

Signature

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Notations

Traditional name

Levi-Civita symbol

Traditional notation

$$\varepsilon_{n_1, n_2, \dots, n_d}$$

Mathematica StandardForm notation

Signature[{ n_1 , n_2 , ..., n_d }]

Primary definition

04.21.02.0001.01

$$\varepsilon_{n_1, n_2, \dots, n_d} = (-1)^t$$

where t is the number of permutations from the sorted version of $\{n_1, n_2, \dots, n_d\}$ to $\{n_1, n_2, \dots, n_d\}$.

04.21.02.0002.01

$$\varepsilon_{n_1, n_2, \dots, n_d} = 0 \text{ ; } n_i = n_j \wedge 1 \leq i \leq d \wedge 1 \leq j \leq d$$

Specific values

Values at fixed points

04.21.03.0001.01

$$\varepsilon_0 = 1$$

04.21.03.0002.01

$$\varepsilon_n = 1$$

04.21.03.0003.01

$$\varepsilon_{1,1} = 0$$

04.21.03.0004.01

$$\varepsilon_{1,2} = 1$$

04.21.03.0005.01

$$\varepsilon_{2,1} = -1$$

04.21.03.0006.01

$$\varepsilon_{z,a} = -1$$

$$04.21.03.0007.01$$

$$\varepsilon_{a,z} = 1$$

$$04.21.03.0008.01$$

$$\varepsilon_{1,1,2} = 0$$

$$04.21.03.0009.01$$

$$\varepsilon_{1,2,3} = 1$$

$$04.21.03.0010.01$$

$$\varepsilon_{1,3,2} = -1$$

$$04.21.03.0011.01$$

$$\varepsilon_{2,3,1} = 1$$

$$04.21.03.0012.01$$

$$\varepsilon_{2,1,3} = -1$$

$$04.21.03.0013.01$$

$$\varepsilon_{3,1,2} = 1$$

$$04.21.03.0014.01$$

$$\varepsilon_{3,2,1} = -1$$

$$04.21.03.0015.01$$

$$\varepsilon_{1,2,3,4} = 1$$

$$04.21.03.0016.01$$

$$\varepsilon_{1,2,4,3} = -1$$

General characteristics

Domain and analyticity

$\varepsilon_{n_1, n_2, \dots, n_d}$ is a nonanalytical function, defined on the set of tuples of integers with possible values $0, \pm 1$.

$$04.21.04.0001.01$$

$$(n_1 * n_2 * \dots * n_d) \rightarrow \varepsilon_{n_1, n_2, \dots, n_d} :: \mathbb{Z}^n \rightarrow \{-1, 0, 1\}$$

Symmetries and periodicities

Quasi-permutation symmetry

$$04.21.04.0002.01$$

$$\varepsilon_{n_1, n_2, \dots, n_d} = -\varepsilon_{n_2, n_1, \dots, n_d}$$

$$04.21.04.0003.01$$

$$\varepsilon_{n_1, n_2, \dots, n_k, \dots, n_j, \dots, n_d} = -\varepsilon_{n_1, n_2, \dots, n_j, \dots, n_k, \dots, n_d}$$

$$04.21.04.0004.01$$

$$\varepsilon_{n_1, n_2, \dots, n_d} = (-1)^{d+1} \varepsilon_{n_2, n_3, \dots, n_d, n_1}$$

Transformations

Products, sums, and powers of the direct function

Products of the direct function

04.21.16.0001.01

$$\varepsilon_{n_1, n_2, \dots, n_d} \varepsilon_{m_1, m_2, \dots, m_d} = - \sum_{\text{permutations}(m_1, m_2, \dots, m_d)} \varepsilon_{m_1, m_2, \dots, m_d} \prod_{k=1}^d \delta_{n_k, m_k}$$

04.21.16.0002.01

$$\varepsilon_{n_1, n_2, \dots, n_{r-1}, n_r, n_{r+1}, \dots, n_d} \varepsilon_{n_1, n_2, \dots, n_{r-1}, m_r, m_{r+1}, \dots, m_d} = - \frac{(d-r)! r!}{d!} \sum_{\text{permutations}(m_r, m_{r+1}, \dots, m_d)} \varepsilon_{m_r, m_{r+1}, \dots, m_d} \prod_{k=r}^d \delta_{n_k, m_k}$$

Complex characteristics

Real part

04.21.19.0001.01

$$\text{Re}(\varepsilon_{n_1, n_2, \dots, n_d}) = \varepsilon_{n_1, n_2, \dots, n_d}$$

Imaginary part

04.21.19.0002.01

$$\text{Im}(\varepsilon_{n_1, n_2, \dots, n_d}) = 0$$

Absolute value

04.21.19.0003.01

$$|\varepsilon_{n_1, n_2, \dots, n_d}| = \sqrt{\varepsilon_{n_1, n_2, \dots, n_d}^2}$$

Argument

04.21.19.0004.01

$$\arg(\varepsilon_{n_1, n_2, \dots, n_d}) = \tan^{-1}(\varepsilon_{n_1, n_2, \dots, n_d}, 0)$$

Conjugate value

04.21.19.0005.01

$$\overline{\varepsilon_{n_1, n_2, \dots, n_d}} = \varepsilon_{n_1, n_2, \dots, n_d}$$

Summation

Finite summation

04.21.23.0001.01

$$\sum_{\tau_1=1}^n \sum_{\tau_2=1}^n \dots \sum_{\tau_r=1}^n \varepsilon_{\tau_1, \tau_2, \dots, \tau_r, \nu_{r+1}, \dots, \nu_n} \varepsilon_{\tau_1, \tau_2, \dots, \tau_r, \mu_{r+1}, \dots, \mu_n} = r! \sum_{\mu_{r+1}=1}^n \sum_{\mu_{r+2}=1}^n \dots \sum_{\mu_n=1}^n \varepsilon_{\mu_{r+1}, \dots, \mu_n} \prod_{k=r+1}^n \delta_{\nu_k, \mu_k}$$

Theorems

The determinant of a matrix with entries elements

The determinant $\det(A)$ of the $n \times n$ matrix A with entries a_{ij} can be expressed as

$$\det(A) = \sum_{k_1=1}^n \sum_{k_2=1}^n \cdots \sum_{k_n=1}^n \varepsilon_{k_1 k_2 \dots k_n} a_{1 k_1} a_{2 k_2} \cdots a_{n k_n}.$$

Antisymmetric Levi-Civita tensor

The set of all $\varepsilon_{n_1, n_2, \dots, n_d}$ /; $1 \leq n_1 \leq d, \dots, 1 \leq n_d \leq d$ forms the completely antisymmetric Levi-Civita tensor of dimension d .

History

–T. Levi-Civita (1896)

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