# MathieuSPrime

## Notations

### Traditional name

Derivative of the odd Mathieu function

### Traditional notation

$\text{Se}'(a, q, z)$

### *Mathematica* StandardForm notation

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MathieuSPrime[a, q, z]
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## Primary definition

11.04.02.0001.01

$$\text{Se}'(a, q, z) == \frac{\partial \text{Se}(a, q, z)}{\partial z}$$

## Specific values

### Specialized values

#### For fixed $a, z$

11.04.03.0001.01

$$\text{Se}'(a, 0, z) == \sqrt{a}\ \cos\!\left(\sqrt{a}\ z\right)$$

## General characteristics

### Domain and analyticity

$\text{Se}'\left(a, q, z\right)$ is an analytical function of $a, q, z$ which is defined in $\mathbb{C}^3$.

11.04.04.0001.01

$$(a * q * z) \longrightarrow \text{Se}'(a, q, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

### Symmetries and periodicities

#### Parity

$\text{Se}'(a, q, z)$ is an even function with respect to $z$.

11.04.04.0002.01

$$\text{Se}'(a, q, -z) == \text{Se}'(a, q, z)$$

### Mirror symmetry

11.04.04.0003.01

$$\text{Se}'(\overline{a}, \overline{q}, \overline{z}) == \overline{\text{Se}'(a, q, z)}$$

### Periodicity

No periodicity

# Series representations

## Generalized power series

### Expansions at generic point $z == z_0$

## For the function itself

11.04.06.0010.01

$$\text{Se}'(a, q, z) \propto \text{Se}^{(0,0,1)}(a, q, z_0) + (2 q \cos(2 z_0) - a) \text{Se}(a, q, z_0) (z - z_0) +$$
$$\frac{1}{2} \left( (2 q \cos(2 z_0) - a) \text{Se}^{(0,0,1)}(a, q, z_0) - 4 q \sin(2 z_0) \text{Se}(a, q, z_0) \right) (z - z_0)^2 +$$
$$\frac{1}{6} \left( \left( a^2 + 4 q \cos(2 z_0) (-a + q \cos(2 z_0) - 2) \right) \text{Se}(a, q, z_0) - 8 q \sin(2 z_0) \text{Se}^{(0,0,1)}(a, q, z_0) \right) (z - z_0)^3 +$$
$$\frac{1}{24} \left( \left( a^2 + 4 q \cos(2 z_0) (-a + q \cos(2 z_0) - 6) \right) \text{Se}^{(0,0,1)}(a, q, z_0) + 16 q (a - 2 q \cos(2 z_0) + 1) \sin(2 z_0) \text{Se}(a, q, z_0) \right) (z - z_0)^4 +$$
$$\frac{1}{120} \left( \left( 2 q \left( \left( 3 q^2 + a (3 a + 28) + 16 \right) \cos(2 z_0) + q (-3 (a + 4) - (3 a + 44) \cos(4 z_0) + q \cos(6 z_0)) \right) - a^3 \right) \text{Se}(a, q, z_0) +$$
$$8 q (3 a - 6 q \cos(2 z_0) + 8) \sin(2 z_0) \text{Se}^{(0,0,1)}(a, q, z_0) \right) (z - z_0)^5 + \dots \; /; (z \to z_0)$$

11.04.06.0011.01

$$\text{Se}'(a, q, z) \propto \text{Se}^{(0,0,1)}(a, q, z_0) (1 + O(z - z_0))$$

### Expansions at $z == 0$

11.04.06.0001.01

$$\text{Se}'(b_{2n+2}(q), q, z) == 2 \sum_{k=0}^{\infty} (k + 1) B_{2k+2}^{2n+2} \cos((2k + 2) z) \; /;$$

$$(b_{2n}(q) - 4) B_2^{2n+2} - q B_4^{2n+2} == 0 \bigwedge \left( b_{2n}(q) - 4 k^2 \right) B_{2k}^{2n} - q \left( B_{2k-2}^{2n+2} + B_{2k+2}^{2n+2} \right) == 0 \bigwedge \sum_{k=0}^{\infty} B_{2k+1}^{2n+1} == 1 \bigwedge n \in \mathbb{Z}$$

11.04.06.0002.01

$$\text{Se}'(b_{2n+1}(q), q, z) == \sum_{k=0}^{\infty} (2k+1) B_{2k+1}^{2n+1} \cos((2k+1)z) \, /;$$

$$(b_{2n+1}(q) - q - 1) B_1^{2n+1} - q B_3^{2n+1} == 0 \bigwedge \left(b_{2n+1}(q) - (2k+1)^2\right) B_{2k+1}^{2n+1} - q \left(B_{2k-1}^{2n+1} + B_{2k+3}^{2n+1}\right) == 0 \bigwedge \sum_{k=0}^{\infty} B_{2k+1}^{2n+1} == 1 \bigwedge n \in \mathbb{Z}$$

## Expansions at $q == 0$

11.04.06.0003.01

$$\text{Se}'(b_r(q), q, z) \propto -r \cos(rz) + \left(\frac{(2-r)\cos((r-2)z)}{4(r-1)} + \frac{(r+2)\cos((r+2)z)}{4(r+1)}\right) q +$$

$$\left(\frac{(4-r)\cos((r-4)z)}{32(r-2)(r-1)} + \frac{r(r^2+1)\cos(rz)}{16(r-1)^2(r+1)^2} - \frac{(r+4)\cos((r+4)z)}{32(r+1)(r+2)}\right) q^2 +$$

$$\left(\frac{(6-r)\cos((r-6)z)}{384(r-3)(r-2)(r-1)} + \left(\left(r^3 - r^2 - r - 11\right)\cos((r-2)z)\right) / \left(128(r-1)^3(r+1)^2\right) - \right.$$

$$\left.\left(\left(r^3 + r^2 - r + 11\right)\cos((r+2)z)\right) / \left(128(r-1)^2(r+1)^3\right) + \left((r+6)\cos((r+6)z)\right) / (384(r+1)(r+2)(r+3))\right) q^3 +$$

$$\left(\left((8-r)\cos((r-8)z)\right) / (6144(r-4)(r-3)(r-2)(r-1)) + \left((r-4)\left(r^3 - r^2 - 7r - 29\right)\cos((r-4)z)\right) / \right.$$

$$\left(1536(r-3)(r-2)(r-1)^3(r+1)^2\right) - \left(r\left(r^8 - 15r^6 - 185r^4 + 675r^2 + 316\right)\cos(rz)\right) /$$

$$\left(1024(r-2)^2(r-1)^4(r+1)^4(r+2)^2\right) + \left((r+4)\left(r^3 + r^2 - 7r + 29\right)\cos((r+4)z)\right) /$$

$$\left.\left(1536(r-1)^2(r+1)^3(r+2)(r+3)\right) - \left((r+8)\cos((r+8)z)\right) / (6144(r+1)(r+2)(r+3)(r+4))\right) q^4 +$$

$$\left(\left((10-r)\cos((r-10)z)\right) / (122\,880(r-5)(r-4)(r-3)(r-2)(r-1)) + \right.$$

$$\left((r-6)\left(r^3 - r^2 - 17r - 55\right)\cos((r-6)z)\right) / \left(24\,576(r-4)(r-3)(r-2)(r-1)^3(r+1)^2\right) -$$

$$\left(\left(r^9 - r^8 - 31r^7 - 5r^6 - 273r^5 + 2457r^4 + 1931r^3 - 6335r^2 - 3572r - 7564\right)\cos((r-2)z)\right) /$$

$$\left(12\,288(r-3)(r-2)(r-1)^5(r+1)^4(r+2)^2\right) +$$

$$\left(\left(r^9 + r^8 - 31r^7 + 5r^6 - 273r^5 - 2457r^4 + 1931r^3 + 6335r^2 - 3572r + 7564\right)\cos((r+2)z)\right) /$$

$$\left(12\,288(r-2)^2(r-1)^4(r+1)^5(r+2)(r+3)\right) -$$

$$\left((r+6)\left(r^3 + r^2 - 17r + 55\right)\cos((r+6)z)\right) / \left(24\,576(r-1)^2(r+1)^3(r+2)(r+3)(r+4)\right) +$$

$$\left.\left((r+10)\cos((r+10)z)\right) / (122\,880(r+1)(r+2)(r+3)(r+4)(r+5))\right) q^5 +$$

$$\left(\left((12-r)\cos((r-12)z)\right) / (2\,949\,120(r-6)(r-5)(r-4)(r-3)(r-2)(r-1)) + \right.$$

$$\left((r-8)\left(r^3 - r^2 - 31r - 89\right)\cos((r-8)z)\right) / \left(491\,520(r-5)(r-4)(r-3)(r-2)(r-1)^3(r+1)^2\right) -$$

$$\left(\left(r^{10} - 3r^9 - 53r^8 + 69r^7 + 145r^6 + 8211r^5 - 16\,879r^4 - 32\,025r^3 + 32\,954r^2 + 16\,404r + 71\,528\right)\cos((r-4)z)\right) /$$

$$\left(196\,608(r-3)(r-2)^3(r-1)^5(r+1)^4(r+2)^2\right) +$$

$$\left(r\left(r^{14} - 72r^{12} + 597r^{10} + 75\,244r^8 - 718\,317r^6 + 153\,312r^4 + 4\,883\,287r^2 + 1\,329\,084\right)\cos(rz)\right) /$$

$$\left(147\,456(r-3)^2(r-2)^2(r-1)^6(r+1)^6(r+2)^2(r+3)^2\right) -$$

$$\left(\left(r^{10} + 3r^9 - 53r^8 - 69r^7 + 145r^6 - 8211r^5 - 16\,879r^4 + 32\,025r^3 + 32\,954r^2 - 16\,404r + 71\,528\right)\cos((r+4)z)\right) /$$

$$\left(196\,608(r-2)^2(r-1)^4(r+1)^5(r+2)^3(r+3)\right) +$$

$$\left((r+8)\left(r^3 + r^2 - 31r + 89\right)\cos((r+8)z)\right) / \left(491\,520(r-1)^2(r+1)^3(r+2)(r+3)(r+4)(r+5)\right) -$$

$$\left.\left((r+12)\cos((r+12)z)\right) / (2\,949\,120(r+1)(r+2)(r+3)(r+4)(r+5)(r+6))\right) q^6 + O(q^7) \, /; \, \neg \, (r \in \mathbb{Z} \wedge -6 \le r \le 6)$$

11.04.06.0004.01

$$\text{Se}'(b_1(q),\, q,\, z) \propto \cos(z) - \frac{3}{8}\cos(3\,z)\,q + \left(-\frac{\cos(z)}{128} + \frac{3}{64}\cos(3\,z) + \frac{5}{192}\cos(5\,z)\right)q^2 +$$

$$\left(\frac{\cos(z)}{512} + \frac{\cos(3\,z)}{1024} - \frac{5\cos(5\,z)}{1152} - \frac{7\cos(7\,z)}{9216}\right)q^3 + \left(-\frac{37\cos(z)}{294\,912} + \frac{7\cos(7\,z)}{49\,152} + \frac{\cos(9\,z)}{81\,920} - \frac{49\cos(3\,z)}{24\,576}\right)q^4 +$$

$$\left(-\frac{121\cos(z)}{1\,769\,472} + \frac{317\cos(3\,z)}{786\,432} + \frac{205\cos(5\,z)}{1\,179\,648} - \frac{\cos(9\,z)}{409\,600} - \frac{7\cos(7\,z)}{5\,898\,240} - \frac{11\cos(11\,z)}{88\,473\,600}\right)q^5 +$$

$$\left(\frac{8105\cos(z)}{339\,738\,624} + \frac{103\cos(3\,z)}{18\,874\,368} + \frac{\cos(9\,z)}{31\,457\,280} + \frac{11\cos(11\,z)}{424\,673\,280} + \frac{13\cos(13\,z)}{14\,863\,564\,800} - \frac{731\cos(5\,z)}{18\,874\,368} - \frac{2653\cos(7\,z)}{471\,859\,200}\right)q^6 + O[q]^{11}$$

11.04.06.0005.01

$$\text{Se}'(b_2(q),\, q,\, z) \propto 2\cos(2\,z) - \frac{1}{3}\cos(4\,z)\,q + \left(\frac{1}{64}\cos(6\,z) - \frac{1}{144}\cos(2\,z)\right)q^2 + \left(\frac{1}{384}\cos(4\,z) - \frac{\cos(8\,z)}{2880}\right)q^3 +$$

$$\left(\frac{119\cos(2\,z)}{1\,327\,104} - \frac{19\cos(6\,z)}{122\,880} + \frac{\cos(10\,z)}{221\,184}\right)q^4 + \left(-\frac{1373\cos(4\,z)}{39\,813\,120} + \frac{\cos(8\,z)}{259\,200} - \frac{\cos(12\,z)}{25\,804\,800}\right)q^5 +$$

$$\left(-\frac{1003\cos(2\,z)}{707\,788\,800} + \frac{8941\cos(6\,z)}{4\,246\,732\,800} - \frac{\cos(10\,z)}{18\,579\,456} + \frac{\cos(14\,z)}{4\,246\,732\,800}\right)q^6 + O\left(q^{11}\right)$$

11.04.06.0006.01

$$\text{Se}'(b_3(q),\, q,\, z) \propto 3\cos(3\,z) + \left(\frac{\cos(z)}{8} - \frac{5}{16}\cos(5\,z)\right)q +$$

$$\left(-\frac{\cos(z)}{64} - \frac{15}{512}\cos(3\,z) + \frac{7}{640}\cos(7\,z)\right)q^2 + \left(-\frac{\cos(z)}{4096} + \frac{3}{512}\cos(3\,z) + \frac{11\cos(5\,z)}{8192} - \frac{\cos(9\,z)}{5120}\right)q^3 +$$

$$\left(\frac{21\cos(z)}{32\,768} - \frac{4863\cos(3\,z)}{13\,107\,200} - \frac{5\cos(5\,z)}{16\,384} - \frac{77\cos(7\,z)}{2\,949\,120} + \frac{11\cos(11\,z)}{5\,160\,960}\right)q^4 +$$

$$\left(-\frac{14\,061\cos(z)}{104\,857\,600} - \frac{27\cos(3\,z)}{131\,072} + \frac{12\,329\cos(5\,z)}{377\,487\,360} + \frac{21\cos(7\,z)}{3\,276\,800} + \frac{\cos(9\,z)}{3\,670\,016} - \frac{13\cos(13\,z)}{825\,753\,600}\right)q^5 +$$

$$\left(-\frac{699\cos(z)}{838\,860\,800} + \frac{13\,050\,583\cos(3\,z)}{181\,193\,932\,800} + \frac{533\cos(5\,z)}{41\,943\,040} - \frac{76\,679\cos(7\,z)}{75\,497\,472\,000} - \frac{17\cos(9\,z)}{235\,929\,600} - \frac{11\cos(11\,z)}{6\,606\,028\,800} + \frac{\cos(15\,z)}{11\,890\,851\,840}\right)$$

$$q^6 + O\left(q^{11}\right)$$

11.04.06.0007.01

$$\text{Se}'(b_4(q),\, q,\, z) \propto$$

$$4\cos(4\,z) + \left(\frac{1}{6}\cos(2\,z) - \frac{3}{10}\cos(6\,z)\right)q + \left(\frac{1}{120}\cos(8\,z) - \frac{17}{900}\cos(4\,z)\right)q^2 + \left(-\frac{1}{800}\cos(2\,z) + \frac{29\cos(6\,z)}{48\,000} - \frac{\cos(10\,z)}{8064}\right)q^3 +$$

$$\left(\frac{18\,839\cos(4\,z)}{103\,680\,000} - \frac{\cos(8\,z)}{112\,000} + \frac{\cos(12\,z)}{860\,160}\right)q^4 + \left(\frac{40\,159\cos(2\,z)}{2\,488\,320\,000} - \frac{65\,633\cos(6\,z)}{9\,676\,800\,000} + \frac{67\cos(10\,z)}{928\,972\,800} - \frac{\cos(14\,z)}{132\,710\,400}\right)q^5 +$$

$$\left(-\frac{1\,933\,949\cos(4\,z)}{677\,376\,000\,000} + \frac{23\,351\cos(8\,z)}{199\,065\,600\,000} - \frac{\cos(12\,z)}{3\,096\,576\,000} + \frac{\cos(16\,z)}{27\,869\,184\,000}\right)q^6 + O\left(q^{11}\right)$$

11.04.06.0008.01

$$\text{Se}'(b_5(q), q, z) \propto 5\cos(5\,z) + \left(\frac{3}{16}\cos(3\,z) - \frac{7}{24}\cos(7\,z)\right)q +$$

$$\left(\frac{\cos(z)}{384} - \frac{65\cos(5\,z)}{4608} + \frac{3}{448}\cos(9\,z)\right)q^2 + \left(-\frac{\cos(z)}{9216} - \frac{7\cos(3\,z)}{24\,576} + \frac{13\cos(7\,z)}{36\,864} - \frac{11\cos(11\,z)}{129\,024}\right)q^3 +$$

$$\left(-\frac{\cos(z)}{589\,824} - \frac{\cos(3\,z)}{49\,152} + \frac{4015\cos(5\,z)}{2\,080\,899\,072} - \frac{\cos(9\,z)}{229\,376} + \frac{13\cos(13\,z)}{18\,579\,456}\right)q^4 +$$

$$\left(-\frac{5\cos(z)}{42\,467\,328} - \frac{457\cos(3\,z)}{11\,098\,128\,384} + \frac{25\cos(5\,z)}{7\,077\,888} + \frac{1157\cos(7\,z)}{7\,134\,511\,104} + \frac{55\cos(11\,z)}{1\,783\,627\,776} - \frac{\cos(15\,z)}{247\,726\,080}\right)q^5 + \left(\frac{3407\cos(z)}{799\,065\,243\,648} + \right.$$

$$\left. \frac{49\cos(3\,z)}{226\,492\,416} - \frac{9145\cos(5\,z)}{456\,608\,710\,656} - \frac{7\cos(7\,z)}{56\,623\,104} - \frac{1273\cos(9\,z)}{310\,747\,594\,752} - \frac{13\cos(13\,z)}{101\,921\,587\,200} + \frac{17\cos(17\,z)}{980\,995\,276\,800}\right)q^6 + O(q^{11})$$

11.04.06.0009.01

$$\text{Se}'(b_6(q), q, z) \propto 6\cos(6\,z) + \left(\frac{1}{5}\cos(4\,z) - \frac{2}{7}\cos(8\,z)\right)q + \left(\frac{1}{320}\cos(2\,z) - \frac{111\cos(6\,z)}{9800} + \frac{5}{896}\cos(10\,z)\right)q^2 +$$

$$\left(-\frac{163\cos(4\,z)}{784\,000} + \frac{257\cos(8\,z)}{1\,097\,600} - \frac{\cos(12\,z)}{16\,128}\right)q^3 + \left(-\frac{233\cos(2\,z)}{50\,176\,000} + \frac{286\,737\cos(6\,z)}{98\,344\,960\,000} - \frac{239\cos(10\,z)}{94\,832\,640} + \frac{\cos(14\,z)}{2\,211\,840}\right)q^4 +$$

$$\left(-\frac{256\,941\cos(4\,z)}{983\,449\,600\,000} + \frac{170\,407\cos(8\,z)}{18\,587\,197\,440\,000} + \frac{41\cos(12\,z)}{2\,528\,870\,400} - \frac{\cos(16\,z)}{425\,779\,200}\right)q^5 +$$

$$\left(-\frac{152\,641\cos(2\,z)}{62\,940\,774\,400\,000} + \frac{8\,712\,317\,791\cos(6\,z)}{234\,198\,687\,744\,000\,000} - \frac{553\,237\cos(10\,z)}{951\,664\,508\,928\,000} - \frac{31\cos(14\,z)}{476\,872\,704\,000} + \frac{\cos(18\,z)}{108\,999\,475\,200}\right)q^6 +$$

$$O(q^{11})$$

# Differential equations

## Ordinary linear differential equations and wronskians

### For the direct function itself

11.04.13.0001.01

$$(a - 2\,q\cos(2\,z))\,w''(z) - 4\,q\sin(2\,z)\,w'(z) + (a - 2\,q\cos(2\,z))^2\,w(z) == 0 \;/;\; w(z) == c_1\,\text{Se}'(a, q, z) + c_2\,\text{Ce}'(a, q, z)$$

11.04.13.0002.01

$$W_z\big(\text{Se}'(a, q, z), \text{Ce}'(a, q, z)\big) == (a - 2\,q\cos(2\,z))\big(\text{Ce}'(a, q, 0)\,\text{Se}(a, q, 0) - \text{Ce}(a, q, 0)\,\text{Se}'(a, q, 0)\big)$$

11.04.13.0003.01

$$w''(z) - \left(\frac{4\,q\sin(2\,g(z))\,g'(z)}{a - 2\,q\cos(2\,g(z))} + \frac{g''(z)}{g'(z)}\right)w'(z) + (a - 2\,q\cos(2\,g(z)))\,g'(z)^2\,w(z) = 0 \;/;\; w(z) = c_1\,\text{Se}'(a, q, g(z)) + c_2\,\text{Ce}'(a, q, g(z))$$

11.04.13.0004.01

$$W_z\big(\text{Se}'(a, q, g(z)), \text{Ce}'(a, q, g(z))\big) == g'(z)\,(a - 2\,q\cos(2\,g(z)))\big(\text{Ce}'(a, q, 0)\,\text{Se}(a, q, 0) - \text{Ce}(a, q, 0)\,\text{Se}'(a, q, 0)\big)$$

11.04.13.0005.01

$$w''(z) - \left( \frac{4\,q\,\sin(2\,g(z))\,g'(z)}{a - 2\,q\,\cos(2\,g(z))} + \frac{2\,h'(z)}{h(z)} + \frac{g''(z)}{g'(z)} \right) w'(z) +$$

$$\left( (a - 2\,q\,\cos(2\,g(z)))\,g'(z)^2 + \frac{2\,h'(z)^2}{h(z)^2} + \frac{1}{h(z)} \left( \frac{h'(z)}{g'(z)} \left( \frac{4\,q\,\sin(2\,g(z))\,g'(z)^2}{a - 2\,q\,\cos(2\,g(z))} + g''(z) \right) - h''(z) \right) \right) w(z) = 0 \ /;$$

$$w(z) = c_1\,h(z)\,\mathrm{Se}'(a, q, g(z)) + c_2\,h(z)\,\mathrm{Ce}'(a, q, g(z))$$

11.04.13.0006.01

$$W_z\bigl(h(z)\,\mathrm{Se}'(a, q, g(z)),\ h(z)\,\mathrm{Ce}'(a, q, g(z))\bigr) == h(z)^2\,g'(z)\,(a - 2\,q\,\cos(2\,g(z)))\,\bigl(\mathrm{Ce}'(a, q, 0)\,\mathrm{Se}(a, q, 0) - \mathrm{Ce}(a, q, 0)\,\mathrm{Se}'(a, q, 0)\bigr)$$

11.04.13.0007.01

$$w''(z) - \left( \frac{4\,q\,\sin(2\,g(z))\,g'(z)}{a - 2\,q\,\cos(2\,g(z))} + \frac{2\,h'(z)}{h(z)} + \frac{g''(z)}{g'(z)} \right) w'(z) +$$

$$\left( (a - 2\,q\,\cos(2\,g(z)))\,g'(z)^2 + \frac{2\,h'(z)^2}{h(z)^2} + \frac{1}{h(z)} \left( \frac{h'(z)}{g'(z)} \left( \frac{4\,q\,\sin(2\,g(z))\,g'(z)^2}{a - 2\,q\,\cos(2\,g(z))} + g''(z) \right) - h''(z) \right) \right) w(z) =$$

$$0 \ /; \ w(z) = c_1\,z^s\,\mathrm{Se}'(a, q, b\,z^r) + c_2\,z^s\,\mathrm{Ce}'(a, q, b\,z^r)$$

11.04.13.0008.01

$$W_z\bigl(z^s\,\mathrm{Se}'(a, q, b\,z^r),\ z^s\,\mathrm{Ce}'(a, q, b\,z^r)\bigr) == b\,r\,z^{r+2\,s-1}\,(a - 2\,q\,\cos(2\,b\,z^r))\,\bigl(\mathrm{Ce}'(a, q, 0)\,\mathrm{Se}(a, q, 0) - \mathrm{Ce}(a, q, 0)\,\mathrm{Se}'(a, q, 0)\bigr)$$

11.04.13.0009.01

$$w''(z) - \left( \frac{4\,b\,q\,\log(r)\,\sin(2\,b\,r^z)\,r^z}{a - 2\,q\,\cos(2\,b\,r^z)} + \log(r) + 2\,\log(s) \right) w'(z) +$$

$$\left( \frac{4\,b\,q\,\log(r)\,\log(s)\,\sin(2\,b\,r^z)\,r^z}{a - 2\,q\,\cos(2\,b\,r^z)} + b^2\,(a - 2\,q\,\cos(2\,b\,r^z))\,\log^2(r)\,r^{2\,z} + \log(s)\,(\log(r) + \log(s)) \right) w(z) =$$

$$0 \ /; \ w(z) = c_1\,s^z\,\mathrm{Se}'(a, q, b\,r^z) + c_2\,s^z\,\mathrm{Ce}'(a, q, b\,r^z)$$

11.04.13.0010.01

$$W_z\bigl(s^z\,\mathrm{Se}'(a, q, b\,r^z),\ s^z\,\mathrm{Ce}'(a, q, b\,r^z)\bigr) == b\,r^z\,s^{2\,z}\,(a - 2\,q\,\cos(2\,b\,r^z))\,\log(r)\,\bigl(\mathrm{Ce}'(a, q, 0)\,\mathrm{Se}(a, q, 0) - \mathrm{Ce}(a, q, 0)\,\mathrm{Se}'(a, q, 0)\bigr)$$

# Differentiation

## Low-order differentiation

### With respect to $z$

11.04.20.0001.01

$$\frac{\partial\,\mathrm{Se}'(a, q, z)}{\partial z} == (2\,q\,\cos(2\,z) - a)\,\mathrm{Se}(a, q, z)$$

11.04.20.0002.01

$$\frac{\partial^2\,\mathrm{Se}'(a, q, z)}{\partial z^2} == (2\,q\,\cos(2\,z) - a)\,\mathrm{Se}'(a, q, z) - 4\,q\,\sin(2\,z)\,\mathrm{Se}(a, q, z)$$

# Integration

## Indefinite integration

### Involving only one direct function

11.04.21.0001.01

$$\int \mathrm{Se}'(a, q, z)\, dz == \mathrm{Se}(a, q, z)$$

## Operations

### Limit operation

11.04.25.0001.01

$$\lim_{a \to \infty} \frac{1}{\sqrt{a}} \mathrm{Se}'\left(a, q, \frac{z}{\sqrt{a}}\right) == \cos(z) \;/;\, q \in \mathbb{R}$$

## Representations through equivalent functions

### With related functions

11.04.27.0001.01

$$\mathrm{Se}'(a, q, z) == (-1)^n \, \mathrm{Se}'\left(a, -q, z + \frac{\pi}{2}\right) /;\, a == a_{2n+1}(q) \bigvee a == b_{2n+1}(q) \bigwedge n \in \mathbb{N}$$

## History

  – E. L. Mathieu (1868, 1873)
  – H. Weber (1869)
  – G.W. Hill (1877)
  – E. Heine (1878)
  – G. Floquet (1883)
  – R. C. Maclaurin (1898)
  – J. Dougall (1916, 1926)

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