Sec

Notations

Traditional name

Secant

Traditional notation

$\sec(z)$

Mathematica StandardForm notation

Sec[z]

Primary definition

01.11.02.0001.01

$$\sec(z) = \frac{1}{\cos(z)} = \frac{2}{e^{i z} + e^{-i z}}$$

Specific values

Specialized values

01.11.03.0001.01

$$\sec\left(\pi\left(\frac{1}{2} + m\right)\right) = \tilde{\infty} \;/;\; m \in \mathbb{Z}$$

01.11.03.0002.01

$$\sec(\pi\, m) = (-1)^m \;/;\; m \in \mathbb{Z}$$

Values at fixed points

01.11.03.0003.01

$$\sec(0) = 1$$

01.11.03.0004.01

$$\sec\left(\frac{\pi}{12}\right) = \sqrt{6} - \sqrt{2}$$

01.11.03.0005.01

$$\sec\left(\frac{\pi}{12}\right) = \left(z; z^4 - 16 z^2 + 16\right)_3^{-1}$$

01.11.03.0006.01

$$\sec\left(\frac{\pi}{10}\right) == \sqrt{2 - \frac{2}{\sqrt{5}}}$$

01.11.03.0007.01

$$\sec\left(\frac{\pi}{10}\right) == \left(z;\, 5\,z^4 - 20\,z^2 + 16\right)_3^{-1}$$

01.11.03.0008.01

$$\sec\left(\frac{\pi}{9}\right) == -\frac{4\sqrt[3]{2}}{\left(-1 + i\sqrt{3}\right)^{4/3} + \left(-1 - i\sqrt{3}\right)^{4/3}}$$

01.11.03.0009.01

$$\sec\left(\frac{\pi}{9}\right) == \left(z;\, z^3 + 6\,z^2 - 8\right)_3^{-1}$$

01.11.03.0010.01

$$\sec\left(\frac{\pi}{9}\right) == \frac{2\sqrt[9]{-1}}{1 + (-1)^{2/9}}$$

01.11.03.0011.01

$$\sec\left(\frac{\pi}{8}\right) == \frac{2}{\sqrt{2 + \sqrt{2}}}$$

01.11.03.0012.01

$$\sec\left(\frac{\pi}{8}\right) == \left(z;\, z^4 - 8\,z^2 + 8\right)_3^{-1}$$

01.11.03.0013.01

$$\sec\left(\frac{\pi}{8}\right) == \frac{2\sqrt[8]{-1}}{1 + \sqrt[4]{-1}}$$

01.11.03.0014.01

$$\sec\left(\frac{\pi}{7}\right) == 24 \Bigg/ \left(2\left(1 - i\sqrt{3}\right)\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} + \frac{2\sqrt{7}\left(-i + \sqrt{3}\right)}{\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}}} + \frac{2\sqrt{7}\left(i + \sqrt{3}\right)}{\sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}}} + \sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}}\left(2 + 2\,i\sqrt{3}\right) + 4\right)$$

01.11.03.0015.01

$$\sec\left(\frac{\pi}{7}\right) == \left(z;\, z^3 - 4\,z^2 - 4\,z + 8\right)_2^{-1}$$

01.11.03.0016.01

$$\sec\left(\frac{\pi}{7}\right) = \frac{2\sqrt[7]{-1}}{1+(-1)^{2/7}}$$

01.11.03.0017.01

$$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$$

01.11.03.0018.01

$$\sec\left(\frac{\pi}{5}\right) = \sqrt{5} - 1$$

01.11.03.0019.01

$$\sec\left(\frac{2\pi}{9}\right) = \frac{2\sqrt[3]{2}}{\sqrt[3]{-1+i\sqrt{3}} + \sqrt[3]{-1-i\sqrt{3}}}$$

01.11.03.0020.01

$$\sec\left(\frac{2\pi}{9}\right) = \left(z;\, z^3 - 6z^2 + 8\right)_2^{-1}$$

01.11.03.0021.01

$$\sec\left(\frac{2\pi}{9}\right) = \frac{2(-1)^{2/9}}{1+(-1)^{4/9}}$$

01.11.03.0022.01

$$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

01.11.03.0023.01

$$\sec\left(\frac{2\pi}{7}\right) = \frac{3\, 2^{2/3}\sqrt[3]{7-21\, i\sqrt{3}}}{-\sqrt[3]{\frac{7}{2}\left(1-3\, i\sqrt{3}\right)} + \left(\frac{7}{2}\left(1-3\, i\sqrt{3}\right)\right)^{2/3} + 7}$$

01.11.03.0024.01

$$\sec\left(\frac{2\pi}{7}\right) = \left(z;\, z^3 + 4z^2 - 4z - 8\right)_3^{-1}$$

01.11.03.0025.01

$$\sec\left(\frac{2\pi}{7}\right) = \frac{2(-1)^{2/7}}{1+(-1)^{4/7}}$$

01.11.03.0026.01

$$\sec\left(\frac{3\pi}{10}\right) = \sqrt{2+\frac{2}{\sqrt{5}}}$$

01.11.03.0027.01

$$\sec\left(\frac{3\pi}{10}\right) = \left(z;\, 5z^4 - 20z^2 + 16\right)_4^{-1}$$

01.11.03.0028.01

$$\sec\left(\frac{\pi}{3}\right) = 2$$

01.11.03.0029.01

$$\sec\left(\frac{3\pi}{8}\right) = \sqrt{2\left(2+\sqrt{2}\right)}$$

01.11.03.0030.01

$$\sec\left(\frac{3\pi}{8}\right) = \left(z; z^4 - 8z^2 + 8\right)_4^{-1}$$

01.11.03.0031.01

$$\sec\left(\frac{3\pi}{8}\right) = \frac{2(-1)^{3/8}}{1+(-1)^{3/4}}$$

01.11.03.0032.01

$$\sec\left(\frac{2\pi}{5}\right) = 1+\sqrt{5}$$

01.11.03.0033.01

$$\sec\left(\frac{5\pi}{12}\right) = \sqrt{6}+\sqrt{2}$$

01.11.03.0034.01

$$\sec\left(\frac{5\pi}{12}\right) = \left(z; z^4 - 16z^2 + 16\right)_4^{-1}$$

01.11.03.0035.01

$$\sec\left(\frac{3\pi}{7}\right) = \left(12\, 2^{2/3} \sqrt[3]{7-21 i \sqrt{3}}\right) \Big/ \left(-4 i \sqrt{7} \sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} - 2\sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} + 2\, 2^{2/3}\sqrt[3]{7-21 i\sqrt{3}} - 2\left(14 - i\sqrt{7} - 3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + \left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + \sqrt{3}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\; i\right)$$

01.11.03.0036.01

$$\sec\left(\frac{3\pi}{7}\right) = \left(z; z^3 - 4z^2 - 4z + 8\right)_3^{-1}$$

01.11.03.0037.01

$$\sec\left(\frac{3\pi}{7}\right) = \frac{2(-1)^{3/7}}{1+(-1)^{6/7}}$$

01.11.03.0038.01

$$\sec\left(\frac{4\pi}{9}\right) = \frac{4 i \sqrt[3]{2}}{\sqrt[3]{-1+i\sqrt{3}}\left(-i+\sqrt{3}\right) - \sqrt[3]{-1-i\sqrt{3}}\left(i+\sqrt{3}\right)}$$

01.11.03.0039.01

$$\sec\left(\frac{4\pi}{9}\right) = \left(z; z^3 - 6z^2 + 8\right)_3^{-1}$$

01.11.03.0040.01

$$\sec\left(\frac{4\pi}{9}\right) = \frac{2(-1)^{4/9}}{1+(-1)^{8/9}}$$

01.11.03.0041.01

$$\sec\left(\frac{\pi}{2}\right) = \tilde{\infty}$$

01.11.03.0042.01

$$\sec\left(\frac{5\pi}{9}\right) = -\frac{4\,i\,\sqrt[3]{2}}{\sqrt[3]{-1+i\sqrt{3}}\,\left(-i+\sqrt{3}\right)-\sqrt[3]{-1-i\sqrt{3}}\,\left(i+\sqrt{3}\right)}$$

01.11.03.0043.01

$$\sec\left(\frac{5\pi}{9}\right) = \left(z;\,z^3+6z^2-8\right)_1^{-1}$$

01.11.03.0044.01

$$\sec\left(\frac{5\pi}{9}\right) = -\frac{2(-1)^{5/9}}{-1+\sqrt[9]{-1}}$$

01.11.03.0045.01

$$\sec\left(\frac{4\pi}{7}\right) = -\left(6\,2^{2/3}\,\sqrt[3]{7-21\,i\sqrt{3}}\right)\Big/\left(2^{2/3}\,\sqrt[3]{7-21\,i\sqrt{3}}-i\sqrt{3}\left(\frac{7}{2}-\frac{21\,i\sqrt{3}}{2}\right)^{2/3}+\left(\frac{7}{2}\left(1-3\,i\sqrt{3}\right)\right)^{2/3}+7\sqrt{3}\,i+7\right)$$

01.11.03.0046.01

$$\sec\left(\frac{4\pi}{7}\right) = \left(z;\,z^3+4z^2-4z-8\right)_1^{-1}$$

01.11.03.0047.01

$$\sec\left(\frac{4\pi}{7}\right) = -\frac{2(-1)^{4/7}}{-1+\sqrt[7]{-1}}$$

01.11.03.0048.01

$$\sec\left(\frac{7\pi}{12}\right) = -\sqrt{6}-\sqrt{2}$$

01.11.03.0049.01

$$\sec\left(\frac{7\pi}{12}\right) = \left(z;\,z^4-16z^2+16\right)_1^{-1}$$

01.11.03.0050.01

$$\sec\left(\frac{3\pi}{5}\right) = -1-\sqrt{5}$$

01.11.03.0051.01

$$\sec\left(\frac{5\pi}{8}\right) = -\sqrt{2\left(2+\sqrt{2}\right)}$$

01.11.03.0052.01

$$\sec\left(\frac{5\pi}{8}\right) = \left(z;\,z^4-8z^2+8\right)_1^{-1}$$

01.11.03.0053.01

$$\sec\left(\frac{5\pi}{8}\right) == -\frac{2(-1)^{5/8}}{-1+\sqrt[4]{-1}}$$

01.11.03.0054.01

$$\sec\left(\frac{2\pi}{3}\right) == -2$$

01.11.03.0055.01

$$\sec\left(\frac{7\pi}{10}\right) == -\sqrt{2+\frac{2}{\sqrt{5}}}$$

01.11.03.0056.01

$$\sec\left(\frac{7\pi}{10}\right) == \left(z;\, 5z^4-20z^2+16\right)_1^{-1}$$

01.11.03.0057.01

$$\sec\left(\frac{5\pi}{7}\right) ==$$

$$\left(12\,2^{2/3}\sqrt[3]{7-21\,i\sqrt{3}}\right)\Big/\left(2\sqrt{7}\,i\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}}-2\sqrt{21}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}}+2\,2^{2/3}\sqrt[3]{7-21\,i\sqrt{3}}-\right.$$
$$i\sqrt{3}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}-2\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}+$$
$$\left.\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}+4\sqrt{7}\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}}\;i\right)$$

01.11.03.0058.01

$$\sec\left(\frac{5\pi}{7}\right) == \left(z;\, z^3-4z^2-4z+8\right)_1^{-1}$$

01.11.03.0059.01

$$\sec\left(\frac{5\pi}{7}\right) == -\frac{2(-1)^{5/7}}{-1+(-1)^{3/7}}$$

01.11.03.0060.01

$$\sec\left(\frac{3\pi}{4}\right) == -\sqrt{2}$$

01.11.03.0061.01

$$\sec\left(\frac{7\pi}{9}\right) == -\frac{2\sqrt[3]{2}}{\sqrt[3]{-1+i\sqrt{3}}+\sqrt[3]{-1-i\sqrt{3}}}$$

01.11.03.0062.01

$$\sec\left(\frac{7\pi}{9}\right) == \left(z;\, z^3+6z^2-8\right)_2^{-1}$$

01.11.03.0063.01

$$\sec\left(\frac{7\pi}{9}\right) = -\frac{2(-1)^{7/9}}{-1+(-1)^{5/9}}$$

01.11.03.0064.01

$$\sec\left(\frac{4\pi}{5}\right) = 1-\sqrt{5}$$

01.11.03.0065.01

$$\sec\left(\frac{5\pi}{6}\right) = -\frac{2}{\sqrt{3}}$$

01.11.03.0066.01

$$\sec\left(\frac{6\pi}{7}\right) = -\left(12\,2^{2/3}\sqrt[3]{7-21\,i\sqrt{3}}\right)\Big/\left(2\,2^{2/3}\sqrt[3]{7-21\,i\sqrt{3}}+\sqrt[3]{2}\left(7-21\,i\sqrt{3}\right)^{2/3}+2\sqrt{3}\left(\frac{7}{2}-\frac{21\,i\sqrt{3}}{2}\right)^{2/3}i-14\,i\sqrt{3}+14\right)$$

01.11.03.0067.01

$$\sec\left(\frac{6\pi}{7}\right) = \left(z;\, z^3+4z^2-4z-8\right)_2^{-1}$$

01.11.03.0068.01

$$\sec\left(\frac{6\pi}{7}\right) = -\frac{2(-1)^{6/7}}{-1+(-1)^{5/7}}$$

01.11.03.0069.01

$$\sec\left(\frac{7\pi}{8}\right) = -\frac{2}{\sqrt{2+\sqrt{2}}}$$

01.11.03.0070.01

$$\sec\left(\frac{7\pi}{8}\right) = \left(z;\, z^4-8z^2+8\right)_2^{-1}$$

01.11.03.0071.01

$$\sec\left(\frac{7\pi}{8}\right) = -\frac{2(-1)^{7/8}}{-1+(-1)^{3/4}}$$

01.11.03.0072.01

$$\sec\left(\frac{8\pi}{9}\right) = \frac{4\sqrt[3]{2}}{\left(-1+i\sqrt{3}\right)^{4/3}+\left(-1-i\sqrt{3}\right)^{4/3}}$$

01.11.03.0073.01

$$\sec\left(\frac{8\pi}{9}\right) = \left(z;\, z^3-6z^2+8\right)_1^{-1}$$

01.11.03.0074.01

$$\sec\left(\frac{8\pi}{9}\right) = -\frac{2(-1)^{8/9}}{-1+(-1)^{7/9}}$$

01.11.03.0075.01

$$\sec\left(\frac{9\pi}{10}\right) = -\sqrt{2 - \frac{2}{\sqrt{5}}}$$

01.11.03.0076.01

$$\sec\left(\frac{9\pi}{10}\right) = \left(z;\, 5z^4 - 20z^2 + 16\right)_2^{-1}$$

01.11.03.0077.01

$$\sec\left(\frac{11\pi}{12}\right) = \sqrt{2} - \sqrt{6}$$

01.11.03.0078.01

$$\sec\left(\frac{11\pi}{12}\right) = \left(z;\, z^4 - 16z^2 + 16\right)_2^{-1}$$

01.11.03.0079.01

$$\sec(\pi) = -1$$

01.11.03.0080.01

$$\sec\left(\frac{13\pi}{12}\right) = \sqrt{2} - \sqrt{6}$$

01.11.03.0081.01

$$\sec\left(\frac{13\pi}{12}\right) = \left(z;\, z^4 - 16z^2 + 16\right)_2^{-1}$$

01.11.03.0082.01

$$\sec\left(\frac{11\pi}{10}\right) = -\sqrt{2 - \frac{2}{\sqrt{5}}}$$

01.11.03.0083.01

$$\sec\left(\frac{11\pi}{10}\right) = \left(z;\, 5z^4 - 20z^2 + 16\right)_2^{-1}$$

01.11.03.0084.01

$$\sec\left(\frac{10\pi}{9}\right) = \frac{4\sqrt[3]{2}}{\left(-1 + i\sqrt{3}\right)^{4/3} + \left(-1 - i\sqrt{3}\right)^{4/3}}$$

01.11.03.0085.01

$$\sec\left(\frac{10\pi}{9}\right) = \left(z;\, z^3 - 6z^2 + 8\right)_1^{-1}$$

01.11.03.0086.01

$$\sec\left(\frac{10\pi}{9}\right) = -\frac{2(-1)^{8/9}}{-1 + (-1)^{7/9}}$$

01.11.03.0087.01

$$\sec\left(\frac{9\pi}{8}\right) = -\frac{2}{\sqrt{2 + \sqrt{2}}}$$

01.11.03.0088.01

$$\sec\left(\frac{9\pi}{8}\right) = \left(z;\, z^4 - 8z^2 + 8\right)_2^{-1}$$

01.11.03.0089.01

$$\sec\left(\frac{9\pi}{8}\right) = -\frac{2(-1)^{7/8}}{-1 + (-1)^{3/4}}$$

01.11.03.0090.01

$$\sec\left(\frac{8\pi}{7}\right) =$$

$$-\left(12\, 2^{2/3} \sqrt[3]{7 - 21 i \sqrt{3}}\right) \Big/ \left(2\, 2^{2/3} \sqrt[3]{7 - 21 i \sqrt{3}} + \sqrt[3]{2} \left(7 - 21 i \sqrt{3}\right)^{2/3} + 2\sqrt{3} \left(\frac{7}{2} - \frac{21 i \sqrt{3}}{2}\right)^{2/3} i - 14 i \sqrt{3} + 14\right)$$

01.11.03.0091.01

$$\sec\left(\frac{8\pi}{7}\right) = \left(z;\, z^3 + 4z^2 - 4z - 8\right)_2^{-1}$$

01.11.03.0092.01

$$\sec\left(\frac{8\pi}{7}\right) = -\frac{2(-1)^{6/7}}{-1 + (-1)^{5/7}}$$

01.11.03.0093.01

$$\sec\left(\frac{7\pi}{6}\right) = -\frac{2}{\sqrt{3}}$$

01.11.03.0094.01

$$\sec\left(\frac{6\pi}{5}\right) = 1 - \sqrt{5}$$

01.11.03.0095.01

$$\sec\left(\frac{11\pi}{9}\right) = -\frac{2\sqrt[3]{2}}{\sqrt[3]{-1 + i\sqrt{3}} + \sqrt[3]{-1 - i\sqrt{3}}}$$

01.11.03.0096.01

$$\sec\left(\frac{11\pi}{9}\right) = \left(z;\, z^3 + 6z^2 - 8\right)_2^{-1}$$

01.11.03.0097.01

$$\sec\left(\frac{11\pi}{9}\right) = -\frac{2(-1)^{7/9}}{-1 + (-1)^{5/9}}$$

01.11.03.0098.01

$$\sec\left(\frac{5\pi}{4}\right) = -\sqrt{2}$$

01.11.03.0099.01

$$\sec\left(\frac{9\pi}{7}\right) = $$

$$\left(12\,2^{2/3}\sqrt[3]{7-21\,i\sqrt{3}}\right)\Big/\left(2\sqrt{7}\,i\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}}-2\sqrt{21}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}}+2\,2^{2/3}\sqrt[3]{7-21\,i\sqrt{3}}-\right.$$
$$i\sqrt{3}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}-2\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}+$$
$$\left.\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}+4\sqrt{7}\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}}\,i\right)$$

01.11.03.0100.01

$$\sec\left(\frac{9\pi}{7}\right) = \left(z;\,z^3-4z^2-4z+8\right)_1^{-1}$$

01.11.03.0101.01

$$\sec\left(\frac{9\pi}{7}\right) = -\frac{2(-1)^{5/7}}{-1+(-1)^{3/7}}$$

01.11.03.0102.01

$$\sec\left(\frac{13\pi}{10}\right) = -\sqrt{2+\frac{2}{\sqrt{5}}}$$

01.11.03.0103.01

$$\sec\left(\frac{13\pi}{10}\right) = \left(z;\,5z^4-20z^2+16\right)_1^{-1}$$

01.11.03.0104.01

$$\sec\left(\frac{4\pi}{3}\right) = -2$$

01.11.03.0105.01

$$\sec\left(\frac{11\pi}{8}\right) = -\sqrt{2\left(2+\sqrt{2}\right)}$$

01.11.03.0106.01

$$\sec\left(\frac{11\pi}{8}\right) = \left(z;\,z^4-8z^2+8\right)_1^{-1}$$

01.11.03.0107.01

$$\sec\left(\frac{11\pi}{8}\right) = -\frac{2(-1)^{5/8}}{-1+\sqrt[4]{-1}}$$

01.11.03.0108.01

$$\sec\left(\frac{7\pi}{5}\right) = -1-\sqrt{5}$$

01.11.03.0109.01

$$\sec\left(\frac{17\pi}{12}\right) = -2\sqrt{2+\sqrt{3}}$$

01.11.03.0110.01

$$\sec\left(\frac{17\pi}{12}\right) = \left(z;\, z^4 - 16z^2 + 16\right)_1^{-1}$$

01.11.03.0111.01

$$\sec\left(\frac{10\pi}{7}\right) = -\left(6\, 2^{2/3}\sqrt[3]{7 - 21\, i\sqrt{3}}\right) \Big/ \left(2^{2/3}\sqrt[3]{7 - 21\, i\sqrt{3}} - i\sqrt{3}\left(\frac{7}{2} - \frac{21\, i\sqrt{3}}{2}\right)^{2/3} + \left(\frac{7}{2}\left(1 - 3\, i\sqrt{3}\right)\right)^{2/3} + 7\sqrt{3}\, i + 7\right)$$

01.11.03.0112.01

$$\sec\left(\frac{10\pi}{7}\right) = \left(z;\, z^3 + 4z^2 - 4z - 8\right)_1^{-1}$$

01.11.03.0113.01

$$\sec\left(\frac{10\pi}{7}\right) = -\frac{2(-1)^{4/7}}{-1 + \sqrt[7]{-1}}$$

01.11.03.0114.01

$$\sec\left(\frac{13\pi}{9}\right) = -\frac{4\, i\sqrt[3]{2}}{\sqrt[3]{-1 + i\sqrt{3}}\left(-i + \sqrt{3}\right) - \sqrt[3]{-1 - i\sqrt{3}}\left(i + \sqrt{3}\right)}$$

01.11.03.0115.01

$$\sec\left(\frac{13\pi}{9}\right) = \left(z;\, z^3 + 6z^2 - 8\right)_1^{-1}$$

01.11.03.0116.01

$$\sec\left(\frac{13\pi}{9}\right) = -\frac{2(-1)^{5/9}}{-1 + \sqrt[9]{-1}}$$

01.11.03.0117.01

$$\sec\left(\frac{3\pi}{2}\right) = \tilde{\infty}$$

01.11.03.0118.01

$$\sec\left(\frac{14\pi}{9}\right) = \frac{4\, i\sqrt[3]{2}}{\sqrt[3]{-1 + i\sqrt{3}}\left(-i + \sqrt{3}\right) - \sqrt[3]{-1 - i\sqrt{3}}\left(i + \sqrt{3}\right)}$$

01.11.03.0119.01

$$\sec\left(\frac{14\pi}{9}\right) = \left(z;\, z^3 - 6z^2 + 8\right)_3^{-1}$$

01.11.03.0120.01

$$\sec\left(\frac{14\pi}{9}\right) = \frac{2(-1)^{4/9}}{1 + (-1)^{8/9}}$$

01.11.03.0121.01

$$\sec\left(\frac{11\pi}{7}\right) = \left(12\, 2^{2/3} \sqrt[3]{7 - 21\, i \sqrt{3}}\right) \Big/ \left(-4\, i \sqrt{7} \sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} - 2\sqrt{7}\left(i + \sqrt{3}\right) \sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}} + 2\, 2^{2/3} \sqrt[3]{7 - 21\, i\sqrt{3}} - 2\left(14 - i\sqrt{7} - 3\sqrt{21}\right)^{2/3} \sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}} + \left(14 + i\sqrt{7} + 3\sqrt{21}\right)^{2/3} \sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} + \sqrt{3}\left(14 + i\sqrt{7} + 3\sqrt{21}\right)^{2/3} \sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}}\; i\right)$$

01.11.03.0122.01

$$\sec\left(\frac{11\pi}{7}\right) = \left(z;\, z^3 - 4z^2 - 4z + 8\right)_3^{-1}$$

01.11.03.0123.01

$$\sec\left(\frac{11\pi}{7}\right) = \frac{2\,(-1)^{3/7}}{1 + (-1)^{6/7}}$$

01.11.03.0124.01

$$\sec\left(\frac{19\pi}{12}\right) = \sqrt{6} + \sqrt{2}$$

01.11.03.0125.01

$$\sec\left(\frac{19\pi}{12}\right) = \left(z;\, z^4 - 16z^2 + 16\right)_4^{-1}$$

01.11.03.0126.01

$$\sec\left(\frac{8\pi}{5}\right) = 1 + \sqrt{5}$$

01.11.03.0127.01

$$\sec\left(\frac{13\pi}{8}\right) = \sqrt{2\left(2 + \sqrt{2}\right)}$$

01.11.03.0128.01

$$\sec\left(\frac{13\pi}{8}\right) = \left(z;\, z^4 - 8z^2 + 8\right)_4^{-1}$$

01.11.03.0129.01

$$\sec\left(\frac{13\pi}{8}\right) = \frac{2\,(-1)^{3/8}}{1 + (-1)^{3/4}}$$

01.11.03.0130.01

$$\sec\left(\frac{5\pi}{3}\right) = 2$$

01.11.03.0131.01

$$\sec\left(\frac{17\pi}{10}\right) = \sqrt{2 + \frac{2}{\sqrt{5}}}$$

01.11.03.0132.01

$$\sec\left(\frac{17\pi}{10}\right) == \left(z;\, 5z^4 - 20z^2 + 16\right)_4^{-1}$$

01.11.03.0133.01

$$\sec\left(\frac{12\pi}{7}\right) == \frac{3\, 2^{2/3} \sqrt[3]{7 - 21 i\sqrt{3}}}{-\sqrt[3]{\frac{7}{2}\left(1 - 3 i\sqrt{3}\right)} + \left(\frac{7}{2}\left(1 - 3 i\sqrt{3}\right)\right)^{2/3} + 7}$$

01.11.03.0134.01

$$\sec\left(\frac{12\pi}{7}\right) == \left(z;\, z^3 + 4z^2 - 4z - 8\right)_3^{-1}$$

01.11.03.0135.01

$$\sec\left(\frac{12\pi}{7}\right) == \frac{2(-1)^{2/7}}{1 + (-1)^{4/7}}$$

01.11.03.0136.01

$$\sec\left(\frac{7\pi}{4}\right) == \sqrt{2}$$

01.11.03.0137.01

$$\sec\left(\frac{16\pi}{9}\right) == \frac{2\sqrt[3]{2}}{\sqrt[3]{-1 + i\sqrt{3}} + \sqrt[3]{-1 - i\sqrt{3}}}$$

01.11.03.0138.01

$$\sec\left(\frac{16\pi}{9}\right) == \left(z;\, z^3 - 6z^2 + 8\right)_2^{-1}$$

01.11.03.0139.01

$$\sec\left(\frac{16\pi}{9}\right) == \frac{2(-1)^{2/9}}{1 + (-1)^{4/9}}$$

01.11.03.0140.01

$$\sec\left(\frac{9\pi}{5}\right) == \sqrt{5} - 1$$

01.11.03.0141.01

$$\sec\left(\frac{11\pi}{6}\right) == \frac{2}{\sqrt{3}}$$

01.11.03.0142.01

$$\sec\left(\frac{13\pi}{7}\right) == 24 \Bigg/ \left(2\left(1 - i\sqrt{3}\right)\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} + \frac{2\sqrt{7}\left(-i + \sqrt{3}\right)}{\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}}} + \frac{2\sqrt{7}\left(i + \sqrt{3}\right)}{\sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}}} + \sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}}\left(2 + 2 i\sqrt{3}\right) + 4\right)$$

01.11.03.0143.01

$$\sec\left(\frac{13\pi}{7}\right) = \left(z;\, z^3 - 4z^2 - 4z + 8\right)_2^{-1}$$

01.11.03.0144.01

$$\sec\left(\frac{13\pi}{7}\right) = \frac{2\sqrt[7]{-1}}{1 + (-1)^{2/7}}$$

01.11.03.0145.01

$$\sec\left(\frac{15\pi}{8}\right) = \frac{2}{\sqrt{2 + \sqrt{2}}}$$

01.11.03.0146.01

$$\sec\left(\frac{15\pi}{8}\right) = \left(z;\, z^4 - 8z^2 + 8\right)_3^{-1}$$

01.11.03.0147.01

$$\sec\left(\frac{15\pi}{8}\right) = \frac{2\sqrt[8]{-1}}{1 + \sqrt[4]{-1}}$$

01.11.03.0148.01

$$\sec\left(\frac{17\pi}{9}\right) = -\frac{4\sqrt[3]{2}}{\left(-1 + i\sqrt{3}\right)^{4/3} + \left(-1 - i\sqrt{3}\right)^{4/3}}$$

01.11.03.0149.01

$$\sec\left(\frac{17\pi}{9}\right) = \left(z;\, z^3 + 6z^2 - 8\right)_3^{-1}$$

01.11.03.0150.01

$$\sec\left(\frac{17\pi}{9}\right) = \frac{2\sqrt[9]{-1}}{1 + (-1)^{2/9}}$$

01.11.03.0151.01

$$\sec\left(\frac{19\pi}{10}\right) = \sqrt{2 - \frac{2}{\sqrt{5}}}$$

01.11.03.0152.01

$$\sec\left(\frac{19\pi}{10}\right) = \left(z;\, 5z^4 - 20z^2 + 16\right)_3^{-1}$$

01.11.03.0153.01

$$\sec\left(\frac{23\pi}{12}\right) = \sqrt{6} - \sqrt{2}$$

01.11.03.0154.01

$$\sec\left(\frac{23\pi}{12}\right) = \left(z;\, z^4 - 16z^2 + 16\right)_3^{-1}$$

01.11.03.0155.01

$$\sec(2\pi) = 1$$

01.11.03.0156.01

$$\sec\left(\frac{\pi}{17}\right) == 4 \Big/ \left(\sqrt{\left(\frac{1}{2}\left(\sqrt{\left(2\left(-\sqrt{2\left(17-\sqrt{17}\right)} + 6\sqrt{17} + \sqrt{34\left(17-\sqrt{17}\right)} - 8\sqrt{2\left(17+\sqrt{17}\right)} + 34\right)\right)} + \sqrt{17} + \sqrt{2\left(17-\sqrt{17}\right)} + 15\right)\right)}\right)$$

01.11.03.0157.01

$$\sec\left(\frac{\pi}{30}\right) == \frac{8}{\sqrt{3} + \sqrt{15} + \sqrt{10 - 2\sqrt{5}}}$$

$\sec\left(\frac{n\pi}{m}\right)$ can be expressed using only square roots if $n \in \mathbb{Z}$ and m is a product of a power of 2 and distinct Fermat primes $\{3, 5, 17, 257, \ldots\}$.

Values at infinities

01.11.03.0158.01

$$\sec(i\,\infty) == 0$$

01.11.03.0159.01

$$\sec(-i\,\infty) == 0$$

01.11.03.0160.01

$$\sec(\tilde{\infty}) == ¿$$

General characteristics

Domain and analyticity

$\sec(z)$ is an analytical function of z which is defined over the whole complex z-plane.

01.11.04.0001.01

$$z \longrightarrow \sec(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\sec(z)$ is an even function.

01.11.04.0002.01

$$\sec(-z) == \sec(z)$$

Mirror symmetry

01.11.04.0003.01

$$\sec(\bar{z}) == \overline{\sec(z)}$$

Periodicity

$\sec(z)$ is a periodic function with period 2π.

01.11.04.0010.01

$\sec(z + 2\pi) = \sec(z)$

01.11.04.0004.01

$\sec(z + 2\pi m) = \sec(z) \;/;\; m \in \mathbb{Z}$

01.11.04.0005.01

$\sec(z + \pi m) = (-1)^m \sec(z) \;/;\; m \in \mathbb{Z}$

Poles and essential singularities

The function $\sec(z)$ has an infinite set of singular points:

a) $z = \pi/2 + \pi k \;/;\; k \in \mathbb{Z}$ are the simple poles with residues $(-1)^{k-1}$;

b) $z = \tilde{\infty}$ is an essential singular point.

01.11.04.0006.01

$\mathcal{S}ing_z(\sec(z)) = \left\{\left\{\left\{\frac{\pi}{2} + \pi k, 1\right\} /; k \in \mathbb{Z}\right\}, \{\tilde{\infty}, \infty\}\right\}$

01.11.04.0007.01

$\operatorname{res}_z(\sec(z))\left(\frac{\pi}{2} + \pi k\right) = (-1)^{k-1} \;/;\; k \in \mathbb{Z}$

Branch points

The function $\sec(z)$ does not have branch points.

01.11.04.0008.01

$\mathcal{BP}_z(\sec(z)) = \{\}$

Branch cuts

The function $\sec(z)$ does not have branch cuts.

01.11.04.0009.01

$\mathcal{BC}_z(\sec(z)) = \{\}$

Series representations

Generalized power series

Expansions at $z = z_0$

For the function itself

01.11.06.0019.01

$$\sec(z) \propto \sec(z_0) + \sec(z_0)\tan(z_0)(z - z_0) + 3\sec(z_0)\left(\frac{1}{2} - \frac{1}{3}\cos(2 z_0)\sec^2(z_0)\right)(z - z_0)^2 + \ldots \;/;\; (z \to z_0)$$

01.11.06.0020.01

$$\sec(z) \propto \sec(z_0) + \sec(z_0)\tan(z_0)(z - z_0) + 3\sec(z_0)\left(\frac{1}{2} - \frac{1}{3}\cos(2 z_0)\sec^2(z_0)\right)(z - z_0)^2 + O\left((z - z_0)^3\right)$$

01.11.06.0021.01

$$\sec(z) = \sec(z_0) \sum_{k=0}^{\infty} \left(\delta_k + (k+1) \sum_{m=0}^{k} \sum_{j=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \frac{(-1)^m 2^{1-m} (m-2j)^k \sec^m(z_0)}{(m+1)\, j!\, (m-j)!\, (k-m)!} \cos\left(\frac{\pi k}{2} + (m-2j) z_0 \right) \right) (z-z_0)^k$$

01.11.06.0022.01

$$\sec(z) \propto \sec(z_0)\,(1 + O(z - z_0))$$

Expansions at $z = 0$

For the function itself

01.11.06.0001.02

$$\sec(z) \propto 1 + \frac{z^2}{2} + \frac{5 z^4}{24} + \frac{61 z^6}{720} + \ldots \,/;\, (z \to 0)$$

01.11.06.0023.01

$$\sec(z) \propto 1 + \frac{z^2}{2} + \frac{5 z^4}{24} + \frac{61 z^6}{720} + O\left(z^8\right)$$

01.11.06.0002.01

$$\sec(z) = \sum_{k=0}^{\infty} \frac{(-1)^k E_{2k} z^{2k}}{(2k)!} \,/;\, |z| < \frac{\pi}{2}$$

01.11.06.0003.02

$$\sec(z) \propto 1 + O\left(z^2\right)$$

Expansions at $z = \frac{\pi}{2}$

For the function itself

01.11.06.0004.02

$$\sec(z) \propto -\frac{1}{z - \frac{\pi}{2}} - \frac{1}{6}\left(z - \frac{\pi}{2}\right) - \frac{7}{360}\left(z - \frac{\pi}{2}\right)^3 + \ldots \,/;\, \left(z \to \frac{\pi}{2}\right)$$

01.11.06.0024.01

$$\sec(z) \propto -\frac{1}{z - \frac{\pi}{2}} - \frac{1}{6}\left(z - \frac{\pi}{2}\right) - \frac{7}{360}\left(z - \frac{\pi}{2}\right)^3 + O\left(\left(z - \frac{\pi}{2}\right)^5\right)$$

01.11.06.0005.01

$$\sec(z) = -\frac{1}{z - \frac{\pi}{2}} - \sum_{k=1}^{\infty} \frac{(-1)^{k-1}\, 2\left(2^{2k-1} - 1\right) B_{2k}}{(2k)!} \left(z - \frac{\pi}{2}\right)^{2k-1} \,/;\, \left|z - \frac{\pi}{2}\right| < \pi$$

01.11.06.0006.02

$$\sec(z) \propto -\frac{1}{z - \frac{\pi}{2}} - \frac{1}{6}\left(z - \frac{\pi}{2}\right) + O\left(\left(z - \frac{\pi}{2}\right)^3\right)$$

q-series

01.11.06.0007.01

$$\sec(z) = -2\sum_{k=1}^{\infty}(-1)^k q^{2k-1} \text{ /; } q = e^{iz}$$

Dirichlet series

01.11.06.0008.01

$$\sec(z) = 2 e^{iz}\sum_{k=0}^{\infty}(-1)^k e^{2izk} \text{ /; } \operatorname{Im}(z) > 0$$

01.11.06.0009.01

$$\sec(z) = 2 e^{-iz}\sum_{k=0}^{\infty}(-1)^k e^{-2izk} \text{ /; } \operatorname{Im}(z) < 0$$

Asymptotic series expansions

01.11.06.0010.01

$$\sec(z) \propto 2 e^{iz}\, {}_1F_0\left(1;\, ;\, -e^{2iz}\right) \text{ /; } \operatorname{Im}(z) > 0 \land (|z| \to \infty)$$

01.11.06.0011.01

$$\sec(z) \propto 2 e^{iz}\left(1 + O\left(e^{2iz}\right)\right) \text{ /; } \operatorname{Im}(z) > 0 \land (|z| \to \infty)$$

01.11.06.0012.01

$$\sec(z) \propto 2 e^{-iz}\, {}_1F_0\left(1;\, ;\, -e^{-2iz}\right) \text{ /; } \operatorname{Im}(z) < 0 \land (|z| \to \infty)$$

01.11.06.0013.01

$$\sec(z) \propto 2 e^{-iz}\left(1 + O\left(e^{-2iz}\right)\right) \text{ /; } \operatorname{Im}(z) < 0 \land (|z| \to \infty)$$

01.11.06.0014.01

$$\sec(z) \propto \sec(z) \text{ /; } \operatorname{Im}(z) = 0 \land (|z| \to \infty)$$

01.11.06.0015.01

$$\sec(z) \propto 2 e^{iz} \text{ /; } \left(z \to e^{i\phi}\infty\right) \bigwedge 0 < \phi < \pi$$

01.11.06.0016.01

$$\sec(z) \propto 2 e^{-iz} \text{ /; } \left(z \to e^{i\phi}\infty\right) \bigwedge -\pi < \phi < 0$$

01.11.06.0025.01

$$\sec(z) \propto \begin{cases} 2 e^{-iz} & -\pi < \arg(z) < 0 \\ 2 e^{iz} & 0 < \arg(z) < \pi \\ \sec(z) & \text{True} \end{cases} \text{ /; } (|z| \to \infty)$$

Other series representations

01.11.06.0017.01

$$\sec(z) = \pi\sum_{k=0}^{\infty}\frac{(-1)^k (2k+1)}{\left(k+\frac{1}{2}\right)^2 \pi^2 - z^2} \text{ /; } \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.11.06.0018.01

$$\sec^2(z) = \sum_{k=-\infty}^{\infty}\frac{1}{\left(z + \pi\left(k+\frac{1}{2}\right)\right)^2} \text{ /; } \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

Integral representations

On the real axis

Of the direct function

01.11.07.0001.01

$$\sec(z) = \frac{2}{\pi}\int_0^\infty \frac{t^{\frac{2z}{\pi}}}{t^2+1}\,dt\;/;\;|\mathrm{Re}(z)| < \frac{\pi}{2}$$

Product representations

01.11.08.0001.01

$$\sec(z) = \prod_{k=1}^{\infty} \frac{\pi^2\,(2k-1)^2}{\pi^2\,(2k-1)^2 - 4z^2}$$

Limit representations

01.11.09.0001.01

$$\sec(z) = \lim_{n\to\infty} \sum_{k=-n}^{n} \frac{(-1)^k}{\pi\left(k+\frac{1}{2}\right)-z}\;/;\;\frac{z}{\pi}-\frac{1}{2} \notin \mathbb{Z}$$

Differential equations

Ordinary nonlinear differential equations

01.11.13.0001.01

$$w'(z)^2 - w(z)^4 + w(z)^2 = 0\;/;\;w(z) = \sec(z)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

01.11.16.0001.01

$$\sec(-z) = \sec(z)$$

01.11.16.0002.01

$$\sec(a\,(b\,z^c)^m) = \sec(a\,b^m\,z^{m c})\;/;\;2\,m \in \mathbb{Z}$$

01.11.16.0003.01

$$\mathrm{Sec}\left(\sqrt{z^2}\right) = \sec(z)$$

Argument involving inverse trigonometric and hyperbolic functions

Involving $\sin^{-1}$

01.11.16.0004.01

$$\sec(\sin^{-1}(z)) == \frac{1}{\sqrt{1-z^2}}$$

01.11.16.0016.01

$$\sec\left(\frac{1}{2}\sin^{-1}(z)\right) == \frac{\sqrt{2}}{\sqrt{\sqrt{1-z^2}+1}}$$

01.11.16.0058.01

$$\sec(i\sin^{-1}(z)) = \frac{2\left(i z+\sqrt{1-z^2}\right)^{i}}{\left(i z+\sqrt{1-z^2}\right)^{2i}+1}$$

01.11.16.0059.01

$$\sec(a\sin^{-1}(z)) = \frac{2\left(i z+\sqrt{1-z^2}\right)^{a}}{\left(i z+\sqrt{1-z^2}\right)^{2a}+1}$$

Involving $\cos^{-1}$

01.11.16.0005.01

$$\sec(\cos^{-1}(z)) == \frac{1}{z}$$

01.11.16.0017.01

$$\sec\left(\frac{1}{2}\cos^{-1}(z)\right) == \frac{\sqrt{2}}{\sqrt{z+1}}$$

01.11.16.0060.01

$$\sec(i\cos^{-1}(z)) = \frac{2\,e^{\pi/2}\left(i z+\sqrt{1-z^2}\right)^{i}}{e^{\pi}\left(i z+\sqrt{1-z^2}\right)^{2i}+1}$$

01.11.16.0061.01

$$\sec(a\cos^{-1}(z)) = \frac{2\,e^{\frac{i a \pi}{2}}\left(i z+\sqrt{1-z^2}\right)^{a}}{\left(i z+\sqrt{1-z^2}\right)^{2a}+e^{i a \pi}}$$

Involving $\tan^{-1}$

01.11.16.0006.01

$$\sec(\tan^{-1}(z)) == \sqrt{1+z^2}$$

01.11.16.0062.01

$$\sec\left(\tan^{-1}(x,\, y)\right) = \frac{\sqrt{x^2 + y^2}}{x}$$

01.11.16.0018.01

$$\sec\left(\frac{1}{2}\tan^{-1}(z)\right) = \frac{\sqrt{2}}{\sqrt{1 + \frac{1}{\sqrt{z^2+1}}}}$$

01.11.16.0063.01

$$\sec\left(\frac{1}{2}\tan^{-1}(x,\, y)\right) = \frac{2}{\sqrt{\frac{x+i\,y}{\sqrt{x^2+y^2}}} + \frac{1}{\sqrt{\frac{x+i\,y}{\sqrt{x^2+y^2}}}}}$$

01.11.16.0064.01

$$\sec\left(i\tan^{-1}(z)\right) = \frac{2\left(z^2 + 1\right)^{i/2}}{(i\,z + 1)^i + (1 - i\,z)^i}$$

01.11.16.0065.01

$$\sec\left(i\tan^{-1}(x,\, y)\right) = \frac{2\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^i}{\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^{2i} + 1}$$

01.11.16.0066.01

$$\sec\left(a\tan^{-1}(z)\right) = \frac{2\left(z^2 + 1\right)^{a/2}}{(i\,z + 1)^a + (1 - i\,z)^a}$$

01.11.16.0067.01

$$\sec\left(a\tan^{-1}(x,\, y)\right) = \frac{2\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^a}{\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^{2a} + 1}$$

Involving $\cot^{-1}$

01.11.16.0007.01

$$\sec\left(\cot^{-1}(z)\right) = \sqrt{1 + \frac{1}{z^2}}$$

01.11.16.0019.01

$$\sec\left(\frac{1}{2}\cot^{-1}(z)\right) = \frac{\sqrt{2}}{\sqrt{1 + \frac{1}{\sqrt{1+\frac{1}{z^2}}}}}$$

01.11.16.0068.01

$$\sec\left(i \cot^{-1}(z)\right) = \frac{2\left(1+\frac{1}{z^2}\right)^{i/2}}{\left(\frac{-i+z}{z}\right)^{i} + \left(\frac{i+z}{z}\right)^{i}}$$

01.11.16.0069.01

$$\sec\left(a \cot^{-1}(z)\right) = \frac{2\left(1+\frac{1}{z^2}\right)^{a/2}}{\left(\frac{-i+z}{z}\right)^{a} + \left(\frac{i+z}{z}\right)^{a}}$$

Involving $\csc^{-1}$

01.11.16.0008.01

$$\sec\left(\csc^{-1}(z)\right) = \frac{\sqrt{z^2}}{\sqrt{z^2-1}}$$

01.11.16.0020.01

$$\sec\left(\frac{1}{2}\csc^{-1}(z)\right) = \frac{\sqrt{2}}{\sqrt{\sqrt{1-\frac{1}{z^2}}+1}}$$

01.11.16.0070.01

$$\sec\left(i \csc^{-1}(z)\right) = \frac{2}{\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{-i} + \left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{i}}$$

01.11.16.0071.01

$$\sec\left(a \csc^{-1}(z)\right) = \frac{2}{\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{-a} + \left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{a}}$$

Involving $\sec^{-1}$

01.11.16.0009.01

$$\sec\left(\sec^{-1}(z)\right) = z$$

01.11.16.0021.01

$$\sec\left(\frac{1}{2}\sec^{-1}(z)\right) = \frac{\sqrt{-2z}}{\sqrt{-z-1}}$$

01.11.16.0072.01

$$\sec\left(i \sec^{-1}(z)\right) = \frac{2\, e^{\pi/2}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{i}}{e^{\pi}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2i}+1}$$

01.11.16.0073.01

$$\sec\left(a\sec^{-1}(z)\right)=\frac{2\,e^{\frac{1}{2}(-i)\,a\,\pi}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{a}}{e^{-i\,a\,\pi}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2\,a}+1}$$

Involving $\sinh^{-1}$

01.11.16.0074.01

$$\sec\left(\sinh^{-1}(z)\right)=\frac{2\left(z+\sqrt{z^2+1}\right)^{i}}{\left(z+\sqrt{z^2+1}\right)^{2\,i}+1}$$

01.11.16.0010.01

$$\sec\left(i\sinh^{-1}(z)\right)=\frac{1}{\sqrt{1+z^2}}$$

01.11.16.0022.01

$$\sec\left(\frac{i}{2}\sinh^{-1}(z)\right)=\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+z^2}}}$$

01.11.16.0075.01

$$\sec\left(a\sinh^{-1}(z)\right)=\frac{2\left(z+\sqrt{z^2+1}\right)^{i\,a}}{\left(z+\sqrt{z^2+1}\right)^{2\,i\,a}+1}$$

Involving $\cosh^{-1}$

01.11.16.0076.01

$$\sec\left(\cosh^{-1}(z)\right)=\frac{2\left(z+\sqrt{z-1}\,\sqrt{z+1}\right)^{i}}{\left(z+\sqrt{z-1}\,\sqrt{z+1}\right)^{2\,i}+1}$$

01.11.16.0011.01

$$\sec\left(i\cosh^{-1}(z)\right)=\frac{1}{z}$$

01.11.16.0023.01

$$\sec\left(\frac{i}{2}\cosh^{-1}(z)\right)=\frac{\sqrt{2}}{\sqrt{1+z}}$$

01.11.16.0077.01

$$\sec\left(a\cosh^{-1}(z)\right) = \frac{2\left(z+\sqrt{z-1}\,\sqrt{z+1}\right)^{i\,a}}{\left(z+\sqrt{z-1}\,\sqrt{z+1}\right)^{2\,i\,a}+1}$$

Involving $\tanh^{-1}$

01.11.16.0078.01

$$\sec\left(\tanh^{-1}(z)\right) = \frac{2\left(1-z^2\right)^{i/2}}{(1-z)^i+(z+1)^i}$$

01.11.16.0012.01

$$\sec\left(i\tanh^{-1}(z)\right) == \sqrt{1-z^2}$$

01.11.16.0024.01

$$\sec\left(\frac{i}{2}\tanh^{-1}(z)\right) == \frac{\sqrt{2}}{\sqrt{1+\frac{1}{\sqrt{1-z^2}}}}$$

01.11.16.0079.01

$$\sec\left(a\tanh^{-1}(z)\right) = \frac{2\left(1-z^2\right)^{\frac{i\,a}{2}}}{(1-z)^{i\,a}+(z+1)^{i\,a}}$$

Involving $\coth^{-1}$

01.11.16.0080.01

$$\sec\left(\coth^{-1}(z)\right) = \frac{2\left(1-\frac{1}{z^2}\right)^{i/2}}{\left(1+\frac{1}{z}\right)^i+\left(1-\frac{1}{z}\right)^i}$$

01.11.16.0013.01

$$\sec\left(i\coth^{-1}(z)\right) == \sqrt{1-\frac{1}{z^2}}$$

01.11.16.0025.01

$$\sec\left(\frac{i}{2}\coth^{-1}(z)\right) == \frac{\sqrt{2}}{\sqrt{1+\frac{1}{\sqrt{1-\frac{1}{z^2}}}}}$$

01.11.16.0081.01

$$\sec\left(a\coth^{-1}(z)\right) = \frac{2\left(1-\frac{1}{z^2}\right)^{\frac{i\,a}{2}}}{\left(1+\frac{1}{z}\right)^{i\,a}+\left(1-\frac{1}{z}\right)^{i\,a}}$$

Involving csch^{-1}

01.11.16.0082.01

$$\sec\left(\text{csch}^{-1}(z)\right) = \frac{2\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{i}}{\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2i}+1}$$

01.11.16.0014.01

$$\sec\left(i\,\text{csch}^{-1}(z)\right) = \frac{\sqrt{-z^2}}{\sqrt{-1-z^2}}$$

01.11.16.0026.01

$$\sec\left(\frac{i}{2}\,\text{csch}^{-1}(z)\right) = \frac{\sqrt{2}}{\sqrt{1+\sqrt{1+\frac{1}{z^2}}}}$$

01.11.16.0083.01

$$\sec\left(a\,\text{csch}^{-1}(z)\right) = \frac{2\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{i a}}{\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2 i a}+1}$$

Involving sech^{-1}

01.11.16.0084.01

$$\sec\left(\text{sech}^{-1}(z)\right) = \frac{2}{\left(\sqrt{\frac{1}{z}-1}\,\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{-i}+\left(\sqrt{\frac{1}{z}-1}\,\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{i}}$$

01.11.16.0015.01

$$\sec\left(i\,\text{sech}^{-1}(z)\right) = z$$

01.11.16.0027.01

$$\sec\left(\frac{i}{2}\,\text{sech}^{-1}(z)\right) = \frac{\sqrt{-2z}}{\sqrt{-z-1}}$$

01.11.16.0085.01

$$\sec\left(a\,\text{sech}^{-1}(z)\right) = \frac{2}{\left(\sqrt{\frac{1}{z}-1}\,\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{-i a}+\left(\sqrt{\frac{1}{z}-1}\,\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{i a}}$$

Addition formulas

01.11.16.0028.01

$$\sec(a+b) == \frac{1}{\cos(b)\cos(a) - \sin(a)\sin(b)}$$

01.11.16.0029.01

$$\sec(a-b) == \frac{1}{\cos(a)\cos(b) + \sin(a)\sin(b)}$$

01.11.16.0030.01

$$\sec(a+b\,i) == \frac{2\cos(a)\cosh(b) + 2\,i\sin(a)\sinh(b)}{\cos(2\,a) + \cosh(2\,b)}$$

01.11.16.0031.01

$$\sec(a-i\,b) == \frac{2\cosh(b)\cos(a) - 2\,i\sin(a)\sinh(b)}{\cos(2\,a) + \cosh(2\,b)}$$

Half-angle formulas

01.11.16.0032.02

$$\sec\left(\frac{z}{2}\right) == \frac{\sqrt{2}}{\sqrt{1+\cos(z)}} \,/;\, |\mathrm{Re}(z)| < \pi \lor \mathrm{Re}(z) == -\pi \land \mathrm{Im}(z) > 0 \lor \mathrm{Re}(z) == \pi \land \mathrm{Im}(z) < 0$$

01.11.16.0033.01

$$\sec\left(\frac{z}{2}\right) == (-1)^{\left\lfloor \frac{\mathrm{Re}(z)+\pi}{2\pi} \right\rfloor} \frac{\sqrt{2}}{\sqrt{\cos(z)+1}} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)+\pi}{2\pi} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)+\pi}{2\pi} \right\rfloor}\right) \theta(-\mathrm{Im}(z))\right)$$

Multiple arguments

Argument involving numeric multiples of variable

01.11.16.0034.01

$$\sec(2\,z) == \frac{\sec^2(z)}{2 - \sec^2(z)}$$

01.11.16.0041.01

$$\sec(3\,z) == \frac{\sec^3(z)}{4 - 3\sec^2(z)}$$

Argument involving symbolic multiples of variable

01.11.16.0042.01

$$\sec(n\,z) == \frac{1}{n \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left((-1)^k\,(-k+n-1)!\,2^{-2\,k+n-1}\right)\cos^{n-2\,k}(z)}{k!\,(n-2\,k)!}} \,/;\, n \in \mathbb{N}$$

01.11.16.0035.01

$$\sec(n\,z) == \frac{1}{T_n(\cos(z))}$$

Products, sums, and powers of the direct function

Products of the direct function

01.11.16.0043.01

$$\sec(a)\sec(b) == \frac{2}{\cos(a-b)+\cos(a+b)}$$

Products involving the direct function

01.11.16.0044.01

$$\sec(a)\csc(b) == \frac{2}{\sin(a+b)-\sin(a-b)}$$

Sums of the direct function

01.11.16.0036.01

$$\sec(a)+\sec(b) == 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)\sec(a)\sec(b)$$

01.11.16.0037.01

$$\sec(a)-\sec(b) == 2\sin\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)\sec(a)\sec(b)$$

Sums involving the direct function

Involving other trigonometric functions

Involving csc

01.11.16.0045.01

$$\sec(z)+\csc(z) == \sqrt{2}\cos\left(z-\frac{\pi}{4}\right)\csc(z)\sec(z)$$

01.11.16.0046.01

$$\sec(z)-\csc(z) == -\sqrt{2}\cos\left(z+\frac{\pi}{4}\right)\csc(z)\sec(z)$$

01.11.16.0047.01

$$\sec(a)+\csc(b) == 2\cos\left(\frac{b-a}{2}-\frac{\pi}{4}\right)\cos\left(\frac{a+b}{2}-\frac{\pi}{4}\right)\csc(b)\sec(a)$$

01.11.16.0048.01

$$\sec(a)-\csc(b) == -2\cos\left(\frac{a+b}{2}+\frac{\pi}{4}\right)\cos\left(\frac{b-a}{2}+\frac{\pi}{4}\right)\csc(b)\sec(a)$$

01.11.16.0049.01

$$a\sec(z)+b\csc(z) == 2a\sqrt{\frac{b^2}{a^2}+1}\,\sin\left(z+\tan^{-1}\left(\frac{b}{a}\right)\right)\csc(2z)$$

Involving hyperbolic functions

Involving csch

01.11.16.0050.01

$$\sec(z)+i\operatorname{csch}(z)=2\,i\cos\left(\frac{e^{\frac{i\pi}{4}}z}{\sqrt{2}}+\frac{\pi}{4}\right)\cos\left(\frac{z\,e^{-\frac{1}{4}(i\pi)}}{\sqrt{2}}-\frac{\pi}{4}\right)\operatorname{csch}(z)\sec(z)$$

01.11.16.0051.01

$$\sec(z)-i\operatorname{csch}(z)=-2\,i\cos\left(\frac{e^{-\frac{1}{4}(i\pi)}z}{\sqrt{2}}+\frac{\pi}{4}\right)\cos\left(\frac{z\,e^{\frac{i\pi}{4}}}{\sqrt{2}}-\frac{\pi}{4}\right)\operatorname{csch}(z)\sec(z)$$

01.11.16.0052.01

$$\sec(a)+i\operatorname{csch}(b)=2\,i\cos\left(\frac{1}{2}(a+b\,i)+\frac{\pi}{4}\right)\cos\left(\frac{1}{2}(a-i\,b)-\frac{\pi}{4}\right)\operatorname{csch}(b)\sec(a)$$

01.11.16.0053.01

$$\sec(a)-i\operatorname{csch}(b)=-2\,i\cos\left(\frac{1}{2}(a-i\,b)+\frac{\pi}{4}\right)\cos\left(\frac{1}{2}(a+b\,i)-\frac{\pi}{4}\right)\operatorname{csch}(b)\sec(a)$$

Involving sech

01.11.16.0054.01

$$\sec(z)+\operatorname{sech}(z)=2\cos\left(\frac{z\,e^{-\frac{1}{4}(i\pi)}}{\sqrt{2}}\right)\cos\left(\frac{z\,e^{\frac{i\pi}{4}}}{\sqrt{2}}\right)\sec(z)\operatorname{sech}(z)$$

01.11.16.0055.01

$$\sec(z)-\operatorname{sech}(z)=2\sin\left(\frac{z\,e^{-\frac{1}{4}(i\pi)}}{\sqrt{2}}\right)\sin\left(\frac{z\,e^{\frac{i\pi}{4}}}{\sqrt{2}}\right)\sec(z)\operatorname{sech}(z)$$

01.11.16.0056.01

$$\sec(a)+\operatorname{sech}(b)=2\cos\left(\frac{1}{2}(a-i\,b)\right)\cos\left(\frac{1}{2}(a+b\,i)\right)\sec(a)\operatorname{sech}(b)$$

01.11.16.0057.01

$$\sec(a)-\operatorname{sech}(b)=2\sin\left(\frac{1}{2}(a-i\,b)\right)\sin\left(\frac{1}{2}(a+b\,i)\right)\sec(a)\operatorname{sech}(b)$$

Powers of the direct function

01.11.16.0038.01

$$\sec^2(z)=\frac{2\sec(2\,z)}{\sec(2\,z)+1}$$

Sums of powers involving the direct function

01.11.16.0039.01

$$\sec^2(a)-\sec^2(b)=\sec^2(a)\sec^2(b)\sin(a-b)\sin(a+b)$$

01.11.16.0040.01

$$\sec^2(a)-\csc^2(b)=-\cos(a-b)\cos(a+b)\csc^2(b)\sec^2(a)$$

Identities

Functional identities

01.11.17.0001.01

$$\sec(2z)\left(2-\sec^2(z)\right)=\sec^2(z)$$

01.11.17.0002.01

$$-\sec^2(z_1)\sec^2(z_2)+2\sec(z_1)\sec(z_1+z_2)\sec(z_2)+\left(\sec^2(z_2)\sec^2(z_1)-\sec^2(z_1)-\sec^2(z_2)\right)\sec^2(z_1+z_2)=0$$

Complex characteristics

Real part

01.11.19.0001.01

$$\operatorname{Re}(\sec(x+i\,y))=\frac{2\cos(x)\cosh(y)}{\cos(2x)+\cosh(2y)}$$

Imaginary part

01.11.19.0002.01

$$\operatorname{Im}(\sec(x+i\,y))=\frac{2\sin(x)\sinh(y)}{\cos(2x)+\cosh(2y)}$$

Absolute value

01.11.19.0003.01

$$|\sec(x+i\,y)|=\frac{\sqrt{2}}{\sqrt{\cos(2x)+\cosh(2y)}}$$

Argument

01.11.19.0004.01

$$\arg(\sec(x+i\,y))=\tan^{-1}\left(\frac{\cos(x)\cosh(y)}{\cos(2x)+\cosh(2y)},\frac{\sin(x)\sinh(y)}{\cos(2x)+\cosh(2y)}\right)$$

01.11.19.0005.01

$$\arg(\sec(x+i\,y))=\tan^{-1}(\tan(x)\tanh(y))+\frac{\pi}{2}\operatorname{sgn}\left(\frac{\operatorname{sgn}(\sin(x)\sinh(y))}{\operatorname{sgn}(\cos(2x)+\cosh(2y))}+\frac{1}{2}\right)\left(1-\frac{\operatorname{sgn}(\cos(x)\cosh(y))}{\operatorname{sgn}(\cos(2x)+\cosh(2y))}\right)$$

Conjugate value

01.11.19.0006.01

$$\overline{\sec(x+i\,y)}=\frac{1}{\cos(x)\cosh(y)+i\sin(x)\sinh(y)}$$

Differentiation

Low-order differentiation

01.11.20.0001.01

$$\frac{\partial\sec(z)}{\partial z}=\sec(z)\tan(z)$$

01.11.20.0002.01

$$\frac{\partial^2 \sec(z)}{\partial z^2} = \sec(z)\left(\sec^2(z) + \tan^2(z)\right)$$

Symbolic differentiation

01.11.20.0003.01

$$\frac{\partial^n \sec(z)}{\partial z^n} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k-n)!} E_{2k} z^{2k-n} /; |z| < \frac{\pi}{2} \bigwedge n \in \mathbb{N}^+$$

01.11.20.0006.01

$$\frac{\partial^n \sec(z)}{\partial z^n} = -i^{n+1} \sum_{k=0}^{n} \frac{(-1)^k k!}{2^k} \mathcal{S}_n^{(k)} \left(\left(i \tan\left(\frac{1}{4}(2z+\pi)\right)+1\right)^k \left(i \tan\left(\frac{1}{4}(2z+\pi)\right)-1\right) - 2^n (i \tan(z)+1)^k (i \tan(z)-1)\right) /; n \in \mathbb{N}$$

01.11.20.0004.01

$$\frac{\partial^n \sec(z)}{\partial z^n} = \sec(z)\left(\delta_n + (n+1)! \sum_{k=0}^{n} \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{(-1)^k 2^{1-k} (k-2j)^n \sec^k(z)}{(k+1)\, j!\, (k-j)!\, (n-k)!} \cos\left(\frac{\pi n}{2} + (k-2j)z\right)\right) /; n \in \mathbb{N}$$

01.11.20.0007.01

$$\frac{\partial^n \sec(z)}{\partial z^n} = i^n \sec(z) \sum_{j=0}^{n} \sum_{k=0}^{j} (-1)^k \binom{n}{j} 2^{j-k} k!\, \mathcal{S}_j^{(k)} (i \tan(z)+1)^k /; n \in \mathbb{N}$$

Victor Adamchik (2005)

Fractional integro-differentiation

01.11.20.0005.01

$$\frac{\partial^\alpha \sec(z)}{\partial z^\alpha} = \sum_{k=0}^{\infty} \frac{(-1)^k E_{2k} z^{2k-\alpha}}{\Gamma(2k-\alpha+1)} /; |z| < \frac{\pi}{2}$$

Integration

Indefinite integration

Involving only one direct function

01.11.21.0018.01

$$\int \sec(b + a z)\, dz = \frac{2 \tanh^{-1}\left(\tan\left(\frac{1}{2}(b + a z)\right)\right)}{a}$$

01.11.21.0019.01

$$\int \sec(a z)\, dz = \frac{2 \tanh^{-1}\left(\tan\left(\frac{a z}{2}\right)\right)}{a}$$

01.11.21.0020.01

$$\int \sec(z)\, dz = 2 \tanh^{-1}\left(\tan\left(\frac{z}{2}\right)\right)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Involving z^n and linear arguments

01.11.21.0021.01

$$\int z\sec(b+a\,z)\,dz = \frac{1}{2\,a^2}\Big(-(2\,b+2\,a\,z-\pi)\left(\log\left(1-i\,e^{-i\,(b+a\,z)}\right)-\log\left(1+i\,e^{-i\,(b+a\,z)}\right)\right)+ (2\,b-\pi)\log\left(\tan\left(\frac{1}{4}\,(-2\,b-2\,a\,z+\pi)\right)\right)+2\,i\left(\mathrm{Li}_2\left(-i\,e^{-i\,(b+a\,z)}\right)-\mathrm{Li}_2\left(i\,e^{-i\,(b+a\,z)}\right)\right)\Big)$$

01.11.21.0022.01

$$\int z^n\sec(a\,z)\,dz = n!\,2\,e^{i\,a\,z}\sum_{j=0}^{n}\frac{(-1)^j\,z^{n-j}}{(n-j)!\,(i\,a)^{j+1}}\;{}_{j+2}F_{j+1}\left(\frac{1}{2},\,\ldots,\,\frac{1}{2},\,1;\,\frac{3}{2},\,\ldots,\,\frac{3}{2};\,-e^{2\,i\,a\,z}\right)/;\,n\in\mathbb{N}^+$$

01.11.21.0023.01

$$\int z\sec(a\,z)\,dz = \frac{a\,z\left(\log\left(1-i\,e^{i\,a\,z}\right)-\log\left(1+i\,e^{i\,a\,z}\right)\right)+i\left(\mathrm{Li}_2\left(-i\,e^{i\,a\,z}\right)-\mathrm{Li}_2\left(i\,e^{i\,a\,z}\right)\right)}{a^2}$$

01.11.21.0024.01

$$\int z^2\sec(a\,z)\,dz = \frac{1}{a^3}\left(a^2\left(\log\left(1-i\,e^{i\,a\,z}\right)-\log\left(1+i\,e^{i\,a\,z}\right)\right)z^2+2\,a\,i\left(\mathrm{Li}_2\left(-i\,e^{i\,a\,z}\right)-\mathrm{Li}_2\left(i\,e^{i\,a\,z}\right)\right)z-2\,\mathrm{Li}_3\left(-i\,e^{i\,a\,z}\right)+2\,\mathrm{Li}_3\left(i\,e^{i\,a\,z}\right)\right)$$

01.11.21.0025.01

$$\int z^3\sec(a\,z)\,dz = \frac{1}{64\,a^4}\Big(16\,a^4\,i\,z^4+64\,a^3\log\left(1+i\,e^{-i\,a\,z}\right)z^3-64\,a^3\log\left(1+i\,e^{i\,a\,z}\right)z^3-32\,i\,a^3\,\pi\,z^3+24\,a^2\,i\,\pi^2\,z^2-96\,a^2\,\pi\log\left(1-i\,e^{-i\,a\,z}\right)z^2+ 96\,a^2\,\pi\log\left(1+i\,e^{i\,a\,z}\right)z^2+192\,a^2\,i\,\mathrm{Li}_2\left(-i\,e^{-i\,a\,z}\right)z^2+192\,a^2\,i\,\mathrm{Li}_2\left(-i\,e^{i\,a\,z}\right)z^2-8\,i\,a\,\pi^3\,z+48\,a\,\pi^2\log\left(1-i\,e^{-i\,a\,z}\right)z- 48\,a\,\pi^2\log\left(1+i\,e^{i\,a\,z}\right)z-192\,i\,a\,\pi\,\mathrm{Li}_2\left(-i\,e^{i\,a\,z}\right)z+384\,a\,\mathrm{Li}_3\left(-i\,e^{-i\,a\,z}\right)z-384\,a\,\mathrm{Li}_3\left(-i\,e^{i\,a\,z}\right)z-7\,i\,\pi^4- 8\,\pi^3\log\left(1+i\,e^{-i\,a\,z}\right)+8\,\pi^3\log\left(1+i\,e^{i\,a\,z}\right)+8\,\pi^3\log\left(\cot\left(\frac{1}{4}\,(\pi-2\,a\,z)\right)\right)+48\,i\,\pi\,(\pi-4\,a\,z)\,\mathrm{Li}_2\left(i\,e^{-i\,a\,z}\right)+ 48\,i\,\pi^2\,\mathrm{Li}_2\left(-i\,e^{i\,a\,z}\right)-192\,\pi\,\mathrm{Li}_3\left(i\,e^{-i\,a\,z}\right)+192\,\pi\,\mathrm{Li}_3\left(-i\,e^{i\,a\,z}\right)-384\,i\,\mathrm{Li}_4\left(-i\,e^{-i\,a\,z}\right)-384\,i\,\mathrm{Li}_4\left(-i\,e^{i\,a\,z}\right)\Big)$$

01.11.21.0026.01

$$\int z^4\sec(a\,z)\,dz = \frac{1}{80\,a^5}\Big(16\,a^5\,i\,z^5+80\,a^4\log\left(1+i\,e^{-i\,a\,z}\right)z^4-80\,a^4\log\left(1+i\,e^{i\,a\,z}\right)z^4-40\,i\,a^3\,\pi^2\,z^3+320\,a^3\,i\,\mathrm{Li}_2\left(-i\,e^{-i\,a\,z}\right)z^3+ 320\,a^3\,i\,\mathrm{Li}_2\left(-i\,e^{i\,a\,z}\right)z^3+40\,a^2\,i\,\pi^3\,z^2-120\,a^2\,\pi^2\log\left(1-i\,e^{-i\,a\,z}\right)z^2+120\,a^2\,\pi^2\log\left(1+i\,e^{i\,a\,z}\right)z^2+ 960\,a^2\,\mathrm{Li}_3\left(-i\,e^{-i\,a\,z}\right)z^2-960\,a^2\,\mathrm{Li}_3\left(-i\,e^{i\,a\,z}\right)z^2-15\,i\,a\,\pi^4\,z+80\,a\,\pi^3\log\left(1-i\,e^{-i\,a\,z}\right)z-80\,a\,\pi^3\log\left(1+i\,e^{i\,a\,z}\right)z- 240\,i\,a\,\pi^2\,\mathrm{Li}_2\left(-i\,e^{i\,a\,z}\right)z-1920\,i\,a\,\mathrm{Li}_4\left(-i\,e^{-i\,a\,z}\right)z-1920\,i\,a\,\mathrm{Li}_4\left(-i\,e^{i\,a\,z}\right)z-10\,i\,\pi^5-5\,\pi^4\log\left(1+i\,e^{-i\,a\,z}\right)- 10\,\pi^4\log\left(1-i\,e^{-i\,a\,z}\right)+15\,\pi^4\log\left(1+i\,e^{i\,a\,z}\right)+5\,\pi^4\log\left(\cot\left(\frac{1}{4}\,(\pi-2\,a\,z)\right)\right)+80\,i\,\pi^2\,(\pi-3\,a\,z)\,\mathrm{Li}_2\left(i\,e^{-i\,a\,z}\right)+ 80\,i\,\pi^3\,\mathrm{Li}_2\left(-i\,e^{i\,a\,z}\right)-240\,\pi^2\,\mathrm{Li}_3\left(i\,e^{-i\,a\,z}\right)+240\,\pi^2\,\mathrm{Li}_3\left(-i\,e^{i\,a\,z}\right)-1920\,\mathrm{Li}_5\left(-i\,e^{-i\,a\,z}\right)+1920\,\mathrm{Li}_5\left(-i\,e^{i\,a\,z}\right)\Big)$$

Involving exponential function

Involving exp

Involving $a^{b z}$

01.11.21.0027.01

$$\int a^{b z} \sec(c z) \, dz = -\frac{2 i a^{b z} e^{i c z}}{c - i b \log(a)} \, {}_2F_1\left(\frac{c - i b \log(a)}{2 c}, 1; \frac{3}{2} - \frac{i b \log(a)}{2 c}; -e^{2 i c z}\right)$$

01.11.21.0028.01

$$\int e^{b z} \sec(a z) \, dz = -\frac{2 i e^{(b + i a) z}}{a - i b} \, {}_2F_1\left(\frac{a - i b}{2 a}, 1; \frac{3}{2} - \frac{i b}{2 a}; -e^{2 i a z}\right)$$

01.11.21.0029.01

$$\int e^{a z} \sec(a z) \, dz = \frac{(1 - i) e^{(1 + i) a z}}{a} \, {}_2F_1\left(\frac{1}{2} - \frac{i}{2}, 1; \frac{3}{2} - \frac{i}{2}; -e^{2 i a z}\right)$$

Involving exponential function and a power function

Involving exp and power

Involving $z^n e^{b z}$

01.11.21.0030.01

$$\int z^n e^{b z} \sec(a z) \, dz = n! \, 2 e^{(b + i a) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j}}{(n-j)! \, (b + i a)^{j+1}} \, {}_{j+2}F_{j+1}\left(\frac{a - i b}{2 a}, \ldots, \frac{a - i b}{2 a}, 1; \frac{a - i b}{2 a} + 1, \ldots, \frac{a - i b}{2 a} + 1; -e^{2 i a z}\right) /; n \in \mathbb{N}$$

01.11.21.0031.01

$$\int z^n e^{-i c z} \sec(c z) \, dz = \frac{2 z^{1+n}}{1 + n} - 2 e^{2 i c z} n! \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (i c)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.11.21.0032.01

$$\int z^n e^{-i c z (2 q + 1)} \sec(c z) \, dz = 2 n! \left(\frac{(-1)^q z^{n+1}}{(n+1)!} + (-1)^q e^{2 i c z} \sum_{j=0}^{n} \frac{(-2 i c)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2 i c z}\right) - \sum_{j=0}^{n} \sum_{k=0}^{q-1} \frac{(-1)^k e^{2 i c (k - q) z} (2 i c (q - k))^{-j-1} z^{n-j}}{(n-j)!}\right) /; n \in \mathbb{N} \land q \in \mathbb{N}^+$$

Arguments involving inverse trigonometric functions

Involving $\sin^{-1}$

01.11.21.0033.01

$$\int \sec\left(\sin^{-1}(z)\right) dz = \sin^{-1}(z)$$

01.11.21.0034.01

$$\int \sec\left(a \sin^{-1}(z)\right) dz = -\frac{i\, e^{-i \sin^{-1}(z)}}{a^2-1} \left((a+1)\, e^{i a \sin^{-1}(z)}\, {}_2F_1\left(\frac{a-1}{2a}, 1; \frac{3}{2} - \frac{1}{2a}; -e^{2 i a \sin^{-1}(z)}\right) + (a-1)\, e^{i (a+2) \sin^{-1}(z)}\, {}_2F_1\left(\frac{a+1}{2a}, 1; \frac{1}{2}\left(3 + \frac{1}{a}\right); -e^{2 i a \sin^{-1}(z)}\right)\right)$$

Involving $\cos^{-1}$

01.11.21.0035.01

$$\int \sec\left(\cos^{-1}(z)\right) dz = \log(z)$$

01.11.21.0036.01

$$\int \sec\left(a \cos^{-1}(z)\right) dz = \frac{e^{i (a-1) \cos^{-1}(z)}}{a^2-1} \left((a-1)\, e^{2 i \cos^{-1}(z)}\, {}_2F_1\left(\frac{a+1}{2a}, 1; \frac{1}{2}\left(3 + \frac{1}{a}\right); -e^{2 i a \cos^{-1}(z)}\right) - (a+1)\, {}_2F_1\left(\frac{a-1}{2a}, 1; \frac{3}{2} - \frac{1}{2a}; -e^{2 i a \cos^{-1}(z)}\right)\right)$$

Involving $\tan^{-1}$

01.11.21.0037.01

$$\int \sec\left(\tan^{-1}(z)\right) dz = \frac{1}{2} \sqrt{z^2+1}\, z + \frac{1}{2} \sinh^{-1}(z)$$

Involving $\cot^{-1}$

01.11.21.0038.01

$$\int \sec\left(\cot^{-1}(z)\right) dz = \frac{z \left(\log(z) - \log\left(\sqrt{z^2+1} + 1\right) + \sqrt{z^2+1}\right)}{\sqrt{z^2+1}} \sqrt{1 + \frac{1}{z^2}}$$

Involving $\csc^{-1}$

01.11.21.0039.01

$$\int \sec\left(\csc^{-1}(z)\right) dz = \sqrt{1 - \frac{1}{z^2}}\, z$$

Involving $\sec^{-1}$

01.11.21.0040.01

$$\int \sec\left(\sec^{-1}(z)\right) dz = \frac{z^2}{2}$$

Arguments involving inverse hyperbolic functions

Involving $\sinh^{-1}$

01.11.21.0041.01

$$\int \sec\left(\sinh^{-1}(z)\right) dz = \left(\frac{1}{2}-\frac{i}{2}\right) e^{(-1+i)\sinh^{-1}(z)} \left(e^{2\sinh^{-1}(z)} {}_2F_1\left(\frac{1}{2}-\frac{i}{2}, 1; \frac{3}{2}-\frac{i}{2}; -e^{2 i \sinh^{-1}(z)}\right) - i\, {}_2F_1\left(\frac{1}{2}+\frac{i}{2}, 1; \frac{3}{2}+\frac{i}{2}; -e^{2 i \sinh^{-1}(z)}\right)\right)$$

01.11.21.0042.01

$$\int \sec\left(a \sinh^{-1}(z)\right) dz = -\frac{i\, e^{i (a+i)\sinh^{-1}(z)}}{a^2+1} \left((a+i)\, e^{2\sinh^{-1}(z)} {}_2F_1\left(\frac{a-i}{2a}, 1; \frac{3}{2}-\frac{i}{2a}; -e^{2 i a \sinh^{-1}(z)}\right) + (a-i)\, {}_2F_1\left(\frac{a+i}{2a}, 1; \frac{3}{2}+\frac{i}{2a}; -e^{2 i a \sinh^{-1}(z)}\right)\right)$$

Involving $\cosh^{-1}$

01.11.21.0043.01

$$\int \sec\left(\cosh^{-1}(z)\right) dz = \left(\frac{1}{2}+\frac{i}{2}\right) e^{(-1+i)\cosh^{-1}(z)} \left({}_2F_1\left(\frac{1}{2}+\frac{i}{2}, 1; \frac{3}{2}+\frac{i}{2}; -e^{2 i \cosh^{-1}(z)}\right) - i\, e^{2\cosh^{-1}(z)} {}_2F_1\left(\frac{1}{2}-\frac{i}{2}, 1; \frac{3}{2}-\frac{i}{2}; -e^{2 i \cosh^{-1}(z)}\right)\right)$$

01.11.21.0044.01

$$\int \sec\left(a \cosh^{-1}(z)\right) dz = \frac{e^{i (a+i)\cosh^{-1}(z)}}{a^2+1} \left((1-i a)\, e^{2\cosh^{-1}(z)} {}_2F_1\left(\frac{a-i}{2a}, 1; \frac{3}{2}-\frac{i}{2a}; -e^{2 i a \cosh^{-1}(z)}\right) + (1+i a)\, {}_2F_1\left(\frac{a+i}{2a}, 1; \frac{3}{2}+\frac{i}{2a}; -e^{2 i a \cosh^{-1}(z)}\right)\right)$$

Involving trigonometric functions

Involving sin

Involving $\sin(b z)$

01.11.21.0045.01

$$\int \sin(b z) \sec(c z)\, dz = -\frac{e^{i (c-b) z}}{(b-c)(b+c)} \left((b+c)\, {}_2F_1\left(\frac{c-b}{2c}, 1; \frac{3}{2}-\frac{b}{2c}; -e^{2 i c z}\right) + (b-c)\, e^{2 i b z} {}_2F_1\left(\frac{b+c}{2c}, 1; \frac{b+3c}{2c}; -e^{2 i c z}\right)\right)$$

Involving power of sin

Involving $\sin^m(b z)$

01.11.21.0046.01

$$\int \sin^m(b z) \sec(c z) \, dz = \frac{\left(2^{1-m} (1 - m \bmod 2)\right)}{c} \tanh^{-1}\left(\tan\left(\frac{c z}{2}\right)\right) \binom{m}{\frac{m}{2}} + 2^{1-m} i^{-m-1} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{e^{i (c+b(-2k+m)) z} \, _2F_1\left(1, \frac{c-2bk+bm}{2c}; \frac{3c-2bk+bm}{2c}; -e^{2icz}\right)}{c - 2bk + bm} + \frac{(-1)^m e^{i(c+2bk-bm)z} \, _2F_1\left(1, \frac{c+2bk-bm}{2c}; \frac{3c+2bk-bm}{2c}; -e^{2icz}\right)}{c + 2bk - bm} \right) /; m \in \mathbb{N}^+$$

Involving cos

Involving cos$(b z)$

01.11.21.0047.01

$$\int \cos(b z) \sec(c z) \, dz = \frac{i e^{i(c-b)z}}{(b-c)(b+c)} \left((b+c) \, _2F_1\left(\frac{c-b}{2c}, 1; \frac{3}{2} - \frac{b}{2c}; -e^{2icz}\right) - (b-c) e^{2ibz} \, _2F_1\left(\frac{b+c}{2c}, 1; \frac{b+3c}{2c}; -e^{2icz}\right) \right)$$

Involving power of cos

Involving cos$^m(b z)$

01.11.21.0048.01

$$\int \cos^m(b z) \sec(c z) \, dz = \frac{i \, 2^{1-m} \, (m \bmod 2 - 1)}{c} \tan^{-1}\left(e^{icz}\right) \binom{m}{\frac{m}{2}} - i \, 2^{1-m} \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{s} \left(\frac{e^{i(c-bm+2bs)z} \, _2F_1\left(1, \frac{c-bm+2bs}{2c}; \frac{3c-bm+2bs}{2c}; -e^{2icz}\right)}{c - bm + 2bs} + \frac{e^{i(c+b(m-2s))z} \, _2F_1\left(1, \frac{c+bm-2bs}{2c}; \frac{3c+bm-2bs}{2c}; -e^{2icz}\right)}{c + b(m-2s)} \right) /; m \in \mathbb{N}^+$$

Involving trigonometric and a power functions

Involving sin and power

Involving z^n sin$(a + b z)$

01.11.21.0049.01

$$\int z^n \sin(a+b\,z)\sec(c\,z)\,dz = -i\,e^{i\,(a+(b+c)\,z)}\,n!\sum_{j=0}^{n}\frac{(-1)^j\,(i\,c+i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c+b}{2\,c},\,\ldots,\,\frac{c+b}{2\,c},\,1;\,\frac{c+b}{2\,c}+1,\,\ldots,\,\frac{c+b}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+ i\,e^{-i\,(a+(b-c)\,z)}\,n!\sum_{j=0}^{n}\frac{(-1)^j\,(i\,c-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c-b}{2\,c},\,\ldots,\,\frac{c-b}{2\,c},\,1;\,\frac{c-b}{2\,c}+1,\,\ldots,\,\frac{c-b}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)\,/;\,n\in\mathbb{N}$$

01.11.21.0050.01

$$\int z^n \sin(b\,z)\sec(c\,z)\,dz = -i\,e^{i\,(b+c)\,z}\,n!\sum_{j=0}^{n}\frac{(-1)^j\,(i\,c+i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c+b}{2\,c},\,\ldots,\,\frac{c+b}{2\,c},\,1;\,\frac{c+b}{2\,c}+1,\,\ldots,\,\frac{c+b}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+ i\,e^{-i\,(b-c)\,z}\,n!\sum_{j=0}^{n}\frac{(-1)^j\,(i\,c-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c-b}{2\,c},\,\ldots,\,\frac{c-b}{2\,c},\,1;\,\frac{c-b}{2\,c}+1,\,\ldots,\,\frac{c-b}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)\,/;\,n\in\mathbb{N}$$

Involving power of sin and power

Involving $z^n \sin^m(b\,z)$

01.11.21.0051.01

$$\int z^n \sin^m(b\,z)\sec(c\,z)\,dz = 2^{1-m}\,e^{i\,c\,z}\binom{m}{\frac{m}{2}}\,n!\,(1-m \bmod 2)\sum_{j=0}^{n}\frac{\left((-1)^j\,z^{n-j}\right)}{(n-j)!\,(i\,c)^{j+1}}\,{}_{j+2}F_{j+1}\left(\frac{1}{2},\,\ldots,\,\frac{1}{2},\,1;\,\frac{3}{2},\,\ldots,\,\frac{3}{2};\,-e^{2\,i\,c\,z}\right)+ 2^{1-m}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\Bigg(e^{\frac{i\,\pi\,m}{2}+i\,(c+2\,b\,k-b\,m)\,z}\sum_{j=0}^{n}\frac{\left((-1)^j\,z^{n-j}\right)}{(n-j)!\,(i\,(c+2\,b\,k-b\,m))^{j+1}}\,{}_{j+2}F_{j+1}\left(\frac{c-b\,(m-2\,k)}{2\,c},\,\ldots,\,\frac{c-b\,(m-2\,k)}{2\,c},\,1;\,\frac{c-b\,(m-2\,k)}{2\,c}+1,\,\ldots,\,\frac{c-b\,(m-2\,k)}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+ e^{i\,(c+b\,(m-2\,k))\,z-\frac{i\,m\,\pi}{2}}\sum_{j=0}^{n}\frac{\left((-1)^j\,z^{n-j}\right)}{(n-j)!\,(i\,(c+b\,(m-2\,k)))^{j+1}}\,{}_{j+2}F_{j+1}\left(\frac{c+b\,(m-2\,k)}{2\,c},\,\ldots,\,\frac{c+b\,(m-2\,k)}{2\,c},\,1;\,\frac{c+b\,(m-2\,k)}{2\,c}+1,\,\ldots,\,\frac{c+b\,(m-2\,k)}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)\Bigg)\,/;\,n\in\mathbb{N}^+\land m\in\mathbb{N}^+$$

Involving cos and power

Involving $z^n \cos(a+b\,z)$

01.11.21.0052.01

$$\int z^n \cos(a+b z) \sec(c z)\, dz = e^{i (a+(b+c) z)}\, n! \sum_{j=0}^{n} \frac{(-1)^j (i c + i b)^{-j-1} z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(\frac{c+b}{2c}, \ldots, \frac{c+b}{2c}, 1; \frac{c+b}{2c}+1, \ldots, \frac{c+b}{2c}+1; -e^{2 i c z}\right) + e^{-i (a+(b-c) z)}\, n! \sum_{j=0}^{n} \frac{(-1)^j (i c - i b)^{-j-1} z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(\frac{c-b}{2c}, \ldots, \frac{c-b}{2c}, 1; \frac{c-b}{2c}+1, \ldots, \frac{c-b}{2c}+1; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.11.21.0053.01

$$\int z^n \cos(b z) \sec(c z)\, dz = e^{i (b+c) z}\, n! \sum_{j=0}^{n} \frac{(-1)^j (i c + i b)^{-j-1} z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(\frac{c+b}{2c}, \ldots, \frac{c+b}{2c}, 1; \frac{c+b}{2c}+1, \ldots, \frac{c+b}{2c}+1; -e^{2 i c z}\right) + e^{-i (b-c) z}\, n! \sum_{j=0}^{n} \frac{(-1)^j (i c - i b)^{-j-1} z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(\frac{c-b}{2c}, \ldots, \frac{c-b}{2c}, 1; \frac{c-b}{2c}+1, \ldots, \frac{c-b}{2c}+1; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

Involving power of cos and power

Involving $z^n \cos^m(b z)$

01.11.21.0054.01

$$\int z^n \cos^m(b z) \sec(c z)\, dz = 2^{1-m} e^{i c z} \binom{m}{\frac{m}{2}} n! \,(1 - m \bmod 2) \sum_{j=0}^{n} \frac{\left((-1)^j z^{n-j}\right)}{(n-j)!\, (i c)^{j+1}}\; {}_{j+2}F_{j+1}\left(\frac{1}{2}, \ldots, \frac{1}{2}, 1; \frac{3}{2}, \ldots, \frac{3}{2}; -e^{2 i c z}\right) + 2^{1-m} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \Bigg(e^{i (c+2 b k - b m) z} \sum_{j=0}^{n} \frac{\left((-1)^j z^{n-j}\right)}{(n-j)!\, (i (c + 2 b k - b m))^{j+1}}\; {}_{j+2}F_{j+1}\left(\frac{c - b (m-2k)}{2c}, \ldots, \frac{c - b (m-2k)}{2c}, 1; \frac{c - b (m-2k)}{2c}+1, \ldots, \frac{c - b (m-2k)}{2c}+1; -e^{2 i c z}\right) + e^{i (c+b (m-2k)) z} \sum_{j=0}^{n} \frac{\left((-1)^j z^{n-j}\right)}{(n-j)!\, (i (c + b (m-2k)))^{j+1}}\; {}_{j+2}F_{j+1}\left(\frac{c + b (m-2k)}{2c}, \ldots, \frac{c + b (m-2k)}{2c}, 1; \frac{c + b (m-2k)}{2c}+1, \ldots, \frac{c + b (m-2k)}{2c}+1; -e^{2 i c z}\right)\Bigg) /; n \in \mathbb{N}^+ \land m \in \mathbb{N}^+$$

Involving trigonometric and exponential functions

Involving sin and exp

Involving $e^{p z} \sin(b z)$

01.11.21.0055.01

$$\int e^{p z} \sin(b z) \sec(c z) \, dz = -\frac{e^{(i b+i c+p) z} \, {}_2F_1\left(1, \frac{b+c-i p}{2 c}; \frac{b+3 c-i p}{2 c}; -e^{2 i c z}\right)}{b+c-i p} - \frac{e^{(-i b+i c+p) z} \, {}_2F_1\left(1, -\frac{b-c+i p}{2 c}; -\frac{b-3 c+i p}{2 c}; -e^{2 i c z}\right)}{b-c+i p}$$

01.11.21.0056.01

$$\int e^{i (a-c) z} \sin(a z) \sec(c z) \, dz = i z - \frac{e^{2 i a z}}{2 a} \, {}_2F_1\left(1, \frac{a}{c}; \frac{a+c}{c}; -e^{2 i c z}\right) - \frac{\log\left(1+e^{2 i c z}\right)}{2 c}$$

01.11.21.0057.01

$$\int e^{-i (a+c) z} \sin(a z) \sec(c z) \, dz = -i z + \frac{\log\left(1+e^{2 i c z}\right)}{2 c} - \frac{e^{-2 i a z}}{2 a} \, {}_2F_1\left(1, -\frac{a}{c}; \frac{c-a}{c}; -e^{2 i c z}\right)$$

Involving power of sin and exp

Involving $e^{p z} \sin^m(b z)$

01.11.21.0058.01

$$\int e^{p z} \sin^m(b z) \sec(c z) \, dz = \frac{2^{1-m} (1 - m \bmod 2)}{i c + p} e^{(i c+p) z} \binom{m}{\frac{m}{2}} {}_2F_1\left(1, -\frac{i (i c + p)}{2 c}; \frac{1}{2}\left(3 - \frac{i p}{c}\right); -e^{2 i c z}\right) + 2^{1-m}$$
$$\sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{e^{(i c-2 i b k+p) z-\frac{1}{2} i m (\pi-2 b z)}}{i c - 2 i b k + i b m + p} \, {}_2F_1\left(1, -\frac{i (i c - 2 i b k + i b m + p)}{2 c}; -\frac{i (3 i c - 2 i b k + i b m + p)}{2 c}; -e^{2 i c z}\right) + \right.$$
$$\left. \frac{e^{(i c+2 i b k+p) z+\frac{1}{2} i m (\pi-2 b z)}}{i c + 2 i b k - i b m + p} \, {}_2F_1\left(1, -\frac{i (i c + 2 i b k - i b m + p)}{2 c}; -\frac{i (3 i c + 2 i b k - i b m + p)}{2 c}; -e^{2 i c z}\right) \right) /; m \in \mathbb{N}^+$$

Involving cos and exp

Involving $e^{p z} \cos(b z)$

01.11.21.0059.01

$$\int e^{p z} \cos(b z) \sec(c z) \, dz = \frac{i e^{(-i b+i c+p) z} \, {}_2F_1\left(1, -\frac{b-c+i p}{2 c}; -\frac{b-3 c+i p}{2 c}; -e^{2 i c z}\right)}{b-c+i p} - \frac{i e^{(i b+i c+p) z} \, {}_2F_1\left(1, \frac{b+c-i p}{2 c}; \frac{b+3 c-i p}{2 c}; -e^{2 i c z}\right)}{b+c-i p}$$

01.11.21.0060.01

$$\int e^{i (a-c) z} \cos(a z) \sec(c z) \, dz = z + \frac{i \log\left(1+e^{2 i c z}\right)}{2 c} - \frac{i e^{2 i a z}}{2 a} \, {}_2F_1\left(1, \frac{a}{c}; \frac{a}{c}+1; -e^{2 i c z}\right)$$

01.11.21.0061.01

$$\int e^{-i (a+c) z} \cos(a z) \sec(c z) \, dz = z + \frac{e^{-2 i a z} i}{2 a} \, {}_2F_1\left(1, -\frac{a}{c}; 1-\frac{a}{c}; -e^{2 i c z}\right) + \frac{i \log\left(1+e^{2 i c z}\right)}{2 c}$$

Involving power of cos and exp

Involving $e^{p z} \cos^m(b z)$

01.11.21.0062.01

$$\int e^{p z} \cos^m(b z) \sec(c z)\, dz = \frac{(1 - m \bmod 2)\, 2^{1-m}\, e^{(i c+p) z}}{i c + p} \binom{m}{\frac{m}{2}}\, {}_2F_1\!\left(1, \frac{c - i p}{2 c}; \frac{1}{2}\left(3 - \frac{i p}{c}\right); -e^{2 i c z}\right) - i\, 2^{1-m} \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{s} \left(\frac{e^{(i c+p-i b (m-2 s)) z}}{c - b m - i p + 2 b s}\, {}_2F_1\!\left(1, \frac{c - b m - i p + 2 b s}{2 c}; \frac{3 c - b m - i p + 2 b s}{2 c}; -e^{2 i c z}\right) + \frac{e^{(i c+p+i b (m-2 s)) z}}{c - i p + b (m - 2 s)}\, {}_2F_1\!\left(1, \frac{c + b m - i p - 2 b s}{2 c}; \frac{3 c + b m - i p - 2 b s}{2 c}; -e^{2 i c z}\right)\right) /; m \in \mathbb{N}^+$$

Involving trigonometric, exponential and a power functions

Involving sin, exp and power

Involving $z^n\, e^{p z} \sin(a + b z) \sec(c z)$

01.11.21.0063.01

$$\int z^n e^{p z} \sin(a + b z) \sec(c z)\, dz = i\, e^{-i (a+(b-c+i p) z)}\, n! \sum_{j=0}^{n} \frac{(-1)^j (i c - i b + p)^{-j-1} z^{n-j}}{(n - j)!}\, {}_{j+2}F_{j+1}\!\left(-\frac{b - c + i p}{2 c}, \ldots, -\frac{b - c + i p}{2 c}, 1; 1 - \frac{b - c + i p}{2 c}, \ldots, 1 - \frac{b - c + i p}{2 c}; -e^{2 i c z}\right) - i\, e^{i (a+(b+c-i p) z)}\, n! \sum_{j=0}^{n} \frac{(-1)^j (i c + i b + p)^{-j-1} z^{n-j}}{(n - j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{b + c - i p}{2 c}, \ldots, \frac{b + c - i p}{2 c}, 1; \frac{b + c - i p}{2 c} + 1, \ldots, \frac{b + c - i p}{2 c} + 1; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.11.21.0064.01

$$\int z^n e^{p z} \sin(b z) \sec(c z)\, dz = i\, e^{-i (b-c+i p) z}\, n! \sum_{j=0}^{n} \frac{(-1)^j (i c - i b + p)^{-j-1} z^{n-j}}{(n - j)!}\, {}_{j+2}F_{j+1}\!\left(-\frac{b - c + i p}{2 c}, \ldots, -\frac{b - c + i p}{2 c}, 1; 1 - \frac{b - c + i p}{2 c}, \ldots, 1 - \frac{b - c + i p}{2 c}; -e^{2 i c z}\right) - i\, e^{i (b+c-i p) z}\, n! \sum_{j=0}^{n} \frac{(-1)^j (i c + i b + p)^{-j-1} z^{n-j}}{(n - j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{b + c - i p}{2 c}, \ldots, \frac{b + c - i p}{2 c}, 1; \frac{b + c - i p}{2 c} + 1, \ldots, \frac{b + c - i p}{2 c} + 1; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.11.21.0065.01

$$\int z^n e^{i (b-c) z} \sin(b z) \sec(c z)\, dz = i \left(\frac{z^{n+1}}{n + 1} - e^{2 i c z}\, n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n - j)!}\, {}_{j+2}F_{j+1}\!\left(1, \ldots, 1; 2, \ldots, 2; -e^{2 i c z}\right) - n!\, e^{2 i b z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i b)^{-j-1}}{(n - j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{b}{c}, \ldots, \frac{b}{c}, 1; \frac{b}{c} + 1, \ldots, \frac{b}{c} + 1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

01.11.21.0066.01

$$\int z^n e^{-i(b+c)z}\sin(bz)\sec(cz)\,dz = i\left(-\frac{z^{n+1}}{n+1}+n!\,e^{2icz}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(2ic)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(1,\ldots,1,1;2,\ldots,2;-e^{2icz}\right)-n!\,e^{-2ibz}\sum_{j=0}^{n}\frac{z^{n-j}(2ib)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{b}{c},\ldots,-\frac{b}{c},1;1-\frac{b}{c},\ldots,1-\frac{b}{c};-e^{2icz}\right)\right)/;\,n\in\mathbb{N}$$

Involving powers of sin, exp and power

Involving $z^n e^{pz}\sin^m(bz)\sec(cz)$

01.11.21.0067.01

$$\int z^n e^{pz}\sin^m(bz)\sec(cz)\,dz = 2^{1-m}e^{(p+ic)z}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(p+ic)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c-ip}{2c},\ldots,\frac{c-ip}{2c},1;\frac{c-ip}{2c}+1,\ldots,\frac{c-ip}{2c}+1;-e^{2icz}\right)+$$
$$2^{1-m}\,n!\,e^{icz}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(e^{(p+bi(m-2k))z-\frac{i\pi m}{2}}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(p+bi(m-2k)+ic)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c-2bk+bm-ip}{2c},\ldots,\frac{c-2bk+bm-ip}{2c},1;\frac{c-2bk+bm-ip}{2c}+1,\ldots,\frac{c-2bk+bm-ip}{2c}+1;-e^{2icz}\right)+e^{\frac{i\pi m}{2}+(p-ib(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}(p-ib(m-2k)+ic)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c+2bk-bm-ip}{2c},\ldots,\frac{c+2bk-bm-ip}{2c},1;\frac{c+2bk-bm-ip}{2c}+1,\ldots,\frac{c+2bk-bm-ip}{2c}+1;-e^{2icz}\right)\right)/;\,n\in\mathbb{N}\land m\in\mathbb{N}^+$$

Involving cos, exp and power

Involving $z^n e^{pz}\cos(a+bz)\sec(cz)$

01.11.21.0068.01

$$\int z^n e^{p z} \cos(a + b z) \sec(c z) \, dz = e^{-i a + (i c - i b + p) z} n! \sum_{j=0}^{n} \frac{(-1)^j (i c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(-\frac{b - c + i p}{2 c}, \ldots, -\frac{b - c + i p}{2 c}, 1; 1 - \frac{b - c + i p}{2 c}, \ldots, 1 - \frac{b - c + i p}{2 c}; -e^{2 i c z}\right) + e^{i a + (i c + i b + p) z} n! \sum_{j=0}^{n} \frac{(-1)^j (i c + i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + c - i p}{2 c}, \ldots, \frac{b + c - i p}{2 c}, 1; \frac{b + c - i p}{2 c} + 1, \ldots, \frac{b + c - i p}{2 c} + 1; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.11.21.0069.01

$$\int z^n e^{p z} \cos(b z) \sec(c z) \, dz = n! \, e^{i c z} \left(e^{(-i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (-i b + p + i c)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c - i p - b}{2 c}, \ldots, \frac{c - i p - b}{2 c}, 1; \frac{c - i p - b}{2 c} + 1, \ldots, \frac{c - i p - b}{2 c} + 1; -e^{2 i c z}\right) + e^{(i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i b + p + i c)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c - i p + b}{2 c}, \ldots, \frac{c - i p + b}{2 c}, 1; \frac{c - i p + b}{2 c} + 1, \ldots, \frac{c - i p + b}{2 c} + 1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

01.11.21.0070.01

$$\int z^n e^{i (b - c) z} \cos(b z) \sec(c z) \, dz = \frac{z^{n+1}}{n+1} - e^{2 i c z} n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2 i c z}\right) + e^{2 i b z} n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i b)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b}{c}, \ldots, \frac{b}{c}, 1; \frac{b}{c} + 1, \ldots, \frac{b}{c} + 1; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.11.21.0071.01

$$\int z^n e^{-i (b + c) z} \cos(b z) \sec(c z) \, dz = \frac{z^{n+1}}{n+1} - n! \, e^{2 i c z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2 i c z}\right) - n! \, e^{-2 i b z} \sum_{j=0}^{n} \frac{z^{n-j} (2 i b)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(-\frac{b}{c}, \ldots, -\frac{b}{c}, 1; 1 - \frac{b}{c}, \ldots, 1 - \frac{b}{c}; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

Involving powers of cos, exp and power

Involving $z^n e^{p z} \cos^m(b z) \sec(c z)$

01.11.21.0072.01

$$\int z^n e^{pz} \cos^m(bz) \sec(cz)\, dz = e^{(p+ic)z} \binom{m}{\frac{m}{2}} 2^{1-m} n! (1 - m \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (p+ic)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c-ip}{2c}, \ldots, \frac{c-ip}{2c}, 1; \frac{c-ip}{2c}+1, \ldots, \frac{c-ip}{2c}+1; -e^{2icz}\right) + 2^{1-m} n! e^{icz} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(e^{(p+ib(m-2k))z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (p+ib(m-2k)+ic)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c-2bk+bm-ip}{2c}, \ldots, \frac{c-2bk+bm-ip}{2c}, 1; \frac{c-2bk+bm-ip}{2c}+1, \ldots, \frac{c-2bk+bm-ip}{2c}+1; -e^{2icz}\right) + e^{(p-ib(m-2k))z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (p-ib(m-2k)+ic)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c+2bk-bm-ip}{2c}, \ldots, \frac{c+2bk-bm-ip}{2c}, 1; \frac{c+2bk-bm-ip}{2c}+1, \ldots, \frac{c+2bk-bm-ip}{2c}+1; -e^{2icz}\right)\right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving functions of the direct function

Involving powers of the direct function

Involving powers of sech

Linear argument

01.11.21.0073.02

$$\int \sec^\nu(cz)\, dz = \frac{\cos^2(cz)^{\frac{\nu-1}{2}} \sec^{\nu-1}(cz) \sin(cz)}{c} \, {}_2F_1\left(\frac{1}{2}, \frac{\nu+1}{2}; \frac{3}{2}; \sin^2(cz)\right)$$

01.11.21.0074.01

$$\int \sec^2(cz)\, dz = \frac{\tan(cz)}{c}$$

01.11.21.0075.01

$$\int \sec^3(cz)\, dz = \frac{2\tanh^{-1}\left(\tan\left(\frac{cz}{2}\right)\right) + \sec(cz)\tan(cz)}{2c}$$

01.11.21.0076.01

$$\int \sec^4(cz)\, dz = \frac{\left(\sec^2(cz) + 2\right)\tan(cz)}{3c}$$

01.11.21.0077.01

$$\int \sec^5(cz)\, dz = \frac{6\tanh^{-1}\left(\tan\left(\frac{cz}{2}\right)\right) + \sec(cz)\left(2\sec^2(cz) + 3\right)\tan(cz)}{8c}$$

01.11.21.0078.01

$$\int \sec^6(c z)\, dz = \frac{\left(3 \sec^4(c z) + 4 \sec^2(c z) + 8\right) \tan(c z)}{15 c}$$

01.11.21.0079.01

$$\int \sec^7(c z)\, dz = \frac{30 \tanh^{-1}\left(\tan\left(\frac{c z}{2}\right)\right) + \sec(c z) \left(8 \sec^4(c z) + 10 \sec^2(c z) + 15\right) \tan(c z)}{48 c}$$

01.11.21.0080.01

$$\int \sec^8(c z)\, dz = \frac{\left(5 \sec^6(c z) + 6 \sec^4(c z) + 8 \sec^2(c z) + 16\right) \tan(c z)}{35 c}$$

01.11.21.0157.01

$$\int \sec^{2n}(c z)\, dz = \frac{\sec^{2n-1}(c z) \sin(c z)}{c (2 n - 1)} \sum_{k=0}^{n-1} \frac{\cos^{2k}(c z) (1 - n)_k}{\left(\frac{3}{2} - n\right)_k} /;\ n \in \mathbb{N}^+$$

01.11.21.0158.01

$$\int \sec^{2n+1}(c z)\, dz = \frac{\left(\frac{1}{2}\right)_n}{2 c n!} \left(4 \tanh^{-1}\left(\tan\left(\frac{c z}{2}\right)\right) + \sin(c z) \sum_{k=1}^{n} \frac{\sec^{2k}(c z) (k - 1)!}{\left(\frac{1}{2}\right)_k}\right) /;\ n \in \mathbb{N}$$

01.11.21.0159.01

$$\int \sec^{2n+1}(c z)\, dz = \frac{1}{2 c n!} \left(\frac{1}{2}\right)_n \left(\sin(c z) \sum_{k=1}^{n} \frac{\sec^{2k}(c z) (k - 1)!}{\left(\frac{1}{2}\right)_k} - 2 \left(\log\left(\cos\left(\frac{c z}{2}\right) - \sin\left(\frac{c z}{2}\right)\right) - \log\left(\cos\left(\frac{c z}{2}\right) + \sin\left(\frac{c z}{2}\right)\right)\right)\right) /;\ n \in \mathbb{N}$$

01.11.21.0160.01

$$\int \sec^{2n}(c z)\, dz = \frac{\sin(c z) \sec^{2n-1}(c z)}{c (2 n - 1)}\, {}_2F_1\left(1, 1 - n; \frac{3}{2} - n; \cos^2(c z)\right) /;\ n \in \mathbb{N}^+$$

01.11.21.0161.01

$$\int \sec^{2n+1}(c z)\, dz = -\frac{\sec^{2n+2}(c z) \sin(c z)}{(2 n + 1) c}\, {}_2F_1\left(1, n + 1; n + \frac{3}{2}; \sec^2(c z)\right) - \frac{\left(\frac{1}{2}\right)_n}{c n!} \left(\sin^{-1}(\sec(c z)) \cot(c z) \sqrt{-\tan^2(c z)} - 2 \tanh^{-1}\left(\tan\left(\frac{c z}{2}\right)\right)\right) /;\ n \in \mathbb{N}$$

01.11.21.0081.01

$$\int \sec^{\frac{1}{2}}(c z)\, dz = \frac{2 \cos^{\frac{1}{2}}(c z)\, F\left(\frac{c z}{2} \middle| 2\right) \sec^{\frac{1}{2}}(c z)}{c}$$

01.11.21.0082.01

$$\int \frac{1}{\sec^{\frac{1}{2}}(c z)}\, dz = \frac{2 E\left(\frac{c z}{2} \middle| 2\right)}{c \cos^{\frac{1}{2}}(c z) \sec^{\frac{1}{2}}(c z)}$$

01.11.21.0083.01

$$\int \sec^{12}(c z)\, dz = \frac{1}{693 c} \left(\left(63 \sec^{10}(c z) + 70 \sec^8(c z) + 80 \sec^6(c z) + 96 \sec^4(c z) + 128 \sec^2(c z) + 256\right) \tan(c z)\right)$$

Involving products of the direct functions

01.11.21.0084.01

$$\int \sec(b + a z) \sec(a z) \, dz = \frac{\csc(b) (\log(\cos(a z)) - \log(\cos(b + a z)))}{a}$$

01.11.21.0085.01

$$\int \sec(b - a z) \sec(a z) \, dz = -\frac{\csc(b) (\log(\cos(a z)) - \log(\cos(b - a z)))}{a}$$

Involving rational functions of the direct function

Involving $(a + b \sec(z))^{-n}$

01.11.21.0086.01

$$\int \frac{1}{a + b \sec(z)} \, dz = \frac{1}{a} \left(z + \frac{2 b}{\sqrt{a^2 - b^2}} \tanh^{-1}\left(\frac{(b - a) \tan\left(\frac{z}{2}\right)}{\sqrt{a^2 - b^2}} \right) \right)$$

01.11.21.0087.01

$$\int \frac{1}{(a + b \sec(z))^2} \, dz = \frac{(b + a \cos(z)) \sec(z)}{a^2 (a + b \sec(z))^2} \left(\frac{a \tan(z) \, b^2}{a^2 - b^2} - \frac{2 b \left(b^2 - 2 a^2\right) (b + a \cos(z)) \sec(z)}{\left(a^2 - b^2\right)^{3/2}} \tanh^{-1}\left(\frac{(b - a) \tan\left(\frac{z}{2}\right)}{\sqrt{a^2 - b^2}} \right) + z (a + b \sec(z)) \right)$$

Involving $\left(a + b \sec^2(z)\right)^{-n}$

01.11.21.0088.01

$$\int \frac{1}{a + b \sec^2(z)} \, dz = \frac{1}{a} \left(z - \frac{\sqrt{b}}{\sqrt{a + b}} \tan^{-1}\left(\frac{\sqrt{b} \tan(z)}{\sqrt{a + b}} \right) \right)$$

01.11.21.0089.01

$$\int \frac{1}{\left(a + b \sec^2(z)\right)^2} \, dz = \frac{(\cos(2 z) a + a + 2 b) \sec^4(z)}{8 a^2 \left(b \sec^2(z) + a\right)^2} \left(2 z (\cos(2 z) a + a + 2 b) - \frac{\sqrt{b} (3 a + 2 b) (\cos(2 z) a + a + 2 b)}{(a + b)^{3/2}} \tan^{-1}\left(\frac{\sqrt{b} \tan(z)}{\sqrt{a + b}} \right) - \frac{a b \sin(2 z)}{a + b} \right)$$

Involving algebraic functions of the direct function

Involving $(a + b \sec(c z))^{\beta}$

01.11.21.0090.01

$$\int \sec(c z)\,(a+b\sec(c z))^{\beta}\,dz = \frac{\sqrt{2}\,\cot\left(\frac{c z}{2}\right)(\sec(c z)-1)\,(a+b\sec(c z))^{\beta}\left(\frac{a+b\sec(c z)}{a+b}\right)^{-\beta}}{c\sqrt{\sec(c z)+1}}\,F_1\left(\frac{1}{2};\frac{1}{2},-\beta;\frac{3}{2};\frac{1}{2}(1-\sec(c z)),\frac{b-b\sec(c z)}{a+b}\right)$$

01.11.21.0091.01

$$\int \sec(c z)\sqrt{a+b\sec(c z)}\,dz = -\frac{1}{\sqrt{-\frac{1}{a+b}}\;c\sqrt{\frac{b(\sec(c z)+1)}{b-a}}}\left(2\,i\cot\left(\frac{c z}{2}\right)\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sec(c z)}\right)\middle|\frac{a+b}{a-b}\right)-F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sec(c z)}\right)\middle|\frac{a+b}{a-b}\right)\right)\sqrt{\frac{b-b\sec(c z)}{a+b}}\right)$$

01.11.21.0092.01

$$\int \frac{\sec(c z)}{\sqrt{a+b\sec(c z)}}\,dz = \frac{\cot^2\left(\frac{c z}{2}\right)^{3/2}}{c(\sec(c z)+1)\sqrt{a+b\sec(c z)}}(1-\sec(c z))^{3/2}\tan\left(\frac{c z}{2}\right)\sqrt{\frac{(b+a\cos(c z))\csc^2\left(\frac{c z}{2}\right)}{b}}\;F\left(\sin^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-\sec(c z)}}\right)\middle|\frac{a+b}{2b}\right)$$

Involving $((a+b\sec(c z))^n)^{\beta}$

01.11.21.0093.01

$$\int \sec(c z)\,((a+b\sec(c z))^n)^{\beta}\,dz = \frac{\sqrt{2}\,\cot\left(\frac{c z}{2}\right)(\sec(c z)-1)\,((a+b\sec(c z))^n)^{\beta}}{c\sqrt{\sec(c z)+1}}\left(\frac{a+b\sec(c z)}{a+b}\right)^{-n\beta}F_1\left(\frac{1}{2};\frac{1}{2},-n\beta;\frac{3}{2};\frac{1}{2}(1-\sec(c z)),\frac{b-b\sec(c z)}{a+b}\right)$$

01.11.21.0094.01

$$\int \sec(c\,z)\sqrt{(a+b\sec(c\,z))^3}\,dz = \left(16\cos^2\left(\frac{c\,z}{2}\right)\cos^2(c\,z)\cot\left(\frac{c\,z}{2}\right)\sqrt{1-\sec(c\,z)}\sqrt{(a+b\sec(c\,z))^3}\left(b(\sec(c\,z)-1)(a+b\sec(c\,z))+\frac{1}{\sqrt{\sec(c\,z)+1}\sqrt{-\tan^2(c\,z)}}\left(\sqrt{1-\sec(c\,z)}\left(-3F\left(\sin^{-1}\left(\frac{\sqrt{1-\sec(c\,z)}}{\sqrt{2}}\right)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(c\,z)}{a+b}}\sqrt{-\tan^2(c\,z)}\,a^2-b^2F\left(\sin^{-1}\left(\frac{\sqrt{1-\sec(c\,z)}}{\sqrt{2}}\right)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(c\,z)}{a+b}}\sqrt{-\tan^2(c\,z)}+\frac{1}{\sqrt{\frac{b(\sec(c\,z)+1)}{b-a}}}\left(4a\left((a+b)E\left(\sin^{-1}\left(\sqrt{\frac{a+b\sec(c\,z)}{a-b}}\right)\middle|\frac{a-b}{a+b}\right)-b\,F\left(\sin^{-1}\left(\sqrt{\frac{a+b\sec(c\,z)}{a-b}}\right)\middle|\frac{a-b}{a+b}\right)\right)(\sec(c\,z)+1)\sqrt{\frac{b-b\sec(c\,z)}{a+b}}\sqrt{\frac{a+b\sec(c\,z)}{a-b}}\right)\right)\right)\right)\Big/\left(c\left(4\left(4\,a\,b\cos(c\,z)+3\left(\cos(2\,c\,z)\,a^2+a^2+2\,b^2\right)\right)\sqrt{1-\sec(c\,z)}\cos^2\left(\frac{c\,z}{2}\right)+16\,a\,b\cos^2(c\,z)\sqrt{\sec(c\,z)+1}\sqrt{-\tan^2(c\,z)}\right)\right)$$

01.11.21.0095.01

$$\int \frac{\sec(c\,z)}{\sqrt{(a+b\sec(c\,z))^3}}\,dz = -\frac{2\cot\left(\frac{c\,z}{2}\right)}{(a-b)\,c\sqrt{\sec(c\,z)+1}\sqrt{(a+b\sec(c\,z))^3}}\sqrt{\frac{a+b\sec(c\,z)}{a+b}}\left(b\sqrt{\frac{a+b\sec(c\,z)}{a+b}}\sqrt{\sec(c\,z)+1}(\sec(c\,z)-1)+(b+a\cos(c\,z))\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\sec(c\,z)}}{\sqrt{2}}\right)\middle|\frac{2b}{a+b}\right)\sqrt{1-\sec(c\,z)}\sec(c\,z)\right)$$

Involving $\left(a+b\sec^2(c\,z)\right)^\beta$

01.11.21.0096.01

$$\int \left(a+b\sec^2(c\,z)\right)^\beta dz = \frac{\cot(c\,z)\left(b\sec^2(c\,z)+a\right)^\beta\sqrt{\sin^2(c\,z)}}{2\,c\,\beta-c}F_1\left(\frac{1}{2}-\beta;\frac{1}{2},-\beta;\frac{3}{2}-\beta;\cos^2(c\,z),-\frac{a\cos^2(c\,z)}{b}\right)\left(\frac{a\cos^2(c\,z)}{b}+1\right)^{-\beta}$$

01.11.21.0097.01

$$\int \sqrt{a+b\sec^2(c\,z)}\,dz = \frac{1}{c\,(\cos(2\,c\,z)\,a+a+2\,b)^{3/2}} \left(\sqrt{2}\cos(c\,z)\left(\sqrt{a}\sqrt{\frac{\cos(2\,c\,z)\,a+a+2\,b}{a+b}}\sqrt{a+b}\sqrt{\cos(2\,c\,z)\,a+a+2\,b}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c\,z)}{\sqrt{a+b}}\right)+\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sin(c\,z)}{\sqrt{\cos(2\,c\,z)\,a+a+2\,b}}\right)(\cos(2\,c\,z)\,a+a+2\,b)\right)\sqrt{b\sec^2(c\,z)+a}\right)$$

01.11.21.0098.01

$$\int \frac{1}{\sqrt{a+b\sec^2(c\,z)}}\,dz = \frac{\sqrt{\cos(2\,c\,z)\,a+a+2\,b}\,\sec(c\,z)}{\sqrt{2}\sqrt{a}\,c\sqrt{b\sec^2(c\,z)+a}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sin(c\,z)}{\sqrt{\cos(2\,c\,z)\,a+a+2\,b}}\right)$$

01.11.21.0099.01

$$\int \sec(c\,z)\left(a+b\sec^2(c\,z)\right)^{\beta}\,dz = \frac{\csc(c\,z)\left(b\sec^2(c\,z)+a\right)^{\beta}\left(\frac{b\sec^2(c\,z)}{a}+1\right)^{-\beta}\sqrt{-\tan^2(c\,z)}}{c}\,F_1\left(\frac{1}{2};\frac{1}{2},-\beta;\frac{3}{2};\sec^2(c\,z),-\frac{b\sec^2(c\,z)}{a}\right)$$

01.11.21.0100.01

$$\int \sec(c\,z)\sqrt{a+b\sec^2(c\,z)}\,dz = \frac{1}{c\,(\cos(2\,c\,z)\,a+a+2\,b)}\left(\sqrt{b\sec^2(c\,z)+a}\left(-\sqrt{2}\sqrt{\frac{\cos(2\,c\,z)\,a+a+2\,b}{a+b}}\,(a+b)\cos(c\,z)\,E\left(c\,z\,\middle|\,\frac{a}{a+b}\right)+\sqrt{2}\sqrt{\frac{\cos(2\,c\,z)\,a+a+2\,b}{a+b}}\,(a+b)\cos(c\,z)\,F\left(c\,z\,\middle|\,\frac{a}{a+b}\right)+(\cos(2\,c\,z)\,a+a+2\,b)\sin(c\,z)\right)\right)$$

01.11.21.0101.01

$$\int \frac{\sec(c\,z)}{\sqrt{a+b\sec^2(c\,z)}}\,dz = \frac{\sqrt{\frac{\cos(2\,c\,z)\,a+a+2\,b}{a+b}}\,F\left(c\,z\,\middle|\,\frac{a}{a+b}\right)\sec(c\,z)}{\sqrt{2}\,c\sqrt{b\sec^2(c\,z)+a}}$$

Involving $\left(\left(a+b\sec^2(c\,z)\right)^n\right)^{\beta}$

01.11.21.0102.01

$$\int \left(\left(a+b\sec^2(c\,z)\right)^n\right)^{\beta}\,dz = \frac{\cot(c\,z)\left(\left(b\sec^2(c\,z)+a\right)^n\right)^{\beta}\sqrt{\sin^2(c\,z)}}{c\,(2\,n\,\beta-1)}\,F_1\left(\frac{1}{2}-n\,\beta;\frac{1}{2},-n\,\beta;\frac{3}{2}-n\,\beta;\cos^2(c\,z),-\frac{a\cos^2(c\,z)}{b}\right)\left(\frac{a\cos^2(c\,z)}{b}+1\right)^{-n\,\beta}$$

01.11.21.0103.01

$$\int \sqrt{\left(a+b\sec^2(c z)\right)^3}\, dz = \frac{1}{c\,(\cos(2 c z)\, a+a+2 b)^{3/2}} \left(\cos(c z) \sqrt{\left(b \sec^2(c z)+a\right)^3} \left(2 \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{a} \sin(c z)}{\sqrt{\cos(2 c z)\, a+a+2 b}}\right) \cos^2(c z)\, a^{3/2} + \sqrt{2} \sqrt{b}\, (3 a+b) \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{b} \sin(c z)}{\sqrt{\cos(2 c z)\, a+a+2 b}}\right) \cos^2(c z) + b \sqrt{\cos(2 c z)\, a+a+2 b}\, \sin(c z)\right)\right)$$

01.11.21.0104.01

$$\int \frac{1}{\sqrt{\left(a+b\sec^2(c z)\right)^3}}\, dz = \frac{\sec^2(c z)}{4 a^{3/2} c \sqrt{\left(b \sec^2(c z)+a\right)^3}} \left(\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{a} \sin(c z)}{\sqrt{\cos(2 c z)\, a+a+2 b}}\right) (\cos(2 c z)\, a+a+2 b)^{3/2} \sec(c z) - \frac{2 \sqrt{a}\, b\, (\cos(2 c z)\, a+a+2 b) \tan(c z)}{a+b}\right)$$

01.11.21.0105.01

$$\int \sec(c z) \left(\left(a+b\sec^2(c z)\right)^n\right)^\beta dz = \frac{\csc(c z) \left(\left(b \sec^2(c z)+a\right)^n\right)^\beta \left(\frac{b \sec^2(c z)}{a}+1\right)^{-n \beta} \sqrt{-\tan^2(c z)}}{c} F_1\left(\frac{1}{2}; \frac{1}{2}, -n \beta; \frac{3}{2}; \sec^2(c z), -\frac{b \sec^2(c z)}{a}\right)$$

01.11.21.0106.01

$$\int \sec(c z) \sqrt{\left(a+b\sec^2(c z)\right)^3}\, dz = \frac{1}{3 c\, (\cos(2 c z)\, a+a+2 b)^2} \left(2 \cos^3(c z) \sqrt{\left(b \sec^2(c z)+a\right)^3} \left(-2 \sqrt{2} \sqrt{\frac{\cos(2 c z)\, a+a+2 b}{a+b}} \left(2 a^2+3 b a+b^2\right) E\left(c z \,\middle|\, \frac{a}{a+b}\right) + \sqrt{2} \sqrt{\frac{\cos(2 c z)\, a+a+2 b}{a+b}} \left(3 a^2+5 b a+2 b^2\right) F\left(c z \,\middle|\, \frac{a}{a+b}\right) + \tan(c z) \left(4 a^2 - b \tan^2(c z)\, a + 11 b a + 2 (2 a+b) \cos(2 c z)\, a + 4 b^2 + b (a+2 b) \sec^2(c z)\right)\right)\right)$$

01.11.21.0107.01

$$\int \frac{\sec(c z)}{\sqrt{\left(a+b\sec^2(c z)\right)^3}}\, dz = \frac{(\cos(2 c z)\, a+a+2 b) \sec^3(c z)}{\sqrt{2}\, a\, (a+b)\, c \sqrt{(\cos(2 c z)\, a+a+2 b)^3 \sec^6(c z)}} \left(-\sqrt{2} \sqrt{\frac{\cos(2 c z)\, a+a+2 b}{a+b}}\, (a+b)\, E\left(c z \,\middle|\, \frac{a}{a+b}\right) + \sqrt{2} \sqrt{\frac{\cos(2 c z)\, a+a+2 b}{a+b}}\, (a+b)\, F\left(c z \,\middle|\, \frac{a}{a+b}\right) + a \sin(2 c z)\right)$$

Involving functions of the direct function and a power function

Involving powers of the direct function and a power function

Involving powers of sech and power

Involving z^n and linear arguments

01.11.21.0108.01

$$\int z^n \sec^\nu(c\,z)\,dz = n!\,\sec^\nu(c\,z)\left(1+e^{2 i c z}\right)^\nu \sum_{j=0}^{n} \frac{(-1)^j z^{n-j}}{(n-j)!\,(i\,c\,\nu)^{j+1}}\,{}_{j+2}F_{j+1}\!\left(\frac{\nu}{2}, \ldots, \frac{\nu}{2}, \nu; \frac{\nu}{2}+1, \ldots, \frac{\nu}{2}+1; -e^{2 i c z}\right) /; n \in \mathbb{N}^+$$

01.11.21.0109.01

$$\int z \sec^\nu(c\,z)\,dz = \frac{1}{4 c^2 (1-\nu)}\left(\sec^{\nu-1}(c\,z)\left(2^\nu (\nu-1)\sqrt{\pi}\,\cos(c\,z)\,\Gamma(1-\nu)\,{}_3\tilde{F}_2\!\left(1, 1-\frac{\nu}{2}, 1-\frac{\nu}{2}; \frac{3-\nu}{2}, 2-\frac{\nu}{2}; \cos^2(c\,z)\right) - 4\,c\,z\,{}_2F_1\!\left(1, 1-\frac{\nu}{2}; \frac{3-\nu}{2}; \cos^2(c\,z)\right)\sin(c\,z)\right)\right)$$

01.11.21.0110.01

$$\int z \sec^2(c\,z)\,dz = \frac{\log(\cos(c\,z)) + c\,z\tan(c\,z)}{c^2}$$

01.11.21.0111.01

$$\int z \sec^3(c\,z)\,dz = \frac{1}{2 c^2}\left(-c\,z\log\left(1+i\,e^{i c z}\right) + c\,z\log\left(1-i\,e^{i c z}\right) + i\,\mathrm{Li}_2\left(-i\,e^{i c z}\right) - i\,\mathrm{Li}_2\left(i\,e^{i c z}\right) - \sec(c\,z) + c\,z\sec(c\,z)\tan(c\,z)\right)$$

01.11.21.0112.01

$$\int z \sec^4(c\,z)\,dz = \frac{(2\,c\,z\tan(c\,z) - 1)\sec^2(c\,z) + 4\,(\log(\cos(c\,z)) + c\,z\tan(c\,z))}{6 c^2}$$

01.11.21.0113.01

$$\int z \sec^5(c\,z)\,dz = \frac{1}{24 c^2}\left(6\,c\,z\tan(c\,z)\sec^3(c\,z) - 2\sec^3(c\,z) + 9\,c\,z\tan(c\,z)\sec(c\,z) - 9\sec(c\,z) - 9\,c\,z\log\left(1+i\,e^{i c z}\right) + 9\,c\,z\log\left(1-i\,e^{i c z}\right) + 9\,i\,\mathrm{Li}_2\left(-i\,e^{i c z}\right) - 9\,i\,\mathrm{Li}_2\left(i\,e^{i c z}\right)\right)$$

01.11.21.0114.01

$$\int z^2 \sec^2(c\,z)\,dz = \frac{c\,z\left(-i\,c\,z + c\tan(c\,z)\,z + 2\log\left(1+e^{2 i c z}\right)\right) - i\,\mathrm{Li}_2\left(-e^{2 i c z}\right)}{c^3}$$

01.11.21.0115.01

$$\int z^3 \sec^3(c\,z)\,dz = \frac{1}{128\,c^4}\Big(16\,c^4\,i\,z^4 + 64\,c^3 \log\left(1 + i\,e^{-i\,c\,z}\right) z^3 - 64\,c^3 \log\left(1 + i\,e^{i\,c\,z}\right) z^3 + 64\,c^3 \sec(c\,z)\tan(c\,z)\,z^3 - 32\,i\,c^3\,\pi\,z^3 + 24\,c^2\,i\,\pi^2\,z^2 - 96\,c^2\,\pi \log\left(1 - i\,e^{-i\,c\,z}\right) z^2 + 96\,c^2\,\pi \log\left(1 + i\,e^{i\,c\,z}\right) z^2 + 192\,c^2\,i\,\text{Li}_2\left(-i\,e^{-i\,c\,z}\right) z^2 + 192\,c^2\,i\,\text{Li}_2\left(-i\,e^{i\,c\,z}\right) z^2 - 192\,c^2 \sec(c\,z)\,z^2 - 8\,i\,c\,\pi^3\,z + 48\,c\,\pi^2 \log\left(1 - i\,e^{-i\,c\,z}\right) z - 48\,c\,\pi^2 \log\left(1 + i\,e^{i\,c\,z}\right) z - 384\,c \log\left(1 + i\,e^{i\,c\,z}\right) z + 384\,c \log\left(1 - i\,e^{i\,c\,z}\right) z - 192\,i\,c\,\pi\,\text{Li}_2\left(-i\,e^{i\,c\,z}\right) z + 384\,c\,\text{Li}_3\left(-i\,e^{-i\,c\,z}\right) z - 384\,c\,\text{Li}_3\left(-i\,e^{i\,c\,z}\right) z - 7\,i\,\pi^4 - 8\,\pi^3 \log\left(1 + i\,e^{-i\,c\,z}\right) + 8\,\pi^3 \log\left(1 + i\,e^{i\,c\,z}\right) + 8\,\pi^3 \log\left(\cot\left(\frac{1}{4}(\pi - 2\,c\,z)\right)\right) + 48\,i\,\pi\,(\pi - 4\,c\,z)\,\text{Li}_2\left(i\,e^{-i\,c\,z}\right) + 48\,i\,\pi^2\,\text{Li}_2\left(-i\,e^{i\,c\,z}\right) + 384\,i\,\text{Li}_2\left(-i\,e^{i\,c\,z}\right) - 384\,i\,\text{Li}_2\left(i\,e^{i\,c\,z}\right) - 192\,\pi\,\text{Li}_3\left(i\,e^{-i\,c\,z}\right) + 192\,\pi\,\text{Li}_3\left(-i\,e^{i\,c\,z}\right) - 384\,i\,\text{Li}_4\left(-i\,e^{-i\,c\,z}\right) - 384\,i\,\text{Li}_4\left(-i\,e^{i\,c\,z}\right)\Big)$$

Involving functions of the direct function and exponential function

Involving powers of the direct function and exponential function

Involving exp

Involving $e^{b\,z}$

01.11.21.0116.01

$$\int e^{b\,z} \sec^\nu(c\,z)\,dz = \frac{e^{b\,z}\left(1 + e^{2\,i\,c\,z}\right)^\nu \sec^\nu(c\,z)}{b + i\,c\,\nu}\,{}_2F_1\left(\frac{-i\,b + c\,\nu}{2\,c}, \nu; \frac{1}{2}\left(2 - \frac{i\,b}{c} + \nu\right); -e^{2\,i\,c\,z}\right)$$

01.11.21.0117.01

$$\int e^{-i\,c\,\nu\,z} \sec^\nu(c\,z)\,dz = \frac{i\,e^{-i\,c\,z\,\nu}\left(1 + e^{-2\,i\,c\,z}\right)^\nu \sec^\nu(c\,z)}{2\,c\,\nu}\,{}_2F_1\left(\nu, \nu; \nu + 1; -e^{-2\,i\,c\,z}\right)$$

01.11.21.0118.01

$$\int e^{i\,c\,z} \sec^2(c\,z)\,dz = \frac{2\,i\,e^{i\,c\,z}}{c\left(1 + e^{2\,i\,c\,z}\right)} - \frac{2\,i\tan^{-1}\left(e^{i\,c\,z}\right)}{c}$$

01.11.21.0119.01

$$\int e^{2\,i\,c\,z} \sec^2(c\,z)\,dz = -\frac{2\,i}{c\left(1 + e^{2\,i\,c\,z}\right)} - \frac{2\,i\log\left(1 + e^{2\,i\,c\,z}\right)}{c}$$

01.11.21.0120.01

$$\int e^{2\,i\,c\,z} \sec^4(c\,z)\,dz = \frac{8\,i\left(1 + 3\,e^{2\,i\,c\,z} + 3\,e^{4\,i\,c\,z}\right)}{3\,c\left(1 + e^{2\,i\,c\,z}\right)^3}$$

01.11.21.0121.01

$$\int e^{-2\,i\,c\,z} \sec^4(c\,z)\,dz = -\frac{8\,i\,e^{2\,i\,c\,z}\left(3 + 3\,e^{2\,i\,c\,z} + e^{4\,i\,c\,z}\right)}{3\,c\left(1 + e^{2\,i\,c\,z}\right)^3}$$

Involving functions of the direct function, exponential and a power functions

Involving powers of the direct function, exponential and a power functions

Involving exp and power

Involving $z^n e^{b z}$

01.11.21.0122.01

$$\int z^n e^{b z} \sec^\nu(c z)\, dz = n!\, \sec^\nu(c z) \left(1 + e^{2 i c z}\right)^\nu e^{b z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j}}{(n-j)!\, (b + i c \nu)^{j+1}}\; {}_{j+2}F_{j+1}\!\left(\frac{c \nu - i b}{2 c}, \ldots, \frac{c \nu - i b}{2 c}, \nu; \frac{c \nu - i b}{2 c} + 1, \ldots, \frac{c \nu - i b}{2 c} + 1; -e^{2 i c z}\right) /; n \in \mathbb{N}^+$$

01.11.21.0123.01

$$\int z^n e^{-i c \nu z} \sec^\nu(c z)\, dz = \frac{e^{-i c \nu z} \left(1 + e^{2 i c z}\right)^\nu z^{n+1} \sec^\nu(c z)}{n+1} - \nu\, e^{-i c (\nu-2) z} \left(1 + e^{2 i c z}\right)^\nu \sec^\nu(c z)\, n! \sum_{j=0}^{n} \frac{(-1)^j (2 i c)^{-j-1} z^{n-j}}{(n-j)!}\; {}_{j+3}F_{j+2}\!\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; -e^{2 i c z}\right) /; n \in \mathbb{N}$$

01.11.21.0124.01

$$\int z^n e^{-i c z (2 q + \nu)} \sec^\nu(c z)\, dz = n!\, \left(1 + e^{2 i c z}\right)^\nu \sec^\nu(c z) \left(\frac{(-1)^q e^{-i c z \nu}\, \Gamma(q + \nu)\, z^{n+1}}{(n+1)!\, q!\, \Gamma(\nu)} + \frac{(-1)^q (\nu)_{q+1} e^{-i c z (\nu-2)}}{(q+1)!} \sum_{j=0}^{n} \frac{z^{n-j}}{(-2 i c)^{j+1} (n-j)!}\; {}_{j+3}F_{j+2}\!\left(1, \ldots, 1, q + \nu + 1; 2, \ldots, 2, q + 2; -e^{2 i c z}\right) - \sum_{j=0}^{n} \sum_{k=0}^{q-1} \frac{(-1)^k (\nu)_k\, e^{i c z (2 k - 2 q - \nu)} z^{n-j}}{(2 i c (q-k))^{j+1}\, k!\, (n-j)!}\right) /; n \in \mathbb{N} \land q \in \mathbb{N}^+$$

Involving functions of the direct function and trigonometric functions

Involving powers of the direct function and trigonometric functions

Involving sin

Involving $\sin(b z)$

01.11.21.0125.01

$$\int \sin(b z) \sec^\nu(c z)\, dz = -\frac{e^{-i b z} \left(1 + e^{2 i c z}\right)^\nu \sec^\nu(c z)}{2 \left(b^2 - c^2 \nu^2\right)} \left(e^{2 i b z} (b - c \nu)\, {}_2F_1\!\left(\frac{b}{2 c} + \frac{\nu}{2}, \nu; \frac{b}{2 c} + \frac{\nu}{2} + 1; -e^{2 i c z}\right) + (b + c \nu)\, {}_2F_1\!\left(\frac{\nu}{2} - \frac{b}{2 c}, \nu; -\frac{b}{2 c} + \frac{\nu}{2} + 1; -e^{2 i c z}\right)\right)$$

Involving powers of sin

Involving $\sin^m(b z)$

01.11.21.0126.01

$$\int \sin^m(b z) \sec^\nu(c z)\, dz = \frac{i\, 2^{-m} \left(1+e^{2 i c z}\right)^\nu (m \bmod 2 - 1) \sec^\nu(c z)}{c \nu} \binom{m}{\frac{m}{2}} {}_2F_1\left(\frac{\nu}{2}, \nu; \frac{\nu}{2}+1; -e^{2 i c z}\right) - i\, 2^{-m} \left(1+e^{2 i c z}\right)^\nu \sec^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{e^{i b (m-2k) z - \frac{i m \pi}{2}}\, {}_2F_1\left(-\frac{b k}{c} + \frac{b m}{2 c} + \frac{\nu}{2}, \nu; -\frac{b k}{c} + \frac{b m}{2 c} + \frac{\nu}{2} + 1; -e^{2 i c z}\right)}{b (m - 2 k) + c \nu} + \frac{e^{\frac{i m \pi}{2} - i b (m-2k) z}\, {}_2F_1\left(\frac{b k}{c} + \frac{\nu}{2} - \frac{b m}{2 c}, \nu; \frac{b k}{c} + \frac{\nu}{2} - \frac{b m}{2 c} + 1; -e^{2 i c z}\right)}{c \nu - b (m - 2 k)} \right) /; m \in \mathbb{N}^+$$

Involving cos

Involving $\cos(b z)$

01.11.21.0127.01

$$\int \cos(b z) \sec^\nu(c z)\, dz = \frac{1}{2} i\, e^{-i b z} \left(1+e^{2 i c z}\right)^\nu \sec^\nu(c z) \left(\frac{{}_2F_1\left(\frac{\nu}{2} - \frac{b}{2 c}, \nu; -\frac{b}{2 c} + \frac{\nu}{2} + 1; -e^{2 i c z}\right)}{b - c \nu} - \frac{e^{2 i b z}\, {}_2F_1\left(\frac{b}{2 c} + \frac{\nu}{2}, \nu; \frac{b}{2 c} + \frac{\nu}{2} + 1; -e^{2 i c z}\right)}{b + c \nu} \right)$$

Involving powers of cos

Involving $\cos^m(b z)$

01.11.21.0128.01

$$\int \cos^m(b z) \sec^\nu(c z)\, dz = 2^{-m} \left(1+e^{2 i c z}\right)^\nu \sec^\nu(c z) \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{s} \left(\frac{e^{-i b (m-2s) z}}{i c \nu - i b (m - 2 s)}\, {}_2F_1\left(-\frac{b m}{2 c} + \frac{b s}{c} + \frac{\nu}{2}, \nu; -\frac{b m}{2 c} + \frac{b s}{c} + \frac{\nu}{2} + 1; -e^{2 i c z}\right) + \frac{e^{i b (m-2s) z}}{b i (m - 2 s) + i c \nu}\, {}_2F_1\left(\frac{b m}{2 c} + \frac{\nu}{2} - \frac{b s}{c}, \nu; \frac{b m}{2 c} + \frac{\nu}{2} - \frac{b s}{c} + 1; -e^{2 i c z}\right) \right) + \frac{i\, 2^{-m} (m \bmod 2 - 1) \sec^\nu(c z)}{c \nu} \left(1+e^{2 i c z}\right)^\nu \binom{m}{\frac{m}{2}} {}_2F_1\left(\frac{\nu}{2}, \nu; \frac{\nu}{2} + 1; -e^{2 i c z}\right) /; m \in \mathbb{N}^+$$

Involving functions of the direct function, trigonometric and a power functions

Involving powers of the direct function, trigonometric and a power functions

Involving sin and power

Involving $z^n \sin(a+b\,z)\sec^\nu(c\,z)$

01.11.21.0129.01

$$\int z^n \sin(a+b\,z)\sec^\nu(c\,z)\,dz = -\frac{i}{2}\left(1+e^{2\,i\,c\,z}\right)^\nu \sec^\nu(c\,z)\,n!\left(e^{i\,a+i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}\,(i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+c\,\nu}{2\,c},\ldots,\frac{b+c\,\nu}{2\,c},\nu;\frac{b+c\,\nu}{2\,c}+1,\ldots,\frac{b+c\,\nu}{2\,c}+1;-e^{2\,i\,c\,z}\right)-e^{-i\,a-i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}\,(-i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+c\,\nu}{2\,c},\ldots,\frac{-b+c\,\nu}{2\,c},\nu;\frac{-b+c\,\nu}{2\,c}+1,\ldots,\frac{-b+c\,\nu}{2\,c}+1;-e^{2\,i\,c\,z}\right)\right)\;/;\;n\in\mathbb{N}\land b\neq c\,\nu\land\,b\neq -c\,\nu$$

01.11.21.0130.01

$$\int z^n \sin(b\,z)\sec^\nu(c\,z)\,dz = -\frac{i}{2}\left(1+e^{2\,i\,c\,z}\right)^\nu \sec^\nu(c\,z)\,n!\left(e^{i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}\,(i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+c\,\nu}{2\,c},\ldots,\frac{b+c\,\nu}{2\,c},\nu;\frac{b+c\,\nu}{2\,c}+1,\ldots,\frac{b+c\,\nu}{2\,c}+1;-e^{2\,i\,c\,z}\right)-e^{-i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}\,(-i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+c\,\nu}{2\,c},\ldots,\frac{-b+c\,\nu}{2\,c},\nu;\frac{-b+c\,\nu}{2\,c}+1,\ldots,\frac{-b+c\,\nu}{2\,c}+1;-e^{2\,i\,c\,z}\right)\right)\;/;\;n\in\mathbb{N}\land b\neq c\,\nu\land\,b\neq -c\,\nu$$

01.11.21.0131.01

$$\int z^n \sin(c\,\nu\,z)\sec^\nu(c\,z)\,dz = \frac{1}{2}\,i\left(1+e^{2\,i\,c\,z}\right)^\nu \sec^\nu(c\,z)\left(\frac{e^{-i\,c\,\nu\,z}\,z^{n+1}}{n+1}-\nu\,e^{-i\,c\,(\nu-2)\,z}\,n!\sum_{j=0}^{n}\frac{\left((-1)^j\,(2\,i\,c)^{-j-1}\,z^{n-j}\right)}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1,\ldots,1,\nu+1;2,\ldots,2;-e^{2\,i\,c\,z}\right)-n!\,e^{i\,c\,\nu\,z}\sum_{j=0}^{n}\frac{\left((-1)^j\,z^{n-j}\right)}{(n-j)!\,(2\,i\,c\,\nu)^{j+1}}\,{}_{j+2}F_{j+1}\left(\nu,\ldots,\nu,\nu;\nu+1,\ldots,\nu+1;-e^{2\,i\,c\,z}\right)\right)\;/;\;n\in\mathbb{N}$$

01.11.21.0132.01

$$\int z^n \sin(q\,\nu\,c\,z)\sec^{\nu}(c\,z)\,dz = \frac{1}{2}\left(-(i\,n!)\right)\sec^{\nu}(c\,z)\left(1+e^{2\,i\,c\,z}\right)^{\nu}\left(-\frac{(-1)^{\frac{1}{2}\nu(q-1)}\,\Gamma\left(\frac{1}{2}\nu(q+1)\right)e^{-i\,c\,\nu\,z}\,z^{n+1}}{\Gamma\left(\frac{1}{2}(q-1)\nu+1\right)\Gamma(\nu)\,(n+1)!} - (-1)^{\frac{1}{2}\nu(q-1)}\right.$$

$$\sum_{j=0}^{n}\frac{\left(e^{i\,c\,z\,(2-\nu)}\,(\nu)_{\frac{1}{2}(q-1)\nu+1}\,z^{n-j}\right)}{(n-j)!\,(-2\,i\,c)^{j+1}\left(\frac{1}{2}(q-1)\nu+1\right)!}\,{}_{j+3}F_{j+2}\left(1,\ldots,1,\frac{1}{2}(q+1)\nu+1;2,\ldots,2,\frac{1}{2}(q-1)\nu+2;-e^{2\,i\,c\,z}\right)+e^{i\,q\,\nu\,c\,z}$$

$$\sum_{j=0}^{n}\frac{\left((-1)^j\,z^{n-j}\right)}{(n-j)!\,(i\,c\,\nu\,(q+1))^{j+1}}\,{}_{j+2}F_{j+1}\left(\frac{1}{2}(q+1)\nu,\ldots,\frac{1}{2}(q+1)\nu,\nu;\frac{1}{2}(q+1)\nu+1,\ldots,\frac{1}{2}(q+1)\nu+1;-e^{2\,i\,c\,z}\right)+$$

$$\left.\sum_{j=0}^{n}\sum_{k=0}^{\frac{1}{2}\nu(q-1)-1}\frac{\left((-1)^k\,(\nu)_k\,z^{n-j}\right)e^{i\,c\,z\,(2\,k-q\,\nu)}}{(i\,c\,(-2\,k+q\,\nu-\nu))^{j+1}\,(n-j)!\,k!}\right)/;\, n\in\mathbb{N}\bigwedge\frac{\nu(q-1)}{2}\in\mathbb{N}$$

Involving powers of sin and power

Involving $z^n \sin^m(b\,z)\sec^{\nu}(c\,z)$

01.11.21.0133.01

$$\int z^n \sin^m(b\,z)\sec^{\nu}(c\,z)\,dz =$$

$$\left(1+e^{2\,i\,c\,z}\right)^{\nu}\binom{m}{\frac{m}{2}}2^{-m}\,n!\,(1-m\bmod 2)\sec^{\nu}(c\,z)\sum_{j=0}^{n}\frac{(-1)^j\,z^{n-j}\,(i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{\nu}{2},\ldots,\frac{\nu}{2},\nu;\frac{\nu}{2}+1,\ldots,\frac{\nu}{2}+1;-e^{2\,i\,c\,z}\right)+$$

$$2^{-m}\left(1+e^{2\,i\,c\,z}\right)^{\nu}n!\sec^{\nu}(c\,z)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}$$

$$\left(e^{(i\,b\,(m-2\,k))\,z-\frac{i\,\pi\,m}{2}}\sum_{j=0}^{n}\frac{(-1)^j\,z^{n-j}\,(i\,b\,(m-2\,k)+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c\,\nu+b\,(-2\,k+m)}{2\,c},\ldots,\frac{c\,\nu+b\,(-2\,k+m)}{2\,c},\right.\right.$$

$$\left.\nu;\frac{c\,\nu+b\,(-2\,k+m)}{2\,c}+1,\ldots,\frac{c\,\nu+b\,(-2\,k+m)}{2\,c}+1;-e^{2\,i\,c\,z}\right)+$$

$$e^{\frac{i\,\pi\,m}{2}+(-i\,b\,(m-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,z^{n-j}\,(-i\,b\,(m-2\,k)+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c\,\nu-b\,(-2\,k+m)}{2\,c},\ldots,\right.$$

$$\left.\left.\frac{c\,\nu-b\,(-2\,k+m)}{2\,c},\nu;\frac{c\,\nu-b\,(-2\,k+m)}{2\,c}+1,\ldots,\frac{c\,\nu-b\,(-2\,k+m)}{2\,c}+1;-e^{2\,i\,c\,z}\right)\right)/;\, n\in\mathbb{N}\bigwedge m\in\mathbb{N}^+$$

Involving cos and power

Involving $z^n \cos(a+b\,z)\sec^{\nu}(c\,z)$

01.11.21.0134.01

$$\int z^n \cos(a+b\,z)\sec^\nu(c\,z)\,dz = \frac{1}{2}\left(1+e^{2\,i\,c\,z}\right)^\nu \sec^\nu(c\,z)\,n!\left(e^{i\,a+i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}\,(i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b+c\,\nu}{2\,c},\ldots,\frac{b+c\,\nu}{2\,c},\nu;\frac{b+c\,\nu}{2\,c}+1,\ldots,\frac{b+c\,\nu}{2\,c}+1;-e^{2\,i\,c\,z}\right)+ e^{-i\,a-i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}\,(-i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{-b+c\,\nu}{2\,c},\ldots,\frac{-b+c\,\nu}{2\,c},\nu;\frac{-b+c\,\nu}{2\,c}+1,\ldots,\frac{-b+c\,\nu}{2\,c}+1;-e^{2\,i\,c\,z}\right)\right)\;/;\;n\in\mathbb{N}\land b\neq -i\,c\,\nu\land b\neq i\,c\,\nu$$

01.11.21.0135.01

$$\int z^n \cos(b\,z)\sec^\nu(c\,z)\,dz = \frac{1}{2}\left(1+e^{2\,i\,c\,z}\right)^\nu \sec^\nu(c\,z)\,n!\left(e^{i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}\,(i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b+c\,\nu}{2\,c},\ldots,\frac{b+c\,\nu}{2\,c},\nu;\frac{b+c\,\nu}{2\,c}+1,\ldots,\frac{b+c\,\nu}{2\,c}+1;-e^{2\,i\,c\,z}\right)+ e^{-i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j z^{n-j}\,(-i\,b+i\,c\,\nu)^{-j-1}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{-b+c\,\nu}{2\,c},\ldots,\frac{-b+c\,\nu}{2\,c},\nu;\frac{-b+c\,\nu}{2\,c}+1,\ldots,\frac{-b+c\,\nu}{2\,c}+1;-e^{2\,i\,c\,z}\right)\right)\;/;\;n\in\mathbb{N}\land b\neq -i\,c\,\nu\land b\neq i\,c\,\nu$$

01.11.21.0136.01

$$\int z^n \cos(c\,\nu\,z)\sec^\nu(c\,z)\,dz = \frac{1}{2}\left(1+e^{2\,i\,c\,z}\right)^\nu \sec^\nu(c\,z)\left(\frac{e^{-i\,c\,\nu\,z}\,z^{n+1}}{n+1} - \nu\,e^{-i\,c\,(\nu-2)\,z}\,n!\sum_{j=0}^{n}\frac{\left((-1)^j\,(2\,i\,c)^{-j-1}\,z^{n-j}\right)}{(n-j)!}\,{}_{j+3}F_{j+2}\!\left(1,\ldots,1,\nu+1;2,\ldots,2;-e^{2\,i\,c\,z}\right)+ e^{i\,c\,\nu\,z}\,n!\sum_{j=0}^{n}\frac{\left((-1)^j\,z^{n-j}\right)}{(n-j)!\,(2\,i\,c\,\nu)^{j+1}}\,{}_{j+2}F_{j+1}\!\left(\nu,\ldots,\nu,\nu;\nu+1,\ldots,\nu+1;-e^{2\,i\,c\,z}\right)\right)\;/;\;n\in\mathbb{N}$$

01.11.21.0137.01

$$\int z^n \cos(q\, \nu\, c\, z) \sec^\nu(c\, z)\, dz = \frac{1}{2}\, n!\, \sec^\nu(c\, z) \left(1 + e^{2 i c z}\right)^\nu \left(\frac{(-1)^{\frac{1}{2}\nu(q-1)}\, e^{-i c \nu z}\, \Gamma\left(\frac{1}{2}\nu(q+1)\right) z^{n+1}}{\Gamma\left(\frac{1}{2}(q-1)\nu+1\right)\Gamma(\nu)\,(n+1)!} + (-1)^{\frac{1}{2}\nu(q-1)} \sum_{j=0}^{n} \frac{\left(e^{i c z(2-\nu)}\, (\nu)_{\frac{1}{2}(q-1)\nu+1}\, z^{n-j}\right)}{(n-j)!\,(-2 i c)^{j+1} \left(\frac{1}{2}(q-1)\nu+1\right)!}\, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \frac{1}{2}(q+1)\nu+1; 2, \ldots, 2, \frac{1}{2}(q-1)\nu+2; -e^{2 i c z}\right) + e^{i q \nu c z} \sum_{j=0}^{n} \frac{\left((-1)^j z^{n-j}\right)}{(n-j)!\,(i c \nu (q+1))^{j+1}}\, {}_{j+2}F_{j+1}\left(\frac{1}{2}(q+1)\nu, \ldots, \frac{1}{2}(q+1)\nu, \nu; \frac{1}{2}(q+1)\nu+1, \ldots, \frac{1}{2}(q+1)\nu+1; -e^{2 i c z}\right) - \sum_{j=0}^{n} \sum_{k=0}^{\frac{1}{2}\nu(q-1)-1} \frac{\left((-1)^k (\nu)_k\, z^{n-j}\right) e^{i c z(2k - q\nu)}}{(i c(-2k+q\nu-\nu))^{j+1}\,(n-j)!\,k!} \right) /;\, n \in \mathbb{N} \bigwedge \frac{\nu(q-1)}{2} \in \mathbb{N}$$

Involving powers of cos and power

Involving $z^n \cos^m(b\, z) \sec^\nu(c\, z)$

01.11.21.0138.01

$$\int z^n \cos^m(b\, z) \sec^\nu(c\, z)\, dz = \left(1 + e^{2 i c z}\right)^\nu \binom{m}{\frac{m}{2}} 2^{-m}\, n!\, (1 - m \bmod 2) \sec^\nu(c\, z) \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i c \nu)^{-j-1}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{\nu}{2}, \ldots, \frac{\nu}{2}, \nu; \frac{\nu}{2}+1, \ldots, \frac{\nu}{2}+1; -e^{2 i c z}\right) + 2^{-m} \left(1 + e^{2 i c z}\right)^\nu n!\, \sec^\nu(c\, z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(e^{(i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i b (m-2k) + i c \nu)^{-j-1}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{c\nu + b(-2k+m)}{2c}, \ldots, \frac{c\nu + b(-2k+m)}{2c}, \nu; \frac{c\nu + b(-2k+m)}{2c}+1, \ldots, \frac{c\nu + b(-2k+m)}{2c}+1; -e^{2 i c z}\right) + e^{(-i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (-i b (m-2k) + i c \nu)^{-j-1}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{c\nu - b(-2k+m)}{2c}, \ldots, \frac{c\nu - b(-2k+m)}{2c}, \nu; \frac{c\nu - b(-2k+m)}{2c}+1, \ldots, \frac{c\nu - b(-2k+m)}{2c}+1; -e^{2 i c z}\right) \right) /;\, n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+$$

Involving functions of the direct function, trigonometric and exponential functions

Involving powers of the direct function, trigonometric and exponential functions

Involving sin and exp

Involving $e^{p z} \sin(a\, z) \sec^\nu(c\, z)$

01.11.21.0139.01

$$\int e^{p z} \sin(a z)\, \sec^{\nu}(c z)\, dz = \frac{1}{2(-i a + p + i c \nu)(a - i p + c \nu)} e^{(-i a + p) z} \left(1 + e^{2 i c z}\right)^{\nu} \sec^{\nu}(c z) \left(e^{2 i a z}\, i\,(a + i p - c \nu)\, {}_2F_1\!\left(\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c}, \nu; \frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c} + 1; -e^{2 i c z}\right) + (i a + p + i c \nu)\, {}_2F_1\!\left(-\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c}, \nu; -\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c} + 1; -e^{2 i c z}\right)\right)$$

01.11.21.0140.01

$$\int e^{i(a - c\nu) z} \sin(a z) \sec^{\nu}(c z)\, dz = -\frac{e^{-i c z \nu} \left(1 + e^{2 i c z}\right)^{\nu} \sec^{\nu}(c z)}{4 a c} \left(-2 a c i z + c\, e^{2 i a z}\, {}_2F_1\!\left(\frac{a}{c}, \nu; \frac{a}{c} + 1; -e^{2 i c z}\right) + a\, e^{2 i c z}\, \nu\, {}_3F_2\!\left(1, 1, \nu + 1; 2, 2; -e^{2 i c z}\right)\right)$$

01.11.21.0141.01

$$\int e^{-i(a + c\nu) z} \sin(a z) \sec^{\nu}(c z)\, dz = \frac{e^{-i z (2 a + c \nu)} \left(1 + e^{2 i c z}\right)^{\nu} \sec^{\nu}(c z)}{4 a c} \left(a\, e^{2 i a z} \left(e^{2 i c z}\, \nu\, {}_3F_2\!\left(1, 1, \nu + 1; 2, 2; -e^{2 i c z}\right) - 2 i c z\right) - c\, {}_2F_1\!\left(-\frac{a}{c}, \nu; 1 - \frac{a}{c}; -e^{2 i c z}\right)\right)$$

Involving powers of sin and exp

Involving $e^{p z} \sin^{m}(a z) \sec^{\nu}(c z)$

01.11.21.0142.01

$$\int e^{p z} \sin^{m}(a z) \sec^{\nu}(c z)\, dz = 2^{-m} \left(1 + e^{2 i c z}\right)^{\nu} \sec^{\nu}(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{e^{(a i (m - 2k) + p) z - \frac{i m \pi}{2}}}{a i (m - 2k) + p + i c \nu}\, {}_2F_1\!\left(-\frac{a k}{c} + \frac{a m}{2c} + \frac{\nu}{2} - \frac{i p}{2c}, \nu; -\frac{a k}{c} + \frac{a m}{2c} + \frac{\nu}{2} - \frac{i p}{2c} + 1; -e^{2 i c z}\right) + \frac{e^{\frac{i \pi m}{2} + (p - i a (m - 2k)) z}}{-i a (m - 2k) + p + i c \nu}\, {}_2F_1\!\left(\frac{a k}{c} + \frac{\nu}{2} - \frac{a m}{2c} - \frac{i p}{2c}, \nu; \frac{a k}{c} + \frac{\nu}{2} - \frac{a m}{2c} - \frac{i p}{2c} + 1; -e^{2 i c z}\right)\right) - \frac{2^{-m} e^{p z} \left(1 + e^{2 i c z}\right)^{\nu} (m \bmod 2 - 1) \sec^{\nu}(c z)}{p + i c \nu} \binom{m}{\frac{m}{2}}\, {}_2F_1\!\left(\frac{\nu}{2} - \frac{i p}{2c}, \nu; -\frac{i p}{2c} + \frac{\nu}{2} + 1; -e^{2 i c z}\right) /;\ m \in \mathbb{N}^+$$

Involving cos and exp

Involving $e^{p z} \cos(a z) \sec^{\nu}(c z)$

01.11.21.0143.01

$$\int e^{p z} \cos(a z) \sec^{\nu}(c z)\, dz = \frac{1}{2} \left(1 + e^{2 i c z}\right)^{\nu} \sec^{\nu}(c z) \left(\frac{e^{(i a + p) z}}{i a + p + i c \nu}\, {}_2F_1\!\left(\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c}, \nu; \frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c} + 1; -e^{2 i c z}\right) + \frac{e^{(-i a + p) z}}{-i a + p + i c \nu}\, {}_2F_1\!\left(-\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c}, \nu; -\frac{a}{2c} + \frac{\nu}{2} - \frac{i p}{2c} + 1; -e^{2 i c z}\right)\right)$$

01.11.21.0144.01

$$\int e^{i(a-c\nu)z}\cos(a z)\sec^{\nu}(c z)\,dz = \frac{e^{-icz\nu}\left(1+e^{2icz}\right)^{\nu}\sec^{\nu}(c z)}{4ac}\left(2acz - i c\, e^{2iaz}\,{}_2F_1\!\left(\frac{a}{c},\nu;\frac{a}{c}+1;-e^{2icz}\right)+a\,e^{2icz}\, i\,\nu\,{}_3F_2\!\left(1,1,\nu+1;2,2;-e^{2icz}\right)\right)$$

01.11.21.0145.01

$$\int e^{-i(a+c\nu)z}\cos(a z)\sec^{\nu}(c z)\,dz = \frac{e^{-iz(2a+c\nu)}\left(1+e^{2icz}\right)^{\nu}\sec^{\nu}(c z)}{4ac}\left(c\, i\,{}_2F_1\!\left(-\frac{a}{c},\nu;1-\frac{a}{c};-e^{2icz}\right)+a\,e^{2iaz}\left(2cz+e^{2icz}\, i\,\nu\,{}_3F_2\!\left(1,1,\nu+1;2,2;-e^{2icz}\right)\right)\right)$$

Involving powers of cos and exp

Involving $e^{pz}\cos^{m}(a z)\sec^{\nu}(c z)$

01.11.21.0146.01

$$\int e^{pz}\cos^{m}(a z)\sec^{\nu}(c z)\,dz = 2^{-m}\left(1+e^{2icz}\right)^{\nu}\sec^{\nu}(c z)\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{s}$$
$$\left(\left(e^{(p-ia(m-2s))z}\,{}_2F_1\!\left(-\frac{am}{2c}+\frac{as}{c}+\frac{\nu}{2}-\frac{ip}{2c},\nu;-\frac{am}{2c}+\frac{as}{c}+\frac{\nu}{2}-\frac{ip}{2c}+1;-e^{2icz}\right)\right)\Big/(-ia(m-2s)+p+ic\nu)+\right.$$
$$\left.\left(e^{(ai(m-2s)+p)z}\,{}_2F_1\!\left(\frac{am}{2c}+\frac{\nu}{2}-\frac{as}{c}-\frac{ip}{2c},\nu;\frac{am}{2c}+\frac{\nu}{2}-\frac{as}{c}-\frac{ip}{2c}+1;-e^{2icz}\right)\right)\Big/(a i(m-2s)+p+ic\nu)\right)-$$
$$\frac{2^{-m}e^{pz}\left(1+e^{2icz}\right)^{\nu}(m \bmod 2-1)\sec^{\nu}(c z)}{p+ic\nu}\binom{m}{\frac{m}{2}}\,{}_2F_1\!\left(\frac{\nu}{2}-\frac{ip}{2c},\nu;-\frac{ip}{2c}+\frac{\nu}{2}+1;-e^{2icz}\right)\ /;\ m\in\mathbb{N}^{+}$$

Involving functions of the direct function, trigonometric, exponential and a power functions

Involving powers of the direct function, trigonometric, exponential and a power functions

Involving sin, exp and power

Involving $z^{n}\, e^{pz}\sin(a+b\,z)\sec^{\nu}(c\,z)$

01.11.21.0147.01

$$\int z^n e^{p z} \sin(a + b z) \sec^\nu(c z)\, dz = -\frac{i}{2} \left(1 + e^{2 i c z}\right)^\nu \sec^\nu(c z)\, n! \left(e^{i a+(p+i b) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i b + p + i c \nu)^{-j-1}}{(n-j)!} \right.$$
$$_{j+2}F_{j+1}\left(\frac{b - i p + c \nu}{2 c}, \ldots, \frac{b - i p + c \nu}{2 c}, \nu; \frac{b - i p + c \nu}{2 c} + 1, \ldots, \frac{b - i p + c \nu}{2 c} + 1; -e^{2 i c z}\right) -$$
$$e^{-i a+(p-i b) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (-i b + p + i c \nu)^{-j-1}}{(n-j)!} \, _{j+2}F_{j+1}\left(\frac{-b - i p + c \nu}{2 c}, \ldots, \frac{-b - i p + c \nu}{2 c}, \nu;\right.$$
$$\left.\left.\frac{-b - i p + c \nu}{2 c} + 1, \ldots, \frac{-b - i p + c \nu}{2 c} + 1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N} \bigwedge p + i b \neq -i c \nu \bigwedge p - i b \neq -i c \nu$$

01.11.21.0148.01

$$\int z^n e^{p z} \sin(b z) \sec^\nu(c z)\, dz =$$
$$\frac{1}{2} \left(1 + e^{2 i c z}\right)^\nu n! \sec^\nu(c z) \left(e^{-\frac{1}{2} (i \pi)+(i b+p) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i b + p + i c \nu)^{-j-1}}{(n-j)!} \, _{j+2}F_{j+1}\left(\frac{c \nu - i p + b}{2 c}, \ldots, \frac{c \nu - i p + b}{2 c}, \nu;\right.\right.$$
$$\left.\frac{c \nu - i p + b}{2 c} + 1, \ldots, \frac{c \nu - i p + b}{2 c} + 1; -e^{2 i c z}\right) + e^{\frac{i \pi}{2}+(-i b+p) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (-i b + p + i c \nu)^{-j-1}}{(n-j)!}$$
$$\left.{}_{j+2}F_{j+1}\left(\frac{c \nu - i p - b}{2 c}, \ldots, \frac{c \nu - i p - b}{2 c}, \nu; \frac{c \nu - i p - b}{2 c} + 1, \ldots, \frac{c \nu - i p - b}{2 c} + 1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

01.11.21.0149.01

$$\int z^n e^{i (b - c \nu) z} \sin(b z) \sec^\nu(c z)\, dz =$$
$$\frac{i}{2} \left(1 + e^{2 i c z}\right)^\nu \sec^\nu(c z) \left(\frac{e^{-i c \nu z} z^{n+1}}{n+1} - e^{i c (2-\nu) z} \nu\, n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} \, _{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; -e^{2 i c z}\right) - \right.$$
$$\left. e^{i (2 b - c \nu) z} n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i b)^{-j-1}}{(n-j)!} \, _{j+2}F_{j+1}\left(\frac{b}{c}, \ldots, \frac{b}{c}, \nu; \frac{b}{c} + 1, \ldots, \frac{b}{c} + 1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

01.11.21.0150.01

$$\int z^n e^{-i (b + c \nu) z} \sin(b z) \sec^\nu(c z)\, dz =$$
$$-i \frac{1}{2} \left(1 + e^{2 i c z}\right)^\nu \sec^\nu(c z) \left(\frac{e^{-i c \nu z} z^{n+1}}{n+1} - n!\, \nu\, e^{i c (2-\nu) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} \, _{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; -e^{2 i c z}\right) + \right.$$
$$\left. e^{-i (2 b + c \nu) z} n! \sum_{j=0}^{n} \frac{z^{n-j} (2 i b)^{-j-1}}{(n-j)!} \, _{j+2}F_{j+1}\left(-\frac{b}{c}, \ldots, -\frac{b}{c}, \nu; 1 - \frac{b}{c}, \ldots, 1 - \frac{b}{c}; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

Involving powers of sin, exp and power

Involving $z^n e^{p z} \sin^m(b z) \sec^\nu(c z)$

01.11.21.0151.01

$$\int z^n e^{p z} \sin^m(b z) \sec^\nu(c z)\, dz = 2^{-m} e^{p z} \left(1+e^{2 i c z}\right)^\nu \binom{m}{\frac{m}{2}} n!\, (1 - m \bmod 2) \sec^\nu(c z) \sum_{j=0}^n \frac{(-1)^j z^{n-j} (p + i c \nu)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c\nu - i p}{2c}, \ldots, \frac{c\nu - i p}{2c}, \nu; \frac{c\nu - i p}{2c}+1, \ldots, \frac{c\nu - i p}{2c}+1; -e^{2 i c z}\right) + 2^{-m} \left(1+e^{2 i c z}\right)^\nu n!\, \sec^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(e^{(p + b i (m-2k)) z - \frac{i \pi m}{2}} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (p + b i (m-2k) + i c \nu)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c\nu - i p + b(-2k+m)}{2c}, \ldots, \frac{c\nu - i p + b(-2k+m)}{2c}, \nu; \frac{c\nu - i p + b(-2k+m)}{2c}+1, \ldots, \frac{c\nu - i p + b(-2k+m)}{2c}+1; -e^{2 i c z}\right) + e^{\frac{i \pi m}{2} + (p - i b (m-2k)) z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (p - i b (m-2k) + i c \nu)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c\nu - i p - b(-2k+m)}{2c}, \ldots, \frac{c\nu - i p - b(-2k+m)}{2c}, \nu; \frac{c\nu - i p - b(-2k+m)}{2c}+1, \ldots, \frac{c\nu - i p - b(-2k+m)}{2c}+1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving cos, exp and power

Involving $z^n e^{p z} \cos(a + b z) \sec^\nu(c z)$

01.11.21.0152.01

$$\int z^n e^{p z} \cos(a + b z) \sec^\nu(c z)\, dz = \frac{1}{2} \left(1+e^{2 i c z}\right)^\nu \sec^\nu(c z)\, n! \left(e^{i a + (p + i b) z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (i b + p + i c \nu)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b - i p + c\nu}{2c}, \ldots, \frac{b - i p + c\nu}{2c}, \nu; \frac{b - i p + c\nu}{2c}+1, \ldots, \frac{b - i p + c\nu}{2c}+1; -e^{2 i c z}\right) + e^{-i a + (p - i b) z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (-i b + p + i c \nu)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b - i p + c\nu}{2c}, \ldots, \frac{-b - i p + c\nu}{2c}, \nu; \frac{-b - i p + c\nu}{2c}+1, \ldots, \frac{-b - i p + c\nu}{2c}+1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N} \land p + i b \neq -i c \nu \land p - i b \neq -i c \nu$$

01.11.21.0153.01

$$\int z^n e^{p z} \cos(b z) \sec^\nu(c z)\, dz = \frac{1}{2} \left(1 + e^{2 i c z}\right)^\nu n! \sec^\nu(c z) \left(e^{(-i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (-i b + p + i c \nu)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c \nu - i p - b}{2 c}, \ldots, \frac{c \nu - i p - b}{2 c}, \nu; \frac{c \nu - i p - b}{2 c} + 1, \ldots, \frac{c \nu - i p - b}{2 c} + 1; -e^{2 i c z}\right) + e^{(i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (i b + p + i c \nu)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c \nu - i p + b}{2 c}, \ldots, \frac{c \nu - i p + b}{2 c}, \nu; \frac{c \nu - i p + b}{2 c} + 1, \ldots, \frac{c \nu - i p + b}{2 c} + 1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

01.11.21.0154.01

$$\int z^n e^{i (b - c \nu) z} \cos(b z) \sec^\nu(c z)\, dz = \frac{1}{2} \left(1 + e^{2 i c z}\right)^\nu \sec^\nu(c z) \left(\frac{e^{-i c \nu z} z^{n+1}}{n+1} - n! \, \nu \, e^{i c (2 - \nu) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; -e^{2 i c z}\right) + e^{i (2 b - c \nu) z} n! \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i b)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b}{c}, \ldots, \frac{b}{c}, \nu; \frac{b}{c} + 1, \ldots, \frac{b}{c} + 1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

01.11.21.0155.01

$$\int z^n e^{-i (b + c \nu) z} \cos(b z) \sec^\nu(c z)\, dz = \frac{1}{2} \left(1 + e^{2 i c z}\right)^\nu \sec^\nu(c z) \left(\frac{e^{-i c \nu z} z^{n+1}}{n+1} - n! \, \nu \, e^{i c (2 - \nu) z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (2 i c)^{-j-1}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; -e^{2 i c z}\right) - n! \, e^{-i (2 b + c \nu) z} \sum_{j=0}^{n} \frac{z^{n-j} (2 i b)^{-j-1}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(-\frac{b}{c}, \ldots, -\frac{b}{c}, \nu; 1 - \frac{b}{c}, \ldots, 1 - \frac{b}{c}; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

Involving powers of cos, exp and power

Involving $z^n e^{p z} \cos^m(b z) \sec^\nu(c z)$

01.11.21.0156.01

$$\int z^n e^{pz} \cos^m(bz) \sec^\nu(cz)\, dz = e^{pz} \left(1+e^{2icz}\right)^\nu \binom{m}{\frac{m}{2}} 2^{-m}\, n!\, (1 - m \bmod 2) \sec^\nu(cz) \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (p+icv)^{-j-1}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{cv-ip}{2c}, \ldots, \frac{cv-ip}{2c}, \nu; \frac{cv-ip}{2c}+1, \ldots, \frac{cv-ip}{2c}+1; -e^{2icz}\right) +$$

$$2^{-m} \left(1+e^{2icz}\right)^\nu n!\, \sec^\nu(cz) \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} \binom{m}{k} \left(e^{(p+ib(m-2k))z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (p+ib(m-2k)+icv)^{-j-1}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{cv-ip+b(-2k+m)}{2c}, \ldots, \frac{cv-ip+b(-2k+m)}{2c}, \nu; \frac{cv-ip+b(-2k+m)}{2c}+1, \ldots, \frac{cv-ip+b(-2k+m)}{2c}+1; -e^{2icz}\right) + e^{(p-ib(m-2k))z} \sum_{j=0}^{n} \frac{(-1)^j z^{n-j} (p-ib(m-2k)+icv)^{-j-1}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{cv-ip-b(-2k+m)}{2c}, \ldots, \frac{cv-ip-b(-2k+m)}{2c}, \nu; \frac{cv-ip-b(-2k+m)}{2c}+1, \ldots, \frac{cv-ip-b(-2k+m)}{2c}+1; -e^{2icz}\right)\right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Definite integration

For the direct function itself

01.11.21.0015.01

$$\int_0^{\frac{\pi}{4}} \sec(t)\, dt = \frac{1}{2}\left(\log(2) + \log\left(\sqrt{2}+2\right) - \log\left(4-2\sqrt{2}\right)\right)$$

01.11.21.0016.01

$$\int_0^{\frac{\pi}{4}} t \sec(t)\, dt = \frac{1}{32}\left(8\pi\left(\log\left(1-(-1)^{3/4}\right) - \log\left(1+(-1)^{3/4}\right)\right) + \sqrt[4]{-1}\left(\psi^{(1)}\left(\frac{1}{8}\right) - i\,\psi^{(1)}\left(\frac{3}{8}\right) - \psi^{(1)}\left(\frac{5}{8}\right) + i\,\psi^{(1)}\left(\frac{7}{8}\right)\right) - 64\,C\right)$$

01.11.21.0017.01

$$\int_0^{\frac{\pi}{4}} t^2 \sec(t)\, dt = \frac{1}{64}\left(\pi\left(4\pi\left(-2i\pi + \log\left(1-(-1)^{3/4}\right) - \log\left(1+(-1)^{3/4}\right)\right) + \sqrt[4]{-1}\left(\psi^{(1)}\left(\frac{1}{8}\right) - i\,\psi^{(1)}\left(\frac{3}{8}\right) - \psi^{(1)}\left(\frac{5}{8}\right) + i\,\psi^{(1)}\left(\frac{7}{8}\right)\right)\right) + 128\left(\mathrm{Li}_3\left((-1)^{3/4}\right) - \mathrm{Li}_3\left(-(-1)^{3/4}\right)\right)\right)$$

Summation

Finite summation

01.11.23.0001.01

$$\sum_{k=0}^{n} \sec\left(\frac{2k\pi}{2n+1}\right) = \frac{1}{2}(-1)^n (2n+1) + \frac{1}{2} /; n \in \mathbb{N}$$

01.11.23.0002.01

$$\sum_{k=0}^{n-1} \sec^2\left(\frac{\pi k}{n} + z\right) == n^2 \csc^2\left(\frac{1}{2} n (2 z + \pi)\right) /; n \in \mathbb{N}^+$$

01.11.23.0003.01

$$\sum_{k=0}^{n-1} \sec^2\left(\frac{2 \pi k}{n} + z\right) == \frac{1}{2} (1 - (-1)^n) n^2 \csc^2\left(\frac{1}{2} n (2 z + \pi)\right) + \frac{1}{4} (1 + (-1)^n) n^2 \csc^2\left(\frac{1}{4} n (2 z + \pi)\right) /; n \in \mathbb{N}^+$$

01.11.23.0004.01

$$\sum_{k=1}^{n} \frac{\sec^2\left(\frac{z}{2^k}\right)}{2^{2 k}} == \csc^2(z) - \frac{1}{2^{2 n}} \csc^2\left(\frac{z}{2^n}\right) /; n \in \mathbb{N}$$

01.11.23.0005.01

$$\sum_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \sec^2\left(\frac{k \pi}{n}\right) == \frac{1}{12} \left(4 n^2 - (-1)^n \left(2 n^2 + 1\right) - 7\right) /; n \in \mathbb{N}^+$$

Infinite summation

01.11.23.0006.01

$$\sum_{k=1}^{\infty} \frac{\sec^2\left(\frac{z}{2^k}\right)}{2^{2 k}} == \csc^2(z) - \frac{1}{z^2}$$

Products

Finite products

01.11.24.0001.01

$$\prod_{k=1}^{2 n} \sec\left(\frac{k \pi}{2 n + 1}\right) == (-1)^n 2^{2 n} /; n \in \mathbb{N}$$

01.11.24.0002.01

$$\prod_{k=1}^{n-1} \sec\left(\frac{\pi k}{n} + z\right) == 2^{n-1} \csc\left(n \left(z + \frac{\pi}{2}\right)\right) \cos(z) /; n \in \mathbb{N}^+$$

01.11.24.0003.01

$$\prod_{k=1}^{n-1} \sec\left(\frac{2 \pi k}{n} + z\right) == \frac{(-2)^{n-1} \cos(z)}{\cos(n z) - \cos\left(\frac{n \pi}{2}\right)} /; n \in \mathbb{N}^+$$

01.11.24.0004.01

$$\prod_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \sec\left(\frac{\pi k}{n}\right) == \frac{2^{\frac{n+1}{2}}}{\sqrt{n} \, (1 + (-1)^n) - (-1)^n + 1} /; n \in \mathbb{N}^+$$

Infinite products

01.11.24.0005.01

$$\prod_{k=1}^{\infty} \sec\left(\frac{z}{2^k}\right) == z \csc(z)$$

Representations through more general functions

Through hypergeometric functions

01.11.26.0029.01

$$\sec(z) = \frac{4\pi}{\pi^2 - 4z^2} \, {}_4F_3\left(1, \frac{3}{2}, \frac{1}{2} - \frac{z}{\pi}, \frac{z}{\pi} + \frac{1}{2}; \frac{1}{2}, \frac{3}{2} - \frac{z}{\pi}, \frac{z}{\pi} + \frac{3}{2}; -1\right)$$

Brychkov Yu.A. (2005)

01.11.26.0001.01

$$\sec(z) = \frac{1}{{}_0F_1\left(; \frac{1}{2}; -\frac{z^2}{4}\right)}$$

Through Meijer *G*

Classical cases for the direct function itself

01.11.26.0002.01

$$\sec(z) = \frac{1}{\sqrt{\pi} \, G_{0,2}^{1,0}\left(\frac{z^2}{4} \,\middle|\, 0, \frac{1}{2}\right)}$$

Generalized cases for the direct function itself

01.11.26.0003.01

$$\sec(z) = \frac{1}{\sqrt{\pi} \, G_{0,2}^{1,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, 0, \frac{1}{2}\right)}$$

Through other functions

Involving Bessel functions

01.11.26.0004.01

$$\sec(z) = \frac{1}{\sqrt{\frac{\pi z}{2}} \, J_{-\frac{1}{2}}(z)}$$

01.11.26.0005.01

$$\sec(z) = \frac{1}{\sqrt{\frac{\pi i z}{2}} \, I_{-\frac{1}{2}}(i z)}$$

01.11.26.0006.01

$$\sec(z) = -\frac{1}{\sqrt{\frac{\pi z}{2}} \, Y_{\frac{1}{2}}(z)}$$

Involving Jacobi functions

01.11.26.0007.01

$$\sec(z) = \frac{1}{\operatorname{cd}(z \mid 0)}$$

01.11.26.0008.01

$$\sec(z) = \frac{1}{\operatorname{cn}(z \mid 0)}$$

01.11.26.0009.01

$$\sec(z) = \operatorname{cn}(i z \mid 1)$$

01.11.26.0010.01

$$\sec(z) = i \operatorname{cs}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)$$

01.11.26.0011.01

$$\sec(z) = \operatorname{dc}(z \mid 0)$$

01.11.26.0012.01

$$\sec(z) = \operatorname{dn}(i z \mid 1)$$

01.11.26.0013.01

$$\sec(z) = \operatorname{ds}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)$$

01.11.26.0014.01

$$\sec(z) = i \operatorname{ds}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)$$

01.11.26.0015.01

$$\sec(z) = \operatorname{nc}(z \mid 0)$$

01.11.26.0016.01

$$\sec(z) = \frac{1}{\operatorname{nc}(i z \mid 1)}$$

01.11.26.0017.01

$$\sec(z) = \frac{1}{\operatorname{nd}(i z \mid 1)}$$

01.11.26.0018.01

$$\sec(z) = \operatorname{ns}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)$$

01.11.26.0019.01

$$\sec(z) = \frac{i}{\operatorname{sc}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)}$$

01.11.26.0020.01

$$\sec(z) = \frac{i}{\operatorname{sd}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)}$$

01.11.26.0021.01

$$\sec(z) = \frac{1}{\operatorname{sd}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)}$$

01.11.26.0022.01

$$\sec(z) = \frac{1}{\operatorname{sn}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)}$$

Involving Mathieu functions

01.11.26.0023.01

$$\sec\left(\sqrt{a}\, z\right) = \frac{\sqrt{a}}{\mathrm{Se}_z(a, 0, z)}$$

01.11.26.0024.01

$$\sec\left(\sqrt{a}\, z\right) = \frac{1}{\mathrm{Ce}(a, 0, z)}$$

Involving some hypergeometric-type functions

01.11.26.0025.01

$$\sec(\pi z) = \frac{1}{\pi} \Gamma\left(\frac{1}{2} - z\right) \Gamma\left(z + \frac{1}{2}\right)$$

01.11.26.0026.01

$$\sec(z) = \frac{1}{1 - \sqrt{\frac{\pi z}{2}}\, \boldsymbol{H}_{\frac{1}{2}}(z)}$$

01.11.26.0027.01

$$\sec(z) = \frac{1}{1 + \sqrt{\frac{\pi i z}{2}}\, \boldsymbol{L}_{\frac{1}{2}}(i z)}$$

01.11.26.0028.01

$$\sec(n z) = \frac{1}{T_n(\cos(z))}$$

Representations through equivalent functions

With inverse function

01.11.27.0001.01

$$\sec\left(\sec^{-1}(z)\right) = z$$

01.11.27.0003.02

$$\sec^{-1}(\sec(z)) = z \;/;\; 0 < \mathrm{Re}(z) < \pi \lor \mathrm{Re}(z) = 0 \land \mathrm{Im}(z) \geq 0 \lor \mathrm{Re}(z) = \pi \land \mathrm{Im}(z) \leq 0$$

01.11.27.0062.01

$$\sec^{-1}(\sec(z)) = -z \;/;\; -\pi < \mathrm{Re}(z) < 0 \lor (\mathrm{Re}(z) = 0 \land \mathrm{Im}(z) \leq 0) \lor (\mathrm{Re}(z) = -\pi \land \mathrm{Im}(z) \geq 0)$$

01.11.27.0002.02

$$\sec^{-1}(\sec(z)) = \sqrt{z^2} \;/;\; |\mathrm{Re}(z)| < \pi \lor \mathrm{Re}(z) = -\pi \land \mathrm{Im}(z) \geq 0 \lor \mathrm{Re}(z) = \pi \land \mathrm{Im}(z) \leq 0$$

01.11.27.0063.01

$$\sec^{-1}(\sec(z)) = \frac{\pi}{2}\left(1 + (-1)^k\right) + (-1)^k (z - \pi (k+1)) \;/;$$
$$(k \pi < \mathrm{Re}(z) < (k+1) \pi \bigvee (\mathrm{Re}(z) = k \pi \bigwedge \mathrm{Im}(z) \geq 0) \bigvee (\mathrm{Re}(z) = (k+1) \pi \bigwedge \mathrm{Im}(z) \leq 0)) \bigwedge k \in \mathbb{Z}$$

01.11.27.0004.01

$$\sec^{-1}(\sec(z)) = \frac{\pi}{2}\left(1 - (-1)^{\left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) + (-1)^{\left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(\left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(\mathrm{Im}(z)) - 1\right) \left(z + \pi \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor\right)$$

01.11.27.0064.01

$$\sec^{-1}(\sec(z)) = \begin{cases} (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(-z + \pi \left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor - \frac{\pi}{2}\right) + \frac{\pi}{2} & \frac{\mathrm{Re}(z)}{\pi} \in \mathbb{Z} \bigwedge \mathrm{Im}(z) \leq 0 \\ (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(z - \pi \left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor - \frac{\pi}{2}\right) + \frac{\pi}{2} & \text{True} \end{cases}$$

With related functions

Involving exp

01.11.27.0005.01

$$\sec(z) = \frac{2}{e^{iz} + e^{-iz}}$$

01.11.27.0006.01

$$\sec(z) = \frac{2\, e^{iz}}{e^{2iz} + 1}$$

Involving sin

01.11.27.0007.01

$$\sec(z) = \frac{1}{\sin\left(\frac{\pi}{2} - z\right)}$$

01.11.27.0008.01

$$\sec(z) = \frac{1}{\sin\left(\frac{\pi}{2} + z\right)}$$

01.11.27.0009.01

$$\sec(z) = \frac{1}{\sqrt{1 - \sin^2(z)}} \;/;\; |\mathrm{Re}(z)| < \frac{\pi}{2}$$

01.11.27.0010.01

$$\sec(z) = \frac{1}{\sqrt{1 - \sin^2(z)}} (-1)^{\left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(\mathrm{Im}(z))\right)$$

01.11.27.0011.01

$$\sec^2(z) = \frac{1}{1 - \sin^2(z)}$$

01.11.27.0012.01

$$\sec\left(\frac{\pi}{2} + z\right) = -\frac{1}{\sin(z)}$$

01.11.27.0013.01

$$\sec\left(\frac{\pi}{2} - z\right) = \frac{1}{\sin(z)}$$

Involving cos

01.11.27.0014.01

$$\sec(z) = \frac{1}{\cos(z)}$$

Involving tan

01.11.27.0015.01

$$\sec(z) = \frac{1+\tan^2\left(\frac{z}{2}\right)}{1-\tan^2\left(\frac{z}{2}\right)}$$

01.11.27.0016.01

$$\sec(z) = \sqrt{1+\tan^2(z)} \;/;\; |\text{Re}(z)| < \frac{\pi}{2}$$

01.11.27.0017.01

$$\sec(z) = \sqrt{1+\tan^2(z)}\, (-1)^{\left\lfloor \frac{1}{2} - \frac{\text{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\text{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\text{Im}(z))\right)$$

01.11.27.0018.01

$$\sec^2(z) = 1 + \tan^2(z)$$

Involving cot

01.11.27.0019.01

$$\sec(z) = \frac{\cot^2\left(\frac{z}{2}\right)+1}{\cot^2\left(\frac{z}{2}\right)-1}$$

01.11.27.0020.01

$$\sec(z) = z\sqrt{\frac{1}{z^2}}\, \frac{\sqrt{\cot^2(z)+1}}{\cot(z)} \;/;\; |\text{Re}(z)| < \pi$$

01.11.27.0021.01

$$\sec(z) = \frac{\sqrt{\cot^2(z)+1}}{\cot(z)} \;/;\; 0 < \text{Re}(z) < \pi$$

01.11.27.0022.01

$$\sec(z) = \frac{\sqrt{\cot^2(z)+1}}{\cot(z)} (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\text{Re}(z)}{\pi} \right\rfloor}\right) \theta(\text{Im}(z))\right)$$

01.11.27.0023.01

$$\sec^2(z) = \frac{\cot^2(z)+1}{\cot^2(z)}$$

Involving csc

01.11.27.0024.01

$$\sec(z) = \csc\left(\frac{\pi}{2} - z\right)$$

01.11.27.0025.01

$$\sec(z) = \csc\left(\frac{\pi}{2} + z\right)$$

01.11.27.0026.01

$$\sec(z) = z\sqrt{\frac{1}{z^2}}\, \frac{\csc(z)}{\sqrt{\csc^2(z)-1}} \;/;\; |\text{Re}(z)| < \frac{\pi}{2}$$

01.11.27.0027.01

$$\sec(z) = \frac{\csc(z)}{\sqrt{\csc^2(z)-1}} \;/;\; 0 < \operatorname{Re}(z) < \frac{\pi}{2}$$

01.11.27.0028.01

$$\sec(z) = \frac{\csc(z)}{\sqrt{\csc^2(z)-1}} (-1)^{\left\lfloor \frac{2\operatorname{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\right) \theta(\operatorname{Im}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(-\operatorname{Im}(z))\right)$$

01.11.27.0029.01

$$\sec^2(z) = \frac{\csc^2(z)}{\csc^2(z)-1}$$

01.11.27.0030.01

$$\csc^2(z) + \sec^2(z) = 4\csc^2(2z)$$

01.11.27.0031.01

$$\sec^2(z) - \csc^2(z) = \frac{\sec^2(z)\left(\sec^2(z)-2\right)}{\sec^2(z)-1}$$

01.11.27.0032.01

$$i\csc(z) + \sec(z) = 2\,i\,e^{-iz}\csc(2z)$$

01.11.27.0033.01

$$\sec(z) - i\csc(z) = -2\,i\,e^{iz}\csc(2z)$$

01.11.27.0034.01

$$\sec(z) + \csc(z) = \frac{2\sqrt{2}\,\csc(2z)}{\csc\left(z+\frac{\pi}{4}\right)}$$

01.11.27.0035.01

$$\sec(z) - \csc(z) = \frac{2\sqrt{2}\,\csc(2z)}{\csc\left(z-\frac{\pi}{4}\right)}$$

01.11.27.0036.01

$$a\sec(z) + b\csc(z) = 2\sqrt{\frac{a^2}{b^2}+1}\; b\cos\left(z - \tan^{-1}\left(\frac{a}{b}\right)\right)\csc(2z)$$

01.11.27.0037.01

$$\sec\left(\frac{\pi}{2}+z\right) = -\csc(z)$$

01.11.27.0038.01

$$\sec\left(\frac{\pi}{2}-z\right) = \csc(z)$$

Involving sinh

01.11.27.0039.01

$$\sec(z) = \frac{i}{\sinh\left(\frac{\pi i}{2} - i z\right)}$$

01.11.27.0040.01

$$\sec(z) = \frac{i}{\sinh\left(\frac{\pi i}{2} + i z\right)}$$

01.11.27.0041.01

$$\sec(z) = \frac{1}{\sqrt{1 + \sinh^2(i z)}} \;/;\; |\mathrm{Re}(z)| < \frac{\pi}{2}$$

01.11.27.0042.01

$$\sec(z) = \frac{1}{\sqrt{1 + \sinh^2(i z)}} (-1)^{\left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(\mathrm{Im}(z))\right)$$

01.11.27.0043.01

$$\sec^2(z) = \frac{1}{1 + \sinh^2(i z)}$$

Involving cosh

01.11.27.0044.01

$$\sec(z) = \frac{1}{\cosh(i z)}$$

01.11.27.0045.01

$$\sec(i z) = \frac{1}{\cosh(z)}$$

Involving tanh

01.11.27.0046.01

$$\sec(z) = \frac{1 - \tanh^2\left(\frac{i z}{2}\right)}{1 + \tanh^2\left(\frac{i z}{2}\right)}$$

01.11.27.0047.01

$$\sec(z) = \sqrt{1 - \tanh^2(i z)} \;/;\; |\mathrm{Re}(z)| < \frac{\pi}{2}$$

01.11.27.0048.01

$$\sec(z) = \sqrt{1 - \tanh^2(i z)}\, (-1)^{\left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\mathrm{Im}(z))\right)$$

01.11.27.0049.01

$$\sec^2(z) = 1 - \tanh^2(i z)$$

Involving coth

01.11.27.0050.01

$$\sec(z) = \frac{\coth^2\left(\frac{i z}{2}\right) - 1}{\coth^2\left(\frac{i z}{2}\right) + 1}$$

01.11.27.0051.01

$$\sec(z) = -\frac{i \sqrt{1 - \coth^2(i z)}}{\coth(i z)} \;/;\; 0 < \mathrm{Re}(z) < \pi$$

01.11.27.0052.01

$$\sec(z) = -\frac{i\sqrt{1-\coth^2(i z)}}{\coth(i z)} (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\text{Re}(z)}{\pi} \right\rfloor}\right) \theta(\text{Im}(z))\right)$$

01.11.27.0053.01

$$\sec^2(z) = \frac{\coth^2(i z) - 1}{\coth^2(i z)}$$

Involving csch

01.11.27.0054.01

$$\sec(z) = i \operatorname{csch}\left(\frac{\pi i}{2} - i z\right)$$

01.11.27.0055.01

$$\sec(z) = i \operatorname{csch}\left(\frac{\pi i}{2} + i z\right)$$

01.11.27.0056.01

$$\sec(z) = -\frac{i\sqrt{-z^2}}{z} \frac{\operatorname{csch}(i z)}{\sqrt{\operatorname{csch}^2(i z) + 1}} \;/;\; \text{Im}(z) \neq 0$$

01.11.27.0057.01

$$\sec(z) = -\frac{\operatorname{csch}(i z)}{\sqrt{\operatorname{csch}^2(i z) + 1}} \;/;\; \text{Im}(z) > 0$$

01.11.27.0058.01

$$\sec^2(z) = \frac{\operatorname{csch}^2(i z)}{\operatorname{csch}^2(i z) + 1}$$

Involving sech

01.11.27.0059.01

$$\sec(z) = \operatorname{sech}(i z)$$

01.11.27.0060.01

$$\sec(i z) = \operatorname{sech}(z)$$

Involving trigonometric and hyperbolic functions

01.11.27.0061.01

$$\sec^2(z) - \csc^2(z) = -4 \cot(2 z) \csc(2 z)$$

Inequalities

01.11.29.0001.01

$$\sec(x) > x \csc(x) \;/;\; 0 < x < \frac{\pi}{2} \bigwedge x \in \mathbb{R}$$

Other information

Value properties

01.11.33.0001.01

$$(x \in \mathbb{Q} \bigwedge \sec(x^{\circ}) \in \mathbb{Q}) \Rightarrow \sec(x) == 1 \lor \sec(x) == -1 \lor \sec(x) == 2 \lor \sec(x) == -2$$

History

– The word "secant" appears for the first time around 800 by Alhaba■Alhãsib
– T. Finck (1583) finalized the use of the modern word "secant"
– A. Magini (1592)
– B. Cavalieri (1643)
– J. Kresa (1720) used symbol "sec"
– L. Euler (1748)

The function sec is encountered often in mathematics and the natural sciences.

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