

# NevilleThetaC

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## Notations

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### Traditional name

Neville theta function  $\vartheta_c$

### Traditional notation

$\vartheta_c(z | m)$

### Mathematica StandardForm notation

NevilleThetaC[ $z$ ,  $m$ ]

## Primary definition

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09.09.02.0001.01

$$\vartheta_c(z | m) = \frac{\sqrt{2\pi} \sqrt[4]{q(m)}}{\sqrt[4]{m} \sqrt{K(m)}} \sum_{k=0}^{\infty} q(m)^{k(k+1)} \cos\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

## Specific values

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### Specialized values

#### For fixed $z$

09.09.03.0001.01

$$\vartheta_c(z | 0) = \cos(z)$$

09.09.03.0002.01

$$\vartheta_c\left(z + \frac{\pi}{2} \middle| 0\right) = -\sin(z)$$

09.09.03.0003.01

$$\vartheta_c(z | 1) = 1$$

#### For fixed $m$

09.09.03.0004.01

$$\vartheta_c(0 | m) = 1$$

## General characteristics

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### Domain and analyticity

$\vartheta_c(z | m)$  is an analytical meromorphic function of  $z$  and  $m$  which is defined over  $\mathbb{C}^2$ .

09.09.04.0001.01

$$(z * m) \rightarrow \vartheta_c(z | m) :: (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

## Symmetries and periodicities

### Parity

$\vartheta_c(z | m)$  is an even function with respect to  $z$ .

09.09.04.0002.01

$$\vartheta_c(-z | m) = \vartheta_c(z | m)$$

### Mirror symmetry

09.09.04.0003.01

$$\vartheta_c(\bar{z} | \bar{m}) = \overline{\vartheta_c(z | m)}$$

### Periodicity

$\vartheta_c(z | m)$  is a periodic function with respect to  $z$  with period  $4K(m)$ .

09.09.04.0004.01

$$\vartheta_c(z + 2K(m) | m) = -\vartheta_c(z | m)$$

09.09.04.0005.01

$$\vartheta_c(z + 4K(m) | m) = \vartheta_c(z | m)$$

09.09.04.0006.01

$$\vartheta_c(z + 2rK(m) | m) = (-1)^r \vartheta_c(z | m) ; r \in \mathbb{Z}$$

## Branch points

Branch points locations: complicated

## Branch cuts

Branch cut locations: complicated

## Series representations

### Generalized power series

09.09.06.0001.01

$$\vartheta_c(z | m) = \frac{\sqrt{2\pi} \sqrt[4]{q(m)}}{\sqrt[4]{m} \sqrt{K(m)}} \sum_{k=0}^{\infty} q(m)^{k(k+1)} \cos\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

## Product representations

09.09.08.0001.01

$$\vartheta_c(z | m) = \frac{2^{2/3} \sqrt[12]{1-m} \sqrt[6]{q(m)}}{\sqrt[6]{m}} \cos\left(\frac{\pi z}{2 K(m)}\right) \prod_{k=1}^{\infty} \left(2 \cos\left(\frac{\pi z}{K(m)}\right) q(m)^{2k} + q(m)^{4k} + 1\right)$$

## Differential equations

### Partial differential equations

09.09.13.0001.01

$$K(m) \frac{\partial^2 \vartheta_c(z | m)}{\partial z^2} + 2z(E(m) + (m-1)K(m)) \frac{\partial \vartheta_c(z | m)}{\partial z} - 4(m-1)m K(m) \frac{\partial \vartheta_c(z | m)}{\partial m} + E(m) \vartheta_c(z | m) = 0$$

09.09.13.0002.01

$$\begin{aligned} & -32zm^3 \frac{\partial^2 \vartheta_c(z | m)}{\partial m^2} \frac{\partial \vartheta_c(z | m)}{\partial z} - 24(m-1)^2 m^2 \left(\frac{\partial \vartheta_c(z | m)}{\partial m}\right)^2 - \\ & 4z^2 m^2 \left(\frac{\partial \vartheta_c(z | m)}{\partial z}\right)^2 + 16(m^2+1)zm^2 \frac{\partial \vartheta_c(z | m)}{\partial z} \frac{\partial^2 \vartheta_c(z | m)}{\partial m^2} + \\ & 4zm^2 \frac{\partial^2 \vartheta_c(z | m)}{\partial z^2} \frac{\partial^2 \vartheta_c(z | m)}{\partial z \partial m} + 4z^2 m \left(\frac{\partial \vartheta_c(z | m)}{\partial z}\right)^2 - 4zm \frac{\partial^2 \vartheta_c(z | m)}{\partial z \partial m} \frac{\partial^2 \vartheta_c(z | m)}{\partial z^2} + \\ & 2(m-1)m \frac{\partial \vartheta_c(z | m)}{\partial m} \left(5 \frac{\partial^2 \vartheta_c(z | m)}{\partial z^2} + 2z \left(9m-5\right) \frac{\partial \vartheta_c(z | m)}{\partial z} - 4(m-1)m \frac{\partial^2 \vartheta_c(z | m)}{\partial z \partial m}\right) - \\ & 4(m-1)zm \frac{\partial \vartheta_c(z | m)}{\partial z} \frac{\partial^3 \vartheta_c(z | m)}{\partial z^2 \partial m} - \left(\frac{\partial^2 \vartheta_c(z | m)}{\partial z^2}\right)^2 + (m-1)\vartheta_c(z | m)^2 + \\ & 2(m-1) \left(\frac{\partial^2 \vartheta_c(z | m)}{\partial z^2} + m \left(4m \frac{\partial \vartheta_c(z | m)}{\partial m} + 2(m-1) \left(2m \frac{\partial^2 \vartheta_c(z | m)}{\partial m^2} - z \frac{\partial^2 \vartheta_c(z | m)}{\partial z \partial m}\right) - \frac{\partial^3 \vartheta_c(z | m)}{\partial z^2 \partial m}\right)\right) \vartheta_c(z | m) = 0 \end{aligned}$$

## Transformations

### Transformations and argument simplifications

#### Argument involving basic arithmetic operations

09.09.16.0001.01

$$\vartheta_c(z + K(m) | m) = -\sqrt[4]{1-m} \vartheta_s(z | m)$$

09.09.16.0002.01

$$\vartheta_c(z + (2r+1)K(m) | m) = (-1)^{r-1} \sqrt[4]{1-m} \vartheta_s(z | m) /; r \in \mathbb{Z}$$

## Differentiation

### Low-order differentiation

With respect to  $z$

09.09.20.0001.01

$$\frac{\partial \vartheta_c(z | m)}{\partial z} = - \frac{\pi^{3/2} \sqrt[4]{q(m)}}{\sqrt{2} \sqrt[4]{m} K(m)^{3/2}} \sum_{k=0}^{\infty} (2k+1) q(m)^{k(k+1)} \sin\left(\frac{\pi z(2k+1)}{2K(m)}\right)$$

09.09.20.0002.01

$$\frac{\partial^2 \vartheta_c(z | m)}{\partial z^2} = - \frac{\pi^{5/2} \sqrt[4]{q(m)}}{2\sqrt{2} \sqrt[4]{m} K(m)^{5/2}} \sum_{k=0}^{\infty} (2k+1)^2 q(m)^{k(k+1)} \cos\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

## Symbolic differentiation

With respect to  $z$

09.09.20.0003.01

$$\frac{\partial^n \vartheta_c(z | m)}{\partial z^n} = \frac{2^{\frac{1}{2}-n} \pi^{n+\frac{1}{2}} K(m)^{-n-\frac{1}{2}} \sqrt[4]{q(m)}}{\sqrt[4]{m}} \sum_{k=0}^{\infty} (2k+1)^n q(m)^{k(k+1)} \cos\left(\frac{\pi n}{2} + \frac{(2k+1)\pi z}{2K(m)}\right); n \in \mathbb{N}^+$$

## Fractional integro-differentiation

With respect to  $z$

09.09.20.0004.01

$$\frac{\partial^\alpha \vartheta_c(z | m)}{\partial z^\alpha} = \frac{2^{\alpha+\frac{1}{2}} \pi z^{-\alpha} \sqrt[4]{q(m)}}{\sqrt[4]{m} \sqrt{K(m)}} \sum_{k=0}^{\infty} q(m)^{k(k+1)} {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{(2k+1)^2 \pi^2 z^2}{16K(m)^2}\right)$$

## Integration

### Indefinite integration

Involving only one direct function

09.09.21.0001.01

$$\int \vartheta_c(z | m) dz = \frac{2}{\sqrt[4]{m}} \sqrt{\frac{2}{\pi}} \sqrt{K(m)} \sqrt[4]{q(m)} \sum_{k=0}^{\infty} \frac{q(m)^{k(k+1)}}{2k+1} \sin\left(\frac{\pi(2kz+z)}{2K(m)}\right)$$

## Representations through equivalent functions

### With related functions

Involving Jacobi and other Neville functions

09.09.27.0001.01

$$\vartheta_c(z | m) = \text{cd}(z | m) \vartheta_d(z | m)$$

09.09.27.0002.01

$$\vartheta_c(z | m) = \frac{\vartheta_d(z | m)}{\text{dc}(z | m)}$$

09.09.27.0003.01

$$\vartheta_c(z | m) = \text{cn}(z | m) \vartheta_n(z | m)$$

09.09.27.0004.01

$$\vartheta_c(z | m) = \frac{\vartheta_n(z | m)}{\operatorname{nc}(z | m)}$$

09.09.27.0005.01

$$\vartheta_c(z | m) = \operatorname{cs}(z | m) \vartheta_s(z | m)$$

09.09.27.0006.01

$$\vartheta_c(z | m) = \frac{\vartheta_s(z | m)}{\operatorname{sc}(z | m)}$$

09.09.27.0007.01

$$\vartheta_c(z | m) = \sqrt[4]{1-m} \vartheta_s(K(m) - z | m)$$

### Involving theta functions

09.09.27.0008.01

$$\vartheta_c(z | m) = \frac{\sqrt{\pi}}{\sqrt{2} \sqrt[4]{m} \sqrt{K(m)}} \vartheta_2\left(\frac{\pi z}{2K(m)}, q(m)\right)$$

09.09.27.0009.01

$$\vartheta_c(z | m) = \frac{1}{\vartheta_2(0, q(m))} \vartheta_2\left(\frac{\pi z}{2K(m)}, q(m)\right)$$

## History

- K. Weierstrass (1894)
- E. N. Neville (1944)

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