RamanujanTauTheta

Notations

Traditional name

Ramanujan tau theta function

Traditional notation

$\tau\theta(z)$

Mathematica StandardForm notation

```
RamanujanTauTheta[z]
```

Primary definition

10.12.02.0001.01

$$\tau\theta(z) = -\log(2\,\pi)\,z - \frac{i}{2}\,(\log\Gamma(6 + i\,z) - \log\Gamma(6 - i\,z))$$

Specific values

Specialized values

10.12.03.0001.01

$$\tau\theta(x) = \operatorname{Im}(\log\Gamma(i\,x + 6)) - \log(2\,\pi)\,x \;/;\; x \in \mathbb{R}$$

10.12.03.0002.01

$$\tau\theta(-5\,i + i\,n) = \frac{1}{2}\,i\left(2\,(5 - n)\log(2\,\pi) + \log\left(\frac{n!}{(10 - n)!}\right)\right) \;/;\; n \in \mathbb{Z} \land 0 \leq n \leq 10$$

10.12.03.0003.01

$$\tau\theta(6\,i + i\,n) = -i\,\infty \;/;\; n \in \mathbb{N}$$

10.12.03.0004.01

$$\tau\theta(-6\,i - i\,n) = i\,\infty \;/;\; n \in \mathbb{N}$$

10.12.03.0005.01

$$\tau\theta\left(6\,i - \frac{i\,n}{2}\right) = \frac{1}{2}\,i\left((n - 12)\log(2\,\pi) + \log\left(\frac{\left(11 - \frac{n}{2}\right)!}{\left(\frac{n}{2} - 1\right)!}\right)\right) \;/;\; n \in \mathbb{Z} \land 0 < n \leq 23$$

10.12.03.0006.01

$$\tau\theta\left(-\frac{11\,i}{2} - i\,n\right) = \frac{1}{2}\,i\left(\log\left(16\,\pi^2\right)n - i\,\pi\,n + 11\log(4\,\pi) - \sum_{k=1}^{n}\log(2\,k - 1) - \sum_{k=1}^{n+11}\log(2\,k - 1)\right) \;/;\; n \in \mathbb{N}$$

10.12.03.0007.01

$$\tau\theta\left(\frac{11\,i}{2}+i\,n\right)=\frac{1}{2}\,i\left(-\log\left(16\,\pi^2\right)n+i\,\pi\,n-11\log(4\,\pi)+\sum_{k=1}^{n}\log(2\,k-1)+\sum_{k=1}^{n+11}\log(2\,k-1)\right)/;\,n\in\mathbb{N}$$

10.12.03.0008.01

$$\tau\theta\left(-\frac{17\,i}{3}-i\,n\right)=\frac{1}{6}\,i\left(6\log(6\,\pi)\,n-3\,i\,\pi\,n+31\log(2\,\pi)+\frac{69\log(3)}{2}+6\log\left(\Gamma\left(\frac{1}{3}\right)\right)-3\sum_{k=1}^{n}\log(3\,k-1)-3\sum_{k=1}^{n+11}\log(3\,k-1)\right)/;\,n\in\mathbb{N}$$

10.12.03.0009.01

$$\tau\theta\left(-\frac{17\,i}{3}+i\,n\right)=\frac{1}{6}\,i\left(\frac{69\log(3)}{2}+31\log(2\,\pi)-6\,n\log(6\,\pi)+6\log\left(\Gamma\left(\frac{1}{3}\right)\right)+3\sum_{k=1}^{n}\log(3\,k-2)-3\sum_{k=1}^{11-n}\log(3\,k-1)\right)/;\,n\in\mathbb{N}\land n\leq 11$$

10.12.03.0010.01

$$\tau\theta\left(-\frac{16\,i}{3}-i\,n\right)=\frac{1}{6}\,i\left(6\log(6\,\pi)\,n-3\,i\,\pi\,n+35\log(2\,\pi)+\frac{63\log(3)}{2}-6\log\left(\Gamma\left(\frac{1}{3}\right)\right)-3\sum_{k=1}^{n}\log(3\,k-2)-3\sum_{k=1}^{n+11}\log(3\,k-2)\right)/;\,n\in\mathbb{N}$$

10.12.03.0011.01

$$\tau\theta\left(-\frac{16\,i}{3}+i\,n\right)=\frac{1}{6}\,i\left(\frac{63\log(3)}{2}+35\log(2\,\pi)-6\,n\log(6\,\pi)-6\log\left(\Gamma\left(\frac{1}{3}\right)\right)-3\sum_{k=1}^{11-n}\log(3\,k-2)+3\sum_{k=1}^{n}\log(3\,k-1)\right)/;\,n\in\mathbb{N}\land n\leq 11$$

10.12.03.0012.01

$$\tau\theta\left(\frac{16\,i}{3}-i\,n\right)=\frac{1}{6}\,i\left(-\frac{63\log(3)}{2}-35\log(2\,\pi)+6\,n\log(6\,\pi)+6\log\left(\Gamma\left(\frac{1}{3}\right)\right)+3\sum_{k=1}^{11-n}\log(3\,k-2)-3\sum_{k=1}^{n}\log(3\,k-1)\right)/;\,n\in\mathbb{N}\land n\leq 11$$

10.12.03.0013.01

$$\tau\theta\left(\frac{16\,i}{3}+i\,n\right)=\frac{1}{6}\,i\left(-6\log(6\,\pi)\,n+3\,i\,\pi\,n-35\log(2\,\pi)-\frac{63\log(3)}{2}+6\log\left(\Gamma\left(\frac{1}{3}\right)\right)+3\sum_{k=1}^{n}\log(3\,k-2)+3\sum_{k=1}^{n+11}\log(3\,k-2)\right)/;\,n\in\mathbb{N}$$

10.12.03.0014.01

$$\tau\theta\left(\frac{17\,i}{3}-i\,n\right)=\frac{1}{6}\,i\left(-\frac{69\log(3)}{2}-31\log(2\,\pi)+6\,n\log(6\,\pi)-6\log\left(\Gamma\left(\frac{1}{3}\right)\right)-3\sum_{k=1}^{n}\log(3\,k-2)+3\sum_{k=1}^{11-n}\log(3\,k-1)\right)/;\,n\in\mathbb{N}\land n\leq 11$$

10.12.03.0015.01

$$\tau\theta\left(\frac{17\,i}{3}+i\,n\right)=\frac{1}{6}\,i\left(-6\log(6\,\pi)\,n+3\,i\,\pi\,n-31\log(2\,\pi)-\frac{69\log(3)}{2}-6\log\left(\Gamma\left(\frac{1}{3}\right)\right)+3\sum_{k=1}^{n}\log(3\,k-1)+3\sum_{k=1}^{n+11}\log(3\,k-1)\right)/;\,n\in\mathbb{N}$$

10.12.03.0016.01

$$\tau\theta\left(-\frac{23\,i}{4}-i\,n\right)=\frac{1}{4}\,i\left(4\log(8\,\pi)\,n-2\,i\,\pi\,n+66\log(2)+21\log(\pi)+4\log\left(\Gamma\left(\frac{1}{4}\right)\right)-2\sum_{k=1}^{n}\log(4\,k-1)-2\sum_{k=1}^{n+11}\log(4\,k-1)\right)\,/;\,n\in\mathbb{N}$$

10.12.03.0017.01

$$\tau\theta\left(-\frac{23\,i}{4}+i\,n\right)=\frac{1}{4}\,i\left(66\log(2)+21\log(\pi)-4\,n\log(8\,\pi)+4\log\left(\Gamma\left(\frac{1}{4}\right)\right)+2\sum_{k=1}^{n}\log(4\,k-3)-2\sum_{k=1}^{11-n}\log(4\,k-1)\right)\,/;\,n\in\mathbb{N}\land n\leq 11$$

10.12.03.0018.01

$$\tau\theta\left(-\frac{21\,i}{4}-i\,n\right)=\frac{i}{4}\left(4\log(8\,\pi)\,n-2\,i\,\pi\,n+66\log(2)+23\log(\pi)-4\log\left(\Gamma\left(\frac{1}{4}\right)\right)-2\sum_{k=1}^{n}\log(4\,k-3)-2\sum_{k=1}^{n+11}\log(4\,k-3)\right)\,/;\,n\in\mathbb{N}$$

10.12.03.0019.01

$$\tau\theta\left(-\frac{21\,i}{4}+i\,n\right)=\frac{1}{4}\,i\left(66\log(2)+23\log(\pi)-4\,n\log(8\,\pi)-4\log\left(\Gamma\left(\frac{1}{4}\right)\right)-2\sum_{k=1}^{11-n}\log(4\,k-3)+2\sum_{k=1}^{n}\log(4\,k-1)\right)\,/;\,n\in\mathbb{N}\land n\leq 11$$

10.12.03.0020.01

$$\tau\theta\left(\frac{21\,i}{4}-i\,n\right)=\frac{1}{4}\,i\left(-66\log(2)-23\log(\pi)+4\,n\log(8\,\pi)+4\log\left(\Gamma\left(\frac{1}{4}\right)\right)+2\sum_{k=1}^{11-n}\log(4\,k-3)-2\sum_{k=1}^{n}\log(4\,k-1)\right)\,/;\,n\in\mathbb{N}\land n\leq 11$$

10.12.03.0021.01

$$\tau\theta\left(\frac{21\,i}{4}+i\,n\right)=\frac{i}{4}\left(-4\log(8\,\pi)\,n+2\,i\,\pi\,n-66\log(2)-23\log(\pi)+4\log\left(\Gamma\left(\frac{1}{4}\right)\right)+2\sum_{k=1}^{n}\log(4\,k-3)+2\sum_{k=1}^{n+11}\log(4\,k-3)\right)\,/;\,n\in\mathbb{N}$$

10.12.03.0022.01

$$\tau\theta\left(\frac{23\,i}{4}-i\,n\right)=\frac{1}{4}\,i\left(-66\log(2)-21\log(\pi)+4\,n\log(8\,\pi)-4\log\left(\Gamma\left(\frac{1}{4}\right)\right)-2\sum_{k=1}^{n}\log(4\,k-3)+2\sum_{k=1}^{11-n}\log(4\,k-1)\right)\,/;\,n\in\mathbb{N}\land n\leq 11$$

10.12.03.0023.01

$$\tau\theta\left(\frac{23\,i}{4}+i\,n\right)=\frac{1}{4}\,i\left(-4\log(8\,\pi)\,n+2\,i\,\pi\,n-66\log(2)-21\log(\pi)-4\log\left(\Gamma\left(\frac{1}{4}\right)\right)+2\sum_{k=1}^{n}\log(4\,k-1)+2\sum_{k=1}^{n+11}\log(4\,k-1)\right)\,/;\,n\in\mathbb{N}$$

10.12.03.0024.01

$$\tau\theta\left(i\left(-5-\frac{p}{q}+n\right)\right)=\frac{i}{2\,q}\left(2\,(p-(n-5)\,q)\log(2\,\pi)-(2\,n-11)\,q\log(q)+q\log\Gamma\left(1-\frac{p}{q}\right)-q\log\Gamma\left(\frac{p}{q}\right)+q\sum_{k=1}^{n}\log(q\,k-p)-q\sum_{k=1}^{11-n}\log(q\,k-q+p)\right)\,/;\,n\in\mathbb{Z}\land 0\leq n\leq 11\land p\in\mathbb{N}^{+}\land q\in\mathbb{N}^{+}\land p<q$$

10.12.03.0025.01

$$\tau\theta\left(i\left(-5-\frac{p}{q}-n\right)\right)=\frac{i}{2q}\left(q(2n+11)\log(q)-i n\pi q+2(p+(n+5)q)\log(2\pi)+q\log\Gamma\left(1-\frac{p}{q}\right)-q\log\Gamma\left(\frac{p}{q}\right)-q\sum_{k=1}^{n}\log(p+kq-q)-q\sum_{k=1}^{n+11}\log(p+kq-q)\right)\text{/; } n\in\mathbb{N}\land p\in\mathbb{N}^+\land q\in\mathbb{N}^+\land p<q$$

10.12.03.0026.01

$$\tau\theta\left(i\left(5+\frac{p}{q}+n\right)\right)=\frac{i}{2q}\left(i n\pi q-q(2n+11)\log(q)-2(p+(n+5)q)\log(2\pi)-q\log\Gamma\left(1-\frac{p}{q}\right)+q\log\Gamma\left(\frac{p}{q}\right)+q\sum_{k=1}^{n}\log(p+kq-q)+q\sum_{k=1}^{n+11}\log(p+kq-q)\right)\text{/; } n\in\mathbb{N}\land p\in\mathbb{N}^+\land q\in\mathbb{N}^+\land p<q$$

Values at fixed points

10.12.03.0027.01

$$\tau\theta(-10\, i)=i\,\infty$$

10.12.03.0028.01

$$\tau\theta\left(-\frac{19\, i}{2}\right)=2\pi-\frac{1}{2}\, i\log\left(\frac{649\,979\,752\,131\,084\,375}{274\,877\,906\,944\,\pi^{19}}\right)$$

10.12.03.0029.01

$$\tau\theta(-9\, i)=i\,\infty$$

10.12.03.0030.01

$$\tau\theta\left(-\frac{17\, i}{2}\right)=\frac{1}{2}\left(3\pi-i\log\left(\frac{3\,201\,870\,700\,153\,125}{17\,179\,869\,184\,\pi^{17}}\right)\right)$$

10.12.03.0031.01

$$\tau\theta(-8\, i)=i\,\infty$$

10.12.03.0032.01

$$\tau\theta\left(-\frac{15\, i}{2}\right)=\pi-\frac{1}{2}\, i\log\left(\frac{23\,717\,560\,741\,875}{1\,073\,741\,824\,\pi^{15}}\right)$$

10.12.03.0033.01

$$\tau\theta(-7\, i)=i\,\infty$$

10.12.03.0034.01

$$\tau\theta\left(-\frac{13\, i}{2}\right)=\frac{1}{2}\left(\pi-i\log\left(\frac{316\,234\,143\,225}{67\,108\,864\,\pi^{13}}\right)\right)$$

10.12.03.0035.01

$$\tau\theta(-6\, i)=i\,\infty$$

10.12.03.0036.01

$$\tau\theta\left(-\frac{11\, i}{2}\right)=-\frac{1}{2}\, i\log\left(\frac{13\,749\,310\,575}{2^{22}\,\pi^{11}}\right)$$

10.12.03.0037.01

$$\tau\theta(-5\, i)=-\frac{1}{2}\, i\log\left(\frac{14\,175}{4\,\pi^{10}}\right)$$

10.12.03.0038.01

$$\tau\theta\left(-\frac{9\,i}{2}\right) = -\frac{1}{2}\,i\log\left(\frac{654\,729\,075}{2^{18}\,\pi^9}\right)$$

10.12.03.0039.01

$$\tau\theta(-4\,i) = -\frac{1}{2}\,i\log\left(\frac{2835}{2\,\pi^8}\right)$$

10.12.03.0040.01

$$\tau\theta\left(-\frac{7\,i}{2}\right) = -\frac{1}{2}\,i\log\left(\frac{11\,486\,475}{2^{14}\,\pi^7}\right)$$

10.12.03.0041.01

$$\tau\theta(-3\,i) = -\frac{1}{2}\,i\log\left(\frac{315}{\pi^6}\right)$$

10.12.03.0042.01

$$\tau\theta\left(-\frac{5\,i}{2}\right) = -\frac{1}{2}\,i\log\left(\frac{135\,135}{2^{10}\,\pi^5}\right)$$

10.12.03.0043.01

$$\tau\theta(-2\,i) = -\frac{1}{2}\,i\log\left(\frac{105}{2\,\pi^4}\right)$$

10.12.03.0044.01

$$\tau\theta\left(-\frac{3\,\pi}{2}\right) = -\frac{1}{2}\,i\log\left(\frac{1287}{64\,\pi^3}\right)$$

10.12.03.0045.01

$$\tau\theta(-i) = -\frac{1}{2}\,i\log\left(\frac{15}{2\,\pi^2}\right)$$

10.12.03.0046.01

$$\tau\theta\left(-\frac{i}{2}\right) = -\frac{1}{2}\,i\log\left(\frac{11}{4\,\pi}\right)$$

10.12.03.0047.01

$$\tau\theta(0) = 0$$

10.12.03.0048.01

$$\tau\theta\left(\frac{i}{2}\right) = \frac{1}{2}\,i\log\left(\frac{11}{4\,\pi}\right)$$

10.12.03.0049.01

$$\tau\theta(i) = \frac{1}{2}\,i\log\left(\frac{15}{2\,\pi^2}\right)$$

10.12.03.0050.01

$$\tau\theta\left(\frac{3\,i}{2}\right) = \frac{1}{2}\,i\log\left(\frac{1287}{64\,\pi^3}\right)$$

10.12.03.0051.01

$$\tau\theta(2\,i) = \frac{1}{2}\,i\log\left(\frac{105}{2\,\pi^4}\right)$$

10.12.03.0052.01

$$\tau\theta\left(\frac{5\,i}{2}\right) = \frac{1}{2}\,i\log\left(\frac{135\,135}{2^{10}\,\pi^5}\right)$$

10.12.03.0053.01

$$\tau\theta(3\,i) = \frac{1}{2}\,i\log\left(\frac{315}{\pi^6}\right)$$

10.12.03.0054.01

$$\tau\theta\left(\frac{7\,i}{2}\right) = \frac{1}{2}\,i\log\left(\frac{11\,486\,475}{2^{14}\,\pi^7}\right)$$

10.12.03.0055.01

$$\tau\theta(4\,i) = \frac{1}{2}\,i\log\left(\frac{2835}{2\,\pi^8}\right)$$

10.12.03.0056.01

$$\tau\theta\left(\frac{9\,i}{2}\right) = \frac{1}{2}\,i\log\left(\frac{654\,729\,075}{2^{18}\,\pi^9}\right)$$

10.12.03.0057.01

$$\tau\theta(5\,i) = \frac{1}{2}\,i\log\left(\frac{14\,175}{4\,\pi^{10}}\right)$$

10.12.03.0058.01

$$\tau\theta\left(\frac{11\,i}{2}\right) = \frac{1}{2}\,i\log\left(\frac{13\,749\,310\,575}{2^{22}\,\pi^{11}}\right)$$

10.12.03.0059.01

$$\tau\theta(6\,i) = -i\,\infty$$

10.12.03.0060.01

$$\tau\theta\left(\frac{13\,i}{2}\right) = \frac{1}{2}\left(-\pi + i\log\left(\frac{316\,234\,143\,225}{67\,108\,864\,\pi^{13}}\right)\right)$$

10.12.03.0061.01

$$\tau\theta(7\,i) = -i\,\infty$$

10.12.03.0062.01

$$\tau\theta\left(\frac{15\,i}{2}\right) = -\pi + \frac{1}{2}\,i\log\left(\frac{23\,717\,560\,741\,875}{1\,073\,741\,824\,\pi^{15}}\right)$$

10.12.03.0063.01

$$\tau\theta(8\,i) = -i\,\infty$$

10.12.03.0064.01

$$\tau\theta\left(\frac{17\,i}{2}\right) = \frac{1}{2}\left(-3\,\pi + i\log\left(\frac{3\,201\,870\,700\,153\,125}{17\,179\,869\,184\,\pi^{17}}\right)\right)$$

10.12.03.0065.01

$$\tau\theta(9\,i) = -i\,\infty$$

10.12.03.0066.01

$$\tau\theta\left(\frac{19\,i}{2}\right) = -2\,\pi + \frac{1}{2}\,i\log\left(\frac{649\,979\,752\,131\,084\,375}{274\,877\,906\,944\,\pi^{19}}\right)$$

10.12.03.0067.01

$\tau\theta(10\,i) = -i\,\infty$

Values at infinities

10.12.03.0068.01

$\tau\theta(\infty) = \infty$

10.12.03.0069.01

$\tau\theta(-\infty) = -\infty$

10.12.03.0070.01

$\tau\theta(i\,\infty) = \tilde{\infty}$

10.12.03.0071.01

$\tau\theta(-i\,\infty) = \tilde{\infty}$

10.12.03.0072.01

$\tau\theta(\tilde{\infty}) = \tilde{\infty}$

General characteristics

Domain and analyticity

$\tau\theta(z)$ is an analytical function of z which is defined over the whole complex z-plane.

10.12.04.0001.01

$z \longrightarrow \tau\theta(z) :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

$\tau\theta(z)$ is an odd function.

10.12.04.0002.01

$\tau\theta(-z) = -\tau\theta(z)$

Mirror symmetry

10.12.04.0003.01

$\tau\theta(\bar{z}) == \overline{\tau\theta(z)} \;/;\; i\,z \notin (-\infty, -6) \bigwedge i\,z \notin (6, \infty)$

Periodicity

No periodicity

Poles and essential singularities

The function $\tau\theta(z)$ does not have poles and essential singularities.

10.12.04.0004.01

$\mathcal{S}ing_z(\tau\theta(z)) == \{\}$

Branch points

The function $\tau\theta(z)$ has infinitely many branch points: $z == \pm i\,(6+k)$ /; $k \in \mathbb{N}$ and $z == \tilde{\infty}$. All these are logarithmic branch points.

10.12.04.0005.01

$$\mathcal{BP}_z(\tau\theta(z)) == \{\{6\,i + k\,i\ /;\ k \in \mathbb{N}\},\ \{-6\,i - k\,i\ /;\ k \in \mathbb{N}\},\ \tilde{\infty}\}$$

10.12.04.0006.01

$$\mathcal{R}_z(\tau\theta(z),\ 6\,i + k\,i) == \log\ /;\ k \in \mathbb{N}$$

10.12.04.0007.01

$$\mathcal{R}_z(\tau\theta(z),\ -6\,i - k\,i) == \log\ /;\ k \in \mathbb{N}$$

10.12.04.0008.01

$$\mathcal{R}_z(\tau\theta(z),\ \tilde{\infty}) == \log$$

Branch cuts

The function $\tau\theta(z)$ is a single-valued function on the z-plane cut along the intervals $\{-i\,\infty,\ -6\,i\}$ and $\{6\,i,\ i\,\infty\}$. At $i\,z \in \{-i\,\infty,\ -6\,i\} \bigvee i\,z \in \{6\,i,\ i\,\infty\}$ potentially multiple branch cuts are situated over each other (at $i\,z$ there are $\lfloor 6 + i\,z \rfloor$, respectively $\lfloor 6 - i\,z \rfloor$ branch cuts overlapping).

The function $\tau\theta(z)$ is continuous from the left on the interval $\{-i\,\infty,\ -6\,i\}$ and from the right on the interval $\{6\,i,\ i\,\infty\}$.

10.12.04.0009.01

$$\mathcal{BC}_z(\tau\theta(z)) == \{\{\{-i\,\infty,\ -6\,i\},\ 1\},\ \{\{6\,i,\ i\,\infty\},\ -1\}\}$$

10.12.04.0010.01

$$\lim_{\epsilon \to +0} \tau\theta(x - \epsilon) == \tau\theta(x)\ /;\ i\,x > 6$$

10.12.04.0011.01

$$\lim_{\epsilon \to +0} \tau\theta(x + \epsilon) == \tau\theta(x) + \pi\,\lfloor 6 - i\,x \rfloor\ /;\ i\,x > 6$$

10.12.04.0012.01

$$\lim_{\epsilon \to +0} \tau\theta(x - \epsilon) == \tau\theta(x)\ /;\ i\,x < -6$$

10.12.04.0013.01

$$\lim_{\epsilon \to +0} \tau\theta(x - \epsilon) == \tau\theta(x) - \pi\,\lfloor 6 + i\,x \rfloor\ /;\ i\,x < -6$$

Series representations

Generalized power series

Expansions on branch cuts

For the function itself

In the lower half-plane

10.12.06.0001.01

$$\tau\theta(z) \propto \tau\theta(z_0) - \pi\left(6 + \lfloor -i\, z_0 \rfloor\right) \left\lfloor \frac{\arg(-i\,(z - z_0))}{2\pi} \right\rfloor - \frac{1}{2}\left(2\log(2\pi) - \psi(i\, z_0 + 6) - \psi(6 - i\, z_0)\right)(z - z_0) + \frac{1}{4}\, i\left(\psi^{(1)}(i\, z_0 + 6) - \psi^{(1)}(6 - i\, z_0)\right)(z - z_0)^2 + \ldots \;/;\; (z \to z_0) \bigwedge i\, z_0 \in \mathbb{R} \bigwedge i\, z_0 > 6 \bigwedge i\, z_0 \notin \mathbb{Z}$$

10.12.06.0002.01

$$\tau\theta(z) \propto \tau\theta(z_0) - \pi\left(6 + \lfloor -i\, z_0 \rfloor\right) \left\lfloor \frac{\arg(-i\,(z - z_0))}{2\pi} \right\rfloor - \frac{1}{2}\left(2\log(2\pi) - \psi(i\, z_0 + 6) - \psi(6 - i\, z_0)\right)(z - z_0) + \frac{1}{4}\, i\left(\psi^{(1)}(i\, z_0 + 6) - \psi^{(1)}(6 - i\, z_0)\right)(z - z_0)^2 + O\left((z - z_0)^3\right) \;/;\; i\, z_0 \in \mathbb{R} \bigwedge i\, z_0 > 6 \bigwedge i\, z_0 \notin \mathbb{Z}$$

10.12.06.0003.01

$$\tau\theta(z) = \tau\theta(z_0) - \pi \left\lfloor \frac{\arg(-i\,(z - z_0))}{2\pi} \right\rfloor \left(6 + \lfloor -i\, z_0 \rfloor\right) - \frac{1}{2}\, i \sum_{k=1}^{\infty} \frac{(-i)^k}{k!} \left(2\,\delta_{k-1} \log(2\pi) + (-1)^k\, \psi^{(k-1)}(6 + i\, z_0) - \psi^{(k-1)}(6 - i\, z_0)\right)(z - z_0)^k \;/;\; i\, z_0 \in \mathbb{R} \bigwedge i\, z_0 > 6 \bigwedge i\, z_0 \notin \mathbb{Z}$$

10.12.06.0004.01

$$\tau\theta(z) \propto \left(\tau\theta(z_0) - \pi\left(\lfloor -i\, z_0 \rfloor + 6\right) \left\lfloor \frac{\arg(-i\,(z - z_0))}{2\pi} \right\rfloor\right)\left(1 + O(z - z_0)\right) \;/;\; i\, z_0 \in \mathbb{R} \bigwedge i\, z_0 > 6 \bigwedge i\, z_0 \notin \mathbb{Z}$$

In the upper half-plane

10.12.06.0005.01

$$\tau\theta(z) \propto \tau\theta(z_0) + \pi \left\lfloor \frac{\arg(i\,(z - z_0))}{2\pi} \right\rfloor \left(6 + \lfloor i\, z_0 \rfloor\right) - \frac{1}{2}\left(2\log(2\pi) - \psi(i\, z_0 + 6) - \psi(6 - i\, z_0)\right)(z - z_0) - \frac{i}{4}\left(\psi^{(1)}(6 - i\, z_0) - \psi^{(1)}(6 + i\, z_0)\right)(z - z_0)^2 + \ldots \;/;\; (z \to z_0) \bigwedge i\, z_0 \in \mathbb{R} \bigwedge i\, z_0 < -6 \bigwedge i\, z_0 \notin \mathbb{Z}$$

10.12.06.0006.01

$$\tau\theta(z) \propto \tau\theta(z_0) + \pi \left\lfloor \frac{\arg(i\,(z - z_0))}{2\pi} \right\rfloor \left(6 + \lfloor i\, z_0 \rfloor\right) - \frac{1}{2}\left(2\log(2\pi) - \psi(i\, z_0 + 6) - \psi(6 - i\, z_0)\right)(z - z_0) - \frac{i}{4}\left(\psi^{(1)}(6 - i\, z_0) - \psi^{(1)}(6 + i\, z_0)\right)(z - z_0)^2 + O\left((z - z_0)^3\right) \;/;\; i\, z_0 \in \mathbb{R} \bigwedge i\, z_0 < -6 \bigwedge i\, z_0 \notin \mathbb{Z}$$

10.12.06.0007.01

$$\tau\theta(z) = \tau\theta(z_0) + \pi \left\lfloor \frac{\arg(i\,(z - z_0))}{2\pi} \right\rfloor \left(6 + \lfloor i\, z_0 \rfloor\right) + \frac{1}{2}\, i \sum_{k=1}^{\infty} \frac{i^k}{k!} \left(2\,\delta_{k-1} \log(2\pi) - \psi^{(k-1)}(6 + i\, z_0) + (-1)^k\, \psi^{(k-1)}(6 - i\, z_0)\right)(z - z_0)^k \;/;\; i\, z_0 \in \mathbb{R} \bigwedge i\, z_0 < -6 \bigwedge i\, z_0 \notin \mathbb{Z}$$

10.12.06.0008.01

$$\tau\theta(z) \propto \left(\tau\theta(z_0) + \pi \left\lfloor \frac{\arg(i\,(z - z_0))}{2\pi} \right\rfloor \left(\lfloor i\, z_0 \rfloor + 6\right)\right)\left(1 + O(z - z_0)\right) \;/;\; i\, z_0 \in \mathbb{R} \bigwedge i\, z_0 < -6 \bigwedge i\, z_0 \notin \mathbb{Z}$$

Expansions at $z == 0$

10.12.06.0009.01

$$\tau\theta(z) \propto \left(-\log(2\pi) + \frac{137}{60} - \gamma\right) z - \frac{1}{6}\, \psi^{(2)}(6)\, z^3 + \frac{1}{120}\, \psi^{(4)}(6)\, z^5 + \ldots \;/;\; (z \to 0)$$

10.12.06.0010.01

$$\tau\theta(z) \propto \left(-\log(2\pi) + \frac{137}{60} - \gamma\right) z - \frac{1}{3}\left(\frac{256103}{216000} - \zeta(3)\right) z^3 + \frac{1}{5}\left(\frac{806108207}{777600000} - \zeta(5)\right) z^5 + O\left(z^7\right)$$

10.12.06.0011.01

$$\tau\theta(z) = z \sum_{k=0}^{\infty} \left(\frac{(-1)^k \psi^{(2k)}(6)}{(2k+1)!} - \log(2\pi)\,\delta_k\right) z^{2k} \;/;\; |z| < 6$$

10.12.06.0012.01

$$\tau\theta(z) \propto \left(-\log(2\pi) + \frac{137}{60} - \gamma\right) z \left(1 + O\left(z^2\right)\right) \;/;\; (z \to 0)$$

Expansions at $z == 6\,i$

10.12.06.0013.01

$$\tau\theta(z) \propto \frac{1}{2} i \log(i(z - 6i)) + \frac{1}{2} i \log(39916800) - 6 i \log(2\pi) + \frac{1}{2}\left(-2\log(2\pi) + \frac{83711}{27720} - 2\gamma\right)(z - 6i) + \frac{239437889\, i}{614718720}(z - 6i)^2 + \ldots \;/;\; (z \to 6i)$$

10.12.06.0014.01

$$\tau\theta(z) \propto \frac{1}{2} i \log(i(z - 6i)) + \frac{1}{2} i \log(39916800) - 6 i \log(2\pi) + \frac{1}{2}\left(-2\log(2\pi) + \frac{83711}{27720} - 2\gamma\right)(z - 6i) + \frac{239437889\, i}{614718720}(z - 6i)^2 + O\left((z - 6i)^3\right)$$

10.12.06.0015.01

$$\tau\theta(z) == \frac{i}{2}\log(i(z - 6i)) - 6 i \log(2\pi) - \frac{i}{2} \sum_{k=0}^{\infty} \frac{i^k \left(-2\,\delta_{1-k}\log(2\pi) + \psi^{(k-1)}(1) - (-1)^k \psi^{(k-1)}(12)\right)}{k!}(z - 6i)^k \;/;\; |z - 6i| < 1$$

10.12.06.0016.01

$$\tau\theta(z) \propto \frac{1}{2} i \log(i(z - 6i)) + i\left(\frac{1}{2}\log(11!) - 6\log(2\pi)\right)(1 + O(z - 6i))$$

Expansions at $z == -6\,i$

10.12.06.0017.01

$$\tau\theta(z) \propto -\frac{1}{2} i \log(-i(z + 6i)) - \frac{1}{2} i \log(39916800) + 6 i \log(2\pi) + \frac{1}{2}\left(-2\log(2\pi) + \frac{83711}{27720} - 2\gamma\right)(z + 6i) - \frac{239437889\, i}{614718720}(z + 6i)^2 + \ldots \;/;\; (z \to -6i)$$

10.12.06.0018.01

$$\tau\theta(z) \propto -\frac{1}{2} i \log(-i(z + 6i)) - \frac{1}{2} i \log(39916800) + 6 i \log(2\pi) + \frac{1}{2}\left(-2\log(2\pi) + \frac{83711}{27720} - 2\gamma\right)(z + 6i) - \frac{239437889\, i}{614718720}(z + 6i)^2 + O\left((z + 6i)^3\right)$$

10.12.06.0019.01

$$\tau\theta(z) = -\frac{i}{2}\log(-i(z + 6i)) + 6 i \log(2\pi) + \frac{i}{2} \sum_{k=0}^{\infty} \frac{(-i)^k \left(-2\,\delta_{1-k}\log(2\pi) + \psi^{(k-1)}(1) - (-1)^k \psi^{(k-1)}(12)\right)}{k!}(z + 6i)^k \;/;\; |z + 6i| < 1$$

10.12.06.0020.01

$$\tau\theta(z) \propto -\frac{1}{2} i \log(-i (z + 6 i)) + i \left(6 \log(2 \pi) - \frac{\log(11!)}{2}\right) (1 + O(z + 6 i))$$

Expansions at $z == z_0$ /; $z_0 \neq \pm 6 i \pm i n$

10.12.06.0021.01

$$\tau\theta(z) \propto \tau\theta(z_0) - \frac{1}{2} (2 \log(2 \pi) - \psi(i z_0 + 6) - \psi(6 - i z_0)) (z - z_0) + \frac{1}{4} i \left(\psi^{(1)}(i z_0 + 6) - \psi^{(1)}(6 - i z_0)\right) (z - z_0)^2 + \ldots /; (z \to z_0) \bigwedge z_0^2 \neq -(k+6)^2 \bigwedge k \in \mathbb{Z}$$

10.12.06.0022.01

$$\tau\theta(z) \propto \tau\theta(z_0) - \frac{1}{2} (2 \log(2 \pi) - \psi(i z_0 + 6) - \psi(6 - i z_0)) (z - z_0) + \frac{1}{4} i \left(\psi^{(1)}(i z_0 + 6) - \psi^{(1)}(6 - i z_0)\right) (z - z_0)^2 + O\left((z - z_0)^3\right) /; z_0^2 \neq -(k+6)^2 \bigwedge k \in \mathbb{Z}$$

10.12.06.0023.01

$$\tau\theta(z) = \tau\theta(z_0) - \log(2 \pi) (z - z_0) - \frac{1}{2} i \sum_{k=1}^{\infty} \frac{i^k \left(\psi^{(k-1)}(6 + i z_0) - (-1)^k \psi^{(k-1)}(6 - i z_0)\right)}{k!} (z - z_0)^k /; (z \to z_0) \bigwedge z_0^2 \neq -(k+6)^2 \bigwedge k \in \mathbb{Z}$$

10.12.06.0024.01

$$\tau\theta(z) \propto \tau\theta(z_0) (1 + O(z - z_0)) /; z_0^2 \neq -(k+6)^2 \bigwedge k \in \mathbb{Z}$$

Expansions at $z == 6 i + i n$

10.12.06.0025.01

$$\tau\theta(z) \propto \frac{i}{2} \log(i (z - i (n+6))) - n \pi \left\lfloor \frac{\arg(i (z - i (n+6)))}{2 \pi} \right\rfloor - i (n+6) \log(2 \pi) + \frac{1}{2} i \log(n! (n+11)!) - \frac{n \pi}{2} + \frac{1}{2} (-2 \log(2 \pi) + \psi(n+1) + \psi(n+12)) (z - i (n+6)) + \frac{i}{4} \left(\frac{\pi^2}{3} - \psi^{(1)}(n+1) - \psi^{(1)}(n+12)\right) (z - i (n+6))^2 + \ldots /; (z \to i (n+6)) \land n \in \mathbb{N}$$

10.12.06.0026.01

$$\tau\theta(z) \propto \frac{i}{2} \log(i (z - i (n+6))) - n \pi \left\lfloor \frac{\arg(i (z - i (n+6)))}{2 \pi} \right\rfloor - i (n+6) \log(2 \pi) + \frac{1}{2} i \log(n! (n+11)!) - \frac{n \pi}{2} + \frac{1}{2} (-2 \log(2 \pi) + \psi(n+1) + \psi(n+12)) (z - i (n+6)) + \frac{i}{4} \left(\frac{\pi^2}{3} - \psi^{(1)}(n+1) - \psi^{(1)}(n+12)\right) (z - i (n+6))^2 + O\left((z - i (n+6))^3\right) /; n \in \mathbb{N}$$

10.12.06.0027.01

$$\tau\theta(z) == \frac{i}{2} \log(i (z - i (n+6))) - \pi \left\lfloor \frac{\arg(i (z - i (n+6)))}{2 \pi} \right\rfloor n - \frac{\pi n}{2} - i (n+6) \log(2 \pi) - \frac{i}{2} \sum_{k=0}^{\infty} \frac{i^k \left(-2 \delta_{k-1} \log(2 \pi) + \left(1 + (-1)^k\right) \psi^{(k-1)}(1) - (-1)^k \left(\psi^{(k-1)}(n+1) + \psi^{(k-1)}(n+12)\right)\right)}{k!} (z - i (n+6))^k /; |z - i (n+6)| < 1 \land n \in \mathbb{N}$$

10.12.06.0028.01

$$\tau\theta(z) \propto \frac{1}{2} i \log(i (z - i (n+6))) + \left(-\pi \left\lfloor \frac{\arg(i (z - i (n+6)))}{2\pi} \right\rfloor n - \frac{\pi n}{2} - i (n+6) \log(2\pi) + \frac{i}{2} \log(n! (n+11)!)\right) (1 + O(z - i (n+6))) /; n \in \mathbb{N}$$

Expansions at $z == -6 i - i n$

10.12.06.0029.01

$$\tau\theta(z) \propto -\frac{i}{2} \log(-i (z + i (n+6))) + n \pi \left\lfloor \frac{\arg(-i (z + i (n+6)))}{2\pi} \right\rfloor + i (n+6) \log(2\pi) - \frac{i}{2} \log(n! (n+11)!) + \frac{n\pi}{2} + \frac{1}{2} (-2 \log(2\pi) + \psi(n+1) + \psi(n+12)) (z + i (n+6)) - \frac{i}{4} \left(-\psi^{(1)}(n+1) + \frac{\pi^2}{3} - \psi^{(1)}(n+12)\right) (z + i (n+6))^2 + \ldots /; (z \to -i (n+6)) \land n \in \mathbb{N}$$

10.12.06.0030.01

$$\tau\theta(z) \propto -\frac{i}{2} \log(-i (z + i (n+6))) + n \pi \left\lfloor \frac{\arg(-i (z + i (n+6)))}{2\pi} \right\rfloor + i (n+6) \log(2\pi) - \frac{i}{2} \log(n! (n+11)!) + \frac{n\pi}{2} + \frac{1}{2} (-2 \log(2\pi) + \psi(n+1) + \psi(n+12)) (z + i (n+6)) - \frac{i}{4} \left(-\psi^{(1)}(n+1) + \frac{\pi^2}{3} - \psi^{(1)}(n+12)\right) (z + i (n+6))^2 + O\left((z + i (n+6))^3\right) /; n \in \mathbb{N}$$

10.12.06.0031.01

$$\tau\theta(z) == -\frac{i}{2} \log(-i (z + i (n+6))) + \pi \left\lfloor \frac{\arg(-i (z + i (n+6)))}{2\pi} \right\rfloor n + \frac{\pi n}{2} + i (n+6) \log(2\pi) + \frac{i}{2} \sum_{k=0}^{\infty} \frac{(-i)^k \left(-2 \delta_{k-1} \log(2\pi) + \left(1 + (-1)^k\right) \psi^{(k-1)}(1) - (-1)^k \left(\psi^{(k-1)}(n+1) + \psi^{(k-1)}(n+12)\right)\right)}{k!} (z + i (n+6))^k /; |i (n+6) + z| < 1 \land n \in \mathbb{N}$$

10.12.06.0032.01

$$\tau\theta(z) \propto -\frac{i}{2} \log(-i (z + i (n+6))) + \left(\pi \left\lfloor \frac{\arg(-i (z + i (n+6)))}{2\pi} \right\rfloor n + \frac{\pi n}{2} + i (n+6) \log(2\pi) - \frac{1}{2} i \log(n! (n+11)!)\right) (1 + O(z + i (n+6))) /; n \in \mathbb{N}$$

Asymptotic series expansions

10.12.06.0033.01

$$\tau\theta(z) \propto z \log(z) + \left(i \pi \left\lfloor \frac{\pi - 2 \arg(z)}{4\pi} \right\rfloor + i \pi \left\lfloor \frac{3}{4} - \frac{\arg(z)}{2\pi} \right\rfloor - \log(2\pi) - 1\right) z + \frac{11\pi}{4} + \frac{11\pi}{2} \left\lfloor \frac{\pi - 2 \arg(z)}{4\pi} \right\rfloor - \frac{11\pi}{2} \left\lfloor \frac{3}{4} - \frac{\arg(z)}{2\pi} \right\rfloor - \frac{1}{z} \sum_{k=0}^{\infty} (-1)^k 6^{2k} \left(\sum_{j=0}^{k} \frac{6^{-2j} (2k)! B_{2j+1}}{2 (j+1) (2j+1)! (2k-2j)!} - \frac{18}{k+1} + \frac{33}{2k+1}\right) z^{-2k} /; (|z| \to \infty)$$

10.12.06.0034.01

$$\tau\theta(z) \propto z\log(z) + \left(i\pi\left\lfloor\frac{\pi-2\arg(z)}{4\pi}\right\rfloor + i\pi\left\lfloor\frac{3}{4}-\frac{\arg(z)}{2\pi}\right\rfloor - \log(2\pi) - 1\right)z + \frac{11\pi}{4} + \frac{11\pi}{2}\left\lfloor\frac{\pi-2\arg(z)}{4\pi}\right\rfloor - \frac{11\pi}{2}\left\lfloor\frac{3}{4}-\frac{\arg(z)}{2\pi}\right\rfloor - \frac{59}{4z}\left(1+O\left(\frac{1}{z^2}\right)\right) /; (|z| \to \infty)$$

10.12.06.0035.01

$$\tau\theta(z) \propto \begin{cases} (-1-\log(2\pi)+i\pi+\log(z))\,z & \arg(z) \le -\frac{\pi}{2} \\ (-\log(2\pi)+\log(z)-1)\,z & -\frac{\pi}{2} < \arg(z) \le -\frac{\pi}{2} \\ (-1-\log(2\pi)-i\pi+\log(z))\,z & \text{True} \end{cases} /; (|z| \to \infty)$$

Other series representations

10.12.06.0036.01

$$\tau\theta(z) = \frac{1}{4}\left(-4z(\log(2\pi)+1) + (-11i+2z)\log(iz+6) + (11i+2z)\log(6-iz) - i\sum_{k=2}^{\infty}\frac{(k-1)\zeta(k, iz+7)-\zeta(k, 7-iz)}{k(k+1)}\right) /; |\text{Im}(z)| < 6$$

Integral representations

On the real axis

Of the direct function

10.12.07.0001.01

$$\tau\theta(z) = \frac{1}{2}(-i)\int_{6-iz}^{iz+6}\psi(t)\,dt - z\log(2\pi)$$

10.12.07.0002.01

$$\tau\theta(z) = -\frac{i}{2}\int_0^{\infty}\frac{e^{-t}}{t(e^t-1)}\left(2e^t iz - 2iz - e^{it(4i+z)} + e^{t(-iz-4)}\right)dt - z\log(2\pi) /; |\text{Im}(z)| < 6$$

10.12.07.0003.01

$$\tau\theta(z) = \frac{1}{4}(-4z(\log(2\pi)+1) + (-11i+2z)\log(iz+6) + (11i+2z)\log(6-iz)) - \frac{i}{2}\int_0^{\infty}\frac{1}{t}\left(e^{-t(iz+6)} - e^{it(6i+z)}\right)\left(\frac{1}{2}-\frac{1}{t}+\frac{1}{e^t-1}\right)dt /; |\text{Im}(z)| < 6$$

10.12.07.0004.01

$$\tau\theta(z) = -\frac{1}{2}i\int_0^{\infty}\frac{1}{t}\left(\frac{(t+1)^{-iz-6}-(t+1)^{iz-6}}{\log(t+1)} + 2e^{-t}iz\right)dt + z(-\log(2\pi)) /; |\text{Im}(z)| < 6$$

10.12.07.0005.01

$$\tau\theta(z) = -\log(2\pi)z - z - i\int_0^{\infty}\frac{\tan^{-1}\left(\frac{t}{iz+6}\right) - i\tanh^{-1}\left(\frac{t}{6i+z}\right)}{e^{2\pi t}-1}dt + \frac{1}{2}z\log(iz+6) + \frac{1}{4}((11i+2z)\log(6-iz) - 11i\log(iz+6)) /; |\text{Im}(z)| < 6$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

10.12.13.0001.01

$$\frac{\partial w(z)}{\partial z} = \frac{1}{2}\,(\psi(i\,z+6)+\psi(6-i\,z)) - \log(2\,\pi)\,/;\, w(z) = \tau\theta(z)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

10.12.16.0001.01

$$\tau\theta(-z) = -\tau\theta(z)$$

10.12.16.0002.01

$$\tau\theta(z+i) = \tau\theta(z) + \frac{1}{2}\,i\,(\log(5+i\,z)+\log(6-i\,z)) - i\log(2\,\pi)$$

10.12.16.0003.01

$$\tau\theta(z-i) = \tau\theta(z) + i\log(2\,\pi) - \frac{1}{2}\,i\,(\log(i\,z+6)+\log(5-i\,z))$$

10.12.16.0004.01

$$\tau\theta(z+i\,n) = \tau\theta(z) - i\,n\log(2\,\pi) + \frac{1}{2}\,i\left(\sum_{k=1}^{n}\log(6-k+i\,z) + \sum_{k=0}^{n-1}\log(6+k-i\,z)\right)/;\, n\in\mathbb{N}$$

10.12.16.0005.01

$$\tau\theta(z-i\,n) = \tau\theta(z) + i\,n\log(2\,\pi) - \frac{1}{2}\,i\left(\sum_{k=0}^{n-1}\log(6+k+i\,z) + \sum_{k=1}^{n}\log(6-k-i\,z)\right)/;\, n\in\mathbb{N}$$

Multiple arguments

Argument involving numeric multiples of variable

10.12.16.0006.01

$$\tau\theta(2\,z) = \tau\theta(z) + \tau\theta\left(z+\frac{i}{2}\right) + \frac{1}{2}\,i\left(\log(2\,\pi) + \sum_{k=0}^{2}\log(3+k+i\,z) - \sum_{k=0}^{2}\log(3+k-i\,z) + \sum_{k=0}^{1}\log\left(\frac{7}{2}+k+i\,z\right) - \sum_{k=0}^{2}\log\left(\frac{7}{2}+k-i\,z\right)\right) + 2\,z\log(2)$$

10.12.16.0007.01

$$\tau\theta(3\,z) = \tau\theta(z) + \tau\theta\left(z+\frac{i}{3}\right) + \tau\theta\left(z+\frac{2\,i}{3}\right) + \frac{1}{2}\,i\left(2\log(2\,\pi) - \sum_{k=2}^{5}\log(k-i\,z) + \sum_{k=2}^{5}\log(k+i\,z) + \sum_{k=2}^{4}\log\left(k+i\,z+\frac{1}{3}\right) + \sum_{k=2}^{4}\log\left(k+i\,z+\frac{2}{3}\right) - \sum_{k=2}^{5}\log\left(k-i\,z+\frac{1}{3}\right) - \sum_{k=2}^{5}\log\left(k-i\,z+\frac{2}{3}\right)\right) + 3\,z\log(3)$$

Identities

Recurrence identities

Consecutive neighbors

10.12.17.0001.01

$$\tau\theta(z) = \tau\theta(z + i) + i \log(2\pi) - \frac{1}{2} i (\log(5 + i z) + \log(6 - i z))$$

10.12.17.0002.01

$$\tau\theta(z) = \tau\theta(z - i) - i \log(2\pi) + \frac{1}{2} i (\log(i z + 6) + \log(5 - i z))$$

Distant neighbors

10.12.17.0003.01

$$\tau\theta(z) = \tau\theta(z + i n) - \frac{1}{2} i \left(\sum_{k=1}^{n} \log(-k + i z + 6) + \sum_{k=0}^{n-1} \log(k - i z + 6) \right) + i n \log(2\pi) \;/;\; n \in \mathbb{N}$$

10.12.17.0004.01

$$\tau\theta(z) = \tau\theta(z - i n) + \frac{1}{2} i \left(\sum_{k=0}^{n-1} \log(k + i z + 6) + \sum_{k=1}^{n} \log(-k - i z + 6) \right) - i n \log(2\pi) \;/;\; n \in \mathbb{N}$$

Complex characteristics

Real part

10.12.19.0001.01

$$\operatorname{Re}(\tau\theta(x + i y)) = -\gamma x - \log(2\pi) x - \frac{1}{2} \tan^{-1}(6 - y, x) + \frac{1}{2} \tan^{-1}(y + 6, -x) + \frac{1}{2} \sum_{k=1}^{\infty} \left(\frac{2x}{k} - \tan^{-1}\left(\frac{k - y + 6}{k}, \frac{x}{k} \right) + \tan^{-1}\left(\frac{k + y + 6}{k}, -\frac{x}{k} \right) \right)$$

Imaginary part

10.12.19.0002.01

$$\operatorname{Im}(\tau\theta(x + i y)) = -\gamma y - \log(2\pi) y + \frac{1}{4} \log\left(x^2 + (y - 6)^2\right) - \frac{1}{4} \log\left(x^2 + (y + 6)^2\right) + \frac{1}{4} \sum_{k=1}^{\infty} \left(\frac{4y}{k} + \log\left(\frac{k^2 - 2(y - 6)k + x^2 + (y - 6)^2}{k^2} \right) - \log\left(\frac{x^2 + (y + 6)^2 + 2k(y + 6)}{k^2} + 1 \right) \right)$$

Differentiation

Low-order differentiation

10.12.20.0001.01

$$\frac{\partial \tau\theta(z)}{\partial z} = \frac{1}{2} (-2 \log(2\pi) + \psi(i z + 6) + \psi(6 - i z))$$

10.12.20.0002.01

$$\frac{\partial^2 \tau\theta(z)}{\partial z^2} = \frac{1}{2} i \left(\psi^{(1)}(i z + 6) - \psi^{(1)}(6 - i z) \right)$$

Symbolic differentiation

10.12.20.0003.01

$$\frac{\partial^n \tau\theta(z)}{\partial z^n} = -\log(2\pi)\,(2-n)_n\, z^{1-n} + \frac{1}{2}\, i^{n-1}\left(\psi^{(n-1)}(i\,z+6) + (-1)^{n-1}\,\psi^{(n-1)}(6-i\,z)\right) /;\, n \in \mathbb{N}$$

10.12.20.0004.01

$$\frac{\partial^n \tau\theta(z)}{\partial z^n} = -\log(2\pi)\,(2-n)_n\, z^{1-n} + \frac{1}{2}\, i^{n-1}\,(n-1)!\,((-1)^n\,\zeta(n,\, i\,z+6) - \zeta(n,\, 6-i\,z)) /;\, n \in \mathbb{Z} \land n \geq 2$$

Fractional integro-differentiation

10.12.20.0005.01

$$\frac{\partial^\alpha \tau\theta(z)}{\partial z^\alpha} = \sum_{k=0}^{\infty} \frac{(-1)^k\,\psi^{(2k)}(6)\, z^{2k-\alpha+1}}{\Gamma(2k-\alpha+2)} - \frac{\log(2\pi)\, z^{1-\alpha}}{\Gamma(2-\alpha)}$$

Integration

Indefinite integration

Involving only one direct function

10.12.21.0001.01

$$\int \tau\theta(z)\,dz = -\frac{1}{2}\log(2\pi)\, z^2 - \frac{1}{2}\,\psi^{(-2)}(i\,z+6) - \frac{1}{2}\,\psi^{(-2)}(6-i\,z)$$

10.12.21.0002.01

$$\int \tau\theta(z)\,dz = \sum_{k=0}^{\infty} \frac{1}{2k+1}\left(\frac{(-1)^k\,\psi^{(2k)}(6)}{(2k+1)!} - \log(2\pi)\,\delta_k\right) z^{2k+1}$$

Involving one direct function and elementary functions

Involving power function

10.12.21.0003.01

$$\int z^n\,\tau\theta(z)\,dz = -\frac{\log(2\pi)\, z^{n+2}}{n+2} - \frac{n!\, z^n}{2} \sum_{k=0}^{n} \frac{(i\,z)^{-k}}{(n-k)!}\left((-1)^k\,\psi^{(-k-2)}(i\,z+6) + \psi^{(-k-2)}(6-i\,z)\right) /;\, n \in \mathbb{N}$$

10.12.21.0004.01

$$\int z^n\,\tau\theta(a+b\,z)\,dz =$$

$$-\log(2\pi)\left(\frac{a}{n+1} + \frac{b\,z}{n+2}\right) z^{n+1} - \frac{z^n\, n!}{2\,b} \sum_{k=0}^{n} \frac{(i\,b\,z)^{-k}\left((-1)^k\,\psi^{(-k-2)}(6+i\,a+i\,b\,z) + \psi^{(-k-2)}(6-i\,a-i\,b\,z)\right)}{(n-k)!} /;\, n \in \mathbb{N}$$

Definite integration

For the direct function itself

10.12.21.0005.01

$$\int_z^{z+1} \tau\theta(t)\,dt = \frac{1}{2}\left(-2\,z\log(2\,\pi) - \log(2\,\pi) - \psi^{(-2)}(-i\,(z+(1+6\,i))) + \psi^{(-2)}(6+i\,z) - \psi^{(-2)}((6+i)+i\,z) + \psi^{(-2)}(6-i\,z)\right) /; \; i\,z \notin (-\infty, -6) \bigwedge i\,z \notin (6, \infty)$$

Representations through more general functions

Through other functions

10.12.26.0001.01

$$\tau\theta(z) = -z\log(2\,\pi) - \frac{1}{2}\,i\left(\zeta^{(1,0)}(0,\,6+i\,z) - \zeta^{(1,0)}(0,\,6-i\,z)\right) /; \; |\mathrm{Im}(z)| < 6$$

Representations through equivalent functions

With related functions

10.12.27.0001.01

$$\tau\theta(z) = -z\log(2\,\pi) - \frac{1}{2}\,i\,(\log(\Gamma(i\,z+6)) - \log(\Gamma(6-i\,z))) /; \; |\mathrm{Re}(z)| \le \frac{9}{5} \bigwedge |\mathrm{Im}(z)| < 6$$

10.12.27.0002.01

$$\tau\theta(z) \to -i\log\left(\frac{\tau Z(z)}{\tau L(i\,z+6)}\right)$$

10.12.27.0003.01

$$\tau\theta(z) = \frac{1}{4}\left(-4\,z\log(2) - 23\,i\log(\pi) + 2\,i\log\Gamma(6-i\,z) - 2\,i\log\Gamma\left(-i\,z - \frac{11}{2}\right) - 4\,\vartheta\left(\frac{23\,i}{2} - 2\,z\right)\right)$$

10.12.27.0004.01

$$\tau\theta(z) = -i\log\left(\frac{e^{\frac{1}{2}(\log\Gamma(i\,z+6) - \log\Gamma(6-i\,z))}\,(2\,\pi)^{-i\,z}\sqrt{\Gamma(6-i\,z)}}{\sqrt{i\,z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\Gamma(-i\,z-5)}}\right)$$

History

Copyright