

MathieuS

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Notations

Traditional name

Odd Mathieu function

Traditional notation

$Se(a, q, z)$

Mathematica StandardForm notation

`MathieuS[a, q, z]`

Primary definition

11.02.02.0001.01

$Se(a, q, z)$

$Se(a, q, z)$ is the odd Mathieu function with characteristic value a and parameter q . It is defined as the odd in z solution $w(z) = Se(a, q, z)$ of the Mathieu differential equation $w''(z) + (a - 2q \cos(2z))w(z) = 0$, which satisfies for $q = 0$ relation $Se(a, 0, z) = \sin(\sqrt{a} z)$. It is analytical function in the variables a, q and z . It is a periodical only in z function for special values of parameter a (so called characteristic values $a = b_r(q)$ with integer or rational numbers r , which makes odd solutions of the form $e^{irz} f(z)$ where $f(z)$ is an odd function of z with period 2π).

Specific values

Specialized values

For fixed a, z

11.02.03.0001.01

$Se(a, 0, z) = \sin(\sqrt{a} z)$

General characteristics

Domain and analyticity

$Se(a, q, z)$ is an analytical function of a, q, z which is defined in \mathbb{C}^3 .

11.02.04.0001.01

$$(a * q * z) \rightarrow \text{Se}(a, q, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\text{Se}(a, q, z)$ is an odd function with respect to z .

11.02.04.0002.01

$$\text{Se}(a, q, -z) = -\text{Se}(a, q, z)$$

Mirror symmetry

11.02.04.0003.01

$$\text{Se}(\bar{a}, \bar{q}, \bar{z}) = \overline{\text{Se}(a, q, z)}$$

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

11.02.06.0010.01

$$\begin{aligned} \text{Se}(a, q, z) \propto & \text{Se}(a, q, z_0) + \text{Se}^{(0,0,1)}(a, q, z_0) (z - z_0) + \frac{1}{2} (2q \cos(2z_0) - a) \text{Se}(a, q, z_0) (z - z_0)^2 + \\ & \frac{1}{6} ((2q \cos(2z_0) - a) \text{Se}^{(0,0,1)}(a, q, z_0) - 4q \sin(2z_0) \text{Se}(a, q, z_0)) (z - z_0)^3 + \\ & \frac{1}{24} ((a^2 + 4q \cos(2z_0) (-a + q \cos(2z_0) - 2)) \text{Se}(a, q, z_0) - 8q \sin(2z_0) \text{Se}^{(0,0,1)}(a, q, z_0)) (z - z_0)^4 + \\ & \frac{1}{120} ((a^2 + 4q \cos(2z_0) (-a + q \cos(2z_0) - 6)) \text{Se}^{(0,0,1)}(a, q, z_0) + 16q (a - 2q \cos(2z_0) + 1) \sin(2z_0) \text{Se}(a, q, z_0)) \\ & (z - z_0)^5 + \dots /; (z \rightarrow z_0) \end{aligned}$$

11.02.06.0011.01

$$\text{Se}(a, q, z) \propto \text{Se}(a, q, z_0) (1 + O(z - z_0))$$

Expansions at $z = 0$

11.02.06.0001.01

$$\text{Se}(b_{2n+2}(q), q, z) = \sum_{k=0}^{\infty} B_{2k+2}^{2n+2} \sin((2k+2)z) /;$$

$$(b_{2n}(q) - 4) B_2^{2n+2} - q B_4^{2n+2} = 0 \wedge (b_{2n}(q) - 4k^2) B_{2k}^{2n} - q (B_{2k-2}^{2n+2} + B_{2k+2}^{2n+2}) = 0 \wedge \sum_{k=0}^{\infty} B_{2k+1}^{2n+1} = 1 \wedge n \in \mathbb{Z}$$

11.02.06.0002.01

$$\text{Se}(b_{2n+1}(q), q, z) = \sum_{k=0}^{\infty} B_{2k+1}^{2n+1} \sin((2k+1)z) /;$$

$$(b_{2n+1}(q) - q - 1) B_1^{2n+1} - q B_3^{2n+1} = 0 \wedge (b_{2n+1}(q) - (2k+1)^2) B_{2k+1}^{2n+1} - q (B_{2k-1}^{2n+1} + B_{2k+3}^{2n+1}) = 0 \wedge \sum_{k=0}^{\infty} B_{2k+1}^{2n+1} = 1 \wedge n \in \mathbb{Z}$$

Expansions at $q = 0$

11.02.06.0003.01

$$\text{Se}(b_r(q), q, z) \propto$$

$$\begin{aligned} & \sin(rz) + \frac{1}{4} \left(\frac{\sin((r-2)z)}{r-1} - \frac{\sin((r+2)z)}{r+1} \right) q + \frac{1}{32} \left(\frac{\sin((r-4)z)}{(r-2)(r-1)} - \frac{2(r^2+1)\sin(rz)}{(r-1)^2(r+1)^2} + \frac{\sin((r+4)z)}{(r+1)(r+2)} \right) q^2 + \frac{1}{384} \\ & \left(\frac{\sin((r-6)z)}{(r-3)(r-2)(r-1)} - \frac{3(r^3-r^2-r-11)\sin((r-2)z)}{(r-2)(r-1)^3(r+1)^2} + \frac{3(r^3+r^2-r+11)\sin((r+2)z)}{(r-1)^2(r+1)^3(r+2)} - \frac{\sin((r+6)z)}{(r+1)(r+2)(r+3)} \right) q^3 + \\ & \frac{1}{6144} \left(\frac{\sin((r-8)z)}{(r-4)(r-3)(r-2)(r-1)} - \frac{4(r^3-r^2-7r-29)\sin((r-4)z)}{(r-3)(r-2)(r-1)^3(r+1)^2} + \frac{6(r^8-15r^6-185r^4+675r^2+316)\sin(rz)}{(r-2)^2(r-1)^4(r+1)^4(r+2)^2} - \right. \\ & \left. \frac{4(r^3+r^2-7r+29)\sin((r+4)z)}{(r-1)^2(r+1)^3(r+2)(r+3)} + \frac{\sin((r+8)z)}{(r+1)(r+2)(r+3)(r+4)} \right) q^4 + \\ & \frac{1}{122880} \left(\frac{\sin((r-10)z)}{(r-5)(r-4)(r-3)(r-2)(r-1)} - \frac{5(r^3-r^2-17r-55)\sin((r-6)z)}{(r-4)(r-3)(r-2)(r-1)^3(r+1)^2} + \right. \\ & \left. \frac{(10(r^9-r^8-31r^7-5r^6-273r^5+2457r^4+1931r^3-6335r^2-3572r-7564)\sin((r-2)z))}{(r-3)(r-2)^2(r-1)^5(r+1)^4(r+2)^2} - \right. \\ & \left. \frac{(10(r^9+r^8-31r^7+5r^6-273r^5-2457r^4+1931r^3+6335r^2-3572r+7564)\sin((r+2)z))}{(r-2)^2(r-1)^4} \right. \\ & \left. + \frac{5(r^3+r^2-17r+55)\sin((r+6)z)}{(r-1)^2(r+1)^3(r+2)(r+3)(r+4)} - \frac{\sin((r+10)z)}{(r+1)(r+2)(r+3)(r+4)(r+5)} \right) q^5 + \\ & \frac{1}{2949120} \left(\frac{\sin((r-12)z)}{(r-6)(r-5)(r-4)(r-3)(r-2)(r-1)} - \frac{6(r^3-r^2-31r-89)\sin((r-8)z)}{(r-5)(r-4)(r-3)(r-2)(r-1)^3(r+1)^2} + \right. \\ & \left. \frac{(15(r^{10}-3r^9-53r^8+69r^7+145r^6+8211r^5-16879r^4-32025r^3+32954r^2+16404r+71528)\sin((r-4)z))}{(r-4)(r-3)(r-2)^3(r-1)^5(r+1)^4(r+2)^2} - \right. \\ & \left. \frac{(20(r^{14}-72r^{12}+597r^{10}+75244r^8-718317r^6+153312r^4+4883287r^2+1329084)\sin(rz))}{(r-3)^2(r-2)^2(r-1)^6(r+1)^6(r+2)^2(r+3)^2} + \right. \\ & \left. \frac{(15(r^{10}+3r^9-53r^8-69r^7+145r^6-8211r^5-16879r^4+32025r^3+32954r^2-16404r+71528)\sin((r+4)z))}{(r-2)^2(r-1)^4(r+1)^5(r+2)^3(r+3)(r+4)} - \frac{6(r^3+r^2-31r+89)\sin((r+8)z)}{(r-1)^2(r+1)^3(r+2)(r+3)(r+4)(r+5)} + \right. \\ & \left. \frac{\sin((r+12)z)}{(r+1)(r+2)(r+3)(r+4)(r+5)(r+6)} \right) q^6 + O(q^7) /; \neg (r \in \mathbb{Z} \wedge -6 \leq r \leq 6) \end{aligned}$$

11.02.06.0004.01

$$\begin{aligned}
 \text{Se}(b_1(q), q, z) \propto & \left(-\frac{4\,539\,285\,691 \sin(z)}{125\,241\,246\,351\,360\,000} + \frac{10\,304\,813 \sin(5z)}{5\,844\,591\,496\,396\,800} + \frac{48\,740\,801 \sin(7z)}{20\,456\,070\,237\,388\,800} + \frac{7\,516\,703 \sin(9z)}{61\,368\,210\,712\,166\,400} + \right. \\
 & \frac{\sin(17z)}{204\,560\,702\,373\,888\,000} + \frac{\sin(19z)}{1\,534\,205\,267\,804\,160\,000} + \frac{\sin(21z)}{151\,886\,321\,512\,611\,840\,000} - \\
 & \left. \frac{1\,282\,939\,901 \sin(3z)}{12\,524\,124\,635\,136\,000} - \frac{30\,773 \sin(11z)}{61\,368\,210\,712\,166\,400} - \frac{25\,379 \sin(13z)}{184\,104\,632\,136\,499\,200} - \frac{2\,839 \sin(15z)}{1\,104\,627\,792\,818\,995\,200} \right) q^{10} + \\
 & \left(\frac{11\,221\,967 \sin(z)}{32\,614\,907\,904\,000} + \frac{506\,831 \sin(3z)}{104\,367\,705\,292\,800} + \frac{16\,013 \sin(9z)}{319\,626\,097\,459\,200} + \frac{3\,167 \sin(11z)}{142\,056\,043\,315\,200} + \right. \\
 & \frac{481 \sin(13z)}{852\,336\,259\,891\,200} - \frac{310\,133 \sin(5z)}{3\,131\,031\,158\,784} - \frac{5\,039\,101 \sin(7z)}{547\,930\,452\,787\,200} - \\
 & \left. \frac{\sin(15z)}{767\,102\,633\,902\,080} - \frac{\sin(17z)}{4\,314\,952\,315\,699\,200} - \frac{\sin(19z)}{345\,196\,185\,255\,936\,000} \right) q^9 + \\
 & \left(-\frac{1\,237\,783 \sin(z)}{1\,449\,551\,462\,400} + \frac{940\,781 \sin(3z)}{543\,581\,798\,400} + \frac{322\,897 \sin(5z)}{815\,372\,697\,600} + \frac{\sin(13z)}{3\,805\,072\,588\,800} + \frac{\sin(15z)}{15\,220\,290\,355\,200} + \right. \\
 & \left. \frac{\sin(17z)}{958\,878\,292\,377\,600} - \frac{2\,143 \sin(9z)}{845\,571\,686\,400} - \frac{9413 \sin(7z)}{3\,261\,490\,790\,400} - \frac{1\,229 \sin(11z)}{13\,317\,754\,060\,800} \right) q^8 + \\
 & \left(-\frac{481 \sin(z)}{226\,492\,416} + \frac{659 \sin(5z)}{11\,324\,620\,800} + \frac{12\,677 \sin(7z)}{67\,947\,724\,800} + \frac{181 \sin(9z)}{16\,986\,931\,200} - \frac{102\,547 \sin(3z)}{13\,589\,544\,960} - \right. \\
 & \left. \frac{\sin(11z)}{26\,424\,115\,200} - \frac{\sin(13z)}{69\,363\,302\,400} - \frac{\sin(15z)}{3\,329\,438\,515\,200} \right) q^7 + \\
 & \left(\frac{8105 \sin(z)}{339\,738\,624} + \frac{103 \sin(3z)}{56\,623\,104} + \frac{\sin(9z)}{283\,115\,520} + \frac{\sin(11z)}{424\,673\,280} + \frac{\sin(13z)}{14\,863\,564\,800} - \frac{731 \sin(5z)}{94\,371\,840} - \frac{379 \sin(7z)}{471\,859\,200} \right) q^6 + \\
 & \left(-\frac{121 \sin(z)}{1\,769\,472} + \frac{317 \sin(3z)}{2\,359\,296} + \frac{41 \sin(5z)}{1\,179\,648} - \frac{\sin(9z)}{3\,686\,400} - \frac{\sin(7z)}{5\,898\,240} - \frac{\sin(11z)}{88\,473\,600} \right) q^5 + \\
 & \left(-\frac{37 \sin(z)}{294\,912} + \frac{\sin(7z)}{49\,152} + \frac{\sin(9z)}{737\,280} - \frac{49 \sin(3z)}{73\,728} \right) q^4 + \left(\frac{\sin(z)}{512} + \frac{\sin(3z)}{3072} - \frac{\sin(5z)}{1152} - \frac{\sin(7z)}{9216} \right) q^3 + \\
 & \left(-\frac{\sin(z)}{128} + \frac{1}{64} \sin(3z) + \frac{1}{192} \sin(5z) \right) q^2 - \frac{1}{8} \sin(3z) q + O(q^{11}) + \sin(z)
 \end{aligned}$$

11.02.06.0005.01

$$\begin{aligned}
 \text{Se}(b_2(q), q, z) \propto & \sin(2z) - \frac{1}{12} \sin(4z)q + \left(\frac{1}{384} \sin(6z) - \frac{1}{288} \sin(2z) \right) q^2 + \left(\frac{\sin(4z)}{1536} - \frac{\sin(8z)}{23040} \right) q^3 + \\
 & \left(\frac{119 \sin(2z)}{2654208} - \frac{19 \sin(6z)}{737280} + \frac{\sin(10z)}{2211840} \right) q^4 + \left(-\frac{1373 \sin(4z)}{159252480} + \frac{\sin(8z)}{2073600} - \frac{\sin(12z)}{309657600} \right) q^5 + \\
 & \left(-\frac{1003 \sin(2z)}{1415577600} + \frac{8941 \sin(6z)}{25480396800} - \frac{\sin(10z)}{185794560} + \frac{\sin(14z)}{59454259200} \right) q^6 + \\
 & \left(\frac{123379 \sin(4z)}{917294284800} - \frac{1591 \sin(8z)}{237817036800} + \frac{67 \sin(12z)}{1664719257600} - \frac{\sin(16z)}{14982473318400} \right) q^7 + \\
 & \left(\frac{21365639 \sin(2z)}{1761205026816000} - \frac{1128997 \sin(6z)}{205473919795200} + \frac{17 \sin(10z)}{224737099776} - \frac{89 \sin(14z)}{410947839590400} + \frac{\sin(18z)}{4794391461888000} \right) q^8 + \\
 & \left(-\frac{335495173 \sin(4z)}{147941222252544000} + \frac{9073499 \sin(8z)}{86299046313984000} - \right. \\
 & \left. \frac{6571 \sin(12z)}{11506539508531200} + \frac{19 \sin(16z)}{21574761578496000} - \frac{\sin(20z)}{1898579018907648000} \right) q^9 + \\
 & \left(-\frac{13475627293 \sin(2z)}{62135313346068480000} + \frac{3067835983 \sin(6z)}{33138833784569856000} - \frac{19755653 \sin(10z)}{16569416892284928000} + \right. \\
 & \left. \frac{7327 \sin(14z)}{2367059556040704000} - \frac{71 \sin(18z)}{25314386918768640000} + \frac{\sin(22z)}{911317929075671040000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.02.06.0006.01

$$\begin{aligned}
 \text{Se}(b_3(q), q, z) \propto & \sin(3z) + \left(\frac{\sin(z)}{8} - \frac{1}{16} \sin(5z) \right) q + \left(-\frac{\sin(z)}{64} - \frac{5}{512} \sin(3z) + \frac{1}{640} \sin(7z) \right) q^2 + \\
 & \left(-\frac{\sin(z)}{4096} + \frac{1}{512} \sin(3z) + \frac{11 \sin(5z)}{40960} - \frac{\sin(9z)}{46080} \right) q^3 + \left(\frac{21 \sin(z)}{32768} - \frac{1621 \sin(3z)}{13107200} - \frac{\sin(5z)}{16384} - \frac{11 \sin(7z)}{2949120} + \frac{\sin(11z)}{5160960} \right) q^4 + \\
 & \left(-\frac{14061 \sin(z)}{104857600} - \frac{9 \sin(3z)}{131072} + \frac{12329 \sin(5z)}{1887436800} + \frac{3 \sin(7z)}{3276800} + \frac{\sin(9z)}{33030144} - \frac{\sin(13z)}{825753600} \right) q^5 + \\
 & \left(-\frac{699 \sin(z)}{838860800} + \frac{13050583 \sin(3z)}{543581798400} + \frac{533 \sin(5z)}{209715200} - \frac{76679 \sin(7z)}{528482304000} - \frac{17 \sin(9z)}{2123366400} - \right. \\
 & \left. \frac{\sin(11z)}{6606028800} + \frac{\sin(15z)}{178362777600} \right) q^6 + \left(\frac{31826419 \sin(z)}{4348654387200} - \frac{70123 \sin(3z)}{33554432000} - \frac{326021051 \sin(5z)}{304405807104000} - \right. \\
 & \left. \frac{1319 \sin(7z)}{27179089920} + \frac{7831 \sin(9z)}{4227858432000} + \frac{37 \sin(11z)}{832359628800} + \frac{\sin(13z)}{2283043553280} - \frac{\sin(17z)}{49941577728000} \right) q^7 + \\
 & \left(-\frac{300245939 \sin(z)}{173946175488000} - \frac{1193766593741 \sin(3z)}{1363738015825920000} + \frac{10194121 \sin(5z)}{86973087744000} + \frac{55617547 \sin(7z)}{2435246456832000} + \frac{30253 \sin(9z)}{53271016243200} - \right. \\
 & \left. \frac{28361 \sin(11z)}{1826434842624000} - \frac{17 \sin(13z)}{106542032486400} - \frac{\sin(15z)}{3196260974592000} + \frac{\sin(19z)}{17579435360256000} \right) q^8 + \\
 & \left(\frac{64306539779 \sin(z)}{10909904126607360000} + \frac{1087026917 \sin(3z)}{3131031158784000} + \frac{767116375621 \sin(5z)}{21819808253214720000} - \frac{468897223 \sin(7z)}{170467251978240000} - \right. \\
 & \left. \frac{21665887 \sin(9z)}{75144747810816000} - \frac{7663 \sin(11z)}{1704672519782400} + \frac{189053 \sin(13z)}{204560702373888000} + \right. \\
 & \left. \frac{23 \sin(15z)}{6903923705187200} - \frac{\sin(17z)}{281270965764096000} - \frac{\sin(21z)}{7594316075630592000} \right) q^9 + \\
 & \left(\frac{78221421983189 \sin(z)}{785513097115729920000} - \frac{3555989290829 \sin(3z)}{99747694871838720000} - \frac{1595308088447 \sin(5z)}{98189137139466240000} - \right. \\
 & \left. \frac{16802983705367 \sin(7z)}{23565392913471897600000} + \frac{50000417 \sin(9z)}{1363738015825920000} + \frac{44442089 \sin(11z)}{18410463213649920000} + \right. \\
 & \left. \frac{113461 \sin(13z)}{4418511171275980800} - \frac{74069 \sin(15z)}{180013418089021440000} - \frac{\sin(17z)}{13807847410237440000} + \right. \\
 & \left. \frac{\sin(19z)}{48603622884035788800} + \frac{\sin(23z)}{3949044359327907840000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.02.06.0007.01

$$\begin{aligned}
 \operatorname{Se}(b_4(q), q, z) \propto & \sin(3z) + \left(\frac{\sin(z)}{8} - \frac{1}{16} \sin(5z) \right) q + \left(-\frac{\sin(z)}{64} - \frac{5}{512} \sin(3z) + \frac{1}{640} \sin(7z) \right) q^2 + \\
 & \left(-\frac{\sin(z)}{4096} + \frac{1}{512} \sin(3z) + \frac{11 \sin(5z)}{40960} - \frac{\sin(9z)}{46080} \right) q^3 + \left(\frac{21 \sin(z)}{32768} - \frac{1621 \sin(3z)}{13107200} - \frac{\sin(5z)}{16384} - \frac{11 \sin(7z)}{2949120} + \frac{\sin(11z)}{5160960} \right) q^4 + \\
 & \left(-\frac{14061 \sin(z)}{104857600} - \frac{9 \sin(3z)}{131072} + \frac{12329 \sin(5z)}{1887436800} + \frac{3 \sin(7z)}{3276800} + \frac{\sin(9z)}{33030144} - \frac{\sin(13z)}{825753600} \right) q^5 + \\
 & \left(-\frac{699 \sin(z)}{838860800} + \frac{13050583 \sin(3z)}{543581798400} + \frac{533 \sin(5z)}{209715200} - \frac{76679 \sin(7z)}{528482304000} - \frac{17 \sin(9z)}{2123366400} - \right. \\
 & \quad \left. \frac{\sin(11z)}{6606028800} + \frac{\sin(15z)}{178362777600} \right) q^6 + \left(\frac{31826419 \sin(z)}{4348654387200} - \frac{70123 \sin(3z)}{33554432000} - \frac{326021051 \sin(5z)}{304405807104000} - \right. \\
 & \quad \left. \frac{1319 \sin(7z)}{27179089920} + \frac{7831 \sin(9z)}{4227858432000} + \frac{37 \sin(11z)}{832359628800} + \frac{\sin(13z)}{2283043553280} - \frac{\sin(17z)}{49941577728000} \right) q^7 + \\
 & \left(-\frac{300245939 \sin(z)}{173946175488000} - \frac{1193766593741 \sin(3z)}{1363738015825920000} + \frac{10194121 \sin(5z)}{86973087744000} + \frac{55617547 \sin(7z)}{2435246456832000} + \frac{30253 \sin(9z)}{53271016243200} - \right. \\
 & \quad \left. \frac{28361 \sin(11z)}{1826434842624000} - \frac{17 \sin(13z)}{106542032486400} - \frac{\sin(15z)}{3196260974592000} + \frac{\sin(19z)}{17579435360256000} \right) q^8 + \\
 & \left(\frac{64306539779 \sin(z)}{10909904126607360000} + \frac{1087026917 \sin(3z)}{3131031158784000} + \frac{767116375621 \sin(5z)}{21819808253214720000} - \frac{468897223 \sin(7z)}{170467251978240000} - \right. \\
 & \quad \frac{21665887 \sin(9z)}{75144747810816000} - \frac{7663 \sin(11z)}{1704672519782400} + \frac{189053 \sin(13z)}{204560702373888000} + \\
 & \quad \left. \frac{23 \sin(15z)}{6903923705187200} - \frac{\sin(17z)}{281270965764096000} - \frac{\sin(21z)}{7594316075630592000} \right) q^9 + \\
 & \left(\frac{78221421983189 \sin(z)}{785513097115729920000} - \frac{3555989290829 \sin(3z)}{99747694871838720000} - \frac{1595308088447 \sin(5z)}{98189137139466240000} - \right. \\
 & \quad \frac{16802983705367 \sin(7z)}{23565392913471897600000} + \frac{50000417 \sin(9z)}{1363738015825920000} + \frac{44442089 \sin(11z)}{18410463213649920000} + \\
 & \quad \frac{113461 \sin(13z)}{4418511171275980800} - \frac{74069 \sin(15z)}{180013418089021440000} - \frac{\sin(17z)}{13807847410237440000} + \\
 & \quad \left. \frac{\sin(19z)}{48603622884035788800} + \frac{\sin(23z)}{3949044359327907840000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.02.06.0008.01

$$\begin{aligned}
 \text{Se}(b_5(q), q, z) \propto & \sin(3z) + \left(\frac{\sin(z)}{8} - \frac{1}{16} \sin(5z) \right) q + \left(-\frac{\sin(z)}{64} - \frac{5}{512} \sin(3z) + \frac{1}{640} \sin(7z) \right) q^2 + \\
 & \left(-\frac{\sin(z)}{4096} + \frac{1}{512} \sin(3z) + \frac{11 \sin(5z)}{40960} - \frac{\sin(9z)}{46080} \right) q^3 + \left(\frac{21 \sin(z)}{32768} - \frac{1621 \sin(3z)}{13107200} - \frac{\sin(5z)}{16384} - \frac{11 \sin(7z)}{2949120} + \frac{\sin(11z)}{5160960} \right) q^4 + \\
 & \left(-\frac{14061 \sin(z)}{104857600} - \frac{9 \sin(3z)}{131072} + \frac{12329 \sin(5z)}{1887436800} + \frac{3 \sin(7z)}{3276800} + \frac{\sin(9z)}{33030144} - \frac{\sin(13z)}{825753600} \right) q^5 + \\
 & \left(-\frac{699 \sin(z)}{838860800} + \frac{13050583 \sin(3z)}{543581798400} + \frac{533 \sin(5z)}{209715200} - \frac{76679 \sin(7z)}{528482304000} - \frac{17 \sin(9z)}{2123366400} - \right. \\
 & \left. \frac{\sin(11z)}{6606028800} + \frac{\sin(15z)}{178362777600} \right) q^6 + \left(\frac{31826419 \sin(z)}{4348654387200} - \frac{70123 \sin(3z)}{33554432000} - \frac{326021051 \sin(5z)}{304405807104000} - \right. \\
 & \left. \frac{1319 \sin(7z)}{27179089920} + \frac{7831 \sin(9z)}{4227858432000} + \frac{37 \sin(11z)}{832359628800} + \frac{\sin(13z)}{2283043553280} - \frac{\sin(17z)}{49941577728000} \right) q^7 + \\
 & \left(-\frac{300245939 \sin(z)}{173946175488000} - \frac{1193766593741 \sin(3z)}{1363738015825920000} + \frac{10194121 \sin(5z)}{86973087744000} + \frac{55617547 \sin(7z)}{2435246456832000} + \frac{30253 \sin(9z)}{53271016243200} - \right. \\
 & \left. \frac{28361 \sin(11z)}{1826434842624000} - \frac{17 \sin(13z)}{106542032486400} - \frac{\sin(15z)}{3196260974592000} + \frac{\sin(19z)}{17579435360256000} \right) q^8 + \\
 & \left(\frac{64306539779 \sin(z)}{10909904126607360000} + \frac{1087026917 \sin(3z)}{3131031158784000} + \frac{767116375621 \sin(5z)}{21819808253214720000} - \frac{468897223 \sin(7z)}{170467251978240000} - \right. \\
 & \left. \frac{21665887 \sin(9z)}{75144747810816000} - \frac{7663 \sin(11z)}{1704672519782400} + \frac{189053 \sin(13z)}{204560702373888000} + \right. \\
 & \left. \frac{23 \sin(15z)}{6903923705187200} - \frac{\sin(17z)}{281270965764096000} - \frac{\sin(21z)}{7594316075630592000} \right) q^9 + \\
 & \left(\frac{78221421983189 \sin(z)}{785513097115729920000} - \frac{3555989290829 \sin(3z)}{99747694871838720000} - \frac{1595308088447 \sin(5z)}{98189137139466240000} - \right. \\
 & \left. \frac{16802983705367 \sin(7z)}{23565392913471897600000} + \frac{50000417 \sin(9z)}{1363738015825920000} + \frac{44442089 \sin(11z)}{18410463213649920000} + \right. \\
 & \left. \frac{113461 \sin(13z)}{4418511171275980800} - \frac{74069 \sin(15z)}{180013418089021440000} - \frac{\sin(17z)}{13807847410237440000} + \right. \\
 & \left. \frac{\sin(19z)}{48603622884035788800} + \frac{\sin(23z)}{3949044359327907840000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.02.06.0009.01

$$\begin{aligned}
 \text{Se}(b_6(q), q, z) \propto & \sin(3z) + \left(\frac{\sin(z)}{8} - \frac{1}{16} \sin(5z) \right) q + \left(-\frac{\sin(z)}{64} - \frac{5}{512} \sin(3z) + \frac{1}{640} \sin(7z) \right) q^2 + \\
 & \left(-\frac{\sin(z)}{4096} + \frac{1}{512} \sin(3z) + \frac{11 \sin(5z)}{40960} - \frac{\sin(9z)}{46080} \right) q^3 + \left(\frac{21 \sin(z)}{32768} - \frac{1621 \sin(3z)}{13107200} - \frac{\sin(5z)}{16384} - \frac{11 \sin(7z)}{2949120} + \frac{\sin(11z)}{5160960} \right) q^4 + \\
 & \left(-\frac{14061 \sin(z)}{104857600} - \frac{9 \sin(3z)}{131072} + \frac{12329 \sin(5z)}{1887436800} + \frac{3 \sin(7z)}{3276800} + \frac{\sin(9z)}{33030144} - \frac{\sin(13z)}{825753600} \right) q^5 + \\
 & \left(-\frac{699 \sin(z)}{838860800} + \frac{13050583 \sin(3z)}{543581798400} + \frac{533 \sin(5z)}{209715200} - \frac{76679 \sin(7z)}{528482304000} - \frac{17 \sin(9z)}{2123366400} - \right. \\
 & \left. \frac{\sin(11z)}{6606028800} + \frac{\sin(15z)}{178362777600} \right) q^6 + \left(\frac{31826419 \sin(z)}{4348654387200} - \frac{70123 \sin(3z)}{33554432000} - \frac{326021051 \sin(5z)}{304405807104000} - \right. \\
 & \left. \frac{1319 \sin(7z)}{27179089920} + \frac{7831 \sin(9z)}{4227858432000} + \frac{37 \sin(11z)}{832359628800} + \frac{\sin(13z)}{2283043553280} - \frac{\sin(17z)}{49941577728000} \right) q^7 + \\
 & \left(-\frac{300245939 \sin(z)}{173946175488000} - \frac{1193766593741 \sin(3z)}{1363738015825920000} + \frac{10194121 \sin(5z)}{86973087744000} + \frac{55617547 \sin(7z)}{2435246456832000} + \frac{30253 \sin(9z)}{53271016243200} - \right. \\
 & \left. \frac{28361 \sin(11z)}{1826434842624000} - \frac{17 \sin(13z)}{106542032486400} - \frac{\sin(15z)}{3196260974592000} + \frac{\sin(19z)}{17579435360256000} \right) q^8 + \\
 & \left(\frac{64306539779 \sin(z)}{10909904126607360000} + \frac{1087026917 \sin(3z)}{3131031158784000} + \frac{767116375621 \sin(5z)}{21819808253214720000} - \frac{468897223 \sin(7z)}{170467251978240000} - \right. \\
 & \left. \frac{21665887 \sin(9z)}{75144747810816000} - \frac{7663 \sin(11z)}{1704672519782400} + \frac{189053 \sin(13z)}{204560702373888000} + \right. \\
 & \left. \frac{23 \sin(15z)}{6903923705187200} - \frac{\sin(17z)}{281270965764096000} - \frac{\sin(21z)}{7594316075630592000} \right) q^9 + \\
 & \left(\frac{78221421983189 \sin(z)}{785513097115729920000} - \frac{3555989290829 \sin(3z)}{99747694871838720000} - \frac{1595308088447 \sin(5z)}{98189137139466240000} - \right. \\
 & \left. \frac{16802983705367 \sin(7z)}{23565392913471897600000} + \frac{50000417 \sin(9z)}{1363738015825920000} + \frac{44442089 \sin(11z)}{18410463213649920000} + \right. \\
 & \left. \frac{113461 \sin(13z)}{4418511171275980800} - \frac{74069 \sin(15z)}{180013418089021440000} - \frac{\sin(17z)}{13807847410237440000} + \right. \\
 & \left. \frac{\sin(19z)}{48603622884035788800} + \frac{\sin(23z)}{3949044359327907840000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

11.02.13.0001.01

$$w''(z) + (a - 2q \cos(2z)) w(z) = 0; w(z) = c_1 \text{Se}(a, q, z) + c_2 \text{Ce}(a, q, z)$$

11.02.13.0002.01

$$W_z(\text{Se}(a, q, z), \text{Ce}(a, q, z)) = \text{Se}(a, q, 0) \text{Ce}'(a, q, 0) - \text{Se}'(a, q, 0) \text{Ce}(a, q, 0)$$

11.02.13.0003.01

$$w''(z) - \frac{g''(z)}{g'(z)} w'(z) + (a - 2q \cos(2g(z))) g'(z)^2 w(z) = 0 ; w(z) = c_1 \operatorname{Se}(a, q, g(z)) + c_2 \operatorname{Ce}(a, q, g(z))$$

11.02.13.0004.01

$$W_z(\operatorname{Se}(a, q, g(z)), \operatorname{Ce}(a, q, g(z))) = g'(z) (\operatorname{Ce}'(a, q, 0) \operatorname{Se}(a, q, 0) - \operatorname{Ce}(a, q, 0) \operatorname{Se}'(a, q, 0))$$

11.02.13.0005.01

$$h(z) w''(z) + \left(-2h'(z) - \frac{h(z)g''(z)}{g'(z)} \right) w'(z) + \left((a - 2q \cos(2g(z))) h(z) g'(z)^2 + \frac{2h'(z)^2}{h(z)} - h''(z) + \frac{h'(z)g''(z)}{g'(z)} \right) w(z) = 0 ;$$

$$w(z) = c_1 h(z) \operatorname{Se}(a, q, g(z)) + c_2 h(z) \operatorname{Ce}(a, q, g(z))$$

11.02.13.0006.01

$$W_z(h(z) \operatorname{Se}(a, q, g(z)), h(z) \operatorname{Ce}(a, q, g(z))) = h(z)^2 g'(z) (\operatorname{Ce}'(a, q, 0) \operatorname{Se}(a, q, 0) - \operatorname{Ce}(a, q, 0) \operatorname{Se}'(a, q, 0))$$

11.02.13.0007.01

$$w''(z) + \frac{1-r-2s}{z} w'(z) + \left(b^2 r^2 (a - 2q \cos(2bz^r)) z^{2r-2} + \frac{s(r+s)}{z^2} \right) w(z) = 0 ; w(z) = c_1 z^s \operatorname{Se}(a, q, bz^r) + c_2 z^s \operatorname{Ce}(a, q, bz^r)$$

11.02.13.0008.01

$$W_z(z^s \operatorname{Se}(a, q, bz^r), z^s \operatorname{Ce}(a, q, bz^r)) = br z^{r+2s-1} (\operatorname{Ce}'(a, q, 0) \operatorname{Se}(a, q, 0) - \operatorname{Ce}(a, q, 0) \operatorname{Se}'(a, q, 0))$$

11.02.13.0009.01

$$w''(z) - (\log(r) + 2 \log(s)) w'(z) + (b^2 (a - 2q \cos(2br^z)) \log^2(r) r^{2z} + \log(s) (\log(r) + \log(s))) w(z) = 0 ;$$

$$w(z) = c_1 s^z \operatorname{Se}(a, q, br^z) + c_2 s^z \operatorname{Ce}(a, q, br^z)$$

11.02.13.0010.01

$$W_z(s^z \operatorname{Ce}(a, q, br^z), s^z \operatorname{Se}(a, q, br^z)) = br^z s^{2z} \log(r) (\operatorname{Ce}'(a, q, 0) \operatorname{Se}(a, q, 0) - \operatorname{Ce}(a, q, 0) \operatorname{Se}'(a, q, 0))$$

Differentiation

Low-order differentiation

With respect to z

11.02.20.0001.01

$$\frac{\partial \operatorname{Se}(a, q, z)}{\partial z} = \operatorname{Se}'(a, q, z)$$

11.02.20.0002.01

$$\frac{\partial^2 \operatorname{Se}(a, q, z)}{\partial z^2} = (2q \cos(2z) - a) \operatorname{Se}(a, q, z)$$

Integration

Definite integration

Involving the direct function

11.02.21.0001.01

$$\int_{-\pi}^{\pi} \operatorname{Se}(b_n(q), q, t) \operatorname{Se}(b_m(q), q, t) dt = \pi \delta_{n,m} ; n \in \mathbb{Z} \wedge m \in \mathbb{Z} \wedge n \neq 0 \wedge m \neq 0 \wedge q \in \mathbb{R}$$

Operations

Limit operation

$$\lim_{a \rightarrow \infty} \operatorname{Se}\left(a, q, \frac{z}{\sqrt{a}}\right) = \sin(z) \quad /; q \in \mathbb{R}$$

Representations through equivalent functions

With related functions

$$\operatorname{Se}(a, q, z) = (-1)^n \operatorname{Ce}\left(a, -q, z - \frac{\pi}{2}\right) \quad /; a = a_{2n+1}(q) \vee a = b_{2n+1}(q) \wedge n \in \mathbb{N}$$

Other information

$$\partial \frac{\int_{-\pi}^{\pi} e^{2iq \sin(t) \sin(z)} \operatorname{Se}(b_r(q), q, t) dt}{\operatorname{Se}(b_r(q), q, z)} = 0 \quad /; r \in \mathbb{Q}$$

History

- E. L. Mathieu (1868, 1873)
- H. Weber (1869)
- G. W. Hill (1877)
- E. Heine (1878)
- G. Floquet (1883)
- R. C. Maclaurin (1898)
- J. Dougall (1916, 1926)

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