq 0$$

01.04.16.0050.01

$$\log(-z) = \log(z) + \frac{\pi \sqrt{-z}}{\sqrt{z}}$$

01.04.16.0051.01

$$\log(-z) = i \pi + 2 i \pi \left\lfloor -\frac{\arg(z)}{2 \pi} \right\rfloor + \log(z)$$

01.04.16.0052.01

$$\log(i z) = \frac{i \pi}{2} + \log(z) \;/;\; \arg(z) \leq \frac{\pi}{2}$$

01.04.16.0053.01

$$\log(i z) = -\frac{3 i \pi}{2} + \log(z) \;/;\; \arg(z) > \frac{\pi}{2}$$

01.04.16.0054.01

$$\log(i z) = \log(z) - \frac{i \pi}{2} + \frac{\sqrt[4]{-1}\, \pi \sqrt{i z}}{\sqrt{z}}$$

01.04.16.0055.01

$$\log(i z) = \frac{i \pi}{2} + 2 i \pi \left\lfloor \frac{1}{4} - \frac{\arg(z)}{2 \pi} \right\rfloor + \log(z)$$

01.04.16.0056.01

$$\log(-i z) = -\frac{i \pi}{2} + \log(z) \;/;\; \arg(z) > -\frac{\pi}{2}$$

01.04.16.0057.01

$$\log(-i z) = \frac{3 i \pi}{2} + \log(z) \;/;\; \arg(z) \leq -\frac{\pi}{2}$$

01.04.16.0058.01

$$\log(-i z) = \log(z) + \frac{i \pi}{2} - \frac{(-1)^{3/4} \pi \sqrt{-i z}}{\sqrt{z}}$$

01.04.16.0059.01

$$\log(-i z) = -\frac{i \pi}{2} + 2 i \pi \left\lfloor \frac{3}{4} - \frac{\arg(z)}{2 \pi} \right\rfloor + \log(z)$$

01.04.16.0009.01

$$\log\left(\frac{1}{z}\right) = -\log(z) \;/;\; z \notin (-\infty, 0)$$

01.04.16.0060.01

$$\log\left(\frac{1}{z}\right) = -\log(z) \;/;\; \arg(z) \neq \pi$$

01.04.16.0061.01

$$\log\left(\frac{1}{z}\right) = 2\,i\,\pi - \log(z) \;/;\; \arg(z) = \pi$$

01.04.16.0062.01

$$\log\left(-\frac{1}{z}\right) = -\log(z) - \pi\,i\left(\sqrt{\frac{1}{z}}\,\sqrt{z} - 1\right)$$

01.04.16.0063.01

$$\log\left(\frac{1}{z}\right) = 2\,i\,\pi\left\lfloor\frac{\arg(z)}{2\,\pi} + \frac{1}{2}\right\rfloor - \log(z)$$

01.04.16.0010.01

$$\log\left(-\frac{1}{z}\right) = i\,\pi - \log(z) \;/;\; \operatorname{Im}(z) \geq 0$$

01.04.16.0011.01

$$\log\left(-\frac{1}{z}\right) = -i\,\pi - \log(z) \;/;\; \operatorname{Im}(z) < 0$$

01.04.16.0064.01

$$\log\left(-\frac{1}{z}\right) = \pi\sqrt{-\frac{1}{z}}\,\sqrt{z} - \log(z)$$

01.04.16.0065.01

$$\log\left(-\frac{1}{z}\right) = i\,\pi + 2\,i\,\pi\left\lfloor\frac{\arg(z)}{2\,\pi}\right\rfloor - \log(z)$$

01.04.16.0066.01

$$\log\left(\frac{i}{z}\right) = \frac{i\,\pi}{2} - \log(z) \;/;\; \arg(z) \geq -\frac{\pi}{2}$$

01.04.16.0067.01

$$\log\left(\frac{i}{z}\right) = -\frac{3\,i\,\pi}{2} - \log(z) \;/;\; \arg(z) < -\frac{\pi}{2}$$

01.04.16.0068.01

$$\log\left(\frac{i}{z}\right) = \frac{i\,\pi}{2} + 2\,i\,\pi\left\lfloor\frac{\arg(z)}{2\,\pi} + \frac{1}{4}\right\rfloor - \log(z)$$

01.04.16.0069.01

$$\log\left(-\frac{i}{z}\right) = -\frac{\pi\,i}{2} - \log(z) \;/;\; \arg(z) < \frac{\pi}{2}$$

01.04.16.0070.01

$$\log\left(-\frac{i}{z}\right) = \frac{3\,i\,\pi}{2} - \log(z) \;/;\; \arg(z) \geq \frac{\pi}{2}$$

01.04.16.0071.01

$$\log\left(-\frac{i}{z}\right) = -\frac{1}{2}(i\,\pi) + 2\,i\,\pi\left\lfloor\frac{\arg(z)}{2\,\pi} + \frac{3}{4}\right\rfloor - \log(z)$$

Addition formulas

01.04.16.0012.01

$$\log(a+z) = 2\tanh^{-1}\left(\frac{z}{2a+z}\right) + \log(a) + \log\left(\frac{1}{2a+z}\right) + \log(2a+z) \,/;\, a > 0$$

01.04.16.0072.01

$$\log(x+i\,y) = \frac{1}{2}\log\left(x^2+y^2\right) + i\arg(x+i\,y)$$

Half-angle formulas

01.04.16.0013.01

$$\log\left(\frac{z}{2}\right) = \log(z) - \log(2)$$

Multiple arguments

For products

01.04.16.0014.01

$$\log(a\,z) = \log(a) + \log(z) \,/;\, a > 0$$

01.04.16.0015.01

$$\log(z_1\,z_2) = \log(z_1) + \log(z_2) \,/;\, z_1 + z_2 \geq 0$$

01.04.16.0016.01

$$\log\left(z - z^2\right) = \log(1-z) + \log(z)$$

01.04.16.0017.01

$$\log\left(-z^2 - z\right) = \log(-z) + \log(z+1)$$

01.04.16.0073.01

$$\log(z_1\,z_2) = \log(z_1) + \log(z_2) \,/;\, \arg(z_1) \leq 0 \bigwedge -\arg(z_1) - \pi < \arg(z_2) \bigvee \arg(z_1) \geq 0 \bigwedge \arg(z_2) \leq \pi - \arg(z_1)$$

01.04.16.0074.01

$$\log(z_1\,z_2) = \log(z_1) + \log(z_2) - 2\,i\,\pi \,/;\, \arg(z_1) \geq 0 \bigwedge \arg(z_2) > \pi - \arg(z_1)$$

01.04.16.0075.01

$$\log(z_1\,z_2) = \log(z_1) + \log(z_2) + 2\,i\,\pi \,/;\, \arg(z_1) \leq 0 \bigwedge \arg(z_2) \leq -\arg(z_1) - \pi$$

01.04.16.0018.01

$$\log(z_1\,z_2) = \log(z_1) + \log(z_2) + 2\,i\,\pi\left\lfloor\frac{\pi - \arg(z_1) - \arg(z_2)}{2\pi}\right\rfloor$$

01.04.16.0076.01

$$\log\left(\prod_{k=1}^{n} z_k\right) = \sum_{k=1}^{n}\log(z_k) + 2\,i\,\pi\left\lfloor\frac{\pi - \sum_{k=1}^{n}\arg(z_k)}{2\pi}\right\rfloor \,/;\, n \in \mathbb{N}^+$$

For quotients

01.04.16.0019.01

$$\log\left(\frac{z}{z+1}\right) = \log(z) - \log(z+1)$$

01.04.16.0020.01

$$\log\left(\frac{z}{z-1}\right) = \log(-z) - \log(1-z)$$

01.04.16.0021.01

$$\log\left(\frac{z_1}{z_2}\right) = \log\left(\frac{z_1}{z_2 - z_1}\right) - \log\left(\frac{z_2}{z_2 - z_1}\right) /; z_2 - z_1 \in \mathbb{R} \land z_2 \neq z_1$$

01.04.16.0022.01

$$\log\left(\frac{z_1}{z_2}\right) = \log(z_1) - \log(z_2) /; z_2 - z_1 \geq 0$$

01.04.16.0023.01

$$\log\left(\frac{z_1}{z_2}\right) = \log(-z_1) - \log(-z_2) /; z_2 - z_1 < 0$$

01.04.16.0077.01

$$\log\left(\frac{z_1}{z_2}\right) = \log(z_1) - \log(z_2) /; \arg(z_1) \leq 0 \land \arg(z_2) < \arg(z_1) + \pi \lor \arg(z_1) > 0 \land \arg(z_2) \geq \arg(z_1) - \pi$$

01.04.16.0078.01

$$\log\left(\frac{z_1}{z_2}\right) = \log(z_1) - \log(z_2) - 2\,i\,\pi /; \arg(z_1) \geq 0 \land \arg(z_2) < \arg(z_1) - \pi$$

01.04.16.0079.01

$$\log\left(\frac{z_1}{z_2}\right) = \log(z_1) - \log(z_2) + 2\,i\,\pi /; \arg(z_1) \leq 0 \land \arg(z_2) \geq \arg(z_1) + \pi$$

01.04.16.0024.01

$$\log\left(\frac{z_1}{z_2}\right) = \log(z_1) - \log(z_2) + 2\,i\,\pi \left\lfloor \frac{\pi - \arg(z_1) + \arg(z_2)}{2\,\pi} \right\rfloor$$

01.04.16.0025.01

$$\log\left(\frac{a+z}{b+z}\right) = \log\left(\frac{a+z}{b-a}\right) - \log\left(\frac{b+z}{b-a}\right) /; a - b \in \mathbb{R} \land a \neq b$$

01.04.16.0026.01

$$\log\left(\frac{z}{1-z}\right) = \log(z) + \log\left(\frac{1}{1-z}\right)$$

01.04.16.0027.01

$$\log\left(-\frac{z}{z+1}\right) = \log(-z) + \log\left(\frac{1}{z+1}\right)$$

01.04.16.0028.01

$$\log\left(\frac{z_1}{z_2}\right) = \log\left(\frac{z_1 + z_2}{z_2}\right) + \log\left(\frac{z_1}{z_1 + z_2}\right) /; z_1 + z_2 \in \mathbb{R} \land z_1 + z_2 \neq 0$$

01.04.16.0029.01

$$\log\left(\frac{z_1}{z_2}\right) = \log(z_1) + \log\left(\frac{1}{z_2}\right) /; z_1 + z_2 \geq 0$$

01.04.16.0030.01

$$\log\left(\frac{z_1}{z_2}\right) = \log(-z_1) + \log\left(-\frac{1}{z_2}\right) /; z_1 + z_2 < 0$$

01.04.16.0031.01

$$\log\left(\frac{a+z}{b-z}\right) = \log\left(\frac{a+z}{a+b}\right) + \log\left(\frac{a+b}{b-z}\right) /; a + b \in \mathbb{R} \land a + b \neq 0$$

Power of arguments

01.04.16.0032.01

$$\log\left(\sqrt{z}\right) = \frac{\log(z)}{2}$$

01.04.16.0033.01

$$\log\left(\sqrt{z^2}\right) = \log(z) /; \operatorname{Re}(z) > 0 \lor \operatorname{Re}(z) = 0 \land \operatorname{Im}(z) > 0$$

01.04.16.0080.01

$$\log\left(\sqrt{z^2}\right) = \log(z) - \pi i /; \frac{\pi}{2} < \arg(z) \leq \pi$$

01.04.16.0081.01

$$\log\left(\sqrt{z^2}\right) = \log(z) + \pi i /; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.04.16.0082.01

$$\log\left(\sqrt{z^2}\right) = \log(z) + \frac{\pi\left(\sqrt{z^2} - z\right)}{2\sqrt{-z^2}}$$

01.04.16.0034.01

$$\log\left(z^{1/n}\right) = \frac{\log(z)}{n} /; n \in \mathbb{Z} \land n \neq 0 \land n \neq -1$$

01.04.16.0083.01

$$\log\left(z^2\right) = 2\log(z) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.04.16.0084.01

$$\log\left(z^2\right) = 2\log(z) + 2\pi i /; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.04.16.0085.01

$$\log\left(z^2\right) = 2\log(z) - 2\pi i /; \frac{\pi}{2} < \arg(z) \leq \pi$$

01.04.16.0086.01

$$\log\left(z^2\right) = 2\log(z) + 2 i \pi \left\lfloor \frac{1}{2} - \frac{\arg(z)}{\pi} \right\rfloor$$

01.04.16.0035.01

$$\log\left(z^2\right) = \log(-i z) + \log(i z)$$

01.04.16.0087.01

$$\log\left(-z^2\right) = \log(-z) + \log(z)$$

01.04.16.0036.01

$$\log(z^a) = a \log(z) /; a \in \mathbb{R} \bigwedge -\pi < a \arg(z) \leq \pi$$

01.04.16.0088.01

$$\log(z^a) = 2 i \pi k + a \log(z) /; a \in \mathbb{R} \bigwedge -\pi - 2\pi k < a \arg(z) \leq \pi - 2\pi k \bigwedge k \in \mathbb{Z}$$

01.04.16.0037.01

$$\log(z^a) = a \log(z) /; -\pi < \operatorname{Im}(a \log(z)) \leq \pi$$

01.04.16.0089.01

$$\log(z^a) == 2\,i\,\pi\,k + a\log(z)\ /;\ -2\,\pi\,k - \pi < \operatorname{Im}(a\log(z)) \leq \pi - 2\,\pi\,k \bigwedge k \in \mathbb{Z}$$

01.04.16.0038.01

$$\log(z^a) == a\log(z) + 2\,i\,\pi\left\lfloor \frac{\pi - \operatorname{Im}(a\log(z))}{2\,\pi} \right\rfloor$$

Exponent of arguments

01.04.16.0090.01

$$\log(e^z) == z\ /;\ -\pi < \operatorname{Im}(z) \leq \pi$$

01.04.16.0091.01

$$\log(e^z) == 2\,i\,\pi\,k + z\ /;\ -2\,\pi\,k - \pi < \operatorname{Im}(z) \leq \pi - 2\,\pi\,k \bigwedge k \in \mathbb{Z}$$

01.04.16.0092.01

$$\log(e^z) = z + 2\,i\,\pi\left\lfloor \frac{\pi - \operatorname{Im}(z)}{2\,\pi} \right\rfloor$$

01.04.16.0093.01

$$\log\left(e^{i\,z}\right) == i\,z + 2\,i\,\pi\left\lfloor \frac{\pi - \operatorname{Re}(z)}{2\,\pi} \right\rfloor$$

01.04.16.0094.01

$$\log(e^z) == \operatorname{Re}(z) + i\,\pi - i\,((\pi - \operatorname{Im}(z)) \bmod (2\,\pi))$$

01.04.16.0095.01

$$\log\left(e^{i\,z}\right) == i\,\pi - \operatorname{Im}(z) - i\,((\pi - \operatorname{Re}(z)) \bmod (2\,\pi))$$

Some functions of arguments

01.04.16.0096.01

$$\log(c\,z^a) == \log(c) + a\log(z) + 2\,i\,\pi\left\lfloor \frac{\pi - \arg(c) - \operatorname{Im}(a\log(z))}{2\,\pi} \right\rfloor$$

01.04.16.0097.01

$$\log(c\,e^z) == z + \log(c) + 2\,i\,\pi\left\lfloor \frac{\pi - \arg(c) - \operatorname{Im}(z)}{2\,\pi} \right\rfloor$$

01.04.16.0098.01

$$\log\left(x^a\,y^b\right) = a\log(x) + b\log(y) + 2\,i\,\pi\left\lfloor \frac{\pi - \operatorname{Im}(a\log(x)) - \operatorname{Im}(b\log(y))}{2\,\pi} \right\rfloor$$

01.04.16.0099.01

$$\log\left(x^a\,y^b\,z^c\right) = a\log(x) + b\log(y) + c\log(z) + 2\,i\,\pi\left\lfloor \frac{\pi - \operatorname{Im}(a\log(x)) - \operatorname{Im}(b\log(y)) - \operatorname{Im}(c\log(z)) + \pi}{2\,\pi} \right\rfloor$$

01.04.16.0100.01

$$\log\left(\prod_{k=1}^{n} z_k^{a_k}\right) = \sum_{k=1}^{n} a_k\log(z_k) + 2\,i\,\pi\left\lfloor \frac{\pi - \sum_{k=1}^{n}\operatorname{Im}(a_k\log(z_k))}{2\,\pi} \right\rfloor$$

01.04.16.0101.01

$$\log((-1)^r\,z^r) = \log(z)\,r + i\,\pi\,r\ /;\ 0 < r \leq \frac{1}{2}$$

Products, sums, and powers of the direct function

Products of the direct function

01.04.16.0039.01

$$\log(z_1)\log(z_2) == \log\left(z_1^{\log(z_2)}\right) /; z_1 > 0 \bigwedge z_2 > 0$$

01.04.16.0040.01

$$\log(z_1)\log(z_2) == \log\left(z_2^{\log(z_1)}\right) /; z_1 > 0 \bigwedge z_2 > 0$$

01.04.16.0041.01

$$\log(z_1)\log(z_2) == \log\left(z_1^{\log(z_2)}\right) - 2\,i\,\pi \left\lfloor \frac{\pi - \operatorname{Im}(\log(z_1)\log(z_2))}{2\,\pi} \right\rfloor$$

Sums of the direct function

01.04.16.0042.01

$$\log(z_1) + \log(z_2) == \log(z_1\,z_2) /; z_1 + z_2 \geq 0$$

01.04.16.0102.01

$$\log(z_1) + \log(z_2) == \log(z_1\,z_2) /; \arg(z_1) \leq 0 \bigwedge -\arg(z_1) - \pi < \arg(z_2) \bigvee \arg(z_1) \geq 0 \bigwedge \arg(z_2) \leq \pi - \arg(z_1)$$

01.04.16.0103.01

$$\log(z_1) + \log(z_2) == \log(z_1\,z_2) + 2\,i\,\pi /; \arg(z_1) \geq 0 \bigwedge \arg(z_2) > \pi - \arg(z_1)$$

01.04.16.0104.01

$$\log(z_1) + \log(z_2) == \log(z_1\,z_2) - 2\,i\,\pi /; \arg(z_1) \leq 0 \bigwedge \arg(z_2) \leq -\arg(z_1) - \pi$$

01.04.16.0043.01

$$\log(z_1) + \log(z_2) == \log(z_1\,z_2) - 2\,i\,\pi \left\lfloor \frac{\pi - \arg(z_1) - \arg(z_2)}{2\,\pi} \right\rfloor$$

01.04.16.0105.01

$$\sum_{k=1}^{n} \log(z_k) == \log\left(\prod_{k=1}^{n} z_k\right) - 2\,\pi\,i \left\lfloor \frac{\pi - \sum_{k=1}^{n} \arg(z_k)}{2\,\pi} \right\rfloor /; n \in \mathbb{N}^+$$

Differences of the direct function

01.04.16.0044.01

$$\log(z_1) - \log(z_2) == \log\left(\frac{z_1}{z_2}\right) /; z_2 - z_1 \geq 0$$

01.04.16.0106.01

$$\log(z_1) - \log(z_2) == \log\left(\frac{z_1}{z_2}\right) /; \arg(z_1) \leq 0 \bigwedge \arg(z_2) < \arg(z_1) + \pi \bigvee \arg(z_1) > 0 \bigwedge \arg(z_2) \geq \arg(z_1) - \pi$$

01.04.16.0107.01

$$\log(z_1) - \log(z_2) == \log\left(\frac{z_1}{z_2}\right) + 2\,i\,\pi /; \arg(z_1) \geq 0 \bigwedge \arg(z_2) < \arg(z_1) - \pi$$

01.04.16.0108.01

$$\log(z_1) - \log(z_2) == \log\left(\frac{z_1}{z_2}\right) - 2\,i\,\pi /; \arg(z_1) \leq 0 \bigwedge \arg(z_2) \geq \arg(z_1) + \pi$$

01.04.16.0045.01

$$\log(z_1) - \log(z_2) == \log\left(\frac{z_1}{z_2}\right) - 2\,i\,\pi \left\lfloor \frac{\pi - \arg(z_1) + \arg(z_2)}{2\,\pi} \right\rfloor$$

Linear combinations of the direct function

01.04.16.0109.01

$$a\log(x)+b\log(y)=\log\left(x^a\,y^b\right)-2\,i\,\pi\left\lfloor\frac{-\operatorname{Im}(a\log(x))-\operatorname{Im}(b\log(y))+\pi}{2\,\pi}\right\rfloor$$

01.04.16.0110.01

$$a\log(x)+b\log(y)+c\log(z)=\log\left(x^a\,y^b\,z^c\right)-2\,i\,\pi\left\lfloor\frac{-\operatorname{Im}(a\log(x))-\operatorname{Im}(b\log(y))-\operatorname{Im}(c\log(z))+\pi}{2\,\pi}\right\rfloor$$

01.04.16.0111.01

$$\sum_{k=1}^{n}a_k\log(z_k)=\log\left(\prod_{k=1}^{n}z_k^{a_k}\right)-2\,i\,\pi\left\lfloor\frac{\pi-\sum_{k=1}^{n}\operatorname{Im}(a_k\log(z_k))}{2\,\pi}\right\rfloor$$

Related transformations

Sums involving the direct function

Involving $\sin^{-1}(z)$

01.04.16.0112.01

$$\log(x)+\sin^{-1}(y)=\log\left(x\left(i\,y+\sqrt{1-y^2}\right)^{-i}\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(\log\left(i\,y+\sqrt{1-y^2}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)$$

01.04.16.0113.01

$$\log(x)+i\sin^{-1}(y)=\log\left(x\left(i\,y+\sqrt{1-y^2}\right)\right)-2\,i\,\pi\left\lfloor\frac{-\arg(x)-\arg\left(i\,y+\sqrt{1-y^2}\right)+\pi}{2\,\pi}\right\rfloor$$

Involving $\cos^{-1}(z)$

01.04.16.0114.01

$$\log(x)+\cos^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{i}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,y+\sqrt{1-y^2}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(x\left(i\,y+\sqrt{1-y^2}\right)^{i}\right)+\frac{\pi}{2}$$

01.04.16.0115.01

$$\log(x)+i\cos^{-1}(y)=-2\,i\,\pi\left\lfloor\frac{-\arg(x)+\arg\left(i\,y+\sqrt{1-y^2}\right)+\pi}{2\,\pi}\right\rfloor+\log\left(\frac{x}{i\,y+\sqrt{1-y^2}}\right)+\frac{i\,\pi}{2}$$

Involving $\tan^{-1}(z)$

01.04.16.0116.01

$$\log(x)+\tan^{-1}(y)= -2\,i\,\pi\left(\left\lfloor\frac{-\arg\left((i\,y+1)^{-\frac{i}{2}}\right)-\arg\left(x\,(1-i\,y)^{i/2}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}(\log(i\,y+1))+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x\,(1-i\,y)^{i/2}\right)\right)}{2\,\pi}\right\rfloor\right)- 2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left((1-i\,y)^{i/2}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}(\log(1-i\,y))}{2\,\pi}\right\rfloor\right)+\log\left(x\,(1-i\,y)^{i/2}\,(i\,y+1)^{-\frac{i}{2}}\right)$$

01.04.16.0117.01

$$\log(x)+i\tan^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)+\frac{1}{2}\arg(1-i\,y)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(\log(1-i\,y))+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)- 2\,i\,\pi\left(\left\lfloor\frac{-\frac{1}{2}\arg(i\,y+1)-\arg\left(\frac{x}{\sqrt{1-i\,y}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(\log(i\,y+1))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\frac{x}{\sqrt{1-i\,y}}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\frac{x\sqrt{i\,y+1}}{\sqrt{1-i\,y}}\right)$$

Involving $\cot^{-1}(z)$

01.04.16.0118.01

$$\log(x)+\cot^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{i}{y}\right)^{-\frac{i}{2}}\right)-\arg\left(x\left(1-\frac{i}{y}\right)^{i/2}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}\left(\log\left(1+\frac{i}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x\left(1-\frac{i}{y}\right)^{i/2}\right)\right)}{2\,\pi}\right\rfloor\right)- 2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left(\left(1-\frac{i}{y}\right)^{i/2}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}\left(\log\left(1-\frac{i}{y}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(x\left(1-\frac{i}{y}\right)^{i/2}\left(1+\frac{i}{y}\right)^{-\frac{i}{2}}\right)$$

01.04.16.0119.01

$$\log(x)+i\cot^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)+\frac{1}{2}\arg\left(1-\frac{i}{y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(\log\left(1-\frac{i}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)- 2\,i\,\pi\left(\left\lfloor\frac{-\frac{1}{2}\arg\left(1+\frac{i}{y}\right)-\arg\left(\frac{x}{\sqrt{1-\frac{i}{y}}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(\log\left(1+\frac{i}{y}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\frac{x}{\sqrt{1-\frac{i}{y}}}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\frac{x\sqrt{1+\frac{i}{y}}}{\sqrt{1-\frac{i}{y}}}\right)$$

Involving $\csc^{-1}(z)$

01.04.16.0120.01

$$\log(x) + \csc^{-1}(y) = \log\left(x\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i}\right) - 2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i}\right)+\pi}{2\,\pi}\right\rfloor + \left\lfloor\frac{\operatorname{Re}\left(\log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor + \left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)$$

01.04.16.0121.01

$$\log(x) + i\csc^{-1}(y) = \log\left(x\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right) - 2\,i\,\pi\left\lfloor\frac{-\arg(x)-\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)+\pi}{2\,\pi}\right\rfloor$$

Involving $\sec^{-1}(z)$

01.04.16.0122.01

$$\log(x) + \sec^{-1}(y) = -2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i}\right)+\pi}{2\,\pi}\right\rfloor + \left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor + \left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)}{2\,\pi}\right\rfloor\right) + \log\left(x\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i}\right) + \frac{\pi}{2}$$

01.04.16.0123.01

$$\log(x) + i\sec^{-1}(y) = -2\,i\,\pi\left\lfloor\frac{-\arg(x)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)+\pi}{2\,\pi}\right\rfloor + \log\left(\frac{x}{\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}}\right) + \frac{i\,\pi}{2}$$

Involving $\sinh^{-1}(z)$

01.04.16.0124.01

$$\log(x) + \sinh^{-1}(y) = \log\left(x\left(y+\sqrt{y^2+1}\right)\right) - 2\,i\,\pi\left\lfloor\frac{-\arg\left(y+\sqrt{y^2+1}\right)-\arg(x)+\pi}{2\,\pi}\right\rfloor$$

Involving $\cosh^{-1}(z)$

01.04.16.0125.01

$$\log(x)+\cosh^{-1}(y)=\log\left(x\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)-2\,i\,\pi\left\lfloor\frac{-\arg\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)-\arg(x)+\pi}{2\,\pi}\right\rfloor$$

Involving $\tanh^{-1}(z)$

01.04.16.0126.01

$$\log(x)+\tanh^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)+\frac{1}{2}\arg(1-y)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(\log(1-y))+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\frac{1}{2}\arg(y+1)-\arg\left(\frac{x}{\sqrt{1-y}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(\log(y+1))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\frac{x}{\sqrt{1-y}}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\frac{x\sqrt{y+1}}{\sqrt{1-y}}\right)$$

Involving $\coth^{-1}(z)$

01.04.16.0127.01

$$\log(x)+\coth^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)+\frac{1}{2}\arg\left(1-\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(\log\left(1-\frac{1}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\frac{1}{2}\arg\left(1+\frac{1}{y}\right)-\arg\left(\frac{x}{\sqrt{1-\frac{1}{y}}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(\log\left(1+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\frac{x}{\sqrt{1-\frac{1}{y}}}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\frac{x\sqrt{1+\frac{1}{y}}}{\sqrt{1-\frac{1}{y}}}\right)$$

Involving $\operatorname{csch}^{-1}(z)$

01.04.16.0128.01

$$\log(x)+\operatorname{csch}^{-1}(y)=\log\left(x\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)-2\,i\,\pi\left\lfloor\frac{-\arg(x)-\arg\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor$$

Involving $\operatorname{sech}^{-1}(z)$

01.04.16.0129.01

$$\log(x)+\operatorname{sech}^{-1}(y)=\log\left(x\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)-2\,i\,\pi\left\lfloor\frac{-\arg(x)-\arg\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor$$

Differences involving the direct function

Involving $\sin^{-1}(z)$

01.04.16.0130.01

$$\log(x)-\sin^{-1}(y)=$$

$$\log\left(x\left(i\,y+\sqrt{1-y^2}\right)^i\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^i\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,y+\sqrt{1-y^2}\right)\right)}{2\,\pi}\right\rfloor\right)$$

01.04.16.0131.01

$$\log(x)-i\sin^{-1}(y)=\log\left(\frac{x}{i\,y+\sqrt{1-y^2}}\right)-2\,i\,\pi\left\lfloor\frac{-\arg(x)+\arg\left(i\,y+\sqrt{1-y^2}\right)+\pi}{2\,\pi}\right\rfloor$$

Involving $\cos^{-1}(z)$

01.04.16.0132.01

$$\log(x)-\cos^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(\log\left(i\,y+\sqrt{1-y^2}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)+$$

$$\log\left(x\left(i\,y+\sqrt{1-y^2}\right)^{-i}\right)-\frac{\pi}{2}$$

01.04.16.0133.01

$$\log(x)-i\cos^{-1}(y)=-2\,i\,\pi\left\lfloor\frac{-\arg(x)-\arg\left(i\,y+\sqrt{1-y^2}\right)+\pi}{2\,\pi}\right\rfloor+\log\left(x\left(i\,y+\sqrt{1-y^2}\right)\right)-\frac{i\,\pi}{2}$$

Involving $\tan^{-1}(z)$

01.04.16.0134.01

$$\log(x)-\tan^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left((1-i\,y)^{-\frac{i}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}(\log(1-i\,y))+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left((i\,y+1)^{i/2}\right)-\arg\left(x\,(1-i\,y)^{-\frac{i}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x\,(1-i\,y)^{-\frac{i}{2}}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}(\log(i\,y+1))}{2\,\pi}\right\rfloor\right)+\log\left(x\,(1-i\,y)^{-\frac{i}{2}}\,(i\,y+1)^{i/2}\right)$$

01.04.16.0135.01

$$\log(x)-i\tan^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{\frac{1}{2}\arg(i\,y+1)-\arg\left(x\sqrt{1-i\,y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(\log(i\,y+1))+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x\sqrt{1-i\,y}\right)\right)}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\frac{1}{2}\arg(1-i\,y)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(\log(1-i\,y))}{2\,\pi}\right\rfloor\right)+\log\left(\frac{x\sqrt{1-i\,y}}{\sqrt{i\,y+1}}\right)$$

Involving $\cot^{-1}(z)$

01.04.16.0136.01

$$\log(x)-\cot^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\arg\left(\left(1-\frac{i}{y}\right)^{-\frac{i}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}\left(\log\left(1-\frac{i}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{i}{y}\right)^{i/2}\right)-\arg\left(x\left(1-\frac{i}{y}\right)^{-\frac{i}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x\left(1-\frac{i}{y}\right)^{-\frac{i}{2}}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}\left(\log\left(1+\frac{i}{y}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(x\left(1-\frac{i}{y}\right)^{-\frac{i}{2}}\left(1+\frac{i}{y}\right)^{i/2}\right)$$

01.04.16.0137.01

$$\log(x)-i\cot^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{\frac{1}{2}\arg\left(1+\frac{i}{y}\right)-\arg\left(x\sqrt{1-\frac{i}{y}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(\log\left(1+\frac{i}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x\sqrt{1-\frac{i}{y}}\right)\right)}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\frac{1}{2}\arg\left(1-\frac{i}{y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(\log\left(1-\frac{i}{y}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\frac{x\sqrt{1-\frac{i}{y}}}{\sqrt{1+\frac{i}{y}}}\right)$$

Involving $\csc^{-1}(z)$

01.04.16.0138.01

$$\log(x) - \csc^{-1}(y) = \log\left(x \left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right)^{i}\right) - 2\, i\, \pi \left(\left\lfloor \frac{-\arg(x) - \arg\left(\left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right)^{i}\right) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}(\log(x))}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Re}\left(\log\left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right)\right)}{2\pi} \right\rfloor\right)$$

01.04.16.0139.01

$$\log(x) - i \csc^{-1}(y) = \log\left(\frac{x}{\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}}\right) - 2\, i\, \pi \left\lfloor \frac{-\arg(x) + \arg\left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right) + \pi}{2\pi} \right\rfloor$$

Involving $\sec^{-1}(z)$

01.04.16.0140.01

$$\log(x) - \sec^{-1}(y) = -2\, i\, \pi \left(\left\lfloor \frac{-\arg(x) - \arg\left(\left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right)^{-i}\right) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\operatorname{Re}\left(\log\left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right)\right) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}(\log(x))}{2\pi} \right\rfloor\right) + \log\left(x \left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right)^{-i}\right) - \frac{\pi}{2}$$

01.04.16.0141.01

$$\log(x) - i \sec^{-1}(y) = -2\, i\, \pi \left\lfloor \frac{-\arg(x) - \arg\left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right) + \pi}{2\pi} \right\rfloor + \log\left(x \left(\sqrt{1 - \frac{1}{y^2}} + \frac{i}{y}\right)\right) - \frac{i \pi}{2}$$

Involving $\sinh^{-1}(z)$

01.04.16.0142.01

$$\log(x) - \sinh^{-1}(y) = \log\left(\frac{x}{y + \sqrt{y^2 + 1}}\right) - 2\, i\, \pi \left\lfloor \frac{\arg\left(y + \sqrt{y^2 + 1}\right) - \arg(x) + \pi}{2\pi} \right\rfloor$$

Involving $\cosh^{-1}(z)$

01.04.16.0143.01

$$\log(x)-\cosh^{-1}(y)=\log\left(\frac{x}{y+\sqrt{y-1}\,\sqrt{y+1}}\right)-2\,i\,\pi\left\lfloor\frac{\arg\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)-\arg(x)+\pi}{2\,\pi}\right\rfloor$$

Involving $\tanh^{-1}(z)$

01.04.16.0144.01

$$\log(x)-\tanh^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{\frac{1}{2}\arg(y+1)-\arg\left(x\sqrt{1-y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(\log(y+1))+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x\sqrt{1-y}\right)\right)}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\frac{1}{2}\arg(1-y)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(\log(1-y))}{2\,\pi}\right\rfloor\right)+\log\left(\frac{x\sqrt{1-y}}{\sqrt{y+1}}\right)$$

Involving $\coth^{-1}(z)$

01.04.16.0145.01

$$\log(x)-\coth^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{\frac{1}{2}\arg\left(1+\frac{1}{y}\right)-\arg\left(x\sqrt{1-\frac{1}{y}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(\log\left(1+\frac{1}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x\sqrt{1-\frac{1}{y}}\right)\right)}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x)-\frac{1}{2}\arg\left(1-\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(\log\left(1-\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\frac{x\sqrt{1-\frac{1}{y}}}{\sqrt{1+\frac{1}{y}}}\right)$$

Involving $\operatorname{csch}^{-1}(z)$

01.04.16.0146.01

$$\log(x)-\operatorname{csch}^{-1}(y)=\log\left(\frac{x}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)-2\,i\,\pi\left\lfloor\frac{-\arg(x)+\arg\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor$$

Involving $\operatorname{sech}^{-1}(z)$

01.04.16.0147.01

$$\log(x)-\operatorname{sech}^{-1}(y)=\log\left(\frac{x}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}\right)-2\,i\,\pi\left\lfloor\frac{-\arg(x)+\arg\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor$$

Linear combinations involving the direct function

Involving $\sin^{-1}(z)$

01.04.16.0148.01

$$a\log(x)+b\sin^{-1}(y)=\log\left(x^a\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(b\log\left(i\,y+\sqrt{1-y^2}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\,\pi}\right\rfloor\right)$$

01.04.16.0149.01

$$a\log(x)+b\sin^{-1}(y)=$$

$$i\,\pi\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}x^a+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(x^a\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\right)}{2\,\pi}\right\rfloor}\right)+i\,(-1)^{\left\lfloor\frac{\arg\left(x^a\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}x^a+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(x^a\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\right)}{\pi}\right\rfloor}$$

$$\left(\sin^{-1}\left(\frac{1}{2}\,x^{-a}\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\left(\left(i\,y+\sqrt{1-y^2}\right)^{-2\,i\,b}x^{2\,a}+1\right)\right)-\frac{\pi}{2}\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(b\log\left(i\,y+\sqrt{1-y^2}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\,\pi}\right\rfloor\right)$$

Involving $\cos^{-1}(z)$

01.04.16.0150.01

$$a\log(x)+b\cos^{-1}(y)=$$

$$\frac{\pi\,b}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(b\log\left(i\,y+\sqrt{1-y^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$

$$\log\left(x^a\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\right)$$

01.04.16.0151.01

$$a\log(x)+b\cos^{-1}(y)=$$

$$\frac{\pi b}{2}-2i\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(iy+\sqrt{1-y^2}\right)^{ib}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(b\log\left(iy+\sqrt{1-y^2}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(iy+\sqrt{1-y^2}\right)^{ib}x^a+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(x^a\left(iy+\sqrt{1-y^2}\right)^{ib}\right)}{2\pi}\right\rfloor}\right)\pi-i(-1)^{\left\lfloor\frac{\arg\left(x^a\left(iy+\sqrt{1-y^2}\right)^{ib}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(iy+\sqrt{1-y^2}\right)^{ib}x^a+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(x^a\left(iy+\sqrt{1-y^2}\right)^{ib}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}x^{-a}\left(iy+\sqrt{1-y^2}\right)^{-ib}\left(\left(iy+\sqrt{1-y^2}\right)^{2ib}x^{2a}+1\right)\right)$$

Involving $\tan^{-1}(z)$

01.04.16.0152.01

$$a\log(x)+b\tan^{-1}(y)=$$

$$-2i\pi\left(\left\lfloor\frac{-\arg\left((iy+1)^{-\frac{1}{2}(ib)}\right)-\arg\left(x^a(1-iy)^{\frac{ib}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}(b\log(iy+1))+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x^a(1-iy)^{\frac{ib}{2}}\right)\right)}{2\pi}\right\rfloor\right)-$$

$$2i\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left((1-iy)^{\frac{ib}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}(b\log(1-iy))}{2\pi}\right\rfloor\right)+\log\left(x^a(1-iy)^{\frac{ib}{2}}(iy+1)^{-\frac{1}{2}(ib)}\right)$$

01.04.16.0153.01

$$a\log(x)+b\tan^{-1}(y)=$$

$$-2i\pi\left(\left\lfloor\frac{-\arg\left((iy+1)^{-\frac{1}{2}(ib)}\right)-\arg\left(x^a(1-iy)^{\frac{ib}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}(b\log(iy+1))+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x^a(1-iy)^{\frac{ib}{2}}\right)\right)}{2\pi}\right\rfloor\right)-$$

$$2i\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left((1-iy)^{\frac{ib}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}(b\log(1-iy))}{2\pi}\right\rfloor\right)+$$

$$i\pi\left(1-(-1)^{\left\lfloor\frac{\arg\left((iy+1)^{-\frac{1}{2}(ib)}x^a(1-iy)^{\frac{ib}{2}}+1\right)}{2\pi}+\frac{1}{2}\right\rfloor}\right)+2i\tan^{-1}\left(\frac{i\left(1-x^a(1-iy)^{\frac{ib}{2}}(iy+1)^{-\frac{1}{2}(ib)}\right)}{(iy+1)^{-\frac{1}{2}(ib)}x^a(1-iy)^{\frac{ib}{2}}+1}\right)$$

Involving $\cot^{-1}(z)$

01.04.16.0154.01

$$a\log(x)+b\cot^{-1}(y)=$$
$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\right)-\arg\left(x^a\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}\left(b\log\left(1+\frac{i}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x^a\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)\right)}{2\,\pi}\right\rfloor\right)-$$
$$2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}\left(b\log\left(1-\frac{i}{y}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(x^a\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\right)$$

01.04.16.0155.01

$$a\log(x)+b\cot^{-1}(y)=$$
$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\right)-\arg\left(x^a\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}\left(b\log\left(1+\frac{i}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x^a\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)\right)}{2\,\pi}\right\rfloor\right)-$$
$$2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}\left(b\log\left(1-\frac{i}{y}\right)\right)}{2\,\pi}\right\rfloor\right)+$$
$$i\,\pi\left(1-(-1)^{\left\lfloor\frac{\arg\left(x^a\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}+1\right)}{2\,\pi}+\frac{1}{2}\right\rfloor}\right)+2\,i\cot^{-1}\left(\frac{i\left(x^a\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}+1\right)}{x^a\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}-1}\right)$$

Involving $\csc^{-1}(z)$

01.04.16.0156.01

$$a\log(x)+b\csc^{-1}(y)=\log\left(x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)-2i\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(b\log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor\right)$$

01.04.16.0157.01

$$a\log(x)+b\csc^{-1}(y)=i\pi\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}x^a+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)}{2\pi}\right\rfloor}\right)+$$

$$i(-1)^{\left\lfloor\frac{\arg\left(x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}x^a+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)}{\pi}\right\rfloor}\left(\csc^{-1}\left(\frac{2x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}}{\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-2 i b}x^{2a}+1}\right)-\frac{\pi}{2}\right)-$$

$$2i\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(b\log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor\right)$$

Involving $\sec^{-1}(z)$

01.04.16.0158.01

$$a\log(x)+b\sec^{-1}(y)=$$

$$\frac{\pi b}{2}-2i\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(b\log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$\log\left(x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\right)$$

01.04.16.0159.01

$$a\log(x)+b\sec^{-1}(y)=$$

$$\frac{\pi b}{2}-2i\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{ib}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(b\log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{ib}x^a+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{ib}\right)}{2\pi}\right\rfloor}\right)\pi-$$

$$i(-1)^{\left\lfloor\frac{\arg\left(x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{ib}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{ib}x^a+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{ib}\right)}{\pi}\right\rfloor}\sec^{-1}\left(\frac{2x^a\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{ib}}{\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{2ib}x^{2a}+1}\right)$$

Involving $\sinh^{-1}(z)$

01.04.16.0160.01

$$a\log(x)+b\sinh^{-1}(y)=\log\left(x^a\left(y+\sqrt{y^2+1}\right)^b\right)-$$

$$2i\pi\left(\left\lfloor\frac{-\arg\left(\left(y+\sqrt{y^2+1}\right)^b\right)-\arg(x^a)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(y+\sqrt{y^2+1}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor\right)$$

01.04.16.0161.01

$$a \log(x) + b \sinh^{-1}(y) =$$

$$(-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b\right)}{\pi} \right\rfloor - \left\lfloor -\frac{\arg\left(i\, x^a \left(y+\sqrt{y^2+1}\right)^b - 1\right)}{2\pi} + \frac{1}{2} - \frac{\arg\left(\left(y+\sqrt{y^2+1}\right)^b (-i)\, x^a - 1\right)}{2\pi} \right\rfloor} \sinh^{-1}\left(\frac{1}{2} x^{-a} \left(y + \sqrt{y^2+1}\right)^{-b} \left(x^{2a} \left(y + \sqrt{y^2+1}\right)^{2b} - 1\right)\right) -$$

$$\frac{1}{2} i \,(-1)^{-\left\lfloor -\frac{\arg\left(i\, x^a \left(y+\sqrt{y^2+1}\right)^b - 1\right)}{2\pi} + \frac{1}{2} - \frac{\arg\left(\left(y+\sqrt{y^2+1}\right)^b (-i)\, x^a - 1\right)}{2\pi} \right\rfloor - \left\lfloor -\frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b\right)}{\pi} \right\rfloor} \Bigg(1 - (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b\right)}{\pi} \right\rfloor} +$$

$$(-1)^{\left\lfloor -\frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b\right)}{2\pi} + \frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b - i\right)}{2\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\arg\left(i\, x^a \left(y+\sqrt{y^2+1}\right)^b - 1\right)}{2\pi} + \frac{1}{2} - \frac{\arg\left(\left(y+\sqrt{y^2+1}\right)^b (-i)\, x^a - 1\right)}{2\pi} \right\rfloor + \left\lfloor \frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b\right)}{2\pi} + \frac{1}{2} - \frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b - i\right)}{2\pi} \right\rfloor + \left\lfloor -\cdots\right.}$$

$$-$$

$$(-1)^{\left\lfloor -\frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b\right)}{2\pi} + \frac{\arg\left(\left(y+\sqrt{y^2+1}\right)^b x^a + i\right)}{2\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\arg\left(i\, x^a \left(y+\sqrt{y^2+1}\right)^b - 1\right)}{2\pi} + \frac{1}{2} - \frac{\arg\left(\left(y+\sqrt{y^2+1}\right)^b (-i)\, x^a - 1\right)}{2\pi} \right\rfloor + \left\lfloor \frac{\arg\left(x^a \left(y+\sqrt{y^2+1}\right)^b\right)}{2\pi} + \frac{1}{2} - \frac{\arg\left(\left(y+\sqrt{y^2+1}\right)^b x^a + i\right)}{2\pi} \right\rfloor + \left\lfloor -\cdots\right.}$$

$$\Bigg)\pi - 2 i \pi \left(\left\lfloor \frac{-\arg\left(\left(y + \sqrt{y^2+1}\right)^b\right) - \arg(x^a) + \pi}{2\pi} \right\rfloor + \right.$$

$$\left.\left\lfloor \frac{\pi - \operatorname{Im}\left(b \log\left(y + \sqrt{y^2+1}\right)\right)}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}(a \log(x))}{2\pi} \right\rfloor\right)$$

Involving $\cosh^{-1}(z)$

01.04.16.0162.01

$$a \log(x) + b \cosh^{-1}(y) = \log\left(x^a \left(y + \sqrt{y-1}\,\sqrt{y+1}\right)^b\right) -$$

$$2 i \pi \left(\left\lfloor \frac{-\arg\left(\left(y + \sqrt{y-1}\,\sqrt{y+1}\right)^b\right) - \arg(x^a) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}\left(b \log\left(y + \sqrt{y-1}\,\sqrt{y+1}\right)\right)}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}(a \log(x))}{2\pi} \right\rfloor\right)$$

01.04.16.0163.01

$$a\log(x)+b\cosh^{-1}(y)=i\,\pi\left(1-(-1)^{\left\lfloor -\frac{\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}x^{a}+1\right)}{2\pi}\right\rfloor-\left\lfloor -\frac{\arg\left(x^{a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\right)}{2\pi}\right\rfloor}\right)+$$

$$(-1)^{\left\lfloor \frac{\arg\left(x^{a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\right)}{\pi}-\frac{2\arg\left(x^{a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}-1\right)}{\pi}\right\rfloor+\left\lfloor -\frac{\arg\left(x^{a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\right)}{\pi}\right\rfloor-\left\lfloor -\frac{\arg\left(x^{a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}-1\right)}{2\pi}\right\rfloor+\left\lfloor -\frac{\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}x^{a}+1\right)}{2\pi}\right\rfloor}$$

$$\cosh^{-1}\left(\frac{1}{2}x^{-a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{-b}\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{2b}x^{2a}+1\right)\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\right)-\arg(x^{a})+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor\right)$$

Involving $\tanh^{-1}(z)$

01.04.16.0164.01

$$a\log(x)+b\tanh^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^{a})-\arg\left((1-y)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(b\log(1-y))+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left((y+1)^{b/2}\right)-\arg\left(x^{a}(1-y)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(b\log(y+1))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x^{a}(1-y)^{-\frac{b}{2}}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$\log\left(x^{a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}\right)$$

01.04.16.0165.01

$$a\log(x)+b\tanh^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^{a})-\arg\left((1-y)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(b\log(1-y))+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left((y+1)^{b/2}\right)-\arg\left(x^{a}(1-y)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(b\log(y+1))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x^{a}(1-y)^{-\frac{b}{2}}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$i\,\pi\left(1-(-1)^{\left\lfloor\frac{\arg\left(x^{a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}+1\right)}{2\pi}+\frac{1}{2}\right\rfloor}\right)+2\tanh^{-1}\left(\frac{x^{a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}-1}{x^{a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}+1}\right)$$

Involving $\coth^{-1}(z)$

01.04.16.0166.01

$$a\log(x)+b\coth^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(b\log\left(1-\frac{1}{y}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{1}{y}\right)^{b/2}\right)-\arg\left(x^a\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(b\log\left(1+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x^a\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$\log\left(x^a\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}\right)$$

01.04.16.0167.01

$$a\log(x)+b\coth^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(b\log\left(1-\frac{1}{y}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{1}{y}\right)^{b/2}\right)-\arg\left(x^a\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(b\log\left(1+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(x^a\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$i\,\pi\left(1-(-1)^{\left\lfloor\frac{\arg\left(x^a\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}+1\right)}{2\pi}+\frac{1}{2}\right\rfloor}\right)+2\coth^{-1}\left(\frac{x^a\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}+1}{x^a\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}-1}\right)$$

Involving $\operatorname{csch}^{-1}(z)$

01.04.16.0168.01

$$a\log(x)+b\,\operatorname{csch}^{-1}(y)=\log\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor\right)$$

01.04.16.0169.01

$$a \log(x)+b \operatorname{csch}^{-1}(y)=(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{\pi}\right\rfloor-\left\lfloor -\frac{\arg\left(-i\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b x^a-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(i x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b-1\right)}{2\pi}\right\rfloor} \operatorname{csch}^{-1}\left(\frac{2 x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b}{x^{2a}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^{2b}-1}\right)-$$

$$\frac{1}{2} i (-1)^{-\left\lfloor -\frac{\arg\left(-i\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b x^a-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(i x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b-1\right)}{2\pi}\right\rfloor-\left\lfloor -\frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{\pi}\right\rfloor} \left(1-(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{\pi}\right\rfloor}+\right.$$

$$(-1)^{\left\lfloor -\frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b-i\right)}{2\pi}+\frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{2\pi}+\frac{1}{2}\right\rfloor+\left\lfloor -\frac{\arg\left(-i\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b x^a-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(i x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b-1\right)}{2\pi}\right\rfloor+\left\lfloor \frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b-i\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{2\pi}\right\rfloor}$$

$$-$$

$$(-1)^{\left\lfloor -\frac{\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b x^a+i\right)}{2\pi}+\frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{2\pi}+\frac{1}{2}\right\rfloor+\left\lfloor -\frac{\arg\left(-i\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b x^a-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(i x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b-1\right)}{2\pi}\right\rfloor+\left\lfloor \frac{\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b x^a+i\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(x^a\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{2\pi}\right\rfloor}$$

$$\left.\right)\pi-2 i \pi\left(\left\lfloor \frac{-\arg(x^a)-\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)+\pi}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor \frac{\pi-\operatorname{Im}(a \log(x))}{2\pi}\right\rfloor+\left\lfloor \frac{\pi-\operatorname{Im}\left(b \log\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor\right)$$

Involving $\operatorname{sech}^{-1}(z)$

01.04.16.0170.01

$$a\log(x)+b\,\operatorname{sech}^{-1}(y)=\log\left(x^a\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor\right)$$

01.04.16.0171.01

$$a\log(x)+b\,\operatorname{sech}^{-1}(y)=i\,\pi\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b x^a+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(x^a\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b\right)}{2\pi}\right\rfloor}\right)+(-1)^{\left\lfloor\frac{\arg\left(x^a\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b\right)}{\pi}-\frac{2\arg\left(x^a\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b-1\right)}{\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(x^a\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b\right)}{\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(x^a\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b-1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b x^a+1\right)}{2\pi}\right\rfloor}\operatorname{sech}^{-1}\left(\frac{2\,x^a\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b}{\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{2b}x^{2a}+1}\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg(x^a)-\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^b\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(a\log(x))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor\right)$$

Identities

Functional identities

Univariate functional identities

01.04.17.0001.01

$$\log(z)=\log(|z|)+i\arg(z)$$

01.04.17.0003.01

$$\log(z)=2\log\left(\sqrt{z}\right)$$

01.04.17.0004.01

$$\log(z) = \log\left(\sqrt{z^2}\right) - \frac{\pi\left(\sqrt{z^2} - z\right)}{2\sqrt{-z^2}}$$

01.04.17.0005.01

$$\log(z) = n \log\left(z^{1/n}\right) /; z \in \mathbb{Z}$$

01.04.17.0006.01

$$\log(z) = \log(-z) - \frac{\pi\sqrt{-z}}{\sqrt{z}}$$

01.04.17.0007.01

$$\log(z) = -\log\left(\frac{1}{z}\right) + \pi i \left(1 - \sqrt{\frac{1}{z}}\sqrt{z}\right)$$

01.04.17.0008.01

$$\log(z) = -\log\left(-\frac{1}{z}\right) + \pi\sqrt{-\frac{1}{z}}\sqrt{z}$$

01.04.17.0009.01

$$\log(z) = \left\lfloor \frac{\arg(z-a)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{a}\right) + \log(a)\right) + \log\left(\frac{z}{a}\right) + \log(a)$$

01.04.17.0010.01

$$\log(z) = \log(a) + \log\left(\frac{z}{a}\right) + 2 i \pi \left\lfloor \frac{\arg(z-a)}{2\pi} \right\rfloor /; a < 0$$

Bivariate functional identities

01.04.17.0002.01

$$g(x)\, g(y) = g(x) + g(y) /; g(x) = c \log(x) \land c \in \mathbb{R}^+ \land c > 0 \land x \in \mathbb{R}^+ \land y \in \mathbb{R}^+$$

Complex characteristics

Real part

01.04.19.0001.01

$$\text{Re}(\log(x + i y)) = \frac{1}{2}\log\left(x^2 + y^2\right)$$

01.04.19.0006.01

$$\text{Re}(\log(z)) = \frac{1}{2}\log\left(\text{Im}(z)^2 + \text{Re}(z)^2\right)$$

01.04.19.0007.01

$$\text{Re}(\log(z)) = \log(|z|)$$

Imaginary part

01.04.19.0002.01

$$\text{Im}(\log(x + i y)) = \tan^{-1}(x, y)$$

01.04.19.0008.01

$$\operatorname{Im}(\log(z)) = \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))$$

01.04.19.0009.01

$$\operatorname{Im}(\log(z)) = \arg(z)$$

Absolute value

01.04.19.0003.01

$$|\log(x + i\, y)| = \sqrt{\tan^{-1}(x, y)^2 + \frac{1}{4} \log^2\left(x^2 + y^2\right)}$$

01.04.19.0010.01

$$|\log(z)| = \sqrt{\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))^2 + \frac{1}{4} \log^2\left(\operatorname{Im}(z)^2 + \operatorname{Re}(z)^2\right)}$$

Argument

01.04.19.0004.01

$$\arg(\log(x + i\, y)) = \tan^{-1}\left(\frac{1}{2} \log\left(x^2 + y^2\right), \tan^{-1}(x, y)\right)$$

01.04.19.0011.01

$$\arg(\log(z)) = \tan^{-1}\left(\frac{1}{2} \log\left(\operatorname{Im}(z)^2 + \operatorname{Re}(z)^2\right), \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\right)$$

Conjugate value

01.04.19.0005.01

$$\overline{\log(x + i\, y)} = \frac{1}{2} \log\left(x^2 + y^2\right) - i \tan^{-1}(x, y)$$

01.04.19.0012.01

$$\overline{\log(z)} = \log(z) - 2\, i \arg(z)$$

01.04.19.0013.01

$$\overline{\log(z)} = \frac{1}{2} \log\left(\operatorname{Im}(z)^2 + \operatorname{Re}(z)^2\right) - i \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))$$

Signum value

01.04.19.0014.01

$$\operatorname{sgn}(\log(x + i\, y)) = \frac{i \tan^{-1}(x, y) + \frac{1}{2} \log\left(x^2 + y^2\right)}{\sqrt{\tan^{-1}(x, y)^2 + \frac{1}{4} \log^2\left(x^2 + y^2\right)}}$$

01.04.19.0015.01

$$\operatorname{sgn}(\log(z)) = \frac{i \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)) + \frac{1}{2} \log\left(\operatorname{Im}(z)^2 + \operatorname{Re}(z)^2\right)}{\sqrt{\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))^2 + \frac{1}{4} \log^2\left(\operatorname{Im}(z)^2 + \operatorname{Re}(z)^2\right)}}$$

Differentiation

Low-order differentiation

01.04.20.0001.01

$$\frac{\partial \log(z)}{\partial z} = \frac{1}{z}$$

01.04.20.0002.01

$$\frac{\partial^2 \log(z)}{\partial z^2} = -\frac{1}{z^2}$$

Symbolic differentiation

01.04.20.0012.01

$$\frac{\partial^n \log(z)}{\partial z^n} = S_n^{(1)} z^{-n} + \delta_n \log(z) /; n \in \mathbb{N}$$

01.04.20.0003.01

$$\frac{\partial^n \log(z)}{\partial z^n} = (-1)^{n-1} (n-1)! \, z^{-n} /; n \in \mathbb{N}^+$$

01.04.20.0004.01

$$\frac{\partial^n \log(z)}{\partial z^n} = (z-1)^{1-n} \, {}_2\tilde{F}_1(1, 1; 2-n; 1-z) /; n \in \mathbb{N}$$

01.04.20.0005.02

$$\frac{\partial^n \log^a(z)}{\partial z^n} = z^{-n} \sum_{k=0}^{n} (a-k+1)_k \, S_n^{(k)} \log^{a-k}(z) /; n \in \mathbb{N}$$

01.04.20.0006.01

$$\frac{\partial^n \log(f(z))}{\partial z^n} = \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k \, f(z)^k} \binom{n}{k} \frac{\partial^n f(z)^k}{\partial z^n} /; n \in \mathbb{N}^+$$

Fractional integro-differentiation

01.04.20.0007.01

$$\frac{\partial^\alpha \log(1+z)}{\partial z^\alpha} = z^{1-\alpha} \, {}_2\tilde{F}_1(1, 1; 2-\alpha; -z)$$

01.04.20.0008.02

$$\frac{\partial^\alpha \log(b+a\,z)}{\partial z^\alpha} = \frac{a\,z^{1-\alpha}}{b} \, {}_2\tilde{F}_1\left(1, 1; 2-\alpha; -\frac{a\,z}{b}\right) + \frac{1}{\Gamma(1-\alpha)} \left(2\,i\,\pi \left\lfloor \frac{\pi - \arg\left(\frac{a\,z}{b}+1\right) - \arg(b)}{2\,\pi} \right\rfloor + \log(b)\right) z^{-\alpha}$$

01.04.20.0009.02

$$\frac{\partial^\alpha \log\left(a\,z^2+b\right)}{\partial z^\alpha} = -\frac{i\sqrt{a}\,z^{1-\alpha}}{\sqrt{b}} \, {}_2\tilde{F}_1\left(1, 1; 2-\alpha; \frac{i\sqrt{a}\,z}{\sqrt{b}}\right) +$$

$$\frac{i\sqrt{a}\,z^{1-\alpha}}{\sqrt{b}} \, {}_2\tilde{F}_1\left(1, 1; 2-\alpha; -\frac{i\sqrt{a}\,z}{\sqrt{b}}\right) + \frac{1}{\Gamma(1-\alpha)} \left(2\,i\,\pi \left\lfloor \frac{\pi - \arg\left(\frac{a\,z}{b}+1\right) - \arg(b)}{2\,\pi} \right\rfloor + \log(b)\right) z^{-\alpha}$$

01.04.20.0010.01

$$\frac{\partial^{\alpha} \log(z)}{\partial z^{\alpha}} == \mathcal{FC}_{\log}^{(\alpha)}(z)\, z^{-\alpha}$$

01.04.20.0013.01

$$\frac{\partial^{\alpha} \log(z)}{\partial z^{\alpha}} = \begin{cases} (-1)^{\alpha-1}\, (\alpha-1)!\, z^{-\alpha} & \alpha \in \mathbb{N}^{+} \\ \frac{(\log(z)-\psi(1-\alpha)-\gamma)\, z^{-\alpha}}{\Gamma(1-\alpha)} & \text{True} \end{cases}$$

01.04.20.0014.01

$$\frac{\partial^{\alpha} \log^{n}(z)}{\partial z^{\alpha}} = \mathcal{FC}_{\log}^{(\alpha)}(z, 0, n)\, z^{-\alpha} \;/;\; n \in \mathbb{N}^{+}$$

01.04.20.0015.01

$$\frac{\partial^{\alpha} \log^{n}(z)}{\partial z^{\alpha}} = \begin{cases} (-1)^{\alpha-1}\, z^{-\alpha}\, n! \sum_{u=0}^{n-1} \sum_{i=0}^{u} \frac{\log^{u-i}(z)\, \Gamma^{(n-u-1)}(1)}{(u-i)!\, (n-u-1)!} \sum_{v=0}^{i} a_{v}\, b_{i-v} & \alpha \in \mathbb{N}^{+} \\ \frac{\partial^{n} \frac{\Gamma(a+1)\, z^{a-\alpha}}{\Gamma(a-\alpha+1)}}{\partial a^{n}} \;/;\; a = 0 & \text{True} \end{cases} \;/;$$

$$n \in \mathbb{N}^{+} \bigwedge a_{2k} = \frac{(-1)^{k}\, \pi^{2k}}{(2k+1)!} \bigwedge a_{2k+1} = 0 \bigwedge b_{k} = \frac{(-1)^{k}\, \Gamma^{(k)}(\alpha)}{k!} \bigwedge k \in \mathbb{N}^{+}$$

01.04.20.0016.01

$$\frac{\partial^{\alpha} \log^{2}(z)}{\partial z^{\alpha}} = \begin{cases} 2\, (-1)^{\alpha-1}\, z^{-\alpha}\, (\alpha-1)!\, (\log(z) - \psi(\alpha) - \gamma) & \alpha \in \mathbb{N}^{+} \\ \frac{z^{-\alpha}}{\Gamma(1-\alpha)} \left((\log(z) - \psi(1-\alpha) - \gamma)^{2} + \frac{\pi^{2}}{6} - \psi^{(1)}(1-\alpha) \right) & \text{True} \end{cases}$$

01.04.20.0017.01

$$\frac{\partial^{\alpha} \log^{3}(z)}{\partial z^{\alpha}} =$$

$$\begin{cases} (-1)^{\alpha}\, z^{-\alpha}\, \Gamma(\alpha) \left(-3\, (\log(z) - \psi(\alpha) - \gamma)^{2} + \frac{\pi^{2}}{2} - 3\, \psi^{(1)}(\alpha) \right) \\ \frac{z^{-\alpha}}{\Gamma(1-\alpha)} \left(\log^{3}(z) + 3\, (-\psi(1-\alpha) - \gamma) \log^{2}(z) + 3 \left(\psi(1-\alpha)^{2} + 2\, \gamma\, \psi(1-\alpha) + \gamma^{2} + \frac{\pi^{2}}{6} - \psi^{(1)}(1-\alpha) \right) \log(z) - \gamma^{3} - \psi(1-\alpha)^{3} - 3\, \gamma\, \psi(\right. \end{cases}$$

01.04.20.0011.01

$$\frac{\partial^{\alpha} (z^{a} \log(z))}{\partial z^{\alpha}} == \mathcal{FC}_{\log}^{(\alpha)}(z, a)\, z^{a-\alpha}$$

01.04.20.0018.01

$$\frac{\partial^{\alpha} (z^{a} \log(z))}{\partial z^{\alpha}} = \begin{cases} (-1)^{-a+\alpha-1}\, \Gamma(a+1)\, (-a+\alpha-1)!\, z^{a-\alpha} & \alpha - a \in \mathbb{N}^{+} \\ \frac{(-1)^{a-1} \left(\log^{2}(z) + 2\, (\psi(-a) - \psi(a-\alpha+1)) \log(z) + \frac{\pi^{2}}{3} + (\psi(-a) - \psi(a-\alpha+1))^{2} - \psi^{(1)}(-a) - \psi^{(1)}(a-\alpha+1) \right)}{2\, (-a-1)!\, \Gamma(a-\alpha+1)}\, z^{a-\alpha} & -a \in \mathbb{N}^{+} \\ \frac{\Gamma(a+1)\, (\log(z) + \psi(a+1) - \psi(a-\alpha+1))\, z^{a-\alpha}}{\Gamma(a-\alpha+1)} & \text{True} \end{cases}$$

01.04.20.0019.01

$$\frac{\partial^{\alpha} (z^{a} \log^{n}(z))}{\partial z^{\alpha}} = \mathcal{FC}_{\log}^{(\alpha)}(z, a, n)\, z^{a-\alpha} \;/;\; n \in \mathbb{N}^{+}$$

01.04.20.0020.01

$$\frac{\partial^{\alpha} (z^{a} \log^{n}(z))}{\partial z^{\alpha}} = \frac{\partial^{n} \left(\mathcal{FC}_{\exp}^{(\alpha)}(z, a)\, z^{a-\alpha} \right)}{\partial a^{n}} \;/;\; n \in \mathbb{N}^{+}$$

01.04.20.0021.01

$$\frac{\partial^{\alpha}(z^{a}\log^{n}(z))}{\partial z^{\alpha}} = \frac{\partial^{n-1}\left(\mathcal{FC}_{\log}^{(\alpha)}(z,a)\,z^{a-\alpha}\right)}{\partial a^{n-1}} \;/;\; n \in \mathbb{N}^{+}$$

01.04.20.0022.01

$$\frac{\partial^{\alpha}(z^{a}\log^{n}(z))}{\partial z^{\alpha}} =$$

$$\begin{cases} (-1)^{-a+\alpha-1}\, z^{a-\alpha}\, n!\, \sum_{u=0}^{n-1}\sum_{i=0}^{u} \frac{\log^{u-i}(z)\,\Gamma^{(n-u-1)}(a+1)\sum_{v=0}^{i} a_v\, b_{i-v}}{(u-i)!\,(n-u-1)!} & \alpha - a \in \mathbb{N}^{+} \\ \frac{\pi(-1)^{a-1}\, n!\, z^{a-\alpha}}{(-a-1)!\,\Gamma(a-\alpha+1)} \sum_{u=0}^{n+1}\sum_{i=0}^{u} \frac{\log^{n-u+1}(z)\,(-i+u+1)}{(n-u+1)!} \left(\sum_{v=0}^{i}(v+1)\sum_{r=0}^{v}\frac{(-1)^{r+v}\binom{v}{r} p_{r,v}\, c_{i-v-1}}{r+1}\right) \sum_{s=0}^{u-i}\frac{(-1)^{s}\binom{u-i}{s} q_{s,u-i}}{s+1} & -a \in \mathbb{N}^{+} \\ \frac{\partial^{n}\frac{\Gamma(a+1)\,z^{a-\alpha}}{\Gamma(a-\alpha+1)}}{\partial a^{n}} & \text{True} \end{cases} \;/;$$

$$n \in \mathbb{N}^{+} \bigwedge a_{2k} = \frac{(-1)^{k}\pi^{2k}}{(2k+1)!} \bigwedge a_{2k+1} = 0 \bigwedge b_{k} = \frac{(-1)^{k}\,\Gamma^{(k)}(\alpha-a)}{k!} \bigwedge k \in \mathbb{N}^{+} \bigwedge p_{j,0} = 1 \bigwedge$$

$$p_{j,k} = \frac{1}{k}\sum_{i=1}^{k}(j\,i+i-k)\,e_i\,p_{j,k-i} \bigwedge e_k = \frac{\Gamma^{(k)}(-a)}{(-a-1)!\,k!} \bigwedge q_{j,0} = 1 \bigwedge q_{j,k} = \frac{1}{k}\sum_{i=1}^{k}(j\,i+i-k)\,d_i\,q_{j,k-i} \bigwedge$$

$$d_k = \frac{\Gamma^{(k)}(a-\alpha+1)}{\Gamma(a-\alpha+1)\,k!} \bigwedge c_{2k+1} = \frac{(-1)^{k}\,2\left(2^{2k+1}-1\right)B_{2k+2}\,\pi^{2k+1}}{(2k+2)!} \bigwedge c_{2k} = 0 \bigwedge k \in \mathbb{N}^{+}$$

01.04.20.0023.01

$$\frac{\partial^{\alpha}\left(z^{a}\log^{2}(z)\right)}{\partial z^{\alpha}} =$$

$$\begin{cases} 2(-1)^{-a+\alpha-1}(-a+\alpha-1)!\,\Gamma(a+1)\,(\log(z)-\psi(\alpha-a)+\psi(a+1))\,z^{a-\alpha} \\ \frac{(-1)^{a}\left(-\log^{3}(z)+3(\psi(a-\alpha+1)-\psi(-a))\log^{2}(z)+\left(-3\psi(-a)^{2}+6\psi(a-\alpha+1)\psi(-a)-\pi^{2}-3\psi(a-\alpha+1)^{2}+3\psi^{(1)}(-a)+3\psi^{(1)}(a-\alpha+1)\right)\log(z)-\psi(-a)^{3}+\psi(a-\alpha+1)^{3}+3\psi(-a)^{2}\psi(a-\alpha+1)+\psi \cdots}{3(-a-1)!\,\Gamma(a-\alpha+1)} \\ \frac{\Gamma(a+1)\left((\log(z)+\psi(a+1)-\psi(a-\alpha+1))^{2}+\psi^{(1)}(a+1)-\psi^{(1)}(a-\alpha+1)\right)z^{a-\alpha}}{\Gamma(a-\alpha+1)} \end{cases}$$

01.04.20.0024.01

$$\frac{\partial^{\alpha}\left(z^{a}\log^{3}(z)\right)}{\partial z^{\alpha}} =$$

$$\begin{cases} (-1)^{\alpha-a}\,\Gamma(a+1)\,\Gamma(\alpha-a)\left(-3(\log(z)+\psi(a+1)-\psi(\alpha-a))^{2}+\pi^{2}-3\psi^{(1)}(a+1)-3\psi^{(1)}(\alpha-a)\right)z^{a-\alpha} \\ \frac{(-1)^{a-1}\left(15\log^{4}(z)+60(\psi(-a)-\psi(a-\alpha+1))\log^{3}(z)+30\left(3\psi(-a)^{2}-6\psi(a-\alpha+1)\psi(-a)+\pi^{2}+3\psi(a-\alpha+1)^{2}-3\psi^{(1)}(-a)-3\psi^{(1)}(a-\alpha+1)\right)\log^{2}(z)+60\left(\psi(-a)^{3}-3\psi(a-\alpha+1)\psi(-a)^{2}+(3\cdots\right.\right.}{\cdots} \\ \frac{\Gamma(a+1)\left(\log^{3}(z)+3(\psi(a+1)-\psi(a-\alpha+1))\log^{2}(z)+3\left(\psi(a+1)^{2}-2\psi(a-\alpha+1)\psi(a+1)+\psi(a-\alpha+1)^{2}+\psi^{(1)}(a+1)-\psi^{(1)}(a-\alpha+1)\right)\log(z)+\psi(a+1)^{3}-\psi(a-\alpha+1)^{3}-3\psi(a+1)^{2}\psi(a-\alpha+1)-3\psi(\cdots\right.}{\Gamma(a-\alpha+1)} \end{cases}$$

01.04.20.0025.01

$$\frac{\partial^\alpha \log(f(z))}{\partial z^\alpha} == \left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{f(z)}{f(0)}\right) - \arg(f(0))}{2\pi} \right\rfloor + \log(f(0))\right) \frac{z^{-\alpha}}{\Gamma(1-\alpha)} + \sum_{s=0}^{\infty} \sum_{k=0}^{\frac{s}{u}} \frac{(-1)^k \Gamma(s+u+1)}{(k+1)\Gamma(s+u-\alpha+1)} \left(\frac{f^{(u)}(0)}{f(0)\, u!}\right)^{k+1} p_{k+1,s-u k}\, z^{s+u-\alpha} /;$$

$$f(0) \neq 0 \bigwedge \left(f^{(k)}(0) == 0 /; 1 \leq k \leq u-1\right) \bigwedge f^{(u)}(0) \neq 0 \bigwedge p_{j,0} == 1 \bigwedge$$

$$p_{j,k} == \frac{u!}{f^{(u)}(z_0)\, k} \sum_{m=1}^{k} \frac{j m + m - k}{(m+u)!\,(m+1)!} f^{(m+u)}(z_0)\, p_{j,k-m} \bigwedge k \in \mathbb{N}^+$$

Integration

Indefinite integration

For the direct function itself

01.04.21.0001.01

$$\int \log(z)\, dz == z \log(z) - z$$

01.04.21.0002.01

$$\int z^{\alpha-1} \log(z)\, dz == \frac{z^\alpha (\alpha \log(z) - 1)}{\alpha^2}$$

Involving the direct function

01.04.21.0003.01

$$\int \frac{1}{z} \log\left(\frac{b + a z}{d + c z}\right) dz == \left(-\log\left(\frac{a z}{b} + 1\right) + \log\left(\frac{c z}{d} + 1\right) + \log\left(\frac{b + a z}{d + c z}\right)\right) \log(z) - \text{Li}_2\left(-\frac{a z}{b}\right) + \text{Li}_2\left(-\frac{c z}{d}\right)$$

01.04.21.0004.01

$$\int \frac{\log(z)}{\sqrt{c z^2 + d}} dz == \frac{1}{2 \sqrt{\frac{c}{d}} \sqrt{c z^2 + d}} \left(\sqrt{\frac{c z^2}{d} + 1} \left(-\sinh^{-1}\left(\sqrt{\frac{c}{d}}\, z\right)^2 - 2 \log\left(1 - e^{-2 \sinh^{-1}\left(\sqrt{\frac{c}{d}}\, z\right)}\right) \sinh^{-1}\left(\sqrt{\frac{c}{d}}\, z\right) + 2 \log\left(\sqrt{\frac{c}{d}}\, z + \sqrt{\frac{c z^2}{d} + 1}\right) \log(z) + \text{Li}_2\left(e^{-2 \sinh^{-1}\left(\sqrt{\frac{c}{d}}\, z\right)}\right)\right)\right)$$

01.04.21.0005.01

$$\int \frac{\log(b + a z)}{d + c z} dz == \frac{1}{c} \left(\log\left(1 - \frac{c(b + a z)}{b c - a d}\right) \log(b + a z) + \text{Li}_2\left(\frac{c(b + a z)}{b c - a d}\right)\right)$$

01.04.21.0006.01

$$\int \frac{\log\left(a z^2 + b z + e\right)}{d + c z}\, dz = \frac{1}{c}\left(\log\left(\frac{b + 2 a z + \sqrt{b^2 - 4 a e}}{2 a}\right)\log\left(-\frac{2 a (d + c z)}{b c + \sqrt{b^2 - 4 a e}\, c - 2 a d}\right) + \log\left(\frac{2 a (d + c z)}{-b c + \sqrt{b^2 - 4 a e}\, c + 2 a d}\right) \log\left(\frac{b - \sqrt{b^2 - 4 a e}}{2 a} + z\right) - \log\left(\frac{b + 2 a z + \sqrt{b^2 - 4 a e}}{2 a}\right)\log(d + c z) - \log\left(\frac{b - \sqrt{b^2 - 4 a e}}{2 a} + z\right)\log(d + c z) + \log(d + c z)\log(e + z (b + a z)) + \mathrm{Li}_2\left(\frac{c\left(-b - 2 a z + \sqrt{b^2 - 4 a e}\right)}{-b c + \sqrt{b^2 - 4 a e}\, c + 2 a d}\right) + \mathrm{Li}_2\left(\frac{c\left(b + 2 a z + \sqrt{b^2 - 4 a e}\right)}{b c + \sqrt{b^2 - 4 a e}\, c - 2 a d}\right)\right)$$

01.04.21.0007.01

$$\int \frac{\log^2(z)}{1 - z}\, dz = -\log(1 - z)\log^2(z) - 2\,\mathrm{Li}_2(z)\log(z) + 2\,\mathrm{Li}_3(z)$$

01.04.21.0008.01

$$\int \frac{\log^2(z + 1)}{z}\, dz = \log(-z)\log^2(z + 1) + 2\,\mathrm{Li}_2(z + 1)\log(z + 1) - 2\,\mathrm{Li}_3(z + 1)$$

01.04.21.0009.01

$$\int \frac{\log^2(a + b z)}{e + f z}\, dz = \frac{1}{f}\left(\log\left(\frac{b (e + f z)}{b e - a f}\right)\log^2(a + b z) + 2\,\mathrm{Li}_2\left(\frac{f (a + b z)}{a f - b e}\right)\log(a + b z) - 2\,\mathrm{Li}_3\left(\frac{f (a + b z)}{a f - b e}\right)\right)$$

01.04.21.0010.01

$$\int \frac{1}{z} \log^2\left(\frac{a+b z}{c+d z}\right) dz = \log\left(-\frac{b z}{a}\right) \log^2\left(\frac{a}{b}+z\right) + 2 \operatorname{Li}_2\left(\frac{b z}{a}+1\right) \log\left(\frac{a}{b}+z\right) +$$
$$\log\left(-\frac{d z}{c}\right) \log^2\left(\frac{c}{d}+z\right) + \log(z) \left(-\log\left(\frac{a}{b}+z\right) + \log\left(\frac{c}{d}+z\right) + \log\left(\frac{a+b z}{c+d z}\right)\right)^2 +$$
$$2 \log\left(\frac{c}{d}+z\right) \operatorname{Li}_2\left(\frac{d z}{c}+1\right) + 2 \left(-\log\left(\frac{a}{b}+z\right) + \log\left(\frac{c}{d}+z\right) + \log\left(\frac{a+b z}{c+d z}\right)\right)$$
$$\left(\left(-\log\left(\frac{b z}{a}+1\right) + \log\left(\frac{d z}{c}+1\right) + \log\left(\frac{a}{b}+z\right) - \log\left(\frac{c}{d}+z\right)\right) \log(z) - \operatorname{Li}_2\left(-\frac{b z}{a}\right) + \operatorname{Li}_2\left(-\frac{d z}{c}\right)\right) -$$
$$2 \operatorname{Li}_3\left(\frac{b z}{a}+1\right) - 2 \left(\frac{1}{2} \left(\log\left(-\frac{b z}{a}\right) + \log\left(\frac{a d-b c}{d (a+b z)}\right) - \log\left(\frac{b c z-a d z}{a c+b z c}\right)\right) \log^2\left(\frac{a (c+d z)}{c (a+b z)}\right) +\right.$$
$$\log\left(\frac{d z}{c}+1\right) \left(\log\left(-\frac{d z}{c}\right) - \log\left(-\frac{b z}{a}\right)\right) \log\left(\frac{a (c+d z)}{c (a+b z)}\right) + \left(\operatorname{Li}_2\left(\frac{b (c+d z)}{d (a+b z)}\right) - \operatorname{Li}_2\left(\frac{a (c+d z)}{c (a+b z)}\right)\right) \log\left(\frac{a (c+d z)}{c (a+b z)}\right) +$$
$$\frac{1}{2} \log\left(\frac{d z}{c}+1\right) \left(\log\left(\frac{d z}{c}+1\right) - 2 \log\left(\frac{a}{b}+z\right)\right) \left(\log\left(-\frac{b z}{a}\right) - \log\left(-\frac{d z}{c}\right)\right) + \log\left(-\frac{b z}{a}\right) \log\left(\frac{a}{b}+z\right) \log\left(\frac{c}{d}+z\right) +$$
$$\left(\log\left(\frac{c}{d}+z\right) - \log\left(\frac{a (c+d z)}{c (a+b z)}\right)\right) \operatorname{Li}_2\left(\frac{b z}{a}+1\right) + \left(\log\left(\frac{a}{b}+z\right) + \log\left(\frac{a (c+d z)}{c (a+b z)}\right)\right) \operatorname{Li}_2\left(\frac{d z}{c}+1\right) -$$
$$\left.\operatorname{Li}_3\left(\frac{b z}{a}+1\right) - \operatorname{Li}_3\left(\frac{d z}{c}+1\right) + \operatorname{Li}_3\left(\frac{a (c+d z)}{c (a+b z)}\right) - \operatorname{Li}_3\left(\frac{b (c+d z)}{d (a+b z)}\right)\right) - 2 \operatorname{Li}_3\left(\frac{d z}{c}+1\right)$$

01.04.21.0011.01

$$\int \frac{\log(z) \log(1-z)}{z} dz = \operatorname{Li}_3(z) - \log(z) \operatorname{Li}_2(z)$$

01.04.21.0012.01

$$\int \frac{\log(a+b z) \log(c+d z)}{e+f z} dz =$$
$$\frac{1}{f} \left(\frac{1}{2} \left(\log\left(\frac{b (e+f z)}{b e-a f}\right) + \log\left(\frac{a d-b c}{d (a+b z)}\right) - \log\left(\frac{(a d-b c) (e+f z)}{(d e-c f) (a+b z)}\right)\right) \log^2\left(\frac{(b e-a f) (c+d z)}{(d e-c f) (a+b z)}\right) +\right.$$
$$\log\left(\frac{f (c+d z)}{c f-d e}\right) \left(\log\left(\frac{d (e+f z)}{d e-c f}\right) - \log\left(\frac{b (e+f z)}{b e-a f}\right)\right) \log\left(\frac{(b e-a f) (c+d z)}{(d e-c f) (a+b z)}\right) +$$
$$\left(\operatorname{Li}_2\left(\frac{b (c+d z)}{d (a+b z)}\right) - \operatorname{Li}_2\left(\frac{(b e-a f) (c+d z)}{(d e-c f) (a+b z)}\right)\right) \log\left(\frac{(b e-a f) (c+d z)}{(d e-c f) (a+b z)}\right) + \frac{1}{2} \log\left(\frac{f (c+d z)}{c f-d e}\right)$$
$$\left(\log\left(\frac{f (c+d z)}{c f-d e}\right) - 2 \log(a+b z)\right) \left(\log\left(\frac{b (e+f z)}{b e-a f}\right) - \log\left(\frac{d (e+f z)}{d e-c f}\right)\right) + \log\left(\frac{b (e+f z)}{b e-a f}\right) \log(a+b z) \log(c+d z) +$$
$$\left(\log(c+d z) - \log\left(\frac{(b e-a f) (c+d z)}{(d e-c f) (a+b z)}\right)\right) \operatorname{Li}_2\left(\frac{f (a+b z)}{a f-b e}\right) + \left(\log(a+b z) + \log\left(\frac{(b e-a f) (c+d z)}{(d e-c f) (a+b z)}\right)\right) \operatorname{Li}_2\left(\frac{f (c+d z)}{c f-d e}\right) -$$
$$\left.\operatorname{Li}_3\left(\frac{f (a+b z)}{a f-b e}\right) - \operatorname{Li}_3\left(\frac{f (c+d z)}{c f-d e}\right) - \operatorname{Li}_3\left(\frac{b (c+d z)}{d (a+b z)}\right) + \operatorname{Li}_3\left(\frac{(b e-a f) (c+d z)}{(d e-c f) (a+b z)}\right)\right)$$

01.04.21.0013.01

$$\int \frac{\log(z) \log(1-z)}{z} dz = \operatorname{Li}_3(z) - \log(z) \operatorname{Li}_2(z)$$

01.04.21.0014.01

$$\int \frac{\log(z)\log(z-1)}{z}\,dz = \frac{1}{2}(\log(z-1)-\log(1-z))\log^2(z)-\mathrm{Li}_2(z)\log(z)+\mathrm{Li}_3(z)$$

01.04.21.0015.01

$$\int \frac{\log^3(z)}{1-z}\,dz = -\log(1-z)\log^3(z)-3\,\mathrm{Li}_2(z)\log^2(z)+6\,\mathrm{Li}_3(z)\log(z)-6\,\mathrm{Li}_4(z)$$

01.04.21.0016.01

$$\int \frac{\log^2(z)\log(1-z)}{z}\,dz = 2\,\mathrm{Li}_3(z)\log(z)-\log^2(z)\,\mathrm{Li}_2(z)-2\,\mathrm{Li}_4(z)$$

01.04.21.0017.01

$$\int \frac{\log^2(z)\log(1-z)}{1-z}\,dz =$$
$$\frac{1}{12}\left(-\log^4\left(\frac{1}{z}-1\right)+4\left(\log(1-z)+\log\left(\frac{1}{z}\right)\right)\log^3\left(\frac{1}{z}-1\right)-6\log^2(1-z)\log^2\left(\frac{1}{z}-1\right)+12\,\mathrm{Li}_2\left(\frac{z-1}{z}\right)\log^2\left(\frac{1}{z}-1\right)+\right.$$
$$4\log^2(1-z)(\log(1-z)+3\log(z))\log\left(\frac{1}{z}-1\right)-24\,\mathrm{Li}_3\left(\frac{z-1}{z}\right)\log\left(\frac{1}{z}-1\right)-$$
$$\log^4(1-z)-6\log^2(1-z)\log^2(z)+4\log^2(1-z)\log(z)(3\log(z)-2\log(1-z))-$$
$$12\log(1-z)\left(-2\log\left(\frac{1}{z}-1\right)+\log(1-z)-2\log(z)\right)\mathrm{Li}_2(1-z)-12\left(\log\left(\frac{1}{z}-1\right)-\log(1-z)\right)^2\mathrm{Li}_2(z)-$$
$$\left.24\left(\log\left(\frac{1}{z}-1\right)+\log(z)\right)\mathrm{Li}_3(1-z)+24\left(\log(1-z)-\log\left(\frac{1}{z}-1\right)\right)\mathrm{Li}_3(z)+24\left(\mathrm{Li}_4(1-z)+\mathrm{Li}_4\left(\frac{z-1}{z}\right)-\mathrm{Li}_4(z)\right)\right)$$

01.04.21.0018.01

$$\int \log(\sin(z))\,dz = -z\log\left(1-e^{2iz}\right)+z\log(\sin(z))+\frac{i}{2}\left(z^2+\mathrm{Li}_2\left(e^{2iz}\right)\right)$$

01.04.21.0019.01

$$\int \log(\cos(z))\,dz = \frac{i}{2}\left(z\left(z+2\,i\log\left(1+e^{2iz}\right)-2\,i\log(\cos(z))\right)+\mathrm{Li}_2\left(-e^{2iz}\right)\right)$$

01.04.21.0020.01

$$\int \log(\tan(z))\,dz = \frac{i}{2}(\log(\tan(z))(\log(1-i\tan(z))-\log(i\tan(z)+1))-\mathrm{Li}_2(-i\tan(z))+\mathrm{Li}_2(i\tan(z)))$$

01.04.21.0021.01

$$\int \log(\sinh(z))\,dz = \frac{1}{2}\left(\mathrm{Li}_2\left(e^{-2z}\right)-z\left(z+2\log\left(1-e^{-2z}\right)-2\log(\sinh(z))\right)\right)$$

01.04.21.0022.01

$$\int \log(\cosh(z))\,dz = \frac{1}{2}\left(\mathrm{Li}_2\left(-e^{-2z}\right)-z\left(z+2\log\left(1+e^{-2z}\right)-2\log(\cosh(z))\right)\right)$$

01.04.21.0023.01

$$\int \log(\tanh(z))\,dz = \frac{1}{2}(\log(\tanh(z))\log(\tanh(z)+1)+\mathrm{Li}_2(1-\tanh(z))+\mathrm{Li}_2(-\tanh(z)))$$

01.04.21.0024.01

$$\int \log(\sin(z))\tan(z)\,dz = \frac{1}{2}(-\log(\sin(z))\log(\sin(z)+1)+\mathrm{Li}_2(1-\sin(z))-\mathrm{Li}_2(-\sin(z)))$$

01.04.21.0025.01

$$\int \log(\cos(z))\tan(z)\,dz = -\frac{1}{2}\log^2(\cos(z))$$

01.04.21.0026.01

$$\int \frac{\log(\cos(z))}{\sin(2z)}\,dz = \frac{1}{4}\left(\log(\cos(z))\left(\log(\cos(z)+1)-\log(\cos(z))\right)-\mathrm{Li}_2(1-\cos(z))+\mathrm{Li}_2(-\cos(z))\right)$$

01.04.21.0027.01

$$\int \log(\tan(z))\tan(z)\,dz = \frac{1}{2}\left(\log(\tan(z))\left(\log(i\tan(z)+1)+\log(1-i\tan(z))\right)+\mathrm{Li}_2(-i\tan(z))+\mathrm{Li}_2(i\tan(z))\right)$$

01.04.21.0028.01

$$\int \log(\cot(z))\cot(z)\,dz = \frac{1}{2}\left(-\log(\cot(z))\log(i\cot(z)+1)+\log(\cot(z))\left(-\log(1-i\cot(z))\right)-\mathrm{Li}_2(-i\cot(z))-\mathrm{Li}_2(i\cot(z))\right)$$

01.04.21.0029.01

$$\int \log(a+\sin(z))\,dz = \frac{1}{8}\left(-i(\pi-2z)^2-4\log(a+\sin(z))(\pi-2z)+32\,i\sin^{-1}\left(\frac{\sqrt{a+1}}{\sqrt{2}}\right)\tan^{-1}\left(\frac{(a-1)\tan\left(\frac{1}{4}(\pi-2z)\right)}{\sqrt{a^2-1}}\right)+\right.$$
$$4\left(-2z+4\sin^{-1}\left(\frac{\sqrt{a+1}}{\sqrt{2}}\right)+\pi\right)\log\left(1+i\left(a-\sqrt{a^2-1}\right)e^{-iz}\right)+4\left(-2z-4\sin^{-1}\left(\frac{\sqrt{a+1}}{\sqrt{2}}\right)+\pi\right)$$
$$\left.\log\left(1+i\left(a+\sqrt{a^2-1}\right)e^{-iz}\right)-8\,i\left(\mathrm{Li}_2\left(i\left(\sqrt{a^2-1}-a\right)e^{-iz}\right)+\mathrm{Li}_2\left(-i\left(a+\sqrt{a^2-1}\right)e^{-iz}\right)\right)\right)$$

01.04.21.0030.01

$$\int \log(a+\cos(z))\,dz =$$
$$\frac{i z^2}{2}+\log(a+\cos(z))\,z-4\,i\sin^{-1}\left(\frac{\sqrt{a+1}}{\sqrt{2}}\right)\tan^{-1}\left(\frac{(a-1)\tan\left(\frac{z}{2}\right)}{\sqrt{a^2-1}}\right)-\left(z+2\sin^{-1}\left(\frac{\sqrt{a+1}}{\sqrt{2}}\right)\right)\log\left(e^{iz}\left(a-\sqrt{a^2-1}\right)+1\right)-$$
$$\left(z-2\sin^{-1}\left(\frac{\sqrt{a+1}}{\sqrt{2}}\right)\right)\log\left(e^{iz}\left(a+\sqrt{a^2-1}\right)+1\right)+i\left(\mathrm{Li}_2\left(\left(\sqrt{a^2-1}-a\right)e^{iz}\right)+\mathrm{Li}_2\left(-\left(a+\sqrt{a^2-1}\right)e^{iz}\right)\right)$$

01.04.21.0031.01

$$\int \log(a+\tan(z))\,dz = \frac{1}{2}\,i\left(\left(\log\left(-\frac{i+\tan(z)}{a-i}\right)-\log\left(1-\frac{a+\tan(z)}{a+i}\right)\right)\log(a+\tan(z))+\mathrm{Li}_2\left(\frac{a+\tan(z)}{a-i}\right)-\mathrm{Li}_2\left(\frac{a+\tan(z)}{a+i}\right)\right)$$

01.04.21.0032.01

$$\int \log\left(\sin^2(z)+a\right)dz = \frac{1}{2}\left(2\,i z^2+2\log\left(\sin^2(z)+a\right)z+4\,i\sin^{-1}\left(\sqrt{-a}\right)\tan^{-1}\left(\frac{(a+1)\tan(z)}{\sqrt{a(a+1)}}\right)-\right.$$
$$2\left(z-\sin^{-1}\left(\sqrt{-a}\right)\right)\log\left(e^{2iz}\left(-2a+2\sqrt{a(a+1)}-1\right)+1\right)-2\left(z+\sin^{-1}\left(\sqrt{-a}\right)\right)$$
$$\left.\log\left(1-\left(2a+2\sqrt{a(a+1)}+1\right)e^{2iz}\right)+i\left(\mathrm{Li}_2\left(\left(2a+2\sqrt{a(a+1)}+1\right)e^{2iz}\right)+\mathrm{Li}_2\left(\left(2a-2\sqrt{a(a+1)}+1\right)e^{2iz}\right)\right)\right)$$

01.04.21.0033.01

$$\int z\log\left(2\sin\left(\frac{z}{2}\right)\right)dz = \frac{i}{12}\left(-z^3+6\,i\log\left(1-e^{-iz}\right)z^2-6\,i\log\left(2\sin\left(\frac{z}{2}\right)\right)z^2-12\,\mathrm{Li}_2\left(e^{-iz}\right)z+\pi^3+12\,i\,\mathrm{Li}_3\left(e^{-iz}\right)\right)$$

01.04.21.0075.01

$$\int \frac{\log^n(z)}{z(1-z)}\, dz - \left(\frac{\log^{n+1}(z)}{n+1} + \frac{1}{6}\pi^2 \delta_{n-1} + \sum_{j=0}^{n} \binom{n}{j} (-1)^j\, j!\, \log^{n-j}(z)\, \mathrm{Li}_{j+1}(z)\right) /;\ n \in \mathbb{N}^+$$

01.04.21.0076.01

$$\int \frac{z^{\alpha-1} \log^n(z)}{1-z}\, dz = z^\alpha \sum_{j=0}^{n} \binom{n}{j} (-1)^j\, j!\, \log^{n-j}(z)\, \alpha^{-j-1}\ {}_{j+2}F_{j+1}(1, \alpha, \ldots, \alpha, 1; \alpha+1, \ldots, \alpha+1; z) /;\ n \in \mathbb{N}$$

Definite integration

For the direct function itself

01.04.21.0034.01

$$\int_0^1 \log(t)\, dt = -1$$

Involving the direct function

01.04.21.0035.01

$$\int_0^1 \frac{\log\left(t^2 - 2\cos(z)\, t + 1\right)}{t}\, dt = \frac{\pi^2}{6} - \frac{1}{2}(\pi - z)^2 /;\ \mathrm{Im}(z) > 0 \lor z > 0$$

01.04.21.0036.01

$$\int_0^1 \log(t) \log(t+1)\, dt = -2\log(2) - \frac{\pi^2}{12} + 2$$

01.04.21.0037.01

$$\int_0^\infty \frac{\log(t+1) \log\left(1 + \frac{1}{t^2}\right)}{t}\, dt = C\pi - \frac{3\zeta(3)}{8}$$

01.04.21.0038.01

$$\int_0^\infty \log(t+1) \log\left(1 + \frac{1}{t^2}\right) dt = \frac{\pi}{2}\log(2) + \frac{5\pi^2}{24} - \pi + 2C$$

01.04.21.0039.01

$$\int_0^\infty \frac{\log(a\, t + 1) \log\left(\frac{z}{t^2} + 1\right)}{t}\, dt =$$

$$\frac{1}{24\, a}\left(6\pi \sqrt{\frac{1}{z}}\, \Phi\left(-\frac{1}{a^2 z}, 2, \frac{1}{2}\right) - a\left(\log^3\left(\frac{1}{a^2 z}\right) + 5\pi^2 \log\left(\frac{1}{a^2 z}\right) + 6\,\mathrm{Li}_2\left(-\frac{1}{a^2 z}\right)\log\left(\frac{1}{a^2 z}\right) - 12\,\mathrm{Li}_3\left(-\frac{1}{a^2 z}\right)\right)\right)$$

01.04.21.0040.01

$$\int_0^1 \frac{\log(t)\log^2(1-t)}{t}\, dt = -\frac{\pi^4}{180}$$

01.04.21.0041.01

$$\int_0^{\frac{1}{2}} \frac{\log^4(t(1-t))}{1-t}\, dt = \frac{16\log^5(2)}{5} - 4\pi^2 \zeta(3) + 72\zeta(5)$$

01.04.21.0042.01

$$\int_0^{\frac{1}{2}} \frac{\log^5(t(1-t))}{1-t}\, dt = \vdots -\frac{1}{3}\ 16\log^6(2) + 120\,\zeta(3)^2 - \frac{79\pi^6}{252}$$

01.04.21.0043.01

$$\int_0^{\frac{1}{2}} \frac{\log^n(t(1-t))}{1-t} \, dt = \frac{1}{2} \lim_{p \to 0} \frac{\partial^n \frac{\frac{\Gamma(p+1)^2}{\Gamma(2p+1)} - 1}{p}}{\partial p^n} - \frac{2^n \log^{n+1}\left(\frac{1}{2}\right)}{n+1} \;/;\; n \in \mathbb{N}^+$$

01.04.21.0044.01

$$\int_0^{\frac{1}{2}} \frac{\log^n\left(\frac{t}{1-t}\right)}{1-t} \, dt = (-1)^n \, n! \, \zeta(n+1) \left(1 - 2^{-n}\right) \;/;\; n \in \mathbb{N}^+$$

01.04.21.0045.01

$$\int_0^1 \frac{\log^m(t) \log^n(1-t)}{t} \, dt = \lim_{\lambda \to 0} \lim_{\mu \to 0} \frac{\partial^n \frac{\partial^m \frac{\exp\left(-\sum_{p=1}^{n+1} \sum_{q=1}^{m+1} \frac{(-1)^{p+q} \lambda^p \mu^q \, (p+q-1)! \, \zeta(p+q)}{p! \, q!}\right)}{\mu}}{\partial \mu^m}}{\partial \lambda^n} \;/;\; m \in \mathbb{N} \bigwedge n \in \mathbb{N}^+$$

01.04.21.0046.01

$$\int_0^z \log\left(2 \sin\left(\frac{t}{2}\right)\right) dt = -\frac{i \pi^2}{6} - z \log\left(1 - e^{i z}\right) + z \log\left(2 \sin\left(\frac{z}{2}\right)\right) + \frac{i}{4} \left(z^2 + 4 \operatorname{Li}_2\left(e^{i z}\right)\right)$$

Clausen's integral

01.04.21.0047.01

$$\int_0^\pi \log\left(2 \sin\left(\frac{t}{2}\right)\right) dt = 0$$

01.04.21.0048.01

$$\int_0^\pi \log^2\left(2 \sin\left(\frac{t}{2}\right)\right) dt = \frac{\pi^3}{12}$$

01.04.21.0049.01

$$\int_0^\pi \log^3\left(2 \sin\left(\frac{t}{2}\right)\right) dt = -\frac{3\pi}{2} \zeta(3)$$

01.04.21.0050.01

$$\int_0^\pi \log^4\left(2 \sin\left(\frac{t}{2}\right)\right) dt = \frac{19 \pi^5}{240}$$

01.04.21.0051.01

$$\int_0^\pi \log^5\left(2 \sin\left(\frac{t}{2}\right)\right) dt = -\frac{5}{4} \left(\pi^3 \zeta(3) + 18 \pi \zeta(5)\right)$$

01.04.21.0052.01

$$\int_0^\pi \log^6\left(2 \sin\left(\frac{t}{2}\right)\right) dt = \frac{45}{2} \pi \zeta(3)^2 + \frac{275 \pi^7}{1344}$$

01.04.21.0053.01

$$\int_0^\pi \log^7\left(2 \sin\left(\frac{t}{2}\right)\right) dt = -\frac{7}{32} \left(19 \pi^5 \zeta(3) + 180 \pi^3 \zeta(5) + 3240 \pi \zeta(7)\right)$$

01.04.21.0054.01

$$\int_0^\pi \log^8\left(2 \sin\left(\frac{t}{2}\right)\right) dt = \frac{604\,800 \pi^3 \zeta(3)^2 + 21\,772\,800 \pi \zeta(5) \zeta(3) + 11\,813 \pi^9}{11\,520}$$

01.04.21.0055.01

$$\int_0^{\pi} \log^9\left(2\sin\left(\frac{t}{2}\right)\right) dt == -\frac{3}{32}\left(10\,080\,\pi\,\zeta(3)^3 + 275\,\pi^7\,\zeta(3) + 2394\,\pi^5\,\zeta(5) + 22\,680\,\pi^3\,\zeta(7) + 428\,400\,\pi\,\zeta(9)\right)$$

01.04.21.0056.01

$$\int_0^{\pi} \log^{10}\left(2\sin\left(\frac{t}{2}\right)\right) dt == \frac{45\left(93\,632\,\pi^5\,\zeta(3)^2 + 1\,774\,080\,\pi^3\,\zeta(5)\,\zeta(3) + 31\,933\,440\,\pi\,\zeta(7)\,\zeta(3) + 2117\,\pi^{11} + 15\,966\,720\,\pi\,\zeta(5)^2\right)}{11\,264}$$

01.04.21.0057.01

$$\int_0^{\frac{\pi}{2}} \log^n\left(\frac{\sin(t)}{2}\right) \tan\left(\frac{t}{2}\right) dt == \frac{1}{2^{n+1}} \lim_{p\to 0} \frac{\partial^n \frac{\frac{\Gamma(p+1)^2}{\Gamma(2p+1)}-1}{p}}{\partial p^n} - \frac{\log^{n+1}\left(\frac{1}{2}\right)}{n+1} \;/;\; n \in \mathbb{N}^+$$

01.04.21.0058.01

$$\int_0^{\frac{\pi}{2}} \log(\tan(t))\, dt == 0$$

01.04.21.0059.01

$$\int_0^{\frac{\pi}{2}} \log^2(\tan(t))\, dt == \frac{\pi^3}{8}$$

01.04.21.0060.01

$$\int_0^{\frac{\pi}{2}} \log^3(\tan(t))\, dt == 0$$

01.04.21.0061.01

$$\int_0^{\frac{\pi}{2}} \log^4(\tan(t))\, dt == \frac{5\,\pi^5}{32}$$

01.04.21.0062.01

$$\int_0^{\frac{\pi}{2}} \log^5(\tan(t))\, dt == 0$$

01.04.21.0063.01

$$\int_0^{\frac{\pi}{2}} \log^6(\tan(t))\, dt == \frac{61\,\pi^7}{128}$$

01.04.21.0064.01

$$\int_0^{\frac{\pi}{2}} \log^7(\tan(t))\, dt == 0$$

01.04.21.0065.01

$$\int_0^{\frac{\pi}{2}} \log^8(\tan(t))\, dt == \frac{1385\,\pi^9}{512}$$

01.04.21.0066.01

$$\int_0^{\frac{\pi}{2}} \log^9(\tan(t))\, dt == 0$$

01.04.21.0067.01

$$\int_0^{\frac{\pi}{2}} \log^{10}(\tan(t))\, dt == \frac{50\,521\,\pi^{11}}{2048}$$

01.04.21.0068.01

$$\int_0^{\frac{\pi}{4}} \log^n(\tan(t)) \tan(t) \, dt = (-1)^n \, 2^{-n-1} \, (1 - 2^{-n}) \, n! \, \zeta(n+1) \, /; \, n \in \mathbb{N}^+$$

01.04.21.0069.01

$$\int_0^{\frac{\pi}{2}} \log^m(\sin(t)) \log^n(\cos(t)) \cot(t) \, dt = 2^{-m-n-1} \lim_{\lambda \to 0} \lim_{\mu \to 0} \frac{\partial^n \dfrac{\partial^m \dfrac{\exp\left(-\sum_{p=1}^{n+1} \sum_{q=1}^{m+1} \frac{\lambda^p \, \mu^q \, \left((-1)^{p+q} \, (p+q-1)!\right) \zeta(p+q)}{p! \, q!}\right)}{\mu}}{\partial \mu^m}}{\partial \lambda^n} \, /; \, m \in \mathbb{N} \land n \in \mathbb{N}^+$$

01.04.21.0070.01

$$\int_0^{\frac{\pi}{3}} t \log^2\left(2 \sin\left(\frac{t}{2}\right)\right) dt = \frac{17 \pi^4}{6480}$$

01.04.21.0071.01

$$\int_0^{2\pi} t^2 \log^2\left(2 \sin\left(\frac{t}{2}\right)\right) dt = \frac{13 \pi^5}{45}$$

01.04.21.0072.01

$$\int_0^{2\pi} t^n \log^m\left(2 \sin\left(\frac{t}{2}\right)\right) dt = \lim_{\nu \to 0} \lim_{\mu \to 0} 2 \pi \, e^{-\frac{\pi i n}{2}} \frac{\partial^m \dfrac{\partial^n \dfrac{e^{\pi i \nu} \, \Gamma(\mu+1)}{\Gamma\left(\frac{\mu}{2}+\nu+1\right) \Gamma\left(\frac{\mu}{2}-\nu+1\right)}}{\partial \nu^n}}{\partial \mu^m} \, /; \, m \in \mathbb{N} \land n \in \mathbb{N}^+$$

01.04.21.0073.01

$$\int_0^{\pi} t^{2m} \log^n\left(2 \cos\left(\frac{t}{2}\right)\right) dt = \lim_{\lambda \to 0} \lim_{\mu \to 0} \pi \, (-1)^m \frac{\partial^n \dfrac{\partial^{2m} e^{-\sum_{q=1}^{n+1} \mu^q \sum_{r=1}^{m+1} \frac{(-1)^r \left(1+(-1)^q\right) (q+r-1)! \, \lambda^r \, \zeta(q+r)}{2^r \, r! \, q!}}}{\partial \mu^{2m}}}{\partial \lambda^n} \, /; \, m \in \mathbb{N} \land n \in \mathbb{N}^+$$

Involving related functions

01.04.21.0074.01

$$\int_0^1 e^{-t} \log(t) \, dt = \text{Ei}(-1) - \gamma$$

01.04.21.0077.01

$$\int_0^{\infty} e^{-t} \log^n(t) \, dt = (n-1)! \sum_{k=2}^{n} \frac{(-1)^k \zeta(k)}{(n-k)!} \int_0^{\infty} e^{-t} \log^{n-k}(t) \, dt - \gamma \int_0^{\infty} e^{-t} \log^{n-1}(t) \, dt \, /; \, n \in \mathbb{N}^+$$

Double integrals

01.04.21.0078.01

$$\int_0^1 \int_0^1 \frac{x-1}{(1-x y) \log(x y)} \, dx \, dy = \gamma$$

01.04.21.0079.01

$$\int_0^1 \int_0^1 \frac{x-1}{(x y+1) \log(x y)} \, dx \, dy = \log\left(\frac{4}{\pi}\right)$$

Integral transforms

Laplace transforms

01.04.22.0001.01

$$\mathcal{L}_t[\log(t)](z) == -\frac{\log(z)+\gamma}{z}$$

Summation

Infinite summation

01.04.23.0001.01

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\log(2k+1)}{2k+1} == \frac{1}{4}\pi\left(\log\left(\frac{2\pi\,\Gamma\left(\frac{3}{4}\right)^2}{\Gamma\left(\frac{1}{4}\right)^2}\right)+\gamma\right)$$

01.04.23.0002.01

$$\sum_{k=1}^{\infty}\log\left(1-e^{kz}\right)=\log\left(\eta\left(-\frac{iz}{2\pi}\right)\right)-\frac{z}{24}$$

Operations

Limit operation

01.04.25.0001.01

$$\lim_{z\to 0} z^a \log(z) == 0 \;/;\; \operatorname{Re}(a) > 0$$

01.04.25.0002.01

$$\lim_{z\to \infty} z^{-a} \log(z) == 0 \;/;\; \operatorname{Re}(a) > 0$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_2F_1$

01.04.26.0001.01

$$\log(z) == (z-1)\,{}_2F_1(1,1;2;1-z)$$

Through Meijer *G*

Classical cases for the direct function itself

01.04.26.0002.01

$$\log(z) == G_{2,2}^{1,2}\left(z-1\,\middle|\,\begin{matrix}1,1\\1,0\end{matrix}\right)$$

01.04.26.0003.01

$$\log(1+z) == G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}1,1\\1,0\end{matrix}\right)$$

01.04.26.0090.01

$$\log(z+1)-\sum_{k=1}^{n}\frac{(-1)^{k-1}z^k}{k}=(-1)^n G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}1,\,n+1\\ n+1,\,0\end{matrix}\right)\;/;\;n\in\mathbb{N}^+$$

Classical cases involving algebraic functions

01.04.26.0004.01

$$\frac{\log(z)}{z-1}=G_{2,2}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\,0\\ 0,\,0\end{matrix}\right)$$

01.04.26.0005.01

$$\frac{\log(z)}{z+1}=-\pi\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\,0.0001,\,\frac{1}{2}\\ 0,\,0,\,\frac{1}{2}\end{matrix}\right)\;/;\;z\notin(-1,\,0)$$

01.04.26.0006.01

$$\frac{1}{z-1}\log\left(\frac{1}{z}\right)=-G_{2,2}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\,0\\ 0,\,0\end{matrix}\right)\;/;\;z\notin(-\infty,\,0)$$

01.04.26.0007.01

$$\frac{1}{z+1}\log\left(\frac{1}{z}\right)=\pi\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}0,\,0,\,\frac{1}{2}\\ 0,\,0,\,\frac{1}{2}\end{matrix}\right)\;/;\;z\notin(-\infty,\,-1)$$

01.04.26.0091.01

$$\frac{\log(z)}{z-d}=\frac{1}{d}G_{2,2}^{2,2}\left(\frac{z}{d}\,\middle|\,\begin{matrix}0,\,0\\ 0,\,0\end{matrix}\right)-\frac{\log(d)}{d}\pi\,G_{2,2}^{1,1}\left(\frac{z}{d}\,\middle|\,\begin{matrix}0,\,\frac{1}{2}\\ 0,\,\frac{1}{2}\end{matrix}\right)\;/;\;d>0$$

01.04.26.0092.01

$$\frac{\log(z)}{d+z}=\frac{\log(d)}{d}G_{1,1}^{1,1}\left(\frac{z}{d}\,\middle|\,\begin{matrix}0\\ 0\end{matrix}\right)-\frac{\pi}{d}G_{3,3}^{2,2}\left(\frac{z}{d}\,\middle|\,\begin{matrix}0,\,0,\,\frac{1}{2}\\ 0,\,0,\,\frac{1}{2}\end{matrix}\right)\;/;\;d>0\land z\notin(-d,\,0)$$

01.04.26.0093.01

$$\frac{1}{z-d}\log\left(\frac{1}{z}\right)=-\frac{1}{d}G_{2,2}^{2,2}\left(\frac{z}{d}\,\middle|\,\begin{matrix}0,\,0\\ 0,\,0\end{matrix}\right)+\frac{\log(d)}{d}\pi\,G_{2,2}^{1,1}\left(\frac{z}{d}\,\middle|\,\begin{matrix}0,\,\frac{1}{2}\\ 0,\,\frac{1}{2}\end{matrix}\right)\;/;\;d>0\land z\notin(-\infty,\,0)$$

01.04.26.0094.01

$$\frac{1}{d+z}\log\left(\frac{1}{z}\right)=-\frac{\log(d)}{d}G_{1,1}^{1,1}\left(\frac{z}{d}\,\middle|\,\begin{matrix}0\\ 0\end{matrix}\right)+\frac{\pi}{d}G_{3,3}^{2,2}\left(\frac{z}{d}\,\middle|\,\begin{matrix}0,\,0,\,\frac{1}{2}\\ 0,\,0,\,\frac{1}{2}\end{matrix}\right)\;/;\;d>0\land z\notin(-d,\,0)$$

Classical cases involving algebraic functions in the arguments

01.04.26.0008.01

$$\log\left(\sqrt{z+1}+\sqrt{z}\right)=\frac{1}{2\sqrt{\pi}}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}1,\,1\\ \frac{1}{2},\,0\end{matrix}\right)$$

01.04.26.0009.01

$$\log\left(\sqrt{z+1}-\sqrt{z}\right)=-\frac{1}{2\sqrt{\pi}}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}1,\,1\\ \frac{1}{2},\,0\end{matrix}\right)$$

01.04.26.0010.01

$$\log\left(\frac{1}{\sqrt{z+1}+\sqrt{z}}\right) = -\frac{1}{2\sqrt{\pi}} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0011.01

$$\log\left(\frac{1}{\sqrt{z+1}-\sqrt{z}}\right) = \frac{1}{2\sqrt{\pi}} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0012.01

$$\log\left(\frac{\sqrt{z+1}+1}{\sqrt{z}}\right) = -\frac{1}{2\sqrt{\pi}} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, 1, \frac{1}{2} \\ 0, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0013.01

$$\log\left(\frac{\sqrt{z+1}-1}{\sqrt{z}}\right) = \frac{1}{2\sqrt{\pi}} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, 1, \frac{1}{2} \\ 0, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0014.01

$$\log\left(\frac{\sqrt{z}}{\sqrt{1+z}+1}\right) = \frac{1}{2\sqrt{\pi}} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, 1, \frac{1}{2} \\ 0, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0015.01

$$\log\left(\frac{\sqrt{z}}{\sqrt{1+z}-1}\right) = -\frac{1}{2\sqrt{\pi}} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, 1, \frac{1}{2} \\ 0, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0016.01

$$\log\left(\frac{\sqrt{z+1}+1}{2}\right) = \frac{1}{2\sqrt{\pi}} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 1, 1, \frac{1}{2} \\ 1, 0, 0 \end{matrix}\right)$$

01.04.26.0017.01

$$\log\left(\frac{\sqrt{z+1}+\sqrt{z}}{2\sqrt{z}}\right) = \frac{1}{2\sqrt{\pi}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 0, 1, 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0018.01

$$\frac{1}{\sqrt{z+1}} \log\left(\sqrt{z}+\sqrt{z+1}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0019.01

$$\frac{1}{\sqrt{z+1}} \log\left(\sqrt{z}-\sqrt{z+1}\right) = -\frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0020.01

$$\frac{1}{\sqrt{z+1}} \log\left(\frac{1}{\sqrt{1+z}+\sqrt{z}}\right) = -\frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0021.01

$$\frac{1}{\sqrt{z+1}} \log\left(\frac{1}{\sqrt{1+z}-\sqrt{z}}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0022.01

$$\frac{1}{\sqrt{z+1}} \log\left(\frac{\sqrt{z+1}+1}{\sqrt{z}}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z \middle| \begin{matrix} 0, 0, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

01.04.26.0023.01

$$\frac{1}{\sqrt{z+1}} \log\left(\frac{\sqrt{z+1}-1}{\sqrt{z}}\right) = -\frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z \middle| \begin{matrix} 0, 0, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

01.04.26.0024.01

$$\frac{1}{\sqrt{z+1}} \log\left(\frac{\sqrt{z}}{\sqrt{1+z}+1}\right) = -\frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z \middle| \begin{matrix} 0, 0, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

01.04.26.0025.01

$$\frac{1}{\sqrt{z+1}} \log\left(\frac{\sqrt{z}}{\sqrt{1+z}-1}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z \middle| \begin{matrix} 0, 0, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

Classical cases involving powers of log involving algebraic functions

01.04.26.0026.01

$$\frac{1}{z+1} \log^2(z) = 2\, G_{3,3}^{3,3}\left(z \middle| \begin{matrix} 0, 0, 0 \\ 0, 0, 0 \end{matrix}\right) - \pi^2\, G_{1,1}^{1,1}\left(z \middle| \begin{matrix} 0 \\ 0 \end{matrix}\right)$$

01.04.26.0027.01

$$\frac{1}{z+1} \log^2\left(\frac{1}{z}\right) = 2\, G_{3,3}^{3,3}\left(z \middle| \begin{matrix} 0, 0, 0 \\ 0, 0, 0 \end{matrix}\right) - \pi^2\, G_{1,1}^{1,1}\left(z \middle| \begin{matrix} 0 \\ 0 \end{matrix}\right) /; z \notin (-\infty, 0)$$

01.04.26.0095.01

$$\frac{\log^2(z)}{z-1} = -2\pi\, G_{4,4}^{3,3}\left(z \middle| \begin{matrix} 0, 0, 0, \frac{1}{2} \\ 0, 0, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0096.01

$$\frac{1}{z-1} \log^2\left(\frac{1}{z}\right) = -2\pi\, G_{4,4}^{3,3}\left(z \middle| \begin{matrix} 0, 0, 0, \frac{1}{2} \\ 0, 0, 0, \frac{1}{2} \end{matrix}\right)$$

Classical cases for powers of log involving algebraic functions in the arguments

01.04.26.0028.01

$$\log^2\left(\sqrt{z+1}+\sqrt{z}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(z \middle| \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0029.01

$$\log^2\left(\sqrt{z+1}-\sqrt{z}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(z \middle| \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0030.01

$$\log^2\left(\frac{1}{\sqrt{z+1}+\sqrt{z}}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(z \middle| \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0031.01

$$\log^2\left(\frac{1}{\sqrt{z+1}-\sqrt{z}}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(z \middle| \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0032.01

$$\log^2\left(\frac{\sqrt{z+1}+1}{\sqrt{z}}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{3,1}\left(z \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0 \end{matrix}\right)$$

01.04.26.0033.01

$$\log^2\left(\frac{\sqrt{z+1}-1}{\sqrt{z}}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{3,1}\left(z \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0 \end{matrix}\right)$$

01.04.26.0034.01

$$\log^2\left(\frac{\sqrt{z}}{\sqrt{z+1}+1}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{3,1}\left(z \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0 \end{matrix}\right)$$

01.04.26.0035.01

$$\log^2\left(\frac{\sqrt{z}}{\sqrt{z+1}-1}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{3,1}\left(z \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0 \end{matrix}\right)$$

Classical cases involving unit step θ

01.04.26.0036.01

$$\theta(1-|z|)\log(z) = -G_{2,2}^{2,0}\left(z \middle| \begin{matrix} 1, 1 \\ 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

01.04.26.0037.01

$$\theta(|z|-1)\log(z) = G_{2,2}^{0,2}\left(z \middle| \begin{matrix} 1, 1 \\ 0, 0 \end{matrix}\right)$$

01.04.26.0038.01

$$\theta(1-|z|)\log\left(\frac{1}{z}\right) = G_{2,2}^{2,0}\left(z \middle| \begin{matrix} 1, 1 \\ 0, 0 \end{matrix}\right)$$

01.04.26.0039.01

$$\theta(|z|-1)\log\left(\frac{1}{z}\right) = -G_{2,2}^{0,2}\left(z \middle| \begin{matrix} 1, 1 \\ 0, 0 \end{matrix}\right) /; z \notin (-\infty, -1)$$

01.04.26.0040.01

$$\theta(1-|z|)\log(1-z) + \theta(|z|-1)\log(z-1) = \pi\, G_{3,3}^{1,2}\left(z \middle| \begin{matrix} 1, 1, \frac{1}{2} \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

Classical cases for powers of log involving unit step θ

01.04.26.0041.01

$$\theta(1-|z|)\log^2(z) = 2\, G_{3,3}^{3,0}\left(z \middle| \begin{matrix} 1, 1, 1 \\ 0, 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

01.04.26.0042.01

$$\theta(|z|-1)\log^2(z) = 2\, G_{3,3}^{0,3}\left(z \middle| \begin{matrix} 1, 1, 1 \\ 0, 0, 0 \end{matrix}\right)$$

01.04.26.0043.01

$$\theta(1-|z|)\log^2\left(\frac{1}{z}\right) == 2\,G_{3,3}^{3,0}\left(z\,\middle|\,\begin{matrix}1,1,1\\0,0,0\end{matrix}\right)$$

01.04.26.0044.01

$$\theta(|z|-1)\log^2\left(\frac{1}{z}\right) == 2\,G_{3,3}^{0,3}\left(z\,\middle|\,\begin{matrix}1,1,1\\0,0,0\end{matrix}\right)\,/;\,z\notin(-\infty,-1)$$

01.04.26.0045.01

$$\theta(1-|z|)\log^{m-1}(z) == (-1)^{m-1}(m-1)!\,G_{m,m}^{m,0}\left(z\,\middle|\,\begin{matrix}1,\ldots,1\\0,\ldots,0\end{matrix}\right)\,/;\,z\notin(-1,0)$$

01.04.26.0046.01

$$\theta(|z|-1)\log^{n-1}(z) == (n-1)!\,G_{n,n}^{0,n}\left(z\,\middle|\,\begin{matrix}1,\ldots,1\\0,\ldots,0\end{matrix}\right)$$

01.04.26.0047.01

$$\theta(1-|z|)\log^{m-1}\left(\frac{1}{z}\right) == (m-1)!\,G_{m,m}^{m,0}\left(z\,\middle|\,\begin{matrix}1,\ldots,1\\0,\ldots,0\end{matrix}\right)$$

01.04.26.0048.01

$$\theta(|z|-1)\log^{n-1}\left(\frac{1}{z}\right) == (-1)^{n-1}(n-1)!\,G_{n,n}^{0,n}\left(z\,\middle|\,\begin{matrix}1,\ldots,1\\0,\ldots,0\end{matrix}\right)\,/;\,z\notin(-\infty,-1)$$

Classical cases involving algebraic functions in the arguments and unit step θ

01.04.26.0049.01

$$\theta(|z|-1)\log\left(\sqrt{z}+\sqrt{z-1}\right) == \frac{\sqrt{\pi}}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}1,1\\0,\frac{1}{2}\end{matrix}\right)$$

01.04.26.0050.01

$$\theta(|z|-1)\log\left(\sqrt{z}-\sqrt{z-1}\right) == -\frac{\sqrt{\pi}}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}1,1\\0,\frac{1}{2}\end{matrix}\right)$$

01.04.26.0051.01

$$\theta(1-|z|)\log\left(\frac{1+\sqrt{1-z}}{\sqrt{z}}\right) == \frac{\sqrt{\pi}}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1\\0,0\end{matrix}\right)\,/;\,z\notin(-1,0)$$

01.04.26.0052.01

$$\theta(1-|z|)\log\left(\frac{1-\sqrt{1-z}}{\sqrt{z}}\right) == -\frac{\sqrt{\pi}}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1\\0,0\end{matrix}\right)\,/;\,z\notin(-1,0)$$

Classical cases involving Abs in the arguments

01.04.26.0053.01

$$\log(|1-z|) == \pi\,G_{3,3}^{1,2}\left(z\,\middle|\,\begin{matrix}1,1,\frac{1}{2}\\1,0,\frac{1}{2}\end{matrix}\right)\,/;\,z>-1$$

01.04.26.0054.01

$$\log\left(\left|\frac{1+z}{1-z}\right|\right) == \pi\,G_{2,2}^{1,1}\left(z^2\,\middle|\,\begin{matrix}\frac{1}{2},1\\\frac{1}{2},0\end{matrix}\right)\,/;\,z>0$$

01.04.26.0055.01

$$\log\left(\left|\frac{1-z}{1+z}\right|\right) = -\pi\, G_{2,2}^{1,1}\left(z^2 \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{2}, 0 \end{matrix}\right) /;\ z > 0$$

Classical cases involving sgn in the arguments

01.04.26.0056.01

$$\log\left(\frac{1+\sqrt{z}}{\operatorname{sgn}(1-|z|)\left(1-\sqrt{z}\right)}\right) = \pi\, G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{2}, 0 \end{matrix}\right) /;\ z \notin (-1, 0)$$

Generalized cases for the direct function itself

01.04.26.0097.01

$$\log(z) = G_{2,2}^{1,2}\left(z, 1 \,\middle|\, \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right) - G_{2,2}^{1,2}\left(z, -1 \,\middle|\, \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right) /;\ z \notin (-1, 0)$$

Generalized cases involving algebraic functions in the arguments

01.04.26.0057.01

$$\log\left(\sqrt{z^2+1} + z\right) = \frac{1}{2\sqrt{\pi}}\, G_{2,2}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0058.01

$$\log\left(\sqrt{z^2+1} - z\right) = -\frac{1}{2\sqrt{\pi}}\, G_{2,2}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0059.01

$$\log\left(\frac{1}{\sqrt{z^2+1} + z}\right) = -\frac{1}{2\sqrt{\pi}}\, G_{2,2}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0060.01

$$\log\left(\frac{1}{\sqrt{z^2+1} - z}\right) = \frac{1}{2\sqrt{\pi}}\, G_{2,2}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0061.01

$$\log\left(\frac{\sqrt{z^2+1} + 1}{z}\right) = -\frac{1}{2\sqrt{\pi}}\, G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1, \frac{1}{2} \\ 0, \frac{1}{2}, 0 \end{matrix}\right) /;\ \operatorname{Re}(z) > 0$$

01.04.26.0062.01

$$\log\left(\frac{\sqrt{z^2+1} - 1}{z}\right) = \frac{1}{2\sqrt{\pi}}\, G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1, \frac{1}{2} \\ 0, \frac{1}{2}, 0 \end{matrix}\right) /;\ \operatorname{Re}(z) > 0$$

01.04.26.0063.01

$$\log\left(\frac{z}{\sqrt{z^2+1} + 1}\right) = \frac{1}{2\sqrt{\pi}}\, G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1, \frac{1}{2} \\ 0, \frac{1}{2}, 0 \end{matrix}\right) /;\ \operatorname{Re}(z) > 0$$

01.04.26.0064.01

$$\log\left(\frac{z}{\sqrt{z^2+1}-1}\right) = -\frac{1}{2\sqrt{\pi}} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, 1, \frac{1}{2} \\ 0, \frac{1}{2}, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0065.01

$$\log\left(\frac{\sqrt{z^2+1}+1}{2}\right) = \frac{1}{2\sqrt{\pi}} G_{3,3}^{1,3}\left(z, \frac{1}{2} \middle| \begin{matrix} 1, 1, \frac{1}{2} \\ 1, 0, 0 \end{matrix}\right)$$

01.04.26.0066.01

$$\log\left(\frac{\sqrt{z^2+1}+z}{2z}\right) = \frac{1}{2\sqrt{\pi}} G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, 1, 1 \\ 0, 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0067.01

$$\frac{1}{\sqrt{z^2+1}} \log\left(z+\sqrt{z^2+1}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0068.01

$$\frac{1}{\sqrt{z^2+1}} \log\left(\sqrt{z^2+1}-z\right) = -\frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0069.01

$$\frac{1}{\sqrt{z^2+1}} \log\left(\frac{1}{\sqrt{z^2+1}+z}\right) = -\frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0070.01

$$\frac{1}{\sqrt{z^2+1}} \log\left(\frac{1}{\sqrt{z^2+1}-z}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

01.04.26.0071.01

$$\frac{1}{\sqrt{z^2+1}} \log\left(\frac{\sqrt{z^2+1}+1}{z}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, 0, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0072.01

$$\frac{1}{\sqrt{z^2+1}} \log\left(\frac{\sqrt{z^2+1}-1}{z}\right) = -\frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, 0, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0073.01

$$\frac{1}{\sqrt{z^2+1}} \log\left(\frac{z}{\sqrt{z^2+1}+1}\right) = -\frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, 0, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0074.01

$$\frac{1}{\sqrt{z^2+1}} \log\left(\frac{z}{\sqrt{z^2+1}-1}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, 0, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

Generalized cases for powers of log involving algebraic functions in the arguments

01.04.26.0075.01

$$\log^2\left(\sqrt{z^2+1}+z\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(z, \frac{1}{2} \middle| \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0076.01

$$\log^2\left(\sqrt{z^2+1}-z\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(z, \frac{1}{2} \middle| \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0077.01

$$\log^2\left(\frac{1}{\sqrt{z^2+1}+z}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(z, \frac{1}{2} \middle| \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0078.01

$$\log^2\left(\frac{1}{\sqrt{z^2+1}-z}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(z, \frac{1}{2} \middle| \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

01.04.26.0079.01

$$\log^2\left(\frac{\sqrt{z^2+1}+1}{z}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0 \end{matrix}\right) /; /; \operatorname{Re}(z) > 0$$

01.04.26.0080.01

$$\log^2\left(\frac{\sqrt{z^2+1}-1}{z}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0081.01

$$\log^2\left(\frac{z}{\sqrt{z^2+1}+1}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0082.01

$$\log^2\left(\frac{z}{\sqrt{z^2+1}-1}\right) = \frac{\sqrt{\pi}}{2} G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{matrix} 0, \frac{1}{2}, 1 \\ 0, 0, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

Generalized cases involving algebraic functions in the arguments and unit step θ

01.04.26.0083.01

$$\theta(|z|-1)\log\left(z+\sqrt{z^2-1}\right) = \frac{\sqrt{\pi}}{2} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} 1, 1 \\ 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0084.01

$$\theta(|z|-1)\log\left(z-\sqrt{z^2-1}\right) = -\frac{\sqrt{\pi}}{2} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} 1, 1 \\ 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

01.04.26.0085.01

$$\theta(1-|z|)\log\left(\frac{1+\sqrt{1-z^2}}{z}\right) = \frac{\sqrt{\pi}}{2} G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, 1 \\ 0, 0 \end{matrix}\right)$$

01.04.26.0086.01

$$\theta(1-|z|)\log\left(\frac{1-\sqrt{1-z^2}}{z}\right) = -\frac{\sqrt{\pi}}{2} G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, 1 \\ 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

Generalized cases involving sgn in the arguments

01.04.26.0087.01

$$\log\left(\frac{z+1}{\operatorname{sgn}(1-|z|)(1-z)}\right) = \pi\, G_{2,2}^{1,1}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

Through other functions

01.04.26.0088.01

$$\log(z) = -\mathrm{Li}_1(1-z)$$

01.04.26.0098.01

$$\log(z) = -S_{0,1}(1-z)$$

01.04.26.0089.01

$$F(z \mid 1) = \log(\sec(z)+\tan(z)) /; |\mathrm{Re}(z)| < \frac{\pi}{2}$$

Representations through equivalent functions

With inverse function Log

01.04.27.0062.01

$$\log(e^z) = z + 2\, i\, \pi \left\lfloor \frac{\pi - \mathrm{Im}(z)}{2\pi} \right\rfloor$$

The left side of above formula corresponds to composition $f\left(f^{(-1)}(z)\right) /;\ f(z) = \log(z)$, which generically does not equal to z.

01.04.27.0063.01

$$\log(e^z) = z /; -\pi < \mathrm{Im}(z) \leq \pi$$

The left side of above formula corresponds to composition $f\left(f^{(-1)}(z)\right) /;\ f(z) = \log(z)$, which equal to z under restriction $-\pi < \mathrm{Im}(z) \leq \pi$.

01.04.27.0064.01

$$e^{\log(z)} = z$$

The left side of above formula corresponds to composition $f^{(-1)}(f(z)) /;\ f(z) = \log(z)$, which generically equal to z.

With related functions

Involving $\sin^{-1}$

01.04.27.0004.01

$$\log(z) = \frac{\pi i}{2} - i \sin^{-1}\left(\frac{z^2+1}{2z}\right) /; \operatorname{Im}(z) > 0 \lor z \leq -1 \lor 0 < z \leq 1$$

01.04.27.0005.01

$$\log(z) = i \sin^{-1}\left(\frac{z^2+1}{2z}\right) - \frac{\pi i}{2} /; \operatorname{Im}(z) < 0 \lor z \geq 1$$

01.04.27.0006.01

$$\log(z) = i \sin^{-1}\left(\frac{z^2+1}{2z}\right) + \frac{3\pi i}{2} /; -1 < z < 0$$

01.04.27.0007.01

$$\log(z) = \frac{\sqrt{-z^2}}{z} \sqrt{\frac{z-1}{z+1}} \sqrt{\frac{z+1}{z-1}} \left(\sin^{-1}\left(\frac{z^2+1}{2z}\right) - \frac{\pi}{2}\right) + \pi i \left(1 - \frac{\sqrt{-z-1}\,\sqrt{z}}{\sqrt{-z}\,\sqrt{z+1}}\right)$$

Involving $\cos^{-1}$

01.04.27.0008.01

$$\log(z) = i \cos^{-1}\left(\frac{z^2+1}{2z}\right) /; \operatorname{Im}(z) > 0 \lor z \leq -1 \lor 0 < z \leq 1$$

01.04.27.0009.01

$$\log(z) = -i \cos^{-1}\left(\frac{z^2+1}{2z}\right) /; \operatorname{Im}(z) < 0 \lor z \geq 1$$

01.04.27.0010.01

$$\log(z) = -i \cos^{-1}\left(\frac{z^2+1}{2z}\right) + 2\pi i /; -1 < z < 0$$

01.04.27.0011.01

$$\log(z) = \pi i \left(1 - \frac{\sqrt{-z-1}\,\sqrt{z}}{\sqrt{-z}\,\sqrt{z+1}}\right) - \frac{\sqrt{-z^2}}{z} \sqrt{\frac{z-1}{z+1}} \sqrt{\frac{z+1}{z-1}} \cos^{-1}\left(\frac{z^2+1}{2z}\right)$$

Involving $\tan^{-1}$

01.04.27.0012.01

$$\log(z) = 2 i \tan^{-1}\left(\frac{i(1-z)}{z+1}\right) /; z \notin (-\infty, -1)$$

01.04.27.0013.01

$$\log(z) = 2 i \tan^{-1}\left(\frac{i(1-z)}{z+1}\right) + 2\pi i /; z < -1$$

01.04.27.0014.01

$$\log(z) = 2 i \tan^{-1}\left(\frac{i(1-z)}{z+1}\right) + \pi i \left(1 - \sqrt{z+1} \sqrt{\frac{1}{z+1}}\right)$$

Involving $\cot^{-1}$

01.04.27.0015.01

$$\log(z) = 2 i \cot^{-1}\left(\frac{i(z+1)}{z-1}\right) /; z \notin (-\infty, -1)$$

01.04.27.0016.01

$$\log(z) = 2\,i\cot^{-1}\left(\frac{i\,(z+1)}{z-1}\right) + 2\,\pi\,i\;/;\;z < -1$$

01.04.27.0017.01

$$\log(z) = 2\,i\cot^{-1}\left(\frac{i\,(z+1)}{z-1}\right) + \pi\,i\left(1 - \sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\right)$$

Involving $\csc^{-1}$

01.04.27.0018.01

$$\log(z) = \frac{\pi\,i}{2} - i\csc^{-1}\left(\frac{2\,z}{1+z^2}\right)\;/;\;\operatorname{Im}(z) > 0 \lor z \le -1 \lor 0 < z \le 1$$

01.04.27.0019.01

$$\log(z) = i\csc^{-1}\left(\frac{2\,z}{1+z^2}\right) - \frac{\pi\,i}{2}\;/;\;\operatorname{Im}(z) < 0 \lor z \ge 1$$

01.04.27.0020.01

$$\log(z) = i\csc^{-1}\left(\frac{2\,z}{1+z^2}\right) + \frac{3\,\pi\,i}{2}\;/;\;-1 < z < 0$$

01.04.27.0021.01

$$\log(z) = \frac{\sqrt{-z^2}}{z}\sqrt{\frac{z-1}{z+1}}\sqrt{\frac{z+1}{z-1}}\left(\csc^{-1}\left(\frac{2\,z}{1+z^2}\right) - \frac{\pi}{2}\right) + \pi\,i\left(1 - \frac{\sqrt{-z-1}\,\sqrt{z}}{\sqrt{-z}\,\sqrt{z+1}}\right)$$

Involving $\sec^{-1}$

01.04.27.0022.01

$$\log(z) = i\sec^{-1}\left(\frac{2\,z}{1+z^2}\right)\;/;\;\operatorname{Im}(z) > 0 \lor z \le -1 \lor 0 < z \le 1$$

01.04.27.0023.01

$$\log(z) = -i\sec^{-1}\left(\frac{2\,z}{1+z^2}\right)\;/;\;\operatorname{Im}(z) < 0 \lor z \ge 1$$

01.04.27.0024.01

$$\log(z) = -i\sec^{-1}\left(\frac{2\,z}{1+z^2}\right) + 2\,\pi\,i\;/;\;-1 < z < 0$$

01.04.27.0025.01

$$\log(z) = \pi\,i\left(1 - \frac{\sqrt{-z-1}\,\sqrt{z}}{\sqrt{-z}\,\sqrt{z+1}}\right) - \frac{\sqrt{-z^2}}{z}\sqrt{\frac{z-1}{z+1}}\sqrt{\frac{z+1}{z-1}}\sec^{-1}\left(\frac{2\,z}{1+z^2}\right)$$

Involving $\sinh^{-1}$

01.04.27.0026.01

$$\log(z) = \sinh^{-1}\left(\frac{z^2-1}{2\,z}\right)\;/;\;\operatorname{Re}(z) > 0 \lor 0 < i\,z < 1 \lor i\,z < -1$$

01.04.27.0027.01

$$\log(z) = \pi\,i - \sinh^{-1}\left(\frac{z^2-1}{2\,z}\right)\;/;\;\operatorname{Re}(z) < 0 \land \operatorname{Im}(z) > 0 \lor -1 \le i\,z < 0 \lor z < 0$$

01.04.27.0028.01

$$\log(z) = -\pi i - \sinh^{-1}\left(\frac{z^2-1}{2z}\right) /; \operatorname{Re}(z) < 0 \land \operatorname{Im}(z) < 0 \lor i z \geq 1$$

01.04.27.0029.01

$$\log(z) = \frac{\pi i}{2}\left(\sqrt{\frac{z}{z+i}}\sqrt{\frac{z+i}{z}} - \sqrt{\frac{z}{z-i}}\sqrt{\frac{z-i}{z}}\right) + \frac{\sqrt{-i z-1}\sqrt{i z-1}}{\sqrt{z^2+1}}\left(\frac{\pi\left(z-\sqrt{z^2}\right)}{2\sqrt{-z^2}} + \frac{\sqrt{z^2}}{z}\sinh^{-1}\left(\frac{z^2-1}{2z}\right)\right)$$

Involving $\cosh^{-1}$

01.04.27.0030.01

$$\log(z) = \cosh^{-1}\left(\frac{z^2+1}{2z}\right) /; |z| > 1 \bigvee |z| = 1 \bigwedge 0 < \arg(z) \leq \pi$$

01.04.27.0031.01

$$\log(z) = -\cosh^{-1}\left(\frac{z^2+1}{2z}\right) /; |z| < 1 \bigwedge z \notin (-1, 0) \bigvee |z| = 1 \bigwedge -\pi < \arg(z) \leq 0$$

01.04.27.0032.01

$$\log(z) = 2\pi i - \cosh^{-1}\left(\frac{z^2+1}{2z}\right) /; -1 < z < 0$$

01.04.27.0033.01

$$\log(z) = \pi i\left(1 - \frac{\sqrt{-z-1}\sqrt{z}}{\sqrt{-z}\sqrt{z+1}}\right) - \frac{\sqrt{-z-1}\sqrt{z-1}\sqrt{-z^2}}{(1-z)^{5/2}\sqrt{z+1}}\sqrt{-\frac{(1-z)^4}{z^2}}\cosh^{-1}\left(\frac{z^2+1}{2z}\right)$$

Involving $\tanh^{-1}$

01.04.27.0034.01

$$\log(z) = 2\tanh^{-1}\left(\frac{z-1}{z+1}\right) /; z \notin (-\infty, -1)$$

01.04.27.0035.01

$$\log(z) = 2\pi i + 2\tanh^{-1}\left(\frac{z-1}{z+1}\right) /; z < -1$$

01.04.27.0036.01

$$\log(z) = 2\tanh^{-1}\left(\frac{z-1}{z+1}\right) + \pi i\left(1 - \sqrt{z+1}\sqrt{\frac{1}{z+1}}\right)$$

01.04.27.0037.01

$$\log(z) = \tanh^{-1}\left(\frac{z^2-1}{z^2+1}\right) /; \operatorname{Re}(z) > 0 \lor i z > 1 \lor -1 < i z < 0$$

01.04.27.0038.01

$$\log(z) = \tanh^{-1}\left(\frac{z^2-1}{z^2+1}\right) + \pi i /; \operatorname{Re}(z) < 0 \land \operatorname{Im}(z) > 0 \lor i z < -1 \lor z < 0$$

01.04.27.0039.01

$$\log(z) = \tanh^{-1}\left(\frac{z^2-1}{z^2+1}\right) - \pi i /; \operatorname{Re}(z) < 0 \land \operatorname{Im}(z) < 0 \lor 0 < i z < 1$$

01.04.27.0040.01

$$\log(z) = \tanh^{-1}\left(\frac{z^2-1}{z^2+1}\right) + \frac{\pi\left(z-\sqrt{z^2}\right)}{2\sqrt{-z^2}} + \frac{\pi i}{2}\left(2 - \sqrt{\frac{1}{i z+1}}\sqrt{i z+1} - \sqrt{\frac{1}{1-i z}}\sqrt{1-i z}\right)$$

Involving $\coth^{-1}$

01.04.27.0041.01

$$\log(z) = 2\coth^{-1}\left(\frac{z+1}{z-1}\right) /; z \notin (-\infty, -1)$$

01.04.27.0042.01

$$\log(z) = 2\coth^{-1}\left(\frac{z+1}{z-1}\right) + 2\pi i /; z < -1$$

01.04.27.0043.01

$$\log(z) = 2\coth^{-1}\left(\frac{z+1}{z-1}\right) + \pi i\left(1 - \sqrt{z+1}\sqrt{\frac{1}{z+1}}\right)$$

01.04.27.0044.01

$$\log(z) = \coth^{-1}\left(\frac{z^2+1}{z^2-1}\right) /; \mathrm{Re}(z) > 0 \lor i z > 1 \lor -1 < i z < 0$$

01.04.27.0045.01

$$\log(z) = \coth^{-1}\left(\frac{z^2+1}{z^2-1}\right) + \pi i /; \mathrm{Re}(z) < 0 \land \mathrm{Im}(z) > 0 \lor i z < -1 \lor z < 0$$

01.04.27.0046.01

$$\log(z) = \coth^{-1}\left(\frac{z^2+1}{z^2-1}\right) - \pi i /; \mathrm{Re}(z) < 0 \land \mathrm{Im}(z) < 0 \lor 0 < i z < 1$$

01.04.27.0047.01

$$\log(z) = \coth^{-1}\left(\frac{z^2+1}{z^2-1}\right) + \frac{\pi\left(z-\sqrt{z^2}\right)}{2\sqrt{-z^2}} + \frac{\pi i}{2}\left(2 - \sqrt{\frac{1}{i z+1}}\sqrt{i z+1} - \sqrt{\frac{1}{1-i z}}\sqrt{1-i z}\right)$$

Involving csch^{-1}

01.04.27.0048.01

$$\log(z) = \operatorname{csch}^{-1}\left(\frac{2z}{z^2-1}\right) /; \mathrm{Re}(z) > 0 \lor 0 < i z < 1 \lor i z < -1$$

01.04.27.0049.01

$$\log(z) = \pi i - \operatorname{csch}^{-1}\left(\frac{2z}{z^2-1}\right) /; \mathrm{Re}(z) < 0 \land \mathrm{Im}(z) > 0 \lor -1 \leq i z < 0 \lor z < 0$$

01.04.27.0050.01

$$\log(z) = -\pi i - \operatorname{csch}^{-1}\left(\frac{2z}{z^2-1}\right) /; \mathrm{Re}(z) < 0 \land \mathrm{Im}(z) < 0 \lor i z \geq 1$$

01.04.27.0051.01

$$\log(z) = \frac{\pi i}{2}\left(\sqrt{\frac{z}{z+i}}\sqrt{\frac{z+i}{z}} - \sqrt{\frac{z}{z-i}}\sqrt{\frac{z-i}{z}}\right) + \frac{\sqrt{-i z-1}\sqrt{i z-1}}{\sqrt{z^2+1}}\left(\frac{\pi\left(z-\sqrt{z^2}\right)}{2\sqrt{-z^2}} + \frac{\sqrt{z^2}}{z}\operatorname{csch}^{-1}\left(\frac{2z}{z^2-1}\right)\right)$$

Involving sech^{-1}

01.04.27.0052.01

$$\log(z) = \operatorname{sech}^{-1}\left(\frac{2z}{1+z^2}\right) /; |z| > 1 \lor |z| = 1 \land 0 < \arg(z) \leq \pi$$

01.04.27.0053.01

$$\log(z) = -\operatorname{sech}^{-1}\left(\frac{2z}{1+z^2}\right) /; |z| < 1 \land z \notin (-1, 0) \lor |z| = 1 \land -\pi < \arg(z) \leq 0$$

01.04.27.0054.01

$$\log(z) = 2\pi i - \operatorname{sech}^{-1}\left(\frac{2z}{1+z^2}\right) /; -1 < z < 0$$

01.04.27.0055.01

$$\log(z) = \pi i\left(1 - \frac{\sqrt{-z-1}\sqrt{z}}{\sqrt{-z}\sqrt{z+1}}\right) - \frac{\sqrt{-z-1}\sqrt{z-1}\sqrt{-z^2}}{(1-z)^{5/2}\sqrt{z+1}}\sqrt{-\frac{(1-z)^4}{z^2}}\operatorname{sech}^{-1}\left(\frac{2z}{1+z^2}\right)$$

Involving $\log_a(z)$

01.04.27.0056.01

$$\log(z) = \log_e(z)$$

01.04.27.0057.01

$$\log(z) = \log(a)\log_a(z)$$

Involving other related functions

01.04.27.0058.01

$$z^{\log(a)} = a^{\log(z)}$$

01.04.27.0059.01

$$\log(W_k(z)) = 2 i \pi k + \log(z) - W_k(z)$$

01.04.27.0060.01

$$\log(W_{-1}(z)) = \log(z) - W_{-1}(z) - 2 i \pi /; z \notin \left(-\infty, -\frac{1}{e}\right) \bigwedge z \notin (0, \infty)$$

01.04.27.0061.01

$$\log(W_{-1}(x))\, 1 = \log(x) - W_{-1}(x) /; -\frac{1}{e} < x < 0$$

Inequalities

01.04.29.0001.01

$$\frac{x}{x+1} < \log(1+x) < x /; x > -1 \land x \neq 0$$

01.04.29.0002.01

$$x < -\log(1-x) < \frac{x}{1-x} \;/;\; x < 1 \land x \neq 0$$

01.04.29.0003.01

$$-\frac{3}{2}\, x < \log(1-x) < \frac{3\,x}{2} \;/;\; 0 < x \leq 0.5828$$

01.04.29.0004.01

$$\log(x) \leq x - 1 \;/;\; x > 0$$

01.04.29.0005.01

$$\log(x) \leq n\left(x^{1/n} - 1\right) \;/;\; n > 0 \land x > 0$$

01.04.29.0006.01

$$\log(1 - |x|) \leq \log(x+1) \leq -\log(1-|x|) \;/;\; -1 < x < 1$$

Zeros

01.04.30.0001.01

$$\log(z) == 0 \;/;\; z == 1$$

Theorems

The Shannon information content of a message

The Shannon information content of a message is $H == -\sum_i p_i \log(p_i)$, where p_i is the probability for the symbol ξ_i.

The entropy of a (classical) physical system

The entropy S of a (classical) physical system is $S == k_B \log(\Gamma(E))$, where k_B is the Boltzmann constant and $\Gamma(E)$ is the phase space volume occupied by the system at energy E.

Transcendentality of a sum

If $\alpha == \sum_{i=1}^{n} \gamma_i \log(\beta_i) \neq 0$, where the β_i, γ_i are algebraic numbers, then α is transcendental.

Conformal mapping from the half-plane to the strip

The conformal map from the upper half w -plane to the strip $-\infty < \mathrm{Im}(z) < 0,\ 0 < \mathrm{Re}(z) < \pi$ is given by $z(w) == \log(w) + \left(1 - w^2\right)/2$.

Differential-Algebraic Constants

To ensure the correctness of many formulas given in this collection over the whole complex plane, it is often necessary to work with expressions of the form $\log(z^2) == \log(i\, z) + \log(-i\, z)$, $\log(z^2) - 2\log(z)$, $\log(z) + \log(1/z)$, $\log(-z^2) == \log(z) + \log(-z)$, etc.. While in a textbook-mathematics setting these expressions are often simplified to $2\log(z)$, 0, 0, $2\log(z) + \pi\, i$, etc, this cannot be done inside *Mathematica*. From a complex function point of view the Riemann surface of such functions are made from disconnected sheets. Inside *Mathematica* all branch cuts of all functions (that have branch cuts) follow uniquely from the branch cut of the power function (the logarithm function respectively). As a result the branch cuts related to functions such as $\log(z^2)$, $\log(z^2) - 2\log(z)$, $\log(z) + \log(1/z)$, $\log(-z^2)$ exist, although they do not start and end at branch points.

In details we have:

$$\mathcal{BC}_z(\log(z^2)) == \mathcal{BC}_z(\log(i\, z) + \log(-i\, z)) == \{\{(-i\,\infty, 0), 1\}, \{(0, i\,\infty), -1\}\}$$

$$\mathcal{BC}_z(\log(z^2) - 2\log(z)) == \{\{(-i\,\infty, 0), 1\}, \{(0, i\,\infty), -1\}, \{(-\infty, 0), -i\}\}$$

$$\mathcal{BC}_z(\log(z) + \log(1/z)) == \{\{(-\infty, 0), \{\}\}\}$$

An expression of the form $\log(z^2) - 2\log(z)$, $\log(z) + \log(1/z)$ are called differential-algebraic constants because their derivative vanishes generically everywhere as a complex function (but not as a generalized function).

History

– J. Napier (1614) published the first tables and used word Log
– H. Briggs (1617) published the first tables in base 10 and found logarithms of the first 25 primes
– J. Burgi (1620)
– J. Kepler (1624)
– B. Cavalieri (1632)
– J. Gregory (1668) found series expansion for log
– N. Mercator (1668) used "Log naturalis"
– J. N. Lambert (1770) and J.-L. Lagrange (1776) found continued fraction representations for Log
– L. Euler (1749) found that log was multivalued

The function log is encountered often in mathematics and the natural sciences.

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