NevilleThetaC

Notations

Traditional name

Neville theta function ϑ_c

Traditional notation

$\vartheta_c(z \mid m)$

Mathematica StandardForm notation

`NevilleThetaC[z, m]`

Primary definition

09.09.02.0001.01

$$\vartheta_c(z \mid m) = \frac{\sqrt{2\pi}\,\sqrt[4]{q(m)}}{\sqrt[4]{m}\,\sqrt{K(m)}} \sum_{k=0}^{\infty} q(m)^{k(k+1)} \cos\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

Specific values

Specialized values

For fixed z

09.09.03.0001.01

$$\vartheta_c(z \mid 0) = \cos(z)$$

09.09.03.0002.01

$$\vartheta_c\left(z + \frac{\pi}{2} \,\middle|\, 0\right) = -\sin(z)$$

09.09.03.0003.01

$$\vartheta_c(z \mid 1) = 1$$

For fixed m

09.09.03.0004.01

$$\vartheta_c(0 \mid m) = 1$$

General characteristics

Domain and analyticity

$\vartheta_c(z \mid m)$ is an analytical meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.09.04.0001.01

$$(z * m) \longrightarrow \vartheta_c(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\vartheta_c(z \mid m)$ is an even function with respect to z.

09.09.04.0002.01

$$\vartheta_c(-z \mid m) = \vartheta_c(z \mid m)$$

Mirror symmetry

09.09.04.0003.01

$$\vartheta_c(\bar{z} \mid \overline{m}) = \overline{\vartheta_c(z \mid m)}$$

Periodicity

$\vartheta_c(z \mid m)$ is a periodic function with respect to z with period $4\,K(m)$.

09.09.04.0004.01

$$\vartheta_c(z + 2\,K(m) \mid m) = -\vartheta_c(z \mid m)$$

09.09.04.0005.01

$$\vartheta_c(z + 4\,K(m) \mid m) = \vartheta_c(z \mid m)$$

09.09.04.0006.01

$$\vartheta_c(z + 2\,r\,K(m) \mid m) = (-1)^r\,\vartheta_c(z \mid m) \,/;\, r \in \mathbb{Z}$$

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

09.09.06.0001.01

$$\vartheta_c(z \mid m) = \frac{\sqrt{2\pi}\,\sqrt[4]{q(m)}}{\sqrt[4]{m}\,\sqrt{K(m)}} \sum_{k=0}^{\infty} q(m)^{k(k+1)} \cos\left(\frac{(2k+1)\pi z}{2\,K(m)}\right)$$

Product representations

09.09.08.0001.01

$$\vartheta_c(z \mid m) = \frac{2^{2/3} \sqrt[12]{1-m} \sqrt[6]{q(m)}}{\sqrt[6]{m}} \cos\left(\frac{\pi z}{2 K(m)}\right) \prod_{k=1}^{\infty} \left(2 \cos\left(\frac{\pi z}{K(m)}\right) q(m)^{2k} + q(m)^{4k} + 1\right)$$

Differential equations

Partial differential equations

09.09.13.0001.01

$$K(m) \frac{\partial^2 \vartheta_c(z \mid m)}{\partial z^2} + 2 z (E(m) + (m-1) K(m)) \frac{\partial \vartheta_c(z \mid m)}{\partial z} - 4 (m-1) m\, K(m) \frac{\partial \vartheta_c(z \mid m)}{\partial m} + E(m) \vartheta_c(z \mid m) = 0$$

09.09.13.0002.01

$$-32 z m^3 \frac{\partial^2 \vartheta_c(z \mid m)}{\partial m^2} \frac{\partial \vartheta_c(z \mid m)}{\partial z} - 24 (m-1)^2 m^2 \left(\frac{\partial \vartheta_c(z \mid m)}{\partial m}\right)^2 -$$
$$4 z^2 m^2 \left(\frac{\partial \vartheta_c(z \mid m)}{\partial z}\right)^2 + 16 \left(m^2+1\right) z\, m^2 \frac{\partial \vartheta_c(z \mid m)}{\partial z} \frac{\partial^2 \vartheta_c(z \mid m)}{\partial m^2} +$$
$$4 z\, m^2 \frac{\partial^2 \vartheta_c(z \mid m)}{\partial z^2} \frac{\partial^2 \vartheta_c(z \mid m)}{\partial z \partial m} + 4 z^2 m \left(\frac{\partial \vartheta_c(z \mid m)}{\partial z}\right)^2 - 4 z\, m \frac{\partial^2 \vartheta_c(z \mid m)}{\partial z \partial m} \frac{\partial^2 \vartheta_c(z \mid m)}{\partial z^2} +$$
$$2 (m-1) m \frac{\partial \vartheta_c(z \mid m)}{\partial m} \left(5 \frac{\partial^2 \vartheta_c(z \mid m)}{\partial z^2} + 2 z \left((9 m - 5) \frac{\partial \vartheta_c(z \mid m)}{\partial z} - 4 (m-1) m \frac{\partial^2 \vartheta_c(z \mid m)}{\partial z \partial m}\right)\right) -$$
$$4 (m-1) z m \frac{\partial \vartheta_c(z \mid m)}{\partial z} \frac{\partial^3 \vartheta_c(z \mid m)}{\partial z^2 \partial m} - \left(\frac{\partial^2 \vartheta_c(z \mid m)}{\partial z^2}\right)^2 + (m-1) \vartheta_c(z \mid m)^2 +$$
$$2 (m-1) \left(\frac{\partial^2 \vartheta_c(z \mid m)}{\partial z^2} + m \left(4 m \frac{\partial \vartheta_c(z \mid m)}{\partial m} + 2 (m-1) \left(2 m \frac{\partial^2 \vartheta_c(z \mid m)}{\partial m^2} - z \frac{\partial^2 \vartheta_c(z \mid m)}{\partial z \partial m}\right) - \frac{\partial^3 \vartheta_c(z \mid m)}{\partial z^2 \partial m}\right)\right) \vartheta_c(z \mid m) = 0$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.09.16.0001.01

$$\vartheta_c(z + K(m) \mid m) = -\sqrt[4]{1-m}\, \vartheta_s(z \mid m)$$

09.09.16.0002.01

$$\vartheta_c(z + (2 r + 1) K(m) \mid m) = (-1)^{r-1} \sqrt[4]{1-m}\, \vartheta_s(z \mid m) /; r \in \mathbb{Z}$$

Differentiation

Low-order differentiation

With respect to *z*

09.09.20.0001.01

$$\frac{\partial \vartheta_c(z \mid m)}{\partial z} = -\frac{\pi^{3/2} \sqrt[4]{q(m)}}{\sqrt{2} \sqrt[4]{m} K(m)^{3/2}} \sum_{k=0}^{\infty} (2k+1) q(m)^{k(k+1)} \sin\left(\frac{\pi z (2k+1)}{2K(m)}\right)$$

09.09.20.0002.01

$$\frac{\partial^2 \vartheta_c(z \mid m)}{\partial z^2} = -\frac{\pi^{5/2} \sqrt[4]{q(m)}}{2\sqrt{2} \sqrt[4]{m} K(m)^{5/2}} \sum_{k=0}^{\infty} (2k+1)^2 q(m)^{k(k+1)} \cos\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

Symbolic differentiation

With respect to z

09.09.20.0003.01

$$\frac{\partial^n \vartheta_c(z \mid m)}{\partial z^n} = \frac{2^{\frac{1}{2}-n} \pi^{n+\frac{1}{2}} K(m)^{-n-\frac{1}{2}} \sqrt[4]{q(m)}}{\sqrt[4]{m}} \sum_{k=0}^{\infty} (2k+1)^n q(m)^{k(k+1)} \cos\left(\frac{\pi n}{2} + \frac{(2k+1)\pi z}{2K(m)}\right) /; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to z

09.09.20.0004.01

$$\frac{\partial^\alpha \vartheta_c(z \mid m)}{\partial z^\alpha} = \frac{2^{\alpha+\frac{1}{2}} \pi z^{-\alpha} \sqrt[4]{q(m)}}{\sqrt[4]{m} \sqrt{K(m)}} \sum_{k=0}^{\infty} q(m)^{k(k+1)} {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{(2k+1)^2 \pi^2 z^2}{16 K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.09.21.0001.01

$$\int \vartheta_c(z \mid m) \, dz = \frac{2}{\sqrt[4]{m}} \sqrt{\frac{2}{\pi}} \sqrt{K(m)} \sqrt[4]{q(m)} \sum_{k=0}^{\infty} \frac{q(m)^{k(k+1)}}{2k+1} \sin\left(\frac{\pi(2kz+z)}{2K(m)}\right)$$

Representations through equivalent functions

With related functions

Involving Jacobi and other Neville functions

09.09.27.0001.01

$$\vartheta_c(z \mid m) = \operatorname{cd}(z \mid m) \vartheta_d(z \mid m)$$

09.09.27.0002.01

$$\vartheta_c(z \mid m) = \frac{\vartheta_d(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.09.27.0003.01

$$\vartheta_c(z \mid m) = \operatorname{cn}(z \mid m) \vartheta_n(z \mid m)$$

09.09.27.0004.01

$$\vartheta_c(z \mid m) = \frac{\vartheta_n(z \mid m)}{\mathrm{nc}(z \mid m)}$$

09.09.27.0005.01

$$\vartheta_c(z \mid m) = \mathrm{cs}(z \mid m)\, \vartheta_s(z \mid m)$$

09.09.27.0006.01

$$\vartheta_c(z \mid m) = \frac{\vartheta_s(z \mid m)}{\mathrm{sc}(z \mid m)}$$

09.09.27.0007.01

$$\vartheta_c(z \mid m) = \sqrt[4]{1-m}\; \vartheta_s(K(m) - z \mid m)$$

Involving theta functions

09.09.27.0008.01

$$\vartheta_c(z \mid m) = \frac{\sqrt{\pi}}{\sqrt{2}\, \sqrt[4]{m}\, \sqrt{K(m)}}\, \vartheta_2\!\left(\frac{\pi z}{2 K(m)}, q(m)\right)$$

09.09.27.0009.01

$$\vartheta_c(z \mid m) = \frac{1}{\vartheta_2(0, q(m))}\, \vartheta_2\!\left(\frac{\pi z}{2 K(m)}, q(m)\right)$$

History

– K. Weierstrass (1894)

– E. N. Neville (1944)

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