Mod

Notations

Traditional name

Congruence function

Traditional notation

$m \bmod n$

Mathematica StandardForm notation

`Mod[m, n]`

Primary definition

04.06.02.0001.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor$$

$m \bmod n$ is the remainder on division of m by n. The sign of $m \bmod n$ for real m, n is always the same as the sign of n.

Examples: $5 \bmod 2 = 1$, $8 \bmod 3 = 2$, $-5 \bmod 3 = 1$, $(7\pi) \bmod 3 = -21 + 7\pi$, $(27 - 3i) \bmod 4 = 3 + i$, $\mathrm{frac}(-\pi) = 3 - \pi$, $(2.7 - 3i) \bmod 5 = 2.7 + 2i$.

Specific values

Specialized values

04.06.03.0001.01

$$0 \bmod n = 0 \;/;\; n \neq 0$$

04.06.03.0002.01

$$m \bmod 1 = 0 \;/;\; m \in \mathbb{Z}$$

04.06.03.0003.01

$$1 \bmod n = n + 1 \;/;\; -n \in \mathbb{N}^+$$

04.06.03.0004.01

$$1 \bmod n = 1 \;/;\; n \in \mathbb{Z} \land n > 1$$

04.06.03.0005.01

$$m \bmod n = m \;/;\; m \in \mathbb{N} \land n \in \mathbb{Z} \land m < n$$

04.06.03.0006.01

$$m \bmod n = m - n \;/;\; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+ \land n \leq m < 2\,n$$

04.06.03.0007.01

$$m \bmod n = m - k\,n \;/;\; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+ \land k \in \mathbb{N}^+ \land k\,n \leq m < (k+1)\,n$$

04.06.03.0008.01

$$n \bmod n = 0$$

04.06.03.0009.01

$$(2\,n) \bmod n = 0$$

04.06.03.0010.01

$$(p-1)! \bmod p = p - 1 \;/;\; p \in \mathbb{P}$$

04.06.03.0011.01

$$\binom{2\,p-1}{p-1} \bmod p^3 = 1 \;/;\; p \in \mathbb{P} \land p > 3$$

04.06.03.0012.01

$$|B_{2\,n}| \bmod 1 = \delta_{\frac{n+1}{2} \bmod 1,0} + (-1)^n \left(\left(\sum_{k=3}^{2\,n+1} \frac{1}{k}\, \chi_{\mathbb{Z}}\!\left(\frac{2\,n}{k-1} \right) \chi_{\mathbb{P}}(k) + \frac{1}{2} \right) \bmod 1 \right)$$

Values at fixed points

04.06.03.0013.01

$$0 \bmod 1 = 0$$

04.06.03.0014.01

$$1 \bmod 2 = 1$$

04.06.03.0015.01

$$1 \bmod 3 = 1$$

04.06.03.0016.01

$$2 \bmod 3 = 2$$

04.06.03.0017.01

$$3 \bmod 3 = 0$$

04.06.03.0018.01

$$4 \bmod 3 = 1$$

04.06.03.0019.01

$$5 \bmod 3 = 2$$

04.06.03.0020.01

$$12 \bmod 8 = 4$$

04.06.03.0021.01

$$-3 \bmod -2 = -1$$

04.06.03.0022.01

$$-\frac{27}{10} \bmod \frac{23}{5} = \frac{19}{10}$$

04.06.03.0023.01

$$(2\,\pi) \bmod e = 2\,\pi - 2\,e$$

04.06.03.0024.01

$-\pi \bmod 2 == 4 - \pi$

04.06.03.0025.01

$\pi \bmod e == \pi - e$

04.06.03.0026.01

$(-3 + \pi\, i) \bmod (-2 - 3\, i\, e) == -3 + (1 + i)\,(-2 - 3\, i\, e) + i\, \pi$

04.06.03.0027.01

$5.2 \bmod 3.1 == 2.1$

General characteristics

Domain and analyticity

$m \bmod n$ is a nonanalytical function; it is a piecewise continuous function which is defined over $\mathbb{C}^2$.

04.06.04.0001.01

$(m * n) \longrightarrow m \bmod n :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

$m \bmod n$ is an odd function.

04.06.04.0002.01

$-m \bmod -n == -(m \bmod n)$

Mirror symmetry

04.06.04.0005.01

$\lfloor \bar{z} \rfloor == \overline{\lfloor z \rfloor} - i\,(1 - \chi_{\mathbb{Z}}(\mathrm{Im}(z)))$

Periodicity

$m \bmod n$ is a periodic function with respect to m with period n.

04.06.04.0006.01

$(m + n) \bmod n = m \bmod n$

04.06.04.0007.01

$(m + k\, n) \bmod n == m \bmod n \;/;\; k \in \mathbb{Z}$

Sets of discontinuity

The function $m \bmod n$ is a piecewise continuous function with jumps on the curves $\mathrm{Re}\left(\frac{m}{n}\right) = k \bigvee \mathrm{Im}\left(\frac{m}{n}\right) = l \;/;\; k,\, l \in \mathbb{Z}$. The functional property $m \bmod n == n\left(\frac{m}{n} \bmod 1\right) == n\left(\frac{m}{n} - \left\lfloor \frac{m}{n} \right\rfloor\right)$ makes the behaviour of the $m \bmod n$ similar to the behaviour of $\left\lfloor \frac{m}{n} \right\rfloor$.

04.06.04.0003.01

$\mathcal{DS}_m(m \bmod n) == \{\{\{(n\, k - i\, \infty,\, n\, k + i\, \infty),\, -1\} \;/;\; k \in \mathbb{Z}\},\, \{\{(i\, n\, k - \infty,\, i\, n\, k + \infty),\, -i\} \;/;\; k \in \mathbb{Z}\}\}$

04.06.04.0004.01

$$\mathcal{DS}_n(m \bmod n) = \left\{\left\{\left\{\frac{m}{(k - i\,\infty,\, k + i\,\infty)},\, -1\right\} /;\, k \in \mathbb{Z}\right\},\, \left\{\left\{\frac{m}{(i\,k - \infty,\, i\,k + \infty)},\, -i\right\} /;\, k \in \mathbb{Z}\right\}\right\}$$

04.06.04.0008.01

$$\lim_{\epsilon \to +0} (m + \epsilon) \bmod n = m \bmod n \;/;\; \mathrm{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge n \in \mathbb{R} \bigwedge n > 0$$

04.06.04.0009.01

$$\lim_{\epsilon \to +0} (m - \epsilon) \bmod n = m \bmod n + n \;/;\; \mathrm{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge n \in \mathbb{R} \bigwedge n > 0$$

04.06.04.0010.01

$$\lim_{\epsilon \to +0} (m + i\,\epsilon) \bmod n = m \bmod n \;/;\; \mathrm{Im}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge n \in \mathbb{R} \bigwedge n > 0$$

04.06.04.0011.01

$$\lim_{\epsilon \to +0} (m - i\,\epsilon) \bmod n = i\,n + m \bmod n \;/;\; \mathrm{Im}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge n \in \mathbb{R} \bigwedge n > 0$$

Series representations

Exponential Fourier series

04.06.06.0001.01

$$m \bmod n = \frac{n}{2} - \frac{n}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin\left(\frac{2\,\pi\,k\,m}{n}\right) /;\; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

Other series representations

04.06.06.0002.01

$$m \bmod n = \frac{n}{2} - \frac{1}{2} \sum_{k=1}^{n-1} \sin\left(\frac{2\,\pi\,k\,m}{n}\right) \cot\left(\frac{\pi\,k}{n}\right) /;\; m \in \mathbb{Z} \bigwedge n - 1 \in \mathbb{N}^+ \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

04.06.16.0001.01

$$(-m) \bmod (-n) = -(m \bmod n)$$

04.06.16.0002.01

$$m \bmod -n = m \bmod n + \chi_{\mathbb{Z}}\left(\frac{m}{n}\right) n - n \;/;\; m \in \mathbb{R} \land n \in \mathbb{R}$$

04.06.16.0003.01

$$m \bmod -n = m \bmod n - n\left(1 - \chi_{\mathbb{Z}}\left(\mathrm{Re}\left(\frac{m}{n}\right)\right)\right) \mathrm{sgn}\left(\left|\mathrm{Re}\left(\frac{m}{n}\right)\right|\right) - i\,n\left(1 - \chi_{\mathbb{Z}}\left(\mathrm{Im}\left(\frac{m}{n}\right)\right)\right) \mathrm{sgn}\left(\left|\mathrm{Im}\left(\frac{m}{n}\right)\right|\right)$$

04.06.16.0004.01

$$-m \bmod n = n - m \bmod n \;/;\; m \in \mathbb{R} \bigwedge n \in \mathbb{R} \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

04.06.16.0005.01

$$-m \bmod n = -\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) n + n - m \bmod n \;/;\; m \in \mathbb{R} \land n \in \mathbb{R}$$

04.06.16.0006.01

$$-m \bmod n = -(m \bmod n) + n\left(1 - \chi_{\mathbb{Z}}\left(\mathrm{Re}\left(\frac{m}{n}\right)\right)\right)\mathrm{sgn}\left(\left|\mathrm{Re}\left(\frac{m}{n}\right)\right|\right) + i\, n\left(1 - \chi_{\mathbb{Z}}\left(\mathrm{Im}\left(\frac{m}{n}\right)\right)\right)\mathrm{sgn}\left(\left|\mathrm{Im}\left(\frac{m}{n}\right)\right|\right)$$

04.06.16.0007.01

$$(i\, m) \bmod (i\, n) = i\,(m \bmod n)$$

04.06.16.0008.01

$$(i\, m) \bmod n = n - n\, \chi_{\mathbb{Z}}\left(\mathrm{Im}\left(\frac{m}{n}\right)\right) + i\,(m \bmod n)$$

04.06.16.0009.01

$$(-i\, m) \bmod n = -i\, n\left(\chi_{\mathbb{Z}}\left(\mathrm{Re}\left(\frac{m}{n}\right)\right) - 1\right) - i\,(m \bmod n)$$

04.06.16.0010.01

$$m \bmod (i\, n) = m \bmod n + \left(\chi_{\mathbb{Z}}\left(\mathrm{Re}\left(\frac{m}{n}\right)\right) - 1\right) n$$

04.06.16.0011.01

$$m \bmod (-i\, n) = m \bmod n + \left(\chi_{\mathbb{Z}}\left(\mathrm{Im}\left(\frac{m}{n}\right)\right) - 1\right) i\, n$$

04.06.16.0012.01

$$\frac{m}{n} \bmod 1 = \frac{m \bmod n}{n}$$

Argument involving related functions

04.06.16.0016.01

$$\lfloor m \rfloor \bmod n = \lfloor m \rfloor - n\left\lfloor \frac{\lfloor m \rfloor}{n} \right\rfloor$$

04.06.16.0017.01

$$\lfloor m \rfloor \bmod 1 = 0$$

04.06.16.0018.01

$$\lfloor m \rceil \bmod n = \lfloor m \rceil - n\left\lfloor \frac{\lfloor m \rceil}{n} \right\rfloor$$

04.06.16.0019.01

$$\lfloor m \rceil \bmod 1 = 0$$

04.06.16.0020.01

$$\lceil m \rceil \bmod n = \lceil m \rceil - n\left\lfloor \frac{\lceil m \rceil}{n} \right\rfloor$$

04.06.16.0021.01

$$\lceil m \rceil \bmod 1 = 0$$

04.06.16.0022.01

$$\mathrm{int}(m) \bmod n = \mathrm{int}(m) - n\left\lfloor \frac{\mathrm{int}(m)}{n} \right\rfloor$$

04.06.16.0023.01

$$\operatorname{int}(m) \bmod 1 = 0$$

04.06.16.0024.01

$$\operatorname{frac}(m) \bmod n = \operatorname{frac}(m) - n \left\lfloor \frac{\operatorname{frac}(m)}{n} \right\rfloor$$

04.06.16.0025.01

$$(m \bmod n) \bmod n = m \bmod n$$

04.06.16.0026.01

$$(m \bmod n) \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor$$

04.06.16.0027.01

$$\operatorname{quotient}(m, n) \bmod n = \left\lfloor \frac{m}{n} \right\rfloor - n \left\lfloor \frac{1}{n} \left\lfloor \frac{m}{n} \right\rfloor \right\rfloor$$

04.06.16.0028.01

$$\operatorname{quotient}(m, 1) \bmod 1 = 0$$

Addition formulas

04.06.16.0029.01

$$(m + k\, n) \bmod n = m \bmod n \;/;\; k \in \mathbb{Z}$$

Multiple arguments

04.06.16.0013.01

$$(k\, m) \bmod n = k\, (m \bmod n) - n \sum_{j=0}^{k-1} j\, \theta\!\left(\frac{m}{n} - \frac{j}{k} - \operatorname{quotient}(m, n)\right) \left(1 - \theta\!\left(\frac{m}{n} - \frac{j+1}{k} - \operatorname{quotient}(m, n)\right)\right) /;\; k \in \mathbb{N} \bigwedge \frac{m}{n} \in \mathbb{R}$$

Related transformations

04.06.16.0015.01

$$a = b \bmod \operatorname{lcm}(n, m)] \;/;\; a = b \bmod n \land a = b \bmod m \land a \in \mathbb{N}^+ \land b \in \mathbb{N}^+ \land n \in \mathbb{N}^+ \land m \in \mathbb{N}^+$$

Identities

Functional identities

04.06.17.0001.01

$$\frac{m}{n} \bmod 1 = \frac{m \bmod n}{n}$$

04.06.17.0002.01

$$(a + c) \bmod n = (b + d) \bmod n \;/;\; a \bmod n = b \bmod n \land c \bmod n = d \bmod n \land a \in \mathbb{R} \land b \in \mathbb{R} \land c \in \mathbb{R} \land d \in \mathbb{R} \land n \in \mathbb{R}$$

Complex characteristics

Real part

04.06.19.0001.01

$$\mathrm{Re}(m \bmod n) = \mathrm{Re}(m) + \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)$$

Imaginary part

04.06.19.0002.01

$$\mathrm{Im}(m \bmod n) = \mathrm{Im}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)$$

Absolute value

04.06.19.0003.01

$$|m \bmod n| = \sqrt{\left(\left(\mathrm{Im}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)\right)^2 + \left(\left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) + \mathrm{Re}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)\right)^2\right)}$$

Argument

04.06.19.0004.01

$$\arg(m \bmod n) = \tan^{-1}\left(\mathrm{Re}(m) + \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n), \mathrm{Im}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)\right)$$

Conjugate value

04.06.19.0005.01

$$\overline{m \bmod n} = \mathrm{Re}(m) + \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n) - i\left(\mathrm{Im}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)\right)$$

Signum value

04.06.19.0006.01

$$\mathrm{sgn}(m \bmod n) = \left(\left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) + \mathrm{Re}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n) + i\left(\mathrm{Im}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)\right)\right) \Big/ \left(\sqrt{\left(\left(\mathrm{Im}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)\right)^2 + \left(\left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Re}(n) - \mathrm{Im}(n)\,\mathrm{Re}(m)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Im}(n) + \mathrm{Re}(m) - \left\lfloor \frac{\mathrm{Im}(m)\,\mathrm{Im}(n) + \mathrm{Re}(m)\,\mathrm{Re}(n)}{\mathrm{Im}(n)^2 + \mathrm{Re}(n)^2} \right\rfloor \mathrm{Re}(n)\right)^2\right)}\right)$$

04.06.19.0007.01

$\operatorname{sgn}(m \bmod n) == \operatorname{sgn}(n) \,/;\, m \in \mathbb{R} \land n \in \mathbb{R}$

Differentiation

Low-order differentiation

With respect to *m*

04.06.20.0001.01

$$\frac{\partial (m \bmod n)}{\partial m} == 1$$

04.06.20.0002.01

$$\frac{\partial^2 (m \bmod n)}{\partial m^2} == 0$$

In a distributional sense for $x \in \mathbb{R}$.

04.06.20.0003.01

$$\frac{\partial (x \bmod n)}{\partial x} == x + \sum_{k=-\infty}^{\infty} \delta(x - k\, n)$$

With respect to *n*

04.06.20.0004.01

$$\frac{\partial (m \bmod n)}{\partial n} == -\left\lfloor \frac{m}{n} \right\rfloor$$

In a distributional sense for $x \in \mathbb{R}$.

04.06.20.0005.01

$$\frac{\partial (m \bmod x)}{\partial x} == \operatorname{sgn}(x) \left(\operatorname{int}\left(\frac{m}{x}\right) - \frac{m}{x^2} \sum_{k=-\infty}^{\infty} \delta_{k,0}\, \delta\left(\frac{m}{x} - k\right) \right)$$

Fractional integro-differentiation

With respect to *m*

04.06.20.0006.01

$$\frac{\partial^\alpha (m \bmod n)}{\partial m^\alpha} == \frac{\alpha\, m^{1-\alpha}}{\Gamma(2-\alpha)} + \frac{(m \bmod n)\, m^{-\alpha}}{\Gamma(1-\alpha)}$$

With respect to *n*

04.06.20.0007.01

$$\frac{\partial^\alpha (m \bmod n)}{\partial n^\alpha} == \frac{n^{-\alpha}\, (m \bmod n)}{\Gamma(2-\alpha)} - \frac{m\, \alpha\, n^{-\alpha}}{\Gamma(2-\alpha)}$$

Integration

Indefinite integration

Involving only one direct function with respect to *m*

04.06.21.0001.01

$$\int m \bmod n \, dm = m\,(m \bmod n) - \frac{m^2}{2}$$

Involving one direct function and elementary functions with respect to *m*

Involving power function

04.06.21.0002.01

$$\int m^{\alpha-1}\,(m \bmod n)\, dm = \frac{m^\alpha}{\alpha\,(\alpha+1)}\,((\alpha+1)\,(m \bmod n) - m)$$

04.06.21.0003.01

$$\int \frac{m \bmod n}{m}\, dm = m\,(1 - \log(m)) + \log(m)\,(m \bmod n)$$

Involving only one direct function with respect to *n*

04.06.21.0004.01

$$\int m \bmod n \, dn = \frac{1}{2}\, n\,(m + m \bmod n)$$

Involving one direct function and elementary functions with respect to *n*

Involving power function

04.06.21.0005.01

$$\int n^{\alpha-1}\,(m \bmod n)\, dn = \frac{n^\alpha}{\alpha\,(\alpha+1)}\,(m + \alpha\,(m \bmod n))$$

04.06.21.0006.01

$$\int \frac{m \bmod n}{n}\, dn = (\log(n) - 1)\, m + m \bmod n$$

Definite integration

For the direct function with respect to *m*

In the following formulas $a \in \mathbb{R}$.

04.06.21.0007.01

$$\int_0^a t \bmod n \, dt = \frac{1}{2}\left((a \bmod n)^2 - n\,(a \bmod n) + a\, n\right)$$

04.06.21.0008.01

$$\int_0^a t^{\alpha-1}\,(t \bmod n)\, dt = \frac{a^{\alpha+1}}{\alpha+1} - \frac{1}{\alpha}\left(-(a \bmod n)\, a^\alpha + a^{\alpha+1} - n^{\alpha+1}\,\zeta(-\alpha) + n^{\alpha+1}\,\zeta\left(-\alpha, \frac{a+n-a \bmod n}{n}\right)\right) /;\ \mathrm{Re}(\alpha) > -1$$

04.06.21.0009.01

$$\int_a^\infty t^{\alpha-1}\,(t \bmod n)\, dt = \frac{1}{\alpha\,(\alpha+1)}\left(-(\alpha+1)\,(a \bmod n)\, a^\alpha + a^{\alpha+1} + n^{\alpha+1}\,(\alpha+1)\,\zeta\left(-\alpha, \frac{a+n-a \bmod n}{n}\right)\right) /;\ \mathrm{Re}(\alpha) < 0$$

04.06.21.0010.01

$$\int_0^{\infty} t^{\alpha-1}\,(t \bmod n)\,dt = \frac{n^{\alpha+1}\,\zeta(-\alpha)}{\alpha}\ /;\ -1 < \operatorname{Re}(\alpha) < 0$$

04.06.21.0011.01

$$\int_{-a}^{a} t \bmod n\,dt = a\,n$$

For the direct function with respect to *n*

In the following formulas $a \in \mathbb{R}$.

04.06.21.0012.01

$$\int_0^{a} m \bmod t\,dt = \frac{1}{2}\left(-\psi^{(1)}\left(\frac{a+m-m \bmod a}{a}\right)m^2 + a\,m + a\,(m \bmod a)\right)$$

04.06.21.0013.01

$$\int_0^{a} t^{\alpha-1}\,(m \bmod t)\,dt = \frac{1}{\alpha^2+\alpha}\left(m\,a^{\alpha} + \alpha\,(m \bmod a)\,a^{\alpha} - m^{\alpha+1}\,\alpha\,\zeta\left(\alpha+1,\ \frac{a+m-m \bmod a}{a}\right)\right)\ /;\ \operatorname{Re}(\alpha) > -1$$

04.06.21.0014.01

$$\int_a^{\infty} t^{\alpha-1}\,(m \bmod t)\,dt = -\frac{1}{\alpha\,(\alpha+1)}\left(\alpha\,(m \bmod a)\,a^{\alpha} + m\left(a^{\alpha} + m^{\alpha}\,\alpha\,\zeta(\alpha+1) - m^{\alpha}\,\alpha\,\zeta\left(\alpha+1,\ \frac{a+m-m \bmod a}{a}\right)\right)\right)\ /;\ \operatorname{Re}(\alpha) < 0$$

04.06.21.0015.01

$$\int_0^{\infty} t^{\alpha-1}\,(m \bmod t)\,dt = -\frac{m^{\alpha+1}\,\zeta(\alpha+1)}{\alpha+1}\ /;\ -1 < \operatorname{Re}(\alpha) < 0$$

Integral transforms

Fourier exp transforms

04.06.22.0001.01

$$\mathcal{F}_t[t \bmod n](z) = n\sqrt{\frac{\pi}{2}}\,\delta(z) - \frac{i\,n}{\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{1}{k}\left(\delta\left(\frac{2\,k\,\pi}{n} - z\right) - \delta\left(\frac{2\,\pi\,k}{n} + z\right)\right)$$

Fourier cos transforms

04.06.22.0002.01

$$\mathcal{F}c_t[t \bmod n](z) = \frac{1}{\sqrt{2\pi}\,z^2}\left(n\,z\cot\left(\frac{n\,z}{2}\right) - 2\right) + \sqrt{\frac{\pi}{2}}\,n\,\delta(z)$$

Fourier sin transforms

04.06.22.0003.01

$$\mathcal{F}s_t[t \bmod n](z) = \frac{n}{\sqrt{2\pi}\,z} - \frac{n}{\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{1}{k}\left(\delta\left(\frac{2\,k\,\pi}{n} - z\right) - \delta\left(\frac{2\,\pi\,k}{n} + z\right)\right)$$

Laplace transforms

04.06.22.0004.01

$$\mathcal{L}_t[t \bmod n](z) = \frac{1}{z^2}\left(1 - \frac{n z}{e^{n z} - 1}\right) /; \operatorname{Re}(n z) > 0$$

Mellin transforms

04.06.22.0005.01

$$\mathcal{M}_t[t \bmod n](z) = \frac{n^{z+1} \zeta(-z)}{z} /; -1 < \operatorname{Re}(z) < 0$$

04.06.22.0006.01

$$\mathcal{M}_t[m \bmod t](z) = -\frac{m^{z+1} \zeta(z+1)}{z+1} /; -1 < \operatorname{Re}(z) < 0$$

Representations through equivalent functions

With related functions

With Floor

04.06.27.0001.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor$$

With Round

For real arguments

04.06.27.0008.01

$$m \bmod n = m - n \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rceil /; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m+n}{2 n} \notin \mathbb{Z}$$

04.06.27.0009.01

$$m \bmod n = m - n - n \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rceil /; \frac{m+n}{2 n} \in \mathbb{Z}$$

04.06.27.0010.01

$$m \bmod n = m - n \left(\chi_{\mathbb{Z}}\left(\frac{m+n}{2 n}\right) + \left\lfloor \frac{m}{n} - \frac{1}{2} \right\rceil\right) /; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.06.27.0003.01

$$m \bmod n = m + n \left(\left\lfloor \frac{1+i}{2} - \frac{m}{n} \right\rceil - \chi_{\mathbb{Z}}\left(\frac{1}{2}\left(\operatorname{Re}\left(\frac{m}{n}\right) + 1\right)\right) - i\, \chi_{\mathbb{Z}}\left(\frac{1}{2}\left(\operatorname{Im}\left(\frac{m}{n}\right) + 1\right)\right)\right)$$

With Ceiling

For real arguments

04.06.27.0011.01

$$m \bmod n = m + n - n \left\lceil \frac{m}{n} \right\rceil \;/;\; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

04.06.27.0012.01

$$m \bmod n = m - n \left\lceil \frac{m}{n} \right\rceil \;/;\; \frac{m}{n} \in \mathbb{Z}$$

04.06.27.0013.01

$$m \bmod n = m + n - n \left\lceil \frac{m}{n} \right\rceil - n\, \theta\!\left(\chi_{\mathbb{Z}}\!\left(\frac{m}{n}\right) - 1\right) \;/;\; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.06.27.0014.01

$$m \bmod n = m + n - n \left\lceil \frac{m}{n} \right\rceil + i\, n \;/;\; \operatorname{Re}\!\left(\frac{m}{n}\right) \notin \mathbb{Z} \bigwedge \operatorname{Im}\!\left(\frac{m}{n}\right) \notin \mathbb{Z}$$

04.06.27.0015.01

$$m \bmod n = m - n \left\lceil \frac{m}{n} \right\rceil + n \;/;\; \operatorname{Re}\!\left(\frac{m}{n}\right) \notin \mathbb{Z} \bigwedge \operatorname{Im}\!\left(\frac{m}{n}\right) \in \mathbb{Z}$$

04.06.27.0016.01

$$m \bmod n = m - n \left\lceil \frac{m}{n} \right\rceil + i\, n \;/;\; \operatorname{Re}\!\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge \operatorname{Im}\!\left(\frac{m}{n}\right) \notin \mathbb{Z}$$

04.06.27.0017.01

$$m \bmod n = m - n \left\lceil \frac{m}{n} \right\rceil \;/;\; \operatorname{Re}\!\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge \operatorname{Im}\!\left(\frac{m}{n}\right) \in \mathbb{Z}$$

04.06.27.0018.01

$$m \bmod n = m - n \left\lceil \frac{m}{n} \right\rceil - n\, \theta\!\left(\chi_{\mathbb{Z}}\!\left(\operatorname{Re}\!\left(\frac{m}{n}\right)\right) - 1\right) + n\, i\, \theta\!\left(-\chi_{\mathbb{Z}}\!\left(\operatorname{Im}\!\left(\frac{m}{n}\right)\right)\right) + n$$

04.06.27.0002.01

$$m \bmod n = m + n \left\lceil -\frac{m}{n} \right\rceil$$

With IntegerPart

For real arguments

04.06.27.0019.01

$$m \bmod n = m - n \operatorname{int}\!\left(\frac{m}{n}\right) \;/;\; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} > 0 \bigvee \frac{m}{n} \in \mathbb{Z}$$

04.06.27.0020.01

$$m \bmod n = m - n \left(\operatorname{int}\!\left(\frac{m}{n}\right) - 1\right) \;/;\; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} < 0 \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

04.06.27.0021.01

$$m \bmod n = m - n \left(\operatorname{int}\!\left(\frac{m}{n}\right) + \operatorname{sgn}\!\left(\chi_{\mathbb{Z}}\!\left(\frac{m}{n}\right) + \theta\!\left(\frac{m}{n}\right)\right) - 1\right) \;/;\; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.06.27.0022.01

$$m \bmod n = m - n\,\mathrm{int}\left(\frac{m}{n}\right) /;\ \mathrm{Re}\left(\frac{m}{n}\right) \geq 0 \bigwedge \mathrm{Im}\left(\frac{m}{n}\right) \geq 0 \bigvee \frac{m}{n} \in \mathbb{Z} \bigvee \frac{i\,m}{n} \in \mathbb{Z}$$

04.06.27.0023.01

$$m \bmod n = m - n\left(\mathrm{int}\left(\frac{m}{n}\right) - 1\right) /;\ \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} < 0 \bigwedge \frac{m}{n} \notin \mathbb{Z} \bigvee \mathrm{Re}\left(\frac{m}{n}\right) < 0 \bigwedge \mathrm{Im}\left(\frac{m}{n}\right) > 0$$

04.06.27.0024.01

$$m \bmod n = m - n\left(\mathrm{int}\left(\frac{m}{n}\right) - i\right) /;\ \frac{i\,m}{n} \in \mathbb{R} \bigwedge \frac{i\,m}{n} > 0 \bigwedge \frac{i\,m}{n} \notin \mathbb{Z} \bigvee \mathrm{Re}\left(\frac{m}{n}\right) > 0 \bigwedge \mathrm{Im}\left(\frac{m}{n}\right) < 0$$

04.06.27.0025.01

$$m \bmod n = m - n\left(\mathrm{int}\left(\frac{m}{n}\right) - 1 - i\right) /;\ \mathrm{Re}\left(\frac{m}{n}\right) < 0 \bigwedge \mathrm{Im}\left(\frac{m}{n}\right) < 0$$

04.06.27.0004.01

$$m \bmod n = m + n\left(-\mathrm{int}\left(\frac{m}{n}\right) + 1 + i - i\,\mathrm{sgn}\left(\chi_{\mathbb{Z}}\left(\mathrm{Im}\left(\frac{m}{n}\right)\right) + \theta\left(\mathrm{Im}\left(\frac{m}{n}\right)\right)\right) - \mathrm{sgn}\left(\chi_{\mathbb{Z}}\left(\mathrm{Re}\left(\frac{m}{n}\right)\right) + \theta\left(\mathrm{Re}\left(\frac{m}{n}\right)\right)\right)\right)$$

With FractionalPart

For real arguments

04.06.27.0026.01

$$m \bmod n = n\,\mathrm{frac}\left(\frac{m}{n}\right) /;\ \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} > 0 \bigvee \frac{m}{n} \in \mathbb{Z}$$

04.06.27.0027.01

$$m \bmod n = n\left(\mathrm{frac}\left(\frac{m}{n}\right) + 1\right) /;\ \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} < 0 \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

04.06.27.0028.01

$$m \bmod n = n\left(\mathrm{frac}\left(\frac{m}{n}\right) - \mathrm{sgn}\left(\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) + \theta\left(\frac{m}{n}\right)\right) + 1\right) /;\ \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.06.27.0029.01

$$m \bmod n = n\,\mathrm{frac}\left(\frac{m}{n}\right) /;\ \mathrm{Re}\left(\frac{m}{n}\right) \geq 0 \bigwedge \mathrm{Im}\left(\frac{m}{n}\right) \geq 0 \bigvee \frac{m}{n} \in \mathbb{Z} \bigvee \frac{i\,m}{n} \in \mathbb{Z}$$

04.06.27.0030.01

$$m \bmod n = n\left(\mathrm{frac}\left(\frac{m}{n}\right) + 1\right) /;\ \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} < 0 \bigwedge \frac{m}{n} \notin \mathbb{Z} \bigvee \mathrm{Re}\left(\frac{m}{n}\right) < 0 \bigwedge \mathrm{Im}\left(\frac{m}{n}\right) > 0$$

04.06.27.0031.01

$$m \bmod n = n\left(\mathrm{frac}\left(\frac{m}{n}\right) + i\right) /;\ \frac{i\,m}{n} \in \mathbb{R} \bigwedge \frac{i\,m}{n} > 0 \bigwedge \frac{i\,m}{n} \notin \mathbb{Z} \bigvee \mathrm{Re}\left(\frac{m}{n}\right) > 0 \bigwedge \mathrm{Im}\left(\frac{m}{n}\right) < 0$$

04.06.27.0032.01

$$m \bmod n = n\left(\mathrm{frac}\left(\frac{m}{n}\right) + 1 + i\right) /;\ \mathrm{Re}\left(\frac{m}{n}\right) < 0 \bigwedge \mathrm{Im}\left(\frac{m}{n}\right) < 0$$

04.06.27.0005.01

$$m \bmod n = n\left(\mathrm{frac}\left(\frac{m}{n}\right) + 1 + i - i\,\mathrm{sgn}\left(\chi_{\mathbb{Z}}\left(\mathrm{Im}\left(\frac{m}{n}\right)\right) + \theta\left(\mathrm{Im}\left(\frac{m}{n}\right)\right)\right) - \mathrm{sgn}\left(\chi_{\mathbb{Z}}\left(\mathrm{Re}\left(\frac{m}{n}\right)\right) + \theta\left(\mathrm{Re}\left(\frac{m}{n}\right)\right)\right)\right)$$

With Quotient

04.06.27.0006.01

$m \bmod n = m - n\,\mathrm{quotient}(m, n)$

With elementary functions

04.06.27.0007.01

$m \bmod n = \frac{n}{2} - \frac{n}{\pi} \tan^{-1}\left(\cot\left(\frac{\pi m}{n}\right)\right) /;\ \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} \notin \mathbb{Z}$

Zeros

04.06.30.0001.01

$m \bmod n = 0 /;\ m = 0 \wedge n \neq 0$

Theorems

Linear congruential random number generator

A sequence of pseudorandom numbers r_k is generated by $r_{k+1} = (a\, r_k + c) \bmod m$, with $a, c, m \in \mathbb{N}$, $m > 0$, $0 \leq a < m$, $0 \leq c < m$, $0 \leq r_0 < m$.

Chinese remainder theorem

Let $m_1, m_2, \ldots, m_n$ be pairwise relatively prime integers ($\gcd(m_i, m_k) = 1 /; i \neq k$). Then for given integers $z_1, z_2, \ldots, z_n$ there exists a unique (mod m_k) integer z such that $z \bmod m_k = z_k$.

Legendre theorem

If $a, b, c \in \mathbb{N}^+ \bigwedge \gcd(a, b) = \gcd(a, c) = \gcd(b, c) = 0 \bigwedge \sqrt{a}, \sqrt{b}, \sqrt{c} \notin \mathbb{N}$, then the equation $a x^2 + b x^2 + c z^2 = 0$ has nontrivial solutions for x, y, z if and only if the equations $x^2 - b\, c \bmod a = 0 \wedge y^2 - a\, c \bmod b = 0 \wedge z^2 - a\, b \bmod c = 0$ are solvable.

Gauss' Easter formula

Easter Sunday is the $i + j + 1$ th day after the 21st of March, where

$i = 28\, \delta_{h,29} + 27\, \delta_{h,28}\, \theta(a - 11) + 11\, (1 - \delta_{h,29})\, (1 - \delta_{h,28}\, \theta(a - 11))$, $j = (2\, b + 4\, c + 6\, i + g) \bmod 7$, $h = (f + 19\, a) \bmod 30$, $a = \mathrm{year} \bmod 19$, $b = \mathrm{year} \bmod 4$, $c = \mathrm{year} \bmod 7$,
$d = \left\lfloor \frac{(8[\mathit{year}/100] + 13)}{25} \right\rfloor - 2$, $e = \left\lfloor \frac{\mathit{year}}{100} \right\rfloor - \left\lfloor \frac{\mathit{year}}{400} \right\rfloor - 2$, $f = (15 + e - d) \bmod 30$, $g = (6 + e) \bmod 7$.

History

– C. F. Gauss (1801) introduced the symbol mod

Applications include pseudo-random number generation.

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