StirlingS1

Notations

Traditional name

Stirling number of the first kind

Traditional notation

$S_n^{(m)}$

Mathematica StandardForm notation

`StirlingS1[n, m]`

Primary definition

04.14.02.0001.01

$$S_n^{(m)} = (-1)^n \left([t^m]\ (-t)_n\right) /; m \in \mathbb{N} \land n \in \mathbb{N}$$

$(-1)^{n-m}\, S_n^{(m)}$ is the number of ways of partitioning a set of n elements which contain exactly m permutation disjoint cycles.

Example: The six permuations {{2, 3, 4, 1}, {2, 4, 1, 3}, {3, 1, 4, 2}, {3, 4, 2, 1}, {4, 1, 2, 3}, {4, 3, 1, 2}} of the list of four numbers {1, 2, 3, 4} have all exactly one cycle:
{2, 3, 4, 1} : 2 –> 3 –> 4 –> 1 → 2; {2, 4, 1, 3} : 1 → 3 → 4 → 2 → 1; {3, 1, 4, 2} : 1 –> 2 –> 4 –> 3 –> 1;…
The eleven permutaions {{1, 3, 4, 2}, {1, 4, 2, 3}, {2, 1, 4, 3}, {2, 3, 1, 4}, {2, 4, 3, 1}, {3, 1, 2, 4}, {3, 2, 4, 1}, {3, 4, 1, 2}, {4, 1, 3, 2}, {4, 2, 1, 3}, {4, 3, 2, 1}} of the list of four numbers {1,2,3,4} have all exactly two cycles:
{1, 3, 4, 2} : 1 and 3 –> 4 –> 2 –> 3; …
By these reasons $S_4^{(4)} = 1$, $S_4^{(3)} = -6$, $S_4^{(2)} = 11$, and $S_4^{(1)} = -6$.

Specific values

Specialized values

For fixed *n*

04.14.03.0001.01

$$S_n^{(0)} = \delta_n /; n \in \mathbb{N}$$

04.14.03.0002.01

$$S_n^{(1)} = (-1)^{n-1}\,(n-1)! /; n \in \mathbb{N}^+$$

04.14.03.0003.01

$$\mathcal{S}_n^{(2)} = (-1)^n\,(n-1)!\,H_{n-1}\,/;\,n\in\mathbb{N}^+$$

04.14.03.0004.01

$$\mathcal{S}_n^{(3)} = \frac{1}{2}\,(-1)^{n-1}\,(n-1)!\,\left(H_{n-1}^2 - H_{n-1}^{(2)}\right)\,/;\,n\in\mathbb{N}^+$$

04.14.03.0030.01

$$\mathcal{S}_n^{(4)} = \frac{1}{6}\,(-1)^n\,(n-1)!\,\left(H_{n-1}^3 - 3\,H_{n-1}^{(2)}\,H_{n-1} + 2\,H_{n-1}^{(3)}\right)\,/;\,n\in\mathbb{N}^+$$

04.14.03.0031.01

$$\mathcal{S}_n^{(5)} = \frac{1}{24}\,(-1)^{n-1}\,(n-1)!\,\left(H_{n-1}^4 - 6\,H_{n-1}^{(2)}\,H_{n-1}^2 + 8\,H_{n-1}^{(3)}\,H_{n-1} + 3\left(H_{n-1}^{(2)}\right)^2 - 6\,H_{n-1}^{(4)}\right)\,/;\,n\in\mathbb{N}^+$$

04.14.03.0032.01

$$\mathcal{S}_n^{(6)} = \frac{1}{120}\,(-1)^n\,(n-1)!\,\left(H_{n-1}^5 - 10\,H_{n-1}^{(2)}\,H_{n-1}^3 + 20\,H_{n-1}^{(3)}\,H_{n-1}^2 + 15\left(\left(H_{n-1}^{(2)}\right)^2 - 2\,H_{n-1}^{(4)}\right)H_{n-1} - 20\,H_{n-1}^{(2)}\,H_{n-1}^{(3)} + 24\,H_{n-1}^{(5)}\right)\,/;\,n\in\mathbb{N}^+$$

04.14.03.0033.01

$$\mathcal{S}_n^{(7)} = \frac{1}{720}\,(-1)^{n-1}\,(n-1)!\,\Bigg(H_{n-1}^6 - 15\,H_{n-1}^{(2)}\,H_{n-1}^4 + 40\,H_{n-1}^{(3)}\,H_{n-1}^3 + 45\left(\left(H_{n-1}^{(2)}\right)^2 - 2\,H_{n-1}^{(4)}\right)H_{n-1}^2 - 24\left(5\,H_{n-1}^{(2)}\,H_{n-1}^{(3)} - 6\,H_{n-1}^{(5)}\right)H_{n-1} + 5\left(-3\left(H_{n-1}^{(2)}\right)^3 + 18\,H_{n-1}^{(4)}\,H_{n-1}^{(2)} + 8\left(\left(H_{n-1}^{(3)}\right)^2 - 3\,H_{n-1}^{(6)}\right)\right)\Bigg)\,/;\,n\in\mathbb{N}^+$$

04.14.03.0034.01

$$\mathcal{S}_n^{(8)} = \frac{1}{5040}\Bigg((-1)^n\,(n-1)!\,\Bigg(H_{n-1}^7 - 21\,H_{n-1}^{(2)}\,H_{n-1}^5 + 70\,H_{n-1}^{(3)}\,H_{n-1}^4 + 105\left(\left(H_{n-1}^{(2)}\right)^2 - 2\,H_{n-1}^{(4)}\right)H_{n-1}^3 - 84\left(5\,H_{n-1}^{(2)}\,H_{n-1}^{(3)} - 6\,H_{n-1}^{(5)}\right)H_{n-1}^2 - 35\left(3\left(H_{n-1}^{(2)}\right)^3 - 18\,H_{n-1}^{(4)}\,H_{n-1}^{(2)} - 8\left(H_{n-1}^{(3)}\right)^2 + 24\,H_{n-1}^{(6)}\right)H_{n-1} + 6\left(35\,H_{n-1}^{(3)}\left(H_{n-1}^{(2)}\right)^2 - 84\,H_{n-1}^{(5)}\,H_{n-1}^{(2)} - 70\,H_{n-1}^{(3)}\,H_{n-1}^{(4)} + 120\,H_{n-1}^{(7)}\right)\Bigg)\Bigg)\,/;\,n\in\mathbb{N}^+$$

04.14.03.0035.01

$$\mathcal{S}_n^{(9)} = \frac{1}{40\,320}\Bigg((-1)^{n-1}\,(n-1)!\,\Bigg(H_{n-1}^8 - 28\,H_{n-1}^{(2)}\,H_{n-1}^6 + 112\,H_{n-1}^{(3)}\,H_{n-1}^5 + 210\left(\left(H_{n-1}^{(2)}\right)^2 - 2\,H_{n-1}^{(4)}\right)H_{n-1}^4 - 224\left(5\,H_{n-1}^{(2)}\,H_{n-1}^{(3)} - 6\,H_{n-1}^{(5)}\right)H_{n-1}^3 - 140\left(3\left(H_{n-1}^{(2)}\right)^3 - 18\,H_{n-1}^{(4)}\,H_{n-1}^{(2)} - 8\left(H_{n-1}^{(3)}\right)^2 + 24\,H_{n-1}^{(6)}\right)H_{n-1}^2 + 48\left(35\,H_{n-1}^{(3)}\left(H_{n-1}^{(2)}\right)^2 - 84\,H_{n-1}^{(5)}\,H_{n-1}^{(2)} - 70\,H_{n-1}^{(3)}\,H_{n-1}^{(4)} + 120\,H_{n-1}^{(7)}\right)H_{n-1} + 7\left(15\left(H_{n-1}^{(2)}\right)^4 - 180\,H_{n-1}^{(4)}\left(H_{n-1}^{(2)}\right)^2 - 160\left(\left(H_{n-1}^{(3)}\right)^2 - 3\,H_{n-1}^{(6)}\right)H_{n-1}^{(2)} + 12\left(15\left(H_{n-1}^{(4)}\right)^2 + 32\,H_{n-1}^{(3)}\,H_{n-1}^{(5)} - 60\,H_{n-1}^{(8)}\right)\right)\Bigg)\Bigg)\,/;\,n\in\mathbb{N}^+$$

04.14.03.0036.01

$$\mathcal{S}_n^{(10)} = \frac{1}{362\,880}\left((-1)^n (n-1)!\left(H_{n-1}^9 - 36 H_{n-1}^{(2)} H_{n-1}^7 + 168 H_{n-1}^{(3)} H_{n-1}^6 + 378\left(\left(H_{n-1}^{(2)}\right)^2 - 2 H_{n-1}^{(4)}\right) H_{n-1}^5 - 504\left(5 H_{n-1}^{(2)} H_{n-1}^{(3)} - 6 H_{n-1}^{(5)}\right) H_{n-1}^4 - 420\left(3\left(H_{n-1}^{(2)}\right)^3 - 18 H_{n-1}^{(4)} H_{n-1}^{(2)} - 8\left(H_{n-1}^{(3)}\right)^2 + 24 H_{n-1}^{(6)}\right) H_{n-1}^3 + 216\left(35 H_{n-1}^{(3)}\left(H_{n-1}^{(2)}\right)^2 - 84 H_{n-1}^{(5)} H_{n-1}^{(2)} - 70 H_{n-1}^{(3)} H_{n-1}^{(4)} + 120 H_{n-1}^{(7)}\right) H_{n-1}^2 + 63\left(15\left(H_{n-1}^{(2)}\right)^4 - 180 H_{n-1}^{(4)}\left(H_{n-1}^{(2)}\right)^2 - 160\left(\left(H_{n-1}^{(3)}\right)^2 - 3 H_{n-1}^{(6)}\right) H_{n-1}^{(2)} + 12\left(15\left(H_{n-1}^{(4)}\right)^2 + 32 H_{n-1}^{(3)} H_{n-1}^{(5)} - 60 H_{n-1}^{(8)}\right)\right) H_{n-1} - 8\left(315 H_{n-1}^{(3)}\left(H_{n-1}^{(2)}\right)^3 - 1134 H_{n-1}^{(5)}\left(H_{n-1}^{(2)}\right)^2 - 270\left(7 H_{n-1}^{(3)} H_{n-1}^{(4)} - 12 H_{n-1}^{(7)}\right) H_{n-1}^{(2)} - 28\left(10\left(H_{n-1}^{(3)}\right)^3 - 90 H_{n-1}^{(6)} H_{n-1}^{(3)} - 81 H_{n-1}^{(4)} H_{n-1}^{(5)} + 180 H_{n-1}^{(9)}\right)\right)\right)\right) /; n \in \mathbb{N}^+$$

04.14.03.0005.01

$$S_n^{(n-1)} = -\binom{n}{2} /; n \in \mathbb{N}^+$$

04.14.03.0006.01

$$S_n^{(n)} = 1 /; n \in \mathbb{N}$$

04.14.03.0007.01

$$S_n^{(m)} = 0 /; n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+ \bigwedge m > n$$

For fixed *m*

04.14.03.0008.01

$$S_0^{(m)} = 0 /; m \in \mathbb{N}^+$$

04.14.03.0009.01

$$S_0^{(m)} = \delta_m /; m \in \mathbb{N}$$

04.14.03.0037.01

$$S_1^{(m)} = \delta_{m-1} /; m \in \mathbb{N}$$

04.14.03.0038.01

$$S_2^{(m)} = \delta_{m-2} - \delta_{m-1} /; m \in \mathbb{N}$$

04.14.03.0039.01

$$S_3^{(m)} = \delta_{m-3} - 3\,\delta_{m-2} + 2\,\delta_{m-1} /; m \in \mathbb{N}$$

04.14.03.0040.01

$$S_4^{(m)} = \delta_{m-4} - 6\,\delta_{m-3} + 11\,\delta_{m-2} - 6\,\delta_{m-1} /; m \in \mathbb{N}$$

04.14.03.0041.01

$$S_5^{(m)} = \delta_{m-5} - 10\,\delta_{m-4} + 35\,\delta_{m-3} - 50\,\delta_{m-2} + 24\,\delta_{m-1} /; m \in \mathbb{N}$$

04.14.03.0042.01

$$S_6^{(m)} = \delta_{m-6} - 15\,\delta_{m-5} + 85\,\delta_{m-4} - 225\,\delta_{m-3} + 274\,\delta_{m-2} - 120\,\delta_{m-1} /; m \in \mathbb{N}$$

04.14.03.0043.01

$$S_7^{(m)} = \delta_{m-7} - 21\,\delta_{m-6} + 175\,\delta_{m-5} - 735\,\delta_{m-4} + 1624\,\delta_{m-3} - 1764\,\delta_{m-2} + 720\,\delta_{m-1} /; m \in \mathbb{N}$$

04.14.03.0044.01

$$S_8^{(m)} = \delta_{m-8} - 28\,\delta_{m-7} + 322\,\delta_{m-6} - 1960\,\delta_{m-5} + 6769\,\delta_{m-4} - 13\,132\,\delta_{m-3} + 13\,068\,\delta_{m-2} - 5040\,\delta_{m-1} /; m \in \mathbb{N}$$

04.14.03.0045.01

$$S_9^{(m)} = \delta_{m-9} - 36\,\delta_{m-8} + 546\,\delta_{m-7} - 4536\,\delta_{m-6} + 22\,449\,\delta_{m-5} - 67\,284\,\delta_{m-4} + 118\,124\,\delta_{m-3} - 109\,584\,\delta_{m-2} + 40\,320\,\delta_{m-1} \;/;\; m \in \mathbb{N}$$

04.14.03.0046.01

$$S_{10}^{(m)} = \delta_{m-10} - 45\,\delta_{m-9} + 870\,\delta_{m-8} - 9450\,\delta_{m-7} + 63\,273\,\delta_{m-6} - 269\,325\,\delta_{m-5} + 723\,680\,\delta_{m-4} - 1\,172\,700\,\delta_{m-3} + 1\,026\,576\,\delta_{m-2} - 362\,880\,\delta_{m-1} \;/;\; m \in \mathbb{N}$$

04.14.03.0047.01

$$S_{m+1}^{(m)} = -\frac{1}{2}\, m\,(m+1) \;/;\; m \in \mathbb{N}$$

04.14.03.0048.01

$$S_{m+2}^{(m)} = \frac{1}{24}\, m\,(m+1)\,(m+2)\,(3\,m+5) \;/;\; m \in \mathbb{N}$$

04.14.03.0049.01

$$S_{m+3}^{(m)} = -\frac{1}{48}\, m\,(m+1)\,(m+2)^2\,(m+3)^2 \;/;\; m \in \mathbb{N}$$

04.14.03.0050.01

$$S_{m+4}^{(m)} = \frac{m\,(m+1)\,(m+2)\,(m+3)\,(m+4)\left(15\,m^3 + 150\,m^2 + 485\,m + 502\right)}{5760} \;/;\; m \in \mathbb{N}$$

04.14.03.0051.01

$$S_{m+5}^{(m)} = -\frac{m\,(m+1)\,(m+2)\,(m+3)\,(m+4)^2\,(m+5)^2\left(3\,m^2 + 23\,m + 38\right)}{11\,520} \;/;\; m \in \mathbb{N}$$

04.14.03.0052.01

$$S_{m+6}^{(m)} = \frac{(m)_7\left(63\,m^5 + 1575\,m^4 + 15\,435\,m^3 + 73\,801\,m^2 + 171\,150\,m + 152\,696\right)}{2\,903\,040} \;/;\; m \in \mathbb{N}$$

04.14.03.0053.01

$$S_{m+7}^{(m)} = -\frac{(m)_8\,(m+6)\,(m+7)\left(9\,m^4 + 198\,m^3 + 1563\,m^2 + 5182\,m + 6008\right)}{5\,806\,080} \;/;\; m \in \mathbb{N}$$

04.14.03.0054.01

$$S_{m+8}^{(m)} = \frac{1}{1\,393\,459\,200}\,(m)_9 \left(135\,m^7 + 6300\,m^6 + 124\,110\,m^5 + 1\,334\,760\,m^4 + 8\,437\,975\,m^3 + 31\,231\,500\,m^2 + 62\,333\,204\,m + 51\,360\,816\right) \;/;\; m \in \mathbb{N}$$

04.14.03.0055.01

$$S_{m+9}^{(m)} = -\frac{1}{2\,786\,918\,400}\,(m)_{10}\,(m+8)\,(m+9) \left(15\,m^6 + 645\,m^5 + 11\,265\,m^4 + 101\,807\,m^3 + 499\,176\,m^2 + 1\,249\,444\,m + 1\,234\,224\right) \;/;\; m \in \mathbb{N}$$

04.14.03.0056.01

$$S_{m+10}^{(m)} = \frac{1}{367\,873\,228\,800}\,(m)_{11}\left(99\,m^9 + 7425\,m^8 + 244\,530\,m^7 + 4\,634\,322\,m^6 + 55\,598\,235\,m^5 + 436\,886\,945\,m^4 + 2\,242\,194\,592\,m^3 + 7\,220\,722\,828\,m^2 + 13\,175\,306\,672\,m + 10\,307\,425\,152\right) \;/;\; m \in \mathbb{N}$$

Values at fixed points

04.14.03.0010.01

$$S_0^{(0)} = 1$$

04.14.03.0011.01

$S_0^{(1)} == 0$

04.14.03.0012.01

$S_1^{(0)} == 0$

04.14.03.0013.01

$S_1^{(1)} == 1$

04.14.03.0014.01

$S_1^{(2)} == 0$

04.14.03.0015.01

$S_2^{(0)} == 0$

04.14.03.0016.01

$S_2^{(1)} == -1$

04.14.03.0017.01

$S_2^{(2)} == 1$

04.14.03.0018.01

$S_2^{(3)} == 0$

04.14.03.0019.01

$S_3^{(0)} == 0$

04.14.03.0020.01

$S_3^{(1)} == 2$

04.14.03.0021.01

$S_3^{(2)} == -3$

04.14.03.0022.01

$S_3^{(3)} == 1$

04.14.03.0023.01

$S_3^{(4)} == 0$

04.14.03.0024.01

$S_4^{(0)} == 0$

04.14.03.0025.01

$S_4^{(1)} == -6$

04.14.03.0026.01

$S_4^{(2)} == 11$

04.14.03.0027.01

$S_4^{(3)} == -6$

04.14.03.0028.01

$S_4^{(4)} == 1$

04.14.03.0029.01

$S_4^{(5)} == 0$

General characteristics

Domain and analyticity

$S_n^{(m)}$ is a nonanalytical function which is defined only for nonnegative integers n, m.

04.14.04.0001.01

$$(n * m) \longrightarrow S_n^{(m)} :: (\mathbb{N} \otimes \mathbb{N}) \longrightarrow \mathbb{Z}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

04.14.06.0001.01

$$S_n^{(m)} = \frac{(-1)^{n-m}\, n!}{m!} \sum_{r_1=1}^{n} \cdots \sum_{r_m=1}^{n} \frac{1}{\prod_{j=1}^{m} r_j}\, \delta_{n, \sum_{j=1}^{m} r_j}$$

Asymptotic series expansions

04.14.06.0004.01

$$S_n^{(m)} \propto (n-1)! \left(\sum_{k=0}^{m-1} \frac{\left([t^k] \frac{1}{\Gamma(t+1)} \right)}{(m-k-1)!} \log^{m-k-1}(n) + O\left(\frac{\log^{m-2}(n)}{n^2} \right) \right) /; (n \to \infty) \land m \in \mathbb{N} \land n \in \mathbb{N}$$

Residue representations

04.14.06.0002.01

$$S_n^{(m)} = \operatorname{res}_z\left((-1)^{m+n} z^{-m-1} (z)_n \right)(0)$$

04.14.06.0003.01

$$S_n^{(m)} = \frac{n!}{m!} \operatorname{res}_z\left(z^{-n-1} \log^m(z+1) \right)(0)$$

Integral representations

Contour integral representations

04.14.07.0001.01

$$S_n^{(m)} = \frac{n!}{2 \pi i\, m!} \int_{|z|=1} z^{-n-1} \log^m(z+1)\, dz$$

Limit representations

04.14.09.0001.01

$$S_n^{(m)} = (-1)^{n-1} \lim_{a \to n} \frac{a^{-m}}{\Gamma(1-a)} \, {}_{m+1}F_m(a, a_1, a_2, \ldots, a_m; a_1+1, a_2+1, \ldots, a_m+1; 1) \,/;\, a_1 = a_2 = \ldots = a_m = a \bigwedge m \in \mathbb{N}^+$$

Generating functions

04.14.11.0001.01

$$S_n^{(m)} = (-1)^n \left([t^m]\ (-t)_n\right) /;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.11.0002.01

$$S_n^{(m)} = \left([t^m]\ (-n+t+1)_n\right) /;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.11.0003.01

$$S_n^{(m)} = (-1)^n\, n! \left([t^m] \binom{n-t-1}{n}\right) /;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.11.0004.01

$$S_n^{(m)} = \left([t^m]\ t^n \prod_{k=1}^{n-1} \left(1 - \frac{k}{t}\right)\right) /;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.11.0005.01

$$S_n^{(m)} = \frac{n!}{m!} \left([t^n]\ \log^m(t+1)\right) /;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.11.0006.01

$$\mathcal{S}_n^{(m)} = n!\ \left([z^n, w^m]\,(z+1)^w\right) /;\, m \in \mathbb{N} \land n \in \mathbb{N}$$

Identities

Recurrence identities

04.14.17.0001.01

$$S_{n+1}^{(m)} = S_n^{(m-1)} - n\, S_n^{(m)} /;\, m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+ \bigwedge m \leq n$$

Functional identities

04.14.17.0002.01

$$S_n^{(m)} = \sum_{k=m}^{n} n^{k-m}\, S_{n+1}^{(k+1)} /;\, m \in \mathbb{N}^+$$

Identities involving determinants

04.14.17.0003.01

$$\left| \left(\frac{(x\,k)!\, \mathcal{S}_{l+k\,x}^{(x\,k)}}{(l+k\,x)!} \right)_{\substack{0 \leq k \leq n \\ 0 \leq l \leq n}} \right| = \left(-\frac{x}{2}\right)^{\binom{n+1}{2}} /;\, n \in \mathbb{N}^+$$

Complex characteristics

Real part

04.14.19.0001.01

$$\operatorname{Re}\left(S_n^{(m)}\right) == S_n^{(m)}$$

Imaginary part

04.14.19.0002.01

$$\operatorname{Im}\left(S_n^{(m)}\right) == 0$$

Absolute value

04.14.19.0003.01

$$\left|S_n^{(m)}\right| == \sqrt{\left(S_n^{(m)}\right)^2}$$

Argument

04.14.19.0004.01

$$\arg\left(S_n^{(m)}\right) == \tan^{-1}\left(S_n^{(m)},\, 0\right)$$

Conjugate value

04.14.19.0005.01

$$\overline{S_n^{(m)}} == S_n^{(m)}$$

Signum value

04.14.19.0006.01

$$\operatorname{sgn}\left(S_n^{(m)}\right) = \begin{cases} 1 & \left(1 \leq m \leq n \bigwedge \frac{m+n}{2} \in \mathbb{Z}\right) \bigvee m = n = 0 \\ -1 & 1 \leq m \leq n \vee m = n = 0 \\ 0 & \text{True} \end{cases}$$

Summation

Finite summation

Not involving Stirling numbers of the second kind

04.14.23.0001.01

$$\sum_{k=0}^{n} S_n^{(k)} z^k == (-1)^n (-z)_n$$

04.14.23.0002.01

$$\sum_{k=m}^{n} S_{n+1}^{(k+1)} n^{k-m} == S_n^{(m)} \;/;\; m \in \mathbb{N}^+$$

04.14.23.0003.01

$$\sum_{k=0}^{n} (-1)^k \binom{k+m+n-1}{k+n} \binom{m+2n}{n-k} S_{k+n}^{(k)} = \mathcal{S}_{m+n}^{(m)} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.23.0004.01

$$\sum_{k=m}^{n} \binom{n+r}{k} S_{n-k+r}^{(r)} S_k^{(m)} = \binom{m+r}{r} S_{n+r}^{(m+r)} /; r \in \mathbb{N} \land m \in \mathbb{N} \land n \in \mathbb{N}$$

Involving Stirling numbers of the second kind

04.14.23.0005.01

$$\sum_{k=0}^{\max(m,n)+1} S_k^{(m)} \mathcal{S}_n^{(k)} = \delta_{m,n} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.23.0006.01

$$\sum_{k=0}^{\max(m,n)+1} S_m^{(k)} \mathcal{S}_k^{(n)} = \delta_{m,n} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.23.0007.01

$$\sum_{k=m}^{n} \sum_{j=0}^{k} \mathcal{S}_k^{(j)} S_n^{(k)} S_j^{(m)} = \mathcal{S}_n^{(m)} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.23.0008.01

$$\sum_{k=m}^{n} \sum_{j=0}^{k} S_k^{(j)} S_n^{(k)} \mathcal{S}_j^{(m)} = S_n^{(m)} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

Infinite summation

04.14.23.0009.01

$$\sum_{k=m}^{\infty} \frac{1}{k!} S_k^{(m)} z^k = \frac{\log^m(z+1)}{m!}$$

04.14.23.0010.01

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{n!} S_n^{(m)} z^n w^m = (z+1)^w$$

Representations through equivalent functions

With related functions

04.14.27.0001.01

$$S_n^{(m)} = \sum_{k=0}^{n-m} (-1)^k \binom{k+n-1}{k-m+n} \binom{2n-m}{-k-m+n} \mathcal{S}_{k-m+n}^{(k)} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.27.0002.01

$$S_n^{(m)} = \sum_{k=m}^{n} \sum_{j=0}^{k} S_k^{(j)} S_n^{(k)} \mathcal{S}_j^{(m)} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

04.14.27.0003.01

$$S_n^{(m)} = \binom{n-1}{n-m} B_{n-m}^{(n)} /; n \in \mathbb{N}^+ \bigwedge m \in \mathbb{N} \bigwedge m < n$$

Theorems

Disjoint cycles in permutations

$S_n^{(m)}$ is the number of permutations of m symbols that have n disjoint cycles.

Converting derivatives to finite differences

With the finite difference operator $\Delta_{x,h}^k$ defined by $\Delta_{x,h}^k f(x) = \Delta_{x,h}^{k-1}(\Delta_{x,h} f(x))$, $\Delta_{x,h} f(x) = f(x+h) - f(x)$ the kth derivative of a function $f(x)$ can be expressed as

$$\frac{\partial^k f(x)}{\partial x^k} = \sum_{j=0}^{\infty} \frac{h^{-k}}{(k+1)_j} S_{j+k}^{(k)} \Delta_{x,h}^{k+j} f(x).$$

History

–J. Stirling (1730, 1749)

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