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SpheroidalQSPrime

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Notations

Traditional name

Derivative of the angular spheroidal function of the second kind

Traditional notation

$QS_{\nu,\mu}'(\gamma, z)$

Mathematica StandardForm notation

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SpheroidalQSPrime[ν, μ, γ, z]
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Primary definition

11.13.02.0001.01

$$QS_{\nu,\mu}'(\gamma, z) = \frac{\partial QS_{\nu,\mu}(\gamma, z)}{\partial z}$$

Specific values

Specialized values

For fixed ν, μ, z

11.13.03.0001.01

$$QS_{\nu,\mu}'(0, z) = \frac{(-\mu + \nu + 1)\, Q_{\nu+1}^{\mu}(z) - z\,(\nu + 1)\, Q_{\nu}^{\mu}(z)}{z^2 - 1}$$

For fixed ν, γ, z

11.13.03.0002.01

$$QS_{\nu,\frac{1}{2}}'(\gamma, z) = -\frac{\pi}{2\sqrt{2\pi}\,(1 - z^2)^{5/4}} \left(z\,\mathrm{Se}\left(b_{\nu+\frac{1}{2}}\left(\frac{\gamma^2}{4}\right), \frac{\gamma^2}{4}, \cos^{-1}(z)\right) - 2\sqrt{1 - z^2}\,\mathrm{Se}'\left(b_{\nu+\frac{1}{2}}\left(\frac{\gamma^2}{4}\right), \frac{\gamma^2}{4}, \cos^{-1}(z)\right)\right)$$

For fixed ν, z

11.13.03.0003.01

$$QS_{\nu,0}'(0, z) = \frac{(\nu + 1)\, Q_{\nu+1}(z) - z\,(\nu + 1)\, Q_{\nu}(z)}{z^2 - 1}$$

11.13.03.0004.01

$$QS_{\nu,\frac{1}{2}}'(0,z)=\frac{\sqrt{\frac{\pi}{2}}}{2\left(1-z^2\right)^{5/4}}\left(\sqrt{1-z^2}\;(2\nu+1)\cos\left(\left(\nu+\frac{1}{2}\right)\cos^{-1}(z)\right)-z\sin\left(\left(\nu+\frac{1}{2}\right)\cos^{-1}(z)\right)\right)$$

Series representations

Generalized power series

Expansions at $\gamma == 0$

11.13.06.0001.01

$$QS_{\nu,\mu}'(\gamma,z)\propto \frac{(-\mu+\nu+1)\,Q_{\nu+1}^{\mu}(z)-z\,(\nu+1)\,Q_{\nu}^{\mu}(z)}{z^2-1}+\frac{1}{2\left(z^2-1\right)(2\nu+1)}\left(\frac{(\mu+\nu-1)\,(\mu+\nu)\left(z\,(\nu-1)\,Q_{\nu-2}^{\mu}(z)+(\mu-\nu+1)\,Q_{\nu-1}^{\mu}(z)\right)}{(1-2\nu)^2}+\frac{(-\mu+\nu+1)\,(-\mu+\nu+2)\left((-\mu+\nu+3)\,Q_{\nu+3}^{\mu}(z)-z\,(\nu+3)\,Q_{\nu+2}^{\mu}(z)\right)}{(2\nu+3)^2}\right)\gamma^2-\frac{1}{8\left(z^2-1\right)}\left(\frac{(\mu+\nu-3)\,(\mu+\nu-2)\,(\mu+\nu-1)\,(\mu+\nu)\left(z\,(\nu-3)\,Q_{\nu-4}^{\mu}(z)+(\mu-\nu+3)\,Q_{\nu-3}^{\mu}(z)\right)}{\left(4\nu^2-8\nu-5\right)\left(4\nu^2-8\nu+3\right)^2}+\frac{8\left(4\mu^2-1\right)\left(\mu^2+(2\nu-1)\,\mu+(\nu-1)\,\nu\right)\left(z\,(\nu-1)\,Q_{\nu-2}^{\mu}(z)+(\mu-\nu+1)\,Q_{\nu-1}^{\mu}(z)\right)}{(1-2\nu)^4\left(8\nu^3-4\nu^2-34\nu-15\right)}+\frac{1}{2\nu+1}\left(\frac{(\mu-\nu)\,(\mu-\nu+1)\,(\mu+\nu-1)\,(\mu+\nu)}{(1-2\nu)^4\,(2\nu-3)}+\frac{(-\mu+\nu+1)\,(-\mu+\nu+2)\,(\mu+\nu+1)\,(\mu+\nu+2)}{(2\nu+3)^4\,(2\nu+5)}\right)\left((-\mu+\nu+1)\,Q_{\nu+1}^{\mu}(z)-z\,(\nu+1)\,Q_{\nu}^{\mu}(z)\right)+\frac{(\mu-\nu-4)\,(\mu-\nu-3)\,(\mu-\nu-2)\,(\mu-\nu-1)\left(z\,(\nu+5)\,Q_{\nu+4}^{\mu}(z)+(\mu-\nu-5)\,Q_{\nu+5}^{\mu}(z)\right)}{\left(4\nu^2+16\nu+7\right)\left(4\nu^2+16\nu+15\right)^2}-\frac{8\left(4\mu^2-1\right)\left(\mu^2-(2\nu+3)\,\mu+\nu^2+3\nu+2\right)\left(z\,(\nu+3)\,Q_{\nu+2}^{\mu}(z)+(\mu-\nu-3)\,Q_{\nu+3}^{\mu}(z)\right)}{(2\nu+3)^4\left(8\nu^3+28\nu^2-2\nu-7\right)}\right)\gamma^4+\ldots\;/;\;(\gamma\to 0)$$

11.13.06.0002.01

$$QS_{\nu,\mu}'(\gamma,z)\propto \frac{(-\mu+\nu+1)\,Q_{\nu+1}^{\mu}(z)-z\,(\nu+1)\,Q_{\nu}^{\mu}(z)}{z^2-1}\left(1+O\left(\gamma^2\right)\right)$$

Expansions at generic point $z == z_0$

11.13.06.0003.01

$$QS_{\nu,\mu}'(\gamma, z) \propto QS_{\nu,\mu}'(\gamma, z_0) + \frac{1}{1-z_0^2}\left(2\, QS_{\nu,\mu}'(\gamma, z_0)\, z_0 + QS_{\nu,\mu}(\gamma, z_0)\left(\left(z_0^2-1\right)\gamma^2 - \lambda_{\nu,\mu}(\gamma) + \frac{\mu^2}{1-z_0^2}\right)\right)(z-z_0) + \frac{1}{2\left(z_0^2-1\right)^3}\left(2\, QS_{\nu,\mu}(\gamma, z_0)\, z_0\left(-3\mu^2 + \gamma^2\left(z_0^2-1\right)^2 - 2\lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right)\right) - QS_{\nu,\mu}'(\gamma, z_0)\left(z_0^2-1\right)\left(\gamma^2 z_0^4 - 2\left(\gamma^2+3\right)z_0^2 + \gamma^2 - \mu^2 - \lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right) - 2\right)\right)(z-z_0)^2 + \frac{1}{6\left(z_0^2-1\right)^4}\left(4\, QS_{\nu,\mu}'(\gamma, z_0)\, z_0\left(z_0^2-1\right)\left(\gamma^2 z_0^4 - 2\left(\gamma^2+3\right)z_0^2 + \gamma^2 - 3\mu^2 - 2\lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right) - 6\right) + QS_{\nu,\mu}(\gamma, z_0)\left(\gamma^4\left(z_0^2-1\right)^4 + \lambda_{\nu,\mu}(\gamma)^2\left(z_0^2-1\right)^2 - 2\gamma^2\left(\mu^2 + 4z_0^2 + 2\right)\left(z_0^2-1\right)^2 - 2\lambda_{\nu,\mu}(\gamma)\left(\gamma^2 z_0^4 - \left(2\gamma^2+9\right)z_0^2 + \gamma^2 - \mu^2 - 3\right)\left(z_0^2-1\right) + \mu^2\left(\mu^2 + 36z_0^2 + 8\right)\right)\right)(z-z_0)^3 + \ldots \;/;\; (z \to z_0)$$

11.13.06.0004.01

$$QS_{\nu,\mu}'(\gamma, z) \propto QS_{\nu,\mu}'(\gamma, z_0)\,(1 + O(z-z_0))$$

Expansions at $z == 0$

11.13.06.0005.01

$$QS_{\nu,\mu}'(\gamma, z) \propto QS_{\nu,\mu}'(\gamma, 0) - \left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma)\right) QS_{\nu,\mu}(\gamma, 0)\, z + -\frac{1}{2}\left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma) - 2\right) QS_{\nu,\mu}'(\gamma, 0)\, z^2\, \frac{1}{6}\left(\gamma^4 - 2\left(\mu^2+2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma)^2 + \mu^2\left(\mu^2+8\right) + 2\left(\gamma^2 - \mu^2 - 3\right)\lambda_{\nu,\mu}(\gamma)\right) QS_{\nu,\mu}(\gamma, 0)\, z^3 + \ldots \;/;\; (z \to 0)$$

11.13.06.0006.01

$$QS_{\nu,\mu}'(\gamma, z) \propto QS_{\nu,\mu}'(\gamma, 0)\,(1 + O(z))$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

11.13.13.0001.01

$$\left(1-z^2\right) w''(z) - 2z\left(\frac{\left(1-z^2\right)^2\gamma^2 + \mu^2}{\mu^2 - \left(1-z^2\right)\left(\left(1-z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma)\right)} + 2\right) w'(z) + \left(\frac{4\left(\left(1-z^2\right)^2\gamma^2 + \mu^2\right)z^2}{\left(1-z^2\right)\left(-\left(1-z^2\right)^2\gamma^2 + \mu^2 - \left(1-z^2\right)\lambda_{\nu,\mu}(\gamma)\right)} + \left(1-z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1-z^2} - 2\right) w(z) = 0 \;/;\; w(z) = c_1\, QS_{\nu,\mu}'(\gamma, z) + c_2\, PS_{\nu,\mu}'(\gamma, z)$$

11.13.13.0002.01

$$W_z\left(QS_{\nu,\mu}'(\gamma, z), PS_{\nu,\mu}'(\gamma, z)\right) == \left(\frac{\gamma^2}{1-z^2} - \frac{\mu^2}{\left(1-z^2\right)^3} + \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-z^2\right)^2}\right)\left(PS_{\nu,\mu}'(\gamma, 0)\, QS_{\nu,\mu}(\gamma, 0) - PS_{\nu,\mu}(\gamma, 0)\, QS_{\nu,\mu}'(\gamma, 0)\right)$$

11.13.13.0003.01

$$\left(1-g(z)^2\right) w''(z)+\left(\left(g(z)^2-1\right)\frac{g''(z)}{g'(z)}-2\,g(z)\,g'(z)\right) w'(z)+\left(\frac{\left(\gamma\,g(z)^2+\mu-\gamma\right)\left(-\gamma\,g(z)^2+\mu+\gamma\right) g'(z)^2}{g(z)^2-1}+\lambda_{\nu,\mu}(\gamma)\,g'(z)^2\right) w(z)=0\;/;\; w(z)=c_1\,QS_{\nu,\mu}{}'(\gamma,\,g(z))\;+c_2\,PS_{\nu,\mu}{}'(\gamma,\,g(z))$$

11.13.13.0004.01

$$W_z\left(QS_{\nu,\mu}{}'(\gamma,\,g(z)),\,PS_{\nu,\mu}{}'(\gamma,\,g(z))\right)= g'(z)\left(\frac{\gamma^2}{1-g(z)^2}+\frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-g(z)^2\right)^2}-\frac{\mu^2}{\left(1-g(z)^2\right)^3}\right)\left(PS_{\nu,\mu}{}'(\gamma,\,0)\,QS_{\nu,\mu}(\gamma,\,0)-PS_{\nu,\mu}(\gamma,\,0)\,QS_{\nu,\mu}{}'(\gamma,\,0)\right)$$

11.13.13.0005.01

$$\left(1-g(z)^2\right) w''(z)+\left(\left(g(z)^2-1\right)\left(\frac{2\,h'(z)}{h(z)}+\frac{g''(z)}{g'(z)}\right)-2\,g(z)\,g'(z)\right) w'(z)+ \left(\frac{\left(\gamma\,g(z)^2+\mu-\gamma\right)\left(-\gamma\,g(z)^2+\mu+\gamma\right) g'(z)^2}{g(z)^2-1}+\lambda_{\nu,\mu}(\gamma)\,g'(z)^2+\frac{2\,g(z)\,h'(z)\,g'(z)}{h(z)}+\frac{\left(g(z)^2-1\right)\left(h(z)\,h''(z)-2\,h'(z)^2\right)}{h(z)^2}- \frac{\left(g(z)^2-1\right) h'(z)\,g''(z)}{h(z)\,g'(z)}\right) w(z)=0\;/;\; w(z)=c_1\,h(z)\,QS_{\nu,\mu}{}'(\gamma,\,g(z))\;+c_2\,h(z)\,PS_{\nu,\mu}{}'(\gamma,\,g(z))$$

11.13.13.0006.01

$$W_z\left(h(z)\,QS_{\nu,\mu}{}'(\gamma,\,g(z)),\,h(z)\,PS_{\nu,\mu}{}'(\gamma,\,g(z))\right)= h(z)^2\,g'(z)\left(\frac{\gamma^2}{1-g(z)^2}+\frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-g(z)^2\right)^2}-\frac{\mu^2}{\left(1-g(z)^2\right)^3}\right)\left(PS_{\nu,\mu}{}'(\gamma,\,0)\,QS_{\nu,\mu}(\gamma,\,0)-PS_{\nu,\mu}(\gamma,\,0)\,QS_{\nu,\mu}{}'(\gamma,\,0)\right)$$

11.13.13.0007.01

$$\left(1-a^2\,z^{2\,r}\right) w''(z)-\frac{a^2\,(r-2\,s+1)\,z^{2\,r}+r+2\,s-1}{z}\,w'(z)+ \left(a^2\,r^2\,\lambda_{\nu,\mu}(\gamma)\,z^{2\,r-2}+\frac{-a^2\,r^2\left(\left(a^2\,z^{2\,r}-1\right)^2\gamma^2-\mu^2\right)z^{2\,r}-s^2\left(a^2\,z^{2\,r}-1\right)^2+r\,s\left(a^4\,z^{4\,r}-1\right)}{z^2\left(a^2\,z^{2\,r}-1\right)}\right) w(z)=0\;/;\; w(z)=c_1\,z^s\,QS_{\nu,\mu}{}'(\gamma,\,a\,z^r)\;+c_2\,z^s\,PS_{\nu,\mu}{}'(\gamma,\,a\,z^r)$$

11.13.13.0008.01

$$W_z\left(z^s\,QS_{\nu,\mu}{}'(\gamma,\,a\,z^r),\,z^s\,PS_{\nu,\mu}{}'(\gamma,\,a\,z^r)\right)= a\,r\,z^{r+2\,s-1}\left(\frac{\gamma^2}{1-a^2\,z^{2\,r}}+\frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-a^2\,z^{2\,r}\right)^2}-\frac{\mu^2}{\left(1-a^2\,z^{2\,r}\right)^3}\right)\left(PS_{\nu,\mu}{}'(\gamma,\,0)\,QS_{\nu,\mu}(\gamma,\,0)-PS_{\nu,\mu}(\gamma,\,0)\,QS_{\nu,\mu}{}'(\gamma,\,0)\right)$$

11.13.13.0009.01

$$\left(1-a^2\,r^{2\,z}\right) w''(z)+\left(-a^2\log(r)\,r^{2\,z}+2\,a^2\log(s)\,r^{2\,z}-\log(r)-2\log(s)\right) w'(z)+ \left(a^2\log^2(r)\,\lambda_{\nu,\mu}(\gamma)\,r^{2\,z}+\frac{1}{a^2\,r^{2\,z}-1}\left(-a^2\left(\left(a^2\,r^{2\,z}-1\right)^2\gamma^2-\mu^2\right)\log^2(r)\,r^{2\,z}-\left(a^2\,r^{2\,z}-1\right)^2\log^2(s)+\left(a^4\,r^{4\,z}-1\right)\log(r)\log(s)\right)\right) w(z)=0\;/;\; w(z)=c_1\,s^z\,QS_{\nu,\mu}{}'(\gamma,\,a\,r^z)\;+c_2\,s^z\,PS_{\nu,\mu}{}'(\gamma,\,a\,r^z)$$

11.13.13.0010.01

$$W_z\left(s^z\, QS_{\nu,\mu}'(\gamma,\, a\, r^z),\, s^z\, PS_{\nu,\mu}'(\gamma,\, a\, r^z)\right) = a\, s^{2z}\, r^z \log(r) \left(\frac{\gamma^2}{1-a^2\, r^{2z}} - \frac{\mu^2}{\left(1-a^2\, r^{2z}\right)^3} + \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-a^2\, r^{2z}\right)^2}\right) \left(PS_{\nu,\mu}'(\gamma,\, 0)\, QS_{\nu,\mu}(\gamma,\, 0) - PS_{\nu,\mu}(\gamma,\, 0)\, QS_{\nu,\mu}'(\gamma,\, 0)\right)$$

Differentiation

Low-order differentiation

With respect to z

11.13.20.0001.01

$$\frac{\partial QS_{\nu,\mu}'(\gamma,\, z)}{\partial z} = \frac{1}{1-z^2}\left(2\, z\, QS_{\nu,\mu}'(\gamma,\, z) - \left(\left(1-z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1-z^2}\right) QS_{\nu,\mu}(\gamma,\, z)\right)$$

11.13.20.0002.01

$$\frac{\partial^2 QS_{\nu,\mu}'(\gamma,\, z)}{\partial z^2} = \frac{1}{\left(z^2-1\right)^3}\left(2\, z\left(\left(z^2-1\right)^2 \gamma^2 - 3\,\mu^2 - 2\left(z^2-1\right)\lambda_{\nu,\mu}(\gamma)\right) QS_{\nu,\mu}(\gamma,\, z) - \left(z^2-1\right)\left(\gamma^2\, z^4 - 2\left(\gamma^2+3\right) z^2 + \gamma^2 - \mu^2 - \left(z^2-1\right)\lambda_{\nu,\mu}(\gamma) - 2\right) QS_{\nu,\mu}'(\gamma,\, z)\right)$$

Integration

Indefinite integration

Involving only one direct function

11.13.21.0001.01

$$\int QS_{\nu,\mu}'(\gamma,\, z)\, dz = QS_{\nu,\mu}(\gamma,\, z)$$

Operations

Representations through equivalent functions

Theorems

History

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