Quotient

Notations

Traditional name

Integer part of the quotient

Traditional notation

quotient(m, n)

Mathematica StandardForm notation

`Quotient[m, n]`

Primary definition

04.07.02.0001.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} \right\rfloor$$

quotient(m, n) is the integer quotient of m and n.

Examples: $\text{quotient}(5, 2) = 2$, $\text{quotient}(13, 3) = 4$, $\text{quotient}(-4, 3) = -2$, $\text{quotient}(\pi, 2) = 1$, $\text{quotient}(27 - 3\,i, 5) = 5 - i$, $\text{quotient}(-\pi, 2) = -2$, $\text{quotient}(2.7 - 3\,i, 5) = -i$.

Specific values

Specialized values

For fixed *m*

04.07.03.0001.01

$$\text{quotient}(m, 1) = m \;/;\; m \in \mathbb{Z}$$

For fixed *n*

04.07.03.0002.01

$$\text{quotient}(0, n) = 0 \;/;\; n \neq 0$$

04.07.03.0003.01

$$\text{quotient}(1, n) = -1 \;/;\; n \in \mathbb{Z} \land n < 0$$

04.07.03.0004.01

$$\text{quotient}(1, n) = 0 \;/;\; n \in \mathbb{Z} \land n > 1$$

04.07.03.0005.01

$$\text{quotient}(m, n) = 0 \;/;\; m \in \mathbb{N} \land n \in \mathbb{N}^+ \land m < n$$

04.07.03.0006.01

$$\text{quotient}(m, n) = 1 \;/;\; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+ \land n \leq m < 2\,n$$

04.07.03.0007.01

$$\text{quotient}(m, n) = k \;/;\; m \in \mathbb{N}^+ \land n \in \mathbb{N}^+ \land k \in \mathbb{N}^+ \land k\,n \leq m < (k+1)\,n$$

04.07.03.0008.01

$$\text{quotient}(n, n) = 1$$

04.07.03.0009.01

$$\text{quotient}(2\,n, n) = 2$$

04.07.03.0010.01

$$\text{quotient}((p-1)!, p) = \frac{(p-1)! + 1 - p}{p} \;/;\; p \in \mathbb{P}$$

04.07.03.0011.01

$$\text{quotient}\left(\binom{2\,p-1}{p-1}, p^3\right) = \frac{1}{p^3}\left(\binom{2\,p-1}{p-1} - 1\right) /;\; p \in \mathbb{P} \land p > 3$$

Values at fixed points

04.07.03.0012.01

$$\text{quotient}(0, 1) = 0$$

04.07.03.0013.01

$$\text{quotient}(1, 2) = 0$$

04.07.03.0014.01

$$\text{quotient}(1, 3) = 0$$

04.07.03.0015.01

$$\text{quotient}(2, 3) = 0$$

04.07.03.0016.01

$$\text{quotient}(3, 3) = 1$$

04.07.03.0017.01

$$\text{quotient}(4, 3) = 1$$

04.07.03.0018.01

$$\text{quotient}(5, 3) = 1$$

04.07.03.0019.01

$$\text{quotient}(12, 8) = 1$$

04.07.03.0020.01

$$\text{quotient}(-3, -2) = 1$$

04.07.03.0021.01

$$\text{quotient}\left(-\frac{27}{10}, \frac{23}{5}\right) = -1$$

04.07.03.0022.01

$$\text{quotient}(2\,\pi, e) = 2$$

04.07.03.0023.01

$\text{quotient}(-\pi, 2) = -2$

04.07.03.0024.01

$\text{quotient}(\pi, e) = 1$

04.07.03.0025.01

$\text{quotient}(-3 + \pi i, -2 - 3 i e) = -1 - i$

04.07.03.0026.01

$\text{quotient}(5.2, 3.1) = 1$

Values at infinities

04.07.03.0027.01

$\text{quotient}(n, \infty) = 0$

04.07.03.0028.01

$\text{quotient}(n, -\infty) = 0$

04.07.03.0029.01

$\text{quotient}(n, i \infty) = 0$

04.07.03.0030.01

$\text{quotient}(n, -i \infty) = 0$

04.07.03.0031.01

$\text{quotient}(n, \tilde{\infty}) = 0$

General characteristics

Domain and analyticity

$\text{quotient}(m, n)$ is a nonanalytical function; it is a piecewise continuous function which is defined over $\mathbb{C}^2$.

04.07.04.0001.01

$(m * n) \longrightarrow \text{quotient}(m, n) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

$\text{quotient}(m, n)$ is an even function.

04.07.04.0002.01

$\text{quotient}(-m, -n) = \text{quotient}(m, n)$

Periodicity

No periodicity

Sets of discontinuity

The function quotient(m, n) is a piecewise continuous function with jumps on the curves $\mathrm{Re}\left(\frac{m}{n}\right) = k \bigvee \mathrm{Im}\left(\frac{m}{n}\right) = l \text{ /; } k, l \in \mathbb{Z}$. The functional property $\text{quotient}(m, n) = \text{quotient}\left(\frac{m}{n}, 1\right) = \left\lfloor \frac{m}{n} \right\rfloor$ makes the behaviour of the quotient(m, n) similar to behavior of $\left\lfloor \frac{m}{n} \right\rfloor$.

04.07.04.0003.01

$$\mathcal{DS}_m(\text{quotient}(m, n)) = \{\{\{(n\,k - i\,\infty, n\,k + i\,\infty), -1\} \text{ /; } k \in \mathbb{Z}\}, \{\{(i\,n\,k - \infty, i\,n\,k + \infty), -i\} \text{ /; } k \in \mathbb{Z}\}\}$$

04.07.04.0004.01

$$\mathcal{DS}_n(\text{quotient}(m, n)) = \left\{\left\{\left\{\frac{m}{(k - i\,\infty, k + i\,\infty)}, -1\right\} \text{ /; } k \in \mathbb{Z}\right\}, \left\{\left\{\frac{m}{(i\,k - \infty, i\,k + \infty)}, -i\right\} \text{ /; } k \in \mathbb{Z}\right\}\right\}$$

04.07.04.0005.01

$$\lim_{\epsilon \to +0} \text{quotient}(m + \epsilon, n) = \text{quotient}(m, n) \text{ /; } \mathrm{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge n \in \mathbb{R} \bigwedge n > 0$$

04.07.04.0006.01

$$\lim_{\epsilon \to +0} \text{quotient}(m - \epsilon, n) = \text{quotient}(m, n) - 1 \text{ /; } \mathrm{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge n \in \mathbb{R} \bigwedge n > 0$$

04.07.04.0007.01

$$\lim_{\epsilon \to +0} \text{quotient}(m + i\,\epsilon, n) = \text{quotient}(m, n) \text{ /; } \mathrm{Im}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge n \in \mathbb{R} \bigwedge n > 0$$

04.07.04.0008.01

$$\lim_{\epsilon \to +0} \text{quotient}(m - i\,\epsilon, n) = \text{quotient}(m, n) - i \text{ /; } \mathrm{Im}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge n \in \mathbb{R} \bigwedge n > 0$$

Series representations

Exponential Fourier series

04.07.06.0001.01

$$\text{quotient}(m, n) = \frac{m}{n} + \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin\left(\frac{2\,\pi\,k\,m}{n}\right) - \frac{1}{2} \text{ /; } \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

Other series representations

04.07.06.0002.01

$$\text{quotient}\,(m, n) = \frac{m}{n} + \frac{1}{2\,n} \sum_{k=1}^{n-1} \sin\left(\frac{2\,\pi\,k\,m}{n}\right) \cot\left(\frac{\pi\,k}{n}\right) - \frac{1}{2} \text{ /; } m \in \mathbb{Z} \bigwedge n - 1 \in \mathbb{N}^+ \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

04.07.16.0001.01

$$\text{quotient}(-m, -n) = \text{quotient}(m, n)$$

04.07.16.0002.01

$$\text{quotient}(m, -n) = -\text{quotient}(m, n) + \chi_{\mathbb{Z}}\left(\frac{m}{n}\right) - 1 \;/;\; m \in \mathbb{R} \land n \in \mathbb{R}$$

04.07.16.0003.01

$$\text{quotient}(m, -n) = -\text{quotient}(m, n) - \left(1 - \chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right)\right) \text{sgn}\left(\left|\text{Re}\left(\frac{m}{n}\right)\right|\right) - i\left(1 - \chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right)\right) \text{sgn}\left(\left|\text{Im}\left(\frac{m}{n}\right)\right|\right)$$

04.07.16.0004.01

$$\text{quotient}(-m, n) = \chi_{\mathbb{Z}}\left(\frac{m}{n}\right) - 1 - \text{quotient}(m, n) \;/;\; m \in \mathbb{R} \land n \in \mathbb{R}$$

04.07.16.0005.01

$$\text{quotient}(-m, n) = -\text{quotient}(m, n) - \left(1 - \chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right)\right) \text{sgn}\left(\left|\text{Re}\left(\frac{m}{n}\right)\right|\right) - i\left(1 - \chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right)\right) \text{sgn}\left(\left|\text{Im}\left(\frac{m}{n}\right)\right|\right)$$

04.07.16.0006.01

$$\text{quotient}(i\, m, i\, n) = \text{quotient}(m, n)$$

04.07.16.0007.01

$$\text{quotient}(i\, m, n) = i\, \text{quotient}(m, n) + \chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right) - 1$$

04.07.16.0008.01

$$\text{quotient}(-i\, m, n) = -i\, \text{quotient}(m, n) + i\left(\chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right) - 1\right)$$

04.07.16.0009.01

$$\text{quotient}(m, i\, n) = -i\, \text{quotient}(m, n) + i\left(\chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right) - 1\right)$$

04.07.16.0010.01

$$\text{quotient}(m, -i\, n) = \chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right) + i\, \text{quotient}(m, n) - 1$$

04.07.16.0011.01

$$\text{quotient}\left(\frac{m}{n}, 1\right) = \text{quotient}(m, n)$$

Argument involving related functions

04.07.16.0014.01

$$\text{quotient}(\lfloor m \rfloor, n) = \left\lfloor \frac{\lfloor m \rfloor}{n} \right\rfloor$$

04.07.16.0015.01

$$\text{quotient}(\lfloor m \rfloor, 1) = \lfloor m \rfloor$$

04.07.16.0016.01

$$\text{quotient}(\lfloor m \rceil, n) = \left\lfloor \frac{\lfloor m \rceil}{n} \right\rfloor$$

04.07.16.0017.01

$$\text{quotient}(\lfloor m \rceil, 1) = \lfloor m \rceil$$

04.07.16.0018.01

$$\text{quotient}(\lceil m \rceil, n) = \left\lfloor \frac{\lceil m \rceil}{n} \right\rfloor$$

04.07.16.0019.01

$$\text{quotient}(\lceil m \rceil, 1) = \lceil m \rceil$$

04.07.16.0020.01

$$\text{quotient}(\text{int}(m), n) = \left\lfloor \frac{\text{int}(m)}{n} \right\rfloor$$

04.07.16.0021.01

$$\text{quotient}(\text{int}(m), 1) = \text{int}(m)$$

04.07.16.0022.01

$$\text{quotient}(\text{frac}(m), n) = \left\lfloor \frac{\text{frac}(m)}{n} \right\rfloor$$

04.07.16.0023.01

$$\text{quotient}(m \bmod n, n) = \left\lfloor \frac{m \bmod n}{n} \right\rfloor$$

04.07.16.0024.01

$$\text{quotient}(m \bmod 1, 1) = 0$$

04.07.16.0025.01

$$\text{quotient}(\text{quotient}(m, n), n) = \left\lfloor \frac{1}{n} \left\lfloor \frac{m}{n} \right\rfloor \right\rfloor$$

04.07.16.0026.01

$$\text{quotient}(\text{quotient}(m, 1), 1) = \lfloor m \rfloor$$

Addition formulas

04.07.16.0027.01

$$\text{quotient}(m + k\, n, n) = \text{quotient}(m, n) + k \;/;\; k \in \mathbb{Z}$$

Multiple arguments

04.07.16.0012.01

$$\text{quotient}(k\, m, n) = k\, \text{quotient}(m, n) + \sum_{j=0}^{k-1} j \left(\theta\left(\frac{m}{n} \bmod 1 - \frac{j}{k} \right) \left(1 - \theta\left(\frac{m}{n} \bmod 1 - \frac{j+1}{k} \right) \right) \right) /;\; k \in \mathbb{N} \bigwedge \frac{m}{n} \in \mathbb{R}$$

Related transformations

04.07.16.0013.01

$$\text{quotient}(m, n) = \left\lfloor \text{Re}\left(\frac{m}{n}\right) \right\rfloor + i \left\lfloor \text{Im}\left(\frac{m}{n}\right) \right\rfloor$$

Identities

Functional identities

04.07.17.0001.01

$$\text{quotient}\left(\frac{m}{n}, 1\right) = \text{quotient}(m, n)$$

Complex characteristics

Real part

04.07.19.0001.01

$$\operatorname{Re}(\operatorname{quotient}(m, n)) = \left\lfloor \frac{\operatorname{Re}(m)\operatorname{Re}(n) + \operatorname{Im}(m)\operatorname{Im}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor$$

Imaginary part

04.07.19.0002.01

$$\operatorname{Im}(\operatorname{quotient}(m, n)) = \left\lfloor \frac{\operatorname{Im}(m)\operatorname{Re}(n) - \operatorname{Im}(n)\operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor$$

Absolute value

04.07.19.0003.01

$$|\operatorname{quotient}(m, n)| = \sqrt{\left\lfloor \frac{\operatorname{Re}(m)\operatorname{Re}(n) + \operatorname{Im}(m)\operatorname{Im}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor^2 + \left\lfloor \frac{\operatorname{Im}(m)\operatorname{Re}(n) - \operatorname{Im}(n)\operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor^2}$$

Argument

04.07.19.0004.01

$$\arg(\operatorname{quotient}(m, n)) = \tan^{-1}\left(\left\lfloor \frac{\operatorname{Im}(m)\operatorname{Im}(n) + \operatorname{Re}(m)\operatorname{Re}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor, \left\lfloor \frac{\operatorname{Im}(m)\operatorname{Re}(n) - \operatorname{Im}(n)\operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor\right)$$

Conjugate value

04.07.19.0005.01

$$\overline{\operatorname{quotient}(m, n)} = \left\lfloor \frac{\operatorname{Im}(m)\operatorname{Im}(n) + \operatorname{Re}(m)\operatorname{Re}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor - i \left\lfloor \frac{\operatorname{Im}(m)\operatorname{Re}(n) - \operatorname{Im}(n)\operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor$$

Signum value

04.07.19.0006.01

$$\operatorname{sgn}(\operatorname{quotient}(m, n)) = \frac{i \left\lfloor \frac{\operatorname{Im}(m)\operatorname{Re}(n) - \operatorname{Im}(n)\operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor + \left\lfloor \frac{\operatorname{Im}(m)\operatorname{Im}(n) + \operatorname{Re}(m)\operatorname{Re}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor}{\sqrt{\left\lfloor \frac{\operatorname{Im}(m)\operatorname{Re}(n) - \operatorname{Im}(n)\operatorname{Re}(m)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor^2 + \left\lfloor \frac{\operatorname{Im}(m)\operatorname{Im}(n) + \operatorname{Re}(m)\operatorname{Re}(n)}{\operatorname{Im}(n)^2 + \operatorname{Re}(n)^2} \right\rfloor^2}}$$

04.07.19.0007.01

$$\operatorname{sgn}(\operatorname{quotient}(m, n)) = \operatorname{sgn}\left(\frac{m}{n}\right) /; m \in \mathbb{R} \land n \in \mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *m*

04.07.20.0001.01

$$\frac{\partial \operatorname{quotient}(m, n)}{\partial m} = 0$$

In a distributional sense for $x \in \mathbb{R}$.

04.07.20.0002.01

$$\frac{\partial \operatorname{quotient}(x, n)}{\partial x} = \operatorname{sgn}(n) \sum_{k=-\infty}^{\infty} \delta(x - k\, n)$$

With respect to *n*

04.07.20.0003.01

$$\frac{\partial \operatorname{quotient}(m, n)}{\partial n} = 0$$

In a distributional sense for $x \in \mathbb{R}$.

04.07.20.0004.01

$$\frac{\partial \operatorname{quotient}(m, x)}{\partial x} = -\frac{\operatorname{sgn}(m)\, m}{x^2} \sum_{k=-\infty}^{\infty} \delta_{k,0}\, \delta\!\left(\frac{m}{x} - k\right)$$

Fractional integro-differentiation

With respect to *m*

04.07.20.0005.01

$$\frac{\partial^\alpha \operatorname{quotient}(m, n)}{\partial m^\alpha} = \frac{\operatorname{quotient}(m, n)\, m^{-\alpha}}{\Gamma(1-\alpha)}$$

With respect to *n*

04.07.20.0006.01

$$\frac{\partial^\alpha \operatorname{quotient}(m, n)}{\partial n^\alpha} = \frac{\operatorname{quotient}(m, n)\, n^{-\alpha}}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

Involving only one direct function with respect to *m*

04.07.21.0001.01

$$\int \operatorname{quotient}(m, n)\, dm = m \operatorname{quotient}(m, n)$$

Involving one direct function and elementary functions with respect to *m*

Involving power function

04.07.21.0002.01

$$\int m^{\alpha-1} \operatorname{quotient}(m, n)\, dm = \frac{m^\alpha \operatorname{quotient}(m, n)}{\alpha}$$

04.07.21.0003.01

$$\int \frac{\text{quotient}(m, n)}{m} \, dm = \log(m) \, \text{quotient}(m, n)$$

Involving only one direct function with respect to *n*

04.07.21.0004.01

$$\int \text{quotient}(m, n) \, dn = n \, \text{quotient}(m, n)$$

Involving one direct function and elementary functions with respect to *n*

Involving power function

04.07.21.0005.01

$$\int n^{\alpha-1} \, \text{quotient}(m, n) \, dn = \frac{n^{\alpha} \, \text{quotient}(m, n)}{\alpha}$$

04.07.21.0006.01

$$\int \frac{\text{quotient}(m, n)}{n} \, dn = \log(n) \, \text{quotient}(m, n)$$

Definite integration

For the direct function with respect to *m*

04.07.21.0007.01

$$\int_0^a \text{quotient}(t, n) \, dt = \frac{1}{2} \, \text{quotient}(a, n) \, (2 \, a - n - n \, \text{quotient}(a, n))$$

04.07.21.0008.01

$$\int_0^a t^{\alpha-1} \, \text{quotient}(t, n) \, dt = \frac{1}{\alpha} \left(\text{quotient}(a, n) \, a^{\alpha} + n^{\alpha} \left(\zeta(-\alpha, \text{quotient}(a, n) + 1) - \zeta(-\alpha)\right)\right)$$

04.07.21.0009.01

$$\int_a^{\infty} t^{\alpha-1} \, \text{quotient}(t, n) \, dt = -\frac{\text{quotient}(a, n) \, a^{\alpha} + n^{\alpha} \, \zeta(-\alpha, \text{quotient}(a, n) + 1)}{\alpha} \, /; \, \text{Re}(\alpha) < -1$$

04.07.21.0010.01

$$\int_0^{\infty} t^{\alpha-1} \, \text{quotient}(t, n) \, dt = -\frac{n^{\alpha}}{\alpha} \, \zeta(-\alpha) \, /; \, \text{Re}(\alpha) < -1$$

04.07.21.0011.01

$$\int_{-a}^a \text{Quotient}(t, n) \, dt = -a$$

For the direct function with respect to *n*

In the following formulas $a \in \mathbb{R}$.

04.07.21.0012.01

$$\int_0^a t^{\alpha-1} \, \text{quotient}(m, t) \, dt = \frac{\text{quotient}(m, a) \, a^{\alpha} + m^{\alpha} \, \zeta(\alpha, \text{quotient}(m, a) + 1)}{\alpha} \, /; \, \text{Re}(\alpha) > 1$$

04.07.21.0013.01

$$\int_a^\infty t^{\alpha-1}\,\text{quotient}(m,t)\,dt = \frac{1}{\alpha}\left(m^\alpha\left(\zeta(\alpha)-\zeta(\alpha,\text{quotient}(m,a)+1)\right)-a^\alpha\,\text{quotient}(m,a)\right)$$

04.07.21.0014.01

$$\int_0^\infty t^{\alpha-1}\,\text{quotient}(m,t)\,dt = \frac{m^\alpha\,\zeta(\alpha)}{\alpha}\;/;\;\text{Re}(\alpha)>1$$

Integral transforms

Fourier exp transforms

04.07.22.0001.01

$$\mathcal{F}_t[\text{quotient}(t,n)](z) = -\sqrt{\frac{\pi}{2}}\,\delta(z)+\frac{i}{\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{1}{k}\left(\delta\left(\frac{2k\pi}{n}-z\right)-\delta\left(\frac{2\pi k}{n}+z\right)\right)-\frac{i\sqrt{2\pi}}{n}\,\delta'(z)$$

Fourier cos transforms

04.07.22.0002.01

$$\mathcal{F}c_t[\text{quotient}(t,n)](z) = -\frac{1}{\sqrt{2\pi}\,z}\cot\left(\frac{n z}{2}\right)-\sqrt{\frac{\pi}{2}}\,\delta(z)$$

Fourier sin transforms

04.07.22.0003.01

$$\mathcal{F}s_t[\text{quotient}(t,n)](z) = \frac{1}{\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{1}{k}\left(\delta\left(\frac{2k\pi}{n}-z\right)-\delta\left(\frac{2\pi k}{n}+z\right)\right)-\frac{\sqrt{2\pi}\,\delta'(z)}{n}-\frac{1}{\sqrt{2\pi}\,z}$$

Laplace transforms

04.07.22.0004.01

$$\mathcal{L}_t[\text{quotient}(t,n)](z) = \frac{1}{(e^{n z}-1)\,z}\;/;\;\text{Re}(n z)>0$$

Mellin transforms

04.07.22.0005.01

$$\mathcal{M}_t[\text{quotient}(t,n)](z) = -\frac{n^z\,\zeta(-z)}{z}\;/;\;\text{Re}(z)<-1$$

04.07.22.0006.01

$$\mathcal{M}_t[\text{quotient}(m,t)](z) = \frac{m^z\,\zeta(z)}{z}\;/;\;\text{Re}(z)>1$$

Representations through equivalent functions

With related functions

With Floor

04.07.27.0001.01

$$\text{quotient}(m, n) = \left\lfloor \frac{m}{n} \right\rfloor$$

With Round

For real arguments

04.07.27.0008.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} - \frac{1}{2} \right\rceil /; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m+n}{2n} \notin \mathbb{Z}$$

04.07.27.0009.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} - \frac{1}{2} \right\rceil + 1 /; \frac{m+n}{2n} \in \mathbb{Z}$$

04.07.27.0010.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} - \frac{1}{2} \right\rceil + \chi_{\mathbb{Z}}\left(\frac{m+n}{2n}\right) /; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.07.27.0003.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} - \frac{1+i}{2} \right\rceil + \chi_{\mathbb{Z}}\left(\frac{1}{2}\left(\text{Re}\left(\frac{m}{n}\right) + 1\right)\right) + i\, \chi_{\mathbb{Z}}\left(\frac{1}{2}\left(\text{Im}\left(\frac{m}{n}\right) + 1\right)\right)$$

With Ceiling

For real arguments

04.07.27.0011.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil - 1 /; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

04.07.27.0012.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil /; \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0013.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil + \theta\left(\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) - 1\right) - 1 /; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.07.27.0014.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil /; \text{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge \text{Im}\left(\frac{m}{n}\right) \in \mathbb{Z}$$

04.07.27.0015.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil - 1 /; \text{Re}\left(\frac{m}{n}\right) \notin \mathbb{Z} \bigwedge \text{Im}\left(\frac{m}{n}\right) \in \mathbb{Z}$$

04.07.27.0016.01

$$\text{quotient}(m, n) = \left\lceil \frac{m}{n} \right\rceil - i /; \text{Re}\left(\frac{m}{n}\right) \in \mathbb{Z} \bigwedge \text{Im}\left(\frac{m}{n}\right) \notin \mathbb{Z}$$

04.07.27.0017.01

$$\text{quotient}(m,\,n) = \left\lceil \frac{m}{n} \right\rceil - 1 - i \,/;\, \text{Re}\left(\frac{m}{n}\right) \notin \mathbb{Z} \bigwedge \text{Im}\left(\frac{m}{n}\right) \notin \mathbb{Z}$$

04.07.27.0018.01

$$\text{quotient}(m,\,n) = \left\lceil \frac{m}{n} \right\rceil + \theta\left(\chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right) - 1\right) - i\,\theta\left(-\chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right)\right) - 1$$

04.07.27.0002.01

$$\text{quotient}(m,\,n) = -\left\lceil -\frac{m}{n} \right\rceil$$

With IntegerPart

For real arguments

04.07.27.0019.01

$$\text{quotient}(m,\,n) = \text{int}\left(\frac{m}{n}\right) /;\, \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} > 0 \bigvee \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0020.01

$$\text{quotient}(m,\,n) = \text{int}\left(\frac{m}{n}\right) - 1 \,/;\, \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} < 0 \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

04.07.27.0021.01

$$\text{quotient}(m,\,n) = \text{int}\left(\frac{m}{n}\right) - 1 + \text{sgn}\left(\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) + \theta\left(\frac{m}{n}\right)\right) /;\, \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.07.27.0022.01

$$\text{quotient}(m,\,n) = \text{int}\left(\frac{m}{n}\right) /;\, \text{Re}\left(\frac{m}{n}\right) \geq 0 \bigwedge \text{Im}\left(\frac{m}{n}\right) \geq 0 \bigvee \frac{m}{n} \in \mathbb{Z} \bigvee i\,\frac{m}{n} \in \mathbb{Z}$$

04.07.27.0023.01

$$\text{quotient}(m,\,n) = \text{int}\left(\frac{m}{n}\right) - 1 \,/;\, \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} < 0 \bigwedge \frac{m}{n} \notin \mathbb{Z} \bigvee \text{Re}\left(\frac{m}{n}\right) < 0 \bigwedge \text{Im}\left(\frac{m}{n}\right) > 0$$

04.07.27.0024.01

$$\text{quotient}(m,\,n) = \text{int}\left(\frac{m}{n}\right) - i \,/;\, i\,\frac{m}{n} \in \mathbb{R} \bigwedge i\,\frac{m}{n} > 0 \bigwedge i\,\frac{m}{n} \notin \mathbb{Z} \bigvee \text{Re}\left(\frac{m}{n}\right) > 0 \bigwedge \text{Im}\left(\frac{m}{n}\right) < 0$$

04.07.27.0025.01

$$\text{quotient}(m,\,n) = \text{int}\left(\frac{m}{n}\right) - 1 - i \,/;\, \text{Re}\left(\frac{m}{n}\right) < 0 \bigwedge \text{Im}\left(\frac{m}{n}\right) < 0$$

04.07.27.0004.01

$$\text{quotient}(m,\,n) = \text{int}\left(\frac{m}{n}\right) - 1 - i + i\,\text{sgn}\left(\chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right) + \theta\left(\text{Im}\left(\frac{m}{n}\right)\right)\right) + \text{sgn}\left(\chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right) + \theta\left(\text{Re}\left(\frac{m}{n}\right)\right)\right)$$

With FractionalPart

For real arguments

04.07.27.0026.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) /; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} > 0 \bigvee \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0027.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 /; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} < 0 \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

04.07.27.0028.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 + \text{sgn}\left(\chi_{\mathbb{Z}}\left(\frac{m}{n}\right) + \theta\left(\frac{m}{n}\right)\right) /; \frac{m}{n} \in \mathbb{R}$$

For complex arguments

04.07.27.0029.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) /; \text{Re}\left(\frac{m}{n}\right) \geq 0 \bigwedge \text{Im}\left(\frac{m}{n}\right) \geq 0 \bigvee \frac{m}{n} \in \mathbb{Z} \bigvee i \frac{m}{n} \in \mathbb{Z}$$

04.07.27.0030.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 /; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} < 0 \bigwedge \frac{m}{n} \notin \mathbb{Z} \bigvee \text{Re}\left(\frac{m}{n}\right) < 0 \bigwedge \text{Im}\left(\frac{m}{n}\right) > 0$$

04.07.27.0031.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - i /; i \frac{m}{n} \in \mathbb{R} \bigwedge i \frac{m}{n} > 0 \bigwedge i \frac{m}{n} \notin \mathbb{Z} \bigvee \text{Re}\left(\frac{m}{n}\right) > 0 \bigwedge \text{Im}\left(\frac{m}{n}\right) < 0$$

04.07.27.0032.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 - i /; \text{Re}\left(\frac{m}{n}\right) < 0 \bigwedge \text{Im}\left(\frac{m}{n}\right) < 0$$

04.07.27.0005.01

$$\text{quotient}(m, n) = \frac{m}{n} - \text{frac}\left(\frac{m}{n}\right) - 1 - i + i\,\text{sgn}\left(\chi_{\mathbb{Z}}\left(\text{Im}\left(\frac{m}{n}\right)\right) + \theta\left(\text{Im}\left(\frac{m}{n}\right)\right)\right) + \text{sgn}\left(\chi_{\mathbb{Z}}\left(\text{Re}\left(\frac{m}{n}\right)\right) + \theta\left(\text{Re}\left(\frac{m}{n}\right)\right)\right)$$

With Mod

04.07.27.0006.01

$$\text{quotient}(m, n) = \frac{m - m \bmod n}{n}$$

With elementary functions

04.07.27.0007.01

$$\text{quotient}(m, n) = \frac{m}{n} + \frac{1}{\pi} \tan^{-1}\left(\cot\left(\frac{\pi m}{n}\right)\right) - \frac{1}{2} /; \frac{m}{n} \in \mathbb{R} \bigwedge \frac{m}{n} \notin \mathbb{Z}$$

Zeros

04.07.30.0001.01

$$\text{quotient}(m, n) = 0 /; 0 \leq \text{Re}\left(\frac{m}{n}\right) < 1 \bigwedge 0 \leq \text{Im}\left(\frac{m}{n}\right) < 1$$

History

– J. Nemorarius (1237)
– the word "quotient" appears for the first time around 1250 in the writings of Meister Gernadus.

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