WhittakerM

Notations

Traditional name

Whittaker hypergeometric function M

Traditional notation

$M_{\nu,\mu}(z)$

Mathematica StandardForm notation

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WhittakerM[ν, μ, z]
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Primary definition

07.44.02.0001.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}}\, e^{-\frac{z}{2}}\, {}_1F_1\!\left(\mu-\nu+\frac{1}{2};\, 2\,\mu+1;\, z\right)$$

Specific values

Specialized values

For fixed ν, μ

07.44.03.0001.01

$$M_{\nu,\mu}(0) = 0 \;/;\; \mathrm{Re}(\mu) > -\frac{1}{2}$$

07.44.03.0002.01

$$M_{\nu,\mu}(0) = \tilde{\infty} \;/;\; \mathrm{Re}(\mu) < -\frac{1}{2}$$

For fixed ν, z

07.44.03.0003.01

$$M_{\nu,-\frac{1}{2}}(z) = \tilde{\infty}$$

07.44.03.0004.01

$$M_{\nu,-\frac{1}{4}}(z) - \frac{2\,\Gamma\!\left(\frac{3}{4}-\nu\right)}{\Gamma\!\left(\frac{1}{4}-\nu\right)}\, M_{\nu,\frac{1}{4}}(z) = \frac{2^{\frac{1}{2}-2\nu}\,\Gamma\!\left(\frac{3}{4}-\nu\right)}{\sqrt{\pi}}\, e^{-\frac{z}{2}}\, \sqrt[4]{z}\, H_{2\nu-\frac{1}{2}}\!\left(\sqrt{z}\right)$$

07.44.03.0005.01

$$M_{\nu,-\frac{1}{4}}(z)-\frac{2\sqrt{-z}\,\Gamma\left(\nu+\frac{3}{4}\right)}{\sqrt{z}\,\Gamma\left(\nu+\frac{1}{4}\right)}M_{\nu,\frac{1}{4}}(z)=\frac{2^{2\nu+\frac{1}{2}}\,e^{z/2}\,\sqrt[4]{z}\,\Gamma\left(\nu+\frac{3}{4}\right)}{\sqrt{\pi}}H_{-2\nu-\frac{1}{2}}\left(\sqrt{-z}\right)$$

07.44.03.0006.01

$$M_{\nu,0}(z)=\sqrt{z}\,e^{-\frac{z}{2}}L_{\nu-\frac{1}{2}}(z)$$

07.44.03.0007.01

$$M_{\nu,0}(z)=\sqrt{z}\,e^{z/2}L_{-\nu-\frac{1}{2}}(-z)$$

For fixed μ, z

07.44.03.0008.01

$$M_{-\frac{1}{2},\mu}(z)=2^{2\mu-1}\,\Gamma\left(\mu+\frac{1}{2}\right)z\left(I_{\mu+\frac{1}{2}}\left(\frac{z}{2}\right)+I_{\mu-\frac{1}{2}}\left(\frac{z}{2}\right)\right)$$

07.44.03.0009.01

$$M_{-\frac{1}{2},\mu}(z)=2^{2\mu-1}\,\Gamma\left(\mu+\frac{1}{2}\right)(-z)^{\frac{1}{2}-\mu}\,z^{\mu+\frac{1}{2}}\left(I_{\mu-\frac{1}{2}}\left(-\frac{z}{2}\right)-I_{\mu+\frac{1}{2}}\left(-\frac{z}{2}\right)\right)$$

07.44.03.0010.01

$$M_{0,\mu}(z)=4^{\mu}\,\Gamma(\mu+1)\sqrt{z}\,I_{\mu}\left(\frac{z}{2}\right)$$

07.44.03.0011.01

$$M_{0,\mu}(z)=4^{\mu}\,\Gamma(\mu+1)(-z)^{-\mu}\,z^{\mu+\frac{1}{2}}\,I_{\mu}\left(-\frac{z}{2}\right)$$

07.44.03.0012.01

$$M_{0,\mu}(z)=z^{\mu+\frac{1}{2}}\,{}_0F_1\left(;\mu+1;\frac{z^2}{16}\right)$$

07.44.03.0013.01

$$M_{0,\mu}(z)=\Gamma(\mu+1)\,z^{\mu+\frac{1}{2}}\,{}_0\tilde{F}_1\left(;\mu+1;\frac{z^2}{16}\right)$$

07.44.03.0014.01

$$M_{\frac{1}{2},\mu}(z)=2^{2\mu-1}\,\Gamma\left(\mu+\frac{1}{2}\right)z\left(I_{\mu-\frac{1}{2}}\left(\frac{z}{2}\right)-I_{\mu+\frac{1}{2}}\left(\frac{z}{2}\right)\right)$$

07.44.03.0015.01

$$M_{\frac{1}{2},\mu}(z)=2^{2\mu-1}\,\Gamma\left(\mu+\frac{1}{2}\right)(-z)^{\frac{1}{2}-\mu}\,z^{\mu+\frac{1}{2}}\left(I_{\mu+\frac{1}{2}}\left(-\frac{z}{2}\right)+I_{\mu-\frac{1}{2}}\left(-\frac{z}{2}\right)\right)$$

07.44.03.0016.01

$$M_{-\frac{n}{2},\mu}(z)=2^{-n+2\mu-1}\,z^{\frac{n+1}{2}}\,\Gamma\left(\mu-\frac{n}{2}\right)\sum_{k=0}^{n}\frac{(-1)^k\,(2k-n+2\mu)\,(-n)_k\,(2\mu-n)_k}{k!\,(2\mu+1)_k}I_{k+\mu-\frac{n}{2}}\left(\frac{z}{2}\right)\,/;\,n\in\mathbb{N}$$

07.44.03.0017.01

$$M_{-\frac{n}{2},\mu}(z)=2^{n+2\mu-2}\,z^{\frac{1-n}{2}}\left(4\,I_{\frac{n}{2}+\mu}\left(\frac{z}{2}\right)\Gamma\left(\frac{n}{2}+\mu+1\right)\sum_{k=0}^{n}\sum_{j=0}^{\left\lfloor \frac{k}{2}\right\rfloor}\frac{(-1)^{n-j-k}\,2^{-2(2j-k+n)}\,(k-j)!\,(-n)_{n-k}\,(2\mu-n)_{n-k}\,z^{2j-k+n}}{j!\,(k-2j)!\,(n-k)!\left(-\frac{n}{2}-\mu\right)_j\left(\mu-\frac{n}{2}\right)_{n-k}\left(\frac{n}{2}+\mu-k+1\right)_j(2\mu+1)_{n-k}}+\right.$$

$$\left.z\,I_{\frac{n}{2}+\mu+1}\left(\frac{z}{2}\right)\Gamma\left(\frac{n}{2}+\mu\right)\sum_{k=0}^{n}\sum_{j=0}^{\left\lfloor \frac{k-1}{2}\right\rfloor}\frac{(-1)^{n-j-k}\,2^{-2(2j-k+n)}\,(k-j-1)!\,(-n)_{n-k}\,(2\mu-n)_{n-k}\,z^{2j-k+n}}{j!\,(k-2j-1)!\,(n-k)!\left(1-\frac{n}{2}-\mu\right)_j\left(\mu-\frac{n}{2}\right)_{n-k}\left(\frac{n}{2}+\mu-k+1\right)_j(2\mu+1)_{n-k}}\right)\;/;\;n\in\mathbb{N}$$

07.44.03.0018.01

$$M_{\frac{n}{2},\mu}(z)=2^{-n+2\mu-1}\,z^{\frac{n+1}{2}}\,\Gamma\left(\mu-\frac{n}{2}\right)\sum_{k=0}^{n}\frac{(2k-n+2\mu)\,(-n)_k\,(2\mu-n)_k}{k!\,(2\mu+1)_k}\,I_{k+\mu-\frac{n}{2}}\left(\frac{z}{2}\right)\;/;\;n\in\mathbb{N}$$

07.44.03.0019.01

$$M_{\frac{n}{2},\mu}(z)=2^{-n+2\mu-2}\,z^{\frac{n+1}{2}}\left(4\,I_{\mu-\frac{n}{2}}\left(\frac{z}{2}\right)\Gamma\left(-\frac{n}{2}+\mu+1\right)\sum_{k=0}^{n}\sum_{j=0}^{\left\lfloor \frac{k}{2}\right\rfloor}\frac{(-1)^{k-j}\,2^{2k-4j}\,z^{2j-k}\,(k-j)!\,(-n)_k\left(-\frac{n}{2}+\mu+1\right)_k(2\mu-n)_k}{j!\,k!\,(k-2j)!\left(-k+\frac{n}{2}-\mu+1\right)_j\left(\mu-\frac{n}{2}\right)_j(2\mu+1)_k}-\right.$$

$$\left.z\,I_{-\frac{n}{2}+\mu-1}\left(\frac{z}{2}\right)\Gamma\left(\mu-\frac{n}{2}\right)\sum_{k=0}^{n}\sum_{j=0}^{\left\lfloor \frac{k-1}{2}\right\rfloor}\frac{(-1)^{k-j}\,2^{2k-4j}\,z^{2j-k}\,(-j+k-1)!\,(-n)_k\left(-\frac{n}{2}+\mu+1\right)_k(2\mu-n)_k}{j!\,k!\,(-2j+k-1)!\left(-k+\frac{n}{2}-\mu+1\right)_j\left(-\frac{n}{2}+\mu+1\right)_j(2\mu+1)_k}\right)\;/;\;n\in\mathbb{N}$$

07.44.03.0020.01

$$M_{-\mu-\frac{3}{2},\mu}(z)=e^{z/2}\,z^{\frac{1}{2}(2\mu+1)}\left(\frac{z}{2\mu+1}+1\right)$$

07.44.03.0021.01

$$M_{-\mu-\frac{1}{2},\mu}(z)=e^{z/2}\,z^{\frac{1}{2}(2\mu+1)}$$

07.44.03.0022.01

$$M_{\frac{1}{2}-\mu,\mu}(z)=\Gamma(2\mu+1)\,(-z)^{-2\mu}\,z^{\mu+\frac{1}{2}}\,e^{-\frac{z}{2}}\,(1-Q(2\mu,-z))$$

07.44.03.0023.01

$$M_{\frac{1}{2}-\mu,\mu}(z)=2\mu\,(-z)^{-2\mu}\,z^{\mu+\frac{1}{2}}\,e^{-\frac{z}{2}}(\Gamma(2\mu)-\Gamma(2\mu,-z))$$

07.44.03.0024.01

$$M_{\frac{1}{2}-\mu,\mu}(z)=2\mu\,z^{\mu+\frac{1}{2}}\,e^{-\frac{z}{2}}\left((-z)^{-2\mu}\,\Gamma(2\mu)-E_{1-2\mu}(-z)\right)$$

07.44.03.0025.01

$$M_{\frac{3}{2}-\mu,\mu}(z)=(-z)^{-2\mu}\,z^{\mu+\frac{1}{2}}\,e^{-\frac{z}{2}}\left(2\mu\left(e^{z}\,(-z)^{2\mu}+(z+2\mu-1)\,\Gamma(2\mu,-z)\right)-(z+2\mu-1)\,\Gamma(2\mu+1)\right)$$

07.44.03.0026.01

$$M_{n-\mu-\frac{1}{2},\mu}(z)=\frac{(-1)^{n-1}\,(-z)^{-2\mu}\,z^{n+\mu-\frac{1}{2}}}{\mathrm{B}(2\mu-n+1,\,n)}\,e^{-\frac{z}{2}}\sum_{k=0}^{n}z^{-k}\binom{n-1}{k}(\Gamma(k-n+2\mu+1)-\Gamma(k-n+2\mu+1,\,-z))\;/;\;n\in\mathbb{N}^{+}$$

07.44.03.0027.01

$$M_{-\mu-n-\frac{1}{2},\mu}(z)=\frac{(-1)^n\,n!\,z^{\mu+\frac{1}{2}}}{(-n-2\mu)_n}\,e^{z/2}\,L_n^{2\mu}(-z)\;/;\;n\in\mathbb{N}$$

07.44.03.0028.01

$$M_{\mu-\frac{3}{2},\mu}(z)=2\,e^{-\frac{z}{2}}\,z^{\frac{1}{2}(2\mu+1)}\,\mu\left(e^{z}\,(z-2\mu+1)\,\Gamma(2\mu,0,z)\,z^{-2\mu}+1\right)$$

07.44.03.0029.01

$$M_{\mu-\frac{1}{2},\mu}(z)=2\,e^{z/2}\,z^{\frac{1}{2}-\mu}\,\mu\,(\Gamma(2\mu)-\Gamma(2\mu,z))$$

07.44.03.0030.01

$$M_{\mu+\frac{1}{2},\mu}(z)=e^{-\frac{z}{2}}\,z^{\frac{1}{2}(2\mu+1)}$$

07.44.03.0031.01

$$M_{\mu+\frac{3}{2},\mu}(z)=e^{-\frac{z}{2}}\,z^{\frac{1}{2}(2\mu+1)}\left(1-\frac{z}{2\mu+1}\right)$$

07.44.03.0032.01

$$M_{\mu+\frac{5}{2},\mu}(z)=e^{-\frac{z}{2}}\,z^{\frac{1}{2}(2\mu+1)}\left(\frac{z^2}{(2\mu+1)(2\mu+2)}-\frac{2z}{2\mu+1}+1\right)$$

07.44.03.0033.01

$$M_{n+\mu+\frac{1}{2},\mu}(z)=\frac{e^{-\frac{z}{2}}\,z^{\frac{1}{2}(2\mu+1)}\,n!}{(2\mu+1)_n}\,L_n^{2\mu}(z)$$

07.44.03.0034.01

$$M_{\mu-n-\frac{1}{2},\mu}(z)=\frac{2\,\mu}{n!}\,e^{-\frac{z}{2}}\,z^{\frac{1}{2}(2\mu+1)}\,\frac{\partial^n\left(e^{z}\,z^{n-2\mu}\,(\Gamma(2\mu)-\Gamma(2\mu,z))\right)}{\partial z^n}\;/;\;n\in\mathbb{N}$$

07.44.03.0035.01

$$M_{\mu-n-\frac{1}{2},\mu}(z)=\frac{e^{z/2}\,z^{n-\mu+\frac{1}{2}}}{\mathrm{B}(2\mu-n,n+1)}\sum_{k=0}^{n+1}(-z)^{-k}\binom{n}{k}(\Gamma(k-n+2\mu)-\Gamma(k-n+2\mu,z))\;/;\;n\in\mathbb{N}$$

For fixed z and half-integer parameters

For fixed z and $\mu=-m/2$

07.44.03.0036.01

$$M_{0,-m-\frac{1}{2}}(z)=\frac{\sqrt{z}\,m!}{\sqrt{\pi}\,(2m)!}\,K_{m+\frac{1}{2}}\!\left(\frac{z}{2}\right)\;/;\;m\in\mathbb{N}^+$$

07.44.03.0037.01

$$M_{\frac{1-m}{2}+n,-\frac{m}{2}}(z)=e^{-\frac{z}{2}}\,z^{\frac{1-m}{2}}\sum_{k=0}^{m-1}\frac{z^k\,(-n)_k}{k!\,(1-m)_k}\;/;\;n\in\mathbb{N}\land m\in\mathbb{Z}\land m>n$$

07.44.03.0038.01

$$M_{\frac{m-1}{2},-\frac{m}{2}}(z)=e^{z/2}\,z^{\frac{1-m}{2}}\,Q(m,z)\;/;\;m\in\mathbb{N}^+$$

07.44.03.0039.01

$$M_{\frac{m-1}{2},-\frac{m}{2}}(z)=e^{-\frac{z}{2}}\,z^{\frac{1-m}{2}}\sum_{k=0}^{m-1}\frac{z^k}{k!}\;/;\;m\in\mathbb{N}^+$$

For fixed z and $\mu = m/2$

07.44.03.0040.01

$$M_{\frac{m-3}{2},\frac{m}{2}}(z) = e^{-\frac{z}{2}} z^{\frac{m+1}{2}} \left((z-m+1)\, m! \left(e^z - \sum_{k=0}^{m-2} \frac{z^k}{k!}\right) z^{-m} + \frac{(m-1)\, m}{z}\right) /; m \in \mathbb{N}$$

07.44.03.0041.01

$$M_{\frac{m-1}{2},\frac{m}{2}}(z) = e^{-\frac{z}{2}} z^{\frac{1-m}{2}}\, m! \left(e^z - \sum_{k=0}^{m-1} \frac{z^k}{k!}\right) /; m \in \mathbb{N}$$

07.44.03.0042.01

$$M_{\frac{1-m}{2},\frac{m}{2}}(z) = (-1)^m e^{-\frac{z}{2}} z^{\frac{1-m}{2}}\, m! \left(1 - e^z \sum_{k=0}^{m-1} \frac{(-z)^k}{k!}\right) /; m \in \mathbb{N}$$

07.44.03.0043.01

$$M_{\frac{m}{2}-n,\frac{m}{2}}(z) = \frac{2^{-m} z^{\frac{m+1}{2}}\, m!\, n!}{\left(\frac{1}{2}\right)_m \left(\frac{1}{2}\right)_n} \sum_{k=0}^{n} \frac{2^{-k} (-z)^k}{k!} L_{n-k}^{k-\frac{1}{2}}(-z) \sum_{p=0}^{k+m} \left(-\frac{1}{2}\right)^p \binom{k+m}{p} \sum_{j=0}^{p} \binom{p}{j} I_{p-2j}\left(\frac{z}{2}\right) /; n \in \mathbb{N} \land m \in \mathbb{N}$$

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07.44.03.0044.01

$$M_{\frac{m}{2}+n,\frac{m}{2}}(z) = \frac{(-1)^n 2^{-m} z^{\frac{m+1}{2}}\, m!}{\left(\frac{1}{2}\right)_m \left(m+\frac{1}{2}\right)_n} \sum_{k=0}^{n} 2^{-k} z^k \binom{n}{k} \left(\frac{1}{2} - m - n\right)_{n-k} \sum_{p=0}^{k+m} \left(-\frac{1}{2}\right)^p \binom{k+m}{p} \sum_{j=0}^{p} \binom{p}{j} I_{p-2j}\left(\frac{z}{2}\right) /; n \in \mathbb{N} \land m \in \mathbb{N}$$

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07.44.03.0045.01

$$M_{\frac{m+1}{2}-n,\frac{m}{2}}(z) = \frac{(m-1)!\, (-m)_n}{(n-1)!} e^{-\frac{z}{2}} z^{\frac{1-m}{2}} \left(\sum_{k=0}^{m-n} \frac{z^k\, (n-m)_k}{k!\, (1-m)_k} - e^z \sum_{k=0}^{n-1} \frac{(-z)^k\, (1-n)_k}{k!\, (1-m)_k}\right) /; m \in \mathbb{N} \bigwedge n \in \mathbb{N}^+ \bigwedge m \geq n$$

07.44.03.0046.01

$$M_{\frac{m+1}{2}-n,\frac{m}{2}}(z) = e^{z/2} z^{\frac{m+1}{2}} \sum_{k=0}^{n-m-1} \frac{(-z)^k\, (m-n+1)_k}{k!\, (m+1)_k} /; m \in \mathbb{N} \bigwedge n \in \mathbb{N}^+ \bigwedge m < n$$

07.44.03.0047.01

$$M_{\frac{m+1}{2}+n,\frac{m}{2}}(z) = e^{-\frac{z}{2}} z^{\frac{m+1}{2}} \sum_{k=0}^{n} \frac{z^k\, (-n)_k}{k!\, (m+1)_k} /; m \in \mathbb{N} \land n \in \mathbb{N}$$

For fixed z and $\mu = \frac{2m\pm1}{4}$

07.44.03.0048.01

$$M_{\frac{2m+3}{4}+n,\frac{2m+1}{4}}(z) = \frac{(-1)^n e^{-\frac{z}{2}} z^{\frac{1}{4}(2m+3)}\, n!}{\left(-m-n-\frac{1}{2}\right)_n} L_n^{m+\frac{1}{2}}(z) /; n \in \mathbb{N} \land m \in \mathbb{Z}$$

07.44.03.0049.01

$$M_{\frac{2m+1}{4}+n,\frac{2m+1}{4}}(z)=\frac{(-1)^m\sqrt{\pi}}{2}\left(\frac{3}{2}\right)_m \operatorname{erfi}\left(\sqrt{z}\right)e^{-\frac{z}{2}}z^{\frac{1-2m}{4}}\sum_{k=0}^{n}\binom{n}{k}L_{k+m}^{-k-m-\frac{1}{2}}(z)+$$
$$\frac{(-1)^m}{2}\left(\frac{3}{2}\right)_m e^{z/2}z^{\frac{3-2m}{4}}\sum_{k=0}^{n}\binom{n}{k}\sum_{p=1}^{k+m}\frac{1}{p}L_{k+m-p}^{-k-m+p-\frac{1}{2}}(z)L_{p-1}^{\frac{1}{2}-p}(-z)\text{ /; }n\in\mathbb{N}\wedge m\in\mathbb{N}$$

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07.44.03.0050.01

$$M_{\frac{2m+1}{4}-n,\frac{2m-1}{4}}(z)=\frac{2m-1}{2}e^{z/2}z^{\frac{3-2m}{4}}\sum_{k=0}^{n-1}\frac{(-1)^k}{k!}\left(\Gamma\left(k+m-\frac{1}{2}\right)-\Gamma\left(k+m-\frac{1}{2},z\right)\right)L_{n-k-1}^{k}(-z)\text{ /; }n\in\mathbb{N}^+\bigwedge m\in\mathbb{Z}$$

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07.44.03.0051.01

$$M_{\frac{2m+1}{4}-n,\frac{2m-1}{4}}(z)=\frac{2m-1}{2}e^{z/2}z^{\frac{3-2m}{4}}\sum_{k=0}^{n-1}\frac{(-1)^k}{k!}L_{-k+n-1}^{k}(-z)$$
$$\left(\operatorname{erf}\left(\sqrt{z}\right)\Gamma\left(k+m-\frac{1}{2}\right)+e^{-z}\sum_{j=0}^{-k-m}\frac{z^{-j-\frac{1}{2}}}{\left(k+m-\frac{1}{2}\right)_{1-j-k-m}}-e^{-z}\sum_{j=0}^{k+m-2}\frac{z^{j+\frac{1}{2}}}{\left(k+m-\frac{1}{2}\right)_{j-k-m+2}}\right)\text{ /; }n\in\mathbb{N}^+\bigwedge m\in\mathbb{Z}$$

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07.44.03.0052.01

$$M_{\frac{2m+1}{4}-n,\frac{2m-1}{4}}(z)=\frac{(2m-2n+1)\left(m-n+\frac{3}{2}\right)_{n-1}}{2(n-1)!}e^{-\frac{z}{2}}z^{\frac{2m+1}{4}}\left(e^z\sqrt{\pi}\,z^{n-m-\frac{1}{2}}\operatorname{erf}\left(\sqrt{z}\right)\sum_{p=0}^{n-1}(-z)^{-p}\binom{n-1}{p}\left(\frac{1}{2}\right)_{m-n+p}-\right.$$
$$\left.2\sum_{p=0}^{n-1}\frac{(-1)^p\binom{n-1}{p}}{2m-2n+2p+1}\sum_{k=1}^{m-n+p}(-z)^{-k}\left(n-m-p-\frac{1}{2}\right)_k\right)\text{ /; }n\in\mathbb{N}^+\bigwedge m\in\mathbb{Z}\bigwedge m\geq n$$

07.44.03.0053.01

$$M_{\frac{2m+1}{4}-n,\frac{2m-1}{4}}(z)=$$
$$\frac{1}{2}\left(\frac{3}{2}\right)_{m-1}\left(e^{z/2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{z}\right)z^{\frac{3-2m}{4}}\sum_{k=0}^{n-m}\binom{n-m}{k}L_{k+m-1}^{\frac{1}{2}-k-m}(-z)+e^{-\frac{z}{2}}z^{\frac{5-2m}{4}}\sum_{k=0}^{n-m}\binom{n-m}{k}\sum_{p=1}^{k+m-1}\frac{1}{p}L_{k+m-p-1}^{p-k-m+\frac{1}{2}}(-z)L_{p-1}^{\frac{1}{2}-p}(z)\right)\text{ /; }$$
$$n\in\mathbb{Z}\bigwedge m\in\mathbb{N}^+\bigwedge m\leq n$$

07.44.03.0054.01

$$M_{\frac{2m-1}{4}-n,\frac{2m-1}{4}}(z)=\frac{(-1)^{n-m}(n-m)!}{\left(\frac{1}{2}-n\right)_{n-m}}e^{z/2}z^{\frac{2m+1}{4}}L_{n-m}^{m-\frac{1}{2}}(-z)\text{ /; }n\in\mathbb{N}\wedge m\in\mathbb{Z}\wedge m\leq n$$

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07.44.03.0055.01

$$M_{\frac{2m-1}{4}-n,\frac{2m-1}{4}}(z)=\frac{2n+1}{2(m-n-1)!}\left(n+\frac{3}{2}\right)_{m-n-1}e^{-\frac{z}{2}}z^{\frac{2m+1}{4}}\left((-1)^n\sqrt{\pi}\,z^{-n-\frac{1}{2}}\operatorname{erfi}\left(\sqrt{z}\right)\sum_{p=0}^{m-n-1}z^{-p}\binom{m-n-1}{p}\left(\frac{1}{2}\right)_{n+p}-\right.$$
$$\left.2e^z\sum_{p=0}^{m-n-1}\frac{(-1)^p\binom{m-n-1}{p}}{2n+2p+1}\sum_{k=1}^{n+p}z^{-k}\left(-n-p-\frac{1}{2}\right)_k\right)/;\,n\in\mathbb{N}\land m\in\mathbb{Z}\land m>n$$

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07.44.03.0056.01

$$M_{\frac{2m-1}{4}+n,\frac{2m-1}{4}}(z)=e^{-\frac{z}{2}}\left(m-\frac{1}{2}\right)(-z)^{\frac{1}{2}-m}z^{\frac{2m+1}{4}}\sum_{k=0}^{m+n-1}\frac{(-1)^k}{k!}\left(\Gamma\left(k+m-\frac{1}{2}\right)-\Gamma\left(k+m-\frac{1}{2},-z\right)\right)L_{m+n-k-1}^k(z)/;\,n\in\mathbb{N}\land m\in\mathbb{N}$$

07.44.03.0057.01

$$M_{\frac{2m-1}{4}+n,\frac{2m-1}{4}}(z)=(-1)^m\frac{1-2m}{2}e^{-\frac{z}{2}}z^{\frac{5-2m}{4}}\sum_{k=0}^{m+n-1}\frac{(-1)^k}{k!}L_{m+n-k-1}^k(z)\left(\frac{\operatorname{erfi}\left(\sqrt{z}\right)\Gamma\left(k+m-\frac{1}{2}\right)}{\sqrt{z}}-e^z\sum_{j=0}^{k+m-2}\frac{(-z)^j}{\left(k+m-\frac{1}{2}\right)_{j-k-m+2}}\right)/;$$
$$n\in\mathbb{N}\bigwedge m\in\mathbb{N}^+$$

For fixed z and $\mu=-\frac{2m+k}{4}$

07.44.03.0058.01

$$M_{-\frac{2m+5}{4}-n,-\frac{2m+3}{4}}(z)=-\frac{1}{2}e^{z/2}(2m+3)z^{\frac{m}{2}+\frac{5}{4}}\sum_{k=0}^{n}\frac{(-1)^k}{k!}\left(\Gamma\left(k-m-\frac{3}{2}\right)-\Gamma\left(k-m-\frac{3}{2},z\right)\right)L_{n-k}^k(-z)/;\,n\in\mathbb{N}\land m\in\mathbb{Z}$$

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07.44.03.0059.01

$$M_{-\frac{2m-1}{4}-n,-\frac{2m+1}{4}}(z)=-\frac{2m+1}{2}e^{z/2}z^{\frac{1}{4}(2m+3)}\sum_{k=0}^{n-1}\frac{1}{k!}(-1)^kL_{n-k-1}^k(-z)$$
$$\left(\operatorname{erf}\left(\sqrt{z}\right)\Gamma\left(k-m-\frac{1}{2}\right)+e^{-z}\sum_{j=0}^{m-k}\frac{z^{-j-\frac{1}{2}}}{\left(k-m-\frac{1}{2}\right)_{m-j-k+1}}-e^{-z}\sum_{j=0}^{k-m-2}\frac{z^{j+\frac{1}{2}}}{\left(k-m-\frac{1}{2}\right)_{j-k+m+2}}\right)/;\,n\in\mathbb{N}^+\bigwedge m\in\mathbb{Z}$$

Brychkov Yu.A. (2006)

07.44.03.0060.01

$$M_{-\frac{2m-1}{4}-n,-\frac{2m+1}{4}}(z)=\frac{(-1)^m e^{-\frac{z}{2}}\sqrt{\pi}\,z^{\frac{1-2m}{4}}(m+1)!}{\Gamma\left(m+\frac{1}{2}\right)(m+n)!}\left(\frac{e^z\sqrt{\pi}\operatorname{erf}\left(\sqrt{z}\right)}{\sqrt{z}}\sum_{k=0}^{m+1}\frac{(k+m+n)!}{k!}L_{m-k+1}^{k-m-\frac{1}{2}}(z)L_{k+m+n}^{-k-\frac{1}{2}}(-z)+\right.$$
$$\left.\sum_{k=0}^{m+1}\frac{(k+m+n)!}{k!}L_{m-k+1}^{k-m-\frac{1}{2}}(z)\sum_{p=1}^{k+m+n}\frac{1}{p}L_{k+m+n-p}^{p-k-\frac{1}{2}}(-z)L_{p-1}^{\frac{1}{2}-p}(z)\right)/;\,n\in\mathbb{N}\land m\in\mathbb{N}$$

07.44.03.0061.01

$$M_{n-\frac{2m-1}{4},-\frac{2m-1}{4}}(z)=\frac{(-1)^m e^{-\frac{z}{2}} z^{\frac{3-2m}{4}} m!}{2\,n!\left(-\frac{1}{2}\right)_m}\left(\frac{\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{z}\right)}{\sqrt{z}}\sum_{k=0}^{m}\frac{(k+n)!}{k!}L_{m-k}^{k-m+\frac{1}{2}}(-z)\,L_{k+n}^{-k-\frac{1}{2}}(z)+e^z\sum_{k=0}^{m}\frac{(k+n)!}{k!}L_{m-k}^{k-m+\frac{1}{2}}(-z)\sum_{p=1}^{k+n}\frac{1}{p}L_{k+n-p}^{p-k-\frac{1}{2}}(z)\,L_{p-1}^{\frac{1}{2}-p}(-z)\right)/;\ n\in\mathbb{N}\land m\in\mathbb{N}$$

Brychkov Yu.A. (2006)

07.44.03.0062.01

$$M_{n-\frac{2m+1}{4},-\frac{2m+1}{4}}(z)=-\frac{2m+1}{2}e^{-\frac{z}{2}}(-z)^{m+\frac{1}{2}}z^{\frac{1-2m}{4}}\sum_{k=0}^{n-m-1}\frac{(-1)^k}{k!}\left(\Gamma\left(k-m-\frac{1}{2}\right)-\Gamma\left(k-m-\frac{1}{2},-z\right)\right)L_{n-k-m-1}^{k}(z)\ /;$$
$$n\in\mathbb{Z}\land m\in\mathbb{Z}\land n>m$$

07.44.03.0063.01

$$M_{n-\frac{2m+1}{4},-\frac{2m+1}{4}}(z)=(-1)^m\frac{2m+1}{2}e^{-\frac{z}{2}}z^{\frac{2m+5}{4}}\sum_{k=0}^{n-m-1}\frac{(-1)^k}{k!}L_{-k-m+n-1}^{k}(z)\left(\frac{\operatorname{erfi}\left(\sqrt{z}\right)\Gamma\left(k-m-\frac{1}{2}\right)}{\sqrt{z}}-e^z\sum_{j=0}^{k-m-2}\frac{(-z)^j}{\left(k-m-\frac{1}{2}\right)_{j-k+m+2}}+e^z\sum_{j=0}^{m-k}\frac{(-z)^{-j-1}}{\left(k-m-\frac{1}{2}\right)_{-j-k+m+1}}\right)/;\ n\in\mathbb{Z}\land m\in\mathbb{Z}\land n>m$$

For fixed z and $\nu=\frac{k}{4}\pm n$, $\mu=-\frac{1}{4}$

07.44.03.0064.01

$$M_{-n-\frac{1}{4},-\frac{1}{4}}(z)=\frac{(-1)^n e^{z/2}\sqrt[4]{z}\,n!}{(2n)!}H_{2n}\left(\sqrt{-z}\right)/;\ n\in\mathbb{N}$$

07.44.03.0065.01

$$M_{\frac{1}{4}-n,-\frac{1}{4}}(z)=e^{-\frac{z}{2}}\sqrt[4]{z}\left(\frac{e^z\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{z}\right)}{2\sqrt{z}}\left(L_{n-1}^{-\frac{1}{2}}(-z)+2\,n\,L_n^{-\frac{3}{2}}(-z)\right)+\frac{1}{2}\sum_{p=0}^{n-2}\frac{1}{p+1}L_{n-p-2}^{p+\frac{1}{2}}(-z)\,L_p^{-p-\frac{1}{2}}(z)+n\sum_{p=0}^{n-1}\frac{1}{p+1}L_{n-p-1}^{p-\frac{1}{2}}(-z)\,L_p^{-p-\frac{1}{2}}(z)\right)/;\ n\in\mathbb{N}^+$$

Brychkov Yu.A. (2006)

07.44.03.0066.01

$$M_{n+\frac{1}{4},-\frac{1}{4}}(z)=\frac{(-1)^n e^{-\frac{z}{2}}\sqrt[4]{z}\,n!}{(2n)!}H_{2n}\left(\sqrt{z}\right)/;\ n\in\mathbb{N}$$

07.44.03.0067.01

$$M_{n+\frac{3}{4},-\frac{1}{4}}(z) = \frac{1}{2} e^{z/2} \sqrt[4]{z} \left(\sum_{p=0}^{n-1} \frac{1}{p+1} L_{n-p-1}^{p+\frac{1}{2}}(z) L_p^{-p-\frac{1}{2}}(-z) + 2(n+1) \sum_{p=0}^{n} \frac{1}{p+1} L_{n-p}^{p-\frac{1}{2}}(z) L_p^{-p-\frac{1}{2}}(-z) \right) + \frac{e^{-\frac{z}{2}} \sqrt{\pi} \operatorname{erfi}(\sqrt{z})}{2\sqrt[4]{z}} \left(2(n+1) L_{n+1}^{-\frac{3}{2}}(z) + L_n^{-\frac{1}{2}}(z) \right) /; n \in \mathbb{N}$$

For fixed z and $\nu = \frac{k}{4} \pm n$, $\mu = \frac{1}{4}$

07.44.03.0068.01

$$M_{-n-\frac{3}{4},\frac{1}{4}}(z) = \frac{(-1)^{n-1} e^{z/2} \sqrt{-z}\, n!}{2\sqrt[4]{z}\,(2n+1)!} H_{2n+1}\left(\sqrt{-z}\right) /; n \in \mathbb{N}$$

07.44.03.0069.01

$$M_{\frac{3}{4}-n,\frac{1}{4}}(z) = \frac{1}{2} e^{z/2} \sqrt{\pi} \sqrt[4]{z} \sum_{k=0}^{n-1} \frac{(-1)^k \left(\frac{1}{2}\right)_k}{k!} \left(1 - Q\left(k+\frac{1}{2}, z\right)\right) L_{-k+n-1}^k(-z) /; n \in \mathbb{N}^+$$

Brychkov Yu.A. (2006)

07.44.03.0070.01

$$M_{n+\frac{1}{4},\frac{1}{4}}(z) = \frac{e^{-\frac{z}{2}} \sqrt{\pi}\, z^{3/4}}{2\sqrt{-z}} \sum_{k=0}^{n} \frac{(-1)^k \left(\frac{1}{2}\right)_k}{k!} \left(1 - Q\left(k+\frac{1}{2}, -z\right)\right) L_{n-k}^k(z) /; n \in \mathbb{N}$$

07.44.03.0071.01

$$M_{n+\frac{3}{4},\frac{1}{4}}(z) = \frac{(-1)^n e^{-\frac{z}{2}} \sqrt[4]{z}\, n!}{2(2n+1)!} H_{2n+1}\left(\sqrt{z}\right) /; n \in \mathbb{N}$$

General characteristics

Domain and analyticity

$M_{\nu,\mu}(z)$ is an analytical function of ν, μ and z which is defined in $\mathbb{C}^3$.

07.44.04.0001.01

$$(\nu * \mu * z) \longrightarrow M_{\nu,\mu}(z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.44.04.0002.01

$$M_{\overline{\nu},\overline{\mu}}(\overline{z}) = \overline{M_{\nu,\mu}(z)}$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, μ, the function $M_{\nu,\mu}(z)$ has an essential singularity at $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is a branch point for generic μ.

07.44.04.0003.01

$$\mathcal{S}ing_z\left(M_{\nu,\mu}(z)\right) = \{\{\tilde{\infty}, \infty\}\}$$

With respect to μ

For fixed ν, z, the function $M_{\nu,\mu}(z)$ has an infinite set of singular points:

a) $\mu = -\frac{k+1}{2}$ /; $k \in \mathbb{N}$, are the simple poles with residues $\frac{(-1)^k}{2\,k!\,(k+1)!}\left(-\frac{k}{2}-\nu\right)_{k+1} M_{\nu,\frac{k+1}{2}}(z)$;

b) $\mu = \tilde{\infty}$ is an essential singular point.

07.44.04.0004.01

$$\mathcal{S}ing_\mu\left(M_{\nu,\mu}(z)\right) = \left\{\left\{\left\{-\frac{k+1}{2}, 1\right\} /;\ k \in \mathbb{N}\right\}, \{\tilde{\infty}, \infty\}\right\}$$

07.44.04.0005.01

$$\operatorname{res}_\mu\left(M_{\nu,\mu}(z)\right)\left(-\frac{k+1}{2}\right) = \frac{(-1)^k\left(-\frac{k}{2}-\nu\right)_{k+1}}{2\,k!\,(k+1)!} M_{\nu,\frac{k+1}{2}}(z) /;\ k \in \mathbb{N}$$

With respect to ν

For fixed μ, z, the function $M_{\nu,\mu}(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

07.44.04.0006.01

$$\mathcal{S}ing_\nu\left(M_{\nu,\mu}(z)\right) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

With respect to z

For fixed ν, and fixed μ /; $\mu + 1/2 \notin \mathbb{Z}$, the function $M_{\nu,\mu}(z)$ has two branch points: $z = 0$, $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is an essential singularity.

07.44.04.0007.01

$$\mathcal{BP}_z\left(M_{\nu,\mu}(z)\right) = \{0, \tilde{\infty}\} /;\ \mu + \frac{1}{2} \notin \mathbb{Z}$$

07.44.04.0008.01

$$\mathcal{BP}_z\left(M_{\nu,\mu}(z)\right) = \{\} /;\ \mu + \frac{1}{2} \in \mathbb{Z}$$

07.44.04.0009.01

$$\mathcal{R}_z\left(M_{\nu,\mu}(z), 0\right) = \log /;\ \mu \notin \mathbb{Q}$$

07.44.04.0010.01

$$\mathcal{R}_z\left(M_{\nu,\frac{p}{q}}(z), 0\right) = q /;\ p \in \mathbb{Z} \land q - 1 \in \mathbb{N}^+ \land \gcd(p, q) = 1$$

07.44.04.0011.01

$$\mathcal{R}_z\left(M_{\nu,\mu}(z),\, \tilde{\infty}\right) = \log \;/;\; \mu \notin \mathbb{Q}$$

07.44.04.0012.01

$$\mathcal{R}_z\left(M_{\nu,\frac{p}{q}}(z),\, \tilde{\infty}\right) = q \;/;\; p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p,\, q) = 1$$

With respect to μ

The function $M_{\nu,\mu}(z)$ does not have branch points with respect to μ.

07.44.04.0013.01

$$\mathcal{BP}_\mu\left(M_{\nu,\mu}(z)\right) = \{\}$$

With respect to ν

The function $M_{\nu,\mu}(z)$ does not have branch points with respect to ν.

07.44.04.0014.01

$$\mathcal{BP}_\nu\left(M_{\nu,\mu}(z)\right) = \{\}$$

Branch cuts

With respect to z

When $\mu + \frac{1}{2}$ is a nonnegative integer, $M_{\nu,\mu}(z)$ is an entire function of z. For fixed μ /; $\mu + 1/2 \notin \mathbb{Z}$, it has one infinitely long branch cut. In this case it is a single-valued function on the z-plane cut along the interval $(-\infty,\ 0)$, where it is continuous from above.

07.44.04.0015.01

$$\mathcal{BC}_z\left(M_{\nu,\mu}(z)\right) = \{\{(-\infty,\, 0),\, -i\}\} \;/;\; \mu + \frac{1}{2} \notin \mathbb{Z}$$

07.44.04.0016.01

$$\mathcal{BC}_z\left(M_{\nu,\mu}(z)\right) = \{\} \;/;\; \mu + \frac{1}{2} \in \mathbb{Z}$$

07.44.04.0017.01

$$\lim_{\epsilon \to +0} M_{\nu,\mu}(x + i\,\epsilon) = M_{\nu,\mu}(x) \;/;\; x \in \mathbb{R} \land x < 0$$

07.44.04.0018.01

$$\lim_{\epsilon \to +0} M_{\nu,\mu}(x - i\,\epsilon) = -e^{-2\,i\,\pi\,\mu}\, M_{\nu,\mu}(x) \;/;\; x \in \mathbb{R} \land x < 0$$

With respect to μ

The function $M_{\nu,\mu}(z)$ does not have branch cuts with respect to μ.

07.44.04.0019.01

$$\mathcal{BC}_\mu\left(M_{\nu,\mu}(z)\right) = \{\}$$

With respect to ν

The function $M_{\nu,\mu}(z)$ does not have branch cuts with respect to ν.

07.44.04.0020.01

$$\mathcal{BC}_\nu\left(M_{\nu,\mu}(z)\right) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

07.44.06.0001.01

$$M_{\nu,\mu}(z) \propto \left(\frac{1}{z_0}\right)^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor} \left(M_{\nu,\mu}(z_0) + \frac{1}{2}\left(\frac{2\mu - z_0 + 1}{z_0} M_{\nu,\mu}(z_0) + \frac{2\mu - 2\nu + 1}{(2\mu+1)\sqrt{z_0}} M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(z_0)\right)(z - z_0) + \frac{1}{8}\left(\frac{\left(4\mu^2 - 1\right) M_{\nu,\mu}(z_0)}{z_0^2} + M_{\nu,\mu}(z_0) + \frac{1}{2 z_0}\left(\frac{(2\mu - 2\nu + 1)(2\mu - 2\nu + 3)}{2\mu^2 + 3\mu + 1} M_{\nu-1,\mu+1}(z_0) - 4(2\mu+1) M_{\nu,\mu}(z_0)\right) + \frac{2(2\mu - 2\nu + 1)}{z_0^{3/2}} M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(z_0) - \frac{2(2\mu - 2\nu + 1)}{(2\mu+1)\sqrt{z_0}} M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(z_0)\right)(z - z_0)^2 + \ldots\right) /; (z \to z_0)$$

07.44.06.0002.01

$$M_{\nu,\mu}(z) \propto \left(\frac{1}{z_0}\right)^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor} \left(M_{\nu,\mu}(z_0) + \left(\frac{2\mu - z_0 + 1}{2 z_0} M_{\nu,\mu}(z_0) + \frac{2\mu - 2\nu + 1}{2(2\mu+1)\sqrt{z_0}} M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(z_0)\right)(z - z_0) + \frac{1}{16 z_0^2}\left(\frac{(2\mu - 2\nu + 1)(2\mu - 2\nu + 3) z_0}{2\mu^2 + 3\mu + 1} M_{\nu-1,\mu+1}(z_0) + 2\left(4\mu^2 + z_0^2 - 2(2\mu+1) z_0 - 1\right) M_{\nu,\mu}(z_0) + \frac{4(2\mu - 2\nu + 1)(2\mu - z_0 + 1)\sqrt{z_0}}{2\mu+1} M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(z_0)\right)(z - z_0)^2 + O\left((z - z_0)^3\right)\right)$$

07.44.06.0003.01

$$M_{\nu,\mu}(z) = \left(\frac{1}{z_0}\right)^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor} \sum_{k=0}^{\infty} \frac{M_{\nu,\mu}^{(0,0,k)}(z_0)}{k!} (z - z_0)^k$$

07.44.06.0004.01

$$M_{\nu,\mu}(z) = \left(\frac{1}{z_0}\right)^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor} \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{k_1=0}^{k} \sum_{k_2=0}^{k} \sum_{k_3=0}^{k} \frac{\delta_{k,k_1+k_2+k_3} (k_1 + k_2 + k_3; k_1, k_2, k_3)}{(2\mu+1)_{k_3}} \left(-\frac{1}{2}\right)^{k_1} z_0^{-k_2-\frac{k_3}{2}} \left(\mu - k_2 + \frac{3}{2}\right)_{k_2} \left(\mu - \nu + \frac{1}{2}\right)_{k_3} M_{\nu-\frac{k_3}{2},\mu+\frac{k_3}{2}}(z_0) (z - z_0)^k$$

07.44.06.0005.01

$$M_{\nu,\mu}(z) \propto \left(\frac{1}{z_0}\right)^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\left(\mu+\frac{1}{2}\right)\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} M_{\nu,\mu}(z_0)\,(1+O(z-z_0))$$

Expansions on branch cuts

For the function itself

07.44.06.0006.01

$$M_{\nu,\mu}(z) \propto e^{(2\mu+1)\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \Bigg(M_{\nu,\mu}(x) + \frac{1}{2}\left(\frac{-x+2\mu+1}{x} M_{\nu,\mu}(x) + \frac{2\mu-2\nu+1}{(2\mu+1)\sqrt{x}} M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(x) \right)(z-x) + \frac{1}{8}\left(\frac{4\mu^2-1}{x^2} M_{\nu,\mu}(x) + M_{\nu,\mu}(x) + \frac{1}{2x}\left(\frac{(2\mu-2\nu+1)(2\mu-2\nu+3)}{2\mu^2+3\mu+1} M_{\nu-1,\mu+1}(x) - 4(2\mu+1) M_{\nu,\mu}(x) \right) + \frac{2(2\mu-2\nu+1)}{x^{3/2}} M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(x) - \frac{2(2\mu-2\nu+1)}{(2\mu+1)\sqrt{x}} M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(x) \right)(z-x)^2 + \ldots \Bigg) /;\ (z \to x) \land x \in \mathbb{R} \land x < 0$$

07.44.06.0007.01

$$M_{\nu,\mu}(z) = e^{(2\mu+1)\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{M_{\nu,\mu}^{(0,0,k)}(x)}{k!} (z-x)^k /;\ x \in \mathbb{R} \land x < 0$$

07.44.06.0008.01

$$M_{\nu,\mu}(z) = e^{(2\mu+1)\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{k_1=0}^{k} \sum_{k_2=0}^{k} \sum_{k_3=0}^{k} \frac{\delta_{k,k_1+k_2+k_3}\,(k_1+k_2+k_3;\, k_1,\, k_2,\, k_3)}{(2\mu+1)_{k_3}} \left(-\frac{1}{2}\right)^{k_1} x^{-k_2-\frac{k_3}{2}} \left(\mu-k_2+\frac{3}{2}\right)_{k_2} \left(\mu-\nu+\frac{1}{2}\right)_{k_3} M_{\nu-\frac{k_3}{2},\mu+\frac{k_3}{2}}(x)\,(z-x)^k /;\ x \in \mathbb{R} \land x < 0$$

07.44.06.0009.01

$$M_{\nu,\mu}(z) \propto e^{(2\mu+1)\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} M_{\nu,\mu}(x)\,(1+O(z-x)) /;\ x \in \mathbb{R} \land x < 0$$

Expansions at $z == 0$

For the function itself

General case

07.44.06.0010.01

$$M_{\nu,\mu}(z) \propto z^{\mu+\frac{1}{2}} \left(1 - \frac{\nu}{1+2\mu} z + \frac{1+2\mu+4\nu^2}{16(1+\mu)(1+2\mu)} z^2 + \ldots \right) /;\ (z \to 0)$$

07.44.06.0011.01

$$M_{\nu,\mu}(z) \propto z^{\mu+\frac{1}{2}} \left(1 - \frac{\nu}{1+2\mu} z + \frac{1+2\mu+4\nu^2}{16(1+\mu)(1+2\mu)} z^2 + O(z^3) \right)$$

07.44.06.0012.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k!} \, {}_2F_1\left(-k, \mu-\nu+\frac{1}{2}; 2\mu+1; 2\right) z^k$$

07.44.06.0013.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} \sum_{k=0}^{\infty} \left(\sum_{j=0}^{k} \frac{\left((-1)^{k-j} \, 2^{j-k}\right) \left(\mu-\nu+\frac{1}{2}\right)_j}{(k-j)! \left((2\mu+1)_j \, j!\right)} \right) z^k$$

07.44.06.0014.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} \, e^{-\frac{z}{2}} \sum_{k=0}^{\infty} \frac{\left(\mu-\nu+\frac{1}{2}\right)_k z^k}{(2\mu+1)_k \, k!}$$

07.44.06.0015.01

$$M_{\nu,\mu}(z) \propto z^{\mu+\frac{1}{2}} \, (1+O(z))$$

Expansions at $z == \infty$ for polynomial cases

07.44.06.0016.01

$$M_{\nu,\mu}(z) = \frac{(-1)^{\nu-\mu-\frac{1}{2}}}{(2\mu+1)_{\nu-\mu-\frac{1}{2}}} \, z^{\nu} \, e^{-\frac{z}{2}} \, {}_2F_0\left(\frac{1}{2}-\mu-\nu, \mu-\nu+\frac{1}{2};; -\frac{1}{z}\right) /; \nu-\mu-\frac{1}{2} \in \mathbb{N}^+$$

Asymptotic series expansions

07.44.06.0017.01

$$M_{\nu,\mu}(z) \propto e^{-\frac{z}{2}} \, z^{\mu+\frac{1}{2}} \, \Gamma(2\mu+1) \, \mathcal{A}_{\tilde{F}}\left(\begin{matrix} \mu-\nu+\frac{1}{2}; \\ 2\mu+1; \end{matrix} \{z, \tilde{\infty}, \infty\}\right) /; (|z| \to \infty)$$

07.44.06.0018.01

$$M_{\nu,\mu}(z) \propto e^{-\frac{z}{2}} \, z^{\mu+\frac{1}{2}} \, \Gamma(2\mu+1) \left(\mathcal{A}_{\tilde{F}}^{(\text{power})}\left(\begin{matrix} \mu-\nu+\frac{1}{2}; \\ 2\mu+1; \end{matrix} \{z, \tilde{\infty}, \infty\}\right) + \mathcal{A}_{\tilde{F}}^{(\exp)}\left(\begin{matrix} \mu-\nu+\frac{1}{2}; \\ 2\mu+1; \end{matrix} \{z, \tilde{\infty}, \infty\}\right) \right) /; (|z| \to \infty)$$

07.44.06.0019.01

$$M_{\nu,\mu}(z) \propto \Gamma(2\mu+1) \left(\frac{(-z)^{\nu-\mu-\frac{1}{2}} \, z^{\mu+\frac{1}{2}}}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)} \, e^{-\frac{z}{2}} \left(1 + \frac{(1+2\mu-2\nu)(-1+2\mu+2\nu)}{4z} + \frac{(1+2\mu-2\nu)(3+2\mu-2\nu)(-3+2\mu+2\nu)(-1+2\mu+2\nu)}{32z^2} + \ldots\right) + \frac{z^{-\nu}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)} \, e^{\frac{z}{2}} \left(1 - \frac{(-1+2\mu-2\nu)(1+2\mu+2\nu)}{4z} + \frac{(-3+2\mu-2\nu)(-1+2\mu-2\nu)(1+2\mu+2\nu)(3+2\mu+2\nu)}{32z^2} + \ldots\right) \right) /; (|z| \to \infty)$$

07.44.06.0020.01

$$M_{\nu,\mu}(z) \propto \Gamma(2\mu+1)\left(\frac{(-z)^{\nu-\mu-\frac{1}{2}} z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}}}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\left(\sum_{k=0}^{n} \frac{(-1)^k \left(\frac{1}{2}-\mu-\nu\right)_k \left(\mu-\nu+\frac{1}{2}\right)_k z^{-k}}{k!} + O\left(\frac{1}{z^{n+1}}\right)\right) + \frac{z^{-\nu} e^{z/2}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\left(\sum_{k=0}^{n} \frac{\left(\nu-\mu+\frac{1}{2}\right)_k \left(\mu+\nu+\frac{1}{2}\right)_k z^{-k}}{k!} + O\left(\frac{1}{z^{n+1}}\right)\right)\right) /; (|z| \to \infty)$$

07.44.06.0021.01

$$M_{\nu,\mu}(z) \propto \Gamma(2\mu+1) \left(\frac{(-z)^{-\mu+\nu-\frac{1}{2}} z^{\mu+\frac{1}{2}}}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)} e^{-\frac{z}{2}} {}_2F_0\left(-\mu-\nu+\frac{1}{2}, \mu-\nu+\frac{1}{2};; -\frac{1}{z}\right) + \frac{z^{-\nu}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)} e^{\frac{z}{2}} {}_2F_0\left(-\mu+\nu+\frac{1}{2}, \mu+\nu+\frac{1}{2};; \frac{1}{z}\right)\right) /; (|z| \to \infty)$$

07.44.06.0022.01

$$M_{\nu,\mu}(z) \propto \Gamma(2\mu+1)\left(\frac{(-z)^{\nu-\mu-\frac{1}{2}} z^{\mu+\frac{1}{2}}}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)} e^{-\frac{z}{2}}\left(1+O\left(\frac{1}{z}\right)\right) + \frac{z^{-\nu}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)} e^{\frac{z}{2}}\left(1+O\left(\frac{1}{z}\right)\right)\right) /; (|z| \to \infty)$$

Integral representations

On the real axis

Of the direct function

07.44.07.0001.01

$$M_{\nu,\mu}(z) = \frac{\Gamma(2\mu+1)\, z^{\mu+\frac{1}{2}}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)} \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{tz} \left(\frac{1}{2}-t\right)^{\mu+\nu-\frac{1}{2}} \left(t+\frac{1}{2}\right)^{\mu-\nu-\frac{1}{2}} dt /; \mathrm{Re}(\mu+\nu) > -\frac{1}{2} \bigwedge \mathrm{Re}(\mu-\nu) > -\frac{1}{2}$$

07.44.07.0002.01

$$M_{\nu,\mu}(z) = \frac{e^{-\frac{z}{2}} z^{\mu+\frac{1}{2}}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)} \int_0^\infty e^{-t} t^{\mu-\nu-\frac{1}{2}} {}_0F_1(; 2\mu+1; tz)\, dt /; \mathrm{Re}(\mu-\nu) > -\frac{1}{2}$$

Limit representations

07.44.09.0001.01

$$M_{\nu,\mu}(z) = e^{-\frac{z}{2}} z^{\mu+\frac{1}{2}} \left(\lim_{p\to\infty} {}_2F_1\left(\mu-\nu+\frac{1}{2}, p; 2\mu+1; \frac{z}{p}\right)\right)$$

Continued fraction representations

07.44.10.0001.01

$$M_{\nu,\mu}(z) = e^{-\frac{z}{2}} z^{\mu+\frac{1}{2}} \left(1 + \cfrac{\frac{\left(\frac{1}{2}+\mu-\nu\right) z}{2\mu+1}}{\left(1 + \cfrac{-\frac{\left(\frac{3}{2}+\mu-\nu\right) z}{2(2\mu+2)}}{\left(1 + \frac{\left(\frac{3}{2}+\mu-\nu\right) z}{2(2\mu+2)} + \cfrac{-\frac{\left(\frac{5}{2}+\mu-\nu\right) z}{3(2\mu+3)}}{1 + \frac{\left(\frac{5}{2}+\mu-\nu\right) z}{3(2\mu+3)} + \cfrac{-\frac{\left(\frac{7}{2}+\mu-\nu\right) z}{4(2\mu+4)}}{1 + \frac{\left(\frac{7}{2}+\mu-\nu\right) z}{4(2\mu+4)} + \cfrac{-\frac{\left(\frac{9}{2}+\mu-\nu\right) z}{5(2\mu+5)}}{1 + \frac{\left(\frac{9}{2}+\mu-\nu\right) z}{5(2\mu+5)} + \ldots}}} \right)} \right)} \right)$$

07.44.10.0002.01

$$M_{\nu,\mu}(z) = e^{-\frac{z}{2}} z^{\mu+\frac{1}{2}} \left(1 + \frac{z\left(\mu - \nu + \frac{1}{2}\right)}{(2\mu+1)\left(1 + \mathrm{K}_k\left(-\frac{z\left(k+\mu-\nu+\frac{1}{2}\right)}{(k+1)(k+2\mu+1)}, \frac{z\left(k+\mu-\nu+\frac{1}{2}\right)}{(k+1)(k+2\mu+1)} + 1\right)_1^\infty\right)}\right)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

07.44.13.0001.01

$$w''(z) - \frac{\left(z^2 - 4\nu z + 4\mu^2 - 1\right) w(z)}{4z^2} = 0 \;/;\; w(z) = c_1 M_{\nu,\mu}(z) + c_2 W_{\nu,\mu}(z)$$

07.44.13.0002.01

$$W_z\left(M_{\nu,\mu}(z), W_{\nu,\mu}(z)\right) = -\frac{\Gamma(2\mu+1)}{\Gamma\left(\mu - \nu + \frac{1}{2}\right)}$$

07.44.13.0003.01

$$w''(z) - \frac{\left(z^2 - 4\nu z + 4\mu^2 - 1\right) w(z)}{4z^2} = 0 \;/;\; c_1 M_{\nu,\mu}(z) + c_2 W_{-\nu,\mu}(-z)$$

07.44.13.0004.01

$$W_z\left(M_{\nu,\mu}(z), W_{-\nu,\mu}(-z)\right) = (-z)^{-\mu-\frac{1}{2}} z^{\mu+\frac{1}{2}} \frac{\Gamma(2\mu+1)}{\Gamma\left(\mu + \nu + \frac{1}{2}\right)}$$

07.44.13.0005.01

$$w''(z) - \frac{\left(z^2 - 4\nu z + 4\mu^2 - 1\right) w(z)}{4z^2} = 0 \;/;\; c_1 W_{\nu,\mu}(z) + c_2 W_{-\nu,\mu}(-z)$$

07.44.13.0006.01

$$W_z\left(W_{\nu,\mu}(z), W_{-\nu,\mu}(-z)\right) = \csc(2\pi\mu)\left((-z)^{\mu-\frac{1}{2}} z^{\frac{1}{2}-\mu} \cos(\pi(\mu-\nu)) - (-z)^{-\mu-\frac{1}{2}} z^{\mu+\frac{1}{2}} \cos(\pi(\mu+\nu))\right)$$

07.44.13.0007.01

$$w''(z) - \frac{z^2 - 4\nu z + 4\mu^2 - 1}{4z^2} w(z) = 0 \;/;\; w(z) = c_1 M_{\nu,\mu}(z) + c_2 M_{\nu,-\mu}(z) \;/;\; 2\mu \notin \mathbb{Z}$$

07.44.13.0008.01

$$W_z\left(M_{\nu,\mu}(z), M_{\nu,-\mu}(z)\right) = -2\mu$$

07.44.13.0009.01

$$w''(z) - \frac{g''(z)}{g'(z)} w'(z) + \left(-\frac{1}{4} g'(z)^2 + \frac{\nu g'(z)^2}{g(z)} + \frac{\left(1 - 4\mu^2\right) g'(z)^2}{4 g(z)^2}\right) w(z) = 0 \;/;\; w(z) = c_1 M_{\nu,\mu}(g(z)) + c_2 W_{\nu,\mu}(g(z))$$

07.44.13.0010.01

$$W_z\left(M_{\nu,\mu}(g(z)), W_{\nu,\mu}(g(z))\right) = -\frac{g'(z)\,\Gamma(2\mu+1)}{\Gamma\left(\mu - \nu + \frac{1}{2}\right)}$$

07.44.13.0011.01

$$w''(z)-\left(\frac{2\,h'(z)}{h(z)}+\frac{g''(z)}{g'(z)}\right)w'(z)+\left(-\frac{1}{4}\,g'(z)^2+\frac{\nu\,g'(z)^2}{g(z)}+\frac{\left(1-4\,\mu^2\right)g'(z)^2}{4\,g(z)^2}+\frac{2\,h'(z)^2}{h(z)^2}+\frac{h'(z)\,g''(z)}{h(z)\,g'(z)}-\frac{h''(z)}{h(z)}\right)w(z)=0\;/;$$
$$w(z)=c_1\,h(z)\,M_{\nu,\mu}(g(z))+c_2\,h(z)\,W_{\nu,\mu}(g(z))$$

07.44.13.0012.01

$$W_z\big(h(z)\,M_{\nu,\mu}(g(z)),\,h(z)\,W_{\nu,\mu}(g(z))\big)=-\frac{h(z)^2\,g'(z)\,\Gamma(2\,\mu+1)}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)}$$

07.44.13.0013.01

$$4\,w''(z)\,z^2-4\,z\,(r+2\,s-1)\,w'(z)+\left(\left(4\,a\,\nu\,z^r-a^2\,z^{2\,r}-4\,\mu^2+1\right)r^2+4\,s\,r+4\,s^2\right)w(z)=0\;/;$$
$$w(z)=c_1\,z^s\,M_{\nu,\mu}\left(a\,z^r\right)+c_2\,z^s\,W_{\nu,\mu}\left(a\,z^r\right)$$

07.44.13.0014.01

$$W_z\big(z^s\,M_{\nu,\mu}(a\,z^r),\,z^s\,W_{\nu,\mu}(a\,z^r)\big)=-\frac{r\,\sqrt{a}\;z^{\frac{r}{2}+2\,s-1}\,\sqrt{a\,z^r}\;\Gamma(2\,\mu+1)}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)}$$

07.44.13.0015.01

$$4\,w''(z)\,z^2-4\,z\,(r+2\,s-1)\,w'(z)+\left(\left(4\,a\,\nu\,z^r-a^2\,z^{2\,r}-4\,\mu^2+1\right)r^2+4\,s\,r+4\,s^2\right)w(z)=0\;/;$$
$$w(z)=c_1\,z^s\,M_{\nu,\mu}\left(a\,z^r\right)+c_2\,z^s\,W_{\nu,\mu}\left(a\,z^r\right)$$

07.44.13.0016.01

$$W_z\big(s^z\,M_{\nu,\mu}(a\,r^z),\,s^z\,W_{\nu,\mu}(a\,r^z)\big)=-\frac{a\,r^z\,s^{2\,z}\,\log(r)\,\Gamma(2\,\mu+1)}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.44.16.0001.01

$$M_{-\nu,\mu}(z)=z^{\mu+\frac{1}{2}}\,(-z)^{-\mu-\frac{1}{2}}\,M_{\nu,\mu}(-z)$$

07.44.16.0002.01

$$M_{\nu,\mu}(-z)=(-z)^{\mu+\frac{1}{2}}\,z^{-\mu-\frac{1}{2}}\,M_{-\nu,\mu}(z)$$

07.44.16.0003.01

$$M_{-\nu,\mu}(-z)=(-z)^{\mu+\frac{1}{2}}\,z^{-\mu-\frac{1}{2}}\,M_{\nu,\mu}(z)$$

Products, sums, and powers of the direct function

Products of the direct function

07.44.16.0004.01

$$M_{\nu,\mu}(-z)\,M_{\nu,\mu}(z)=\left(-z^2\right)^{\mu+\frac{1}{2}}\,{}_2F_3\left(\mu-\nu+\frac{1}{2},\,\mu+\nu+\frac{1}{2};\,\mu+\frac{1}{2},\,\mu+1,\,2\,\mu+1;\,\frac{z^2}{4}\right)$$

07.44.16.0005.01

$$M_{\nu,\mu}(c\,z)\,M_{\lambda,\gamma}(d\,z) = e^{-\frac{c+d}{2}z}\,(c\,z)^{\mu+\frac{1}{2}}\,(d\,z)^{\gamma+\frac{1}{2}}\sum_{k=0}^{\infty} z^k\,c_k\;/;$$

$$c_k = \frac{d^k\left(\gamma-\lambda+\frac{1}{2}\right)_k}{k!\,(2\,\gamma+1)_k}\,{}_3F_2\!\left(-k,\,-k-2\,\gamma,\,\mu-\nu+\frac{1}{2};\,-k-\gamma+\lambda+\frac{1}{2},\,2\,\mu+1;\,-\frac{c}{d}\right)\bigvee$$

$$c_k = \frac{c^k\left(\mu-\nu+\frac{1}{2}\right)_k}{k!\,(2\,\mu+1)_k}\,{}_3F_2\!\left(-k,\,-k-2\,\mu,\,\gamma-\lambda+\frac{1}{2};\,-k-\mu+\nu+\frac{1}{2},\,2\,\gamma+1;\,-\frac{d}{c}\right)$$

07.44.16.0006.01

$$M_{\nu,\mu}(c\,z)\,M_{\lambda,\gamma}(d\,z) = e^{-\frac{c+d}{2}z}\,(c\,z)^{\mu+\frac{1}{2}}\,(d\,z)^{\gamma+\frac{1}{2}}\sum_{k=0}^{\infty} z^k\,c_k\;/;$$

$$c_k = \frac{d^k\left(\gamma-\lambda+\frac{1}{2}\right)_k}{k!\,(2\,\gamma+1)_k}\,{}_3F_2\!\left(-k,\,-k-2\,\gamma,\,\mu-\nu+\frac{1}{2};\,-k-\gamma+\lambda+\frac{1}{2},\,2\,\mu+1;\,-\frac{c}{d}\right)\bigvee$$

$$c_k = \frac{c^k\left(\mu-\nu+\frac{1}{2}\right)_k}{k!\,(2\,\mu+1)_k}\,{}_3F_2\!\left(-k,\,-k-2\,\mu,\,\gamma-\lambda+\frac{1}{2};\,-k-\mu+\nu+\frac{1}{2},\,2\,\gamma+1;\,-\frac{d}{c}\right)$$

07.44.16.0007.01

$$M_{\lambda,\gamma}(d\,z)\,M_{\nu,\mu}(c\,z) = e^{-\frac{c+d}{2}z}\,(c\,z)^{\mu+\frac{1}{2}}\,(d\,z)^{\gamma+\frac{1}{2}}\sum_{k=0}^{\infty}\sum_{m=0}^{k}\frac{\left(\gamma-\lambda+\frac{1}{2}\right)_{k-m}\left(\mu-\nu+\frac{1}{2}\right)_m}{(k-m)!\,m!\,(2\,\gamma+1)_{k-m}\,(2\,\mu+1)_m}\,c^m\,d^{k-m}\,z^k$$

07.44.16.0008.01

$$M_{\lambda,\gamma}(d\,z)\,M_{\nu,\mu}(c\,z) = e^{\frac{1}{2}(-(c+d))z}\,(c\,z)^{\mu+\frac{1}{2}}\,(d\,z)^{\gamma+\frac{1}{2}}\,F_{0:1;1}^{0:1;1}\!\left(\begin{matrix}:\mu-\nu+\frac{1}{2};\,\gamma-\lambda+\frac{1}{2};\\ :2\,\mu+1;\,2\,\gamma+1;\end{matrix}\;c\,z,\,d\,z\right)$$

Sums of the direct function

07.44.16.0009.01

$$M_{\nu,\mu}(z)+\frac{\Gamma(2\,\mu)\,\Gamma\!\left(\frac{1}{2}-\mu-\nu\right)}{\Gamma(-2\,\mu)\,\Gamma\!\left(\mu-\nu+\frac{1}{2}\right)}\,M_{\nu,-\mu}(z) = \frac{\Gamma\!\left(\frac{1}{2}-\mu-\nu\right)}{\Gamma(-2\,\mu)}\,e^{-\frac{z}{2}}\,z^{\mu+\frac{1}{2}}\,U\!\left(\mu-\nu+\frac{1}{2},\,2\,\mu+1,\,z\right)/;\,2\,\mu\notin\mathbb{Z}$$

Identities

Recurrence identities

Consecutive neighbors

07.44.17.0001.01

$$M_{\nu,\mu}(z) = \frac{2\,z-4\,\nu-4}{2\,\mu-2\,\nu-1}\,M_{\nu+1,\mu}(z)+\frac{2\,\mu+2\,\nu+3}{2\,\mu-2\,\nu-1}\,M_{\nu+2,\mu}(z)$$

07.44.17.0002.01

$$M_{\nu,\mu}(z) = \frac{-2\,z+4\,\nu-4}{2\,\mu+2\,\nu-1}\,M_{\nu-1,\mu}(z)+\frac{2\,\mu-2\,\nu+3}{2\,\mu+2\,\nu-1}\,M_{\nu-2,\mu}(z)$$

07.44.17.0003.01

$$M_{\nu,\mu}(z) = \frac{4\mu^2 + 8\mu - 2z\nu + 3}{z(2\mu+1)(2\mu+3)} M_{\nu,\mu+1}(z) + \frac{(2\mu - 2\nu + 3)(2\mu + 2\nu + 3)}{16(\mu+1)(\mu+2)(2\mu+3)^2} M_{\nu,\mu+2}(z)$$

07.44.17.0004.01

$$M_{\nu,\mu}(z) = -\frac{16(\mu-1)\mu(2\mu-1)\left(4\mu^2 - 8\mu - 2z\nu + 3\right)}{z(2\mu-3)(2\mu-2\nu-1)(2\mu+2\nu-1)} M_{\nu,\mu-1}(z) + \frac{16(1-2\mu)^2(\mu-1)\mu}{(2\mu-2\nu-1)(2\mu+2\nu-1)} M_{\nu,\mu-2}(z)$$

Distant neighbors

07.44.17.0005.01

$$M_{\nu,\mu}(z) = C_n(\nu,\mu,z) M_{n+\nu,\mu}(z) - \frac{2n+2\mu+2\nu+1}{2n-2\mu+2\nu-1} C_{n-1}(\nu,\mu,z) M_{n+\nu+1,\mu}(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{-2z+4\nu+4}{-2\mu+2\nu+1} \bigwedge C_n(\nu,\mu,z) = \frac{2(2n-z+2\nu)}{2n-2\mu+2\nu-1} C_{n-1}(\nu,\mu,z) - \frac{2n+2\mu+2\nu-1}{2n-2\mu+2\nu-3} C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.44.17.0006.01

$$M_{\nu,\mu}(z) = C_n(\nu,\mu,z) M_{\nu-n,\mu}(z) - \frac{2n+2\mu-2\nu+1}{2n-2\mu-2\nu-1} C_{n-1}(\nu,\mu,z) M_{\nu-n-1,\mu}\left(z\right) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = -\frac{2(z-2\nu+2)}{2\mu+2\nu-1} \bigwedge C_n(\nu,\mu,z) = \frac{2(2n+z-2\nu)}{2n-2\mu-2\nu-1} C_{n-1}(\nu,\mu,z) - \frac{2n+2\mu-2\nu-1}{2n-2\mu-2\nu-3} C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

07.44.17.0007.01

$$(2\nu - 2\mu - 1) M_{\nu-1,\mu}(z) + 2(z - 2\nu) M_{\nu,\mu}(z) + (2\mu + 2\nu + 1) M_{\nu+1,\mu}(z) = 0$$

07.44.17.0008.01

$$2(2\mu+1)(z+2\mu) M_{\nu,\mu}(z) - \sqrt{z}\,(2\mu+2\nu+1) M_{\nu+\frac{1}{2},\mu+\frac{1}{2}}(z) - 4\sqrt{z}\,\mu(2\mu+1) M_{\nu-\frac{1}{2},\mu-\frac{1}{2}}(z) = 0$$

07.44.17.0009.01

$$(2z + 2\mu - 2\nu - 1) M_{\nu,\mu}(z) + (2\mu + 2\nu + 1) M_{\nu+1,\mu}(z) - 4\sqrt{z}\,\mu M_{\nu-\frac{1}{2},\mu-\frac{1}{2}}(z) = 0$$

07.44.17.0010.01

$$-(2\mu+1) M_{\nu,\mu}(z) + (2\mu+1) M_{\nu+1,\mu}(z) + \sqrt{z}\, M_{\nu+\frac{1}{2},\mu+\frac{1}{2}}(z) = 0$$

07.44.17.0011.01

$$(-2\mu + 2\nu - 1) M_{\nu-1,\mu}(z) + (-2\mu - 2\nu + 1) M_{\nu,\mu}(z) + 4\sqrt{z}\,\mu M_{\nu-\frac{1}{2},\mu-\frac{1}{2}}(z) = 0$$

07.44.17.0012.01

$$(2\mu+1)(2\mu-2\nu+1) M_{\nu-1,\mu}(z) - (2\mu+1)(2z+2\mu-2\nu+1) M_{\nu,\mu}(z) + \sqrt{z}\,(2\mu+2\nu+1) M_{\nu+\frac{1}{2},\mu+\frac{1}{2}}(z) = 0$$

07.44.17.0013.01

$$-4\sqrt{z}\,(2\mu+1)(z-2\nu-1) M_{\nu,\mu}(z) - \left(4\mu^2 + 8(\nu+1)\mu + 4\nu^2 + 8\nu + 3\right) M_{\nu+\frac{3}{2},\mu+\frac{1}{2}}(z) + (2\mu-2\nu+1)(2z+2\mu-2\nu-1) M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(z) = 0$$

07.44.17.0014.01

$$-4\sqrt{z}\,(\mu+1)(2\mu+1)(2z+2\mu-2\nu+1)\,M_{\nu,\mu}(z)+\sqrt{z}\,(2\mu+2\nu+1)(2\mu+2\nu+3)\,M_{\nu+1,\mu+1}(z)+4(\mu+1)(z+2\mu+1)(2\mu-2\nu+1)\,M_{\nu-\frac{1}{2},\mu+\frac{1}{2}}(z)=0$$

Relations of special kind

07.44.17.0015.01

$$M_{\nu,\mu}(z)=(-z)^{-\mu-\frac{1}{2}}\,z^{\mu+\frac{1}{2}}\,M_{-\nu,\mu}(-z)$$

Division on even and odd parts and generalization

07.44.17.0016.01

$$M_{\nu,\mu}(z)=A^{-}[z]+A^{+}[z]\;/;\;A^{+}[z]=\frac{1}{2}\left(e^{-z}\,z^{\mu+\frac{1}{2}}\,M_{\nu,\mu}(-z)\,(-z)^{-\mu-\frac{1}{2}}+M_{\nu,\mu}(z)\right)\bigwedge A^{-}[z]=\frac{1}{2}\left(M_{\nu,\mu}(z)-e^{-z}\,(-z)^{-\mu-\frac{1}{2}}\,z^{\mu+\frac{1}{2}}\,M_{\nu,\mu}(-z)\right)$$

07.44.17.0017.01

$$M_{\nu,\mu}(z)=A^{-}[z]+A^{+}[z]\;/;\;A^{+}[z]=e^{-\frac{z}{2}}\,z^{\mu+\frac{1}{2}}\,{}_2F_3\left(\frac{\mu-\nu}{2}+\frac{1}{4},\frac{\mu-\nu}{2}+\frac{3}{4};\frac{1}{2},\mu+\frac{1}{2},\mu+1;\frac{z^2}{4}\right)\bigwedge$$
$$A^{-}[z]=\frac{e^{-\frac{z}{2}}\,z^{\mu+\frac{3}{2}}\,(2\mu-2\nu+1)}{2(2\mu+1)}\,{}_2F_3\left(\frac{\mu-\nu}{2}+\frac{3}{4},\frac{\mu-\nu}{2}+\frac{5}{4};\frac{3}{2},\mu+1,\mu+\frac{3}{2};\frac{z^2}{4}\right)$$

07.44.17.0018.01

$$M_{\nu,\mu}(z)=e^{-\frac{z}{2}}\,z^{\mu+\frac{1}{2}}\sum_{k=0}^{n-1}\frac{\left(\mu-\nu+\frac{1}{2}\right)_k z^k}{k!\,(2\mu+1)_k}\,{}_{n+1}F_{2n}\left(1,\frac{k+\mu-\nu}{n}+\frac{1}{2n},\ldots,\frac{k+\mu-\nu}{n}+\frac{n-1}{2n};\frac{k+1}{n},\ldots,\frac{k+n}{n},\frac{k+2\mu+1}{n},\ldots,\frac{k+2\mu+n}{n};n^{-n}z^n\right)$$

Differentiation

Low-order differentiation

With respect to ν

07.44.20.0001.01

$$M_{\nu,\mu}^{(1,0,0)}(z)=\psi\left(\mu-\nu+\frac{1}{2}\right)M_{\nu,\mu}(z)-z^{\mu+\frac{1}{2}}\sum_{k=0}^{\infty}\left(\sum_{j=0}^{k}\frac{\left((-1)^{k-j}\,2^{j-k}\right)\left(\mu-\nu+\frac{1}{2}\right)_j\,\psi\left(j+\mu-\nu+\frac{1}{2}\right)}{(k-j)!\left((2\mu+1)_j\,j!\right)}\right)z^k$$

07.44.20.0002.01

$$M_{\nu,\mu}^{(1,0,0)}(z)=-\frac{1}{2\mu+1}\,e^{-\frac{z}{2}}\,z^{\mu+\frac{3}{2}}\,F_{2,0,1}^{1,1,2}\left(\begin{matrix}\mu-\nu+\frac{3}{2};\,1;\,1,\,\mu-\nu+\frac{1}{2};\\ 2,\,2\mu+2;\,;\,\mu-\nu+\frac{3}{2};\end{matrix}\;z,z\right)$$

With respect to μ

07.44.20.0003.01

$$M_{\nu,\mu}^{(0,1,0)}(z) = z^{\mu+\frac{1}{2}} \sum_{k=0}^{\infty} \left(\sum_{j=0}^{k} \frac{\left((-1)^{k-j}\, 2^{j-k}\right) \left(\mu-\nu+\frac{1}{2}\right)_j \left(\psi\left(j+\mu-\nu+\frac{1}{2}\right) - 2\,\psi(j+2\,\mu+1)\right)}{(k-j)!\, \left((2\,\mu+1)_j\, j!\right)} \right) z^k + \left(\log(z) + 2\,\psi(2\,\mu+1) - \psi\left(\mu-\nu+\frac{1}{2}\right)\right) M_{\nu,\mu}(z)$$

07.44.20.0004.01

$$M_{\nu,\mu}^{(0,1,0)}(z) = e^{-\frac{z}{2}} \left(\frac{z}{2\,\mu+1}\, F_{2,0,1}^{1,1,2}\left(\begin{matrix} \mu-\nu+\frac{3}{2};\, 1;\, 1,\, \mu-\nu+\frac{1}{2}; \\ 2,\, 2\,\mu+2\,;\,;\, \mu-\nu+\frac{1}{2}; \end{matrix} \; z,\, z \right) - \frac{1}{(2\,\mu+1)^2}\, (z\,(2\,\mu-2\,\nu+1))\, F_{2,0,1}^{1,1,2}\left(\begin{matrix} \mu-\nu+\frac{3}{2};\, 1;\, 1,\, 2\,\mu+1; \\ 2,\, 2\,\mu+2\,;\,;\, 2\,\mu+2; \end{matrix} \; z,\, z \right) \right) z^{\mu+\frac{1}{2}} + \log(z)\, M_{\nu,\mu}(z)$$

With respect to z

07.44.20.0005.01

$$\frac{\partial M_{\nu,\mu}(z)}{\partial z} = \left(\frac{1}{2} - \frac{\nu}{z}\right) M_{\nu,\mu}(z) + \frac{2\,\mu+2\,\nu+1}{2\,z}\, M_{\nu+1,\mu}(z)$$

07.44.20.0006.01

$$\frac{\partial^2 M_{\nu,\mu}(z)}{\partial z^2} = \frac{z^2 - 4\,\nu\,z + 4\,\mu^2 - 1}{4\,z^2}\, M_{\nu,\mu}(z)$$

07.44.20.0007.01

$$\frac{\partial^3 M_{\nu,\mu}(z)}{\partial z^3} = \frac{1}{8\,z^3} \left(\left(z^3 - 6\,\nu\,z^2 + \left(4\,\mu^2 + 8\,\nu^2 + 8\,\nu - 1\right) z - 2\left(4\,\mu^2 - 1\right)(\nu+2)\right) M_{\nu,\mu}(z) + (2\,\mu+2\,\nu+1)\left(z^2 - 4\,\nu\,z + 4\,\mu^2 - 1\right) M_{\nu+1,\mu}(z) \right)$$

07.44.20.0008.01

$$\frac{\partial^4 M_{\nu,\mu}(z)}{\partial z^4} = \frac{1}{16\,z^4} \left(\left(z^4 - 8\,\nu\,z^3 + 2\left(4\,\mu^2 + 8\,\nu^2 + 8\,\nu - 1\right) z^2 - 8\,(\nu+1)\left(4\,\mu^2 + 4\,\nu - 1\right) z + \left(4\,\mu^2 - 1\right)\left(4\,\mu^2 + 16\,\nu + 23\right)\right) M_{\nu,\mu}(z) - 8\,(2\,\mu+2\,\nu+1)\left(4\,\mu^2 - 2\,z\,\nu - 1\right) M_{\nu+1,\mu}(z) \right)$$

Symbolic differentiation

With respect to ν

07.44.20.0009.01

$$M_{\nu,\mu}^{(n,0,0)}(z) = z^{\mu+\frac{1}{2}} \sum_{k=0}^{\infty} \left(\sum_{j=0}^{k} \frac{(-1)^{k-j}\, 2^{j-k}}{(k-j)!\, (2\,\mu+1)_j\, j!}\, \frac{\partial^n \left(\mu-\nu+\frac{1}{2}\right)_j}{\partial \nu^n} \right) z^k \;/;\; n \in \mathbb{N}$$

With respect to μ

07.44.20.0010.01

$$M_{\nu,\mu}^{(0,n,0)}(z) = \sqrt{z} \sum_{k=0}^{\infty} \left(\sum_{j=0}^{k} \frac{(-1)^{k-j}\, 2^{j-k}}{(k-j)!\, j!} \frac{\partial^n \frac{\left(\mu-\nu+\frac{1}{2}\right)_j z^{\mu}}{(2\mu+1)_j}}{\partial \mu^n} \right) z^k \,/;\, n \in \mathbb{N}$$

With respect to *z*

07.44.20.0011.01

$$\frac{\partial^n M_{\nu,\mu}(z)}{\partial z^n} = \sum_{k_1=0}^{n} \sum_{k_2=0}^{n} \sum_{k_3=0}^{n} \frac{\delta_{n,k_1+k_2+k_3}\, (k_1+k_2+k_3;\, k_1,\, k_2,\, k_3)}{(2\mu+1)_{k_3}} \left(-\frac{1}{2}\right)^{k_1} z^{-k_2-\frac{k_3}{2}} \left(\mu-k_2+\frac{3}{2}\right)_{k_2} \left(\mu-\nu+\frac{1}{2}\right)_{k_3} M_{\nu-\frac{k_3}{2},\mu+\frac{k_3}{2}}(z) \,/;\, n \in \mathbb{N}$$

07.44.20.0012.01

$$\frac{\partial^n \left(z^{\alpha} M_{\nu,\mu}(z)\right)}{\partial z^n} = \sum_{k_1=0}^{n} \sum_{k_2=0}^{n} \sum_{k_3=0}^{n} \frac{\delta_{n,k_1+k_2+k_3}\, (k_1+k_2+k_3;\, k_1,\, k_2,\, k_3)}{(2\mu+1)_{k_3}} \left(-\frac{1}{2}\right)^{k_1} z^{\alpha-k_2-\frac{k_3}{2}} \left(\mu-\nu+\frac{1}{2}\right)_{k_3} \left(\alpha+\mu-k_2+\frac{3}{2}\right)_{k_2} M_{\nu-\frac{k_3}{2},\mu+\frac{k_3}{2}}(z) \,/;\, n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to *z*

07.44.20.0013.01

$$\frac{\partial^{\alpha} M_{\nu,\mu}(z)}{\partial z^{\alpha}} = \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k!\, \Gamma\left(k-\alpha+\mu+\frac{3}{2}\right)} {}_2F_1\left(-k,\, \mu-\nu+\frac{1}{2};\, 2\mu+1;\, 2\right) \Gamma\left(k+\mu+\frac{3}{2}\right) z^{k-\alpha+\mu+\frac{1}{2}} \,/;\, -\mu-\frac{1}{2} \in \mathbb{N}^+$$

07.44.20.0014.01

$$\frac{\partial^{\alpha} M_{\nu,\mu}(z)}{\partial z^{\alpha}} = \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k!} {}_2F_1\left(-k,\, \mu-\nu+\frac{1}{2};\, 2\mu+1;\, 2\right) \mathcal{FC}_{\exp}^{(\alpha)}\left(z,\, k+\mu+\frac{1}{2}\right) z^{k-\alpha+\mu+\frac{1}{2}}$$

07.44.20.0015.01

$$\frac{\partial^{\alpha} M_{\nu,\mu}(z)}{\partial z^{\alpha}} = \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{(-1)^{k-j}\, 2^{j-k} \left(\mu-\nu+\frac{1}{2}\right)_j}{(k-j)!\, (2\mu+1)_j\, j!} \mathcal{FC}_{\exp}^{(\alpha)}\left(z,\, k+\mu+\frac{1}{2}\right) z^{k-\alpha+\mu+\frac{1}{2}}$$

Integration

Indefinite integration

Involving one direct function and elementary functions

Involving exponential function

07.44.21.0001.01

$$\int e^{-\frac{z}{2}} M_{\nu,\mu}(z)\, dz = z^{\mu+\frac{3}{2}}\, \Gamma\left(\mu+\frac{3}{2}\right) \Gamma(2\mu+1)\, {}_2\tilde{F}_2\left(\mu+\nu+\frac{1}{2},\, \mu+\frac{3}{2};\, 2\mu+1,\, \mu+\frac{5}{2};\, -z\right)$$

07.44.21.0002.01

$$\int e^{z/2} M_{\nu,\mu}(z)\, dz = z^{\mu+\frac{3}{2}}\, \Gamma\left(\mu+\frac{3}{2}\right) \Gamma(2\mu+1)\, {}_2\tilde{F}_2\left(\mu-\nu+\frac{1}{2}, \mu+\frac{3}{2}; 2\mu+1, \mu+\frac{5}{2}; z\right)$$

Involving exponential function and a power function

07.44.21.0003.01

$$\int z^{\alpha-1} e^{-\frac{cz}{2}} M_{\nu,\mu}(c z)\, dz = z^{\alpha} (c z)^{\mu+\frac{1}{2}}\, \Gamma\left(\alpha+\mu+\frac{1}{2}\right) \Gamma(2\mu+1)\, {}_2\tilde{F}_2\left(\mu+\nu+\frac{1}{2}, \alpha+\mu+\frac{1}{2}; 2\mu+1, \alpha+\mu+\frac{3}{2}; -c z\right)$$

07.44.21.0004.01

$$\int z^{\alpha-1} e^{\frac{cz}{2}} M_{\nu,\mu}(c z)\, dz = z^{\alpha} (c z)^{\mu+\frac{1}{2}}\, \Gamma\left(\alpha+\mu+\frac{1}{2}\right) \Gamma(2\mu+1)\, {}_2\tilde{F}_2\left(\mu-\nu+\frac{1}{2}, \alpha+\mu+\frac{1}{2}; 2\mu+1, \alpha+\mu+\frac{3}{2}; c z\right)$$

Definite integration

Involving the direct function

07.44.21.0005.01

$$\int_0^{\infty} t^{\alpha-1} e^{-c t} M_{\nu,\mu}(-t)\, dt = i\, (-1)^{\mu} \left(c-\frac{1}{2}\right)^{-\alpha-\mu-\frac{1}{2}} \Gamma\left(\alpha+\mu+\frac{1}{2}\right) {}_2F_1\left(\mu-\nu+\frac{1}{2}, \alpha+\mu+\frac{1}{2}; 2\mu+1; \frac{2}{1-2c}\right) /;$$
$$\mathrm{Re}(\alpha+\mu) > -\frac{1}{2} \bigwedge \mathrm{Re}(c) > \frac{1}{2}$$

Integral transforms

Laplace transforms

07.44.22.0001.01

$$\mathcal{L}_t\left[M_{\nu,\mu}(t)\right](z) = \left(z-\frac{1}{2}\right)^{-\mu-\frac{3}{2}} \Gamma\left(\mu+\frac{3}{2}\right) {}_2F_1\left(\mu+\frac{3}{2}, \mu+\nu+\frac{1}{2}; 2\mu+1; \frac{2}{1-2z}\right) /; \mathrm{Re}(z) > \frac{1}{2} \bigwedge \mathrm{Re}(\mu) > -\frac{3}{2}$$

Operations

Limit operation

07.44.25.0001.01

$$\lim_{\mu \to -\frac{n}{2}} \frac{M_{\nu,\mu}(z)}{\Gamma(2\mu+1)} = \frac{1}{n!} \left(\frac{1-n}{2} - \nu\right)_n M_{\nu,\frac{n}{2}}(z) /; n \in \mathbb{N}$$

07.44.25.0002.01

$$\lim_{\nu \to \infty} \left(-\frac{z}{\nu}\right)^{-\mu-\frac{1}{2}} M_{\nu,\mu}\left(-\frac{z}{\nu}\right) = z^{-\mu}\, I_{2\mu}\left(2\sqrt{z}\right) \Gamma(2\mu+1)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_1F_1$

07.44.26.0001.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}} {}_1F_1\left(\mu - \nu + \frac{1}{2}; 2\mu + 1; z\right)$$

Involving ${}_1\tilde{F}_1$

07.44.26.0002.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}} \Gamma(2\mu + 1) {}_1\tilde{F}_1\left(\mu - \nu + \frac{1}{2}; 2\mu + 1; z\right)$$

Involving ${}_pF_q$

07.44.26.0003.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}} {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z) /; p = 1 \bigwedge q = 1 \bigwedge a_1 = \mu - \nu + \frac{1}{2} \bigwedge b_1 = 2\mu + 1$$

07.44.26.0004.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}} {}_2F_2\left(\mu - \nu + \frac{1}{2}, a_2; 2\mu + 1, a_2; z\right)$$

07.44.26.0005.01

$$M_{\nu,\mu}(z)\, M_{\nu,\mu}(-z) = \left(-z^2\right)^{\mu+\frac{1}{2}} {}_2F_3\left(\mu - \nu + \frac{1}{2}, \mu + \nu + \frac{1}{2}; \mu + \frac{1}{2}, \mu + 1, 2\mu + 1; \frac{z^2}{4}\right)$$

Involving ${}_pF_q$

07.44.26.0006.01

$$M_{\nu,\mu}(z)\, M_{\nu,\mu}(-z) = 4^{-\mu} \sqrt{\pi} \left(-z^2\right)^{\mu+\frac{1}{2}} \Gamma(2\mu + 1)^2 {}_2\tilde{F}_3\left(\mu - \nu + \frac{1}{2}, \mu + \nu + \frac{1}{2}; \mu + \frac{1}{2}, \mu + 1, 2\mu + 1; \frac{z^2}{4}\right)$$

Through Meijer *G*

Classical cases involving exp

07.44.26.0007.01

$$e^{z/2} M_{\nu,\mu}(z) = \frac{\pi \Gamma(2\mu + 1)}{\Gamma\left(\mu - \nu + \frac{1}{2}\right)} G_{2,3}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu + 1, \mu + 1 \\ \mu + \frac{1}{2}, \frac{1}{2} - \mu, \mu + 1 \end{matrix}\right)$$

07.44.26.0008.01

$$e^{z/2} M_{\nu,\mu}(z) = \frac{\Gamma(2\mu + 1) z^{\mu+\frac{1}{2}}}{\Gamma\left(\mu - \nu + \frac{1}{2}\right)} G_{1,2}^{1,1}\left(-z \,\middle|\, \begin{matrix} \nu - \mu + \frac{1}{2} \\ 0, -2\mu \end{matrix}\right)$$

07.44.26.0009.01

$$e^{z/2} M_{\nu,\mu}(z) == z^{\mu+\frac{1}{2}} - \frac{\pi \Gamma(2\mu + 1)}{\Gamma\left(\mu - \nu + \frac{1}{2}\right)} G_{3,4}^{1,2}\left(z \,\middle|\, \begin{matrix} \mu + \frac{3}{2}, \nu + 1, \mu + 1 \\ \mu + \frac{3}{2}, \mu + \frac{1}{2}, \frac{1}{2} - \mu, \mu + 1 \end{matrix}\right)$$

07.44.26.0010.01

$$e^{z/2} M_{\nu,\mu}(z) == z^{\mu+\frac{1}{2}} - \frac{\pi \Gamma(2\mu + 1) z^{\mu+\frac{1}{2}}}{\Gamma\left(\mu - \nu + \frac{1}{2}\right)} G_{3,4}^{1,2}\left(z \,\middle|\, \begin{matrix} 1, -\mu + \nu + \frac{1}{2}, \frac{1}{2} \\ 1, 0, -2\mu, \frac{1}{2} \end{matrix}\right)$$

07.44.26.0011.01

$$e^{-\frac{z}{2}} M_{\nu,\mu}(z) = \frac{\Gamma(2\mu+1)}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1-\nu \\ \mu+\frac{1}{2}, \frac{1}{2}-\mu \end{matrix}\right)$$

Classical cases involving cosh

07.44.26.0012.01

$$\cosh\left(\frac{z}{2}\right) M_{\nu,\mu}(z) = \frac{\pi\,\Gamma(2\mu+1)}{2\,\Gamma\left(\mu-\nu+\frac{1}{2}\right)} G_{2,3}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+1, \mu+1 \\ \mu+\frac{1}{2}, \frac{1}{2}-\mu, \mu+1 \end{matrix}\right) + \frac{\Gamma(2\mu+1)}{2\,\Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1-\nu \\ \mu+\frac{1}{2}, \frac{1}{2}-\mu \end{matrix}\right)$$

Classical cases involving sinh

07.44.26.0013.01

$$\sinh\left(\frac{z}{2}\right) M_{\nu,\mu}(z) = \frac{\pi\,\Gamma(2\mu+1)}{2\,\Gamma\left(\mu-\nu+\frac{1}{2}\right)} G_{2,3}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+1, \mu+1 \\ \mu+\frac{1}{2}, \frac{1}{2}-\mu, \mu+1 \end{matrix}\right) - \frac{\Gamma(2\mu+1)}{2\,\Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1-\nu \\ \mu+\frac{1}{2}, \frac{1}{2}-\mu \end{matrix}\right)$$

Classical cases for products of WhittakerM *M*

07.44.26.0014.01

$$M_{\nu,\mu}(z)\, M_{\nu,\mu}(-z) = \frac{2\sqrt{\pi}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)\Gamma\left(\mu-\nu+\frac{1}{2}\right)} G_{2,4}^{1,2}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \nu+1, 1-\nu \\ \mu+\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}-\mu \end{matrix}\right)$$

07.44.26.0015.01

$$M_{\nu,\mu}(z)\, M_{\nu,\mu}(-z) = \frac{4^{-\mu}\,\pi^{3/2}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)} \left(-z^2\right)^{\mu+\frac{1}{2}} G_{3,5}^{1,2}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} -\mu+\nu+\frac{1}{2}, -\mu-\nu+\frac{1}{2}, \frac{1}{2} \\ 0, -\mu, \frac{1}{2}-\mu, -2\mu, \frac{1}{2} \end{matrix}\right)$$

07.44.26.0016.01

$$M_{0,\gamma}(-z)\, M_{0,\mu}(z) = \frac{2^{\gamma+\mu}\,\Gamma(\gamma+1)\,\Gamma(\mu+1)}{\sqrt{\pi}} (-z)^{\gamma+\frac{1}{2}} z^{\mu+\frac{1}{2}}\, G_{2,4}^{1,2}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{1-\mu-\gamma}{2}, -\frac{\mu+\gamma}{2} \\ 0, -\gamma-\mu, -\gamma, -\mu \end{matrix}\right)$$

07.44.26.0017.01

$$M_{0,\gamma}(-z)\, M_{0,\mu}(z) = 2^{\gamma+\mu}\sqrt{\pi}\,\Gamma(\gamma+1)\,\Gamma(\mu+1)\,(-z)^{\gamma+\frac{1}{2}} z^{\mu+\frac{1}{2}}\, G_{3,5}^{1,2}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{1-\mu-\gamma}{2}, -\frac{\mu+\gamma}{2}, \frac{1}{2} \\ 0, -\gamma-\mu, -\gamma, -\mu, \frac{1}{2} \end{matrix}\right)$$

07.44.26.0018.01

$$M_{-\nu,\mu}(z)\, M_{\nu,\mu}(z) = \frac{4^{-\mu}\sqrt{\pi}\,\Gamma(2\mu+1)^2 z^{-2\mu-1}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{2,4}^{1,2}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} -\mu+\nu+\frac{1}{2}, -\mu-\nu+\frac{1}{2} \\ 0, -\mu, \frac{1}{2}-\mu, -2\mu \end{matrix}\right)$$

07.44.26.0019.01

$$M_{-\nu,\mu}(z)\, M_{\nu,\mu}(z) = \frac{4^{-\mu}\,\pi^{3/2}\,\Gamma(2\mu+1)^2 z^{2\mu+1}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{3,5}^{1,2}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} -\mu+\nu+\frac{1}{2}, -\mu-\nu+\frac{1}{2}, \frac{1}{2} \\ 0, -\mu, \frac{1}{2}-\mu, -2\mu, \frac{1}{2} \end{matrix}\right)$$

07.44.26.0020.01

$$M_{0,\gamma}(z)\, M_{0,\mu}(z) = \frac{2^{\gamma+\mu}\,\Gamma(\gamma+1)\,\Gamma(\mu+1)\,z^{\gamma+\mu+1}}{\sqrt{\pi}} G_{2,4}^{1,2}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{1}{2}(-\gamma-\mu+1), -\frac{\mu+\gamma}{2} \\ 0, -\gamma-\mu, -\gamma, -\mu \end{matrix}\right)$$

07.44.26.0021.01

$$M_{0,\gamma}(z)\,M_{0,\mu}(z)=2^{\gamma+\mu}\sqrt{\pi}\;\Gamma(\gamma+1)\,\Gamma(\mu+1)\,z^{\gamma+\mu+1}\,G_{3,5}^{1,2}\!\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{1}{2}(-\gamma-\mu+1),\,-\frac{\mu+\gamma}{2},\,\frac{1}{2}\\0,\,-\gamma-\mu,\,-\gamma,\,-\mu,\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0022.01

$$M_{\nu,\mu}\!\left(i\sqrt{z}\right)M_{\nu,\mu}\!\left(-i\sqrt{z}\right)=\frac{2\sqrt{\pi}\;\Gamma(2\mu+1)^2}{\Gamma\!\left(\mu+\nu+\frac{1}{2}\right)\Gamma\!\left(\mu-\nu+\frac{1}{2}\right)}\,G_{2,4}^{1,2}\!\left(\frac{z}{4}\,\middle|\,\begin{matrix}\nu+1,\,1-\nu\\\mu+\frac{1}{2},\,\frac{1}{2},\,1,\,\frac{1}{2}-\mu\end{matrix}\right)$$

07.44.26.0023.01

$$M_{-\nu,\mu}\!\left(\sqrt{z}\right)M_{\nu,\mu}\!\left(\sqrt{z}\right)=\frac{2\,\pi^{3/2}\,\Gamma(2\mu+1)^2}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right)\Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\!\left(\frac{z}{4}\,\middle|\,\begin{matrix}\nu+1,\,1-\nu,\,\mu+1\\\mu+\frac{1}{2},\,\mu+1,\,\frac{1}{2},\,1,\,\frac{1}{2}-\mu\end{matrix}\right)$$

Classical cases involving exp and $_1F_1$

07.44.26.0024.01

$$e^{z/2}\,{}_1F_1\!\left(\mu-\nu+\frac{1}{2};\,2\mu+1;\,-z\right)M_{\nu,\mu}(z)=\frac{4^{-\mu}\sqrt{\pi}\;\Gamma(2\mu+1)^2\,z^{\mu+\frac{1}{2}}}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right)\Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\,G_{2,4}^{1,2}\!\left(-\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\mu\end{matrix}\right)$$

07.44.26.0025.01

$$e^{z/2}\,{}_1F_1\!\left(\mu-\nu+\frac{1}{2};\,2\mu+1;\,-z\right)M_{\nu,\mu}(z)=\frac{4^{-\mu}\,\pi^{3/2}\,\Gamma(2\mu+1)^2\,z^{\mu+\frac{1}{2}}}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right)\Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\!\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2},\,\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\mu,\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0026.01

$$e^{z/2}\,{}_1F_1(c;\,2c;\,-z)\,M_{0,\mu}(z)=\frac{2^{c+\mu-\frac{1}{2}}\;\Gamma\!\left(c+\frac{1}{2}\right)\Gamma(\mu+1)}{\sqrt{\pi}}\,z^{\mu+\frac{1}{2}}\,G_{2,4}^{1,2}\!\left(-\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(-2c-2\mu+3),\,\frac{1}{4}(-2c-2\mu+1)\\0,\,-c-\mu+\frac{1}{2},\,\frac{1}{2}-c,\,-\mu\end{matrix}\right)$$

07.44.26.0027.01

$$e^{z/2}\,{}_1F_1(c;\,2c;\,-z)\,M_{0,\mu}(z)=2^{c+\mu-\frac{1}{2}}\sqrt{\pi}\;z^{\mu+\frac{1}{2}}\,\Gamma\!\left(c+\frac{1}{2}\right)\Gamma(\mu+1)\,G_{3,5}^{1,2}\!\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(-2c-2\mu+3),\,\frac{1}{4}(-2c-2\mu+1),\,\frac{1}{2}\\0,\,-c-\mu+\frac{1}{2},\,\frac{1}{2}-c,\,-\mu,\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0028.01

$$e^{-\frac{z}{2}}\,{}_1F_1\!\left(\mu+\nu+\frac{1}{2};\,2\mu+1;\,z\right)M_{\nu,\mu}(z)=\frac{4^{-\mu}\sqrt{\pi}\;\Gamma(2\mu+1)^2\,z^{\mu+\frac{1}{2}}}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right)\Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\,G_{2,4}^{1,2}\!\left(-\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\mu\end{matrix}\right)$$

07.44.26.0029.01

$$e^{-\frac{z}{2}}\,{}_1F_1\!\left(\mu+\nu+\frac{1}{2};\,2\mu+1;\,z\right)M_{\nu,\mu}(z)=\frac{4^{-\mu}\,\pi^{3/2}\,\Gamma(2\mu+1)^2\,z^{\mu+\frac{1}{2}}}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right)\Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\!\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2},\,\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\mu,\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0030.01

$$e^{-\frac{z}{2}}\,{}_1F_1(c;\,2c;\,z)\,M_{0,\mu}(z)=\frac{2^{c+\mu-\frac{1}{2}}\,\Gamma\!\left(c+\frac{1}{2}\right)\Gamma(\mu+1)\,z^{\mu+\frac{1}{2}}}{\sqrt{\pi}}\,G_{2,4}^{1,2}\!\left(-\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(-2c-2\mu+3),\,\frac{1}{4}(-2c-2\mu+1)\\0,\,-c-\mu+\frac{1}{2},\,\frac{1}{2}-c,\,-\mu\end{matrix}\right)$$

07.44.26.0031.01

$$e^{-\frac{z}{2}}\, {}_1F_1(c;\, 2\,c;\, z)\, M_{0,\mu}(z) = 2^{c+\mu-\frac{1}{2}}\, \sqrt{\pi}\, \Gamma\!\left(c+\frac{1}{2}\right) \Gamma(\mu+1)\, z^{\mu+\frac{1}{2}}\, G_{3,5}^{1,2}\!\left(\frac{z^2}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}(-2\,c-2\,\mu+3),\, \frac{1}{4}(-2\,c-2\,\mu+1),\, \frac{1}{2} \\ 0,\, -c-\mu+\frac{1}{2},\, \frac{1}{2}-c,\, -\mu,\, \frac{1}{2} \end{array}\right)$$

Classical cases involving exp and ${}_1\tilde{F}_1$

07.44.26.0032.01

$$e^{z/2}\, {}_1\tilde{F}_1\!\left(\mu-\nu+\frac{1}{2};\, 2\,\mu+1;\, -z\right) M_{\nu,\mu}(z) = \frac{4^{-\mu}\, \sqrt{\pi}\, \Gamma(2\,\mu+1)\, z^{\mu+\frac{1}{2}}}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right) \Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\, G_{2,4}^{1,2}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\,\mu \end{array}\right)$$

07.44.26.0033.01

$$e^{z/2}\, {}_1\tilde{F}_1\!\left(\mu-\nu+\frac{1}{2};\, 2\,\mu+1;\, -z\right) M_{\nu,\mu}(z) = \frac{4^{-\mu}\, \pi^{3/2}\, \Gamma(2\,\mu+1)\, z^{\mu+\frac{1}{2}}}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right) \Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\, G_{3,5}^{1,2}\!\left(\frac{z^2}{4} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2},\, \frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\,\mu,\, \frac{1}{2} \end{array}\right)$$

07.44.26.0034.01

$$e^{z/2}\, {}_1\tilde{F}_1(c;\, 2\,c;\, -z)\, M_{0,\mu}(z) = \frac{2^{-c+\mu+\frac{1}{2}}\, \Gamma(\mu+1)}{\Gamma(c)}\, z^{\mu+\frac{1}{2}}\, G_{2,4}^{1,2}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}(-2\,c-2\,\mu+3),\, \frac{1}{4}(-2\,c-2\,\mu+1) \\ 0,\, -c-\mu+\frac{1}{2},\, \frac{1}{2}-c,\, -\mu \end{array}\right)$$

07.44.26.0035.01

$$e^{z/2}\, {}_1\tilde{F}_1(c;\, 2\,c;\, -z)\, M_{0,\mu}(z) = \frac{2^{-c+\mu+\frac{1}{2}}\, \pi\, z^{\mu+\frac{1}{2}}\, \Gamma(\mu+1)}{\Gamma(c)}\, G_{3,5}^{1,2}\!\left(\frac{z^2}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}(-2\,c-2\,\mu+3),\, \frac{1}{4}(-2\,c-2\,\mu+1),\, \frac{1}{2} \\ 0,\, -c-\mu+\frac{1}{2},\, \frac{1}{2}-c,\, -\mu,\, \frac{1}{2} \end{array}\right)$$

07.44.26.0036.01

$$e^{-\frac{z}{2}}\, {}_1\tilde{F}_1\!\left(\mu+\nu+\frac{1}{2};\, 2\,\mu+1;\, z\right) M_{\nu,\mu}(z) = \frac{4^{-\mu}\, \sqrt{\pi}\, \Gamma(2\,\mu+1)\, z^{\mu+\frac{1}{2}}}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right) \Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\, G_{2,4}^{1,2}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\,\mu \end{array}\right)$$

07.44.26.0037.01

$$e^{-\frac{z}{2}}\, {}_1\tilde{F}_1\!\left(\mu+\nu+\frac{1}{2};\, 2\,\mu+1;\, z\right) M_{\nu,\mu}(z) = \frac{4^{-\mu}\, \pi^{3/2}\, \Gamma(2\,\mu+1)\, z^{\mu+\frac{1}{2}}}{\Gamma\!\left(\mu-\nu+\frac{1}{2}\right) \Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\, G_{3,5}^{1,2}\!\left(\frac{z^2}{4} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2},\, \frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\,\mu,\, \frac{1}{2} \end{array}\right)$$

07.44.26.0038.01

$$e^{-\frac{z}{2}}\, {}_1\tilde{F}_1(c;\, 2\,c;\, z)\, M_{0,\mu}(z) = \frac{2^{-c+\mu+\frac{1}{2}}\, z^{\mu+\frac{1}{2}}\, \Gamma(\mu+1)}{\Gamma(c)}\, G_{2,4}^{1,2}\!\left(-\frac{z^2}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}(-2\,c-2\,\mu+3),\, \frac{1}{4}(-2\,c-2\,\mu+1) \\ 0,\, -c-\mu+\frac{1}{2},\, \frac{1}{2}-c,\, -\mu \end{array}\right)$$

07.44.26.0039.01

$$e^{-\frac{z}{2}}\, {}_1\tilde{F}_1(c;\, 2\,c;\, z)\, M_{0,\mu}(z) = \frac{2^{-c+\mu+\frac{1}{2}}\, \pi\, z^{\mu+\frac{1}{2}}\, \Gamma(\mu+1)}{\Gamma(c)}\, G_{3,5}^{1,2}\!\left(\frac{z^2}{4} \,\middle|\, \begin{array}{c} \frac{1}{4}(-2\,c-2\,\mu+3),\, \frac{1}{4}(-2\,c-2\,\mu+1),\, \frac{1}{2} \\ 0,\, -c-\mu+\frac{1}{2},\, \frac{1}{2}-c,\, -\mu,\, \frac{1}{2} \end{array}\right)$$

Classical cases involving exp and hypergeometric *U*

07.44.26.0040.01

$$e^{z/2}\,U\left(\mu-\nu+\frac{1}{2},\,2\,\mu+1,\,-z\right)M_{\nu,\mu}(z)=\frac{2^{-2\,\mu-1}\,\Gamma(2\,\mu+1)\,z^{\mu+\frac{1}{2}}}{\sqrt{\pi}\;\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\,G_{2,4}^{3,1}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\,\mu\end{matrix}\right)/;$$
$$\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge-\pi<\arg(z)\leq-\frac{\pi}{2}$$

07.44.26.0041.01

$$e^{-\frac{z}{2}}\,U\left(\mu+\nu+\frac{1}{2},\,2\,\mu+1,\,z\right)M_{\nu,\mu}(z)=\frac{2^{-2\,\mu-1}\,\Gamma(2\,\mu+1)\,z^{\mu+\frac{1}{2}}}{\sqrt{\pi}\;\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{2,4}^{3,1}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu-\nu+\frac{1}{2},\,-\mu+\nu+\frac{1}{2}\\0,\,\frac{1}{2}-\mu,\,-\mu,\,-2\,\mu\end{matrix}\right)/;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.44.26.0042.01

$$e^{-\frac{\sqrt{z}}{2}}\,U\left(\mu-\nu+\frac{1}{2},\,2\,\mu+1,\,\sqrt{z}\right)M_{\nu,\mu}\left(-\sqrt{z}\right)=\frac{2^{-2\,\mu-1}\left(-\sqrt{z}\right)^{\mu+\frac{1}{2}}\,\Gamma(2\,\mu+1)}{\sqrt{\pi}\;\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\,G_{2,4}^{3,1}\left(\frac{z}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\,\mu\end{matrix}\right)$$

07.44.26.0043.01

$$e^{-\frac{\sqrt{z}}{2}}\,U\left(\mu+\nu+\frac{1}{2},\,2\,\mu+1,\,\sqrt{z}\right)M_{\nu,\mu}\left(\sqrt{z}\right)=\frac{2^{-\mu-\frac{1}{2}}\,\Gamma(2\,\mu+1)}{\sqrt{\pi}\;\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{2,4}^{3,1}\left(\frac{z}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(-2\,\mu-4\,\nu+3),\,-\frac{\mu}{2}+\nu+\frac{3}{4}\\\frac{1}{4}(1-2\,\mu),\,\frac{1}{4}(3-2\,\mu),\,\frac{1}{4}(2\,\mu+1),\,\frac{1}{4}(1-6\,\mu)\end{matrix}\right)$$

Classical cases involving exp and Laguerre *L*

07.44.26.0044.01

$$e^{z/2}\,L_{\nu-\frac{1}{2}}(-z)\,M_{\nu,0}(z)=\frac{\cos(\pi\,\nu)\sqrt{z}}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(-\frac{z^2}{4}\,\middle|\,\begin{matrix}\nu+\frac{1}{2},\,\frac{1}{2}-\nu\\0,\,0,\,0,\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0045.01

$$e^{z/2}\,L_{\nu-\frac{1}{2}}(-z)\,M_{\nu,0}(z)=\sqrt{\pi}\,\cos(\pi\,\nu)\sqrt{z}\;G_{3,5}^{1,2}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\nu+\frac{1}{2},\,\frac{1}{2}-\nu,\,\frac{1}{2}\\0,\,0,\,0,\,\frac{1}{2},\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0046.01

$$e^{z/2}\,L_{-\mu+\nu-\frac{1}{2}}^{2\,\mu}(-z)\,M_{\nu,\mu}(z)=\frac{4^{-\mu}\,z^{\mu+\frac{1}{2}}\cos(\pi\,(\mu-\nu))\,\Gamma(2\,\mu+1)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(-\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\,\mu\end{matrix}\right)$$

07.44.26.0047.01

$$e^{z/2}\,L_{-\mu+\nu-\frac{1}{2}}^{2\,\mu}(-z)\,M_{\nu,\mu}(z)=4^{-\mu}\sqrt{\pi}\,\cos(\pi\,(\mu-\nu))\,\Gamma(2\,\mu+1)\,z^{\mu+\frac{1}{2}}\,G_{3,5}^{1,2}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2},\,\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\,\mu,\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0048.01

$$e^{-\frac{z}{2}}\,L_{-\nu-\frac{1}{2}}(z)\,M_{\nu,0}(z)=\sqrt{\pi}\,\cos(\pi\,\nu)\sqrt{z}\;G_{3,5}^{1,2}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{1}{2}-\nu,\,\nu+\frac{1}{2},\,\frac{1}{2}\\0,\,0,\,0,\,\frac{1}{2},\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0049.01

$$e^{-\frac{z}{2}}\,L_{-\mu-\nu-\frac{1}{2}}^{2\,\mu}(z)\,M_{\nu,\mu}(z)=4^{-\mu}\sqrt{\pi}\,\cos(\pi\,(\mu+\nu))\,\Gamma(2\,\mu+1)\,z^{\mu+\frac{1}{2}}\,G_{3,5}^{1,2}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu-\nu+\frac{1}{2},\,-\mu+\nu+\frac{1}{2},\,\frac{1}{2}\\0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\,\mu,\,\frac{1}{2}\end{matrix}\right)$$

Classical cases involving Whittaker *W*

07.44.26.0050.01

$$M_{-\nu,\mu}(z)\,W_{\nu,\mu}(z)=\frac{\Gamma(2\mu+1)}{\sqrt{\pi}\,\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\,G_{2,4}^{3,1}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\nu+1,\,1-\nu\\ \frac{1}{2},\,1,\,\mu+\frac{1}{2},\,\frac{1}{2}-\mu\end{matrix}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.44.26.0051.01

$$M_{\nu,\mu}(-z)\,W_{\nu,\mu}(z)=\frac{2^{-2\mu-1}\,\Gamma(2\mu+1)\left(-z^2\right)^{\mu+\frac{1}{2}}}{\sqrt{\pi}\,\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\,G_{2,4}^{3,1}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\ 0,\,\frac{1}{2}-\mu,\,-\mu,\,-2\mu\end{matrix}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.44.26.0052.01

$$M_{-\nu,\mu}\left(\sqrt{z}\right)W_{\nu,\mu}\left(\sqrt{z}\right)=\frac{\Gamma(2\mu+1)}{\sqrt{\pi}\,\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\,G_{2,4}^{3,1}\left(\frac{z}{4}\,\middle|\,\begin{matrix}\nu+1,\,1-\nu\\ \frac{1}{2},\,1,\,\mu+\frac{1}{2},\,\frac{1}{2}-\mu\end{matrix}\right)$$

07.44.26.0053.01

$$M_{\nu,\mu}\left(-\sqrt{z}\right)W_{\nu,\mu}\left(\sqrt{z}\right)=\frac{2^{-\mu-\frac{1}{2}}\,\Gamma(2\mu+1)\left(-\sqrt{z}\right)^{\mu+\frac{1}{2}}}{\sqrt{\pi}\,\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\,G_{2,4}^{3,1}\left(\frac{z}{4}\,\middle|\,\begin{matrix}-\frac{\mu}{2}+\nu+\frac{3}{4},\,\frac{1}{4}(-2\mu-4\nu+3)\\ \frac{1}{4}(1-2\mu),\,\frac{1}{4}(3-2\mu),\,\frac{1}{4}(2\mu+1),\,\frac{1}{4}(1-6\mu)\end{matrix}\right)$$

Generalized cases involving exp

07.44.26.0054.01

$$e^{z/2}\,M_{\nu,\mu}(z)+e^{-\frac{z}{2}}\,M_{-\nu,\mu}(z)=\frac{2^{1-\nu}\,\pi^{3/2}\,\Gamma(2\mu+1)}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\left(\frac{z}{2},\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\,\frac{\nu+2}{2},\,\frac{1}{4}(2\mu+3)\\ \frac{1}{4}(2\mu+1),\,\frac{1}{4}(2\mu+3),\,\frac{1}{4}(1-2\mu),\,\frac{1}{4}(3-2\mu),\,\frac{1}{4}(2\mu+3)\end{matrix}\right)$$

07.44.26.0055.01

$$e^{z/2}\,M_{\nu,\mu}(z)-e^{-\frac{z}{2}}\,M_{-\nu,\mu}(z)=\frac{2^{1-\nu}\,\pi^{3/2}\,\Gamma(2\mu+1)}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\left(\frac{z}{2},\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\,\frac{\nu+2}{2},\,\frac{1}{4}(2\mu+5)\\ \frac{1}{4}(2\mu+3),\,\frac{1}{4}(2\mu+1),\,\frac{1}{4}(2\mu+5),\,\frac{1}{4}(1-2\mu),\,\frac{1}{4}(3-2\mu)\end{matrix}\right)$$

Generalized cases for products of WhittakerM M

07.44.26.0056.01

$$M_{-\nu,\mu}(z)\,M_{\nu,\mu}(z)=\frac{2\,\pi^{3/2}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\left(\frac{z}{2},\,\frac{1}{2}\,\middle|\,\begin{matrix}\nu+1,\,1-\nu,\,\mu+1\\ \mu+\frac{1}{2},\,\mu+1,\,\frac{1}{2},\,1,\,\frac{1}{2}-\mu\end{matrix}\right)$$

07.44.26.0057.01

$$M_{-\nu,\mu}(z)\,M_{\nu,\mu}(z)=\frac{4^{-\mu}\,\sqrt{\pi}\,z^{2\mu+1}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{2,4}^{1,2}\left(\frac{i z}{2},\,\frac{1}{2}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\ 0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\mu\end{matrix}\right)$$

07.44.26.0058.01

$$M_{0,\mu}(z)\,M_{0,\gamma}(z)=$$
$$2^{2\gamma+2\mu+1}\,\sqrt{\pi}\,\Gamma(\gamma+1)\,\Gamma(\mu+1)\,G_{3,5}^{1,2}\left(\frac{z}{2},\,\frac{1}{2}\,\middle|\,\begin{matrix}1,\,\frac{1}{2},\,\frac{1}{2}(\gamma+\mu+2)\\ \frac{1}{2}(\gamma+\mu+1),\,\frac{1}{2}(-\gamma-\mu+1),\,\frac{1}{2}(-\gamma+\mu+1),\,\frac{1}{2}(\gamma-\mu+1),\,\frac{1}{2}(\gamma+\mu+2)\end{matrix}\right)$$

07.44.26.0059.01

$$M_{0,\mu}(z)\,M_{0,\gamma}(z)=\frac{2^{\gamma+\mu}\,z^{\gamma+\mu+1}\,\Gamma(\gamma+1)\,\Gamma(\mu+1)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(\frac{i z}{2},\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\gamma-\mu}{2},\,-\frac{\gamma+\mu}{2}\\ 0,\,-\gamma-\mu,\,-\gamma,\,-\mu\end{matrix}\right)$$

07.44.26.0060.01

$$M_{\nu,\mu}(-z)\,M_{\nu,\mu}(z)=\frac{4^{-\mu}\sqrt{\pi}\,\left(-z^2\right)^{\mu+\frac{1}{2}}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{2,4}^{1,2}\left(\frac{i z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\ 0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\mu\end{matrix}\right)$$

07.44.26.0061.01

$$M_{\nu,\mu}(-z)\,M_{\nu,\mu}(z)=\frac{2^{\frac{1}{2}-\mu}\,\pi^{3/2}\,(-z)^{\mu+\frac{1}{2}}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}-\frac{\mu}{2}+\nu+\frac{3}{4},\,-\frac{\mu}{2}-\nu+\frac{3}{4},\,\frac{\mu}{2}+\frac{3}{4}\\ \frac{\mu}{2}+\frac{1}{4},\,\frac{1}{4}-\frac{\mu}{2},\,\frac{3}{4}-\frac{\mu}{2},\,\frac{1}{4}-\frac{3\mu}{2},\,\frac{\mu}{2}+\frac{3}{4}\end{matrix}\right)$$

07.44.26.0062.01

$$M_{0,\gamma}(-z)\,M_{0,\mu}(z)=\frac{2^{\gamma+\mu}\,(-z)^{\gamma+\frac{1}{2}}\,z^{\mu+\frac{1}{2}}\,\Gamma(\gamma+1)\,\Gamma(\mu+1)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(\frac{i z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}(-\gamma-\mu+1),\,\frac{1}{2}(-\gamma-\mu)\\ 0,\,-\gamma-\mu,\,-\gamma,\,-\mu\end{matrix}\right)$$

07.44.26.0063.01

$$M_{0,\gamma}(-z)\,M_{0,\mu}(z)=2^{\gamma+2\mu+\frac{1}{2}}\sqrt{\pi}\,(-z)^{\gamma+\frac{1}{2}}\,\Gamma(\gamma+1)\,\Gamma(\mu+1)\,G_{3,5}^{1,2}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4}-\frac{\gamma}{2},\,\frac{1}{4}-\frac{\gamma}{2},\,\frac{\mu}{2}+\frac{3}{4}\\ \frac{\mu}{2}+\frac{1}{4},\,-\gamma-\frac{\mu}{2}+\frac{1}{4},\,-\gamma+\frac{\mu}{2}+\frac{1}{4},\,\frac{1}{4}-\frac{\mu}{2},\,\frac{\mu}{2}+\frac{3}{4}\end{matrix}\right)$$

Generalized cases involving exp and ${}_1F_1$

07.44.26.0064.01

$$e^{z/2}\,{}_1F_1\left(\mu-\nu+\frac{1}{2};2\mu+1;-z\right)M_{\nu,\mu}(z)=\frac{2^{-2\mu}\sqrt{\pi}\,\Gamma(2\mu+1)^2}{z^{\frac{1}{2}(-2\mu-1)}\,\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{2,4}^{1,2}\left(\frac{i z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\ 0,\,-\mu,\,\frac{1}{2}(-2\mu-1)+1,\,-2\mu\end{matrix}\right)$$

07.44.26.0065.01

$$e^{z/2}\,{}_1F_1\left(\mu-\nu+\frac{1}{2};2\mu+1;-z\right)M_{\nu,\mu}(z)=\frac{4^{-\mu}\,\pi^{3/2}\,z^{\mu+\frac{1}{2}}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2},\,\frac{1}{2}\\ 0,\,-\mu,\,\frac{1}{2}(-2\mu-1)+1,\,-2\mu,\,\frac{1}{2}\end{matrix}\right)$$

07.44.26.0066.01

$$e^{z/2}\,{}_1F_1(c;2c;-z)\,M_{0,\mu}(z)=\frac{2^{c+\mu-\frac{1}{2}}\,z^{\mu+\frac{1}{2}}\,\Gamma\left(c+\frac{1}{2}\right)\Gamma(\mu+1)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(\frac{i z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2}\left(-c-\mu-\frac{1}{2}\right)+1,\,\frac{1}{2}\left(-c-\mu+\frac{1}{2}\right)\\ 0,\,-c-\mu+\frac{1}{2},\,\frac{1}{2}-c,\,-\mu\end{matrix}\right)$$

07.44.26.0067.01

$$e^{z/2}\,{}_1F_1(c;2c;-z)\,M_{0,\mu}(z)=$$

$$2^{c+2\mu}\sqrt{\pi}\,\Gamma\left(c+\frac{1}{2}\right)\Gamma(\mu+1)\,G_{3,5}^{1,2}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{c}{2},\,\frac{1-c}{2},\,\frac{1}{4}(2\mu+3)\\ \frac{1}{4}(2\mu+1),\,\frac{1}{4}(-4c-2\mu+3),\,\frac{1}{4}(-4c+2\mu+3),\,\frac{1}{4}(1-2\mu),\,\frac{1}{4}(2\mu+3)\end{matrix}\right)$$

07.44.26.0068.01

$$e^{-\frac{z}{2}}\,{}_1F_1\left(\mu+\nu+\frac{1}{2};2\mu+1;z\right)M_{\nu,\mu}(z)=\frac{4^{-\mu}\sqrt{\pi}\,z^{\mu+\frac{1}{2}}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{2,4}^{1,2}\left(\frac{i z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}-\mu+\nu+\frac{1}{2},\,-\mu-\nu+\frac{1}{2}\\ 0,\,-\mu,\,\frac{1}{2}-\mu,\,-2\mu\end{matrix}\right)$$

07.44.26.0069.01

$$e^{-\frac{z}{2}}\,{}_1F_1\left(\mu+\nu+\frac{1}{2};2\mu+1;z\right)M_{\nu,\mu}(z)=$$

$$\frac{2^{\frac{1}{2}-\mu}\,\pi^{3/2}\,\Gamma(2\mu+1)^2}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\,G_{3,5}^{1,2}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}-\frac{\mu}{2}+\nu+\frac{3}{4},\,\frac{1}{4}(-2\mu-4\nu+3),\,\frac{1}{4}(2\mu+3)\\ \frac{1}{4}(2\mu+1),\,\frac{1}{4}(1-2\mu),\,\frac{1}{4}(3-2\mu),\,\frac{1}{4}(1-6\mu),\,\frac{1}{4}(2\mu+3)\end{matrix}\right)$$

07.44.26.0070.01

$$e^{-\frac{z}{2}} {}_1F_1(c;\, 2\,c;\, z)\, M_{0,\mu}(z) = \frac{2^{c+\mu-\frac{1}{2}} z^{\mu+\frac{1}{2}} \Gamma\left(c+\frac{1}{2}\right) \Gamma(\mu+1)}{\sqrt{\pi}} G_{2,4}^{1,2}\left(\frac{i\,z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}(-2\,c-2\,\mu+3), \frac{1}{4}(-2\,c-2\,\mu+1) \\ 0, -c-\mu+\frac{1}{2}, \frac{1}{2}-c, -\mu \end{matrix}\right)$$

07.44.26.0071.01

$$e^{-\frac{z}{2}} {}_1F_1(c;\, 2\,c;\, z)\, M_{0,\mu}(z) =$$

$$2^{c+2\,\mu} \sqrt{\pi}\, \Gamma\left(c+\frac{1}{2}\right) \Gamma(\mu+1)\, G_{3,5}^{1,2}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 1-\frac{c}{2}, \frac{1-c}{2}, \frac{1}{4}(2\,\mu+3) \\ \frac{1}{4}(2\,\mu+1), \frac{1}{4}(-4\,c-2\,\mu+3), \frac{1}{4}(-4\,c+2\,\mu+3), \frac{1}{4}(1-2\,\mu), \frac{1}{4}(2\,\mu+3) \end{matrix}\right)$$

Generalized cases involving exp and ${}_1\tilde{F}_1$

07.44.26.0072.01

$$e^{z/2} {}_1\tilde{F}_1\left(\mu-\nu+\frac{1}{2};\, 2\,\mu+1;\, -z\right) M_{\nu,\mu}(z) = \frac{2^{-2\,\mu} \sqrt{\pi}\, \Gamma(2\,\mu+1)}{z^{\frac{1}{2}(-2\,\mu-1)} \Gamma\left(\mu-\nu+\frac{1}{2}\right) \Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{2,4}^{1,2}\left(\frac{i\,z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} -\mu+\nu+\frac{1}{2}, -\mu-\nu+\frac{1}{2} \\ 0, -\mu, \frac{1}{2}(-2\,\mu-1)+1, -2\,\mu \end{matrix}\right)$$

07.44.26.0073.01

$$e^{z/2} {}_1\tilde{F}_1\left(\mu-\nu+\frac{1}{2};\, 2\,\mu+1;\, -z\right) M_{\nu,\mu}(z) = \frac{4^{-\mu} \pi^{3/2} z^{\mu+\frac{1}{2}} \Gamma(2\,\mu+1)}{\Gamma\left(\mu-\nu+\frac{1}{2}\right) \Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{3,5}^{1,2}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} -\mu+\nu+\frac{1}{2}, -\mu-\nu+\frac{1}{2}, \frac{1}{2} \\ 0, -\mu, \frac{1}{2}(-2\,\mu-1)+1, -2\,\mu, \frac{1}{2} \end{matrix}\right)$$

07.44.26.0074.01

$$e^{z/2} {}_1\tilde{F}_1(c;\, 2\,c;\, -z)\, M_{0,\mu}(z) = \frac{2^{-c+\mu+\frac{1}{2}} z^{\mu+\frac{1}{2}} \Gamma(\mu+1)}{\Gamma(c)} G_{2,4}^{1,2}\left(\frac{i\,z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}\left(-c-\mu-\frac{1}{2}\right)+1, \frac{1}{2}\left(-c-\mu+\frac{1}{2}\right) \\ 0, -c-\mu+\frac{1}{2}, \frac{1}{2}-c, -\mu \end{matrix}\right)$$

07.44.26.0075.01

$$e^{z/2} {}_1\tilde{F}_1(c;\, 2\,c;\, -z)\, M_{0,\mu}(z) =$$

$$\frac{2^{-c+2\,\mu+1} \pi\, \Gamma(\mu+1)}{\Gamma(c)} G_{3,5}^{1,2}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 1-\frac{c}{2}, \frac{1-c}{2}, \frac{1}{4}(2\,\mu+3) \\ \frac{1}{4}(2\,\mu+1), \frac{1}{4}(-4\,c-2\,\mu+3), \frac{1}{4}(-4\,c+2\,\mu+3), \frac{1}{4}(1-2\,\mu), \frac{1}{4}(2\,\mu+3) \end{matrix}\right)$$

07.44.26.0076.01

$$e^{-\frac{z}{2}} {}_1\tilde{F}_1\left(\mu+\nu+\frac{1}{2};\, 2\,\mu+1;\, z\right) M_{\nu,\mu}(z) = \frac{4^{-\mu} \sqrt{\pi}\, z^{\mu+\frac{1}{2}} \Gamma(2\,\mu+1)}{\Gamma\left(\mu-\nu+\frac{1}{2}\right) \Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{2,4}^{1,2}\left(\frac{i\,z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} -\mu+\nu+\frac{1}{2}, -\mu-\nu+\frac{1}{2} \\ 0, -\mu, \frac{1}{2}-\mu, -2\,\mu \end{matrix}\right)$$

07.44.26.0077.01

$$e^{-\frac{z}{2}} {}_1\tilde{F}_1\left(\mu+\nu+\frac{1}{2};\, 2\,\mu+1;\, z\right) M_{\nu,\mu}(z) =$$

$$\frac{2^{\frac{1}{2}-\mu} \pi^{3/2} \Gamma(2\,\mu+1)}{\Gamma\left(\mu-\nu+\frac{1}{2}\right) \Gamma\left(\mu+\nu+\frac{1}{2}\right)} G_{3,5}^{1,2}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} -\frac{\mu}{2}+\nu+\frac{3}{4}, \frac{1}{4}(-2\,\mu-4\,\nu+3), \frac{1}{4}(2\,\mu+3) \\ \frac{1}{4}(2\,\mu+1), \frac{1}{4}(1-2\,\mu), \frac{1}{4}(3-2\,\mu), \frac{1}{4}(1-6\,\mu), \frac{1}{4}(2\,\mu+3) \end{matrix}\right)$$

07.44.26.0078.01

$$e^{-\frac{z}{2}} {}_1\tilde{F}_1(c;\, 2\,c;\, z)\, M_{0,\mu}(z) = \frac{2^{-c+\mu+\frac{1}{2}} z^{\mu+\frac{1}{2}} \Gamma(\mu+1)}{\Gamma(c)} G_{2,4}^{1,2}\left(\frac{i\,z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}(-2\,c-2\,\mu+3), \frac{1}{4}(-2\,c-2\,\mu+1) \\ 0, -c-\mu+\frac{1}{2}, \frac{1}{2}-c, -\mu \end{matrix}\right)$$

07.44.26.0079.01

$$e^{-\frac{z}{2}}\, {}_1\tilde{F}_1(c;\, 2\, c;\, z)\, M_{0,\mu}(z) = \frac{2^{-c+2\,\mu+1}\, \pi\, \Gamma(\mu+1)}{\Gamma(c)}\, G_{3,5}^{1,2}\!\left(\frac{z}{2},\, \frac{1}{2} \,\middle|\, \begin{array}{c} 1-\frac{c}{2},\, \frac{1-c}{2},\, \frac{1}{4}\,(2\,\mu+3) \\ \frac{1}{4}\,(2\,\mu+1),\, \frac{1}{4}\,(-4\,c-2\,\mu+3),\, \frac{1}{4}\,(-4\,c+2\,\mu+3),\, \frac{1}{4}\,(1-2\,\mu),\, \frac{1}{4}\,(2\,\mu+3) \end{array}\right)$$

Generalized cases involving exp and hypergeometric U

07.44.26.0080.01

$$e^{-\frac{z}{2}}\, U\!\left(\mu+\nu+\frac{1}{2},\, 2\,\mu+1,\, z\right) M_{\nu,\mu}(z) = \frac{2^{-\mu-\frac{1}{2}}\, \Gamma(2\,\mu+1)}{\sqrt{\pi}\, \Gamma\!\left(\mu+\nu+\frac{1}{2}\right)}\, G_{2,4}^{3,1}\!\left(\frac{z}{2},\, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{4}\,(-2\,\mu-4\,\nu+3),\, -\frac{\mu}{2}+\nu+\frac{3}{4} \\ \frac{1}{4}\,(1-2\,\mu),\, \frac{1}{4}\,(3-2\,\mu),\, \frac{1}{4}\,(2\,\mu+1),\, \frac{1}{4}\,(1-6\,\mu) \end{array}\right)$$

07.44.26.0081.01

$$e^{-\frac{z}{2}}\, U\!\left(\mu-\nu+\frac{1}{2},\, 2\,\mu+1,\, z\right) M_{\nu,\mu}(-z) = \frac{2^{-2\,\mu-1}\, (-z)^{\mu+\frac{1}{2}}\, \Gamma(2\,\mu+1)}{\sqrt{\pi}\, \Gamma\!\left(\mu-\nu+\frac{1}{2}\right)}\, G_{2,4}^{3,1}\!\left(\frac{z}{2},\, \frac{1}{2} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2} \\ 0,\, \frac{1}{2}-\mu,\, -\mu,\, -2\,\mu \end{array}\right)$$

Generalized cases involving exp and Laguerre L

07.44.26.0082.01

$$e^{z/2}\, L_{\nu-\frac{1}{2}}(-z)\, M_{\nu,0}(z) = \frac{\cos(\pi\,\nu)\,\sqrt{z}}{\sqrt{\pi}}\, G_{2,4}^{1,2}\!\left(-\frac{1}{2}\,(i\,z),\, \frac{1}{2} \,\middle|\, \begin{array}{c} \nu+\frac{1}{2},\, \frac{1}{2}-\nu \\ 0,\, 0,\, 0,\, \frac{1}{2} \end{array}\right)$$

07.44.26.0083.01

$$e^{z/2}\, L_{\nu-\frac{1}{2}}(-z)\, M_{\nu,0}(z) = \sqrt{\pi}\, \sqrt{z}\, \cos(\pi\,\nu)\, G_{3,5}^{1,2}\!\left(-\frac{z}{2},\, \frac{1}{2} \,\middle|\, \begin{array}{c} \nu+\frac{1}{2},\, \frac{1}{2}-\nu,\, \frac{1}{2} \\ 0,\, 0,\, 0,\, \frac{1}{2},\, \frac{1}{2} \end{array}\right)$$

07.44.26.0084.01

$$e^{-z}\, {}_1F_1(a;\, 1;\, z)\, L_{a-1}(z) = \sqrt{\pi}\, \sin(a\,\pi)\, G_{3,5}^{1,2}\!\left(\frac{z}{2},\, \frac{1}{2} \,\middle|\, \begin{array}{c} a,\, 1-a,\, \frac{1}{2} \\ 0,\, 0,\, 0,\, \frac{1}{2},\, \frac{1}{2} \end{array}\right)$$

07.44.26.0085.01

$$e^{z/2}\, L_{-\mu+\nu-\frac{1}{2}}^{2\,\mu}(-z)\, M_{\nu,\mu}(z) = \frac{4^{-\mu}\, z^{\mu+\frac{1}{2}}\, \cos(\pi\,(\mu-\nu))\, \Gamma(2\,\mu+1)}{\sqrt{\pi}}\, G_{2,4}^{1,2}\!\left(\frac{i\,z}{2},\, \frac{1}{2} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\,\mu \end{array}\right)$$

07.44.26.0086.01

$$e^{z/2}\, L_{-\mu+\nu-\frac{1}{2}}^{2\,\mu}(-z)\, M_{\nu,\mu}(z) = \frac{4^{-\mu}\, z^{\mu+\frac{1}{2}}\, \cos(\pi\,(\mu-\nu))\, \Gamma(2\,\mu+1)}{\sqrt{\pi}}\, G_{2,4}^{1,2}\!\left(-\frac{1}{2}\,(i\,z),\, \frac{1}{2} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\,\mu \end{array}\right)$$

07.44.26.0087.01

$$e^{z/2}\, L_{-\mu+\nu-\frac{1}{2}}^{2\,\mu}(-z)\, M_{\nu,\mu}(z) = 4^{-\mu}\, \sqrt{\pi}\, z^{\mu+\frac{1}{2}}\, \cos(\pi\,(\mu-\nu))\, \Gamma(2\,\mu+1)\, G_{3,5}^{1,2}\!\left(\frac{z}{2},\, \frac{1}{2} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2},\, \frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\,\mu,\, \frac{1}{2} \end{array}\right)$$

07.44.26.0088.01

$$e^{z/2}\, L_{-\mu+\nu-\frac{1}{2}}^{2\,\mu}(-z)\, M_{\nu,\mu}(z) = 4^{-\mu}\, \sqrt{\pi}\, z^{\mu+\frac{1}{2}}\, \cos(\pi\,(\mu-\nu))\, \Gamma(2\,\mu+1)\, G_{3,5}^{1,2}\!\left(-\frac{z}{2},\, \frac{1}{2} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2},\, \frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\,\mu,\, \frac{1}{2} \end{array}\right)$$

07.44.26.0089.01

$$e^{z/2}\, L_{\lambda}^{-2\lambda-1}(-z)\, M_{0,\mu}(z) = \frac{2^{\lambda+\mu+\frac{1}{2}}\, z^{\mu+\frac{1}{2}}\, \Gamma(\mu+1)}{\Gamma(\lambda+1)}\, G_{2,4}^{1,2}\!\left(\frac{i z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{4}(2\lambda-2\mu+3),\, \frac{1}{4}(2\lambda-2\mu+1) \\ 0,\, \lambda-\mu+\frac{1}{2},\, \lambda+\frac{1}{2},\, -\mu \end{array}\right)$$

07.44.26.0090.01

$$e^{z/2}\, L_{\lambda}^{-2\lambda-1}(-z)\, M_{0,\mu}(z) =$$

$$\frac{2^{\lambda+2\mu+1}\, \pi\, \Gamma(\mu+1)}{\Gamma(\lambda+1)}\, G_{3,5}^{1,2}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{\lambda}{2}+1,\, \frac{\lambda+1}{2},\, \frac{1}{4}(2\mu+3) \\ \frac{1}{4}(2\mu+1),\, \frac{1}{4}(4\lambda-2\mu+3),\, \frac{1}{4}(4\lambda+2\mu+3),\, \frac{1}{4}(1-2\mu),\, \frac{1}{4}(2\mu+3) \end{array}\right)$$

07.44.26.0091.01

$$e^{-\frac{z}{2}}\, L_{-\mu-\nu-\frac{1}{2}}^{2\mu}(z)\, M_{\nu,\mu}(z) = \frac{4^{-\mu}\, z^{\mu+\frac{1}{2}}\, \cos(\pi(\mu+\nu))\, \Gamma(2\mu+1)}{\sqrt{\pi}}\, G_{2,4}^{1,2}\!\left(\frac{i z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} -\mu+\nu+\frac{1}{2},\, -\mu-\nu+\frac{1}{2} \\ 0,\, -\mu,\, \frac{1}{2}-\mu,\, -2\mu \end{array}\right)$$

07.44.26.0092.01

$$e^{-\frac{z}{2}}\, L_{-\mu-\nu-\frac{1}{2}}^{2\mu}(z)\, M_{\nu,\mu}(z) = 2^{\frac{1}{2}-\mu}\, \sqrt{\pi}\, \cos(\pi(\mu+\nu))\, \Gamma(2\mu+1)\, G_{3,5}^{1,2}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} -\frac{\mu}{2}+\nu+\frac{3}{4},\, -\frac{\mu}{2}-\nu+\frac{3}{4},\, \frac{\mu}{2}+\frac{3}{4} \\ \frac{\mu}{2}+\frac{1}{4},\, \frac{1}{4}-\frac{\mu}{2},\, \frac{3}{4}-\frac{\mu}{2},\, \frac{1}{4}-\frac{3\mu}{2},\, \frac{\mu}{2}+\frac{3}{4} \end{array}\right)$$

07.44.26.0093.01

$$e^{-\frac{z}{2}}\, L_{\lambda}^{-2\lambda-1}(z)\, M_{0,\mu}(z) = \frac{2^{\lambda+\mu+\frac{1}{2}}\, z^{\mu+\frac{1}{2}}\, \Gamma(\mu+1)}{\Gamma(\lambda+1)}\, G_{2,4}^{1,2}\!\left(\frac{i z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{4}(2\lambda-2\mu+3),\, \frac{1}{4}(2\lambda-2\mu+1) \\ 0,\, \lambda-\mu+\frac{1}{2},\, \lambda+\frac{1}{2},\, -\mu \end{array}\right)$$

07.44.26.0094.01

$$e^{-\frac{z}{2}}\, L_{\lambda}^{-2\lambda-1}(z)\, M_{0,\mu}(z) = \frac{2^{\lambda+2\mu+1}\, \pi\, \Gamma(\mu+1)}{\Gamma(\lambda+1)}\, G_{3,5}^{1,2}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{\lambda}{2}+1,\, \frac{\lambda+1}{2},\, \frac{1}{4}(2\mu+3) \\ \frac{1}{4}(2\mu+1),\, \frac{1}{4}(4\lambda-2\mu+3),\, \frac{1}{4}(4\lambda+2\mu+3),\, \frac{1}{4}(1-2\mu),\, \frac{1}{4}(2\mu+3) \end{array}\right)$$

Generalized cases involving Whittaker *W*

07.44.26.0095.01

$$M_{\nu,\mu}(-z)\, W_{\nu,\mu}(z) = \frac{2^{-\mu-\frac{1}{2}}\, (-z)^{\mu+\frac{1}{2}}\, \Gamma(2\mu+1)}{\sqrt{\pi}\, \Gamma\!\left(\mu-\nu+\frac{1}{2}\right)}\, G_{2,4}^{3,1}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} -\frac{\mu}{2}+\nu+\frac{3}{4},\, \frac{1}{4}(-2\mu-4\nu+3) \\ \frac{1}{4}(1-2\mu),\, \frac{1}{4}(3-2\mu),\, \frac{1}{4}(2\mu+1),\, \frac{1}{4}(1-6\mu) \end{array}\right)$$

07.44.26.0096.01

$$M_{\nu,-\mu}(-z)\, W_{\nu,\mu}(z) = \frac{2^{\mu-\frac{1}{2}}\, (-z)^{\frac{1}{2}-\mu}\, \Gamma(1-2\mu)}{\sqrt{\pi}\, \Gamma\!\left(-\mu-\nu+\frac{1}{2}\right)}\, G_{2,4}^{3,1}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{\mu}{2}+\nu+\frac{3}{4},\, \frac{1}{4}(2\mu-4\nu+3) \\ \frac{1}{4}(2\mu+1),\, \frac{1}{4}(2\mu+3),\, \frac{1}{4}(1-2\mu),\, \frac{1}{4}(6\mu+1) \end{array}\right)$$

07.44.26.0097.01

$$M_{-\nu,\mu}(z)\, W_{\nu,\mu}(z) = \frac{\Gamma(2\mu+1)}{\sqrt{\pi}\, \Gamma\!\left(\mu-\nu+\frac{1}{2}\right)}\, G_{2,4}^{3,1}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \nu+1,\, 1-\nu \\ \frac{1}{2},\, 1,\, \mu+\frac{1}{2},\, \frac{1}{2}-\mu \end{array}\right)$$

07.44.26.0098.01

$$M_{-\nu,\mu}(z)\, W_{\nu,-\mu}(z) = \frac{\Gamma(2\mu+1)}{\sqrt{\pi}\, \Gamma\!\left(\mu-\nu+\frac{1}{2}\right)}\, G_{2,4}^{3,1}\!\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \nu+1,\, 1-\nu \\ \frac{1}{2},\, 1,\, \mu+\frac{1}{2},\, \frac{1}{2}-\mu \end{array}\right)$$

07.44.26.0099.01

$$M_{-\nu,-\mu}(z)\, W_{\nu,\mu}(z) = \frac{\Gamma(1-2\mu)}{\sqrt{\pi}\; \Gamma\left(-\mu-\nu+\frac{1}{2}\right)}\, G_{2,4}^{3,1}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} \nu+1,\, 1-\nu \\ \frac{1}{2},\, 1,\, \frac{1}{2}-\mu,\, \mu+\frac{1}{2} \end{array}\right)$$

Representations through equivalent functions

With related functions

07.44.27.0001.01

$$M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}}\, e^{-\frac{z}{2}}\, {}_1F_1\left(\mu-\nu+\frac{1}{2};\, 2\mu+1;\, z\right)$$

07.44.27.0002.01

$$M_{\nu,\mu}(z) = \frac{e^{-\frac{z}{2}}\, z^{\mu+\frac{1}{2}}\, \Gamma(2\mu+1)\, \Gamma\left(-\mu+\nu+\frac{1}{2}\right)}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\, L_{-\mu+\nu-\frac{1}{2}}^{2\mu}(z) \,/;\, 2\mu \notin \mathbb{Z}$$

07.44.27.0003.01

$$M_{\nu,\mu}(z) = \frac{\pi\, e^{-\frac{z}{2}}\, z^{\mu+\frac{1}{2}}}{\left((-z)^{2\mu} \cos(\pi(\mu-\nu)) - z^{2\mu} \cos(\pi(\mu+\nu))\right) \Gamma(-2\mu)} \left(\frac{e^{z}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)}\, U\left(-\mu+\nu+\frac{1}{2},\, 1-2\mu,\, -z\right) - \frac{1}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\, U\left(-\mu-\nu+\frac{1}{2},\, 1-2\mu,\, z\right)\right) /;\, 2\mu \notin \mathbb{Z}$$

07.44.27.0004.01

$$M_{\nu,\mu}(z) = \frac{e^{-\frac{z}{2}}\, z^{\mu+\frac{1}{2}}\, \Gamma(2\mu+1)\, \Gamma\left(-\mu+\nu+\frac{1}{2}\right)}{\Gamma\left(\mu+\nu+\frac{1}{2}\right)}\, L_{-\mu+\nu-\frac{1}{2}}^{2\mu}(z)$$

07.44.27.0005.01

$$M_{\nu,\mu}(z) = \frac{\pi\, z^{\mu-\frac{1}{2}}}{\left(z^{2\mu} \cos(\pi(\mu+\nu)) - (-z)^{2\mu} \cos(\pi(\mu-\nu))\right) \Gamma(-2\mu)\, \Gamma\left(\mu-\nu+\frac{1}{2}\right) \Gamma\left(\mu+\nu+\frac{1}{2}\right)} \left(\Gamma\left(\mu+\nu+\frac{1}{2}\right) (-z)^{\mu+\frac{1}{2}}\, W_{-\nu,-\mu}(-z) + \Gamma\left(\mu-\nu+\frac{1}{2}\right) z^{\mu+\frac{1}{2}}\, W_{\nu,-\mu}(z)\right) /;\, 2\mu \notin \mathbb{Z}$$

Theorems

History

– E. T. Whittaker (1904);
– C. S. Meijer (1936)

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